Neural Networks for Simulating Cognitive Development: A Case Study in Early Mathematical Abilities

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Mathematical development may be investigated through the observation of behaviour, or alternatively, through computer-based modelling. Connectionist models, often referred to as artificial neural networks, can demonstrate aspects of human cognitive development, owing to their capacity to learn. Mathematical abilities have been explored through computer metaphors by developmental psychologists (Wynn, 1995), and through connectionist approaches by cognitive neuropsychologists (McCloskey and Lindemann, 1992) and neuroscientists (Dehaene and Changeux, 1993). We examine how artificial neural networks can be of benefit in the study of mathematical development amongst infants by describing how two specific aspects of this development can be simulated. First, we attempt to simulate a biologically-endowed mechanism for the discrimination and representation of small numerosities of entities: for this model we use a self-organizing neural network in a process of feature extraction. Second, we present arguments as to why and how the so-called recurrent neural networks can be used to model rhythmicity in a simulation of learning the temporal behaviour of counting. The notion of recurrency is examined in this model through a combination of feedback loops with delay elements.

We suggest that three categories of neural network learning procedures appear to describe the component branches of numerical development. Firstly, an unsupervised learning algorithm appears to simulate the acquisition of the earliest, pre-verbal skills: those which appear to spontaneously emerge, such as numerosity discrimination and the acquisition of ordinal concepts. Secondly, the reinforcement learning algorithm appears to simulate conditioned behaviour, and thirdly a supervised learning algorithm may be used to describe aspects of numerical development determined by linguistic knowledge: those later-occurring abilities which are subject to environmental experience, for example, the acquisition of the linguistic counting system and the arithmetic facts.

Furthermore, we propose that a nervous system-level model, a term coined after Kohonen (1990), is an appropriate architecture for a simulation of mathematical development. Such an architecture enables a range of learning strategies to be simulated within a single model. Nervous system-level (organizational) models are composed of a system of interconnected connectionist networks, where the connections themselves are connectionist networks. Each constituent network in a simulation of development is based upon a different, well-grounded architecture and employs an appropriate learning algorithm, in order to mimic an aspect of the development under consideration. Under this approach, the spectrum of pre-verbal and language-dependent mathematical abilities may be modelled within a single architecture.

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