UCLA

UCLA Previously Published Works

Title

Leaky and bound modes in terahertz metasurfaces made of transmission-line metamaterials

Permalink

<https://escholarship.org/uc/item/8nw8n0zw>

Journal

Journal of Applied Physics, 113(3)

ISSN

0021-8979

Authors

Hon, PWC

Liu, Z

Itoh, T

et al.

Publication Date

2013-01-21

DOI

10.1063/1.4776761

Peer reviewed

Leaky and bound modes in terahertz metasurfaces made of transmission-line metamaterials

Philip W. C. Hon,¹ Zhijun Liu,^{1,2} Tatsuo Itoh,¹ and Benjamin S. Williams^{1,2,a)}

¹Department of Electrical Engineering, University of California, Los Angeles, California 90095, USA

²California NanoSystems Institute (CNSI), University of California, Los Angeles, California 90095, USA

(Received 21 November 2012; accepted 2 January 2013; published online 18 January 2013)

Prism coupling and reflection spectroscopy are used to characterize bound modes within composite right/left handed terahertz metamaterial waveguides. The cavity antenna model is used to understand the polarization dependence of the radiative coupling to TM_{00} and TM_{01} waveguide modes. Furthermore, the cavity model along with transmission-line theory is used to derive a surface impedance model for a waveguide array metasurface. Qualitative agreement with the experiment is observed, including a mode splitting for p-polarized surface waves at the light line and the existence of s-polarized magnetic spoof surface plasmons. © 2013 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4776761>]

I. INTRODUCTION

The prospect of guiding or concentrating electromagnetic energy at length scales smaller than the wavelength has motivated a great deal of recent activity in the electromagnetics and photonics communities. The topic is particularly relevant for the development of terahertz (THz) quantum-cascade (QC) laser sources in the underdeveloped THz frequency range (1–10 THz).^{1,2} These lasers are typically grown epitaxially in GaAs/AlGaAs heterostructure quantum wells to form an active region that is 5–10 μm thick—significantly smaller than the wavelength ($\lambda_0 = 100 \mu\text{m}$ at 3 THz). This renders conventional dielectric waveguides impractical. Hence, two successful waveguides have been demonstrated, the surface plasmon waveguide¹ and the metal-metal waveguide,³ both of which exhibit an enhanced confinement of fields within the subwavelength active volume. Metal-metal waveguides are fabricated by sandwiching the semiconductor intersubband gain medium between two metal cladding layers using wafer bonding.^{3,4} The resulting ridge waveguides are similar in form to parallel plate or microstrip transmission lines (TLs), which are known to support a quasi-TEM mode without a low frequency cutoff. A major challenge for THz QC-laser design is the efficient out-coupling of laser power into a directive beam from a cavity with subwavelength dimensions. The primary techniques have involved the use of Bragg scattering from integrated gratings or photonic crystals to achieve either surface emission^{5–7} or end-fire emission.⁸

We have recently proposed a different approach for beam engineering for THz QC-lasers: the use of 1D composite right/left handed (CRLH) TL metamaterial waveguides as leaky-wave antennas (LWA).^{9,10} The CRLH TL metamaterial concept, originally explored in the microwave regime, has been used to implement a wide variety of guided-wave devices (e.g., multi-band and enhanced bandwidth components, power combiners/splitters, compact resonators, phase shifters, and phased array feed lines), as well as radiated-wave devices

(e.g., 1D and 2D resonant and leaky-wave antennas).^{11–13} A conventional transmission line exhibits right handed (forward wave) propagation, where a unit cell is composed of a shunt capacitance C_R and series inductance L_R . If the line is periodically loaded with distributed series capacitance C_L and shunt inductance L_L , the loaded TL becomes highly dispersive and exhibits regions of left-handed (backward wave) propagation. If operated within the leaky-wave region (i.e., where the propagation constant $|\beta| < \omega/c$), a CRLH waveguide can operate as a leaky-wave antenna. These features allow electromagnetic waveguide and antenna concepts to be implemented in the THz regime. LWAs for THz QC-lasers that exhibit both right hand (RH) only and CRLH operation have been demonstrated.^{14,15}

In this work, we experimentally characterize both bound and leaky-wave modes in passive THz CRLH waveguides. Since it is non-trivial to measure the S-parameters of a single waveguide in the THz range, we instead fabricate a uniaxial metasurface composed of dense arrays of passive THz CRLH waveguides and experimentally map their dispersion relation using angle-resolved reflection spectroscopy. As described in Ref. 16, the waveguides are implemented using metal-insulator-metal topology and exhibit dispersion relations that are well fit by a circuit model. Here, we extend the measurement to include observation of bound surface wave modes using Otto (prism) coupling.¹⁷ Depending upon the incident polarization, either a RH-only, or a CRLH waveguide mode is excited; a behavior which is explained by invoking the microwave cavity antenna model. We have previously used the cavity antenna model to provide a quantitative estimate of the radiative loss, as well as the far-field beam pattern and polarization from metal-metal waveguide cavities and antennas.¹⁰ Here, we use the cavity antenna model along with TL theory to derive an effective surface impedance model to describe CRLH waveguide array metasurfaces. This model is in good qualitative agreement with the experiment and correctly predicts the coupling and mode splitting (i.e., anticrossing) between the p-incident radiation and RH only waveguide modes. These modes are similar to

^{a)}Electronic mail: bswilliams@ucla.edu.

the spoof surface plasmon polariton modes that are supported on structured metal surfaces that exhibit large inductive surface impedance.^{18–20} The CRLH waveguide mode is shown to form s-polarized surface waves which do not exhibit such a splitting. Furthermore, we show that these s-polarized CRLH modes can be considered as a type of magnetic spoof surface plasmon that is supported when the surface impedance of the metasurface becomes large and capacitive.^{21,22}

II. CRLH METAMATERIAL METAL-METAL WAVEGUIDE

A THz CRLH waveguide can be realized in metal-metal waveguide by introducing gaps into the top metallization to create a series capacitance C_L (Figs. 1(a) and 1(b)). Although the shunt inductance L_L can be realized by introducing side-wall or via metallization from the top conductor to the ground plane,⁹ such a structure is difficult to fabricate and requires the use of a lossy virtual ground capacitor if one wishes to apply a DC electrical bias to the top contact.

A preferable approach is to operate the waveguide in its higher-order lateral TM_{01} mode,^{10,14} which can be qualitatively represented using a circuit model with two coupled parallel transmission lines, as shown in Fig. 1(c). The conduction currents in the transverse direction can be represented by an effective shunt inductance L_L . The odd symmetry of the mode introduces a virtual ground plane along the center of the waveguide. Proper choice of dimensions and introduction of other structures (such as holes and overlay capacitors) allows

control of the four circuit parameters for realization of a CRLH metamaterial, which exhibits both forward and backward wave propagation.¹⁰ In general, such a transmission line is “unbalanced” and exhibits a bandgap between the LH and RH ranges bounded by the shunt ($\omega_{sh} = (L_L C_R)^{-1/2}$) and series ($\omega_{se} = (L_R C_L)^{-1/2}$) resonant frequencies (see Fig. 1(d)). For operation at or near $\beta \approx 0$, the shunt or series resonant frequency is associated with stored energy in either the series $L_R C_L$ or shunt $L_L C_R$ tank (field profiles as shown in Figs. 1(e) and 1(f), respectively). The dispersion can be “balanced” by choosing parameter values such that $\omega_{se} = \omega_{sh}$; a balanced line has no bandgap and is characterized by propagating modes at $\beta = 0$. The shunt resonance frequency ω_{sh} corresponds to the standing-wave resonance condition where the waveguide width w is approximately equal to one half of the wavelength in the dielectric ($w \approx \lambda_o/2n$). The theory of CRLH TL metamaterials is described in considerable detail in several reviews and books.^{11–13}

When the waveguide is operating in its fundamental lateral TM_{00} mode (see Fig. 2), the effective shunt inductance L_L does not exist, and only a RH branch of operation in its dispersion relation is observed. If series gap capacitors are present, propagating modes are cut-off below the series resonant frequency ω_{se} .

III. RADIATION MODELING

The cavity antenna model provides a semi-quantitative tool for analyzing the radiation patterns, polarization, and

FIG. 1. (a) A perspective and (b) top view of the metal-metal waveguide operating near $\beta = 0$ with a unit cell size, p , with its x component, E -field profile, and equivalent magnetic current sources $\mathbf{M}_s$, given by double-headed arrows for the higher-order lateral TM_{01} waveguide mode. (c) Circuit model and (d) typical dispersion. (e) Side view of the E -field profile from a full-wave finite element simulation (Ansys's HFSS) of an infinitely long structure operating at the series band edge mode ω_{se} and (f) shunt band edge mode ω_{sh} .

FIG. 2. A perspective and top view of the metal-metal waveguide operating near $\beta = 0$ with a unit cell size, p , with its x component, E-field profile, and equivalent magnetic current sources $\mathbf{M}_s$, given by double-headed arrows for the fundamental TM_{00} waveguide mode with its equivalent circuit model.

quality factors from conventional and CRLH metal-metal waveguides.¹⁰ This model is based upon the field equivalence principle, where volumetric current sources inside the cavity are replaced by effective magnetic currents $\mathbf{M}_s$ on the outer surface (described in detail in Ref. 23). The magnitude of these currents is determined by the transverse electric fields according to $\mathbf{M}_s = -2\hat{\mathbf{n}} \times \mathbf{E}$, where $\hat{\mathbf{n}}$ is the surface normal vector. For our geometry, due to the presence of the ground plane, and the orientation of the magnetic field, the effect of electric currents can be neglected for low-profile structures with minimal fringing fields. Considering operation near $\beta = 0$ for the fundamental lateral TM_{00} waveguide mode (Fig. 2), the equivalent magnetic current sources along opposite sidewalls are out of phase, leading to destructive interference. On the other hand, the equivalent sources in the gaps are in phase, which gives rise to far-field radiation that is polarized in the θ -direction or along the waveguide axis. The E-field is concentrated within the gap capacitor and has a field profile similar to the HFSS field plot shown in (Fig. 1(e)). Hence, for the fundamental mode, coupling to radiation takes place almost entirely through the series gap capacitors—we represent this phenomenologically by placing a radiation conductance G_{rad} across the series capacitance C_L in its circuit model (see Fig. 2).

The case is reversed for the higher-order lateral TM_{01} waveguide mode (Figs. 1(a) and 1(b)). Here, the equivalent magnetic current sources along opposite side walls are in phase and give rise to far-field radiation polarized in the φ -direction or transverse to the waveguide axis. Because the E-field within the series capacitive gaps follows the symmetry of the mode, the equivalent magnetic current sources in the gaps are out of phase, which leads to destructive interference and cancellation of its far field at broadside. Therefore, the coupling to radiation takes place mostly through the fields associated with the shunt capacitors, which we can represent with a shunt radiation conductance G_{rad} across C_R (Fig. 1(c)). This is essentially the same radiation mechanism as a patch antenna.²³ For non-zero values of β , there will be a phase shift ($\sim \beta p$) between $\mathbf{M}_s$ from adjacent unit cells, which leads to both steering of the beam away from surface normal ($\theta = 90^\circ$ relative to coordinate system in Fig. 2(a)) and a change in the radiated power.

The cavity antenna model can be used to derive analytic expressions for the radiation conductance in limiting cases. We use the formalism and approximations that are described in the Appendix of Ref. 10 to calculate the total radiated power P_{rad} from a waveguide. The difference here is that we consider radiation from arrays of waveguides spaced with a

lateral period Λ rather than a single waveguide; we choose this geometry to be consistent with the surface impedance model described next in Sec. IV. We assume the ideal limiting conditions for the ridge width $w \ll \lambda_0$, height $h \ll \lambda_0$, and length $\ell \gg \lambda_0$, and consider only the leaky-wave regime, where the effective modal index obeys $|n_{\text{eff}}| < 1$ (where $n_{\text{eff}} = \beta c / \omega$). A value for the radiation conductance can be derived by invoking the circuit theory relation for time averaged dissipated power $G_{\text{rad}} = 2P_{\text{rad}}/|V_0|^2$. V_0 is the equivalent voltage across C_L , which is approximately related to the E-field E_0 within the gap of length a by the expression $V_0 = E_0 a$. For a single waveguide operating in its TM_{00} mode within $|n_{\text{eff}}| < 1$, the radiation conductance is then given by

$$G_{\text{rad},00} = \frac{1}{\eta_0 \cos \alpha} \frac{w^2}{\Lambda p}, \quad (1)$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, p is the unit cell size, w is the ridge width, and α is the incident angle relative to the surface normal. Similarly, for the TM_{01} mode, we obtain

$$G_{\text{rad},01} = \frac{\cos \alpha}{\eta_0} \frac{1}{F} \frac{2p}{\Lambda}. \quad (2)$$

However, in this case, the voltage appears across the shunt capacitor C_R , so that V_0 is related to the E-field at the waveguide sidewall. The presence of $\cos \alpha$ in these expressions reflects the dependence of the free-space characteristic impedance of a plane wave incident upon a surface with angle of incidence α relative to the surface normal. The factor F appears as a correction factor that accounts for the sinusoidal lateral E and H-field variation along the width of the waveguide ridge ($w \approx \lambda_0/2n$) for the TM_{01} mode. It is necessary to ensure that conservation of energy is satisfied for calculations of the radiative power and is consistent between the field model and the circuit model. For the ideal analytic case, where the top waveguide metallization is continuous in the lateral direction (Figs. 1(a) and 1(b)) then $F = \pi/4$. As the structure becomes more complicated, such as by the introduction of subwavelength holes in the top metallization to increase L_L ,^{10,14} then F will increase—eventually reaching unity in the lumped element limit.

IV. SURFACE IMPEDANCE MODEL

It is convenient to characterize the dispersion characteristics of both bound and leaky-wave modes of CRLH waveguides using angle dependent reflection spectroscopy. If

FIG. 3. (a) Transmission line model for a CRLH metasurface with its fundamental TM_{00} waveguide mode excited by p-polarized incident light. (b) HFSS simulations and surface impedance model predictions for surface reactance for p-polarized incident light at an incident angle $\alpha = 40^\circ$ relative to the surface normal. The surface impedance model qualitatively captures the reactive transitions in the dispersion. The inset maps the regions of capacitive and inductive reactance.

arrays of waveguides are fabricated, spaced with a periodicity $\Lambda < \lambda_0/2$ in the lateral direction, only the zeroth order diffractive component will contribute to the far-field reflected power (i.e., specular reflection). Provided the meta-material unit cell length is $p < \lambda_0/4n$ (where n is the refractive index of the dielectric), we operate sufficiently far from the 1st Brillouin zone edge that Bragg scattering will not significantly contribute to the dispersion, and we can use the previously described TL metamaterial model to describe the propagating waveguide modes. The subwavelength structure of the waveguide will determine the detailed dispersion relation $\beta(\omega)$, both through the fundamental and higher-order Floquet-Bloch terms, whose contribution can be captured via the effective lumped element values for the TL (C_R , L_R , C_L , L_L). However, the higher-order Floquet-Bloch terms contribute only to the evanescent near field and do not radiate into the far field. We can then treat this as a metasurface described by a surface impedance Z_S and in turn calculate the reflection coefficient for incident plane waves. When plane waves are incident upon the metasurface with the plane of incidence along the waveguide axis (z-axis in Fig. 3(a), Fig. 4(a)) and

with incident angle α , incident light will couple with propagating waveguide modes when $\beta(\omega) = k_0 \sin(\alpha) = k_z$, where it is either absorbed (due to dielectric or ohmic losses), or reradiated (specular reflection). Depending upon the polarization of an incident propagating plane wave, either the TM_{00} or TM_{01} waveguide mode will be excited. While at higher frequencies higher order lateral modes can be excited, we will confine our treatment here to the lowest order odd and even modes.

A. P-polarization

As shown in Fig. 2, along with the cavity model discussion in Sec. III, the symmetry of the TM_{00} waveguide mode dictates that radiation from the waveguide is mediated via E-fields in the series capacitor. For a p-incident polarized plane wave, the E-field is polarized along the axis of each waveguide, and by reciprocity couples to the TM_{00} waveguide mode via the series gap capacitors. Hence, we define a transmission line model where we place an input port across the series capacitance C_L in each unit cell, as shown in Fig. 3(a). The incident wave induces a voltage $V'(z)$

FIG. 4. (a) Transmission line model for a CRLH metasurface with its higher order lateral TM_{01} waveguide mode excited by s-polarized incident light. (b) HFSS simulations and surface impedance model predictions for surface reactance for s-polarized incident light at an incident angle $\alpha = 40^\circ$ relative to the surface normal. The surface impedance model qualitatively captures the reactive transitions in the dispersion, either from capacitive to inductive or inductive to capacitive. The inset maps the regions of capacitive and inductive reactance.

$= V'_0 e^{-jk_z z}$ and current $I'(z) = I'_0 e^{-jk_z z}$, which defines an input impedance for the unit cell as $Z_{in,p} = V'(z)/I'(z)$. We obtain

$$Z_{in,p}(k_z, \omega) = Z_1 \frac{Y_1 Z_2 + k_z^2 p^2}{Y_1 Z_1 + Y_1 Z_2 + k_z^2 p^2}, \quad (3)$$

where $Z_1 = 1/(j\omega C_L + G_L)$, $Z_2 = j\omega L_R + R_R$, $Y_1 = j\omega C_R + G_R$. If there are no metal ohmic losses ($R_R = 0$) and no dielectric losses ($G_L = G_R = 0$), we obtain the lossless case where $Z_1 = 1/j\omega C_L$, $Z_2 = j\omega L_R$, $Y_1 = j\omega C_R$. The input impedance is then

$$Z_{in,p}(k_z, \omega) = \frac{-j}{\omega C_L} \frac{\omega^2 - k_z^2 p^2 / L_R C_R}{\omega^2 - \omega_{se}^2 - k_z^2 p^2 / L_R C_R}. \quad (4)$$

It should be noted that $Z_{in,p}$ is different from the characteristic impedance often defined for a CRLH line.¹¹ We now convert $Z_{in,p}$ to a surface impedance $Z_{s,p}$ (units $\Omega/\square$)

$$Z_{s,p} = Z_{in,p} Z_0 G_{rad,00} = Z_{in,p} \frac{w^2}{p\Lambda}. \quad (5)$$

The input impedance was rescaled by multiplying $Z_{in,p}$ by the factor $Z_0 G_{rad}$, where G_{rad} is the radiation conductance in the series branch (Eq. (1)), and Z_0 is the characteristic wave impedance in a vacuum for p-polarized plane waves ($Z_0 = \eta_0 \cos \alpha$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$). The rescaling of Z_{in} to Z_s accounts for both the aspect ratio of the metamaterial unit cell area ($p \times \Lambda$) and cases where ridges of width w may have a fill factor below unity ($w < \Lambda$). The dispersion relation for the waveguide modes, $\beta(\omega)$, can be extracted from the poles of Z_{in} by replacing k_z with β . For normal incidence and unity fill factor ($w = \Lambda$), $Z_{s,p}$ is equivalent to that of a Sievenpiper mushroom surface.²⁴

Results from the analytic surface impedance model were compared with the surface impedance extracted from a HFSS full-wave 3D simulation of a plane-wave reflection coefficient of a lossless CRLH waveguide metasurface. Lumped element circuit parameters for Eq. (3) were extracted from a circuit model fit to the dispersion relation obtained from a full-wave 3D simulation. The details of the simulated CRLH waveguide are not important for this comparison and are left for discussion in Sec. V. All dielectrics were lossless and all metals were simulated as a perfect electric conductor (PEC), so that the surface impedance is purely reactive. For p-polarized incident light at an incident angle $\alpha = 40^\circ$ relative to the surface normal, for example, the metasurface is inductive at low frequencies and is capacitive above ~ 3.6 THz (Fig. 3(b)). The crossover frequency where $Z_s^{-1} = 0$ is the frequency of the propagating mode; the fact that there is only one crossover reflects the fact that p-polarized light excites the TM_{00} waveguide mode, which has only a RH branch of the dispersion.

B. S-polarization

The cavity model predicts that the TM_{01} waveguide mode couples with s-polarized incident light (E-field polarized transverse to the ridge) through the sidewalls' surface

equivalent magnetic current sources. Hence, the incident E-field induces a voltage $V'(z)$ across the shunt capacitor C_R , and we can place the coupling input port to free space across the shunt branch of each unit cell, as shown in Fig. 4(a). Because of the odd symmetry of the TM_{01} mode, the TL shown represents only one branch of the odd-mode line, so that a unit cell occupies area $p/2 \times \Lambda$. We obtain for the input impedance for one unit cell

$$Z_{in,s}(k_z, \omega) = \frac{1}{Y_{sh} + k_z^2 p^2 / Z_{se}}, \quad (6)$$

where $Z_{se} = j\omega L_R + R_R + 1/(j\omega C_L + G_L)$, and $Y_{sh} = j\omega C_R + G_R + 1/(j\omega L_L + R_L)$. Once again, we obtain the lossless relation by setting the resistances and conductances to zero

$$Z_{in,s}(k_z, \omega) = j\omega L_L \frac{\omega_{sh}^2 \omega_{se}^2 (1 - \omega^2 / \omega_{se}^2)}{\omega^4 - \omega^2 (\omega_{sh}^2 + \omega_{se}^2) + \omega_{sh}^2 \omega_{se}^2 + \frac{k_z^2 p^2}{L_R C_R}}.$$

The surface impedance is given by

$$Z_{s,s} = Z_{in,s} Z_0 G_{rad,01} = Z_{in,s} \frac{1}{F} \frac{2p}{\Lambda}. \quad (7)$$

For s-polarized radiation, $Z_0 = \eta_0 / \cos \alpha$ and once again, the rescaling accounts for the non-square aspect ratio of the unit cell as well as the voltage correction factor F necessary to satisfy conservation of energy.

We compare the surface impedance calculated analytically from Eq. (7) with that extracted from full-wave 3D reflectivity simulations. The surface reactance for an incident angle of $\alpha = 40^\circ$ is shown in Fig. 4(b). Unlike the p-polarized case, for s-polarized radiation the metasurface exhibits two crossover frequencies where $Z_s^{-1} = 0$, corresponding to the LH and RH branches of the dispersion, respectively. $Z_s = 0$ at $\omega = \omega_{se}$, where the series resonance shorts out the surface (see inset of Fig. 4(b)). With an accurate circuit model fit of the metasurface's dispersion, the surface impedance model qualitatively captures the metasurface's different reactive regions with good agreement.

V. LARGE AREA PASSIVE CRLH METAMATERIAL

We now apply the surface impedance model to analyze the experimentally realized CRLH waveguide metasurface reported in Ref. 16. Passive, large area ($1 \text{ cm} \times 1 \text{ cm}$) CRLH TL metamaterial arrays were fabricated in metal-insulator-metal waveguide technology using benzocyclobutene (BCB) as the insulating layer (Figs. 5(a) and 5(b)). The series capacitance C_L was realized by gaps in the top conductor of the metal-metal waveguide; C_L was further enhanced by metal top patches that were isolated with a thin layer of SiO_2 in order to achieve a close-to-balanced dispersion. Three sample variations were fabricated, where the series capacitance C_L was increased by increasing the longitudinal dimension of the $6.5 \mu\text{m}$ wide overlay top metallization from $A = 6.8$, 7.6 , and $8.6 \mu\text{m}$ for samples referred to as S1, S2, and S3, respectively (Fig. 5(a)). This allows the series resonance frequency $\omega_{se} = 1/\sqrt{L_R C_L}$ to be varied with respect to the shunt resonance frequency $\omega_{sh} = 1/\sqrt{L_L C_R}$.

FIG. 5. (a) A perspective and (b) cross section side view of the metal-metal waveguide's unit cell of length p , with its equivalent circuit model. (c) Scanning electron microscope images of the fabricated CRLH waveguide array (perspective view) and a top down view of a unit cell (inset).

Circuit parameters in Figs. 3(a) and 4(a) were extracted using circuit model fits to the dispersion relations obtained from HFSS simulations of the unit cell (Figs. 5(a) and 5(b)). Lossy component values G_R, G_L, R_R, R_L were chosen using lumped element approximations for a parallel plate waveguide with appropriate material parameters (i.e., loss tangents and skin depths).

A. Leaky waves

Both the TM_{00} and TM_{01} waveguide modes exhibit a propagating mode with a finite bandwidth that falls within

the light cone ($|\beta| < \omega/c$). The dispersion is experimentally characterized with angle-resolved Fourier transform infrared (FTIR) reflection spectroscopy. In this technique, incident light is coupled into the waveguide array when the in-plane wave-vector of the incident light matches the waveguides' propagation constant β . This coupling condition is marked by an absorption dip in the reflectivity spectrum. The experimental setup is described in detail in Ref. 16.

Absorption spectra were measured at multiple angles of incidence for each of the three samples, and then plotted as a contour plot as a function of β in Figs. 6(a)–6(c). As predicted by the cavity model, s-polarized (E-field transverse to the waveguide axis) incident light couples to the TM_{01} waveguide mode which exhibits the CRLH characteristic dispersion relation. Notably, both left-handed (backward-wave) and right-handed (forward-wave) branches of the dispersion relation are seen. We compare these experimental data with predicted absorption characteristics calculated using the surface impedance model, as shown in Figs. 6(d)–6(f). Good agreement is observed in the dispersion relation, except for slightly broader spectral features observed in the experimental data. This suggests an underestimate of the lossy lumped element values (G_R, G_L, R_R, R_L) used in the surface impedance model.

As larger overlay patches are used to increase the series capacitance C_L , we observe the dispersion transition from nearly balanced (sample S1) to increasingly unbalanced with a significant bandgap opening (sample S2 and S3). We also observe the absorption in the LH branch become relatively weaker for the unbalanced samples, particularly near the $\beta = 0$ point. This can be understood by considering the distribution of energy in an unbalanced CRLH line as a function of β . At $\beta = 0$, the band-edge resonant modes at ω_{sh} and ω_{se} (see Fig. 1(d)) correspond to resonance purely in the shunt $L_L C_R$ tank or series $L_R C_L$ tank circuits respectively. As $|\beta|$ increases above zero, although the energy is no longer solely stored in the shunt or series tank, nevertheless each dispersion branch retains its “shunt-like” or

FIG. 6. (a)–(c) Measured and (d)–(f) surface impedance model predicted absorption contour plots for samples S1, S2, and S3 respectively, when illuminated with s-polarized incident light.

FIG. 7. Simulated H-field averaged over the unit cell of the CRLH metal-metal waveguide (sample S1). Insets show the magnitude of the E-field for a cross section along the direction of propagation for a unit cell and the placement of the polyethylene prism relative to the sample (Otto-coupling configuration) used to map the waveguides' dispersion outside the light cone.

“series-like” character to some degree. The more unbalanced the line, the larger the bandgap, and the larger the value of $|\beta|$ for which this modal character persists. As we described in Sec. III, due to the odd-symmetry of the TM_{01} mode in our CRLH waveguide, the series resonance does not radiate significantly—it is a “dark” mode. Therefore, s-polarized incident light does not efficiently couple to the “series-like” branch of the dispersion relation, a phenomenon which is observed experimentally and is accurately reproduced by the surface impedance model. The fact that the LH branch of the dispersion relation is “series-like” near $|\beta| \approx 0$ for samples S2 and S3 simply reflects the fact that $\omega_{se} < \omega_{sh}$. Were C_L made to be smaller, the case would easily be reversed and the RH branch would be “series-like.” Hence, reflection spectroscopy is a useful technique to identify the band edge frequencies as either ω_{se} or ω_{sh} , something which cannot be determined solely from looking at a dispersion relation.

Although we have not plotted the corresponding absorption data for p-incident polarization, it is too well fit by the surface impedance model.¹⁶

B. Surface waves

In addition to leaky-wave modes within the light cone, we expect the waveguide array to support bound surface waves for operation outside the light cone. As an example, for the TM_{01} , the magnitude of the averaged H-field components across the unit cell for such a bound mode, calculated at 2.97 THz (on the LH branch), is shown in Fig. 7. The field is observed to decay exponentially away from the metasurface as shown in Fig. 7, therefore confirming the existence of the bound surface wave. We also plot for comparison the expected field decay obtained from the perpendicular component of the wavevector $H \sim \exp(-k_x x)$. Significant E-field enhancement is observed in the $1 \mu\text{m}$ BCB layer compared to the evanescent field—the field within the SiO_2 series capacitors is stronger still (Fig. 7 inset).

In order to access the bound-mode region using variable angle FTIR reflection spectroscopy, we used the Otto-coupling configuration,¹⁷ where a prism is mounted close to the sample with an air gap of Δ , as shown in the inset of Fig. 7. Incident waves couple evanescently across the gap to the metasurface at wavenumbers in the range $\omega/c < k_z < n\omega/c$. A polyethylene prism is used, with a 90° apex, 1 cm sides, and a refractive index of $n = 1.52$, which has a critical angle $\theta_{inc} = 41.1^\circ$. The prism position was fixed and the sample was located on a single axis translation stage. Alignment was performed by bringing the prism into contact with the metasurface manually and then varying the coupling distance from $\sim 5 - 300 \mu\text{m}$ with a piezoelectric translation stage. FTIR reflection spectra were measured using a global black-body source with a resolution of 1 cm^{-1} and a silicon composite bolometer detector under a N_2 purge. To obtain a reflectivity spectrum, the reference background spectrum was taken with the prism in place and the metasurface sample several millimeters away so that total internal reflection occurs at the prism face without any evanescent field interaction.

A set of the reflection spectra outside the light cone for p and s-incident polarization is shown in Fig. 8 for a fixed prism-to-sample coupling distance of $\Delta = 10 \mu\text{m}$. Compared to the reflection spectra within the light line,¹⁶ the signal to noise ratio (SNR) is degraded due to insertion loss of the

FIG. 8. Measured spectra for sample S3 for (a) p-incident and (b) s-incident angles with coupling distance between the sample and prism of $10 \mu\text{m}$. Measured absorption for (inset of (a)) p-incident light and (inset of (b)) s-incident light at a prism-to-air gap incident angle of $\theta_{inc} = 41.5^\circ$ for different prism-to-sample coupling distances.

prism. For p-polarization, only a single absorption feature is observed corresponding to the RH branch, while for s-polarization, two features are seen, corresponding to the LH and RH branches. The absorption features shift with varying angle according to the dispersion relation.

Dispersion relations outside the light line were obtained by fitting Lorentzian curves to the reflection spectra to obtain the absorption peak at each angle. The data from the Otto-coupled measurements are plotted in Fig. 9 alongside the data measured within the light cone that were previously published in Ref. 16. The solid lines represent data from HFSS simulations. The prism line designates the calculated dispersion mapping limit for the polyethylene prism. Similar to operation inside the light cone, a tuning of the bound mode dispersion outside the light cone is observed for an increase in series capacitance. For p-incident polarization, an anticrossing gap is observed as the waveguide dispersion crosses the light line, leading to the formation of a bound mode and radiative mode. This gap indicates the formation of a polariton as the TM_{00} waveguide mode becomes strongly coupled with p-polarized plane waves of grazing incidence. This is to be expected since the fundamental TM_{00} mode is characterized by electric fields in the gap capacitors that couple well to p-polarized incident light at grazing incidence. No anticrossing gap is observed for s-polarized incident light interacting with the CRLH TM_{01} mode. This too is expected, since the incident E-field component is polarized transverse to the ground plane such that it “shorts” with its image. This is also reflected in the fact that radiative quality factors for the CRLH modes within the light cone are higher than those for the TM_{00} mode.

The surface impedance model accurately predicts the dispersion, as shown in the insets of Fig. 9 (sample S1 shown). Each inset contour plot is comprised of two calculations: one within the light cone and one outside the light cone. Inside the light cone, the reflection coefficient is calculated simply for a plane wave incident upon the metasurface from free space. Outside the light cone, the presence of the prism and air gap were accounted for by transforming the surface impedance using transmission line formalism. For the TM_{00} waveguide mode, the surface impedance model

predicts a relatively broad resonance compared to the TM_{01} waveguide mode because of a lower radiative quality factor.

We also examined the effect of the prism-to-air gap Δ spacing on the spectra for a fixed incident angle of $\theta_{inc} = 41.5^\circ$, which is very close to the light line. For the p-polarized spectra, there is little effect on the spectrum for gaps up to $\Delta = 50 \mu\text{m}$; for $\Delta = 100 \mu\text{m}$, the peak redshifts slightly, and for $\Delta = 300 \mu\text{m}$ the absorption is highly redshifted, broadened, and nearly too weak to distinguish (see inset in Fig. 8(a)). This redshift with increased coupling distance is also captured by the surface impedance model (not shown). This measurement suggests that at this particular angle of incidence, for prism distances of $\Delta \leq 50 \mu\text{m}$ the prism is slightly overcoupled to the metasurface and perturbs the bound surface wave mode. As Δ increases beyond this, the apparent broadening and redshift of the absorption feature results from the bound mode returning to its unperturbed state and “hugging” the light line at $\beta = \omega/c$ as it becomes more light-like; this is seen in the contour plot (inset of Fig. 9(a)). In contrast, for s-polarized incident light, no clear shift in frequency is observed as a function of coupling distance—at least to the degree distinguishable from noise (inset in Fig. 8(b)). This finding is consistent with the fact that no anticrossing gap is observed at the light line.

C. Comparison to spoof-surface-plasmon structures

It is instructive to compare our CRLH waveguide metasurface with other reported metasurfaces that support spoof surface plasmons. While THz frequencies are too far below the plasma frequencies of noble metals to support true surface plasmon polaritons, so called “spoof surface plasmons” (i.e., p-polarized surface waves) are supported even for perfectly conducting metal surfaces, provided they are structured to present an inductive surface impedance.^{18–20} One example is a metal surface corrugated with a subwavelength lattice of metal grooves or holes slightly less than a quarter wavelength deep so that the impedance of the surface is tuned from zero to be large and inductive.^{25–27} However, at THz frequencies, the necessary groove depth may be as large as several tens to hundreds of microns, which can be

FIG. 9. Measured dispersion and HFSS simulations for all three samples for (a) p-polarized and (b) s-polarized light inside and outside of the light cone. Dispersion captured by the surface impedance model for (inset of (a)) p-polarized incident light with a $40 \mu\text{m}$ prism-to-sample coupling spacing and (inset of (b)) s-polarized incident light with $10 \mu\text{m}$ prism-to-sample coupling spacing.

prohibitive for planar fabrication processes. Compared to such a corrugated surface, our CRLH metasurface has several differences. First, the height of our metasurface is approximately $1.7\ \mu\text{m}$ —approximately 50–60 times smaller than the free-space wavelength. Such thin structures are often advantageous for microfabrication in a planar process compared to a high-aspect ratio quarter-wavelength deep trench. It is made possible using a multi-layer metal-insulator-metal geometry, where a quarter-wave transmission-line transformer has been replaced by low-profile folded circuit elements to achieve the high inductive impedance required to support a p-polarized surface wave. Second, while our CRLH structure exhibits a p-polarized polariton anticrossing signature, it is not proper to describe the p-polarized surface wave as a spoof surface plasmon because the p-polarized surface wave is a coupling of a plane wave to a propagating waveguide mode. At high wavenumbers, the dispersion asymptotically tends to the waveguide dispersion and not a single resonant “plasmon” frequency.

Our CRLH waveguide metasurface is closer in form to the Sievenpiper mushroom surface,²⁴ with the exception of several differences. First, since there is a defined axis for the waveguides, our structure is not isotropic. Second, our structure has no via to the ground plane, which eases fabrication. Otherwise, there are similarities: Sievenpiper surfaces support both p-polarized spoof-surface-plasmon polariton bound modes (below the surface LC resonance frequency where the surface impedance is inductive) and s-polarized surface wave modes (above the LC resonance frequency where the surface is capacitive).²¹ The existence of s-polarized surface waves is expected when a surface exhibits a capacitive surface impedance, and these can be considered to be magnetic spoof surface plasmons.²² Although it was not observed in Ref. 21, the p-polarized surface wave on a Sievenpiper surface can even exhibit LH behavior if the series capacitance is made sufficiently large.²⁸ Our metasurface has a third important difference: its CRLH behavior exists only for s-polarized light, and hence is not characterized by an anticrossing gap. As discussed below, this difference is important for the development of THz CRLH leaky-wave antennas.

VI. CONCLUSION

In this work, we have experimentally demonstrated the existence of bound modes within CRLH THz metamaterial waveguides and mapped their dispersion using prism-coupled angle-resolved FTIR reflection spectroscopy. Two sets of leaky-wave and bound modes were observed: p-polarized RH only modes associated with the waveguide TM_{00} mode and s-polarized CRLH modes associated with the waveguide TM_{01} mode. We have presented a surface impedance model to support analysis of CRLH waveguide metasurfaces and their radiative properties. The surface impedance model quantitatively reproduces the experimentally measured reflection spectra, using as inputs lumped element transmission line circuit parameters extracted from the dispersion relation. Furthermore, this model correctly predicts the surface reactance of the metasurface, which can be used to predict the presence of p- or s-polarized surface waves. It

is computationally non-intensive, allowing for a quick assessment and physical understanding of a design's features: polarization response, dispersion relation, and loss.

These results guide our understanding of these CRLH waveguides for one particular application: as leaky-wave antennas fed by monolithically integrated THz QC-lasers.^{14,15} Endfire operation is particularly interesting, since it allows directive beams to be achieved from THz QC-laser waveguides with subwavelength transverse dimensions.⁸ The TM_{00} waveguide mode is clearly not suitable for LWA operation. It can only achieve RH propagation, so only a forward directed beam is possible. Also, the strong radiative coupling will result in a small radiative quality factor and a large radiation loss coefficient—for a typical $h = 5\ \mu\text{m}$, the power attenuation coefficient (radiation loss) is greater than $500\ \text{cm}^{-1}$ near $\beta = 0$.¹⁰ The fast attenuation leads to a small effective emitting aperture and a non-directive beam. Furthermore, the radiative loss grows even stronger closer to the light line since the group velocity approaches zero. Most importantly, feeding the waveguide structure at a single frequency at $n_{\text{eff}} = 1$ for endfire operation is not possible due to the anticrossing gap between the radiative and bound mode. The TM_{01} waveguide mode is more favorable for LWA operation. First, it can achieve CRLH operation, which allows backward-to-forward beam scanning, and if properly balanced has no stopband at $\beta = 0$.¹² Second, the radiative loss coefficient is more modest with values of $\sim 120\ \text{cm}^{-1}$ for a typical $h = 5\ \mu\text{m}$ design near $\beta = 0$; it can be further reduced if desired by decreasing the height of the waveguide or introducing holes.¹⁰ Third, no anticrossing will occur at the light line, and the waveguide can be excited exactly at the end-fire condition, $n_{\text{eff}} = 1$. However, it must be noted that due to the orientation of the radiating magnetic current dipoles on the waveguide sidewalls, such an end-fire mode from a single waveguide will radiate an azimuthally polarized donut-shaped beam in the far field, rather than a focused spot.

Substitution of QC-laser medium for the BCB insulator here is feasible, and potentially could lead to tuned active leakywave antennas, end-fire antennas, or metamaterial lasers.^{14,15} Alternatively, the E-field enhancement present within the capacitive structures suggests that such metasurfaces may be useful to enhance nonlinear optical effects.

ACKNOWLEDGMENTS

This work was supported by NSF Grant No. ECCS-0901827.

¹R. Kohler, A. Tredicucci, F. Beltram, H. E. Beere, E. Harvey, E. H. Linfield, A. G. Davies, D. A. Ritchie, R. C. Iotti, and F. Rossi, *Nature* **417**, 156 (2002).

²B. S. Williams, *Nat. Photonics* **1**, 517 (2007).

³B. S. Williams, S. Kumar, H. Callebaut, Q. Hu, and J. L. Reno, *Appl. Phys. Lett.* **83**, 2124 (2003).

⁴B. S. Williams, S. Kumar, Q. Hu, and J. L. Reno, *Opt. Express* **13**, 3331 (2005).

⁵J. A. Fan, M. A. Belkin, F. Capasso, S. Khanna, M. Lachab, A. G. Davies, and E. H. Linfield, *Opt. Express* **14**, 11672 (2006).

⁶S. Kumar, B. S. Williams, Q. Qin, A. W. Lee, Q. Hu, and J. L. Reno, *Opt. Express* **15**, 113 (2007).

- ⁷G. Scalari, L. Sirigu, R. Terazzi, C. Walther, M. I. Amanti, M. Giovannini, N. Hoyler, J. Faist, M. L. Sadowski, H. Beere *et al.*, *J. Appl. Phys.* **101**, 081726 (2007).
- ⁸M. I. Amanti, M. Fischer, G. Scalari, M. Beck, and J. Faist, *Nat. Photonics* **3**, 586 (2009).
- ⁹A. A. Tavallae, P. W. C. Hon, K. Mehta, T. Itoh, and B. S. Williams, *IEEE J. Quantum Electron.* **46**, 1091 (2010).
- ¹⁰P. Hon, A. Tavallae, Q.-S. Chen, B. Williams, and T. Itoh, *IEEE Trans. Terahertz Sci. Technol.* **2**, 323 (2012).
- ¹¹C. Caloz and T. Itoh, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications* (Wiley-IEEE, 2005).
- ¹²A. Lai, T. Itoh, and C. Caloz, *IEEE Microw. Mag.* **5**, 34 (2004).
- ¹³G. V. Eleftheriades and K. G. Balmain, *Negative-Refractive Metamaterials* (Wiley-IEEE, 2005).
- ¹⁴A. A. Tavallae, B. S. Williams, P. W. C. Hon, T. Itoh, and Q.-S. Chen, *Appl. Phys. Lett.* **99**, 141115 (2011).
- ¹⁵A. A. Tavallae, P. W. C. Hon, Q.-S. Chen, T. Itoh, and B. S. Williams, *Appl. Phys. Lett.* **102**, 021103 (2013).
- ¹⁶Z. Liu, P. W. C. Hon, A. A. Tavallae, T. Itoh, and B. S. Williams, *Appl. Phys. Lett.* **100**, 071101 (2012).
- ¹⁷A. Otto, *Z. Phys. A: Hadrons Nucl.* **216**, 398 (1968).
- ¹⁸R. Ulrich and M. Tacke, *Appl. Phys. Lett.* **22**, 251 (1973).
- ¹⁹J. B. Pendry, L. Martín-Moreno, and F. J. Garcia-Vidal, *Science* **305**, 847 (2004).
- ²⁰M. J. Lockyear, A. P. Hibbins, and J. R. Sambles, *Phys. Rev. Lett.* **102**, 073901 (2009).
- ²¹A. P. Hibbins, M. J. Lockyear, and J. R. Sambles, *Phys. Rev. B* **84**, 115130 (2011).
- ²²J. N. Gollub, D. R. Smith, D. C. Vier, T. Perram, and J. J. Mock, *Phys. Rev. B* **71**, 195402 (2005).
- ²³C. A. Balanis, *Antenna Theory* (Wiley-Interscience, 2005).
- ²⁴D. Sievenpiper, L. Zhang, R. Broas, N. Alexopolous, and E. Yablonovitch, *IEEE Trans. Microwave Theory Tech.* **47**, 2059 (1999).
- ²⁵A. P. Hibbins, B. R. Evans, and J. R. Sambles, *Science* **308**, 670 (2005).
- ²⁶R. C. Williams, R. S. Andrews, A. S. Maier, I. A. Fernandez-Dominguez, L. Martín-Moreno, and F. J. Garcia-Vidal, *Nat. Photonics* **2**, 175 (2008).
- ²⁷A. Cullen, *Proc. IEE-Part IV: Inst. Monogr.* **101**, 225 (1954).
- ²⁸A. Sanada, C. Caloz, and T. Itoh, *IEEE Trans. Microwave Theory Tech.* **52**, 1252 (2004).