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Land use and transportation: basic theory†

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Abstract. Four partial models of the relationship between transportation and land use are presented, each representing a different slice of a single unified view of the basic theory in the field. Attention is given to transportation as a provider of access and as a consumer of land, both with and without variations in relative levels of capital intensity. The intent is to provide a normative framework for evaluating transportation policy as it affects land use.

1 Introduction

An important relationship has been hypothesized between transportation and land use: namely, transportation shapes land use. Yet the theory dealing explicitly with this relationship is not satisfactorily developed, and empirical evidence remains weak.

The purpose of this paper is to propose a set of four theoretical models analyzing the basic relationship between transportation and land use. An effort has been made to develop especially simple models, which can be carried through to conclusion with a great deal of transparency. This necessarily implies a limited focus on a specific aspect of the relationship, and hence a partial approach to the problem. If the partial models are well designed, deductions from them should be useful as part of a more general theory. Each of the following models is directed at a particular transportation impact or policy issue, and each is offered as a meaningful partial theory, that is, it makes sense to consider each specific cluster of variables and functions while holding the others constant.

For purposes of organization, a distinction is made here between the role of transportation in providing access to nontransportation land-consuming activities, and the role of transportation as a competitor for the use of scarce urban land. In the first pair of models, transportation is purely complementary since it serves other activities without diverting any land away from them. The first model (section 2) develops, from a von Thunen base, the relationship between access and land rent with supply-demand interaction in the central market. The second model (section 3) introduces changes in land use intensity as a consequence of capital-land substitution in production[1]. The approach taken is essentially production oriented, as distinct from the more demand-oriented approaches of Alonso (1965), Muth (1969), and Wingo (1961), which typically rely on consumer theory.

In the second pair of models, land consumption by the transportation sector is introduced, making transportation a competitor for land. The first of these (section 4) defines a balance between transportation and other land use by including both the complementarity and the competition between them. The last model (section 5) achieves a balance between transportation modes by selecting capital

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(1) See also Beckmann (1972), and Lee and Yujnovsky (1971).

intensity according to the relative cost of land consumed. Both models are also supply or production oriented, and can be compared to work by Mills (1969) and Muth (1971). General conclusions are offered at the end.

1.1 *Notation*

(a) Quantities and prices

Q	p	output of the productive activity
K	k	nonland inputs, including capital
L_p	R	land input for production
L_t	R	land input for transportation
x	r	distance and transport rate.

(b) Functional relationships

p	demand for output
$Q = Q(L_p, K_p, T)$	production function for nontransportation goods
$T = T(K_t, L_t)$	production function for transportation services
$r = r_0$	constant transport rate
$k = k_0$	exogenous price of capital inputs.

2 Transportation as access: supply–demand equilibrium

The response of land use and land prices to changes in the transportation system depend to an important degree on demand conditions. Unfortunately, several previous conclusions with respect to the rent gradient have implicitly assumed either perfectly elastic aggregate demand or perfectly inelastic demand for the output of land[(2)]. The first step in formalizing the transportation–land-use relationship is to consider supply and demand in the simplest von Thunen model, with uniform land use intensity. Results derived from this model can be extended to a variety of alterations in the supply side.

2.1 *A single land use*

Demand for areally produced goods at the single dimensionless market is taken to be a linear and downward sloping function of the price,

$$Q_d = \alpha_0 - \alpha_1 p \, , \qquad \alpha_0, \alpha_1 \geqslant 0 \, . \tag{1}$$

With output constant per unit of land, the total supply on the market is simply the area under production. Thus

$$Q_s = Q(L_p) = L_p = \pi x_m^2 \, , \qquad x_m \geqslant 0 \, , \tag{2}$$

represents the supply of areally-produced goods, where x_m is the maximum distance from the market at which production can take place and still cover factor costs[(3)]. Since the only nonzero costs are for transportation, the maximum distance is given by

$$x_m = \frac{p}{r} \, , \qquad r > 0 \, ,$$

and substituting in equation (2) gives

$$Q_s = \pi \frac{p^2}{r^2} \, . \tag{3}$$

(2) Some examples are Haig (1927) and Goldberg (1970).

(3) Factor costs for production can be taken as zero without loss of generality. Units of land, output, and distance are assumed to be measured in such a way that the need for conversion factors, such as yield, is eliminated. Thus, for example, $Q(L)/L = 1$.

Equating supply and demand determines the equilibrium market price p^*, the spatial extent of the market x_m^*, and the total output Q^*, as functions of the parameters of the demand function and the transport rate. Letting

$$\frac{\phi}{r} = \left(\alpha_1^2 + \frac{4\pi\alpha_0}{r^2}\right)^{1/2} \geqslant \alpha_1$$

for simplicity of notation gives

$$p^* = \frac{r^2}{2\pi}\left(-\alpha_1 + \frac{\phi}{r}\right), \tag{4}$$

$$x_m^* = \frac{r}{2\pi}\left(-\alpha_1 + \frac{\phi}{r}\right), \tag{5}$$

$$Q^* = \alpha_0 - \alpha_1 \frac{r^2}{2\pi}\left(-\alpha_1 + \frac{\phi}{r}\right). \tag{6}$$

Each of these quantities will be necessarily equal to or greater than zero.

If location rent is taken to be the residual return to land after other factors have been paid, the rent is

$$R(x) = p^* - rx, \tag{7}$$

and hence is a function of the distance of the unit of land from the market(4). Diagrammatically the result is shown in figure 1. The slope of the rent gradient is the transportation rate.

Comparative static analysis of this simple model yields several conclusions. First, an increase in demand (that is, a shift in the schedule such that more is demanded at

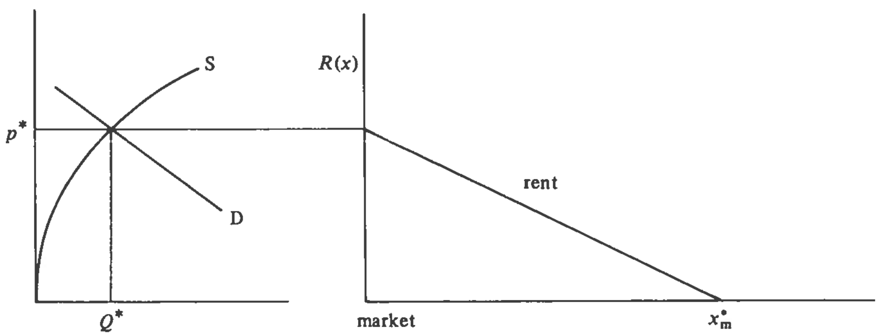

Figure 1. Determination of the rent gradient.

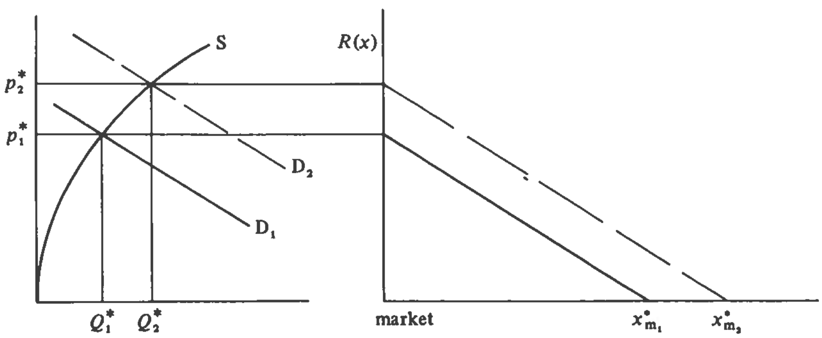

Figure 2. Shift in the demand schedule.

(4) Alternative definitions of rent can be used: see Bronfenbrenner (1971), chapter 14.

each price, as in figure 2) will result in more land in production and higher rents throughout. Second, a decrease in transportation costs will necessarily result in both a decrease in the market price and also an increase in the area under production, as shown in figure 3. Thus rents at some locations will increase and rents at others will decrease. Only if demand is perfectly elastic will all rents increase; only if demand is perfectly inelastic will all rents decrease. In general, rents near the market will decline and those farther away will increase. The reason some rents increase is simple: the output of a unit of land is more valuable because it costs less to get it to market. Other rents decrease because the monopoly strength of the central location is reduced by making distant sites accessible. The overall effect is to flatten the rent gradient and extend the area of production.

From this model it is clear that a decrease in the transportation rate will result in (a) a reduction in p^*, the level of prices in the market, (b) an increase in x^*_{m}, the spatial extent of production, and (c) an increase in Q^*, total output. Only if demand is perfectly inelastic will the size of the area under production remain the same, and only if demand is perfectly elastic will the price level stay constant.

Originally, the von Thunen model was formulated in respect to agricultural rents and agricultural land use. A few centuries ago, consumption took place in markets that were spatially small, and agriculture was easily the largest space-consuming industry. Now, in a contemporary urban context, the model can be given a broader interpretation. For example, firms producing goods or services for sale in or distribution from a single urban center may require land for production, thereby fulfilling the conditions of the model. Another interpretation is to think of the productive activity as being housing, whose occupants commute to centrally located employment.

2.2 *Land use competition*

Instead of a single land use (for example, residential), suppose there were another land use (for example, commercial) for which the transport rate (per unit of output of one unit of land) were higher but the price at the market were also higher. Then the two uses would have to compete for the same land, and the supply curve for each land use would depend upon both the parameters of its own supply and demand and those of the alternative land use. If linear demand schedules are assumed, equilibrium would require:

$$\alpha_0 + \alpha_1 p_{\mathrm{A}} = \pi\left(\frac{p_{\mathrm{A}} - p_{\mathrm{B}}}{r_{\mathrm{A}} - r_{\mathrm{B}}}\right)^2 , \tag{8}$$

$$\beta_0 + \beta_1 p_{\mathrm{B}} = \pi\left[\left(\frac{p_{\mathrm{B}}}{r_{\mathrm{B}}}\right)^2 - \left(\frac{p_{\mathrm{A}} - p_{\mathrm{B}}}{r_{\mathrm{A}} - r_{\mathrm{B}}}\right)^2\right] , \tag{9}$$

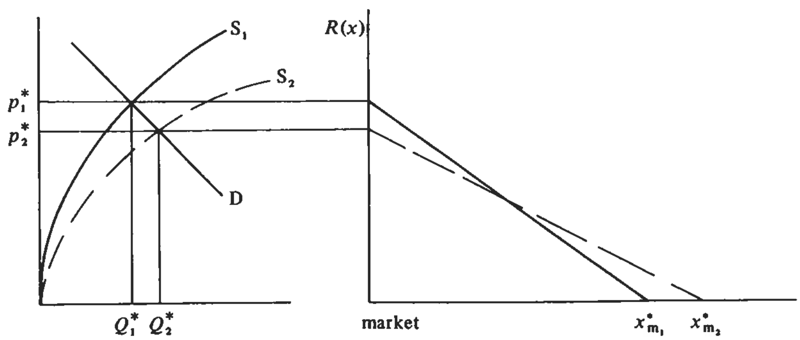

Figure 3. Change in the transport rate.

where the left-hand sides represent the demand for A (commercial) and B (residential) land uses, and the right-hand sides represent the supply curves as functions of prices in the market and transportation rates. Graphically the solution is straightforward as shown in figure 4. Commercial land use outbids residential where the higher demand for commercial output allows it to offer a higher rent, and residential use locates on land not used for commercial or where it can outbid commercial (because of the difference in transportation rates).

If either supply or demand parameters change, the arrival at a new equilibrium can be viewed as a series of iterative steps. For example, assume that the demand for residential use increases. Then residential will bid some land away from commercial, driving the supply curve and the price for commercial upwards, allowing commercial to bid back some of the land it lost. The process is repeated for residential, and then again for commercial, and so on. Either an increase in demand for commercial land use or a decrease in the commercial transport rate would increase the size of the city, since the residential use would pick up some new land at the fringe to replace what it lost to commercial at the margin.

From the above investigation of land use as a producing activity and transportation as an access provider, we have seen that a change in the transportation system alters the supply of areally produced goods and services and also their relative prices. Particularly, a decrease in the cost of transportation will have the effect of extending the area of the city and reducing the dominance of the core, spreading the distribution of rents or land values. More than one homogeneous land use can be considered at a time, but the cost in increased complexity is high [Hoover (1948) and Isard (1956) have extended location theory in this direction]. A better alternative may be to utilize the intensity of land use as a continuous variable, as is done in the following model.

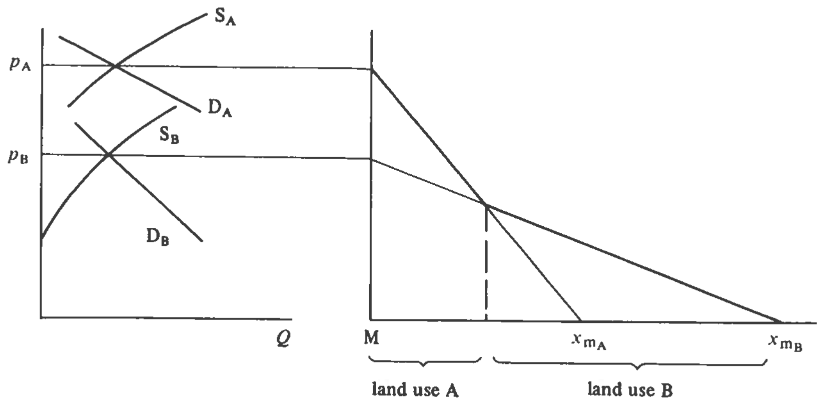

Figure 4. Two competing land uses.

3 Transportation as access: land use intensity

Up to this point, a very simple production function has been used to represent the productive sector, with output constant per unit of land. A Cobb-Douglas function with constant returns to scale is assumed now, allowing substitution between capital and land inputs:

$$Q(x) = K_{\mathrm{p}}^{\alpha}(x)L_{\mathrm{p}}^{\beta}(x) , \quad 0 < \alpha, \beta < 1 \text{ and } \alpha+\beta = 1 . \tag{10}$$

Hence variations may occur in the output produced on a unit of land.

In this model and the ones to follow, aggregate demand is taken as exogenous, represented by the price, p, of a unit of output at the central market [5]. At any given distance x, the entrepreneur's ability to pay for the use of the land is the residual between gross revenues and other factor costs, namely capital and transportation:

$$R(x) = (p-rx)\frac{Q(x)}{L_p(x)} - \frac{kK_p(x)}{L_p(x)} \;. \tag{11}$$

This quantity is also the location rent per unit of land.

If it is assumed that individual entrepreneurs maximize their rent paying ability by adjusting their capital investment at each distance from the center, the optimum level of investment is determined for each location:

$$\max_{K_p} R(x) = (p-rx)K_p^{\alpha}(x)L_p^{\beta-1}(x) - \frac{kK_p(x)}{L_p(x)} \;. \tag{12}$$

The first order condition $\partial R/\partial K_p = 0$ is then

$$\alpha(p-rx)\left[\frac{K_p(x)}{L_p(x)}\right]^{\alpha-1} = k \;, \tag{13}$$

which states that the optimal intensity of land use is achieved when the net value of the marginal product of capital just equals its marginal cost or price, k. Solving for $K_p(x)/L_p(x)$ gives

$$\frac{K_p(x)}{L_p(x)} = \left(\frac{\alpha}{k}\right)^{1/1-\alpha}(p-rx)^{1/1-\alpha} \;. \tag{14}$$

The second order condition for a maximum is satisfied since $0 < \alpha < 1$, that is, decreasing returns to capital inputs K_p.

Intensity of capital per unit of land as a function of distance is thus decreasing at a decreasing rate, and exhibits the concave shape of figure 5. By substituting for $K_p(x)$, the rent function becomes

$$R(x) = \left(\frac{\alpha}{k}\right)^{\alpha/1-\alpha}(1-\alpha)(p-rx)^{1/1-\alpha} \;. \tag{15}$$

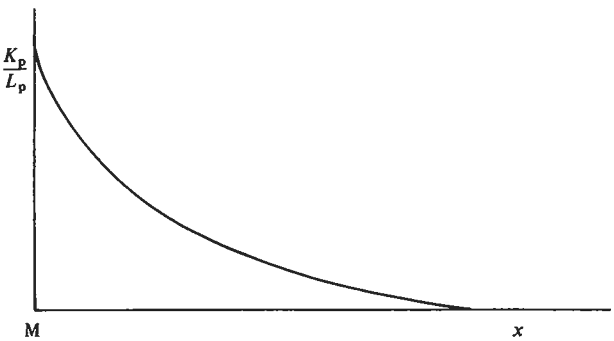

Figure 5. Land use intensity profile.

[5] Using p to represent aggregate demand may be slightly misleading, since supply, and hence actual demand, depends upon the shape of the city. A semicircular city would produce half as much as a circular city, for the same market price. We are assuming that p is determined by some competitive equilibrium that has taken nonproductive land into account (bodies of water, public lands, unusable terrain, etc.), whatever its spatial distribution, and we are now investigating the land use profile or gradient.

Rent is thus also a function of distance, decreasing at a decreasing rate, and exhibits a shape similar to that in figure 5. It can be noted that intensity and rent differ only by a constant factor,

$$\frac{kK_{\mathrm{p}}(x)}{L_{\mathrm{p}}(x)}\bigg/R(x) = \frac{\alpha}{1-\alpha} = \frac{\alpha}{\beta} , \tag{16}$$

which is the ratio of the value of capital improvements to land value; the more efficient capital is relative to land, the higher will be the optimal level of capital improvements relative to land value.

It is now possible to ask under what conditions, within a comparative static framework, will land values increase the most near a particular point of access and less at greater distances from the point. An example might be a rapid transit or subway station. Let x^* be the particular point bestowed with a new level of access that was previously enjoyed by a point located a distance s closer to the market, that is, at $x^* - s$. Then the change in rent as one moves farther out from the new point of access is,

$$\begin{aligned} \Delta R(x) &= R(x-s)-R(x) , && 0 < s < x^* , \\ &= -sR'(x-\theta_1 s) , && 0 < \theta_1 < 1 . \end{aligned} \tag{17}$$

A necessary and sufficient condition, then, for the rent increment $\Delta R(x)$ to be positive is that the rent function $R(x)$ decreases. The increasing or decreasing nature of the rent increment function can be obtained by considering the sign of its derivative,

$$\begin{aligned} \Delta R'(x) &= R'(x-s)-R'(x) \\ &= -sR''(x-\theta_2 s) , \quad \text{with } 0 < \theta_2 < 1 . \end{aligned} \tag{18}$$

Thus a concave intensity gradient is a necessary and sufficient condition for land value increases to be higher nearer a point of access. Diagrammatically the situation can be represented by figure 6. This provides a theoretical hypothesis about the impacts of rapid transit stations on surrounding land values, and warrants empirical testing.

By allowing the intensity of land use to vary in the above model, a simple and direct derivation of the empirically well-known concave rent gradient has been obtained. The same model has also been used to explain the hypothesis that land values should increase proportionately more nearer new rapid transit stations.

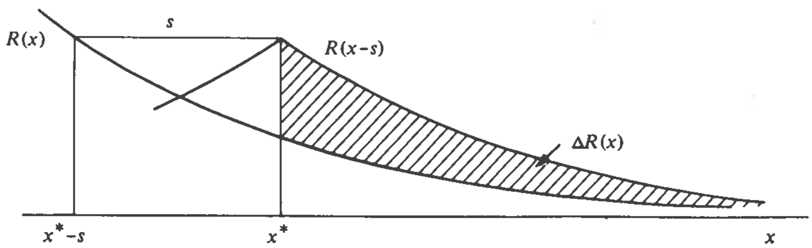

Figure 6. Rent change around a point of access.

4 Transportation as land use: balance between transportation and other land use

On the one hand, transportation is used not as an end in itself, but as a means for consuming or producing other goods and services. As such, the demand for transportation is a derived demand, and transportation is an intermediate commodity which complements the production of other commodities. On the other hand, transportation consumes land and thus competes with other activities for the use of a scarce resource. A policy objective should be to provide enough transportation to enhance the production and consumption of other goods without using up too many resources in the process. One formalization of this concept of balance is offered in the model described below.

Consider an economy with two activities: a productive activity requiring land and transportation as inputs, according to a production function,

$$Q = Q(L_p, T)\,; \tag{19}$$

and a transportation activity requiring land only as an input,

$$T = T(L_t)\,. \tag{20}$$

The total supply of land, $\overline{L}$, is finite and given, such that

$$L_t + L_p = \overline{L}\,. \tag{21}$$

All productive land is assumed to be appropriately served by the transportation service, without any need to explicitly consider distance or a central market.

By taking the maximization of total output as an objective, the problem can be stated as,

$$\max Q(L_p, T)$$

subject to

$$\begin{aligned} &T = T(L_t)\,, \\ &L_t + L_p = \overline{L}\,, \\ &L_t \geqslant 0,\ L_p \geqslant 0\,; \end{aligned} \tag{22}$$

or

$$\max Q[\overline{L} - L_t, T(L_t)]$$

subject to

$$\begin{aligned} &\overline{L} - L_t \geqslant 0\,, \\ &L_t \geqslant 0\,. \end{aligned} \tag{22'}$$

If concavity of the function $Q[\overline{L} - L_t, T(L_t)]$ is assumed, the Kuhn–Tucker conditions are necessary and sufficient for a global maximum. The Lagrangian of this problem is defined to be,

$$\mathcal{L}(L_t, \lambda) = Q[\overline{L} - L_t, T(L_t)] + \lambda[\overline{L} - L_t] \tag{23}$$

(where λ is a Lagrange multiplier), and the Kuhn–Tucker conditions are

$$\begin{aligned} &\frac{\partial \mathcal{L}}{\partial L_t} = -\frac{\partial Q}{\partial L_p} + \frac{\partial Q}{\partial T}\frac{\partial T}{\partial L_t} - \lambda \leqslant 0 \qquad \text{and} = 0 \text{ if } L_t > 0\,, \\ &\frac{\partial \mathcal{L}}{\partial \lambda} = \overline{L} - L_t \geqslant 0 \qquad \text{and} = 0 \text{ if } \lambda > 0\,. \end{aligned} \tag{24}$$

The economic interpretation of the Lagrange multiplier, λ, is clear: it is equal to the change in optimal total output which would occur in response to a unit change in the land constraint, or, in other words, the marginal productivity of land in the optimal city.

If an interior solution is obtained, that is, $0 < L_t < \overline{L}$, the condition for optimality is

$$\frac{\partial Q}{\partial L_p} = \frac{\partial Q}{\partial T}\frac{\partial T}{\partial L_t}\,, \tag{25}$$

stating that the marginal productivity of land in production should be equal to the marginal productivity (in terms of contribution to total output) of land in transportation.

If all land is allocated to the productive activity, that is, $L_{\mathrm{p}} = \bar{L}$, then

$$\frac{\partial \mathcal{L}}{\partial L_{\mathrm{t}}} < 0 ,$$

or equivalently,

$$\frac{\partial Q}{\partial L_{\mathrm{p}}} > \frac{\partial Q}{\partial T}\frac{\partial T}{\partial L_{\mathrm{t}}} ,$$

implying that the marginal productivity of land devoted to production is higher than the marginal productivity of land devoted to transportation, even when no land is used for transportation.

As an example, take

$$Q(L_{\mathrm{p}}, T) = L_{\mathrm{p}}^{\alpha} T^{\beta}$$

and

$$T(L_{\mathrm{t}}) = a + bL_{\mathrm{t}} .$$

Then the share of land given over to transportation in the optimal city is

$$\frac{L_{\mathrm{t}}}{\bar{L}} = \frac{b\beta\bar{L} - \alpha a}{(\alpha+\beta)b} .$$

If b, the (constant) rate at which land can be transformed into transportation services, is allowed to approach infinity, then the above expression approaches

$$\frac{L_{\mathrm{t}}}{\bar{L}} = \frac{\alpha}{\alpha+\beta} .$$

Hence the limit on the optimal share of land in transportation, given a linear production function for transportation, depends upon the relative efficiency of the two factors of production. Should it happen to be the case that $\alpha = \beta$, the limit becomes one-half, or 50% of the city given over to transportation. For a higher share to be optimal, it is necessary to have a high productivity of land in transportation (b large), and either increasing returns in transportation (transportation land becomes more efficient at larger scales) or a high efficiency of transportation in production (α relatively larger than β). Since it appears that the proportion of land taken up by streets and related automobile land uses approaches 60% or higher in portions of some cities, the actual forms and parameters of these production functions is of substantial interest (Sagasti and Ackoff, 1971).

5 Transportation as land use: balance between transportation modes

As a final slice at the transportation and land use relationship, regard the transportation agency as an independent economic unit. At any point in the city it produces transportation services from two inputs, land and capital, and faces a downward sloping demand for its services(6). Prices of the inputs are provided

(6) Labor has been ignored as an input. While clearly a major cost component, it ought to be a separable issue from the land use viewpoint.

exogenously. The two functions,

$$T(x) = T[L(x), K(x)] > 0 \qquad \frac{\partial T}{\partial L}, \frac{\partial T}{\partial K} > 0 ,$$

$$p(x) = p[T(x)] > 0 \qquad \frac{\partial p}{\partial T} < 0 , \tag{26}$$

are production and demand respectively, for transportation at any given distance from the center. Local profits are

$$\Pi(x) = p(x)T(x) - R(x)L(x) - kK(x) , \tag{27}$$

where the price of land, $R(x)$, typically is declining with increasing distance from the point of maximum access, and the price of capital, k, is fixed. By substituting and maximizing profits with respect to the two inputs,

$$\frac{\partial T}{\partial L}\left(p + \frac{\partial p}{\partial T}T\right) = R ,$$

$$\frac{\partial T}{\partial K}\left(p + \frac{\partial p}{\partial T}T\right) = k , \tag{28}$$

which indicates that the marginal value product of each input should equal its price, in equilibrium[7]. If the two equations are combined,

$$\frac{\partial T}{\partial K}\Big/\frac{\partial T}{\partial L} = \frac{k}{R}$$

equates the marginal rate of technical substitution with the price ratio of the inputs.

For example, assuming a Cobb–Douglas production function,

$$T(x) = L^{\beta}(x)K^{\alpha}(x) ,$$

leads to the input substitution relationship,

$$K(x) = \frac{\alpha R(x)}{\beta k}L(x) ,$$

which dictates that more capital be used where the efficiency of capital is high or the cost of land is high, and less capital be used if its own price or the efficiency of land is high. If the price of land is higher at points closer to the center of the city, then the capital intensity of transportation should also be higher. A land intensive mode is appropriate for low density areas, and subways for central business districts.

Because high demand and high land prices are likely to be spatially associated, as in downtown areas, the capital intensity of transportation can be expected to vary widely. One of the major shortcomings of urban transportation is to fail to account for this variation and adapt to it, particularly the variation in land value. The problem is compounded by the difficulty of smoothly adjusting capital–land ratios over space, in response to differing demand and supply conditions. Current technologies require a physical shift in mode in order to obtain higher land utilization, but dual mode systems are an attempt to surmount this obstacle.

[7] Profit maximization may not always be the preferred criterion for public agencies to apply, but it is used here. Second order conditions place some constraints on the production and demand functions; requiring second derivatives of $T(x)$ and $p(x)$ to be negative are sufficient, although not necessary, conditions.

6 Conclusions

Four partial models have been presented as representing important issues in transportation and land use that have not received adequate attention. The first two models dealt with the response of land use to the access-providing aspect of transportation; the second two models treated transportation as a land use, in competition with other activities for the use of scarce resources. Emphasis throughout has been on the supply side, regarding both land users and transportation users as producers requiring inputs, in contrast to the more common consumer demand approach to both land use distribution and transportation services.

Results from partial models must be applied cautiously, but the above models were designed to lead to a number of general conclusions:

1 Aggregate demand normally can be regarded as exogenous for purposes of constructing theory in land use and transportation, but this should not necessarily imply that demand can be taken as constant in a comparative static framework.
2 Intensity of land use increases with increasing access, and at an increasing rate. Land price and density gradients are compatible reflections of this overall structure.
3 Capital intensity in transportation should parallel intensity and rent in adjacent land uses, that is, expensive land should be occupied by heavy capital investment, whether it is for transportation or something else.
4 Transportation should be required to compete for scarce resources, including land, on the basis of the ability of transportation to contribute (indirectly) to aggregate welfare, the same as any other land use.

Public policy has tended to ignore the above proposals, with the result that pricing, taxation, and investment related to transportation have greatly distorted the allocation of land to competing uses and the allocation of resources between modes. Achieving balance in land use and transportation calls for an understanding of the structural interrelationships linking the two together, and the application of efficiency criteria that treat alternative resource uses equivalently.

The models above are admittedly partial, fail to acknowledge the numerous important dynamic aspects of transportation and land use, and ignore the nature of the mixed economy in which both systems, in reality, operate. A partial synthesis into a more general framework has been achieved in Averous and Lee (1973), including a treatment of the mixed economy, but with a concomitant increase in complexity.

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