Title
Exploring the Ways Emergent Bilingual Students' Engagement in Mathematical Practices is Supported Through the Teacher-Curriculum Interaction

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Publication Date
2019

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA SAN DIEGO

SAN DIEGO STATE UNIVERSITY

Exploring the Ways Emergent Bilingual Students' Engagement in Mathematical Practices is Supported Through the Teacher-Curriculum Interaction

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy

in

Mathematics and Science Education

by

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2019

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University of California San Diego

San Diego State University

2019

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LIST OF ABBREVIATIONS

EB	Emergent Bilingual
EL	English Learner
CME	Center for Mathematics Education
CPM	College Preparatory Mathematics (original meaning, now just known as CPM)
MVP	Mathematics Vision Project
TTA	Transition to Algebra
CCSSM	Common Core State Standards for Mathematics
SMP	Standards for Mathematical Practice
HoM	Habits of Mind
IM#	Integrated Mathematics #

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ACKNOWLEDGEMENTS

Many people have supported me during the writing of this dissertation – intellectually and emotionally. Without them, this work would not be what it is today, and completing it would not have been possible. Thank you to everyone who has helped me along the way. You all have contributed more than you may ever realize. For that, I am extremely grateful and blessed beyond measure.

It was truly amazing to be part of a joint doctoral program and to have the support of faculty from two universities to help me grow as a scholar and a researcher. The educational opportunities you have provided not only for me but to all of the students in our program have transformed us. Thank you for the hours you have spent preparing for our classes and providing feedback on our work to help us improve and expand our capabilities. Thank you for providing me with the ability and skills to complete this dissertation and to confidently seek opportunities for the next chapter of my life. Joanne and Jeff, thank you for your leadership and your commitment to this program. Deb and Sherry, thanks for all you do to help us navigate the requirements of two universities.

I will always think fondly of my fellow graduate students who I've met during this journey. I'm thankful I found a program in which my colleagues have faithfully supported each other throughout the process, no matter how hectic our schedules may have been. To those who served as mentors and helped me navigate through this new adventure, I not only appreciate all the help you provided specifically to me but also how you passed along the traditions that make the MSED program special. Molly, Mike, Bridget, DQ, Casey, John, Brooke, C. David: thank you for the examples you set for me in my first year through holding practice talks and brown bag seminars to help me learn what I had signed up for. Hayley, Ray, and Naneh: thank you for always feeling like my second cohort even though I was a year behind you! This program wouldn’t have been the same without you all. I’ve missed you dearly this year. Katie and Carren, my cohort, thank you for the many hours of intellectual conversations, second-year exam studying, commuting together, game nights, friendship, and never-ending encouragement! Matt: you’ve been a great officemate. Thanks for always having a listening ear, encouraging me when times were rough, and sharing an appreciation for LEGOs! Lauren, Amelia, Tina, Adriana, Mike, Antonio, Nicole, Kevin, Ernesto, Ted, Makenna: I appreciate all the support and encouragement you have given me by attending practice talks or by simply asking how things are going. Kevin and Ernesto, it was a pleasure to work with you on the mELd project – thanks for joining this year and making it possible for me to feel like I could take a step back from the day-to-day work, knowing the project was in capable hands!

Deb Escamilla, thank you for sharing your office with so many of us and always listening when we need to vent. I’m amazed you accomplish any work with all of us around! I will miss you and our daily chats dearly. Thank you for the seemingly constant supply of therapy kittens!

Words can’t express how much I have learned from working with my advisor and dissertation chair, Bill Zahner. Thank you for your mentorship and the example you have set as a professor and a researcher. You have modeled the importance of thinking carefully and deeply about how we present our research. Your positivity and respect for others is admirable and contagious. Thank you for all the research opportunities you have provided for me over the last four years and for the countless hours you spent offering feedback on this dissertation. Thank you also to my dissertation committee – Randy, Susan, Chris, and Laura – for the guidance and wisdom you have shared with me through this dissertation study, as well as over the years during research rotations, coursework, and my second-year exam.

Thank you to the teachers who volunteered for this research and allowed me to take a brief snapshot of their practice. I greatly appreciated your participation and how open you were to allow a stranger to come in and observe your classes. You have taught me a lot and gave me much to consider as I wrote this dissertation. I am grateful.

None of this would have been possible without the encouragement and support of my husband, our families, and friends who might as well be family. Chris, thank you for supporting us financially, cooking meals, encouraging me when I was down, understanding when I was too busy to have fun, being patient with me when stress got the better of me, and, in many ways, putting life on hold while I have been in graduate school. I look forward to what the next chapter holds for us! Next, I thank my parents, Stephen and Debra Judy, for instilling in me a love of learning from an early age. I appreciate the value you have always placed on education. Thank you for passing that along to me, and for your reassurance that I could do this. Heather, Eric, and Brooklyn – thank you for providing fun distractions that preserved my sanity throughout this process – trips to the beach, the zoo, Disneyland, Philadelphia, or simple FaceTime calls with Brookie gave me just what I needed to keep pressing on. Ed and Norma Wynn, thank you for your constant support and encouragement as I worked to achieve this goal. I owe you some Hand and Foot games to make up for some of those hours I spent writing research papers at your house! Bethany, Tom, Gabe, Grant, and Cole – you also provided much needed fun time to help me through school, from funny math jokes to building train tracks, to audio messaging with a toddler and watching videos of the kids as they grow up. Thank you to my amazing friend, Deborah Chapman, for the countless hours of prayers you have lifted on my behalf, the lunches we've shared, and for taking on the daunting task of editing this document (any remaining errors are mine!). Your friendship is irreplaceable, and I thank God for bringing you into my life.

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Zahner, W., Wynn, L., & Ulloa, J. S. (2018). Designing and redesigning a lesson for equity and access in a linguistically diverse high school classroom. In D. Y. White, A. Fernandes, & M. Civil (Eds.), *Access & Equity: Promoting high- quality mathematics in grades 9-12* (pp. 107–123). Reston, VA: National Council of Teachers of Mathematics.

Seethaler, S., Czworkowski, J., & Wynn, L. (2017). Analyzing general chemistry texts' treatment of rates of change concepts in reaction kinetics reveals missing conceptual links. *Journal of Chemical Education*, *95*(1), 28-36.

ABSTRACT OF THE DISSERTATION

Exploring the Ways Emergent Bilingual Students' Engagement in Mathematical Practices is Supported Through the Teacher-Curriculum Interaction

by

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This study investigated how eleven secondary mathematics teachers used curriculum resources to plan and enact lessons that support the participation of emergent bilingual (EB) students in mathematical practices in their linguistically diverse classrooms. EB students classified as English Learners often experience mathematics as a set of disconnected procedures. This investigation was rooted in a situated sociocultural theory of learning (Moschkovich, 2002) and a conceptual framework for the teacher-curriculum interaction (Remillard, 2005, 2009). This study was designed to answer the following questions: (1) How do curriculum materials provide supports for engaging emergent bilingual students in mathematical practices? and (2) How do teachers in linguistically diverse classrooms use curriculum materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?

The study used a contrastive design, examining both the official curriculum and teacher-created materials used by teachers in two districts that adopted different Common Core-aligned curricula. The official curricula provided different types of guidance for teachers to support EB students. The eleven participating teachers, six from School A and five from School B, participated in a lesson planning interview (Grossman, 1990), a classroom observation, and a debriefing interview.

Data analysis included inductive and deductive coding, using a priori codes based upon the CCSSM Standards for Mathematical Practice (NGA & CCSSO, 2010), the English Learners Success Forum's *Guidelines for Improving Math Materials for ELs* (ELSF, n.d.), and research on supporting English Learners (Chval, Pinnow, & Thomas, 2015). Some of the main findings of this investigation include the following. (1) The participating teachers held varying interpretations of the eight CCSSM Standards for Mathematical Practice, leading to a wide range of enactments of the same practice standard across classrooms. (2) The participating teachers were not using curriculum materials in their published forms. Teachers cited text difficulty and the difficulty their linguistically diverse students had with reading the textbooks as a rationale for modifying the written curriculum. (3) Despite these variations in enactment, all participating teachers expressed similar care and concern about the success of their students and believed that they were doing exactly what their students need to learn mathematics.

Chapter 1: Introduction

Emergent bilingual (EB) students and their teachers face a distinctive challenge in secondary mathematics as EB students must simultaneously learn mathematics and the language of instruction. This study explored how mathematics teachers in two linguistically diverse school districts used their adopted curricula, particularly examining how curriculum materials support teachers in developing mathematical practices and supporting the mathematics learning of students classified as English Learners[1] (ELs). A better understanding of the ways in which curricular resources support teachers working with EB students will guide those seeking to improve mathematics curricula for instructors in linguistically diverse settings.

Recent mathematics curriculum reforms (e.g., CCSSM, 2010; NCTM, 2014) have changed expectations of mathematics teaching to include the enactment of new mathematical practices. Some of these practices, such as *explaining one's reasoning*, *justifying solutions*, and *critiquing the reasoning of others*, require communication among students and teachers (Moschkovich, 2015). In order to meet the expectations of these ambitious standards, many teachers must reimagine what it is to learn mathematics in their classrooms, possibly without having experienced this type of classroom themselves as a student or teacher education candidate (Selling, 2016). While meeting the communication demands of the CCSSM is challenging for all students and teachers, this emphasis on communication presents a greater challenge for teachers in linguistically diverse classrooms and for EB students (Moschkovich, 2015).

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[1] While schools use the classification "English Learners," I will use the phrase "emergent bilingual students" to highlight the resources these students bring to school.

Motivation

This study was motivated by demographic shifts and curricular realities. The number of EB students in U.S. public schools continues to rise, with about 10% of all U.S. students classified as ELs – and this number is higher, about 25%, in certain areas of the US such as California or major urban areas (U.S. Department of Education, 2013). With regard to curriculum, teachers often inherit their mathematics curriculum from their department or from the district rather than being able to select or design their instructional materials. According to Banilower et al. (2013), more than 80% of teachers use commercially published textbooks to guide their mathematics instruction. Combined, the call for more discussion in mathematics classrooms and the increased presence of students classified as ELs have raised some concerns about how teaching reforms are possible (Ross, 2014). By identifying particular aspects of curricular resources that support teachers in developing mathematical practices in linguistically diverse classrooms, this study will help researchers, curriculum developers, and teacher educators consider ways to support students classified as ELs.

An Illustrative Vignette: The Rectangular Dog Pen

Many people believe that mathematics is a universal language, claiming 2 + 2 = 4 no matter what language you speak. While it is true that 2 + 2 = 4, this arithmetic sentence is not what most students experience as mathematics, especially at the high school level. We even see this commonsense claim reflected in the CME Project textbook (Cuoco & Kerins, 2016)! The CME Project Integrated Mathematics 1 student textbook contains the following image (see Figure 1.1) on the same page as the dog pen problem described below. The translated caption says, “You can understand x, $5x$, $5x + 2$, …, regardless of the language you speak.” This raises the question, “What does it mean to understand x, $5x$, $5x + 2$, …?” However, researchers assert that language and content cannot be separated (Barwell, 2005; Moschkovich, 2015) and I have personally observed the complexity of simultaneously teaching mathematics content while students are learning the language of instruction.

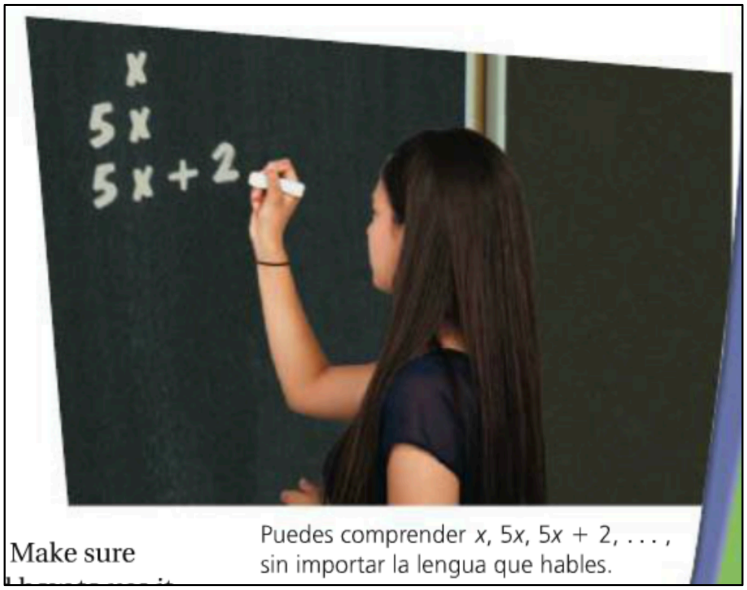


Figure 1.1. An image from the CME Project student textbook (Cuoco & Kerins, 2016, p. 115).

I had the opportunity to visit an Integrated Mathematics 1 class in which all the students were newcomers to the US (all had recently immigrated from several countries including Mexico and Somalia) and spoke very little English. The teacher was doing a demonstration lesson for researchers and curriculum designers. In preparation for our visit, the teacher selected the following problem from her textbook to highlight some of the strategies she used to make a challenging problem accessible to her linguistically diverse group of students:

> To build a rectangular dog pen, Cheng uses a wall of his house for one of the long sides. Let l equal the length of the longer side. Let w equal the length of the shorter side.
>
> a. Write an expression for the amount of fencing Cheng needs to build the pen.
>
> b. How much fencing does Cheng need if he wants a width of 8 feet and a length of 12 feet?
>
> c. How much fencing does Cheng need if he wants a width of 5 feet and a length of 20 feet?

d. Suppose the length is 9 feet more than the width. Use only *one* variable to write an expression for the amount of fencing Cheng needs. (Cuoco & Kerins, 2016, p. 115)

The teacher anticipated that the phrase *dog pen* would be problematic for her students, so she started the discussion of this problem by asking the students if they knew what dog pen meant. Most students recognized *dog*, but the word *pen*, which has a common meaning of a writing instrument, was not as easy. Students motioned writing with a pen, picked up their pens as an example, or said *pluma*. The teacher then displayed several photographs of fenced in areas, many with four sides of fencing, to help the students understand a new meaning for pen (see Figure 1.2). She then asked the students to draw the dog pen described in the problem.

Figure 1.2. Images of dog pens similar to the ones shown in class.

Drawing a representation of the dog pen in the problem was also challenging for the teacher. Mathematically, one has to understand that the pen will only have three sides of fencing, using the wall of the house as the fourth side. Figure 1.3 is a reconstruction of the sequence of pictures that were drawn on the smartboard in order to clarify the next sentence in the problem.

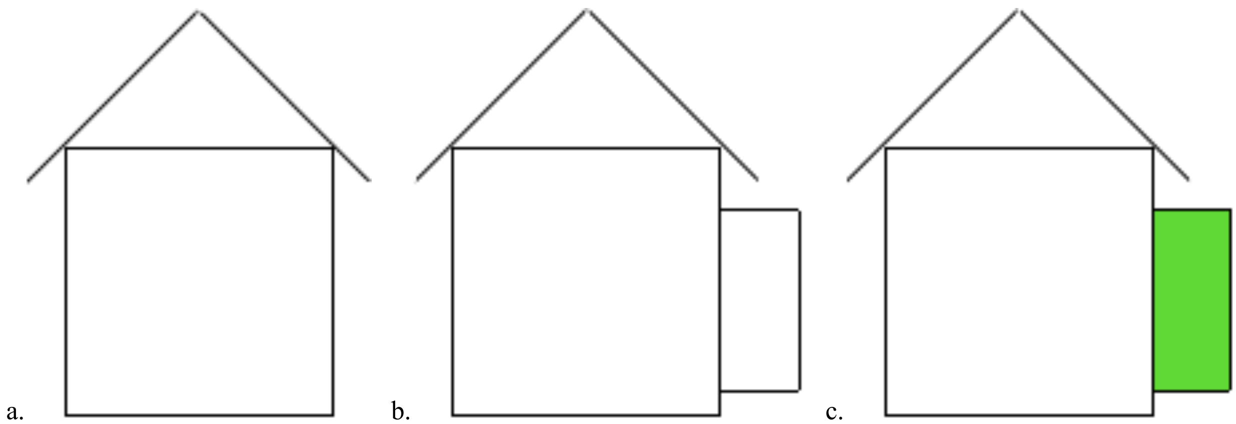


Figure 1.3. The representation of (a) Cheng's house, (b) the house with the dog pen on the side, and (c) the dog pen shaded in to represent the grass.

Figure 1.3 demonstrates that, in addition to making sense of the word *pen*, this problem also presents the challenge of representing a three-dimensional situation in two dimensions. The teacher's drawing started with a house, and then she showed the pen attached to the side of the house. But, due to the perspectives she used, the dog pen appears to be attached to the side of the house, perpendicular to the ground, hanging in the air. The teacher added green in the middle of the pen for grass, to help show that the area being fenced in was on the ground rather than perpendicular to it. This shading then seemed to trigger students' knowledge of the area of a rectangle, evidenced by their equations and solutions to parts a – c, drawing their attention to the inside of the fenced in area (region) and away from the linear amount of fencing needed. Adding to the linguistic complexity, note my use of the word *area* in the last sentence – once as the mathematical term area and once as a location, an everyday term.

These are only two examples of the linguistic challenges represented in this problem. Upon reflecting on this lesson, the researchers and curriculum designers identified some of the other potential issues:

- A fence has more than one dimension (i.e., length and height) even though only the length is needed for this problem.
- The problem used of both of the words *length* and *width* to represent how much fencing is needed, using a one-dimensional measurement of a two-dimensional object.

- The authors used *feet* as the units in this problem, which may not be familiar to students who learned the metric system or could potentially be confused with the dog's paws.
- *Fencing* is also a sport.
- The questions contain the phrase "if he wants…," which may cue an English learner to consider whether Cheng wants to make the fence or not.

Reflecting on this problem in conjunction with the Spanish caption on the same page seemingly minimizing the importance of language further highlights the importance of examining our assumptions about language use in mathematics classrooms. With this vignette illustrating some of the issues, we move into the focus of this research.

Statement of Issue

In this section, I identify four areas of research that are relevant to this study. To consider how secondary mathematics teachers work to simultaneously support students learn the language of instruction and grade-level mathematics content, I draw from research about emergent bilingual students, curriculum use, mathematical practices, and linear and exponential rates of change.

Emergent Bilingual Students

In the local context of this research, the prevalence of students who are classified as ELs cannot be ignored. Researchers have shown that ELs, along with other underserved groups, have not received the same level of mathematics education as their peers, including being denied access to higher level mathematics courses and quality mathematics teachers (e.g., Mosqueda & Maldonado, 2013). Additionally, many experienced, well-qualified teachers have not had adequate training in how to support ELs in their mathematics classrooms in their teacher preparation programs or in ongoing professional development programs (Chval et al., 2015). While many researchers have been working to improve education for ELs, Barwell (2005) argued that "language and content cannot be separated [from mathematics] or studied in isolation" (p. 145). This sentiment is reflected in the design of my study, which revealed some of the ways in which secondary mathematics teachers incorporate language (as well as discourse practices and mathematical practices) in their teaching of content. Not only is the support of EB students a pressing issue of practice, but investigating the interactions of teachers and EB students surrounding content is important to extending the current state of research in the field.

Curriculum

Despite the commonly held idea that curriculum determines what is taught in school, what teachers teach is not determined by what is written in their textbook. Remillard (2005, 2009) proposed a teacher-curriculum interaction framework suggesting that teachers have a participatory relationship with their curriculum (see Figure 1.4). The interaction combines elements of teachers' pedagogical content knowledge, subject matter knowledge, and beliefs about teaching mathematics with features of the curriculum, such as the representations of concepts and tasks, structures, and voice. Furthermore, these components are all situated within a particular classroom context in a specific school within a community. The interaction of these components influences the teacher's planned curriculum, and further interactions with students and the context influence the enacted curriculum. I draw from this framework for my study, recognizing that curriculum developers are communicating their perception of what it means to do mathematics through their materials while the teachers who use the curriculum also have their own perspectives of what it means to do mathematics in their classrooms, influenced by their students and local communities. This study investigated this interaction as teachers planned lessons for their students, particularly attending to the ways in which the written curriculum, planned curriculum, and enacted curriculum supported teachers to engage EB students in mathematical practices.

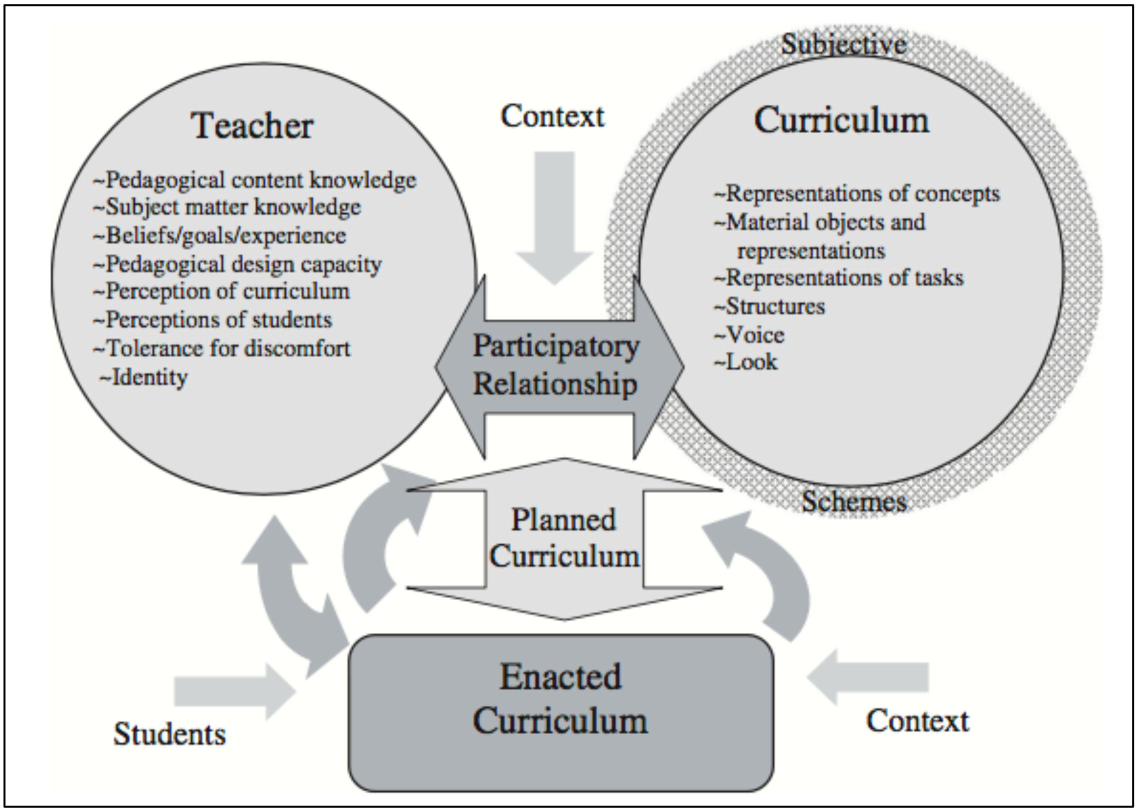


Figure 1.4. Remillard's (2005) framework for the teacher-curriculum interaction (p. 235).

Mathematical Practices

The Standards for Mathematical Practice in the Common Core State Standards (NGA & CCSSO, 2010) describe eight practices intended to develop students' induction into the disciplinary forms of reasoning. This list of practices was informed by NCTM's (National Council of Teachers of Mathematics) process standards (NCTM, 1989, 2000) and a National Research Council (NRC) report (2001) detailing strands of mathematical proficiency. We know from prior research that EB students typically have limited access to rigorous mathematics content and experience mathematics instruction that emphasizes procedures and provides few opportunities to develop mathematical practices (Chval et al., 2015; Moschkovich, 2015; Mosqueda & Maldonado, 2013). In this study, I investigated the ways in which teachers planned to engage their students in mathematical practices and how this was enacted in the classroom. I was particularly interested in how EB students were supported in and encouraged to participate in these practices.

Linear and Exponential Rates of Change

My initial content focus was on the topics of linear and exponential rates of change. Relatively few researchers have examined student learning of exponential rates of change (prominent exceptions are Confrey & Smith, 1995; Ellis, Ozgur, Kulow, Williams, & Amidon, 2012) compared to the relatively large research base on linear rates of change (e.g., Lobato, Ellis, & Muñoz, 2003; Moschkovich, 1996, 1998; Thompson, 1994; Zahner, 2015). Notably, the early introduction of exponential rates of change in ninth grade mathematics reflects a much earlier development of these concepts under the Common Core State Standards than in previous curricular sequences. Figure 1.5 shows that, in California's old standards, exponential functions did not appear until Algebra 2, which was two courses later than they are being introduced in the CCSSM Traditional Pathway.

Additionally, the CCSSM reflect a significant change in thinking about linear functions as growing by "equal differences over equal intervals" versus exponential functions as growing by "equal factors over equal intervals" (CCSSM, 2010, HSF.LE.A.1.A). Not only does this reflect a conceptual change in the forms of mathematical thinking that are expected from students, this short statement highlights the complexity of the interaction of language and mathematics for all students, but particularly for EB students. Words such as *factors* and *differences* have different meanings in different contexts, and specific meanings when used in this standard. This complexity potentially increases the difficulty of understanding the situated language of mathematics. Finally, real-world applications related to exponential rates of change, such as population growth or earning interest on investments, are challenging ideas to discuss in mathematics in general and the language we use around these concepts is subtle. Yet these concepts are critical for using mathematics to make sense of real-world contexts, so it is important to find ways to make this topic accessible to students, particularly EB students.

Algebra 1						
Old CA	Arithmetic Properties	Linear Equations and Inequalities	Linear Functions and Graphing	Quadratics and Polynomials	Rational Expressions and Functions	Logical Arguments
CCSSM	Relationships Between Quantities and Reasoning with Equations	Linear and Exponential Relationships	Descriptive Statistics	Expressions and Equations	Quadratic Functions and Modeling	
Algebra 2						
Old CA	Solving Equations and Inequalities	Polynomials and Factoring	Complex Numbers and Rational Expressions	Quadratics Functions and Graphing	Logarithms and Exponential Functions	Conic Sections and Discrete Mathematics
CCSSM	Polynomial, Rational, and Radical Relationships	Trigonometric Functions	Modeling with Functions	Inferences and Conclusions from Data		

Figure 1.5. Comparison of the sequencing of the Algebra standards in the former California standards and the Common Core content standards.

Research Questions

My research questions highlight the themes of my research: engaging students in the mathematical practices, supporting EB students while they simultaneously learn mathematics and the language of instruction, and positioning EB students as learners of mathematics.

1. In what ways do the locally adopted, CCSSM-aligned, curricula support secondary mathematics teachers to engage their students in mathematical practices?
    a. What supports are provided explicitly to help teachers engage their emergent bilingual students in these practices?
    b. How do the curriculum materials position emergent bilingual students as learners of mathematics?

2. How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?

Significance

This research contributes to the existing body of research in mathematics education by investigating the ways in which research-based, CCSSM-aligned curricula have the potential to support teachers in engaging students, particularly EB students, in the mathematical practices in their linguistically diverse classrooms. Regardless of the types of supports provided in the curriculum, although helpful, ultimately, it is the teacher who determines how mathematics teaching and learning will be enacted in her classroom, with the teacher acting as a mediator between curriculum and students. This study provides additional information about how the participating teachers interpreted and enacted mathematical practices, their beliefs about curriculum, and the ways they help EB students have access to and participate in grade-level mathematics.

Chapter 2: Literature Review

The purpose of this study is to examine how emergent bilingual students' engagement in mathematical practices is supported by the teacher-curriculum interaction. Understanding this entails examining the curriculum materials to determine what supports are (explicitly or implicitly) provided, discerning how teachers take up (or reject) these supports in their lesson planning, and observing how the lesson is enacted with emergent bilingual students in the classroom. While this research is supported by a strong literature base, this study pulls together this knowledge in a new configuration – considering what influence curriculum may have on the ways teachers choose to engage their emergent bilingual students in mathematical practices. In this chapter, I begin with a discussion of the theoretical framework underpinning this work. Next, I present literature on emergent bilingual students, including state policy and research on teachers of linguistically diverse students. This section is followed by a discussion of research on curriculum and the teacher-curriculum interaction. Then I discuss mathematical practices, elaborating on different ways researchers have defined mathematical practices and presenting research on identifying mathematical practices in classrooms. Finally, I describe literature on teaching and learning of linear and exponential rates of change, the content focus for this study.

Theoretical Framework

This study draws upon a situated sociocultural conceptual framework for learning, which highlights that learning mathematics entails learning to participate in the mathematical practices in a particular classroom (Brown, Collins, & Duguid, 1989; Forman, 2003; Moschkovich, 2002; O'Connor, 1998). Since the mathematical practices rely heavily upon student participation in mathematical discourse, this framework is apt for this study. Moschkovich (2015) proposed a definition for *academic literacy in mathematics* which includes three interrelated components:

mathematical proficiency, mathematical practices, and mathematical discourse. In the following sections, I will elaborate upon the theory of situated cognition, sociocultural learning theory, and academic literacy in mathematics.

Situated Cognition

According to the theory of situated cognition, the setting in which a learning activity takes place affects what is learned. The setting includes social interactions, the tools available to the learners, and the culture of the learning environment. Brown, Collins, and Duguid (1989) posited that separating "what is learned from how it is learned" (p. 32) could prevent learning from occurring. The authors viewed situations and activities as an integral part of cognition and learning. In response to the prevalent learning theories adopted in education at the time, Brown, Collins and Duguid suggested that knowledge was given great importance and considered to be independent and transferable, yet knowledge was separated from context and learning activities were deemphasized. The problem with separating knowledge from authentic settings where that knowledge is used is that the artificial settings in which knowledge was transmitted gave students an unrealistic perception of what experts in the field actually do.

In this situated perspective on learning, the tools that are available to the learner and how the learner interacts with these tools impacts how he or she learns. For example, Carraher, Carraher, and Schliemann (1985) analyzed how children in Brazil used math as they sold goods on the street and compared their informal methods of calculating sales totals and change to their understanding of the formal arithmetic methods taught in school. They found that the children were exceptionally good at mental mathematics when the problem was given in a context such as selling coconuts. When asked to use pencil and paper to solve problems, even the same problems they had correctly solved in an informal context, the students tried to use the algorithms they were taught in school with little success.

Not only are the tools important, but so is the setting in which one uses the tools. Säljö and Wyndhamn (1996) conducted a study aiming to demonstrate that the setting in which a problem is presented to students shapes their perception of the problem as well as their solution methods. Eighth- and ninth-grade students in Sweden were given a chart of postage rates for mailing domestic letters and were asked to determine the cost of mailing a letter that weighs 120 grams. Some of the students were given this task in social studies class and the others were given in mathematics class. Säljö and Wyndhamn identified two strategies, *reading off* and *calculating*, used by the students. To find the postage rate, one only needed to find the appropriate weight category and read off the price. Almost 71% of the students in the social studies class read the cost off the chart, but only 43% of the students in the mathematics class used a reading off strategy. The researchers posited that the students' interpretations of the task were directly related to their assumptions about the types of activities that one does in a given situation.

Another feature of situated cognition is that of cognitive apprenticeship, which provides opportunities for enculturating students into the discipline they are studying through the use of authentic activities that reflect what the experts do (Brown, Collins, & Duguid, 1989). Similar to an apprenticeship in a trade such as carpentry, researchers from this perspective argue students should be given the opportunity to learn as an expert would learn, using the same tools for the same purpose as the expert would use the tools. In mathematics, it is unrealistic that problems are always so well defined as we often see in textbooks, so students need to be given the opportunity to solve problems as a mathematician would solve them. Authentic activities are modeled after what a practicing professional does in her profession. Many of the problems solved in a school mathematics classroom, though modeled after realistic situations, are too well-defined and do not resemble what a mathematician does in practice. Brown et al. (1989) caution that these contrived activities can introduce students “to a formalistic, intimidating view of math that encourages a culture of math phobia rather than one of authentic math activity” (pg. 34). Alternatively, participating in a collaborative problem-solving experience, moderated by an expert, gives students the opportunity to engage in mathematical thinking and to recognize that mathematics is a “sense-making pursuit” (Brown et al., 1989, p. 37).

Sociocultural Learning Theories

According to Vygotsky, language is developed through social interaction, which includes gestures and affective responses (Confrey, 1995; Vygotsky, 1987). Thought is developed within the context of the child’s activity. How the child interacts with objects around him and how he uses these tools is an indicator of the development of logical thought. Vygotsky believed that thought and language are intertwined. Schutz (2004) described that, for Vygotsky, language is not just an expression of what the child has learned, but that thought and speech have a fundamental correspondence and work together to form knowledge and personality features. Thus, Vygotsky claimed that language and thought are reflexive. Speaking about something can help clarify one’s thoughts. Conversely, one can express their ideas through language. We use language to define and shape our experiences. Without language, our lived experiences would be very different than how we experience life with language, because language provides a framework for perceiving, experiencing, and acting in the world.

Vygotsky also maintained that all higher thought processes come from mediated activity (Berger, 2005). The mediator can be physical tools, words, mathematical symbols, graphs, maps, dominoes, or anything that helps develop knowledge (Berger, 2005; Confrey, 1995). Vygotsky called all of these items that mediate knowledge "psychological tools," or tools that "work upon" the mind rather than the environment (Friesen, 2012). The mediator is the mechanism by which the social world is internalized. In mathematics, it is possible to form a concept because it can be expressed through some sign or word that already exists and is understood in the social world.

These ideas about the interaction of thought and language relate to the role of discourse in learning mathematics. The following is one example from research that illustrates sociocultural theory in action. Kazemi and Stipek (2001) identified four practices for promoting conceptual thinking in mathematics classrooms – explanations, strategies, errors, and collaborative group work – and claimed students are expressing ideas and formalizing thought through social interaction. This social interaction takes place both within collaborative groups and in the whole classroom setting. The students are expressing their ideas, the teacher is encouraging deeper mathematical reasoning, and the students are learning mathematics as they explain concepts to others and while others explain to them.

Kazemi and Stipek's (2001) study of four upper elementary classrooms focused on promoting conceptual thinking in mathematics was influenced by a sociocultural theory of learning, which emphasizes that it is possible to understand individual learning through analyzing the organization of the social environment and how students participate in the social practices of the learning environment. The authors identified both the social norms and sociomathematical norms (Cobb & Yackel, 1996) that were present in the four classrooms studied, and posited that the differences between the classrooms were rooted in the sociomathematical norms and the degree to which teachers exhibited "press for learning"

(Kazemi & Stipek, 2001, p. 61). Press for learning is a measure of the extent to which a teacher engages students in mathematical thinking, including encouraging persistence, focusing on understanding, supporting students to rely on themselves and their peers, and not emphasizing correct answers.

Each of the classrooms was found to have four social norms in common: (1) students are expected to share their thinking, (2) students are encouraged to come up with multiple solution methods and share them with the class, (3) mistakes are expected and viewed as part of the learning process, and (4) students work together to solve problems (Kazemi & Stipek, 2001). However, the authors argued that while these social norms are necessary for promoting conceptual thinking, they are not sufficient. They identified four sociomathematical norms that distinguished the high-press classes from the low-press classes. First, students were required to provide mathematical justifications for their solutions, not just step-by-step procedures of how they arrived at their answers. Second, students should be encouraged to find mathematical relationships between the multiple solution methods presented and be able to explain mathematically why various strategies are acceptable. Third, errors should be viewed as fodder for productive mathematical discussions and not simply dismissed. Finally, students need to be held individually accountable for learning the material their group is developing collaboratively and when disagreements arise, they need to reach an agreement through mathematical explanation and justification.

Academic Literacy in Mathematics

Moschkovich (2002) adapted the tenets of sociocultural theory to critique the then common perspectives of how emergent bilingual students learn mathematics (e.g., learning vocabulary or learning specialized word meanings and registers of mathematics in isolation). In contrast with these views, Moschkovich argued for researchers to adopt a situated and sociocultural perspective of learning mathematics and language. When teachers focus on vocabulary in instruction for emergent bilingual students, learning mathematics tends to mean computing and solving traditional word problems, and emergent bilingual students are assessed on their ability to use mathematical vocabulary rather than the meanings they are building for these terms or the multiple resources they use to communicate their understanding of the mathematics. Focusing exclusive attention on the multiple meanings of words and the mathematical register (Halliday, as cited in Moschkovich, 2002) reflects the view that mathematics learning is learning how to use multiple meanings of words appropriately within the context (e.g., *set* as a mathematical object rather than *set the table* or a *chess set*). In other words, to learn mathematics means transitioning from informal or everyday language use to more precise technical or mathematical words. Moschkovich emphasized that both registers are necessary, and both registers are resources for communicating mathematically and serve different purposes for students. Supporting first-language or everyday language use while communicating mathematically acknowledges the resources that emergent bilingual students bring to the classroom.

Moschkovich (2002) argued that Gee's notion of Discourses paired with a situated sociocultural view of mathematics education can broaden our view of the resources emergent bilingual students bring to the classroom, rather than focusing on what these students may be lacking as the vocabulary or multiple meanings perspectives can invoke. She contends that a situated sociocultural perspective can be used to tease apart how students use resources such as everyday language and their first language to communicate about mathematics, broadening what counts as "competent mathematical communication" (Moschkovich, 2002, p. 193). Using Gee's definition of Discourse, Moschkovich posited that communication in mathematics (Mathematical Discourses) is far more than writing, speaking, or technical registers, but also involves "mathematical values, beliefs, and points of view of a situation" (p. 198). According to Moschkovich, the situated sociocultural perspective enables researchers also to consider nonverbal resources that participants use in a given situation, including gestures, concrete objects, mathematical artifacts, and everyday experiences, as well as considering the use of their native languages as a resource to communicate their ideas. Moschkovich strongly asserts that if one wishes to support emergent bilingual students' engagement in mathematical discussions, it is far more important to listen to (and watch) the student to determine the mathematics he does know rather than to attempt to uncover where he went wrong.

More recently, Moschkovich (2015) built upon this work and introduced a sociocultural conceptual framework for *academic literacy in mathematics*, which includes three interrelated components: mathematical proficiency, mathematical practices, and mathematical discourse, that are critical for teaching emergent bilingual students. She argued that it is essential for teachers of emergent bilingual students to adopt a complex view of mathematical discourse, which goes beyond spoken and written words. For example, gestures, diagrams, tables, graphs, physical objects, and informal everyday language are all a vital part of the mathematical discourse that is situated in a classroom, and these can be resources for emergent bilingual students and their teachers (Dominguez, 2016; Shein, 2012; Turner, Dominguez, Maldonado, & Empson, 2013). Moschkovich (2015) emphasized the importance of recognizing that students (and teachers) bring different resources to a situation – certain words, contexts, or representations may have different meanings for different people – so it is necessary for shared meanings to be constructed and negotiated through situated interactions.

Moschkovich embraced the current description of mathematical proficiency from the NRC (2001) that consists of five intertwined strands of proficiency (see Figure 2.1), but stressed that ELs typically are only given access to the procedural fluency strand. This tendency to only expose ELs to procedural mathematics also denies ELs access to effective mathematics teaching and higher-level mathematics. Next, Moschkovich generally defined *academic mathematical practices* as "using language and other symbols to think, talk, and participate in the practices that are 'the objective of school learning'" (Moschkovich, 2015, p. 47). She argued that by including mathematical practices in her framework, the focus shifts from a purely cognitive nature of mathematical proficiency towards a sociocultural and discursive nature of mathematics learning.

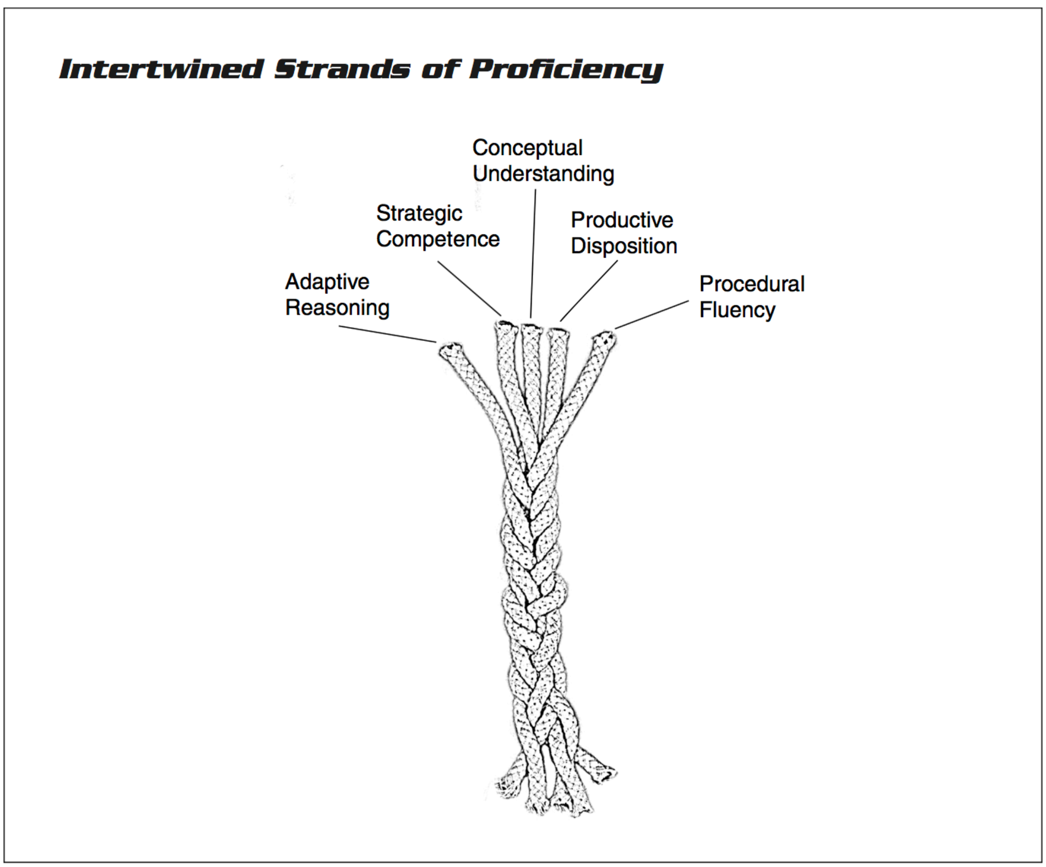


Figure 2.1. Intertwined Strands of Mathematical Proficiency. (National Research Council, 2001, p. 117).

The final component of her framework is *mathematical discourse*, which describes all that is necessary to effectively communicate about mathematics. In other words, mathematical discourse is everything students need in order to engage in mathematical practices. While discourse makes one think of language (or more specifically, technical mathematical terminology), Moschkovich contended that mathematical discourse also includes symbolic systems, artifacts, diagrams, gestures, and informal language. Broadening the view of what counts as mathematical explanations or descriptions supports ELs' progress in simultaneously learning mathematics alongside the language of instruction.

In summary, the situated sociocultural framework I am drawing upon to investigate the ways in which the teacher-curriculum interaction supports emergent bilingual students' engagement in mathematical practices has important implications for my study. It would not be sufficient to simply interview the teachers in the study about how they use their curricular resources to support their EB students' participation in mathematics learning. This learning is situated within a certain group of students, within a particular school, and within a broader community context. Reflecting upon Remillard's framework, the social interactions and sociomathematical norms that are established in the classroom are two of the contexts that influence the enactment of the teachers' plans. Thus, it is necessary also to observe the lessons in the particular context of each classroom. From the sociocultural perspective, the mathematical practices I wish to study are inherently social, emphasizing language-rich activities such as explaining one's reasoning and critiquing the reasoning of others. Therefore, this study includes analysis of how teachers create opportunities for and support such language-rich activities.

Emergent Bilingual Students

As described in Chapter 1, students classified as ELs have been underserved in mathematics classrooms. Mathematics instruction for ELs has been characterized by individual seatwork and worksheets (rather than participation in mathematical discussions), mathematics learning has been dominated by procedural computations rather than conceptual understanding, and emergent bilingual students are not offered access to higher level mathematics content or courses (Gutiérrez, 2002; Mosqueda & Maldonado, 2013; Secada, 1992). According to the Fall 2016 California Language Census, about 21% of the state's student population (1.3 million students) are classified as ELs and 42.6% of the student population speak a language other than English at home. Of the ELs, 28% are in secondary grades (7-12). While the census revealed 65 language groups represented in the state of California, 94% of the EL students fall in the top ten languages shown in Figure 2.2. Given the substantial size of the EL population in California schools, it is imperative that California educators recognize and meet the needs of these emergent bilingual students and their families. In the following sections, I describe the efforts the state has made to serve this population through credentialing requirements and English language development policy, then discuss some of the research regarding teachers who serve linguistically diverse student populations.

State Policy

In California, part of the requirements for obtaining a single subject teaching credential is to complete coursework that authorizes the credential holder to teach English learners within their subject area (State of California Commission on Teacher Credentialing, 2016). This qualifies the teacher to provide English language development and specially designed academic instruction in English within the teacher's subject area authorization and grade level authorization. Prospective single subject teachers (i.e., high school teachers) must also complete a comprehensive reading instruction course that includes the study of phonics; diagnostic and intervention strategies; and literature, language, and comprehension.

Language	Percent
Spanish	83.10%
Vietnamese	2.14%
Mandarin (Putonghua)	1.59%
Arabic	1.40%
Filipino (Pilipino or Tagalog)	1.31%
Cantonese	1.20%
Korean	0.77%
Hmong	0.74%
Punjabi	0.72%
Russian	0.63%

Figure 2.2. The top 10 languages spoken by California students, representing about 94% of the students classified as ELs (California Department of Education, 2017).

In 2014, the California Department of Education released the *English Language Arts/English Language Development Framework for California Public Schools: Kindergarten Through Grade Twelve (ELA/ELD Framework)*, a new policy document that communicates the state's vision for literacy across all content areas. This document provides guidance and support for implementing two sets of standards while promoting an interdisciplinary approach to literacy and language instruction. The standards reflect five key themes (Meaning Making, Language Development, Effective Expression, Content Knowledge, and Foundational Skills) and they emphasize equity and access through a focus on culturally and linguistically responsive teaching. Cultural diversity, multilingualism, and biliteracy are situated as resources rather than challenges. The policy document calls for increased "collaboration and shared responsibility among teachers, specialists, educational leaders, parents, and communities" (Yopp, Spycher, & Brynelson, 2016, p. 10) for language development and literacy.

The authors of the *ELA/ELD Framework*, like Barwell (2005), emphasized that language and content cannot be separated. This policy document called for all teachers, regardless of content area, also to be language teachers. However, one practical limitation that may hinder the uptake of this framework by all teachers is the over 1000 page-long document, and the over 150 page-long chapter devoted to content areas other than ELA/ELD at the high school level. This chapter begins with an introduction to the features of adolescent brain development and the types of motivational factors that are effective with this age group, and then provides an overview of the vision of an integrated and interdisciplinary approach to teaching ELs in this age group, elaborating on each of the five key themes and how the standards of both ELA and other disciplines (history, science, and technical disciplines) call for students to engage in each area. Next, the chapter further emphasizes the skills needed in each of the five key themes for grades 9-10 and again for 11-12, as well as emphasizing ELD goals in each grade span. Vignettes are provided to help clarify and exemplify the goals established in the policy document. With the volume of information provided and the amount of inference a content area teacher must make in order to meet the goals established in this policy document, it is difficult to imagine a secondary mathematics teacher first being aware that they may need to read the *ELA/ELD Framework* and second having the time and resources to engage with the recommendations set forth in the document.

In California, students are classified as English Learners through an assessment. At the time of this study, the state of California was in the process of transitioning from the California English Language Development Test (CELDT) to the English Language Proficiency Assessments for California (ELPAC) for classifying students' language proficiency. This change in testing will also change the categories for students who are classified as English learners. The

CELDT test identified five proficiency levels of English learners – Beginning (B), Early Intermediate (EI), Intermediate (I), Early Advanced (EA), and Advanced (A) – which seemed to highlight what students lack rather than the resources that the students bring to a classroom. The new ELPAC test will have three proficiency levels – emerging, expanding, and bridging – which have a much more positive connotation than the previous labels. See Figure 2.3 below for a more complete comparison of the CELDT and ELPAC tests.

CELDT	ELPAC
Aligned with the 1999 California English Language Development (ELD) Standards with five proficiency levels	Must be aligned with the 2012 California ELD Standards, which have three proficiency levels (Emerging, Expanding, and Bridging)
One test used for two purposes: initial assessment and annual assessment	Two separate tests for two purposes: (1) initial identification; and (2) annual summative assessment. The initial identification will be brief and locally scored.
Paper-pencil tests	Paper-pencil tests with a potential to transition to computer-based tests
July 1–October 31 Annual Assessment window	Annual Summative Assessment window to be a four month period after January 1 (proposed February 1–May 31), allowing for more pre-test instructional time
Five grades/grade spans: K–1, 2, 3–5, 6–8, and 9–12	Seven grades/grade spans: K, 1, 2, 3–5, 6–8, 9–10, and 11–12
Five performance levels	Four performance levels
Reporting domains: Listening, Speaking, Reading, and Writing	Reporting domains: Listening, Speaking, Reading, and Writing

Figure 2.3. A comparison of the CELDT and ELPAC tests (California Department of Education, 2016).

Research on Teachers in Linguistically Diverse Classrooms

In this section, I will discuss five studies involving teachers of emergent bilingual students in mathematics classrooms. These studies highlight the effect teachers, their beliefs about mathematics and ELs, and the local contexts (e.g., school or district policies, state testing) have on the quality of mathematics education for emergent bilingual students. Three of these studies focused directly on the effectiveness of teachers of linguistically diverse students, two involving teachers who were certified to teach English as a Second Language (ESL) and the other involving teachers who were not specifically trained to do so, providing contrasting pictures of how emergent bilingual students may be perceived by their teachers. The fourth study examined how teachers and their linguistically diverse students jointly established shared knowledge through mathematical dialogue. The final study explored the teaching practices of three teachers of linguistically diverse students and how their implementation of the same technology-based lesson impacted the students' conceptual development of ideas related to piecewise linear functions.

ESL certified teachers. De Araujo (2017) conducted a study of three secondary mathematics teachers of English Language Learners (ELLs), selected due to being ESL certified and teaching sheltered ELL mathematics courses. The teachers were asked to fill out a survey to gather background information, beliefs about their teaching practice, and understanding their classroom environment, which included preparation for teaching ELLs and the materials used when teaching ELLs and how they were modified. Each teacher was interviewed and observed daily for about two weeks in their sheltered mathematics classes, and tasks they used in instruction were collected for analysis. De Araujo found that only one of the 42 observed tasks were from the textbooks, with the teachers opting to use software or online resources. The tasks selected by the teachers were repetitive and "consistently decontextualized and low in cognitive demand" (de Araujo, 2017, p. 11), despite having access to curriculum and other online state resources that provided a mix of high and low cognitive demand tasks as well as contextualized and decontextualized tasks. Additionally, each teacher looked for tasks that would help build the students' mathematical vocabulary.

With regard to the teachers' beliefs about ELLs language proficiency, de Araujo characterized each teacher as having deficit beliefs, evidenced both by their selection of mathematical tasks (low level of cognitive demand, few words, and avoidance of word problems or written explanations) and the way they described their own practice. One teacher spoke of not being able to use resources with riddles or jokes because her students don't understand the language well enough to catch the humor, which led to her use of decontextualized mathematical tasks with simplified language. The teachers also focused on the number of words in problems as being particularly problematic rather than the sentence structure or the way content was presented.

Unfortunately, the teachers also seemed to believe their ELLs had lower mathematical proficiency than their English-speaking peers, perhaps conflating students' language development with their ability to do mathematics. De Araujo found that the teachers felt the ELLs lacked the prerequisite skills that the other students have mastered, so they often found resources that began with the basic skills the ELLs were perceived to be lacking rather than engaging in higher-level content, resulting in the overwhelming selection of procedurally oriented tasks. Additionally, the teachers tended to speak of their ELL students as though they were a homogeneous group, often referring to stereotypes about ELLs. When shown an example of a higher-level task designed to prepare students for future coursework, one teacher acknowledged the task was good, but dismissed it as unnecessary for her students, because they won't be taking advanced math courses since they will just work in the local factory. Finally, two of the teachers embraced direct instruction as the best method for teaching ELLs mathematics, while the third liked to do groupworthy tasks because her ELLs responded better if they worked together (but she does not teach her nonsheltered mathematics class the same way).

In Zahner's (2015) study, he purposefully selected a teacher in a bilingual classroom setting to observe based on her "national reputation for teaching conceptually-focused mathematics to her linguistically diverse classes" (p. 19). While one of her stated goals for the linear functions unit was to develop slope conceptually as a rate of change, Zahner reported that the students seemed to have adopted only a procedural understanding of slope. His analysis of the nested activity systems ranging from the students' group discussion, to whole class discussion, to the wider communities of the school and the district revealed that external assessment pressures shaped the teacher's decisions about the slope tasks she selected for the students and affected the way the teacher and students communicated about slope in mathematical discussions. As the district benchmark assessment approached, the teacher seemed to abandon her philosophy of conceptually-focused mathematics and adopted a more traditional, procedural approach to teaching slope (e.g., rise over run) to mirror the types of questions students would see on the assessment. Zahner (2015) emphasized that this shift from conceptually-based to procedurally-based teaching reflected the conflicts teachers face in light of assessment and student performance pressures, and that this shift "was not necessarily a product of the bilingual classroom setting" (p. 36).

Teachers without formal ESL training. Similar to de Araujo and Zahner, Gutiérrez (2002) sought out three high school mathematics teachers of linguistically diverse students, but in this study, they did not have specific training for teaching ELLs. Instead, these were teachers who had been successful in encouraging large numbers of Latino/a students to take higher levels of mathematics courses. At the time of this article, Gutiérrez noted that most of the mathematics education research on Latinos/as fell into three categories: (1) English Language Learners, (2) elementary school mathematics, and (3) middle school math settings. For this study, Gutiérrez focused only on factors that are related to language, but noted that there are other factors that contributed to the teachers' success such as embracing a "strong belief in teaching as a political endeavor and their emancipatory goals" (Gutiérrez, 2002, p. 1081).

Interestingly, Gutiérrez found that many of the strategies that were described in the research literature as effective for ELs in elementary and middle school mathematics were also evident in the work of these "untrained" high school teachers. These teachers had no specific training for working with a diverse language population and developed their strategies mostly through trial and error, perfecting their pedagogy as they learned what worked for their students. One strategy that they used in their classes that is supported by the research literature is encouraging the students to work in their primary language if they so desire. Gutiérrez also found that the teachers knew their students, both in terms of their linguistic and mathematical abilities and preferences. These teachers relied heavily on the background knowledge of their students, developing worksheets that related to problems they had solved in the classroom or those that were of interest to the students, rather than following a textbook. These worksheets were created with the students' linguistic and mathematical backgrounds in mind, but the textbook problems were also brought in occasionally to support students' ability to decipher the language they would encounter on standardized tests and in future college courses. The students regularly worked in groups, discussing problems and explaining strategies while being accountable for the learning of each group member. The teachers also established a sense of community through sharing personal stories with their students, listening to stories about students' interests, and providing food to share with the class.

Shared knowledge through mathematical discussions. Turner, Dominguez, Empson, and Maldonado (2013) studied how teachers and students developed a shared communicative space during an after-school mathematics program for fourth and fifth grade students as they worked with fractions and ratios. The focus of their analysis was *intersubjectivity*, which is the common understanding that "participants achieve through dialogue around joint activity" (Turner et al., 2013, p. 349). In order to study this, the authors employed a temporal analysis to identify that instances of intersubjectivity have a history in prior interactions and will have a future in related activities. Turner et al. (2013) also sought to change the perception that one should avoid mathematical discussions when you have linguistically diverse students, but rather to see such discussions as opportunities to build upon students' convergent and divergent views in order to develop shared understandings.

The authors found that moments of confusion or misinterpretation seemed to spur on the mathematical discourse of the classrooms and opened up different forms of communication, including inscriptions, gestures, and the use of two languages. It appeared to be important to both the teachers and the students to resolve these misunderstandings in order to promote a shared understanding of the mathematical tasks at hand. The use of two languages in these case studies promoted bilingualism as a resource for the students, not a hindrance as one may read about in other literature. Through the temporal analysis of the interactions, Turner et al. (2013) discovered the importance of the teachers' moves to give the students multiple and prolonged opportunities to consider the ideas of their peers as well as finding other consequential moments, such as misunderstandings, that led to extended discussion and promoted shared understandings.

Supporting Latino/a students' conceptual understanding. Zahner, Velazquez, Moschkovich, Vahey, and Lara-Meloy (2012) analyzed how three teachers of linguistically diverse students structured the same technology-based lesson, attending to their use of mathematical discourse practices and whole class discussion to promote students' conceptual understanding. The motivation for their analysis of this lesson was due to the results of a curriculum-aligned assessment, on which the students in two of the classes performed better than those students in the third class. Zahner and colleagues reported on a lesson that involved a road trip taken by a van and a bus. The students were first given a geographical map of the trip, then three distance-time graphs of the road trip (see Figure 2.4). From these graphs, the students were instructed to make predictions about the movements of the van and the bus. They checked their predictions with modeling software, and then told a story about what happened. Overall, Zahner et al. found this task to be beneficial in supporting EB students' conceptual understanding of concepts related to piecewise linear functions. However, they also found that the teachers' instructional moves resulted in different learning opportunities for the students.

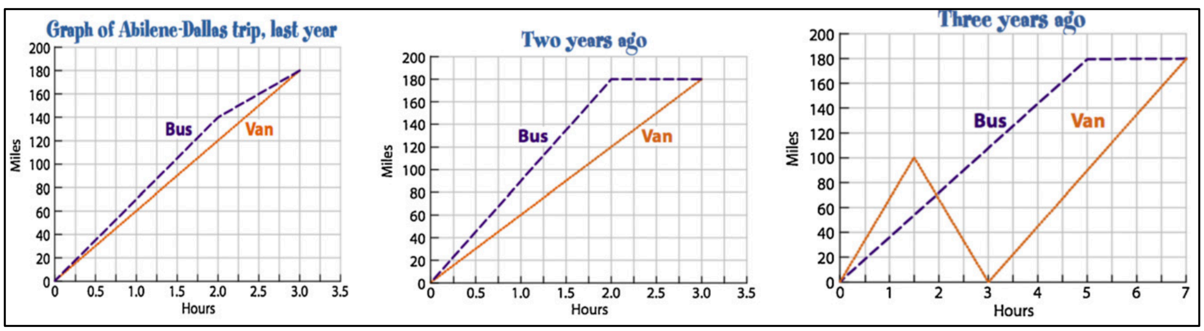


Figure 2.4 The three distance-time graphs representing the road trip (Zahner et al., 2012. p. 436).

Zahner et al.'s (2012) analyses revealed teaching practices that supported EB students' conceptual understanding of the lesson. Their first observation was that all teachers used a combination of teacher-led whole class discussion and student work (alone or in pairs) at their computers. However, the decisions of how to structure this time differed. Two of the teachers elected to alternate between the two formats, allowing students time to explore with the software, talk with a partner, or have individual think time in between whole class discussions. The second teaching practice that supported students' conceptual understanding was *how* teachers introduced content-specific vocabulary. In the classes with the higher student achievement, the teachers carefully developed new terminology first through informal language and then supplied the formal vocabulary after the ideas had been discussed informally by the students. Third, the teachers approached the use of stories differently. One teacher tended to write the stories with (or, perhaps, for?) the students during the whole class discussion, while the other teachers gave the students time to think about and write their own individually or in groups and then share their stories with the whole class. The final teaching practice Zahner and colleagues analyzed was the teachers' response to students' incorrect contributions to discussions of the task. They found that the more successful teachers responded with "higher-level responses to student errors such as repeating inaccurate responses and soliciting more answers, or asking refining questions" (Zahner et al., 2012, p. 444).

In summary, the literature above demonstrates that teachers, their beliefs about the nature of mathematics, their perceptions of and assumptions about emergent bilingual students, their pedagogical decisions, and the local context in which they teach all have an effect on the EB students' opportunities to access meaningful mathematics and to engage in mathematical practices. In these studies, the greater the amount of ESL training the teachers had did not necessarily translate into better opportunities to learn mathematics for their students. The more successful teachers in Gutierrez (2002), for example, had less training, but their belief in the abilities of their emergent bilingual students to engage in mathematical discussions and their willingness to get to know their students and build a sense of community in their classrooms seemed to support student engagement better than the procedurally- and vocabulary-driven techniques of the ESL-trained teachers. Additionally, Zahner's (2015) study revealed that external pressures, such as district benchmark testing, have the capacity to make even a highly-regarded teacher make pedagogical decisions that go against her own philosophy of teaching, which limited the opportunities these particular students had to develop a more conceptual understanding of slope. The implications of this research for my study include being aware of the multiple factors that influence curricular and pedagogical decisions, recognizing that these decisions have the potential to reveal teachers' beliefs about emergent bilingual students, and identifying the ways in which these decisions may enhance or restrict EB students' opportunities to engage in mathematical practices.

Curriculum

Stein, Remillard, and Smith (2007) reviewed the state of research on how curriculum has influenced student learning in mathematics. They found a renewed interest in researching the effectiveness of standards-based curriculum (which they classified as alignment with the 1989 NCTM *Curriculum and Evaluation Standards for School Mathematics* and other features that distinguished them from the traditional textbooks available at that time). Since the creation of some of these curriculum materials was accomplished with funding from the NSF, researchers sought to provide evidence that their materials were achieving the curricular goals and supporting student learning. In an attempt to provide an image of curriculum research, the authors created the framework in Figure 2.5 not only to capture the fact that there is not a direct link between curriculum materials and student learning, but also as a means for providing a structural organization for the chapter. By examining the diagram of the framework, one sees that the *written curriculum* and *student learning* are separated by two components, *intended curriculum* and *enacted curriculum*. The *intended curriculum* represents what the teacher plans to do in the class after interacting with the written curriculum. The *enacted curriculum* is what actually occurs in the classroom when the teacher and students interact with the curriculum materials or tasks. As one can see in the framework, the authors contended that the connection between curriculum and its influence on student learning is complex and cannot be determined without also considering how curriculum is interpreted and intended to be used by teachers and what actually transpires in the classroom during the enactment of the lesson with the students.

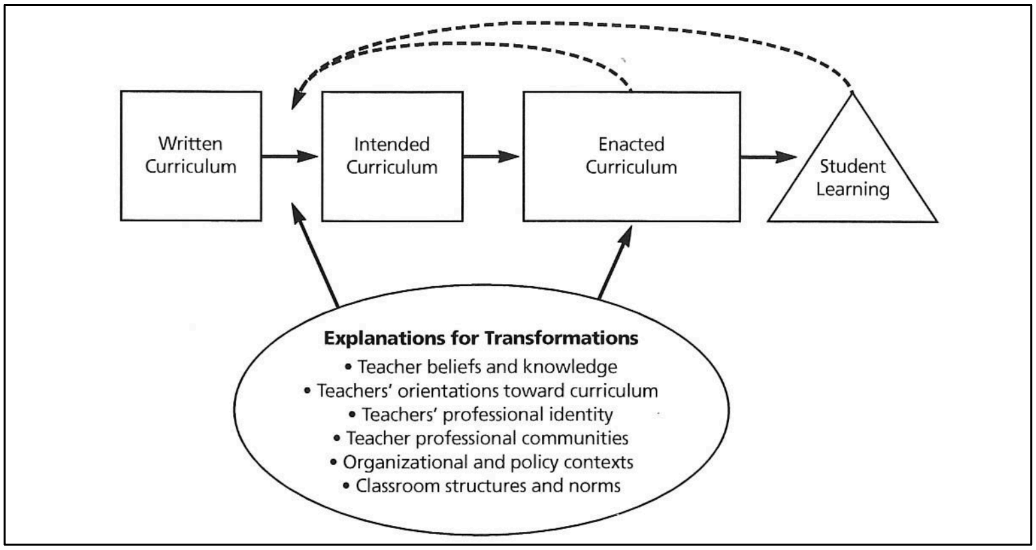


Figure 2.5. Stein et al.'s (2007) temporal phases of curriculum use.

Stein et al. (2007) summarized their findings in five main points. First, they found that curriculum materials vary significantly, not only between standards-based versus conventional curricula, but also by the criteria used to evaluate them which can (and did) result in the same curriculum being rated highly by one scale and very poorly by another. Second, the authors summarized that the differences do impact student learning (e.g., students using standards-based curricula score similarly in procedural knowledge but higher in conceptual knowledge compared to students using conventional curricula), but that this finding highlights the need for discussion of what type of mathematics students should be learning and how. Third, they stressed that no curriculum is "self-enacting", highlighting the ways in which teachers interpret and transform the curriculum to meet their own beliefs about what their students need in the moment. Fourth, the authors acknowledged that it is difficult to enact standards-based curriculum well, noting that the modifications teachers make to the written curriculum may no longer reflect the original intent of the mathematical activity. Finally, the successful enactment of a standards-based curriculum is multifaceted – far more than the curriculum itself needs to be considered: contextual factors (teachers, students, time, local cultures) are also salient when considering the effectiveness of a curriculum.

The Teacher-Curriculum Interaction

Based upon her review of research on curriculum literature spanning 25 years, Remillard (2005) developed a framework intended for "characterizing and studying teachers' interactions with curriculum materials" (p. 211). Before discussing this framework, it is worth noting that Remillard identified underlying, and sometimes conflicting, assumptions in the research on curriculum use and elaborated on the implications of these assumptions. See Figure 2.6 for a summary of her findings for the four conceptions of curriculum use. *Following or subverting* the text referred to curriculum studies in which the degree to which the enacted curriculum is aligned with the written curriculum. *Drawing on* the text referred to studies that perceive the textbook as one of the many resources a teacher may use when planning a lesson, a helpful tool rather than a guide for instruction. *Interpreting* the text acknowledges the impossibility of a teacher enacting the curriculum precisely as intended by the authors of the curriculum, recognizing that the teacher holds her own beliefs and experience about teaching mathematics that will influence how she interprets the intent of the authors. The final conception of curriculum use, *participating with* the text, involved research aimed at determining how teachers use the text and explaining the nature of this teacher-text relationship. Additionally, research in this conception also included studying how teachers can learn from their use of the curricular materials. This participatory relationship between a teacher and curriculum materials is a focal point for my study.

Key assumptions and theoretical perspectives influencing conceptions of curriculum use

Conceptions of curriculum use	Following or subverting	Drawing on	Interpreting	Participating with
Conceptions of curriculum materials	Fixed representation of enacted curriculum	One of many available resources	Representation of tasks and concepts	Artifacts or tools; products of sociocultural evolution
Conceptions of the teacher's role	Enactor of planned curriculum	Active designer of the enacted curriculum	Meaning maker; draws on beliefs and experience to make meaning	Collaborator with curriculum materials to design enacted curriculum
View of teacher–curriculum relationship	Fidelity is possible and a desirable goal	Teacher has agency over curriculum	Fidelity is not possible	Participatory relationship influenced by both teacher and curriculum
Theoretical or epistemological influences	Positivism	Positivism or interpretivism	Interpretivism; reader-response literary theory	Sociocultural analysis
Focus of research: Illustrative research questions	Agency of the text as influencing factor: To what extent and under what circumstances do teachers use the curriculum with fidelity? How can fidelity be increased?	Agency of teachers: What influences the choices that teachers make? How are their choices played out in classrooms?	Nature of interpretations and resulting classroom practices: How do teachers interpret their curriculum resources? How do these inter-pretations play out in mathematics teaching?	The participatory relationship: How do teachers engage with and use curriculum resources? What teacher and curricular factors influence this relationship?

Figure 2.6. Four perspectives underlying curriculum use research and the implications of each (Remillard, 2005, p. 217).

Influenced by the research that fell into the conception *participating with* the text, Remillard (2005) proposed a framework that highlights the participatory relationship a teacher has with curriculum, called the *teacher-curriculum relationship*. There are four principal constructs in her framework: (1) the teacher, (2) the curriculum, (3) the participatory relationship between the teacher and the curriculum, and (4) the planned and enacted curricula that result from this interaction. See Figure 2.7 for a depiction of the connections among these four main constructs. Two underlying assumptions about teaching are represented in the framework: (1) teaching is multifaceted and (2) teachers are involved in curriculum design work. The cycles of design work are reflected in the framework by acknowledging that teachers are involved in making curricular decisions prior to, during, and after teaching a lesson. Additionally, the

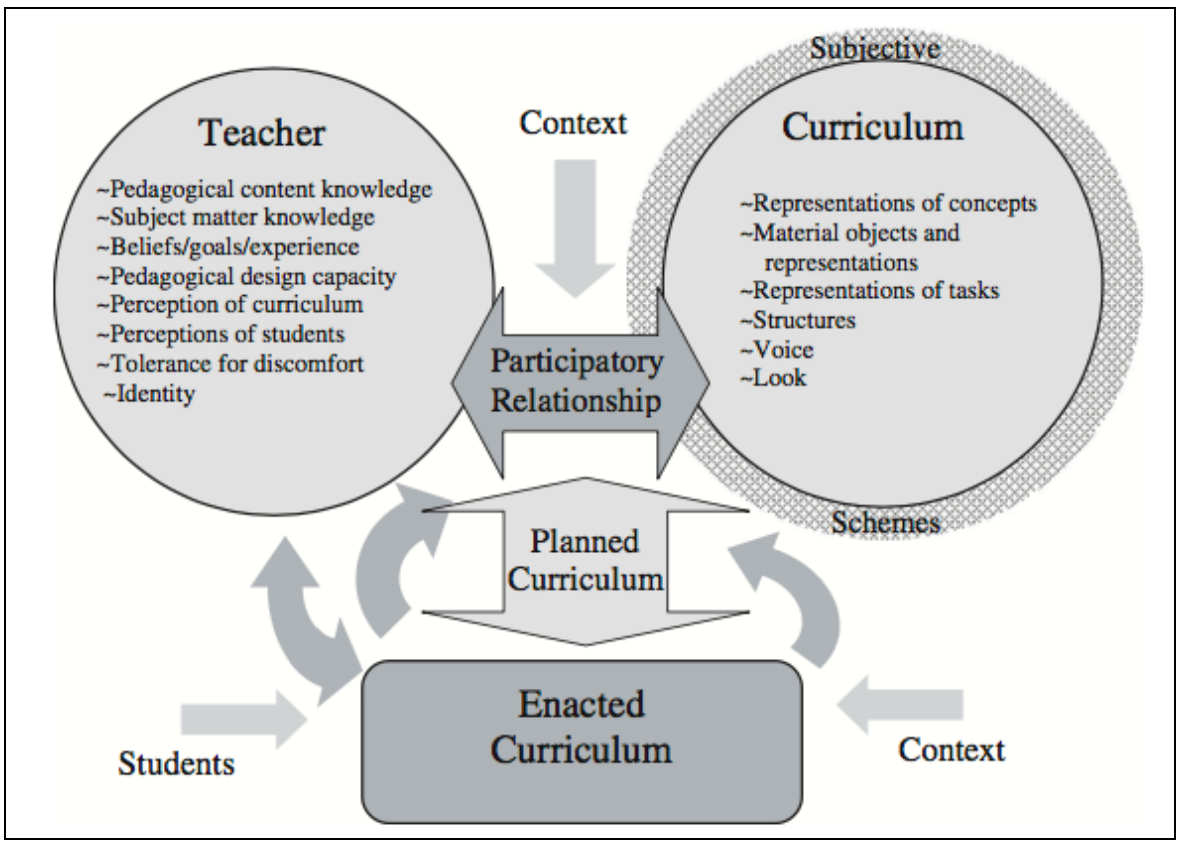


Figure 2.7. Remillard's (2005) framework for the teacher-curriculum interaction (p. 235).

participatory relationship between teacher and curriculum is influenced by the local and global context in which the teacher works. Teachers will make curricular decisions based upon numerous contextual influences, such as schoolwide or districtwide policies or their perceptions of the needs of their students. Students and contextual factors will also influence the enacted curriculum. Teachers may need to adapt their plans in the spur of the moment due to unexpected student responses to the planned tasks or because of classroom interruptions, such as an emergency drill. Notice that the enacted curriculum is not the end of the cycle, rather it serves as a potential change agent as teachers reflect upon and adapt the lessons they just taught and may also influence how they subsequently use the curriculum to plan for future lessons.

The circle on the left reflects the characteristics that the teacher brings to the teacher-curriculum interaction. Different facets of teacher knowledge, beliefs, and experience will affect the way in which the teacher uses and adapts curricular resources for use in their classrooms. Many of these characteristics are well-documented in the research literature about teachers, but others, such as teacher beliefs, need further refining. The teachers' perception of curriculum, whether it is something to be followed or merely a suggestion or something in between, will influence how the teacher interacts with the curriculum materials. Teachers' perceptions of the nature of mathematics and mathematics instruction may or may not align with the authors' perceptions, causing a disconnect between the two, possibly resulting in the teacher disregarding the material in the textbook or in the teacher modifying the suggested mathematical tasks and losing sight of the authors' purpose for the particular tasks.

The circle on the right represents the curriculum being used by the teacher. The outer ring labeled *subjective schemes* reflects how the teacher and others (other teachers, administrators, students, parents, etc.) feel about the particular curriculum and its features, as well as their perceptions about curriculum in general. Less research has been conducted on the curriculum features listed in this circle than those in the teacher circle, thus Remillard indicated that these are tentative and may need further refining. Existing curriculum research tends to focus on structural features like the *mathematical content* or the *pedagogical content* of the texts, but others had begun to look at other nonstructural features such as the *voice* of the text (how the authors communicate with the readers, which is often invisibly) or the *look* of the text (glossy, colorful pages vs. plain pages, use of numerous photos and a variety of fonts vs. few pictures and fewer fonts) and in what ways and to what extent these nonstructural features of texts actually matter in the teacher-curriculum interaction.

Between these two circles lies a horizontal bidirectional arrow representing the participatory relationship, emphasizing the contributions both make in developing the intended or planned curriculum, and noting that the local and global context in which this relationship is forged matters and influences the resulting plans. One may consider it odd that curriculum or a textbook can have an active participatory role until one considers the curriculum from a sociocultural perspective – curriculum materials are artifacts or tools used by participants in the situated community of practice. Brown (2009) studied science teachers' use of curriculum materials and identified three ways in which they were used: (1) offloading, which involved teacher reliance on and almost literal use of the written materials as presented; (2) adapting, which is when teachers used tasks from the curriculum but modified them to align with their beliefs about how the content should be taught, and (3) improvising, when teachers go "off-script" to follow a student's solution method or to create their own materials while still supporting the overall curricular goals. Brown's categories of curriculum use highlight some of the ways in which the curriculum can be an active participant in this relationship (e.g., adapters start with the written curriculum so what appears on the page shapes the initial revision actions).

Below the participatory relationship lies a vertical bidirectional arrow labeled *planned* curriculum, which connects the participatory relationship of teacher and curriculum with the *enacted curriculum*. The planned curriculum is the product of the participatory interaction between the teacher and the curriculum, but the bidirectional arrow also indicates that the planned curriculum may also influence what teachers look for and use in the curriculum. The enacted curriculum reflects what actually occurs in the classroom when the plans are used with students in that particular context. Remillard (2005) described the enacted curriculum as “co-constructed by teachers and students in a particular context” (p. 238). The planned curriculum is often modified during the enactment, and these adaptations may be small or large. Remillard calls for more research in this area, wondering if there may exist consistent or predictable ways that teachers adapt their planned curriculum during enactment and what may influence these changes (curriculum, teacher, or contextual factors).

Curricular Noticing

Influenced in part by Remillard’s (2005) work on the teacher-curriculum interaction and by Jacobs, Lamb, and Philipp's (2010) work on the professional noticing of children’s mathematical thinking (PNCMT), Dietiker, Amador, Earnest, Males, and Stohlmann (2014) have developed a framework for curricular noticing (CN). While PNCMT describes a noticing framework for attending to, interpreting, and responding to student thinking, the CN framework focuses the same activities on curriculum materials. The act of attending to curriculum materials involves familiarizing oneself with the curricula, perusing and noting different aspects. Interpreting refers to the activities a teacher does to make sense of the curriculum materials they have perceived. The teacher then responds, making choices about what will be presented to the students and in what order based upon their interpretations of the aspects of the curriculum to which they have attended. Figure 2.8 is an elaboration of the different activities a teacher may do at each level of engagement with curriculum materials.

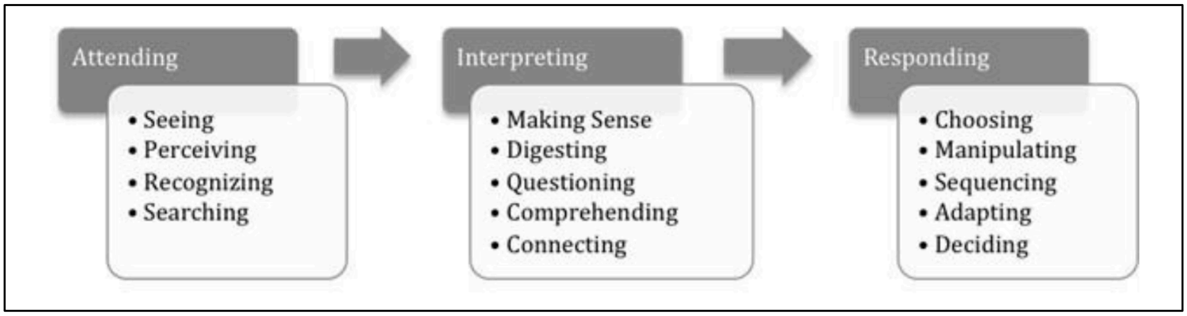


Figure 2.8. The activities involved within each component of the CN framework. (Males et al., 2015, p. 2).

Males, Earnest, Dietiker, and Amador (2015) reported on four exploratory studies designed to test the CN framework. The first was designed to explore which features of task design preservice teachers (PSTs) attend to while working on an optimization task. Two groups were formed, one given an open-strategy version and the other a closed-strategy version of the task. After completing the tasks, the PSTs discussed their strategies and challenges, and then reflected on the affordances and constraints of each task design. The PSTs identified five main themes of task design, providing evidence that this form of task comparison led to interpretations of task design. In the second study, the researchers were exploring how to support PSTs interpretations of mathematical properties present in routine tasks through the use of nonroutine tasks. They found that some mathematical properties were not identified by the PSTs in the routine tasks until they had been identified through interacting with nonroutine tasks, supporting their hypothesis that discussing nonroutine problems would enhance the noticing of and interpreting the mathematical properties of the types routine tasks commonly found in curriculum materials.

The third study examined how PSTs evaluate content in secondary mathematics textbooks. After comparing the three presentations of quadratics and selecting which textbook they would use and why, the PSTs used the CCSSM Curriculum Analysis Tool (CCCAT) (Bush, 2011) to evaluate these materials based upon content, practices, and equity, special needs, and technology. After using this tool, they were again asked which textbook they would use and why. While their overall choice in curriculum did not shift much, the reasons provided for why they would make their selection were much more specific and focused on different features of the material, indicating that the CCCAT may have shifted what the PSTs attended to and how they interpreted curricular materials. The final study involved PSTs decision-making as they created their intended lesson plans, referencing a variety of approaches in different curricular materials. The CN framework helped the researchers find that the decisions PSTs make in lesson planning are closely related to their prior experience with that content, often searching for curricular materials that most closely matched their learning experiences with that particular content. Taken together, the authors concluded that PSTs can learn to attend to the different aspects of curriculum materials through participation in methods course activities that support the development of curricular noticing.

English Learners Success Forum

The English Learners Success Forum (ELSF) is an organization dedicated to improving the quality and availability of instructional materials for both English Language Arts (ELA) and mathematics that consider the needs of students classified as English Learners. The ELSF identified areas of challenge that need to be addressed in U.S. schools, including (1) only 20% of public school teachers are certified to teach ELs, (2) only 24% of K-12 teachers surveyed have had EL-focused professional development over the previous three years, (3) effective language development strategies are rarely found in ELA and mathematics instructional materials, and (4) U.S. demographic shifts point toward most teachers have or will have a student classified as an EL in their classroom. To address these needs, the ELSF brings together EL experts and high-quality content developers to improve materials to address the needs, linguistic and cultural, of ELs. The ELSF team is composed of researchers, teachers, district leaders, and funders. A 501(c)(3) public charity, the New Venture Fund (NVF), is a fiscal sponsor of ELSF and manages all the financial, business, and legal aspects, enabling the ELSF team to focus on achieving their mission of ensuring the availability of high-quality K-12 ELA and mathematics instructional materials that are inclusive of ELs. The guidelines are grounded in the research on EL strategies and best practices and have been strongly influenced by the work of the Understanding Language project at Stanford University, whose major advisers are leaders in EL education including Aida Walqui, Guadalupe Valdés, Kenji Hakuta, Lily Wong Fillmore, and Judit Moschkovich. Additionally, the work of many other researchers including Kathryn Chval, Margaret Heritage, and Marta Civil influenced the creation of these guidelines.

The ELSF website offers resources for ELA and mathematics teachers and content developers. First, there is a self-reflection survey in which one can consider whether ELs are engaging in classroom activities or with the materials used during these activities. Next, they offer two sets of guidelines, one for ELA and one for math, for improving instructional materials for ELs. These guidelines are designed for anyone who develops curriculum or instructional materials, selects curriculum for classroom use, or for those who wish to address the needs of ELs in ELA or math classrooms. Not only have they developed the guidelines themselves, but they also offer a Guidelines Inventory to rate how well current instructional materials meet the needs of ELs for each of the guidelines. Additionally, the ELSF website currently offers two sample lesson plans that they have designed in collaboration with their colleagues at The Math Learning Center. Each lesson is annotated to highlight the ways in which the ELSF guidelines have been incorporated. Finally, ELSF offers a library of tools and resources that include strategies and activities that will support implementation of the ELSF Guidelines.

The *Guidelines for Improving Math Materials for English Learners* were created in order to address the call for high-quality instructional materials that are inclusive of and meet the academic and language demands of ELs. The Guidelines were developed and reviewed by a variety of people with diverse experiences, including researchers, linguists, education leaders, practitioners, and EL experts (among the mathematics reviewers were Judit Moschkovich and Kathryn Chval). The guidelines reflect this collective knowledge of how EL supports should look in curricular resources. ELSF suggests the following audiences for these guidelines: (1) content developers, such as curriculum publishers or teachers who wish to modify current materials; (2) professional learning communities, for practical suggestions about how to plan for and support the needs of ELs; and (3) education leaders tasked with curriculum adoption.

The authors of the guidelines have identified five Areas of Focus for consideration in creating quality supports for ELs. Each of these five Areas of Focus have three Guidelines, for a total of 15 Guidelines (see Figure 2.9). Each Guideline is then broken down into Specifications, between two and four per Guideline, which provide more specific details about how instructional materials have the potential to support ELs. In addition to the Specifications for each Guideline, nine of the Guidelines also have links to examples and additional resources such as research articles or mathematical language routines. In this study, I use the ELSF Guidelines because they provide a distillation of many of the key ideas I have reviewed thus far.

Area of Focus I: Interdependence of Mathematical Content, Practices, and Language	Area of Focus II: Scaffolding and Supports for Simultaneous Development	Area of Focus III: Mathematical Rigor Through Language	Area of Focus IV: Leveraging Students' Assets	Area of Focus V: Assessment of Mathematical Content, Practices, and Language
1. Strategic opportunities to use and refine both language and mathematics over time	4. Opportunities for students to interact with and produce a variety of methods and representations	7. Explicit guidance for teachers to engage students in using mathematical practices	10. Opportunities to draw on and incorporate students' cultural background and lived experiences in mathematics learning	13. Descriptions, illustrations, and examples of quality work and mathematical practices with varying levels of language proficiency
2. Explicit mathematics and language learning goals and pathways	5. Directions for providing specialized individual and small group instruction to ELs	8. Maintain appropriate challenge and high expectations of mathematics learning for EL students	11. Suggestions for incorporating and valuing ELs' written and spoken contributions	14. Assessments able to capture and measure students' mathematics and language progress over time
3. Regular and varying opportunities to learn, reflect upon, and demonstrate learning of mathematics using a variety of modes and forms	6. Guidance for anticipating potential language demands and opportunities in student activities	9. Guidance for facilitating mathematical discussion and co-construction of meaning	12. Encouragement for ELs to use and build on existing language resources	15. Guidance for recognizing and attending to student language produced to inform instructional decisions

Figure 2.9. ELSF Guidelines

In summary, the literature on curriculum discussed in this section highlights the complex nature of curriculum use and evaluation. Remillard (2005) summed up this complexity in her framework for the teacher-curriculum interaction, emphasizing that there is a participatory relationship between the teacher and the curriculum and that this relationship is influenced by students and other local contexts in which this interaction takes place. This framework is important for my study because it draws attention to the fact that I cannot simply assume that a given curriculum is better suited for supporting the engagement of EB students' participation in mathematical practices than another based on the curriculum alone. The teachers in my study will have their own beliefs about teaching mathematics to EB students which will affect the way in which they use the curriculum. Additionally, the students themselves will influence the way in which the teacher enacts the curriculum by responding to them in the moment, which is likely to differ from the planned curriculum.

Mathematical Practices

In this section I will describe two of the ways in which mathematics education researchers have defined mathematical practices. The first, classroom mathematical practices, focuses on the mathematical development of a whole class, using argumentation as a marker for identifying "taken as shared" knowledge. The second use is consistent with the NCTM standards, NRC's mathematical proficiencies, Moschkovich's (2015) characterization of mathematical practices, and the CCSSM Standards for Mathematical Practice. In this study, I will use this second description of mathematical practices.

Classroom Mathematical Practices

A practice is an established way of "operating, arguing, or using tools" (Lobato & Walters, 2017) within a community that no longer needs justification. Cobb and Yackel (1996) documented that it is possible to analyze the mathematical development of a whole class in addition to that of individual students. They believe that the relationship between collective mathematical practices and individual conceptions is reflexive. The students are active contributors to the evolving mathematical practices as they reorganize their own individual mathematical activities and these reorganizations are made possible by (and limited by) the students' participation in the mathematical practices. According to the emergent perspective, learning is a constructive process that happens when the learner is participating in and contributing to the practices of the local community. Furthermore, Cobb and Yackel observed an indirect link between collective and individual processes. They say it is indirect because participating in the collective provides opportunities to learn (or limits them) but there is no guarantee that individual members of the community will learn exactly the same things in exactly the same ways. Cobb and Yackel described participating in the activities of the community as a possibility of learning, whereas van Oers (as cited in Cobb & Yackel, 1996) described Vygotsky's perspective that the qualities of students' thinking is a direct result of the activities in which they have participated.

How do researchers identify a classroom mathematical practice? Stephan and Akyuz (2012) used the parts of Toulmin's model of argumentation to code transcripts of whole-class discussions observed during a teaching experiment. Every instance of *data* (supporting evidence), *claim* (a conclusion), *warrant* (clarification of supporting evidence), and *backing* (justification) were identified, as well as student challenges and the final claim, which may have been revised from its original statement, that became "taken as shared." The researchers then analyzed this argumentation log to identify three criteria. First, they noted when students challenged a claim. Second, they looked at when a claim that had been challenged no longer needed any justification or when students used a previously challenged claim without warrant or backing. Third, they identified when a previous claim was used as data to support a new claim. Stephan and Rasmussen (as cited in Stephan & Akyuz, 2012) posited that these events indicate that a mathematical claim has been "taken as shared." Once all of the mathematical claims that had become "taken as shared" were identified, a list of these mathematical ideas established by the collective were reviewed and organized into classroom mathematical practices.

In the process of discussing a topic, students will make many claims, both correct and incorrect. Simply making a correct claim that the speaker can justify does not make the argument a "taken as shared" classroom mathematical practice. Other students must also use the claim and know why the claim is true. If any student questions why others are using this method, the claim is not yet "taken as shared" and can't be considered a classroom mathematical practice.

Standards for Mathematical Practice

As mentioned in Chapter 1, recent mathematics curriculum reforms (e.g., CCSSM, 2010; NCTM, 2014) have resulted in the call for enacting mathematical practices, most of which require increased communication among students and teachers. Influenced by the NCTM process standards and the NRC report, the CCSSM Standards for Mathematical Practice were developed to highlight the variety of ways mathematical proficiency is developed. The eight Standards for Mathematical Practice are:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (CCSSM, 2010)

Each of these standards are further elaborated in the CCSSM document. A careful reading of these descriptions with emergent bilingual students in mind reveals a significant emphasis on language – explaining, analyzing, making conjectures, planning solution paths, understanding and critiquing the reasoning of others, understanding and using definitions, justifying, communicating and responding, etc. This conception of mathematical practices could be imagined as and interpreted with a situated sociocultural perspective on learning. Mathematical practices are not just ways in which individual people think mathematically, but are also a reflection of social interactions and reflect historically and culturally organized normative practices within the mathematics community. Selling (2016) and Moschkovich (2013) extend Cobb and Yackel's (1996) assertion that classroom mathematical practices are coconstructed by students and teachers by also emphasizing that mathematical practices reflect the disciplinary tools and ways of working used by mathematicians.

Moschkovich (2015) asserted that the above list of standards for mathematical practices, or any other list for that matter, is necessarily incomplete and not a reflection of all the ways in which mathematics can be and have been done, and cautioned that they may be interpreted differently by individuals depending on the theoretical framework one employs. For example, while a sociocultural perspective lends itself to an interactive and dialogic interpretation of many of the standards, she cautioned that a cognitive perspective could be taken and each of the above could be assumed to be individual standards of mathematical proficiency.

Selling (2016) raised the question of how explicit teachers should be in engaging students in mathematical practices. She claimed that students may not realize that a mathematical activity they were involved in was actually a disciplinary practice and students may benefit from it being labeled as such, yet she cited concerns about turning the mathematical practices into prescriptions, similar to using a key word approach to solving word problems, or creating a false dichotomy between the mathematical practices and mathematics content, such as creating a specific unit on proof rather than integrating proof throughout a course. A final concern about explicit teaching of the mathematical practices involves limiting student opportunities to authentically engage in mathematical practices, similar to the ways in which teachers reduce the cognitive demand of mathematical tasks by adapting the tasks or doing some of the problem for the students to get them started. However, Selling asserted that it is possible to be explicit about mathematical practices through *reprising moves*.

Selling (2016) described a *reprising move* as “when the teacher explicitly reflects back on student participation in mathematical practices” (p. 518), and is similar to what Cobb, Boufi, McClain, and Whitenack (1997) referred to as *reflective discourse*, Goodwin's (1994) notion of *highlighting*, or what Lobato, Hohensee, and Rhodehamel (2013) refer to as *mathematical noticing*. As a result of her analysis of three mathematics classes over the span of one month, Selling identified eight types of reprising moves (see Figure 2.10) the teachers used to make mathematical practices explicit in their classes.

Reprising Moves That Make Mathematical Practices Explicit in Classroom Discourse

Types of reprising moves
Naming the mathematical practice (or practices) in which students just engaged (Lobato et al., 2013)
Highlighting aspects of student engagement in mathematical practices (Goodwin, 1994; Lobato et al., 2013)
Evaluating student engagement in mathematical practices
Explaining the goal or rationale for engaging in a mathematical practice
Connecting different students' engagement in mathematical practices
Framing student engagement in mathematical practices expansively (Engle, Lam, Meyer, & Nix. 2012)
Eliciting self-assessment with respect to mathematical practices
Referring to a teaching narrative about mathematical practices

Figure 2.10. Selling's (2016) framework for making mathematical practices explicit (p. 524).

Naming could be the use of an informal or conventional term for an activity students were engaged in, with the intent of making it salient to the participants as well as having a way to recall it in the future. *Highlighting* was used by teachers to draw attention to a mathematical practice that recently transpired and explicitly to point out the particular practice. *Evaluating* typically occurred alongside naming or highlighting moves and offered students an evaluative comment about their engagement in mathematical practices. *Explaining the goal or rationale* involved the teacher explaining why the students needed to engage in a mathematical practice, such as how color coding can help one see in what ways different representations are related. *Connecting* occurred when a teacher drew attention to the ways that different students' mathematical work was similar or related. *Framing* referred to situating engagement in mathematical practices in an expanded frame or setting, such as indicating that a mathematical practice will be needed for a test or a future class. *Eliciting self-assessment* typically occurred when students were asked to decide how well they had understood another student's explanation. *Referring to a teaching narrative* involved the teachers explicitly referring to the students' engagement in mathematical practices and how the teachers specifically designed the task to encourage participation in that particular mathematical practice.

In summary, there is more than one way to conceive of a mathematical practice. In this study, I will use the conception of mathematical practices that is consistent with the use of this term by national organizations such as NCTM and the NRC, and by mathematics education researchers such as Moschkovich and Selling. Acknowledging that no list of mathematical practices will be exhaustive, I will use the CCSSM Standards of Mathematical Practice as a starting point for identifying the mathematical practices in which the teachers plan to engage their students and to determine whether they make these practices explicit to their students using Selling's reprising move framework.

Linear and Exponential Rates of Change

The concept of rate of change has received much attention in the mathematics education literature. At first one may wonder why it seems to be such a popular topic, until one realizes how foundational rate of change is to the study of mathematics – from slopes of linear functions to derivatives in calculus – and the application of mathematical ideas in the physical world. A search of the Common Core State Standards for Mathematics (CCSSM) yielded ten instances in which the phrase "rate of change" appeared. Another search for the word "slope" found thirteen matches, some of which were used in conjunction with rate of change. Additionally, a search for the word "speed" produced four results that were not connected to any of the above instances. From sixth grade forward, mathematics students are being exposed to rates of change.

Informal experiences with rate of change begin long before students are introduced to the formal concepts of speed, slope, and rate of change in their mathematics curriculum. Children are constantly experiencing motion – swinging in a swing, sliding down a slide, riding a bicycle, riding in a car, amusement park rides – so much so that it would be impossible to generate a complete list. It would be surprising to find a sixth grader who doesn't have any familiarity at all with the concept of speed or the fact that cars travel at speeds measured in miles per hour.

As mentioned earlier, one can find an abundance of mathematics education literature devoted to the concept of rate of change. For example, Smith and Thompson (2008) argued for an emphasis on quantitative reasoning in order to help students understand the relationships among the quantities involved in the problems they are solving. Improving this understanding should lead to an improvement in students' ability to "conceptualize, reason about, and operate on quantities and relationships in sensible problem situations" (Smith & Thompson, 2008, p. 95). They claimed that developing students' quantitative reasoning would bolster their ability to utilize, operate on, and reason with variables in algebra.

Lobato, Hohensee, and Rhodehamel (2013) presented a focusing framework for students' mathematical noticing, claiming that "what students notice mathematically has consequences for their subsequent reasoning" (p. 809). To illustrate the use of this focusing framework, Lobato et al. discussed the mathematical activities of two classrooms both focused on learning about slope as a rate of change, but with different types of activities. As proposed, these students developed different ways of reasoning about slope. Students in one class formed a composed unit conception of slope, noting that there was a constant multiplicative relationship between distance and time, while the students in the other class attended to the additive growth between the number of objects in a pattern without considering (or coordinating with) the step number.

As described in Chapter 1, exponential rates of change are being introduced to students earlier than they were previously. Specifically, the California standards in effect before the CCSSM did not introduce exponential functions until Algebra 2, while the new CCSSM standards introduce exponential functions in the ninth grade. Numerous researchers have studied linear rates of change, but fewer have focused on exponential rates of change. Confrey and Smith (1995) and Ellis, Ozgur, Kulow, Dogan, and Amidon (2016) are prominent contributors to the research on exponential rates of change.

Confrey and Smith (1995) proposed a covariational approach to understanding exponential functions, contrasting this with the conventional correspondence approach in most secondary textbooks. The correspondence approach describes functions with a rule, such as $y = a \cdot b^x$, and the authors argued that the approach creates an emphasis on algebraic representations while neglecting the use of tables to assist in the development of an understanding of functions. The covariational approach emphasizes that a function is a “juxtaposition of two sequences, each of which is generated independently through a pattern of data values” (Confrey & Smith, 1995, p. 67). They also proposed that the operations of *splitting* and *additive counting* are essential to understanding exponential functions.

Confrey and Smith (1995) introduced the operation of *splitting* through making a set of four theoretical claims. First, simply thinking of multiplication as repeated addition is problematic for many multiplicative situations. For example, if you knew a computer virus was increasing to nine times its size every hour, a multiplicative rate, how could you determine the rate every half hour? Second, splitting provides an operational basis for multiplication and division. The act of sharing items is an early application of splitting, as well as the processes of halving and doubling, which they have repeatedly seen children spontaneously do as a problem-solving technique. Third, splitting foreshadows the concept of ratio, using similarity as the starting point to developing a sense of invariance of proportion. Finally, they proposed the idea of the *splitting world* which has a different structure and developmental path than the *counting* *world* that was prevalent in mathematics curriculum at that time. See Figure 2.11 for a comparison of the characteristics of the counting and splitting worlds.

Characteristics of Counting and Splitting Worlds

Splitting	Counting
One is the origin	Zero is the origin
Splitting by *n* is the successor action	Adding one is the successor action
The unit (of growth) is *n* or *n*:1	The basic unit is one
Multiplication and division are basic operations	Addition and subtraction are the basic operations
One is the identity element	Zero is the identity element
Reinitializing to one	Reinitializing to zero
Commutativity applies to multiplication	Commutativity applies to addition
Ratio is used to describe the interval between two successive "whole" numbers	Difference is used to describe the interval between two successive "whole" numbers
Composite units are made by raising "splitting units" to a higher power	Composite units are formed by aggregating counts into larger groups
Parts (multiplicative) are created by *n*-rooting	Parts (additive) are created by *n*-splitting
Exponentiation is created as repeated multiplication	Multiplication is contructed as repeated addition
Distributivity is applied to exponentiation over multiplication	Distributivity is applied to multiplication over addition
Rate is the ratio per unit time	Rate is the difference per unit time

Figure 2.11. Parallel structures in the splitting and counting worlds (Confrey & Smith, 1995, p. 75).

Ellis, Ozgur, Kulow, Dogan, and Amidon (2016) proposed an exponential growth learning trajectory (EGLT) following two teaching experiments. Like Confrey and Smith (1995), the authors support a covariation approach to understanding exponential growth. However, they found that many of the students in their studies also developed and relied upon a correspondence view while the teacher-researchers worked to develop understanding of exponential growth with an emphasis on covariation. The authors identified three major stages of conceptual development in the EGLT: prefunctional reasoning, the covariation view, and the correspondence view. See Figure 2.12 for the main component understandings at each of these three stages. Ellis and her colleagues emphasized that students do not move linearly through these component understandings, but that the students in their teaching experiments alternated among the three main stages of conceptual development throughout the tasks. Additionally, the students in their teaching experiments were middle school students who had only seen a repeated multiplication model of exponentiation, so the authors caution that secondary students who may have more exposure to exponential growth and functions will most likely have different entry points in their EGLT.

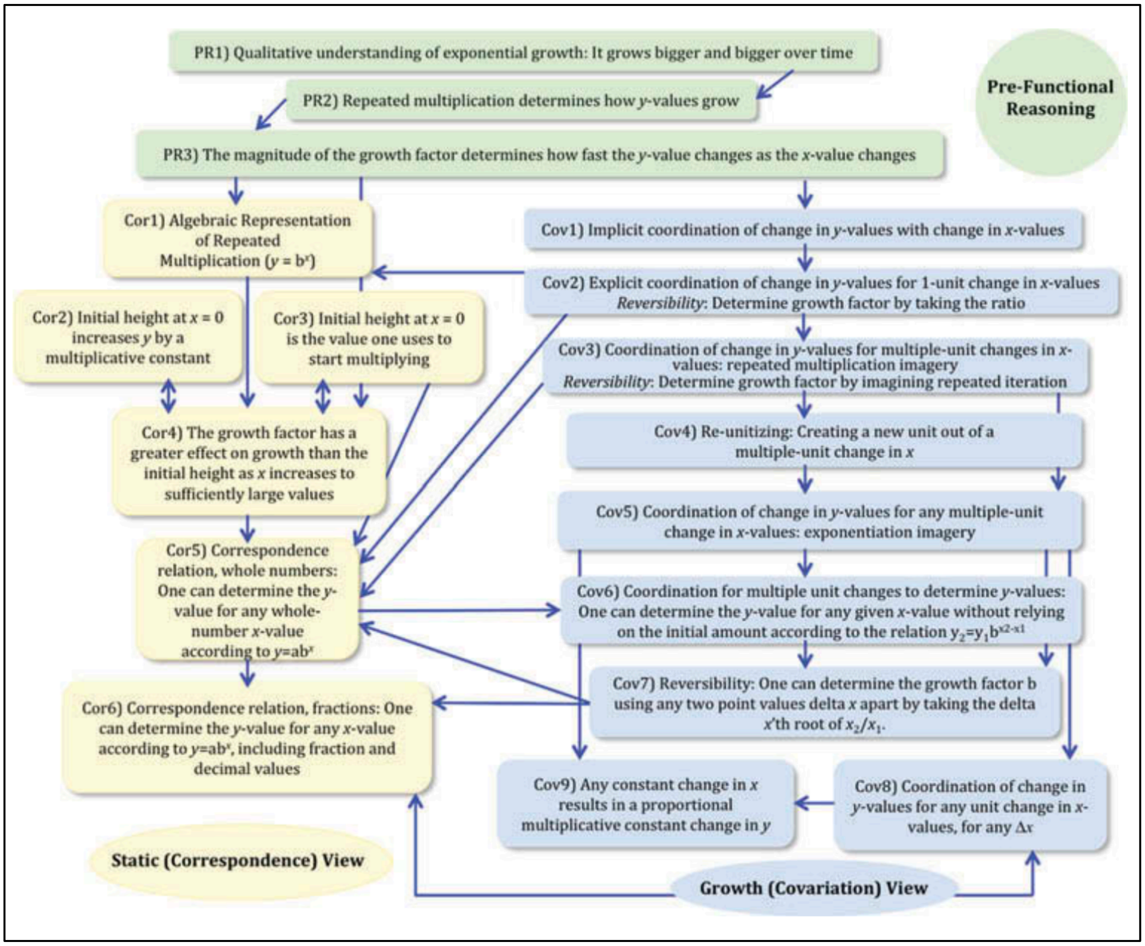


Figure 2.12. Ellis et al.'s (2016) exponential growth learning trajectory (p. 160).

Ellis et al. (2016) found that exploring exponential growth in a context that allowed the students to think about continuously covarying quantities using dynamic software supported the students' ability to coordinate the multiplicative growth in y-values with the additive growth in x-values, which the authors asserted is essential to understanding exponential growth. The context of the tasks was a fictitious plant, called a Jactus, which doubled (or tripled or quadrupled) in size every week. In this context, students developed the ability to coordinate the ratio of plant heights for the corresponding time intervals, noting that the ratio of the growth from week 3 to week 5 would be the same the ratio of the growth from week 53 to week 55. The students were also asked to consider the growth in between the weeks, which led to working with fractional exponents in a context that the students could easily imagine or notice while exploring with the Geogebra script created by the authors.

In summary, the concepts related to linear and exponential rates of change are not trivial, especially exponential rates of change, neither mathematically nor linguistically. While we physically experience rate and may have an intuitive understanding of fast and slow, the coordination of two quantities and forming a rate is not a simple concept. Talking about something that is doubling in size every week is difficult to model in continuous real-life contexts, but Ellis et al.'s (2016) use of the Jactus, a fictitious plant, alongside dynamic software, enabled students to imagine a scenario in which something could be continuously growing. More importantly, the students were able to then think about smaller time frames of growth, developing their understanding of fractional exponents and n^{th} roots. Coordinating the multiplicative growth in y-values with the additive growth in x-values was a key understanding involved in their EGLT. The research on linear and exponential growth provided a content focus for this study.

Summary

My study draws upon a situated sociocultural theoretical framework to analyze the ways in which emergent bilingual students' engagement in mathematics practices is supported by the teacher-curriculum interaction. My research questions are:

1. In what ways do the locally adopted, CCSSM aligned, curricula support secondary mathematics teachers to engage their students in mathematical practices?

    a. What supports are provided explicitly to help teachers engage their emergent bilingual students in these practices?
    b. How do the curriculum materials position emergent bilingual students as learners of mathematics?
2. How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?

The conceptual framework for my study is Remillard (2005) framework for the teacher-curriculum interaction, highlighting the complex and multifaceted nature of teaching by emphasizing the participatory roles both the teacher and the written curriculum play in the planned and enacted curriculum, in addition to the effect the students and the local context have on this interaction. As my study specifically focuses on the support of emergent bilingual students' engagement in mathematical practices, I also draw upon the body of research on effective teaching practices by teachers of EB students. I adopt Moschkovich's (2002, 2015) view that Mathematical Discourses are not limited to spoken or written language or formal technical vocabulary, but include all forms of communication such as gestures, visual representations, diagrams, and everyday informal descriptions of mathematical concepts. As such, my identification of mathematical practices was not necessarily limited to the eight CCSSM standards for mathematical practice, and considered the nuanced ways teachers talked about and supported emergent bilingual students in engaging in mathematical practices.

Chapter 3: Methods

This study investigated how eleven secondary mathematics teachers used curriculum resources to plan and enact lessons that supported the participation of emergent bilingual (EB) students in mathematical practices in their linguistically diverse classrooms. EB students classified as English Learners often experience mathematics as a set of disconnected procedures. This investigation is rooted in a situated sociocultural theory of learning (Moschkovich, 2002) and a conceptual framework for the teacher-curriculum interaction (Remillard, 2005, 2009). This study uses a contrastive design, examining both the official curriculum and teacher-created materials used by teachers in two districts that adopted different Common Core-aligned curricula. The official curricula provide different types of guidance for teachers to support their EB students. A lesson on concepts related to linear rates of changed will be used as a paradigmatic example of the capacity of the curriculum to support emergent bilingual students' engagement in mathematical practices.

Remillard's (2005, 2009) framework for the teacher-curriculum interaction provided a conceptual framework for this study, recognizing that the curriculum developers are communicating their perception of what it means to do mathematics through their materials while the teachers who use the curriculum also have their own perspectives of what it means to do mathematics in their classrooms, influenced by their students and local communities. This study investigated this teacher-curriculum interaction as teachers planned lessons for their students, particularly attending to the ways in which the written curriculum, planned curriculum, and enacted curriculum support EB students. Results of this study provide a better understanding of the ways in which curricular resources support teachers working with EB students and will guide those seeking to improve curricula for instructors in linguistically diverse settings. As a reminder, this study seeks to answer the following research questions:

1. In what ways do the locally adopted, CCSSM aligned, curricula support secondary mathematics teachers to engage their students in mathematical practices?
    a. What supports are provided explicitly to help teachers engage their emergent bilingual students in these practices?
    b. How do the curriculum materials position emergent bilingual students as learners of mathematics?
2. How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?

Settings

Data collection took place in two school districts which had linguistically diverse student populations and had both adopted different Common Core-aligned secondary mathematics curriculum programs. I refer to the districts as District A and District B. See Figure 3.1 for the demographic information about these districts. Data collection took place at two high schools, one in each of the two districts, that were specifically selected due to having a large proportion of students classified as ELs. District A had adopted the CME Project curriculum published by Pearson that was developed by a team of mathematicians and mathematics educators at EDC (Education Development Center). District B had adopted the CPM curriculum published by the CPM Educational Program, a California non-profit that is known for developing secondary mathematics curriculum that uses group work and problem-based instruction. Both of these curricula were described by their authors as responsive to the curricular reforms called for in the CCSSM.

	District A	**District B**
Student Population	131,252 (PS-12)	> 42,000 (7-12)
Ethnic Groups	> 15	
Languages and Dialects	> 60	
Number of High Schools	22	13
Ethnic Diversity of Students	46.5% Hispanic 23.4% White 10.2% African-American 5.4% Filipino 4.9% Indo-Chinese 3.3% Asian .3% Native American .6% Pacific Islander 5.4% Multi Racial/Eth.	76.84% Hispanic/Latino 5.78% White 2.70% African-American 8.26% Filipino 1.37% Asian .2% American Indian .3% Pacific Islander 4.54% 2 or More Races
English Learners	26.5%	22.3%
Eligibility for Free or Reduced Meals	59.4%	61.3%
Special Education Students	14,787	5,143
Students of Military Families	9,156	
Students designated GATE	> 27,230	
Foster youth enrolled	1,344	206
Annual Operating Budget	$1 billion	Base Funding: $359.3 million Supplemental Concentration Funding: $39.4 million

Figure 3.1. Demographics of District A and District B (retrieved from https://data1.cde.ca.gov/dataquest/).

During the academic year of this study, School A had an enrollment of about 1,100 students in grades nine through twelve. Thirty-five percent of the students at School A were classified as ELs and 41% were classified as RFEP (reclassified fluent English proficient). Almost 90% of the student population were reported to be socioeconomically disadvantaged.

Languages spoken at School A include Spanish (41%), Somali (14%), Vietnamese (8%), Burmese (4%), Arabic (3%), Other Non-English Languages (25%), and several others that were each spoken by fewer than 1% of the student population. Spanish is the only language that meets the state threshold of "15% and above" (the percentage is calculated by the number of students classified as ELs divided by the total number of ELs and FEP in a specific language group) that requires schools to provide document translations for parents.

School B had an enrollment of about 2,600 students in grades nine through twelve, with about 25% of their students classified as ELs. Another 51% of students were classified as RFEP. Though the proportions of ELs are different, both schools had roughly 76% bilingual students. However, at School B, there were only two "main" language groups, Spanish (89%) and Filipino (9%), and a handful of other languages spoken by fewer than five students each. Like at School A, documents for parents are only required to be translated into Spanish. About 85% of the students at School B come from socioeconomically disadvantaged homes.

Participants

Participants were identified through criterion sampling (Patton, 2002). The two schools, one CME school and one CPM school, were selected for having a high percentage of students classified as ELs (approximately 25%) and an even higher proportion of former ELs in their student populations. Initially, my study design aimed to include only Integrated Mathematics 1 (IM1) teachers at each of the two schools in the hopes of observing teachers teaching similar mathematics content, exponential rates of change. During the recruitment phase of the study, it became clear that I would need to modify my participant requirements because (1) there were only three IM1 teachers at School A and (2) several IM1 teachers at School B opted not to volunteer for the study because they were too busy during my data collection window. After consulting my committee, it was agreed upon to expand my recruitment to IM2 and IM3 teachers, if possible. Ten of the eleven participating teachers taught an integrated mathematics course, and the eleventh taught honors precalculus. In Table 3.1, I provide a summary of the participating teachers. All names are pseudonyms, and languages are listed in the order of acquisition (i.e., first language, or L1, is listed first). Six teachers were selected as case studies, indicated by the names in bold. An introduction to each of these case study teachers is presented in Chapter 5.

Table 3.1. Participants Educational, Professional, and Language Background

Summary of Participant Information

Teacher	School	Years of Teaching	Education	Fluent Languages
Ms. Ryan	A	2	BS Mathematics M Ed in Secondary Education	English
Mr. Herrera	A	1	BA Mathematics Secondary Education MA Education	Spanish, English
Mr. Leong	A	1	BA Mathematics Secondary Education MA Education	English
Mr. Martin	A	19	BA Biology and Psychology	English
Mr. Hepner	A	15	BA Liberal Studies BA Mathematics	English
Ms. Rainey	A	40	BA Spanish MA Educational Leadership	English, Spanish, French
Ms. Ochoa	B	18	BA Mathematics MA Curriculum	Spanish, English
Ms. Carter	B	1	BA Criminal Justice	English
Mr. Turner	B	12	BA Social Science MA Education	English
Ms. Montez	B	17	BA Mathematics MA Educational Technology	Spanish, English
Mr. Estrada	B	9	BA Mathematics MA Mathematics Education	Spanish, English

Each teacher was asked to participate in a one- to one-and-a-half-hour lesson planning interview (Grossman, 1990) which included gathering some background information about each teacher, including educational background and certifications held, teaching experience, and language(s) spoken by themselves and their students (see Appendix A for the lesson planning interview protocol). The interview was followed by a classroom observation and a debriefing interview (see Appendix B for the debriefing interview protocol). Each teacher received a gift card in appreciation for the time they spent engaging in these activities.

Data Collection

The data collection timeline for this study was envisioned in three phases: a curriculum analysis phase, lesson planning interviews, and classroom observations with debriefing interviews to follow. Phase 1 laid the groundwork for this study by using a text analysis to compare the two written secondary mathematics curricula adopted by the school districts. Focusing on the introduction of exponential rates of change in each IM1 textbook, I noted the supports provided for promoting student engagement in mathematical practices and additional suggestions for EB students in the teacher edition or supplemental materials. A checklist matrix (Miles, Huberman, & Saldaña, 2014) for each section in this unit included explicit features of the teacher resources for engaging students in mathematical practices, explicitly stated and implicit supports for EBs (e.g., images, diagrams, and multiple representations) in the student textbook and in the teacher resources. See Figure 3.2 for a sample of the type of entries in these matrices. I wrote analytic memos (Maxwell, 2013) for each section detailing how the concepts of exponential rates of change were introduced in each curriculum, what explicit and implicit supports for engaging EB students in mathematical practices were present, and how each text introduced the mathematical content.

During phase 2, I conducted interviews with teachers which included a lesson planning interview (Grossman, 1990) to get a sense of how the teachers interact with written curriculum materials, to note the extent to which they plan for inclusion of mathematical practices, and to

	CPM	CME	Notes
Section and Lesson Objective(s)	8.1.3 How does it grow? More Applications of Exponential Functions Students will use what they know about linear and exponential functions to investigate simple and compound interest.	5.16 Compound Interest 1. Use exponential functions to calculate compound interest. 2. Calculate compound interest in a number of different schemes.	CPM teacher resources are all on the same webpage with links to additional materials. CME supports are in the margins of the teacher edition, at the beginning of the chapter, and at the beginning of each investigation (which is a grouping of about three sections on a topic)
CCSSM Standards	A-CED.1, A-CED.2, F-BF.1a, F-IF.6, F-IF.7e, F-LE.1a, F-LE.1c, F-LE.2, F-LE.5	N/A in Teacher Edition Supplemental Document: A.CED.1, F.LE.1, F.LE.1c, F.LE.2, F.LE.5	CME offers a separate document that includes correlations to the CCSSM. It is arranged by CCSSM Standard rather than by the section of the textbook.
Mathematical Practices/Habits and Skills	• make sense of problems and persevere in solving them • reason abstractly and quantitatively • construct viable arguments and critique the reasoning of others • model with mathematics • use appropriate tools strategically • look for and make use of structure	• use a Δ column to find whether a table represents a linear function • understand how and why you can use exponential functions in some specific applications • use tables to answer questions involving recursive rules	CME lists "habits of mind" and skills for Investigations, but they are not a consistent feature in each Section. CPM explicitly calls out the CCSS standards for mathematical practice, while CME uses their own descriptions of similar practices related to the content.
Supports for EB Students	Bank account interest will be new to students. Provide many visuals, such as tables and graphs, for reference during the lesson. Discuss the word compound: "com" together, "pound" place or position. Related words: community, impound, compete	N/A	CME does not explicitly give suggestions about how to support EB students. What implicit supports exist in the text (such as graphs, tables, images, vocabulary, etc.)?

Figure 3.2. Excerpt of the checklist matrix for the Compound Interest sections of CPM and CME.

document what supports they planned to provide for the EB students in their classrooms. The intent was to select lessons from the unit on exponential growth. Participant recruitment proved to be more difficult than I had imagined, and since the participants were no longer restricted to IM1 teachers, it was not possible only to observe lessons on exponential rates of change. Ultimately, I observed planning lessons about the topics of linear and exponential rates of change, polynomials, and circles.

Following the planning portion of the interview, I collected additional information from the teachers which included each teacher's educational background, language background, years and types of teaching experience, teacher preparation and professional development programs, and training in engaging students in the mathematical practices and teaching ELs in a non-sheltered classroom. (See Appendix A for the full text of the interview protocol.) Each interview was video recorded, and copies of all planning materials created by the teachers were preserved (i.e., handwritten plans or handouts were photographed, digital copies were emailed at the end of the interview) for analysis.

In Phase 3, I observed the teachers as they taught the lesson they planned in their interview. The observations were video recorded, and I took detailed field notes. My notes captured a summary of what took place during the lesson, including the tasks assigned, the activity structures, "interesting" moments (very loosely defined and subjective – instances that caught my attention in the moment), and anything that seemed noteworthy for answering my research questions. While in the classrooms, I also noted the presence of posters of the CCSSM SMPs, anchor charts or other visual references, and the general layout of the room. The camera and audio were primarily focused on the teacher and the presentation of the lesson. As soon after

Table 3.2. Classroom Observation Summary: Curriculum and Topic

Summary of Class Observation Data

Teacher	School	Course Observed	Primary Curriculum	Lesson Topic
Ms. Ryan	A	IM1/SIFE	TTA	Graphing Two Related Quantities
Mr. Herrera	A	IM2 Adv	MVP	Completing the Square
Mr. Leong	A	IM3	MVP	Graphing Polynomial Functions
Mr. Martin	A	IM1	Teacher-Created	Graphing Linear Inequalities
Mr. Hepner	A	IM1	Teacher-Created	Graphing Linear Systems of Inequalities
Ms. Rainey	A	Honors Precalculus	Teacher-Created	Geometric Sequences
Ms. Ochoa	B	IM1	CPM	Multiple Representations of Exponential Functions
Ms. Carter	B	IM1	CPM	Writing an Exponential Function Given Two Points
Mr. Turner	B	IM2	Teacher-Created	Arc Length and Sector Area
Ms. Montez	B	IM3	Teacher-Created	Radian Measure and Unit Circle
Mr. Estrada	B	IM1 Bilingual	Teacher-Created	Graphing Linear Systems of Equations and Inequalities

the classroom observations as possible, I recorded my general reflections on the observation and made notes of instances related to engagement in mathematical practices and supporting language access or production. I also noted occurrences that I wondered about or reminders of tasks to do before the next appointment. The classroom observations were followed by a debriefing interview with each teacher, which were also video recorded. (See Appendix B for the full text of the observation debrief protocol.) I provide a summary of the topics of the lessons I observed in Table 3.2. Note also the curriculum resources the teachers used to plan these lessons. The careful reader will note the absence of teachers who used the CME Project curriculum in School A. I discuss how this modified my curriculum analyses in the next section.

Data Analysis

My analysis of teachers' interaction with curriculum materials was qualitative, focusing on teachers' efforts to engage their students in the mathematical practices and the ways in which they support the receptive and productive functions of language for their EB students as they interact with their curricular materials. Data analysis was ongoing (Miles et al., 2014) throughout the data collection phase, starting with the curriculum analysis in Phase 1 and continuing as I started making informal observations about similarities or differences between teacher planning and enactment of lessons. The big picture overview for my data analysis process was to process all of the data in preparation for analysis, to analyze each component (i.e., the texts, the teacher interviews, and the observations) individually and create profiles of each, and to compare within and across curricular materials and teachers (Miles et al., 2014). (For the original design matrix (Maxwell, 2013) for this study that linked each research question to the supporting data, see Figure 3.3.)

Processing the data included transcribing all interviews, lesson planning and debriefing, for every teacher. For the classroom observation videos, I created video content logs of the classroom observations to supplement and enhance my field notes, documenting the types of activity structures (e.g., warm-up problem, group discussion, homework review, etc.) that occurred and when, along with a narrative summary to capture not only what happened during the lesson, but also to begin identifying potential episodes in which the students were engaged in mathematical practices and moments when the teachers' actions supported the emergent bilingual students' participation in the lesson. I elaborate on these analyses in the following sections in which I describe how each question was answered, then provide more specific detail about the analyses in the results chapters that follow.

Research Questions – What do I need to know?	**Sampling decisions – Where will I find this data?**	**Data collection methods – What kind of data will answer these questions?**	**Whom do I contact for access?**	**Data analysis**
In what ways do the locally adopted, CCSSM-aligned, curricula support secondary mathematics teachers to engage their students in mathematical practices? What supports are provided explicitly to help teachers engage their emergent bilingual students in these practices? How do the curriculum materials position emergent bilingual students as learners of mathematics?	CME and CPM student and teacher textbooks and supplemental resources	Textual analysis	N/A	Checklist matrices Analytic memos
How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?	Math teachers	Lesson Planning Interviews Class Observations Observation Debrief	School Principals Math Department Leads	Video of interview Transcription Coding Observation field notes

Figure 3.3. The design matrix created for this study.

Research Question 1

Phase 1 analysis began with the curriculum analysis described above and provided initial guidance on how to answer my first research question: *In what ways do the locally adopted, CCSSM-aligned, curricula support secondary mathematics teachers to engage their students in mathematical practices? (a) What supports are provided specifically to help teachers engage their emergent bilingual students in these practices? (b) How do the curriculum materials position emergent bilingual students as learners of mathematics?* Initially, the primary analysis involved a systematic reading of the introduction of exponential rates of change in both sets of curriculum materials (CME Project and CPM). A checklist matrix (Miles et al., 2014) detailing the explicit supports provided in the curriculum materials for each section was created, as well as an analytic memo (Maxwell, 2013) that described a comparison of both the *mathematical features* (including pedagogical implications) of the content and the *language access and production* opportunities and supports provided in the curriculum materials. (See Appendix C for an example of an analytic memo I wrote detailing the treatments of the concept of graphing exponential functions.) Remillard's (2005) framework provided a lens for this initial curriculum analysis, particularly by considering the components in the curriculum circle. These features of curriculum include the representations of tasks, the representations of concepts, the structure of the curriculum, and the look of the curriculum. Part (b) was to be answered through the Phase 1 data analysis of the curriculum materials, which produced checklist matrices for each section of the exponential growth unit that included the supports for EB students as well as analytic memos that addressed the language access and production strategies identified in the curriculum materials. This analysis was guided by Remillard's (2005) framework, noting how the curriculum structures supports for EB students, the voice of the authors as they write about EB students, and the types of tasks and representations of those tasks that are included in the text. To analyze the voice of the text, I drew upon the work of Herbel-Eisenmann (2007) and considered the use of imperatives (a direct instruction to the reader) and first- and second-person pronouns in the student-facing materials I analyzed. Imperatives can be inclusive or exclusive, which may signal to the reader that they are either an accepted member of the mathematical community or someone who is still seeking reception into that community. First-person personal pronoun usage has the potential to highlight or conceal the presence of the authors (as human contributors to mathematical knowledge) in the mathematical text, while second-person personal pronoun usage directly addresses the reader.

However, as noted in Table 3.2, none of the teachers in School A were using the CME Project curriculum as a resource for planning. I reconsidered my plan for curriculum analysis and concluded that if I were to have any points to conclude about the participatory relationship in the teacher-curriculum interaction, it was necessary to analyze any resource, published curriculum or teacher-created materials, the teachers were using in their enacted lessons. The teacher-created materials didn't really have any accompanying resources and typically only represented one lesson in a unit, so it became necessary to find a means for analyzing the materials in a way that would allow me to draw comparisons across different types of resources. Rather than analyzing a whole unit, I elected to analyze the lessons the teachers taught as well as a lesson on the introduction to linear functions in each of the four published curriculums that the participating teachers had used. The introduction to linear functions lesson was selected because seven of the eleven participating teachers taught lessons that included an aspect of linear growth, such as graphing linear equations (and inequalities) and comparing linear and exponential growth.

Each lesson was coded for the potential to engage students in mathematical practices, using the CCSSM SMPs as a coding scheme. I pulled phrases out of the elaborations of each SMP to create preliminary code descriptors. For example, I looked for evidence that students might be asked to “analyze givens, constraints, or relationships” as one possible indicator for SMP1 (see Figure 3.4 for an excerpt from my code book). Each lesson was scanned and entered into MaxQDA for coding. After coding some of the materials, I returned to a lesson I had coded and found that I would now code some of the tasks differently and realized that my coding

	• Interpret results in context & check feasibility
5. Use appropriate tools strategically Question: Does the task require use of tools?	• Consider available tools • Know affordances and constraints of tools • Explore with technology • Seek out & use external resources • Use technology to deepen conceptual understanding
6. Attend to precision* Question: Does the task require clear communication or attention to detail beyond getting a correct answer?	• Communicate precisely • Use clear definitions • State the meaning of symbols • Use the equal sign appropriately • Specify units • Label axes for clarifying quantities/correspondences • Give carefully formulated explanations • Examine claims & make explicit use of definitions
7. Look for and make use of structure Question: Does the task focus attention on identifying patterns?	• Find patterns or structures • Make use of the pattern or structure • Consider progress & shift perspective as needed • See complicated things as objects
8. Look for and express regularity in repeated reasoning Question: Do the students have the opportunity to do repeated calculations to help them notice general methods or discover a relationship?	• Notice repeated calculations • Look for general methods or shortcuts • Notice big picture and small details simultaneously • Evaluate reasonableness of intermediate results
*For coding SMP6, I intentionally left out the component “calculate accurately and efficiently” since practically every task in the coded documents ultimately has a single correct answer, thereby eliminating the need to code every task that has a correct answer as meeting one of the criteria for an opportunity to engage in SMP6.	

Figure 3.4. An excerpt from my SMP code book.

scheme needed further refining. I added focusing questions to each standard to help clarify when I might apply the code for that practice, such as, “Does the task ask for explanation, justification or conjectures?” for SMP3. After I felt more confident that I was consistently coding the SMPs, I enlisted the help of a second coder. Going through the process of establishing code reliability highlighted the need for me to further refine the codebook as we realized that we interpreted the

SMPs in different ways. After three rounds of discussing the meaning of the SMPs, how we see them in the student materials, and achieving consensus coding after each discussion, our fourth attempt yielded a high level of agreement (91%).

To investigate the presence of supports for EB students in the curriculum, I began with an a priori inductive coding scheme, drawing on the work of researchers such as Moschkovich (2002), Barwell (2003), Khisty and Chval (2002). I started with the list of supports that are beneficial for EB students compiled by Chval and collegues (2015) that was mentioned in Chapter 3. As I worked with and expanded upon this list in my curriculum analysis, I discovered the English Learners Success Forum's *Guidelines for Improving Math Materials for English Learners.* I found that the ELSF Guidelines included the supports I had been considering in my analysis as well as some additional items I had not yet considered, thus adding more structure and nuance to my coding scheme. These guidelines were also developed based on the same literature basis of my a priori scheme and in consultation with such scholars as Moschkovich and Chval. While the ELSF Guidelines were written as recommendations for improving written curriculum materials rather than as a curriculum coding scheme, I decided to try out these Guidelines as my coding scheme for my curriculum analysis.

The Specifications for each of the ELSF Guidelines (see Appendix D for a complete listing of the ELSF Areas of Focus, Guidelines, and Specifications) were used to code the student and teacher materials (when present). Each lesson that was taught by the teachers in this study was digitized and imported into MaxQDA and then coded for the presence of the Specifications. The unit of analysis for the student worksheets or textbooks was by each individual task. For example, if a section had 12 problems in which the student was given a table and asked to find an equation for the data in the table, this section would receive 12 instances for

ELSF 4a, one for each problem. In the teacher materials, the unit of analysis was by section. For example, if a section was devoted to supporting a classroom discussion around making connections between two different representations of a situation, the whole section was marked once for ELSF 4b, rather than multiple times for each suggested question to guide the discussion. I report on these analyses in Chapter 4.

Research Question 2

My second research question, *How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?*, was answered through my analyses of the planning interviews from data collection Phase 2 and the classroom observations and debriefs from Phase 3. The choice to answer this question through multiple data sources was purposeful. "Although interviewing is often an efficient and valid way of understanding someone's perspective, observation can enable you to draw inferences about this perspective that you couldn't obtain by relying on interview data" (Maxwell, 2013, p. 103). For example, by observing the enactment of the planned lesson, I gained a better understanding of the context (i.e., the students, the social norms and sociomathematical norms of the class) and witnessed how the teacher interacts with the students in the moment as the planned lesson unfolds. I had anticipated that the observations may also reveal further ways in which the teachers supported EB students than they may have mentioned in their lesson planning interview (i.e., tacit knowledge, Berliner, 2004), which turned out to be true.

I anticipated that the first part of research question two, *how teachers use curriculum materials while they plan lessons*, would be answered based upon the data from the lesson planning interview. The video recordings, the transcripts of the interviews, and the preserved artifacts of the teachers' plans would serve as the primary data. Through my own observation (in person and on the video) in conjunction with the teachers' self-report of their planning process, I would be able to identify *what* resources (e.g., their textbook or an activity found on the internet) the teachers had used to plan the requested lesson. However, I found that many of the experienced teachers came with prepared lessons they had already planned with other teachers or had taught in previous years. In these cases, it was necessary to rely upon how the teachers reported what they did to create these lessons. When addressing *how* the curriculum materials were used, I drew upon the curricular noticing framework (Males et al., 2015), which offered a lens for discerning what teachers do with their curriculum materials and for examining how they do it, which focused my analysis by *noticing* what the teachers attend to in the written curriculum, how they *interpret* what they have attended to, and how they *respond* to this interpretation through their choice of and sequencing of activities. (Again, some of this analysis was based on teachers' reporting of their interpretations of the curriculum.)

Recognizing that I was creating a somewhat artificial setting by asking the teachers to plan a lesson in my presence (while being video recorded) and that this may cause a change in the teachers' typical lesson planning routine, I included the following questions to be asked after the teachers describe their lesson plan: *Some teachers also use the internet, other books, or shared materials from another teacher. Did you use any of these while you were putting together this lesson? Do you use the internet or other books in your daily lesson plans?* While conducting my pilot interview, the importance of these questions was confirmed when the teacher expressed that she loves to use Pinterest to find creative ideas for teaching content through games or other interactive activities, but she had not consulted any resource other than the student textbook while creating her lesson during the interview. These questions proved to be even more important when the teachers came with a student packet in hand.

Moving beyond what resources the teachers use for planning, the next level of analysis was to identify the types of mathematical practices (NCTM, 2000; NGA & CCSSO, 2010; NRC, 2001) that were evident in the teachers' plans and in the enacted lesson. In the lesson planning interview, teachers were asked to plan as though they would be teaching the lesson on the following day, and to include what activities or problems they would have the students do and how, whether they would use small group or whole class discussions for the activities, etc. During the planning debrief, the teachers were encouraged to share as much detail as possible and were reminded to explain how they planned for students to participate in the lesson. Following this discussion, the teachers were asked about their familiarity with the eight CCSSM practices and in what ways, if any, these practice standards influence their planning and/or teaching.

The artifacts of the teachers' lesson plans, the interview transcripts, the classroom observation content logs, and the observation debrief transcripts were coded for evidence of mathematical practices. Both *deductive* and *inductive coding* (Miles et al., 2014) were used. I used the CCSS Standards for Mathematical Practice for coding, but, as Moschkovich (2015) asserted, the list of CCSS practice standards is not an exhaustive list of all potential mathematical practices. With this assertion and my pilot data in mind, I anticipated the creation of *descriptive codes* (summarizing a mathematical practice in a word or short phrase) when a mathematical practice is implied and *in vivo codes* (codes that use the teachers' own words) when a mathematical practice is explicitly stated during the initial round of inductive coding (Miles et al., 2014). For example, during the pilot interview a teacher referred to *talking* and *explaining* as mathematical practices, which she elaborated as having students talk to each other about mathematics in order to develop their ability to explain their reasoning. Both of these are evident in the practice standards; in fact, one may argue that nearly every practice standard includes talking (or written communication) and explaining. Coding such general statements with multiple codes (such as all eight mathematical practices) would not be useful, so it was important to capture the nuanced ways the teachers express their perceptions of how students might engage in the mathematical practices. A codebook was created that includes both a definition of and an example for each code. As the analysis progressed, the method of constant comparison (Strauss & Corbin, 1994) codes was employed to verify consistent use of the codebook as well as to identify codes which may need to be combined and those that may need to be subdivided.

Anticipating that the teachers may not explicitly identify all of the mathematical practices that they have planned for in the lesson, it was necessary to code the enacted lessons for evidence of participation in SMPs. For example, during the first pilot interview the teacher planned to create a Desmos activity for exploring several transformations of the absolute value function. This activity not only encompassed her mathematical practices of *talking* and *explaining*, but also included at least two of the CCSS mathematical practices: *use appropriate tools strategically* (using dynamic software to explore transformation) and *look for and make use of structure* (making conjectures about the transformations based on what changed and what remained invariant). These inferred mathematical practices were also coded, noting that they were implied, for comparison to mathematical practices enacted during the lesson and those identified by the teacher in the observation debrief.

To fully answer my second research question, I then attended to how the teachers modified the curriculum materials or added supports for EB students. I used the list of research ideas for supporting ELs compiled by Chval, Pinnow, and Thomas (2015) as a starting point for my analysis, again anticipating that there may be additional codes that emerge from the data introduced by the teachers. Chval et al. (2015) compiled the following list that summarizes some of the supports that can be provided by mathematics teachers of EB students that researchers have identified to be beneficial particularly for EB students:

- Connect mathematics with students' life experiences and existing knowledge (Barwell 2003; Secada and De La Cruz 1996).
- Create classroom environments that are rich in language and mathematics content (Anstrom 1997; Khisty and Chval 2002).
- Emphasize meaning and the multiple meanings of words – students may need to communicate meaning through using gestures, drawings, or their first language while they develop command of the target language and mathematics (Moll 1988, 1989; Morales et al. 2003; Moschkovich 2002).
- Use visual supports such as concrete objects, videos, illustrations, and gestures in classroom conversations (Moschkovich 2002; Raborn 1995).
- Connect language with mathematical representations (e.g., pictures, tables, graphs, and equations) (Khisty and Chval 2002).
- Write essential ideas, concepts, representations, and words on the board without erasing so that students can refer back to it throughout the lesson (Stigler et al. 1996).
- Discuss examples of students' mathematical writing and provide opportunities for students to revise their writing (Chval and Khisty 2009). (p. 107)

All of the data collected in Phases 2 and 3 (planning interview transcripts, artifacts of plans, classroom observation content logs, debriefing interview transcripts) were analyzed for supports for EB students. Like the mathematical practices analysis, both explicit and implied supports were included in this analysis.

Curious about whether the ELSF Guidelines could also be used as a coding scheme for identifying supports for EB students, I modified them to be about the teacher instead of the curriculum materials (see Appendix I). This provided a more nuanced analysis of the supports for EB students than the first coding, yet reflected similar profiles of teachers' enactments of supporting EB students, giving me confidence that this application of the ELSF Guidelines was useful. On the first pass coding of the classroom observation videos, the intercoder agreement was high (88%). These analyses are discussed further in Chapter 5.

The final phase of analysis for my second research question involved comparing my findings within and across teachers and curriculum. Beginning with the within-case analyses (Miles et al., 2014), I explored similarities and differences among each teacher's planned lesson and the enactment of his or her planned lesson, focusing on how EB students were engaged in mathematical practices and searching for evidence that these decisions could be traced back to the curricular resources. The cross-case analysis (Miles et al., 2014) involved several components. I considered whether there were discernable differences in the mathematical practices in which students were encouraged to participate and if these could be reliably traced back to the curricular materials, the teachers' plans, the teachers' preparation and experience, or some combination thereof. Next, I considered whether I had evidence that particular teachers appeared to provide more supports for EB students than others, then I investigated whether this can be partially attributed to the curriculum materials, teacher background, or other factors. Finally, though I had expected that the enacted curriculum would typically differ from the planned curriculum, I found that the teachers for the most part enacted the lessons they had planned, perhaps because some of them had already taught the lessons in the past.

Conclusion

In this study, I explored the ways in which the teacher-curriculum interaction supports the engagement of emergent bilingual students in mathematical practices. My data collection and analysis included a text analysis of curriculum materials and teacher-created materials, lesson planning interviews, classroom observations, and a final debriefing interview. In Chapter 4 I address the written curriculum materials and answer RQ1. In Chapter 5 I focus on my interviews and observations of the participation teachers and answer RQ2. This data was collected and analyzed in order to answer the following research questions:

1. In what ways do the locally adopted, CCSSM aligned, curricula support secondary mathematics teachers to engage their students in mathematical practices?
    a. What supports are provided explicitly to help teachers engage their emergent bilingual students in these practices?
    b. How do the curriculum materials position emergent bilingual students as learners of mathematics?
2. How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?

I expand upon and report the results of these analyses in the following chapters.

Chapter 4: Curriculum Analysis

Recall that Banilower and colleagues (2013) found that more than 80% of teachers use commercially published textbooks to guide mathematics instruction. With the widespread adoption of the Common Core content and practice standards, there was a need for curriculum resources that supported the vision of mathematics teaching suggested in the new standards. In some states, these new standards were more rigorous than their previously adopted standards (Rentner & Kober, 2014), sending teachers, schools, and districts in search of new curriculum materials. According to the Center on Education Policy report, two-thirds of the districts surveyed were relying on teacher- or district-created materials to meet the new math standards (Rentner & Kober, 2014).

For my first research question, I investigated curriculum materials, including those adopted by the districts and those in use by the participating teachers, to consider how the curriculum materials may support teachers to engage students, particularly emergent bilingual students, in the mathematical practices. As a reminder, my research question is:

1. In what ways do the locally adopted, CCSSM aligned, curricula support secondary mathematics teachers to engage their emergent bilingual students in mathematical practices?
    a. What supports are provided explicitly to help teachers engage their emergent bilingual students in these practices?
    b. How do the curriculum materials position emergent bilingual students as learners of mathematics?

In this chapter I provide an overview of each curricula and its organization, an analysis of how each curriculum may support teachers in developing lessons that encourage student engagement in mathematical practices, and what additional ways the curriculum may help teachers anticipate and meet the needs of their emergent bilingual students. For the published materials, both the student textbooks and the supplemental teacher materials were analyzed, and the curriculum program websites and introductory materials were consulted to learn additional information about each curriculum. To provide a means for comparison among each curriculum, a lesson from the introduction to linear functions units was selected and will be discussed throughout this chapter (see Appendix E through Appendix H). The introduction to linear functions lesson was selected because seven of the eleven participating teachers taught lessons that included an aspect of linear growth, such as graphing linear equations (and inequalities) and comparing linear and exponential growth. Table 4.1 provides summary information for the curriculum materials used by the teachers in this study.

As shown in Table 4.1, District A had adopted the CME Project's integrated mathematics curriculum (Cuoco & Kerins, 2016). However, during the data collection phase of this study, I found that none of the teachers at School A were using the district adopted curriculum. Two of the 9th grade mathematics teachers had written their own curriculum, and the other 9th grade mathematics teacher was piloting the Transition to Algebra curriculum at the request of the district mathematics coach for School A. The two intern teachers at School A reported primarily using Mathematics Vision Project's (MVP) integrated mathematics curriculum (Hendrickson, Honey, Kuehl, Lemon, & Sutorius, 2016a), which was also supported by the coach. The Honors Precalculus teacher at School A created her own materials, sometimes drawing inspiration from such curriculum resources as MVP, Blitzer's *Precalculus* (Blitzer, 2010), or an older edition of the CPM *Core Connections* textbooks (Dietiker, Baldinger, & Kassarjian, 2014).

Table 4.1. Curriculum Materials

High-level Overview of Curriculum Materials Used by the Teachers in this Study

Curriculum (where used)	Type	Courses Available	Development Funding	Format
CME Project (adopted by District A, not used in School A)	Commercial	Integrated 1, 2, 3	NSF	Printed or Online Textbook
CPM (adopted by District B)	Non-profit	Integrated 1, 2, 3	California Postsecondary Education Commission (Eisenhower-funded block grant)	Printed or Online Textbook
MVP (used by teachers in School A)	Educator-driven, Non-profit (additional resources available for purchase)	Integrated 1, 2, 3	Utah State Office of Education	Online Modules (daily worksheets)
TTA (used by teachers in School A)	Commercial	Supplement to Algebra 1	NSF	Unit Worktexts
Teacher-created materials (used by teachers in School A and School B)	Teacher-created or other worksheets	Used in Integrated 1, 2, and Honors Precalculus	N/A	Worksheets or electronic slides

District B had adopted CPM's integrated mathematics curriculum and were in their third year of implementation at the time of data collection for this study. According to some of the teachers at School B, there had been a big push from the district that all teachers would be using the CPM curriculum, and the district and the publisher had provided training sessions during the first two years of implementing the new curriculum. Data collection for this study took place in the third year after the district adoption, and in this third year, there had been no district-provided CPM training nor time allotted for course-level planning among the teachers. For some teachers, this perceived lack of district guidance regarding continued training or collaborative planning time for the CPM curriculum was interpreted as the freedom to choose the curriculum that worked best for their students, whether that be the CPM materials or the teacher-created materials they had been using prior to the CPM adoption. As such, only two of the five participating teachers from School B were committed to using the CPM curriculum, while the other three reported using a combination of CPM materials and the units they had created in their course-level teams. On the days I observed these three teachers, all were using teacher-created materials.

The CME Project

The CME Project textbooks were authored by a team of mathematicians, teachers, cognitive scientists, education researchers, curriculum developers, educational technology specialists, and teacher educators led by Al Cuoco of the Center for Mathematics Education (CME) at the Education Development Center (EDC). The curriculum development was funded by the NSF. The overarching goal for the development team was to write a curriculum that helps students to develop a deep understanding of mathematics (Cuoco, 2008). Based upon prior research as well as feedback on their earlier textbooks, the team established the following core principles for their design work: (a) Habits of Mind, (b) Experience Before Formality, (c) High Expectations, (d) Textured Emphasis, (e) General-Purpose Tools, and (f) A Mathematical Community. The team gathered feedback during the textbook development process from a variety of sources, including field-testing the curriculum in a number of schools around the country, seeking teacher and student feedback, and demonstrating evidence of increased gains for students as measured by an independent evaluator.

Habits of Mind (HoM) (Cuoco, Goldenberg, & Mark, 1996) was the fundamental organizing principle of the CME Project. HoM refers to how students interpret and solve mathematical problems and see mathematics in the world around them. Cuoco et al. (1996) argue that some HoM, such as *generalizing from examples* and *finding and explaining patterns*, are common across mathematical subdisciplines (e.g., geometry, statistics, and algebra), while others are more specific to different subdisciplines. Geometric/Analytic HoM include *reasoning by continuity* and *looking at extreme cases* and Algebraic HoM include *seeking and specifying structural similarities* and *chunking (changing variables in order to hide complexity)*. By adopting a HoM framework, the authors sought to develop general mathematical skills and forms of reasoning that will be applicable to a wide range of future mathematical endeavors, rather than attempting to guess at the specific mathematical processes, procedures, or techniques that may be required of mathematics students at a later time (a risky proposition given the current pace of technological change). Throughout the CME textbooks, the authors explicitly reference HoM using margin notes and remind students to use HoM in the expository text.

The authors described *Experience Before Formality* as having students grapple with mathematical ideas and problems before presenting them with a formula, procedure, or definition. Because the authors expressed that most students have the ability to think mathematically, they have *High Expectations* for students and have created a mathematically rigorous curriculum to match such expectations. For example, their design was based upon a low-threshold, high ceiling approach, which they describe as starting with activities that are accessible to all students and ending with activities that will challenge advanced students with the goal of seeing how far the students go with the materials. One example of this approach is that the book starts with exploring numerical examples of loan payment situations and then eventually has students derive the generalized payment function for a loan using the sum of a geometric series. When they refer to *Textured Emphasis*, the authors indicated that they were conscientious about separating vocabulary and convention from "matters of mathematical substance" (Cuoco, 2008). Additionally, they carefully crafted practice problems to have a larger mathematical purpose than simply for practice. For example, a problem from the IM1 unit on slope that at first glance appears to be about repeatedly finding the slope contains carefully chosen points. Looking across the parts of the exercise can be a basis for a whole class discussion about collinearity, horizontal and vertical lines, and parallel and perpendicular lines (see Figure 4.1).

1. Find the slope between each pair of points.

a. (2, 1) and (6, 8)
b. (6, 8) and (2, 1)
c. (3, 10) and (12, 2)
d. (3, 10) and $\left(\frac{15}{2}, 6\right)$
e. $\left(\frac{15}{2}, 6\right)$ and (12, 2)
f. (−4, 5) and (0, 0)
g. (5, 4) and (0, 0)
h. (−8, 10) and (0, 0)
i. (−4, 5) and (12, 5)
j. (5, 4) and (−20, 4)
k. (3, 3) and (25, 25)
l. (4, 5) and (4, −7)

Figure 4.1. Slope exercise (Cuoco & Kerins, 2016, p. 250).

Next, the authors emphasized the development of *General-Purpose Tools*, such as the distributive property of multiplication over addition, which support mathematical understanding more broadly than special-purpose techniques, such as FOIL, that only work in specific contexts. An example of this idea can be found in the Integrated 1 textbook when students are introduced to a set of core graphs and perform transformations on these graphs before learning specific names of or details about the graphs. Finally, the CME Project developers sought to build a

Mathematical Community through having a development team from diverse disciplines as well as advisors from a variety of fields.

The textbooks in the CME Project have eight *Chapters* that are each organized around a central mathematical theme. (Note that each of the three integrated texts also have an “Honors Appendix” that includes an additional chapter of advanced college preparatory materials.) Each *Chapter* is broken into major sections called *Investigations* designed to develop the theme of the *Chapter*, and each *Investigation* has several *Lessons*. Each *Investigation* begins with an exploratory lesson that previews the main ideas of the *Investigation*, triggers prior knowledge that is needed for the new topic, and has the potential to provide the teacher with information about what the students already know regarding the topic they are about to learn. The rest of the *Lessons* (see Appendix E for an example) in the *Investigation* include exposition, instruction, and exercises on the core topics of the *Investigation*.

Introduction to Linear Functions

Here we will look at one lesson from the CME Project IM1 textbook as an exemplar. Prior to this lesson in the CME Project Mathematics 1 textbook, students are introduced to the concept that an equation is a “point-tester,” which emphasizes that a graph is the set of all points that make an equation true. Thus, an equation can be used to determine whether a point falls on the graph and, alternatively, finding points that satisfy the equation and plotting them leads to a graph of the equation (and all other points that make the equation true). Additionally, slope is also introduced in the previous chapter. Slope is presented in three ways: slope as steepness (rise over run), slope as rate of change (average rate of change on distance-time graphs), and slope as a property of a line (collinear points have the same slope). The CME Project team then uses the slope formula and the concept of a point-tester to motivate writing the equation of a line without introducing the slope-intercept or point-slope forms of the line (see Appendix E). In the expository text, the authors assert that one only needs to know the slope of a line and one point on the line in order to write the equation of the line. Then two fictional students, Tony and Sasha, have a discussion about how to find the equation of a line if they know two points on a line. They first calculate the slope using the slope formula $m(A, B) = \frac{y_2 - y_1}{x_2 - x_1}$. (Note the function notation structure the CME Project authors have chosen to use for slope which may add additional layers of mathematical complexity for students.) Once the slope is calculated, the two fictional students write an expression for the slope between one of the given points and an arbitrary point $P(x, y)$. For example, if $A(5, -2)$ was given and we found the slope to be $\frac{3}{4}$, then we can write $m(A, P) = \frac{y-(-2)}{x-5} = \frac{3}{4}$. Tony and Sasha then check their work by plugging in their original points, running into an issue (division by zero) when they check the point used in the point-tester equation. To solve this issue of dividing by zero, Sasha multiplies both sides of the point-tester equation by the denominator. That is, $y - (-2) = \frac{3}{4}(x - 5)$ and now the equation can be used to verify both given points without producing an error. (The authors are careful about mathematical accuracy; see the margin note about multiplying or dividing both sides of an equation by an expression containing a variable.)

As mentioned in Chapter 2, written curriculum is one of the four components of Remillard's (2005) teacher-curriculum interaction framework and includes such details as voice, look, and representations of concepts and tasks. In terms of the look, the CME Project curriculum has a fairly “traditional” look and structure, in that each lesson has exposition of content, examples, and exercises. The teacher resources are presented in a “wrap-around” format, meaning commentary appears around an image of the student textbook. Cuoco (2008) described the CME Project textbooks as “a third alternative to the choice between traditional texts driven by basic skill development and more progressive texts that have unfamiliar organizations” (p. iv). The pages are glossy and colorful, and the fonts are altered with italics, bold, or color to provide emphasis.

Remillard (2005) included the voice of the curriculum in her framework, which considers how the authors of the text are represented and how they communicate with the readers (teachers and students). Drawing from the work of Remillard and Herbel-Eisenmann (2007), I considered the use of imperatives (a direct instruction to the reader) and first- and second-person pronouns in the lessons I analyzed. Imperatives can be *inclusive*, embracing the reader as part of the mathematical community, or *exclusive*, signifying that the reader is not yet part of the mathematical community but working towards acceptance. Of the 21 imperatives in the student tasks, seven were inclusive imperatives (e.g., prove, decide, explain) and 14 were exclusive imperatives (e.g., test, use, write, find, sketch). Rotman (as cited in Herbel-Eisenmann, 2007) posited that inclusive imperatives position the reader as a *thinker*, while exclusive imperatives construct the reader as a *scribbler* (one who performs actions). Those who do mathematics need to perform both roles, yet we see in this lesson that the reader is more often assigned the role of the scribbler. This repeated assignation of the scribbler role to the reader (student) emphasizes the authors’ (or textbook’s) position of a mathematical authority figure – the reader is “told what to do and how to do it” (Herbel-Eisenmann, 2007, p. 354).

Within the CME Project text, first-person personal pronouns (such as *I* and *we*) are absent in the expository text portions of the lesson (see Appendix E, p. 257, first four sentences), camouflaging the presence of human contributors to the mathematical knowledge conveyed to the reader and setting up a more formal, distant, relationship between the authors and the readers.

In other portions of the expository text (see Appendix E, p.257, just after the definition of slope of a line) and some of the tasks, the authors address the reader directly through the second-person pronoun *you*. In this lesson, there were 16 instances of *you* used throughout the lesson. According to Herbel-Eisenmann's (2007) analysis of a middle school curriculum, she identified four categories of you-forms. Only two were found in this CME Project lesson, *you* + *verb* (8) and *you* + *modal verb* (8). The purpose of these you-forms were to remind (or perhaps inform) the reader what they had previously learned (or should have learned) and to connect it to what the reader will learn in the current lesson, attempting to establish a common knowledge base upon which to build. The Minds in Action sections (see Appendix E, pp. 257 – 258) provide a contrast to the expository text as these are more conversational and insert the voices of fictional students who are doing mathematics, using pronouns such as *I* and *we* (possibly disrupting or further reinforcing the authority of the text). Perhaps these dialogues provide an opportunity for students to take on the role of vicarious learner (Lobato & Walker, 2019) as they read about Tony and Sasha's problem solving strategies.

Mathematical Practices

The CME Project team organized their textbooks around the development of mathematical Habits of Mind (Cuoco et al., 1996). They explicitly connect these HoM to the Standards for Mathematical Practice. (See Figure 4.2 for the authors' comparison.)
Each HoM is explicitly described as they arise in the student textbook (see Figure 4.3) and opportunities to use a given HoM is specifically called out in the student text as a reminder of strategies the students can use to get started on a new task (see Figure 4.4). The authors have interwoven the HoM throughout the textbook, sequencing tasks to help students develop each HoM over time.

Connecting the Habits of Mind to the Standards for Mathematical Practice

Mathematical Practice Standards	Mathematical Habits of Mind
#1 Make sense of problems and persevere in solving them	• Performing thought experiments • Expecting math to make senses
#2 Reason abstractly and quantitatively	• Finding and explaining patterns • Creating and using representations • Generalizing from examples • "Delayed evaluation" – Seeking form in calculations • Purposefully transforming and interpreting expressions • Seeking and specifying structural similarities
#3 Construct viable arguments and critique the reasoning of others	• Expecting math to make sense • Extending operations to preserve rules for calculating
#4 Model with mathematics	• Creating and using representations • "Delayed evaluation" – Seeking form in calculations
#5 Use appropriate tools strategically	• Seeking and specifying structural similarities • Purposefully transforming and interpreting expressions
#6 Attend to precision	• Expecting mathematics to make sense • Seeking and expressing regularity in repeated calculations
#7 Look for and make use of structure	• "Delayed evaluation" – Seeking form in calculations • "Chunking" (changing variables in order to hide complexity) • Reasoning about and picturing calculations and operations • Extending operations to preserve rules for calculating • Purposefully transforming and interpreting expressions • Seeking and specifying structural similarities
#8 Look for and express regularity in repeated reasoning	• Seeking and expressing regularity in repeated calculations • Generalizing from examples • Finding and explaining patterns • Purposefully transforming and interpreting expressions

Figure 4.2. CME Project's connections between Habits of Mind and Standards for Mathematical Practice (Cuoco, 2008).

However, while the authors of the CME Project have built a vision of school mathematics and HoM into their curriculum, teachers have their own interpretations of the written curriculum and it would be rare to find two identical enactments of the same lesson (Brown, 2009; Remillard, 2005). The textbook may be seen as a starting point for what content to cover, how content may be sequenced, or for potential activities or tasks to include in a lesson. Teachers go through several steps of attending, interpreting, and responding to curriculum (Males et al., 2015) before enacting a lesson with their students. The CME Project authors' chose not to specify pedagogical supports such as classroom organization, use of technology, or content that students may find challenging. Thus, whether the students have the opportunity to develop these HoM over time as intended by the curriculum authors will be largely dependent on the teachers' interactions with and the enactment of the curriculum in their classrooms.

Mathematical habits of mind are the most fundamental concepts and applications that you will take away from your mathematics courses. These habits are the bedrock for serious questioning, good problem solving, and critical analysis.

You will use each of the following behaviors beyond the world of mathematics. Sound habits of mind encourage and support your success in the world.

Be a pattern detective:

Build and see patterns.

Recognize a similar process.

Count without counting.

Look for relationships.

Be an experimenter:

Simplify the problem.

Find and repeat a process.

Try numerical cases.

Identify key characteristics.

Figure 4.3. Excerpt from student textbook about Habits of Mind. (Cuoco & Kerins, 2016, p. 4).

8. Describe how the position of a point changes with each transformation given.

a. Change the sign of its *x*-coordinate, but keep the sign of the *y*-coordinate the same.

b. Change the sign of its *y*-coordinate, but keep the sign of the *x*-coordinate the same.

c. Change the sign of both coordinates.

9. For each part, draw a set of coordinate axes. Shade the quadrants with points that match each description.

a. negative *x*-coordinates

b. positive *y*-coordinates

c. negative *x*-coordinates and positive *y*-coordinates

Habits of Mind

Experiment. Try it with points! Choose points from different quadrants.

Figure 4.4. Excerpt from student textbook encouraging students to use a Habit of Mind (Cuoco & Kerins, 2016, p. 193).

Supports for Emergent Bilingual Students

The CME Project does not have explicit guidance for addressing the needs of emergent bilingual students in either the student or teacher text. However, the textbook authors do include elements that can serve as *implicit* supports in the curriculum that have the potential to support the learning of emergent bilingual students. For example, the student textbook is printed in full color and contains many diagrams, visual representations, and images to accompany the text. Following Moschkovich's (2002, 2015) call to include all forms of communication such as visual representations and diagrams (not just spoken or written language), the visual supports provided in the text have the potential to assist emergent bilingual students in accessing context or engaging with mathematical content. However, not all images are directly connected to the mathematical content. For example, the clip art image in Appendix E of a woman holding a giant pen that is taller than her doesn’t connect to the idea that the slope between any two points on a line is the same no matter what points you choose. Other images are connected to the content on the page, but the subtlety of these connections may not be clear to the students. For example, in Figure 4.5 we see an individual who appears to be hiking and is about to cross a bridge that doesn’t appear safe to cross. Upon closer inspection, the gorge is shaped like a parabola and one can think of the bridge as a line intersecting that parabola, which matches the content of the lesson: solving systems of equations. However, one is left to ponder what this image may convey to emergent bilingual students about the mathematical content or conceptual understanding of the lesson.

The *Minds in Action* features throughout the textbook (see Appendix E, pp. 257 - 258), provide dialogues between fictional students Tony, Sasha, and Derman while they solve mathematical tasks. These fictional students grapple with the same unfamiliar mathematics that

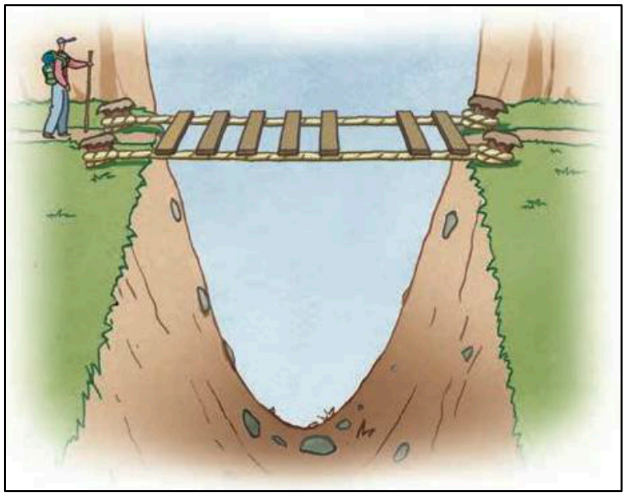


Figure 4.5. A clip art image in the solving systems of equations lesson. (Cuoco & Kerins, 2016, p. 326)

the students who read the text are encountering. The fictional students discuss their struggles, make mistakes, suggest strategies for solving the problems, and talk through their ideas. The authors of the CME Project intended for this modeling of students' productive struggle to help students learn about how to have mathematical discussions and further develop the HoM that are useful in approaching such problems (Cuoco, 2008).

Positioning of Emergent Bilingual Students

While the CME Project teacher resources do not explicitly address emergent bilingual students, there are some references in the *Implementing and Teaching Guide* (Cuoco, 2008) that may be inferred to be a potential reference to emergent bilingual students due to the research that has shown that ELs are underserved in mathematics classrooms and typically don't perform as well as their English-speaking peers (Mosqueda & Maldonado, 2013). While describing their curriculum, the authors discuss their findings that most high school students, even "low-performing students," can and do appreciate the subtlety and beauty of mathematics when they have the opportunity to use a combination of common sense, mathematical knowledge, and technical skills. In a later section entitled "Poorly Performing Students," it is emphasized that due to the CME Project's approach, they have found students with a history of poor performance in math demonstrate a great deal of cleverness and resourcefulness when solving problems. While neither of these statements specifically mention emergent bilingual students, it is not hard to imagine that a teacher looking for guidance on how to meet the needs of EB students may lump emergent bilingual students into this category of students who don't do as well as other students in mathematics. There is a contradiction here in that drawing attention to groups of students in this manner (e.g., *even low-performing students* and *poorly performing students*) reflects a deficit focus. Yet, simultaneously, the authors appear to be attempting to express a positive sentiment about the capabilities of such students.

CPM Educational Program

The CPM Educational Program (originally called College Preparatory Mathematics) is a non-profit organization in California that was started by a group of educators whose curriculum writing work was initially funded by a $600,000 Eisenhower-funded state block grant awarded through the California Postsecondary Education Commission in 1989. The CPM curriculum is aligned to the Common Core content and practice standards, rated by EdReports (EdReports, n.d.-a) as meeting *Alignment* expectations (scores of 15 out of 18 in *Focus & Coherence* and 16 out of 16 in *Rigor & Mathematical Practices*, both measures out of a possible 16) and meeting *Usability* expectations (score of 33 out of 36). Even before the introduction of the Common Core State Standards, CPM aligned with the recommendations of mathematics education researchers and the NCTM *Standards* (e.g., NCTM, 1989), taking a student-centered, problem-solving approach to mathematics curriculum. Student-student interactions are emphasized in the CPM curriculum in order to foster mathematical discussions around a core idea presented in problem-based lessons. Procedural fluency is not meant to be mastered in a single lesson, but instead developed over time with practice problems interwoven throughout the units (mixed, spaced practice) to give students multiple opportunities to practice and learn mathematics. CPM strongly recommends professional development implementation workshops to schools that adopt their curriculum. The 8-day workshops begin in the summer and carry into the school year and are led by current and retired CPM teachers. Additional supports by CPM mentors and coaches are also available if desired by schools and districts that adopt CPM.

The CPM curriculum was organized around a core set of assumptions. The following list appears in the Course Design section of the Program Description in the CPM teacher resources:

- Mathematics is a coherent set of ideas, not a collection of disjointed facts, and needs to be taught in a way that makes this coherence clear.
- A curriculum should allow all students to be successful, including those who struggle or those who excel.
- Teachers teach better when curriculum materials are flexible.
- Structured investigations and lessons are more successful when students understand what they are looking for.
- Students learn more when they solve problems and discuss their thinking with others.
- Teams work more effectively when the work actually requires a team and there is something to talk about.
- Closure is a vital part of the lesson.
- A student's learning is more meaningful and is better retained when the level of understanding necessary to explain and justify thinking is attained.
- A mathematical text should have usable reference elements.
- Rigorous and meaningful mathematical study can strengthen literacy.
- The structure of the lessons and layout of the textbook help students focus on mathematics and eliminate distractions. (Dietiker et al., 2014)

Each chapter in the CPM textbooks have a Guiding Question to motivate interest in the chapter topic and typically serve as a reminder to students to look for connections and meaning among the topics in each section of the chapter. Each lesson in the CPM textbooks consists of a set of Core Problems (see Appendix F, pp. 262 - 264) and a homework set called *Review & Preview* (see Appendix F, pp. 265 - 266). Lessons may also include additional features such as *Math Notes* (see Appendix F, p. 265), *Discussion Points* (see Appendix F, p. 262, questions to think about during the lesson), and *Learning Logs* (see Appendix F, p. 264, 2-39). Some optional extension problems are included in each lesson for students who need additional challenge, and occasionally problems are provided for additional support if students are struggling. Icons are used consistently throughout the text to flag specific types of activities. For example, a stoplight icon (see Appendix F, p. 264, 2-37, and p. 266, 2-42) is used to signal a problem that contains an error in reasoning or procedure that the students need to identify and explain why it is incorrect. A check mark icon is used to identify key homework problems, called *Checkpoints*, that students should have mastered by this point in the course.

Introduction to Linear Functions

The CPM Integrated 1 textbook develops the concept of linear functions through investigating growing tile patterns (linear), identifying the starting value (y-intercept) and growth (slope), and writing a general equation for the number of tiles at any step. Next, the students learn about slope as steepness, relying on a stair climbing analogy to assist students in comparing the relative steepness of lines. After these concepts are developed, the authors focus on using the slope and y-intercept to write the equation of a line (see Appendix F, p. 262, 2-35). The slope-intercept form of the equation of a line ($y = mx + b$) is given, along with new vocabulary (variables, parameters, coefficient, and constant term). Students are asked to reflect on their work with the tile patterns and connect each variable and parameter to what it represented in the equation for a tile pattern. For example, m is the growth of the tile pattern and b is the starting value. Next, students are asked to imagine that they work at a Line Factory processing orders for lines. They are to determine if the customer has provided enough information to create a specific line. If so, they are directed to send the "production department" the equation and graph, but if not, the student must send the "customer" a graph of at least two different lines that meet their request and ask for more information. Third, leaving the Line Factory context, students are given two points and are asked to consider how to find the slope of the line between them without graphing. Next, students consider the steepest line possible, its slope, and why they think that is so. Finally, students are instructed to reflect on what they know about the slopes and *y*-intercepts and how they would find each of these in different representations (i.e., situation, graph, table, and equation).

The look of the CPM student textbook is very different from that of the CME Project's text. The pages are black and white, not glossy (in the print edition), with clip art images. The clip art may signify a recurring feature in the text such as a journal and pencil image to indicate a *Learning Log* entry (see Appendix F, p. 264, 2-39) or an image that relates to the context of the problem such as an image of two workers in hard hats carrying a line out in front of the Line Factory (see Appendix F, p. 263, 2-36). The font is modified with bold (typically key vocabulary) and italics (typically for signifying mathematical symbols), and some headings such as Math Notes (see Appendix F, p. 265) have a different font. The electronic book version of the student textbook replaces bold vocabulary words with blue links to definitions and examples of the word and also includes links to Desmos resources for exploring mathematical tasks (see Figure 4.6).

My initial impression of the voice of the CME Project lesson and that of the CPM lesson was that the CPM authors appeared to talk directly to the reader more often. I was also left with the impression that the mathematics was presented in a less formal (and direct) way, inviting the student to develop their own understanding through carefully selected tasks. However, after performing a more systematic analysis of the grammatical features of the text to identify the voice of the text (Herbel-Eisenmann, 2007), it appeared that the authors' language choices may be communicating a different perception of the reader than I had initially interpreted. In the CPM lesson, I identified 25 imperatives, of which six (24%) were inclusive and 19 (76%) were exclusive. (Recall that in the CME Project lesson, 66% of the imperatives were exclusive.) Additionally, all of the inclusive imperatives in this CPM lesson appeared in the in-class portion of the text; none appeared in the homework exercises. This is an interesting juxtaposition (assuming the textbook materials are used as intended) in that during class the readers are positioned as thinkers and at home the readers are positioned as scribblers. In a future study, it would be interesting to conduct a similar analysis on a larger sample of the CPM and CME Project lessons to see if this distinction is common to other lessons and, if so, reflecting on what this may suggest about the CPM authors' epistemological stance regarding the roles of classwork and homework.

In the introduction to the CPM lesson, there are three instances of the first-person pronoun *we* in the three questions students are asked to consider throughout the lesson (see Appendix F, p. 262). The use of *we* in these questions feels inclusive, inviting the reader to think together with other members of the mathematical community (e.g., classmates, teacher, or the authors) about the answers. There were only eight instances of the second-person pronoun *you* in the lesson, five of the *you + verb* form and three of the *you + modal verb* form (Herbel-Eisenmann, 2007). Like the CME Project lesson, these phrases are used to remind students about things they already "know" or "did" and what they will be doing in this new lesson, serving the dual function of telling the reader something about themselves as well as establishing common knowledge to build upon for the current lesson. Reflecting upon my initial impression on why the CPM lesson may have seemed more conversational in tone than the CME Project lesson, there is very little expository text in the CPM lesson (just the introduction and the Math Notes, see Appendix F, p. 262) and tasks designed to guide the reader to develop understanding of the mathematics. Contrast that with the CME Project lesson (see Appendix E) which had a more formal mathematical structure, such as stating a theorem and leaving the proof to the reader.

Mathematical Practices

The CPM team has been incorporating mathematical practices recommended by NCTM and National Research Council (NRC) from its inception in 1989, long before the creation of the CCSS Standards for Mathematical Practice. Because of this long-held desire to engage students in problem-based lessons while interacting in groups to promote mathematical discourse, CPM asserts that the SMPs are deeply embedded in all aspects of their curriculum. The teacher guide explicitly states which SMPs are included in each lesson (see Appendix F, p. 267, Mathematical Practices), and the suggested lesson activities in the teacher notes offer guidance on how the lesson may be enacted (see Appendix F, pp. 267 - 268, Suggested Lesson Activity).

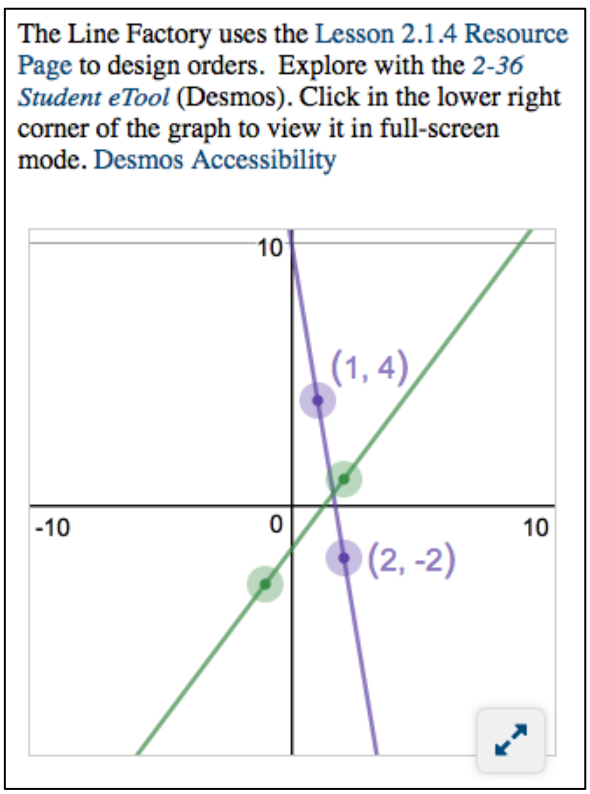


Figure 4.6. CPM eBook link to Desmos eTool for The Line Factory task.

Supports for Emergent Bilingual Students

Of the curricula reviewed, CPM offered the most guidance to teachers of emergent bilingual students. In the teacher notes for each lesson, CPM has included a section, called *Universal Access,* which offers tips and strategies related to the activities in that lesson (see Appendix F, p. 269, Universal Access). This may include calling upon resources like root words to understand vocabulary, identifying concepts that may present a challenge for language learners, and offering recommendations on what visual resources should be available and referred to throughout the lesson to anchor discussions. Additionally, CPM offers a Spanish edition of their student textbooks. The eBook version of the text also includes a link to Google Translate, but the authors caution that sometimes these translations can be incorrect or even comical.

Positioning of Emergent Bilingual Students

While CPM provided the most guidance to teachers about supporting emergent bilingual students in their classrooms, the introductory materials start by highlighting challenges EB students may face. The following is an excerpt from the Universal Access Guidebook of the teacher edition, under the ELL tab:

> English Language Learners have some unique problems in an English only classroom that is usually in a new country. Some may be *embarrassed by their accent or lack of English comprehension*, including some U.S. born students raised in a language island where English is not the dominant language. Others, depending on their immigration story, may be *suffering from trauma or PTSD*, or *living with extended family versus parents*. Many are *poorly educated* in their country of origin and *many are very poor* and/or *unfamiliar with available resources*. Often, *parents and guardians are immigrants also and are intimidated by the education system*. And, no matter what class, ELLs are always learning two things, the content and English. (Dietiker et al., 2014, emphasis added)

While any one (or more) of the above may be true of some emergent bilingual students, this framing may lead teachers to miss that EB students also bring many valuable resources and experiences to the classroom. Students classified as ELs are likely to be a diverse group and range from newcomers (recent immigrants) to long-term English learners who have been in US schools since early elementary school but haven't been reclassified as fully English proficient yet. Note the framing of ELL students in this excerpt – everything listed above focuses on what ELL students may lack when they arrive in a classroom, without noting or acknowledging the cultural and ethnic diversity and resources – especially additional languages - these students bring to a classroom.

Mathematics Vision Project

The Mathematics Vision Project (MVP) is an educator-driven initiative to build a secondary mathematics curriculum from the ground up to meet the Common Core content and practice standards. The MVP curriculum was also rated by EdReports (n.d.-b), meeting expectations for Alignment (scoring 15 out of 18 in Focus & Coherence and 15 out of 16 for Rigor & Mathematical Practices), but only partially meets expectations for Usability (23 out of 36). The MVP author team consists of five educators in Utah, who have held positions ranging from teachers and mathematics specialists to assistant district superintendent and associate teaching professor. The MVP student editions and teacher notes are published under a Creative Commons license and are freely available online. Ancillary materials, such as Enhanced Teacher Notes (teacher resource), Mathematical Practices Prompt Cards (classroom set), and Helps, Hints, & Explanations (parent and student resource), as well as professional development options are available for purchase on their website.

The MVP team relied upon the Comprehensive Mathematics Instructional (CMI) Framework (see Figure 4.7) as they developed their curriculum materials (Hendrickson, Hilton, & Bahr, 2008). The CMI Framework includes three major components: a *Teaching Cycle*, a

Learning Cycle, and a *Continuum of Mathematical Understanding*. The Learning Cycle – Develop, Solidify, Practice – is used as a model for building mathematical knowledge over time, not just in a single lesson but throughout a unit of study. In each module's table of contents, each classroom task is identified as either a Developing Understanding task, a Solidifying Understanding task, or a Practice Understanding task. For each classroom task, the authors envision the following basic structure. Students are first presented with a task and are invited to explore ways of solving it. As students come up with ideas, the teacher coordinates the students' discussions and guides them toward the mathematical goal of the lesson. As the students' ideas are developed amongst the class, they evolve into problem-solving strategies and mathematical habits and practices that have been developed by the students themselves as they work toward a collective body of mathematical knowledge together.

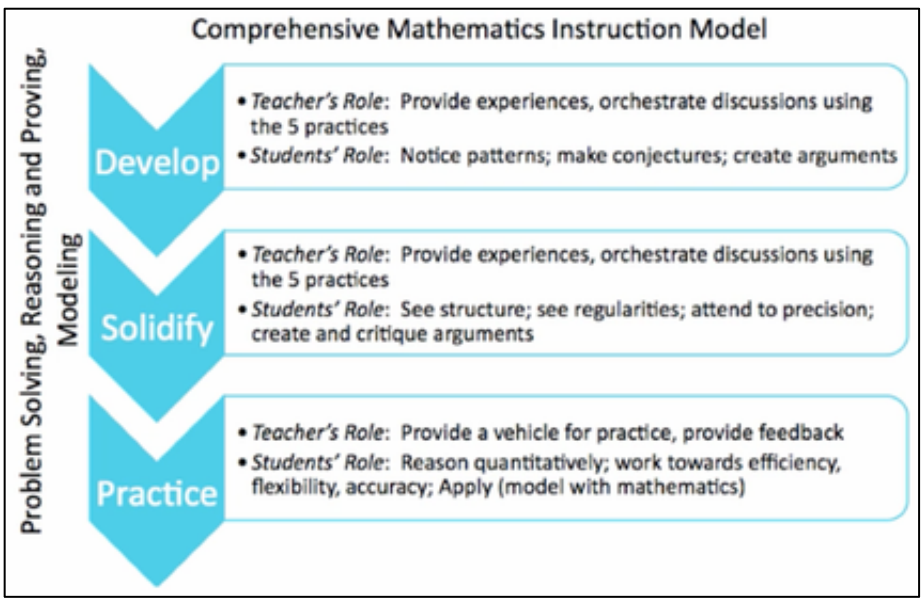


Figure 4.7. The CMI Framework.

The Learning Cycle is used in conjunction with The Teaching Cycle, which is composed of three parts: Launch, Explore, and Discuss. Note that the Teaching Cycle is not new to or a creation of the CMI Framework – it is similar to the Launch-Explore-Summarize (LES) instructional model implemented in the *Connected Mathematics Project* (CMP) curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). Furthermore, CMP also did not invent the LES instructional model – the earliest reference I found was Shroyer (1984). Teachers are encouraged to carefully plan and deliberately implement each step of the teaching cycle. In the Launch phase, the teacher should think about how they will motivate the students to engage with the task and describe to the students what they need to produce to finish the task. During planning for the Explore phase, teachers should think about what they might accept as evidence of student understanding and consider what questions they may need to ask to help students stay focused and move forward on the task. For the discussion phase, teachers need to have a plan for how they will select which students to present their solutions or strategies as well as what strategies or ideas they want to pursue. Other important considerations include how much the teacher should contribute to the discourse and how long to allow the students to struggle during sense-making. Figure 4.8 shows how the two instructional frameworks are related, with the Teaching Cycle occurring daily and the Learning Cycle extending over days or weeks throughout the unit.

Another feature of MVP curriculum is their "multi-tasking approach" to learning and implementing the content standards. They describe this idea as some tasks may focus on a single standard, while others may incorporate several standards. Many standards appear more than once throughout the curriculum. By doing so, the authors claim that a set of interrelated concepts, strategies, and skills can be merged into a coherent product. Finally, the MVP authors considered how to differentiate their curriculum to meet the needs of a variety of students. They accomplished this goal through creating tasks that have low-thresholds, high ceilings, and multiple entry points that tap into students' intuitive understandings and are situated in contexts that promote access for students and are often accompanied by visual representations.

The MVP materials are composed of two main components, the *classroom experience* and the *"Ready, Set, Go!"* homework assignment. The *classroom experience* (see Appendix G, pp. 272 - 273) was described above in which students are challenged to engage with a task and grapple with the mathematics through exploring ideas and discussing them with classmates, with the teacher facilitating the discussion and helping to draw out productive strategies that will lead

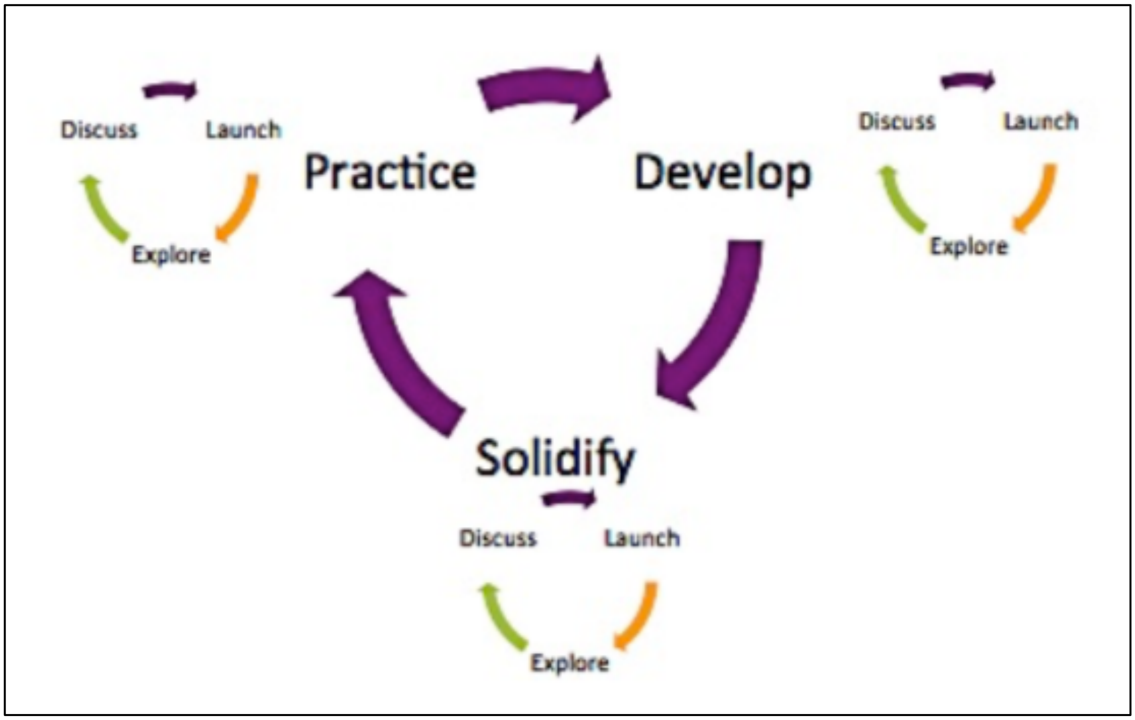


Figure 4.8. The Teaching Cycle and the Learning Cycle

to developing the mathematical goal of the lesson. While the purpose of the *classroom experience* is to develop students' reasoning and sense-making skills, the homework assignments are designed to develop procedural fluency and add structure to the skills learned during class. The homework assignments are broken up into three sections (see Appendix G, pp. 274 - 276). *Ready* and *Go* sections contain a spiral review of content and are designed for practicing procedural fluency. The *Set* section is designed to solidify the mathematical content of the aligned lesson.

Introduction to Linear Functions

In the Common Core State Standards, the introduction to linear functions is actually found in the eighth-grade standards (e.g., 8.EE.B.6 or 8.F.A.3). As such, the authors of the MVP curriculum have not included the development of linear functions in their high school curriculum. Rather, the MVP Secondary Math One curriculum begins with a module on linear and geometric sequences. The first lesson investigates a square floor tile pattern with a checkerboard border. Students are asked to find a way to quickly and efficiently calculate the number of colored tiles needed for the checkerboard border if they know the size of the square set of tiles inside the border. Students are asked to solve the problem first with a specific size, and then generalize their solution to an expression to calculate the number of colored tiles needed. The homework problems include finding a table of values for situations that are modeled by an arithmetic sequence. The second lesson (see Appendix G**,** p. 272 - 273) focuses on a dot pattern that grows by a constant additive rate and the students generalize to an equation that represents the number of dots in the pattern after *t* minutes. As in the checkerboard boarder problem, it is anticipated that students will visualize the growth in different ways and come up with different, yet equivalent, equations. The *Enhanced Teacher Notes* (see Appendix G, pp. 281 - 284, Discuss) offers guidance on how to have a whole class discussion about the different counting methods, representations, and equations the students may have used while solving this task.

The MVP materials are available online, so the way they are presented to students (online or hard copy) may change the appearance of the materials. Online, there is limited color use – the student text is typically black, the MVP logo is three-colored, and the *Ready, Set, Go* section titles in the homework are in color; occasionally color is used in diagrams, such as for the tile checkerboard border described above. Another notable exception is that each new lesson in the student text has an image or photograph at the beginning of the lesson that is in full color. However, when the *Enhanced Teacher Notes* are purchased, the pages are printed without color. It is most likely that students will experience the student text in black and white handouts provided by their teacher. The *classroom experience* materials (see Appendix G, pp. 272 - 273) typically have only tasks, while the homework pages (see Appendix G, pp. 274 - 276) often have expository material that one would expect to see in a more traditional textbook.

Turning to the analysis of voice in this MVP lesson, the *classroom experience* and the *Ready, Set, Go* materials contain two instances (13.3%) of inclusive imperatives (describe and show) and 13 (86.7%) exclusive imperatives. All but one of the exclusive imperatives are in the homework section, which is similar to the construction of the CPM lesson. Note that the *classroom experience* (see Appendix G, pp. 272 - 273) is essentially one task, a growth pattern, with four questions (describe the pattern, find the next number in the pattern, find the 100^{th} number in the pattern, and find the n^{th} number in the pattern). The teacher notes go into great detail (approximately six pages, see Appendix G, pp. 279 - 284) about how to facilitate a class discussion around this task. This level of detail is very different than the other curriculum included in this analysis. I share this here because this MVP lesson heavily relies upon the teacher to orchestrate a discussion of this problem that positions the students as members of the mathematical community. Yet a cursory readthrough of the teacher notes revealed the authors' use of exclusive imperatives throughout the suggested lesson discussion, such as ask, watch for, encourage, monitor, and tell. This concurrence of exclusive imperatives, commands, if you will, and the sole reliance on the teacher to successfully facilitate a prolonged whole class discussion about this task may be sending mixed messages. One in which the teacher is positioned as the mathematical authority, and, simultaneously, the language choices in the teacher notes may unintentionally be perceived as distancing the reader (teacher) from the mathematical community.

While this MVP lesson is very short, there were two instances in which the authors used the *you + verb* form. In both cases, the emphasis on using the second-person pronoun felt a bit different than in the CME Project and CPM lessons. In those lessons, students were often reminded of something they had done or noticed in a previous problem. Here in the MVP lesson, the use of the phrases "you see" and "how you arrived" could be interpreted similarly, where the authors were attempting to define what the user is doing or has done. Another possible interpretation is that the authors were emphasizing that the reader can and perhaps should find their own pattern or method of identifying the pattern and that it's permissible for their answer to be different from another student's pattern or method. In the homework exercises, there were no instances of first- or second- pronoun use, which removes the presence of human beings from the mathematics. However, there is a reference to mathematicians in the opening expository text (see Appendix G, p. 274), the purpose of which appears to be the motivation for why the reader needs to learn function notation – because mathematicians write it this way. Additionally, tasks 14 through 16 involve fictitious students in a class and how they thought about a visual pattern. These two instances reintroduce, if you will, human beings into the practice of doing mathematics. Beyond these two instances, the homework tasks are less personalized, use more formal mathematical language, and reflect traditional practice problems (in the sense Schoenfeld (1992) described such problems as routine exercises).

Mathematical Practices

The MVP curriculum was designed to meet the CCSS content and practice standards. Recall that the student curriculum and standard teacher notes are freely available online (under a Creative Commons license), while the *Enhanced Teacher Notes* and other additional resources (such as sample assessments, core standards tracking tools, and curriculum -aligned professional development) are available for purchase. In the free teacher materials, each lesson has a list of mathematical practices associated with the lesson. There is a description of how the authors envision this lesson unfolding in the class, broken up into three phases – launch, explore, discuss. While these descriptions may give some insight into how the authors saw the indicated Standards for Mathematical Practice enacted in the lesson, they do not provide an explicit connection back to the selected practice standards in these descriptions. However, the enhanced teacher notes do provide more guidance on how teachers can engage their students in the mathematical practices rather than simply listing the practice standards (see Appendix G, pp. 278 - 279, Standards for Mathematical Practice). For example, in a section in which students are investigating a checkerboard border pattern, SMP7, *Look for and make use of structure*, has the following description:

> The focus of the task is for students to use variables to demonstrate different ways of seeing a pattern and to understand that while different expressions may be equivalent, they tell a different story. As various strategies for seeing the pattern are shared in the class discussion, students will see how the structure of the expression written with variables models the structure used for efficiently counting the squares. (Hendrickson, Honey, Kuehl, Lemon, & Sutorius, 2016b)

Additionally, MVP offers the option to purchase a classroom set of *Mathematical Practices Prompt Cards*. There are 40 sets of 8 cards, one for each mathematical practice, which have sentence frames to help students talk about their work in the context of a specific practice. For example, on the card for SMP2, *Reason abstractly and quantitatively*, one option is “There is a relationship between _______ and ______ because ________.”

Supports for Emergent Bilingual Students

Like the supports for mathematical practices, the freely available MVP standard teacher notes do not include supports for emergent bilingual students. The *Enhanced Teacher Notes* (available for purchase – approximately $200 per course) include additional guidance about instructional supports and adaptations, which may include ideas for scaffolding, making contexts accessible, and promoting students’ production of academic language (see Appendix G, pp. 284 - 285). This additional information in the *Enhanced Teacher Notes* is where teachers can find ways to support their emergent bilingual students, but it should be noted that while there is additional information in the *Enhanced Teacher Notes* for every lesson, not every lesson has additional supports explicitly for EB students. However, many of the instructional supports and adaptations suggested may also support emergent bilingual students as many are geared toward supporting academic language access and production. For example, one lesson includes activities to highlight how the use of the phrases “mathematical representation” and “model” may be unfamiliar to students or used differently in a mathematics class than they are in everyday conversation. Finally, the MVP student materials have been translated into Spanish, which has the potential to support the mathematical learning of some emergent bilingual students.

Positioning of Emergent Bilingual Students

It is difficult to say with any certainty how the authors of MVP position emergent bilingual students due to the infrequency with which they mention English learners in the teacher materials. Many of the instructional strategies and interventions that are presented in the enhanced teacher notes are potentially useful as suggestions for supporting EB students, but the authors do not specifically mention English learners very often. When they do call out English learners, it tends to be in service of reminding teachers that the context of or the vocabulary in the problem may need extra explanation for English learners or that sentence frames are a good tool for supporting students, especially English learners, in using academic language. However, these supports are only found in the *Enhanced Teacher Notes*.

Transition to Algebra

Funded by the NSF, the Transition to Algebra (TTA) curriculum is also a product of EDC and is intended to be a supplement to a first-year secondary algebra program (either prior to the course or alongside it). The TTA development team consists of five members whose specialties include mathematics education, curriculum development, and psychology. Notably, June Mark is the lead author of TTA and is a co-author of the Habits of Mind article referenced earlier when discussing the CME Project curriculum (Cuoco et al., 1996). The intent of the TTA curriculum is to provide students, especially those who may struggle with the content of a traditional algebra course, with experiences that not only develop the tools and strategies to be successful in algebra, but to help students gain confidence in their mathematical ability and see mathematics as a coherent and logical system.

Similar to the CME Project, TTA focuses on algebraic habits of mind (Cuoco et al., 1996). Perhaps not surprisingly, like the CME Project team, the TTA team likens habits of mind to the Common Core Standards for Mathematical Practice. The TTA authors focused on five of the standards and gave them names of their own: *puzzling and persevering*, *seeking and using structure*, *using tools strategically*, *describing repeated reasoning*, and *communicating with precision*. In order to successfully develop these habits of mind, the authors of TTA describe how to foster a mathematical classroom culture as well as offer assistance on how to plan professional development workshops that help teachers understand TTA's vision for implementing their materials.

The TTA authors describe the classroom setting as being lively and active spaces in which students work both individually and in small groups to solve problems and share their ideas to develop their mathematical thinking with their peers and their teacher (Mark, Goldenberg, Fries, Kang, & Cordner, 2014a). To help teachers support a mathematical culture in their classrooms, the authors shared the following key ideas that inspired the design of the TTA materials: (1) increase student efficacy, (2) counter a negative image of mathematics, (3) shift toward a problem-solving environment, (4) foster good mathematical discussion, (5) connect arithmetic and algebra, and (6) develop metacognition and executive function (Mark et al., 2014a). By the time students reach an algebra course, some may have come to believe that they aren't good at mathematics, so the authors chose to use a variety of problems and puzzles that offer both a challenge and an entry point to students with a range of prior mathematics skills. By using puzzles and problems that can be solved from a variety of logical approaches, the authors seek to debunk the too often held conception that mathematics isn't supposed to make sense. The use of puzzles not only motivates students, but also helps to shift the focus of a classroom from didactive teaching to a problem-solving environment where participants discusses how students are thinking about mathematics. Classroom discussions are supported by curricular features such as student dialogues, discussion prompts and guiding questions, and examples of potential student contributions or confusions.

TTA has twelve units, each printed in a separate consumable worktext. Each unit consists of between five and eight lessons and most units include two explorations. The lessons have a consistent structure to help facilitate students' independent learning (see Appendix H, pp. 290-

294). Each lesson in the student worktext has three sections: *Important Stuff*, *Stuff to Make You Think*, and *Tough Stuff*. Since the lessons are designed to promote student thinking and discussing mathematics, a variety of problem types and formats are presented within the familiar structure of each lesson. For example, regular features include *Thinking Out Loud* (dialogues among fictional students about their problem-solving similar to the CME Project's *Minds in Action* dialogs), *Discuss & Write What You Think* (prompts to reflect, discuss, and write about their thinking), and *Algebraic Habits of Mind* (boxes that highlight mathematical ways of thinking). The explorations are longer problems that require extended thought and experimentation. While these explorations do have an answer, the focus is on the problem-solving process the students undertake as they engage with the task. Students are challenged to explain their reasoning, make arguments to support how they know they have completed the task, and think about how their strategies could be used to solve related problems.

Introduction to Linear Functions

Since TTA is designed to be a supplemental support for an Algebra 1 course, the authors focus on developing algebraic habits of mind and general strategies rather than specific formulas or procedures. Perhaps as a result of this philosophy, the TTA materials do not explicitly present the slope-intercept form of the equation of the line, but students are given the experience of working with and graphing linear (and nonlinear) equations. Similar to the CME Project, this is accomplished through developing a point-testing approach to equations. Students are instructed to find points that satisfy the equation, plot them, and then use the graph to help find more solutions. The goal is to develop the concept that a graph is a set of all solutions to an equation.

As mentioned above, there is a consistent structure to each lesson in the TTA curriculum. Each of the three main sections is color-coded with a background color: *Important Stuff* is green, *Stuff to Make You Think* is white, and *Tough Stuff* is red (see Appendix H, pp. 290 - 293). The students worktexts are printed in color on regular (non-glossy) paper. Despite being developed around puzzles, games, and visual representations, the voice of the student text has a fairly traditional style of presenting tasks and not talking directly to the reader. In fact, there were 56 instances of imperatives in the TTA lesson, 55 of which were exclusive imperatives. Only once is the reader directly asked to explain something in the tasks, which only positioned the reader as a member of the mathematical community in this one instance. The remainder of the lesson, through the repeated practice of using equations as point-testers, positions the reader as someone who needs to perform these tasks in order to become a member of that mathematical community. It was also noteworthy that imperatives are used more than twice as often in the TTA lesson than in any other lesson included in this analysis. Recall that the TTA curriculum was intended to be a supplemental curriculum to support students who may have struggled or are expected to struggle in a first-year algebra course, so it may be reasonable to suspect that the authors do not yet see their intended readers as full-fledged members of the mathematical community (or this may unintentionally reinforce this idea).

In the Solutions and Point Testing lesson (see Appendix H, p. 290, p. 294), the authors included a *Where Am I?* game in which the first-person pronoun *I* (or *my*) referenced a point (or one of its coordinates) in the coordinate plane. (This occurred 32 times in the 11 *Where Am I?* tasks.) The authors used a first-person personal pronoun to reference an inanimate object. There were 10 instances of the use of the second-person pronoun you in this lesson. Ninety percent of these were assumptions about what the reader had done. The final instance of the word you appeared in the phrase "The clues below will let you…" (Mark, Goldenberg, Fries, Kang, & Cordner, 2014c, p. 30). In this you-form, there is an *inanimate object + animate verb + you* which is a clear example of removing human agency from the text (Herbel-Eisenmann, 2007). Perhaps ironically, this phrase occurs in the only task that included a context involving humans (a student production crew preparing for a play).

Table 4.2. Connecting Habits of Mind and Standards for Mathematical Practice

TTA Habits of Mind and the Corresponding CCSS Standards for Mathematical Practice

TTA Habit of Mind	CCSS Standards for Mathematical Practice
Puzzling and persevering	1) Make sense of problems and persevere in solving them
Seeking and using structure	7) Look for and make use of structure
Using tools strategically	5) Use appropriate tools strategically
Describing repeated reasoning	8) Look for and express regularity in repeated reasoning
Communicating with precision	6) Attend to precision

Mathematical Practices

The TTA authors selected five mathematical habits of mind to emphasize throughout this supplemental course. While they use different descriptors, one can see the similarities between these habits of mind and some of the CCSS Standards for Mathematical Practice. See Table 4.2 for a comparison. In the teacher guide, the module introduction as well as most individual lessons in the modules not only identifies the habits of mind associated with the content, but also provides a description of how that habit of mind connects to the content of the lesson. See Figure 4.9 for an example. The student worktexts explicitly point out instances when the fictitious students in the *Thinking Out Loud* dialogues use a habit of mind or when a student should use a habit of mind while solving problems. At the end of each unit, the teacher guide also describes the ways in which students have focused on HoM throughout the unit, giving specific examples of types of puzzles or tasks they have solved and how this helped the students to develop that HoM.

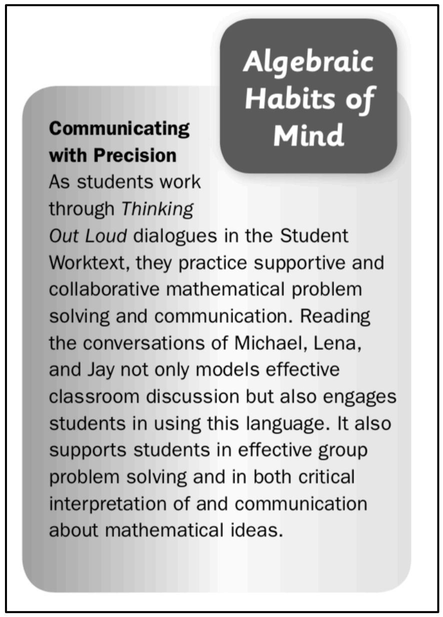


Figure 4.9. An Example of TTA's Teaching Guide Highlighting a Habit of Mind (Mark, Goldenberg, Fries, Kang, & Cordner, 2014b, p. T9)

Supports for Emergent Bilingual Students

The authors of TTA make a few references to English learners in their introductory materials and in some of the teacher guides. In the introductory materials, teachers are encouraged to allow time for student problem solving in partners or small groups. The authors indicate that this collaboration time is especially important for English learners because they may be more comfortable sharing their mathematical ideas with their peers than they are with the teacher. In the teacher guides, the recommendations for English learners also reflect this sentiment as well as encouraging the teacher to look for ways to appreciate the contributions of English learners, no matter how small. While the TTA materials didn't reference Complex Instruction, this suggestion aligns with the idea of assigning competence to students as they interact with other students as well as disrupting inequitable student interactions (Cohen, 2002).

Additionally, the teacher materials emphasize the subtleties of how we use the English language in mathematics. For example, when we use the phrases "divide in two" or "divide in half" we mean the same thing, yet when we say "divide by two" or "divide by one half" these do not indicate the same action. Another example arises in the difference between saying "six less than *d*" and "six is less than *d*." At one point in the teacher guide, teachers are cautioned not to focus on the precise vocabulary for numbers, but to accept anything that is within reason since the mathematical goal of the lesson is not how to properly pronounce a number. Thus, if a student is trying to say, "one thousand two hundred," there is nothing wrong in this context for the student to say, "twelve hundred" or even "one-two-zero-zero." Finally, the authors chose to use consistent lesson formats, puzzles, visual representations, and hands-on activities to develop mathematical habits of mind, paired with modeling and encouraging mathematical discussions. While these design choices were not done solely in support of helping emergent bilingual students, the visual nature of the materials, coupled with the opportunities to produce language through discussing with peers or in writing, certainly support the learning of emergent bilingual students.

Positioning of Emergent Bilingual Students

As mentioned in the previous section, there are few references to emergent bilingual students in the TTA teacher materials. A search of the introductory materials for TTA and all 12 teacher guides produced a total of five references to students whose first language is not English, English learners, or ELLs. One of these references indicated that even students who are English-fluent may struggle with mathematical or academic English. Of the four remaining instances, three times English learners are positioned as being reluctant to talk to teachers and more willing to communicate with their peers. In addition, English learners are described as possibly being overwhelmed with the amount of verbal instruction that takes place in a more traditional mathematics classroom or perhaps feeling anxious about their level of understanding of the material.

The authors emphasize that it is important to provide all students with a combination of whole class discussion, individual think time, and pair or small group work, but indicate that these structures are especially important for students who are learning English to have the opportunity to communicate with other students. In the sections mentioned above in which the teacher materials point out the subtle ways in which we use the English language to mean different ideas in mathematics, these descriptions were presented as potential stumbling blocks for any student and were not explicitly mentioned as supports for emergent bilingual students. The consistent structure of the materials and the recurring puzzle types throughout the curriculum were also built in to help students feel comfortable with guiding their own learning as well as to help build self-efficacy, but, again, these were part of the overall philosophy for the TTA curriculum, not necessarily as an aid to emergent bilingual students. Due to the lack of description about supports for emergent bilingual students, it suggests that perhaps some of the ways in which emergent bilingual students' learning may be supported were a happy accident rather than an intentional decision on the part of the author team.

Teacher-Created Materials

As stated in the introduction, Banilower et al. (2013) reported that over 80% of teachers use commercially published textbooks to guide their mathematics instruction. With the adoption of Common Core Standards, a need to improve curriculum materials arose to meet the demands of the content and practice standards. In the spring of 2014, the Center on Educational Policy (CEP) at The George Washington University conducted a national survey of a representative sample of school districts that had adopted the Common Core. CEP found that while over 80%

of the districts reported teaching math aligned to the Common Core, only about a third of the districts had adopted CCSS-aligned curriculum materials (Rentner & Kober, 2014). This meant that the CCSS-aligned materials used in the schools were being developed either by the district or the teachers. Additionally, 90% of the districts reported that developing or identifying CCSS-aligned materials had been either a major (45%) or minor (45%) challenge. Finally, only one-third of the districts surveyed reported that their teachers were prepared to teach the Common Core (Rentner & Kober, 2014).

The relative lack of availability of Common Core-aligned materials may explain why six of the 11 participating teachers in this study regularly created their own materials rather than using the district-adopted curriculum. Two of these teachers had created an entire ninth grade mathematics course (though they did this before the adoption of the Common Core in California). The other four teachers created worksheets or unit packets that drew upon a variety of resources such as Milliken worksheets (Freeman, 2013), Kuta Software ("KUTA Software," n.d.), internet resources like Pinterest and Teachers Pay Teachers, and a combination of commercially published textbooks. Additionally, two other teachers who had reported using either TTA or MVP as their primary curriculum significantly modified or replaced the written curriculum on the day they were observed. One teacher modified a graph and scenario matching activity from TTA to make it more accessible to her SIFE (Students with Interrupted Formal Education) class, while the other was revisiting a topic from the MVP curriculum to use visual representations of algebra tiles in his second lesson on completing the square. In the following, I will report on the content of the lessons from seven of these teachers (the six who regularly use teacher-created materials and the one who revisited completing the square – I chose not to include the SIFE lesson since the structure of the lesson still resembled that found in TTA).

Four of the seven teachers provided worksheets that were strictly routine practice exercises (Schoenfeld, 1992). Each worksheet ranged from four to eight exercises on each topic that was addressed on the day of the classroom observation. Two of the teachers prepared electronic slides for their lessons – one teacher printed out the slides for the students and the other expected her students to take their own notes from the projected slides. The final lesson was a combination of routine practice exercises, a hands-on investigation of radian measurement, labeling a unit circle, and using the unit circle to answer more routine exercises. Commonalities across the printed worksheets/packets include providing blank space to work out each exercise, answer blanks, boxes, or coordinate grids (or the unit circle) for graphing (or labeling degree and radian measurements and coordinates). Four of these five worksheets had no pronouns and all sentences were instructions for the practice exercises, all written as exclusive imperatives. Not only is there a lack of human beings in these worksheets, the reader is also positioned as not yet accepted into the mathematical community through the use of exclusive imperatives (Herbel-Eisenmann, 2007). The remaining three materials do have a human element through the use of pronouns, yet the reader is still overwhelmingly positioned as someone who needs to do things to gain access to the mathematical community. In Table 4.3, I provide a summary of the use of imperatives and pronouns in the teacher-created worksheets or packets.

Mathematical Practices

There was a great deal of variety in the types of materials created by the teachers for use in the enacted lesson. Because the teacher-created student handouts are not accompanied by corresponding teacher resources like one has with a published curriculum, here I will only address the potential for engagement in mathematical practices based on what is written in the

Table 4.3. Analysis of Voice in the Teacher-created Materials.

Use of imperatives and pronouns in the teacher-created materials.

	Teacher						
	Mr. Estrada	Mr. Turner	Mr. Martin	Mr. Hepner	Ms. Montez	Mr. Herrera	Ms. Rainey
Inclusive Imperatives	0 of 3	0 of 13	0 of 5	0 of 6	1 of 17	1 of 13	1 of 8
Exclusive Imperatives	3 of 3	13 of 13	5 of 5	6 of 6	16 of 17	12 of 13	7 of 8
First-person Pronouns	0	0	0	0	1 of 5	8 of 15	2 of 13
Second-person Pronouns	0	0	0	0	4 of 5	7 of 15	11 of 13
you + verb	N/A	N/A	N/A	N/A	1 of 4	3 of 7	10 of 11
you + modal verb	N/A	N/A	N/A	N/A	3 of 4	4 of 7	1 of 11

student materials, acknowledging that how the teachers chose to present or assign the materials may affect how the students interacted with the mathematical tasks.

Three of the teachers were teaching lessons on graphing linear equations, linear inequalities, systems of linear equations, or systems of linear inequalities. In general, the student materials were in the same format - an equation or inequality (or a system thereof) was given, and the students were expected to produce a graph of the equation(s) or solution set. A generous interpretation is that these materials allowed students to engage in SMP6, attend to precision, but only in terms of producing accurate graphs. These tasks did not include the remainder of SMP6, which highlights opportunities to communicate precisely. Similarly, another teacher was reviewing arc length and sector area with his students. All of the problems solved that day also fell into SMP6, attend to precision, as students were expected to calculate efficiently and accurately, giving both exact and approximate solutions, and to specify units.

One teacher was introducing her students to radian measure. In the warm up, students were presented with two problems that could invoke SMP2, reason abstractly and quantitatively, because there was a need to decontextualize and contextualize a situation in order to solve the given problems. In the first part of the main lesson, students were asked to measure radian lengths on circles of varying size to identify how many radians are in a full circle. This activity supported engagement in SMP5, use appropriate tools strategically, and SMP8, look for and express regularity in repeated reasoning. The tool in this case was a strip of paper used to mark off the length of the radius. The repeated reasoning opportunity came in the form of repeating the process on a larger circle and comparing how many radians were in each full circle. The remainder of the lesson included making connections between radian and degree measures and filling out a unit circle with both measurements and the corresponding coordinate points. Here, students have the opportunity to engage in SMP6, attend to precision, in terms of calculating efficiently, attending to units, and labeling the unit circle.

The last of the teachers who created her own materials was developing the concept of geometric sequences. The warm up problem involved withdrawing money from a bank each day until you empty the account, and the students were asked to write this situation in summation/sigma notation. This offered students the opportunity to engage in SMP1, make sense of problems and persevere in solving them, and SMP4, model with mathematics. Finding the pattern or constant ratio of geometric sequences also allowed a teacher to invoke SMP7, look for and make use of structure. Later tasks included calculating the number of views a YouTube video has and finding the value of a home as it appreciates over time, both of which provided more implied opportunities to develop SMP1, SMP4, and SMP7. Additionally, the teacher included a formal textbook definition of a geometric sequence for the students to compare to her informal definition. This allowed students to engage in SMP6, attend to precision, in the sense of using clear definitions and stating the meaning of symbols. The tasks solved throughout the lesson also engaged students in calculating efficiently and accurately, another component of SMP6.

Supports for Emergent Bilingual Students

As stated above, the absence of teacher guides makes the analysis of the teacher-created materials themselves a bit ambiguous since the student worksheets don't explicitly call out strategies the teachers are using to support the emergent bilingual students in their classes. The most obvious support for EB students in the teacher-designed lessons was found in the lesson designed by the SIFE teacher who was inspired by a TTA graph and scenario matching task. She knew her students were capable of doing the activity, but she anticipated that the language and contexts presented in the original activity would be inaccessible or unfamiliar to most of her students. Without reducing the mathematical rigor of the task, she wrote new scenarios in contexts she knew her students had experienced, shortened the sentences in the scenarios, and avoided unnecessarily complicated vocabulary or linguistic structures.

An activity that used hands-on measurements to develop the construct of a radian was another example of when teacher-created materials had the potential to support the learning of emergent bilingual students. By physically measuring the number of radians around the circle, students had the opportunity to think about what they were measuring (arc lengths), how these measurements related to the radius of the circle, and how this held true for different size circles. This activity also opened up the possibility of communicating through gestures and actions, which tends to be a rare opportunity in secondary mathematics classrooms. Third, one teacher's use of algebra tiles to teach students how to complete the square had the potential to help students visualize this process and ground their learning in concrete manipulatives – a potential support for EB students.

Finally, a few other supports were present in the materials. For example, four teachers included sentence frames in their planned lesson. One teacher planned to provide a vocabulary list for student use. Another teacher defined words, such as *appreciate*, during the lesson, in the context of the problem as needed. In many classes, visual representations, such as tables, graphs, or diagrams, accompanied tasks to help students understand the context of a problem or what type of question they are to answer.

Positioning of Emergent Bilingual Students

Again, since there are no teacher materials or guides, one only has the written student materials to help answer this question. Unfortunately, the majority (more than two-thirds) of the activities planned in these teacher-created lessons involved procedurally-oriented tasks that were devoid of contexts. The students were simply directed to graph lines or calculate sector areas. Following the analysis within de Araujo (2017), the selection of decontextualized, procedurally-oriented tasks with low levels of cognitive demand and the absence of written explanations may signal that the teachers have implicit deficit beliefs about the students classified as ELs in their linguistically diverse classrooms, perhaps conflating language proficiency level with mathematical proficiency level.

Supports Present for Emergent Bilingual Students

When planning for data collection, it was assumed that the two high schools would be using their district-adopted curriculum and I set about analyzing the exponential rates of change units in CME Project Mathematics 1 and CPM Integrated 1 textbooks. Among other things, I compared the presentation of mathematical content and tasks, the presence of tasks that had the potential to engage students in mathematical practices, and the ways in which the curriculum provided supports for emergent bilingual students. As noted in the program overviews above, CPM regularly included explicit suggestions for accommodating EB students in the Universal Access section of the teacher notes, while CME Project did not explicitly provide supports for EBs. Yet it was clear that the authors of the CME Project text had provided supports for EBs implicitly – visual representations, diagrams, modeling mathematical discussions between fictional students – and it became necessary to create a way to code implicit supports.

I began with an a priori inductive coding scheme, drawing on the work of researchers such as Moschkovich (2002), Barwell (2003), Khisty and Chval (2002). I started with the list of supports that are beneficial for EB students compiled by Chval and collegues (2015) that was mentioned in Chapter 3. As I worked with and expanded upon this list in my curriculum analysis, I discovered the English Learners Success Forum's *Guidelines for Improving Math Materials for English Learners.* I found that the ELSF Guidelines included the supports I had been considering in my analysis as well as some additional items I had not yet considered, thus adding more structure and nuance to my coding scheme. These guidelines were also developed based on the same literature basis of my a priori scheme and in consultation with such scholars as Moschkovich and Chval. While the ELSF Guidelines were written as recommendations for improving written curriculum materials rather than as a curriculum coding scheme, I decided to try out these Guidelines as my coding scheme for my curriculum analysis. After deciding to test out the feasibility of the ELSF Guidelines as a coding scheme, I selected the four lessons on linear functions that were discussed in this chapter and coded these lessons for the presence of supports for EB students (where the operationalization of supports was guided by the ELSF Specifications) as well as the potential for engaging students in mathematical practices during these lessons. The teacher-created materials were also coded for supports for engaging EB students in mathematical practices.

In this section, I will limit my report on the results of the ELSF coding to the four lessons about linear functions that have been described in this chapter from the published curriculum resources. I am limiting this reporting to the coding done for these four lessons in order to provide a comparable unit size for comparison. Additionally, the suggested lesson activity length for each of the selected lessons was one class session and the lessons contained similar content (i.e., writing a linear equation). Clearly, coding one lesson per curriculum is not going to capture all of the ways in which the curriculum authors have supported emergent bilingual students, so this comparison is not meant to be comprehensive nor a statement of which curriculum is best for EB students. Because the teacher-created materials vary in form and content as well as by teacher, they are not included in this section. Additionally, some of the ELSF Specifications explicitly refer to units, teacher materials, or assessment items, so it was not possible to look for evidence of these in the single snapshots I had from one observation of each teacher using their own materials. For the curriculum resources that allow access to this information, I will include my observations about the potential to support EBs in these areas but will not include these in the code counts that follow so as to keep a comparable comparison unit size. (Recall from Chapter 3 that the ELSF Guidelines have five components called an Area of Focus. Each Area of Focus has three Guidelines, and each Guideline has between two and four Specifications. There are a total of 46 Specifications.)

For student materials, my primary unit of analysis was by task. That is, if there were five problems about completing a table and writing the equation for the function, each of these five problems received a code for "using multiple representations" (table and equation). Since some problems may also ask for an explanation or a justification, these were also coded as an "opportunity to use language while engaging in a mathematical practice." Thus, it was possible for a task to receive more than one code. For the exposition or narrative sections of the student textbook lessons that were not task based, the unit of analysis was by section, meaning that I looked for breaks in the format such as switching between providing information and asking the reader to complete a task. For example, in the CME Project Lesson 4.02 (see Appendix E, p. 257), everything prior to the *For Discussion* question was considered a codable unit, and everything in between the *For Discussion* and the *Minds in Action* feature was counted as a codable unit. Features such as *Minds in Action* were also coded as a unit. In coding the materials written for teacher guidance, I considered all information about a single task to be a unit. For sections that included narratives, all information related to a particular topic was counted as a unit, regardless of the length of the description. For example, in the CPM Lesson 2.1.4 teacher notes (see Appendix F, p. 269, Team Strategies), two strategies are presented as options for helping the teams when they are stuck, which relates to "providing multiple sensory modalities" for student interaction. This was coded once for the whole paragraph since only one strategy could potentially be used.

A Snapshot of the Four Lessons

In this section, I provide the results of coding for the 46 ELSF Specifications in the selected lessons described in this chapter. It is important to note that these results provide just a small glimpse into the potential for the curricula to support EB students and do not reflect the capacity of the curriculum as a whole to support the learning of EB students. These results are, at best, a minimum measure of the supports the curricula offers teachers with relation to engaging EB students in meaningful mathematics. Noting this, one can see a great deal of variety in the presence of the ELSF Specifications present in these four lessons selected for inclusion in this analysis.

Each lesson, both student materials and teacher guidance, was divided up into codable units as described above. Then the lessons were coded for the presence and frequency of the ELSF Specifications in each codable unit. A codable unit (e.g., a task or section of narrative in the teacher guide) could technically receive anywhere from zero to 46 codes, but in practice, a single codable unit had at most nine codes assigned. Having a high number of codes assigned to the same codable unit occurred infrequently, yet when it did happen, it was most often in the teacher notes for the lesson. The code frequencies for each curriculum were then converted into a ratio, counts divided by codable units, in order to provide a clean comparison among the curricula.

In the ELSF Guidelines, some Specifications explicitly designate student materials, teacher materials, or the unit as a whole. However, some Specifications simply refer to *materials* or *activities* rather than specifying whether these appear in the student or teacher materials. Since I was only coding a single lesson in each curriculum, I elected to err on the side of generosity when coding each Specification. For example, if a Specification explicitly called for a support for EB students to appear in the teacher materials, yet this support was more prominently featured in the student materials (or vice versa), I still coded this as an instance of meeting the Specification regardless of the placement of the support.

It is also worth noting that many of the ELSF Specifications called for an explicit focus on ELs. For curricula such as the CME Project that did not specifically address supports for EB students, the presence of supports for EB students will be disproportionately underspecified in this coding scheme. As mentioned previously, more implicit guidance such as use of visual representations, images, and student dialogues to model mathematical discussions were present in the curricula that were not be captured by coding with the ELSF Specifications.

Figure 4.10 displays the results of coding one linear functions lesson in each of the four curriculum materials. Recall that each of the 15 Guidelines had between two and four Specifications. The blue shading indicates the proportion of the Specifications that were found during the coding of the single lesson. For example, the lesson in the CPM curriculum had at least one instance of each of the Specifications for Guidelines 1 and 3 but only half of the Specifications for Guideline 6. (Note that the shading is all left justified in order to make visual comparisons easier to see. One should not interpret the CPM chart for Guideline 6 to necessarily mean that Specifications 6a and 6b were met, and 6c and 6d were not.) The CPM lesson met 50% of the ELSF Specifications at least once in the lesson, TTA met 28.3%, MVP 26.1%, and CME Project received 10.9% of the possible codes.

A quick glance at these results may lead one to believe that none of the textbooks analyzed were very supportive of EB students. However, this interpretation does not take into consideration the following issues. First, coding only one lesson in an entire textbook will not capture all the ways the authors have included supports for EB students throughout the curriculum. The selected mathematical content may have narrowed the types of supports the authors chose to provide for this particular lesson. Other lessons that include more applications of linear functions (i.e., more contexts to interpret or longer narratives in the tasks) may have more language support suggestions in the teacher guide. Second, several of the ELSF Specifications called for explicit attention to language, including language objectives, stating the purpose for student-student communication within the context of the lesson, and providing examples of mathematical content learning expressed by students of varying language

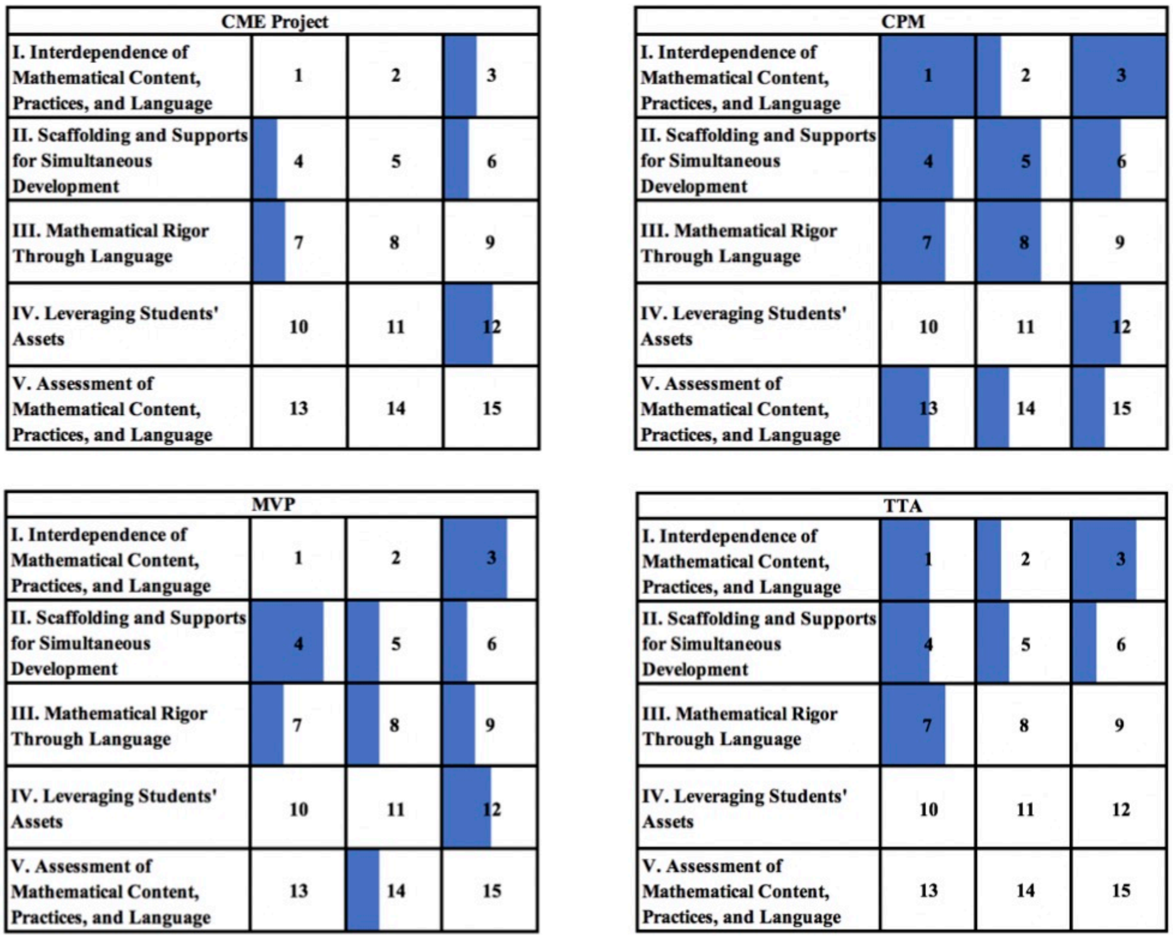


Figure 4.10. ELSF Coding for the introduction to linear functions lessons.

proficiency. Finally, the ELSF Specifications also explicitly called for supports for English Learners – pointing out specific times in the lesson to check in with ELs, what questions to ask ELs, what to listen for in their answers, and how to use lesson-specific ways to support ELs if they are struggling. While these last two suggestions (explicit attention to ELs and language in mathematics textbook teacher materials) are not necessarily new to those advocating for supporting EB students' participation in rigorous mathematics, they do seem to be recommending a new way of thinking about teacher resource materials that we haven't necessarily seen yet. For example, providing examples of student communication at varying levels of language proficiency that show evidence of engagement in mathematical practices that are connected to the current lesson content sounds very different than lesson supports that are primarily focused on mathematical content. However, it isn't hard to imagine that the authors of the CME Project or TTA could expand the cast of fictional students in the dialogues to include an EB student who may have difficulty expressing thoughts verbally, but is great at communicating through diagrams or other semiotic resources.

While many of the Specifications were not present in these four lessons, it was notable that so many of the ELSF Guidelines that reflected the recommendations of educators and researchers over the years were already included in the curriculum materials. Many of the ELSF Guidelines that are absent from the lessons analyzed in this chapter seem to point toward the mathematical community's continued efforts to reform school mathematics in which students are engaged in mathematical discussions around solving actual problems (e.g., not simply a set of practice exercises on the skill learned in class that day; Schoenfeld, 1992). Specific guidance for teachers about how to have these discussions – what to ask, what to listen for in student discussions, how to encourage and honor EB students' contributions, etc. – may have the potential to help teachers reimagine what teaching mathematics looks like through providing sample dialogues between the teacher and the students.

Focusing on ELSF's Five Areas of Focus

As described previously, the 15 ELSF Guidelines are grouped into five Areas of Focus (AOF). In Figure 4.10, one can see the extent to which each of the four lessons analyzed met the five AOF (or the minimum potential of the curriculum for supporting EB students). Two of the lessons coded for this analysis (MVP and CPM) met at least one Specification in each AOF, CME met at least one Specification in four AOF, and TTA met at least one Specification in three AOF. However, for curriculum such as the CME Project that does not explicitly provide supports for EB students in their teacher notes, this snapshot may be closer to the actual coverage than one may expect due to the ELSF Guidelines call for explicit EL supports.

Area of Focus I called for attention to simultaneous advancement of mathematics content, engagement in mathematical practices, and development of language functions (speaking, writing, listening, and reading). All of the lessons studied for this analysis provided opportunities to use language either through writing prompts or engaging in mathematical discussions in small groups or whole class. However, AOF I included specifying not only mathematical content objectives, but also conveying mathematical practice and language objectives to be met during the lesson. None of the lessons studied included specific practice and language objectives stated at the outset of the lesson. Some lessons (MVP and CPM) did convey which SMPs were included in the lesson, but these were not necessarily specified as a learning objective.

Each of the lessons analyzed had elements of Area of Focus II, *Scaffolding and Supports for Simultaneous Development* [of mathematical content, practices, and language]. Due to the nature of mathematics and the mathematical community's focus on multiple representations of functions (graphs, tables, equations, and situations), each lesson provided students with the opportunity to consider different representations of linear functions as well as to make comparisons and connections across the representations. AOF II Specifications that were not commonly found in these four lessons included how and when to provide individual and small group instruction to EB students, how to make sense of mathematical language and English language structures commonly used in mathematics, and guidance for the teacher to anticipate language opportunities and demands in the lessons.

Elements of Area of Focus III, *Mathematical Rigor Through Language*, were also identified in all four lessons. The most common occurrences of AOF III involved specific guidance for engaging students in mathematical practices, mention of common misconceptions that students may hold about the content in the lesson, and the use of visual aids or other resources for students to reference during the lesson. The lessons analyzed for this analysis did not include opportunities for students to revise mathematical writing (their own, a peer's, or a fictitious student's), guidance for teachers to support students' productive struggle before stepping in to help, or why students should have a mathematical discussion and how a teacher can facilitate this discussion and help students build on each other's ideas.

While three of the lessons received at least one code that fell into Area of Focus IV, *Leveraging Students' Assets*, all of the codes were evidence of Guideline 12, *Encouragement for ELs to use and build on existing language resources*. Two of the curricula, CPM and MVP, offered Spanish translations of their student editions. CPM's eBook also provided a direct link to Google Translate as a resource for students but included a caveat in the teacher materials that these translations are not always reliable. In the CPM teacher notes, it was suggested that students may wish to know why the letter *m* is used for slope, and the authors indicated that nobody really knows, but perhaps it would be useful to connect the use of the letter *m* for slope (or the growth) to the word *más* in Spanish for *more* or *add*. This was coded as an attempt by the authors to build on students' existing language resources, yet one does have to wonder how salient or helpful this suggestion may be. (What about a line with a negative slope? Would that imply that *m* may also be menos?) Other teacher notes suggest that the teachers should encourage their students to use and share their own methods for solving problems rather than pushing for all students to solve in the same way. Overwhelmingly missing from all four of these lessons were suggestions for making connections between the mathematics and the students' lives and providing opportunities for the students to draw upon prior knowledge, culture, and experiences. AOF IV also calls for curriculum resources to provide teachers with suggestions of how they can incorporate and value the contributions of EB students. This may be accomplished by providing teachers with sample student responses at varying levels of language proficiency that provide evidence of engagement in mathematical practices and how mathematical knowledge may be communicated informally.

Area of Focus V, *Assessment of Mathematical Content, Practices, and Language*, was perhaps the most challenging set of Specifications to identify within a single lesson. Given the single lesson snapshot of both the curriculum resources I am analyzing in this chapter as well as the single lesson classroom observations, it was necessary to think about assessment in terms of formative assessment – what opportunities were presented in the lesson that had the potential to inform the teacher about what the students may have understood at that moment. Even with specific guidance in the teacher notes regarding formative assessment, one can't be certain how the teacher will take up these suggestions and enact them during the lesson. Due to the nature of the teacher materials, the MVP and CPM lessons had more formative assessment opportunities due to the detailed descriptions of the suggested lesson activities. Additionally, MVP provided guidance on different ways of student thinking to draw out during the discussion, how these ways of thinking should be sequenced during the lesson, and how to help students make connections between representations that were presented. Recall that the CME Project authors chose not to include instructional or pedagogical decisions for the teacher in their notes, but rather to focus on the mathematical content of the lessons and the development of mathematical habits of mind through the provided activities, so there was little to draw upon for this AOF.

Guideline 13 calls for teacher materials to provide examples of quality work and engagement in mathematical practices at varying levels of language proficiency, including examples of teacher-student and student-student interactions. Of the resources that do provide sample interactions or discussion prompts, these examples are typically one-sided, such as what questions you should ask the students to help them when they are stuck or how to lead a discussion about the mathematical content with potential questions provided. Notably, these suggestions do not include ways to support EB students' engagement in these discussions.

Two of the lessons analyzed, CPM and MVP, included tasks that engaged students in mathematical practices through language, such as describing patterns using multiple representations or reflecting on the assumptions one needs to make in order to make a prediction with a proportional equation. However, Guideline 14 also includes an element of monitoring student progress over time, attending to a shift from informal, everyday language to more formal mathematical and academic language over time. With the lack of language objectives in the current curricula, this Guideline would be difficult to operationalize in practice. Finally, Guideline 15 encourages curriculum authors to help teachers recognize and attend to student language production and then make informed instructional decisions. The English Learner Success Forum calls for a variety of formative assessments that allow students to use their existing language resources and for summative assessment tools that describe and measure both language and mathematical success, errors, and misconceptions, along with appropriate scoring guidelines. At present, these elements are absent.

Discussion

As I planned data collection for this study, I had expected that the teachers in the two schools would use the district-adopted curriculum as a basis for planning their lessons. While I didn't presume any teacher would enact these curricula with fidelity, I had not considered the possibility that the district-adopted curriculum would not be in use at all or that there might not be a common textbook sequence used in the same school from Integrated Mathematics 1 through Integrated Mathematics 3. Though this discovery left me wondering about issues of content coverage, continuity, access to quality mathematics materials, and equity, I elected to reserve judgement on such issues and expand my analysis to each curriculum and teacher-created materials that were in use on the days I visited with the teachers. My initial impression of the published curriculum materials was that they were quite different from each other. Based on judging each book by its cover, I may have described the CME Project textbook as a tool for *direct instruction* (Roth McDuffie, Choppin, Drake, Davis, & Brown, 2018) with lots of practice exercises (Schoenfeld, 1992), CPM as a tool for *dialogic instruction* (Roth McDuffie et al., 2018), and MVP would fall somewhere in between them. Through my analyses of these textbook lessons, I found that my initial impressions may have been a bit hasty and there were more commonalities than I originally noticed.

All three of the curriculum series (CME Project, CPM, and MVP) either adopted by the district or selected by the participating teachers reported alignment to the Common Core Mathematics Standards and embraced a vision of mathematics teaching that is rich in discussion. For example, the importance of mathematical discussion can be found in (a) CME Project's *Minds to Action* segments and the authors' intentional selection of exercises that can lead to rich discussions (see Figure 4.1), (b) CPM's emphasis on team tasks and structured interaction strategies, and (c) MVP's detailed teacher notes on how to structure a whole class discussion around tasks and highlighting student thinking. Each curriculum also highlighted the importance of mathematical practices, either through signaling which SMPs students will be engaged in during a lesson or by developing mathematical Habits of Mind over time. Though TTA is a one-year supplemental text to a first-year algebra course rather than a core textbook that is part of a three-course curriculum, the authors of TTA also explicitly value mathematical discussions and developing HoM.

As described in this chapter, each curriculum had a very different look than the others. Three (CPM, MVP, and TTA) provided teacher resource materials separately from the student materials, while the CME Project selected the (perhaps) more familiar look of a wrap-around format in which both the teacher notes and the student text can be viewed simultaneously. This decision to separate teacher and student materials may have provided the authors of CPM, MVP, and TTA the freedom to include more detailed teacher notes than are provided in the CME Project textbooks. However, Cuoco (2008) reported both that the CME Project elected to use a familiar organization for their textbooks and also that they chose not to "prescribe classroom organization, flag mandatory uses of technology, [or] make judgments about what students will find difficult" (p. 93), yet they do offer suggestions about when it may be helpful to utilize group work and share difficulties their field-test students had with certain problems.

Similar to Herbel-Eisenmann's (2007) findings, my analyses on the use of imperatives and pronoun use highlighted that all of the curriculum materials and teacher-created materials adhered to "conventional mathematics and mathematics education discourses" (p. 361) through language choice. The lack of pronoun use in mathematics textbooks masks the presence of human beings (the authors) in mathematics. Each of the lessons studied in the published curriculum inserted human characters in at least one task. In one instance, human agency is handed over to the set of clues that would "help" the reader solve the problem. In another, the first-person pronoun *I* is used to refer to a coordinate point. The choice of language in mathematics texts may be leaving the reader confused about what role he or she fills in mathematics. Our (the mathematical community's) tendency to use exclusive imperatives (i.e., <u>Find</u> the slope of the line, <u>Multiply</u> by five, <u>Sketch</u> the graph) in mathematical tasks has the potential to further alienate the reader by issuing commands to perform an activity, positioning the reader as a scribbler rather than a thinker (Rotman, as cited in Herbel-Eisenmann, 2007). In the lessons analyzed in this chapter, two-thirds (CME Project) to 100% (teacher-created materials) of all imperative statements were in the exclusive form. Herbel-Eisenmann suggested that if we, as a mathematical community, seek to embrace equity, we may need to pay close attention to the voice used in curriculum materials. In addition to Herbel-Eisenmann's general commentary about terminology, I wondered to what extent the conventional voice of mathematics textbooks affects the EB students - are EB students more in tune to the nuances of language since they may be translating at the word level, thereby the lack of pronouns and prevalence of imperatives is highlighted? Finally, reflecting on the TTA lesson and the emphasis on learning mathematics through puzzles, visual representations, and problem-solving, I wonder how well this grammatical analysis applies to TTA and whether a different lens would be better suited to this style of learning.

The four published curriculums vary most widely in the level of support for EB students. Explicit supports for EB students were not present in the CME Project curriculum. There were occasional references to EB students in the TTA teacher guides. The CPM authors devoted a section in their teacher notes for every lesson to address the anticipated additional needs of EB students. The MVP authors have included instructional supports, accommodations, and language activities in the expanded (and only for-purchase) version of the teacher notes. When coding all of the lessons that were taught by the teachers during the classroom observations, overwhelmingly the most often coded ELSF Specification was the use of multiple representations (Focus Area 2, Guideline 4, Specification a). Yet reflecting on the nature of mathematics, we often use tables, graphs, equations, and situations simultaneously (Zahner et al., 2012). Thus, it is not surprising that code was most prevalent, and the use of multiple representations was likely not an intentionally included support explicitly for EB students. The more common types of explicit language supports found in the curriculum were related to breaking down words into their roots, comparing words to other words with the same roots, and providing visual references throughout a lesson. It's curious that so many of the explicit language supports tended to be at the word-level rather than at the level of register or Discourse as has been recommended by researchers such as Moschkovich (2002). Looking at the areas of growth identified in my use of the ELSF Guidelines, it is possible that the ELSF Guidelines will help push this agenda in future curriculum materials.

As a final reflection on curriculum materials, I return to the presence of deficit framing of EB students in the accommodations. It is important to note that deficit language about EB students is part of the dominant discourse that we hear not only in education, but also in everyday conversations. If we are not conscientious and intentional about our word choices or the way we frame discussion about EB students (or adults), it's very easy to perpetuate this discourse. Consider how many times you may have heard a teacher say something like, "Even my ELs got it!" as they excitedly shared about a recent success they had in their mathematics class. Returning to curriculum resources, de Araujo, Smith, and Dwiggins (2018) reviewed three algebra textbooks that had materials that were marketed as EL supports and found that these resources were offered separate from the main textbook, focused on vocabulary support, positioned ELs as being below grade level, and included a high proportion of low cognitive demand tasks. These same resources were also often tagged as appropriate for use with students who are struggling to learn mathematics. We know these deficit framings continue to persist and need to be countered. Yet, deficit discourses are like smog in the sense that they are pervasive, and we may unintentionally contribute to the problem with our everyday activity. This will make countering such discourses as daunting as eliminating smog. Yet, it is possible. Perhaps materials such as the ELSF Guidelines will also be useful in pushing the mathematical community forward as we reconsider instructional and curriculum supports for EB students.

Conclusion

In this chapter I have summarized my analysis of the published curriculum resources as well as the teacher-created materials in use at the two schools in this study. Generally speaking, the published curriculum resources attend to the development of and student engagement in mathematical practices, some explicitly referencing the eight CCSSM Standards for Mathematical Practice and others focusing on the related concept of mathematical Habits of Mind (which the curriculum authors linked to the SMPs). While the opportunities for engaging in mathematical practices are certainly represented in each of the curriculum resources, the degree to which a teacher may attend to, interpret, and respond to these opportunities to plan (Dietiker et al., 2014) and enact lessons that engage students in mathematical practices would seem to depend on how explicit the curriculum supports may be about the intended purpose for the given sequence of tasks (and whether the teacher consults these provided resources). For example, both the CPM and MVP teacher resources give detailed suggestions for lesson activities that include whole class discussions and small group work as well as the purpose(s) for engaging in the lesson (mathematical content and mathematical practices), while the CME Project authors chose to leave such instructional decisions up to the discretion of the teacher. These decisions by the authors may enhance or inhibit opportunities for EB students to engage in mathematical practices.

The teacher-created materials that were not adaptations of a lesson from one of the four published curricula tended to look more like collections of exercises, or routine practice for using a prescribed procedure. For many of these materials, at best there was potential for engagement in SMP6, attend to precision, at the level of calculating accurately and efficiently, attending to units, or labeling and scaling axes of graphs. Some of the teacher-created materials afforded opportunities to engage in other mathematical practices such as use appropriate tools strategically, model with mathematics, and look for and make use of structure during the in-class portion of the lesson, while the homework assignments often resembled the routine practice one would find in a traditional textbook. In Chapter 5, I will describe how the teachers used structured interactions in their lessons that made these routine practice exercise more interactive, requiring more student-student communication than one may have predicted based solely upon the look of the teacher-created worksheets.

In this chapter I have also explored the ways in which emergent bilingual students may be supported through the published curriculum materials. While some authors have elected not to include any explicit supports, modifications, or suggestions for teachers to help emergent bilingual students, there still exist implicit supports in the form of visual models, multiple representations, and the modeling of fictitious student-student dialogue about mathematics. Other curriculum developers have made a substantial effort to include suggestions for supporting emergent bilingual students – attending to potentially confusing language or contexts, providing Spanish translations of their textbooks, and suggesting the use of language supports such as sentence frames. Yet the analysis of each curriculum in light of the ELSF Guidelines suggests that more can be done to help teachers be successful in supporting their EB students. Curriculum materials can be enhanced by providing teachers with specific examples of what student understanding at varying levels of language proficiency may look or sound like. Learning objectives can be expanded from merely the mathematical content to include both mathematical practice and language goals. Students can be provided with a variety of assessment opportunities in varying modalities to provide more occasions and ways to communicate their mathematical proficiency in ways that make sense to them to avoid conflating their language proficiency with math ability – and teachers need examples of what these assessments may look like and how one may score them accordingly.

Chapter 5: Teacher-Curriculum Interaction

For my second research question, I investigated how teachers interact with curriculum resources to plan and enact lessons that engage students, particularly emergent bilingual students, in mathematical practices. As a reminder, the research question I address in this chapter is:

2. How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?

In what follows I will provide an overview and comparison of the curriculum usage in each school by the teachers in this study, discuss my rationale for the selection of the focal teachers for the case studies reported in this chapter, and present my methods for analyzing and synthesizing this data. Next, I will present six case studies, three from each school, focusing on how the teachers planned for and enacted lessons that engaged students in mathematical practices in their linguistically diverse classrooms. Finally, I will synthesize my findings across cases.

Data, Settings, and Analysis

As described in Chapter 3, the data for this study was collected in two California school districts with linguistically diverse student populations. Both districts had adopted Common Core-aligned secondary mathematics curriculum programs. (Refer to Figure 3.1 for demographic information about these districts.) One high school with a large proportion of students classified as ELs was selected from each district as a focal school, and each mathematics teacher in both schools were invited to participate in the study. I will refer to the school from District A as School A and District B as School B. Eleven teachers (six in School A, five in School B) volunteered to be a part of the study. Each teacher participated in a lesson planning interview (Grossman, 1990), a classroom observation, and a debriefing interview.

Comparison of Curriculum Usage

District A had adopted the CME Project's *Integrated Mathematics* curriculum (Cuoco & Kerins, 2016) four years prior to this study. However, despite the district-wide adoption, none of the six teachers studied at School A used this curriculum. In fact, when asked about curriculum, the teachers indicated that they did not have a set curriculum at their school. When specifically asking about the district-adopted CME Project curriculum, I often had to describe it (or, in some cases, point to the classroom set on the shelf) to the teachers. In place of the CMP Project, there were at least three different mathematics curriculum programs in use at the time of this study – *Transition to Algebra* (Mark, Goldenberg, Fries, Kang, & Cordner, 2014b), *Mathematics Vision Project* (Hendrickson et al., 2016b), and a teacher-created ninth grade mathematics curriculum – in addition to other teacher-created materials drawn from a variety of resources.

District B was in their third year of adopting the CPM *Core Connections Integrated* mathematics curriculum (Leslie Dietiker et al., 2014). In the first two years after adopting CPM, teachers were provided with training and the district emphasized their desire for all teachers to implement the CPM curriculum. By spring of year three, I found that three of the five participating teachers at School B were no longer using the CPM materials on a regular basis, and none of the teachers were using the CPM materials strictly as written. Instead, the three teachers reported using a combination of CPM materials and lessons they had developed in their PLCs prior to the CPM adoption. In response to a question about whether a new teacher to the school would get any training on the CPM curriculum, one teacher said,

> Well, to be honest, I don't think the CPM curriculum is strong anymore. Since we stopped having these trainings or professional development geared into CPM, at

> the very beginning was like a big push, everybody has to adopt this, like it or not, you have to do it, right? But then as time went by, then they kind of stop. And CPM is not an easy curriculum, neither for the students nor for the teachers, so I think everybody started to see what worked best again in the classroom and then we started mixing again what we thought it was good about CPM and what things we needed to modify a little bit that really, they weren't exactly, you know, correlating with the CPM. And at this point, I don't think that would be an issue. (Ms. Montez, lesson planning interview)

While their lessons did not always come from the CPM curriculum, evidence of teaching "the CPM way" (as described by one teacher) was visible in the classes. For example, working in teams of four, assigning group roles, using a variety of activity structures to encourage students to discuss mathematics with each other, and adopting CPM terminology (e.g., equal values method or multiple representations web).

During the lesson planning interviews, I found that all of the teachers consulted a student version of the textbook during their lesson planning. In fact, the only time I saw teachers consulting the teacher edition of the textbooks was when I joined the Integrated Mathematics 1 teachers at School B for their two-day unit planning meeting. The teachers had each planned a lesson from the unit, and the teacher edition was consulted to clarify the mathematical content goal of the lesson. (These planning days are described in more detail in case study five.) Additionally, during my classroom observations, I did not see a single student use (or access) a math textbook in any of the 14 classes I observed. (Three teachers invited me to observe their classes more than once.)

Since the students were not directly using textbooks, all of the teachers were creating some form of a worksheet or a unit packet to handout to the students. For those lessons that were created from the written curriculum, common alterations to the written curriculum materials included adding white space for students to show their work for each task, providing answer blanks or boxes, adding lines for students to write down a daily learning target for the lesson, breaking longer text passages into smaller chunks, providing coordinate grids for graphing equations, and adding a variety of activity structures to the lessons (e.g., doing appointment books or challenging teams to do a set of tasks without asking the teacher for help in order to earn bonus points). In addition to creating worksheets, six of the teachers (three from each school) utilized individual student whiteboards in their lessons for exercises in which students practiced procedural fluency.

Rationale for Case Study Selection

While designing this study, criterion sampling (Patton, 2002) was used to identify two schools, one in each district, in which to conduct my research. Each school was selected due to having a high percentage of students classified as ELs (approximately 30%) and an even higher proportion of former ELs in their student populations. All Integrated Mathematics 1 teachers at each school were invited to participate in the study. My goal was to interview six teachers at each school and to identify three teachers at each school to include as case studies. As data collection progressed, I discovered that School A only had three IM1 teachers and my sampling criteria was broadened to include the rest of the math teachers at School A. At School B, while there were plenty of IM1 teachers to invite, only three of the IM1 teachers agreed to participate. (The others indicated that they did not have time to participate.) Again, I extended the invitation to other teachers in the mathematics department, focusing on those who taught Integrated Mathematics 2 or 3. A combination of criterion sampling and opportunistic or emergent sampling (Patton, 2002) resulted in a total of six teachers from School A and five teachers from School B participating in this study.

As I analyzed the data and considered which 6 teachers to include as case studies, I identified conditions for which it would be most likely to find answers to my research questions.

Using an extreme or deviant case sampling approach (Patton, 2002), my first criteria for inclusion as a focal teacher was to choose the classes with the highest proportion of emergent bilingual students at each school. In School A, I observed Ms. Ryan's combination Integrated Math 1 and SIFE (Students with Interrupted Formal Education) class in which all students were classified as ELs, and many were newcomers. Ms. Ryan's students spoke many different languages, including Spanish, Somali, Swahili, Karen, Vietnamese, and Marshallese. In School B, I observed Mr. Estrada's bilingual Integrated Math 1 course in which all students were classified as ELs. In Mr. Estrada's class, all of the students spoke Spanish.

My second criteria for inclusion was to use intensity sampling (Patton, 2002) to choose teachers with a high level of student interaction in hopes of seeing high engagement in mathematical practices. During the course of data collection, one teacher at each school invited me to observe their classes more than once to get a broader sense of their teaching practice. In both cases, each of these teachers was working closely with another participating teacher. Remarkably, in both cases, there was a lot of similarity between the pairs of teachers, so when presenting these cases I will treat each pair as a single case study. In School B, I observed Ms. Ochoa, an Integrated Math 1 teacher (and her student teacher, Ms. Carter), who reported that she was committed to implementing the CPM curriculum and using teamwork. (In fact, Ms. Ochoa had been asked to join the IM1 team this school year to help the team implement the CPM curriculum.) In School A, I observed a pair of experienced Integrated Math 1 teachers, Mr. Martin and Mr. Hepner, who had developed their own curriculum. They both described incorporating mathematical practices into their curriculum development. These teachers were remarkably similar in their enactment of the curriculum, down to sayings they used to describe a process and writing the same notation at each step of problems.

Using extreme case sampling, my final criteria for selection was to select an experienced teacher at each school in hopes of capturing evidence of tacit knowledge, the things the teachers do to support engagement of EB students in mathematical practices without necessarily realizing that they are doing so. At School A, Ms. Rainey had 40 years of teaching experience. She described her teaching style as speaking less and less each year and finding ways to help students "bump into" mathematical ideas. At School B, Ms. Montez had 17 years of teaching experience and she reported that she doesn't really make modifications or accommodations for emergent bilingual students in her classes, yet there was evidence that she did do things to support EB students. (Ms. Ochoa at School B had been teaching for one year longer than Ms. Montez but had already selected as a case study.) Additionally, I observed Ms. Rainey's Honors Precalculus class and Ms. Montez's Integrated Mathematics 3 course, so they also provide contrasting cases of older students and higher-level coursework to the other four cases which were all 9^{th} grade IM1 courses.

Analysis and Synthesis

Each lesson planning interview and debriefing interview was transcribed for analysis. During each classroom observation, I recorded field notes and wrote detailed reflections as soon as possible after the observation. During the analysis phase, I watched each classroom observation video and wrote video content logs of the classroom interactions, which included narrative summaries of what happened during the lesson. In each video content log, the class session was broken up into episodes that reflected different phases of the lesson. In most cases, the boundaries of these episodes were identified first by changes in tasks, then by changes in activity structure. For example, one task may be broken up into three episodes: the launch or introduction of the task (whole class), students working individually or in teams on the task (small group), and the discussion of the task (whole class). While creating these video content logs, I noted instances of student engagement in mathematical practices and supports for emergent bilingual students. Portions of the class that related to supporting emergent bilingual students were transcribed, noting gestures, facial expressions, or visual aids that may have helped communicate the meaning of the interaction. Still images of the video were captured to aid in understanding the mathematical activities or language supports when a verbal description on its own may not be adequate to capture the interaction.

The case studies that follow were compiled using multiple data sources – the transcript of the lesson planning interview, my fieldnotes from the classroom observation, the video content log of the classroom observation videos, the transcript of the observation debrief interview, my reflections written after each classroom observation, and my ethnographic observations from the informal times spent with the teachers before or after the formal interviews or observations. These data sources were coded for the potential and actual engagement of students in mathematical practices and for the additional supports planned for and provided to EB students. Each case study represents the synthesis of this information for the six teachers selected.

Case Studies

In the following section, I will present six case studies, three from each school, which were identified by the criteria described above. The three case studies from School A will be presented first, followed by the three case studies from School B. For each case study, I will include an introduction to the teacher, a report of how the teacher planned for mathematical practices with EB students in mind, the enactment of the lesson, and the teacher's reflections about the lesson. The order of the case studies for each school will follow the order of inclusion as a case study described in the last section – most EB students, most observations, and most years of experience.

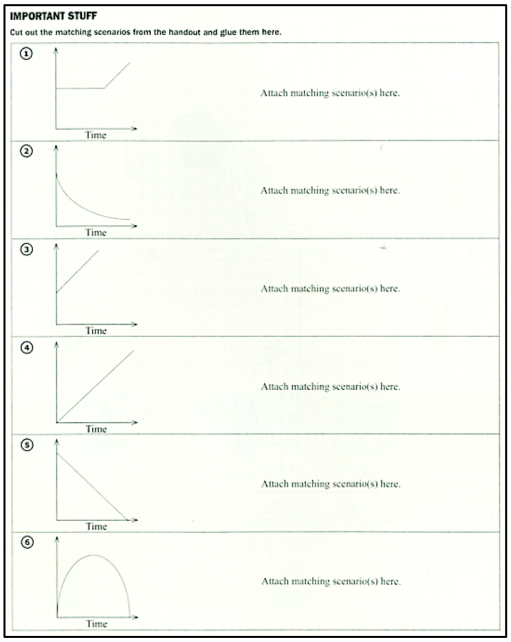
Case Study 1 – Ms. Ryan, School A

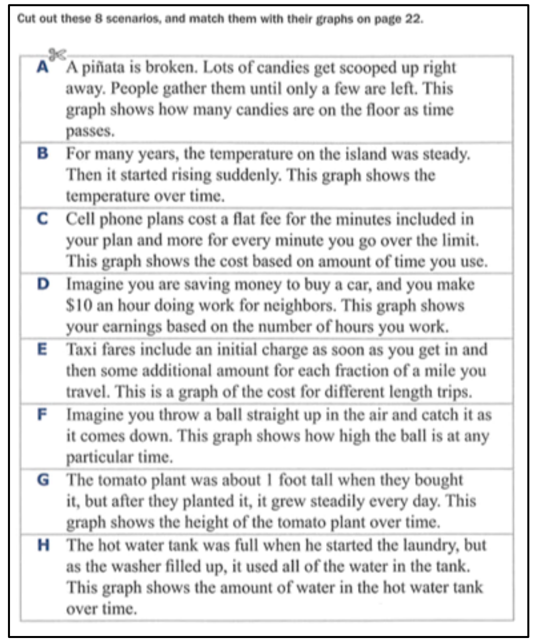
Ms. Ryan was in her first full year of teaching at the time of this study. Prior to teaching, Ms. Ryan earned a bachelor's degree in pure mathematics, a minor in mathematics education, a master's in secondary education, and a teaching credential. After her internship year at a different school in District A, Ms. Ryan was placed in the teaching pool for the district and she placed bids for schools that had a four by four schedule (i.e., four blocks in a day with four quarters) because she believed this schedule is a good fit for students and had read studies that found the four by four schedules are beneficial. During her internship year, Ms. Ryan taught Integrated Mathematics 2 and 3.

At the time of this study, Ms. Ryan was teaching a combination Integrated Mathematics 1 and SIFE (Students with Interrupted Formal Education) class. All of the students in this combination class were classified as ELs. Additionally, most of the students were newcomers, having moved to the US within the last year. Ms. Ryan had a co-teacher for this class, a support teacher for English Language Development. She shared that they work together on lesson planning and debriefing lessons, combining their respective strengths and knowledge to prepare the SIFE students for taking an Integrated Mathematics 1 course the following year. Ms. Ryan also shared that she had taken improv classes in college, which helped her be more expressive and animated in the classroom, using her full body to help communicate meaning to her students.

Planning. Ms. Ryan was asked by the district mathematics coach for School A to participate in the pre-pilot testing of the *Transition to Algebra* curriculum. Due to the visual nature and conceptual emphasis of the materials, Ms. Ryan agreed to use the curriculum in her combination IM1/SIFE class. Ms. Ryan explained that there are no standards for the SIFE class, so her goal was to introduce the students to as much mathematical vocabulary and to build as much mathematical understanding as possible during the year to prepare the students to take high school mathematics the following year. Thus, all of the students were being exposed to IM1 content, but only the students on the IM1 roster were held accountable to learn the material. The SIFE students participated alongside the IM1 students but were not expected to grasp everything that was introduced over the course of the year.

Ms. Ryan used the TTA Unit 6 student worktext (Mark et al., 2014d) as a resource to plan her lesson (see Figure 5.1). The lesson began with a matching activity in which students were to match scenarios to the graph. Ms. Ryan expressed concern about her students being able to understand the scenarios, so she and her co-teacher would likely spend a significant amount of time helping the students read and understand the scenarios through acting them out or building up the vocabulary for the students. She anticipated that she would need to introduce a scenario, help the students understand the context, then ask questions to help the students consider the quantities involved focusing on where the graph should start (e.g., at the origin, above the *x*-axis, etc.). Once the matching graph was identified, she would then move on to the next scenario and repeat this process until they had gone through each scenario. As she described this process, referencing specific scenarios, Ms. Ryan used many gestures to emphasize her words as she spoke. For example, when talking about a piñata problem, she asked how much candy would be on the ground at the beginning, after the piñata was broken, and after the children picked up the candy. Throughout this explanation, she repeatedly gestured in a vertical up and down motion as those she was moving along a *y*-axis to indicate no candy or a lot of candy and whether the graph should be increasing or decreasing.





Cut out these 8 scenarios, and match them with their graphs on page 22.

A A piñata is broken. Lots of candies get scooped up right away. People gather them until only a few are left. This graph shows how many candies are on the floor as time passes.

B For many years, the temperature on the island was steady. Then it started rising suddenly. This graph shows the temperature over time.

C Cell phone plans cost a flat fee for the minutes included in your plan and more for every minute you go over the limit. This graph shows the cost based on amount of time you use.

D Imagine you are saving money to buy a car, and you make $10 an hour doing work for neighbors. This graph shows your earnings based on the number of hours you work.

E Taxi fares include an initial charge as soon as you get in and then some additional amount for each fraction of a mile you travel. This is a graph of the cost for different length trips.

F Imagine you throw a ball straight up in the air and catch it as it comes down. This graph shows how high the ball is at any particular time.

G The tomato plant was about 1 foot tall when they bought it, but after they planted it, it grew steadily every day. This graph shows the height of the tomato plant over time.

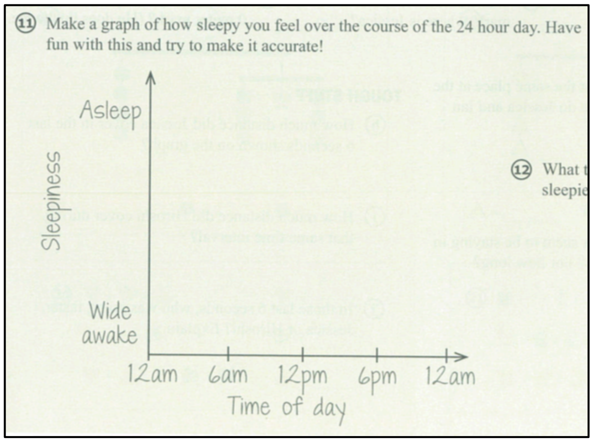
H The hot water tank was full when he started the laundry, but as the washer filled up, it used all of the water in the tank. This graph shows the amount of water in the hot water tank over time.

Figure 5.1. TTA Student Text (Mark et al., 2014d, p. 22 & 49).

Following the matching activity, she planned to have the students work on the additional practice problems which provided students more opportunity to select or create graphs that matched written descriptions of situations. Ms. Ryan reported she and her co-teacher would circulate around the room to help students understand as needed and check their progress on these additional exercises. She commented that they would have to provide a lot of support during this lesson because it is so “language heavy.” Ms. Ryan described the structure of the lesson as a lot of back and forth in which she and the co-teacher would help them understand the scenario, then the students would be asked to find the graph, then she would explain the next scenario, and so on because they had found that the students get overwhelmed if they try to have them do too much at one time and then the students just stop working. Her reflection was that classes flowed better if there were smaller chunks of activity and more interaction, language support, and feedback more often. Throughout the lesson, Ms. Ryan indicated that she and her co-teacher would be incorporating ways to get the students to practice speaking English and saying mathematical words.

When asked if she would go straight into the matching activity or have a warmup, Ms. Ryan indicated that she would plan some type of activity in which she could develop the idea that the graph represents something changing over time. She posited that she might time the students walking across her classroom and plotting distance over time, where distance was measured according to a student's foot size. After the warmup, Ms. Ryan planned to do the matching activity, which she estimated would take ten to fifteen minutes, and the additional practice problems from the worktext. As for the class structure, she anticipated going back and forth repeatedly from whole class instruction to individual, partner, or group work. While reviewing the student workbook, Ms. Ryan was clearly focused on issues of language access and production and how she may support her students to build conceptual understanding of connecting graphs to the situations presented. However, the only clues she gave on how this might happen would be in reference to gesturing and acting out scenarios. Ms. Ryan did not explicitly reference mathematical practices during her planning, but she did emphasize a goal of helping students communicate mathematical vocabulary.

Enactment. Still concerned about the accessibility of the tasks in the TTA curriculum, Ms. Ryan had modified her plans for the day of the observation but maintained her stated goals for the lesson. Her goals for this day were to start developing a conceptual understanding of a function and relating two quantities over time and to have some vocabulary for talking about what's happening in the graph. For her warm-up, she modified a question from the student worktext (see Figure 5.2), asking students to plot how hungry they feel throughout the school day. Note that she changed from clock times in a 24-hour day on the horizontal axis to the school day schedule and added a middle option for the amount of hunger.



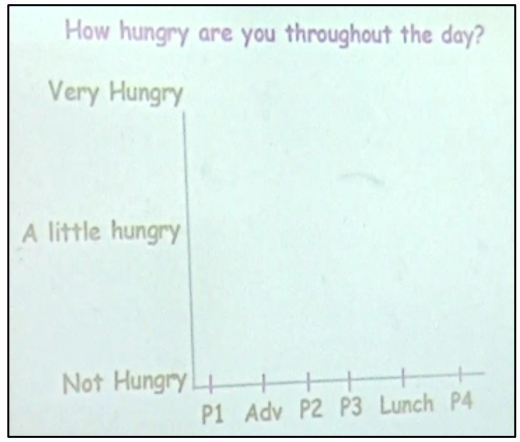


Figure 5.2. TTA worktext question (Mark et al., 2014b, p. 23) and Ms. Ryan's modification for her warmup.

Ms. Ryan’s next activity involved working in pairs to time how long it takes to write each letter of the alphabet on the whiteboard desk writing one letter at a time and erasing it before writing the next letter. One partner wrote the letters, the other used their cell phone as a stopwatch to time how long it took to write each letter and plot the cumulative time on a graph. Ms. Ryan and the co-teacher first demonstrated the activity for the students, then spent time developing language to talk about the task. Ms. Ryan wrote questions down (e.g., What are we measuring?), developed answers with the students (e.g., We are measuring the time it takes to write each letter.), and then called on a student from each group to recite the sentence out loud with her. She and the co-teacher then assigned which partner would write the letters and which would record the times in the graph. Ms. Ryan expressed that she had students who needed to practice writing the letters of the alphabet (recall this was a class for students with interrupted formal education), so their decisions about who would time and who would write were intentional for providing additional letter-writing practice to those students who needed it. After the students completed the activity, Ms. Ryan displayed two student graphs on the document camera (see Figure 5.3) and asked the students, “Who is writing faster? Student A or Student B?” Ms. Ryan then developed vocabulary about the steepness of the line, slope, rate, and speed, again writing out sentences with the students and having different groups of students practice reading the sentences out loud.

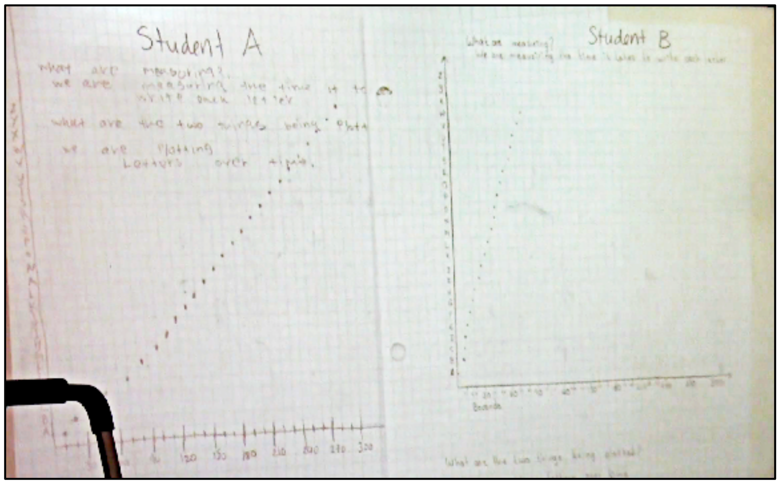


Figure 5.3. Sample student work from the alphabet graphing activity.

The final activity for the class session was to do a matching activity with scenarios and graphs. Recall that Ms. Ryan had been concerned about whether her students would understand the scenarios provided in the TTA curriculum. She elected to write her own scenarios and graphs that she believed were more familiar to her students. In the TTA activity, there were six graphs and eight situations (two graphs were each matched with two scenarios). Ms. Ryan's matching activity had 12 situations and 12 graphs.

Throughout the lesson, Ms. Ryan's students were actively engaged in several mathematical practices. First, they were involved in SMP1, make sense of problems and persevere in solving them. Not only did the students have to make sense of the words in the scenarios, they also needed to consider the relationship of the given quantities over time, make conjectures about the shape of the graph, and analyze the constraints of the situations. Second, the students were engaged in SMP2, reason abstractly and quantitatively, as they considered the quantities and their relationships and created a coherent representation (i.e., a graph) of the relationship. Third, the students were highly involved in SMP6, attend to precision, as Ms. Ryan worked with them to develop conceptual understanding of concepts like steepness, slope, and rate while simultaneously building their academic language to talk about graphs (e.g., increasing, decreasing). Finally, there were three other mathematical practices evident in this lesson, but to a lesser intensity than those mentioned previously. During the warmup and alphabet activities, the students were given the opportunity to share their work and respond to the reasoning of their classmates (SMP3). The alphabet activity also hinted at SMP4, model with mathematics, as the students collected and plotted their own data and interpreted their results in the context of the activity, and SMP5, use appropriate tools strategically, as they used a stopwatch app to time their partner and then used a graph to compare rates.

With a class full of students classified as ELs and a teacher who reported being monolingual, one might imagine that there was necessarily a great deal of strategies used throughout the class period that were intentionally used to provide access not only to the mathematical content, but also to develop the students' language skills. Ms. Ryan and her co-teacher were both very animated, using exaggerated gestures and facial expressions as they communicated with the students – reminiscent of the idea of total physical response in teaching second language classes to beginners. (The students also communicated with each other in this manner. I watched a young man explaining to his classmate that she could not copy his graph of how hungry he was throughout the day, because it was about him, not her, and she had to do her own.) She also spoke slowly and clearly, repeated important concepts multiple times both by herself and in unison with her students, and wrote down sentences as she read them to her students. As Ms. Ryan discussed the graphs with the students, she used informal language, sound effects, and gestures to communicate about the underlying mathematics. While the point of these discussions was to introduce properties of functions (e.g., each input can have only one output), these ideas were expressed informally through the context of the activities. For example, in the warmup, Ms. Ryan asked if it was possible to be both not hungry and very hungry at the same time when a student presented a graph with multiple parts.

Case Study 2 – Mr. Martin, School A

At the time of the study, Mr. Martin had been teaching 9th grade mathematics (Algebra 1 and Integrated Mathematics 1) at School A for 19 years. His first teaching credential was in Biology, but he went back to school to take extra classes in mathematics and got a supplemental authorization in mathematics. Due to the push for smaller class sizes in math when he was searching for a job, he got his first job teaching 9th grade mathematics at School A and had been there ever since. Mr. Martin also earned a master's degree in English and more recently added a supplemental authorization in physical education to his credential. In his words, he is interested in different things. Both he and Mr. Hepner, a math teacher with whom he regularly collaborated, came to teaching after pursuing other careers.

In response to a district-adopted curriculum that they perceived as inaccessible to their student population and professional development activities that drew their attention to mastery-based teaching, he and Mr. Hepner created their own 9th grade mathematics curriculum with the support of a previous principal. They also intended to create materials for the subsequent years as well, but administrative changes prevented them from going further with this effort. Mr. Martin described a researcher coming to the school to talk about other similar schools with 90% of students that were below the poverty line, 90% of the students were minorities, and 90% of the students were testing proficient in mathematics. Mr. Martin reported that the researcher had stated that "mastery-based" teaching was the common thread in these schools. According to Mr. Martin's perception of mastery-based teaching, students should be required to learn fewer standards throughout the school year and have the opportunity to have ample practice time to gain confidence on a standard before moving on to the next one. Mr. Martin and Mr. Hepner asserted that students should receive very focused direct instruction, a lot of guided practice, and immediate feedback. Prior to the adoption of Common Core, Mr. Martin and Mr. Hepner had found great success with their version of a mastery-based curriculum, with the students in their new arrivals program scoring just as well as any other student in the district (on the old state assessment that predated the SBAC).

Mr. Martin and Mr. Hepner shared some of the design principles that guided the development of their curriculum and activity structures. They believed that their students needed fewer standards to master each year and that students need immediate and constant feedback in order to develop self-belief and self-confidence in mathematics, a subject in which Mr. Martin and Mr. Hepner believed their students may not have seen a lot of success in previous schooling. To this end, Mr. Martin and Mr. Hepner selected three of the Standards for Mathematical Practice (1, 4, and 5) to focus on in 9^{th} grade with the intention of adding the others in subsequent years. They also do a lot of practice problems during class, using small whiteboards, which they feel encourages students to try problems more readily than committing to paper and pencil that's not as easy to modify. For class notes, Mr. Martin and Mr. Hepner have created graphic organizers for each unit connecting procedures and vocabulary to the central concept for the lesson (see Figure 5.4). Having also experienced a lot of students switching courses mid-year, it was important to Mr. Martin and Mr. Hepner to continue to plan together and to teach the same lesson on the same day in order to provide a seamless transition from one class to the other for students. They also described that it would not matter which of them I watched because they teach the same way. (Something I doubted but turned out to be a truer statement than I would have imagined to be possible – even down to saying phrases like “chop chop” to indicate changing subtraction to adding the opposite and using the same non-standard symbolic notations to indicate moving a term from one side of the equal sign to the other.)

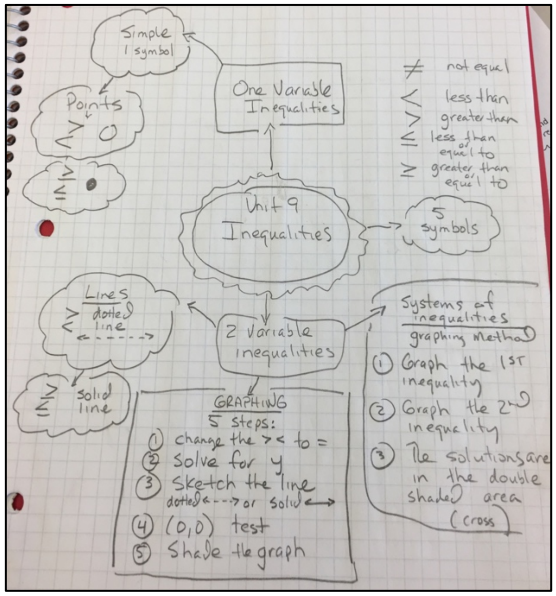


Figure 5.4. Mr. Hepner's Unit 9 notes.

Planning. Mr. Martin and Mr. Hepner requested to do their lesson planning interviews together since this is how they typically prepare for class. Since they had co-developed the curriculum, this planning session was a bit atypical in that they didn’t really need to do anything more than say what content they planned to cover on the days I would be visiting their classes. Mr. Martin expressed that if I really wanted to understand what they do throughout the year, it would be best to see each of them teach at least twice – once for a regular lesson, in which I would witness a fair amount of direct instruction, and once for an end-of-unit review, where students would be involved in much more group work than usual. When asked to describe a typical day, Mr. Hepner described that I would see bell-to-bell instruction with students working from the very beginning of class to the very end because they value instructional minutes and need to take advantage of that time as best as possible. Mr. Martin added that there will be a variety of transitions throughout the period, not just one continuous activity for the whole 80 minutes. He then stressed the importance of opportunities to do guided practice with immediate feedback to build the students' self-belief and self-confidence that they have the skills to do the independent practice or perform on a test when that time comes.

Mr. Martin and Mr. Hepner planned to have me come observe on the days they were teaching two-variable inequalities and systems of inequalities (linear). Again, their familiarity with their curriculum seemed to indicate that they were done planning, but I was still uncertain of what I might observe when I came to their classes. I inquired whether it would be possible to see the slides or handouts or whatever form of materials they might be using with their students. While Mr. Hepner went to retrieve his unit 9 materials, Mr. Martin described the way they do notes in a graphic organizer style with inequalities in the center and branches off to one-variable inequalities, two-variable inequalities, and some general information off to the side about the symbols and what they mean (see Figure 5.4) which is built up as they progress through the unit. Mr. Hepner then pulled up the slides he would use for this unit, which had a sample problem with space to solve below, a coordinate grid for graphing the inequality, and the steps for solving a two-variable inequality. The worksheets they shared had between six and eight practice problems each, and they intended to do two worksheets per period in addition to introducing the new material and doing guided practice on the whiteboards.

When teachers were recruited for my study, they were made aware that I was interested in how EB students were supported in engaging in mathematical practices. Without any prompting during the interview, Mr. Martin shared that they had selected three of the eight SMPs (1, 4, 5) to focus on this year. He described that at the beginning of a unit, they may give their students a word problem and encourage the students to solve it in their own way, then throughout the unit they build up the tools they need to model the situation in multiple representations (e.g., tables, graphs, and equations). It appeared Mr. Martin and Mr. Hepner interpreted "tools" as prior knowledge or mathematical representations rather than the more common interpretation of tools as rulers, calculators, software, etc. They also shared that their entire curriculum was written in a way to support English language learners and develop their self-belief and self-confidence in both the language and mathematics, all the while demonstrating that they (the teachers) are there to support them and help them master the content of their course.

Enactment. I visited Mr. Martin's and Mr. Hepner's classes three times each during the data collection. My first visit was to observe on the chapter review day in which students were instructed to engage in group problem solving. On the day before, the students had taken a pretest, which Mr. Martin and Mr. Hepner commit to scoring by the next day. Based on the pretest scores, Mr. Martin selected seven students to be coaches for the groups and each group was assigned to a table. The eighth group was led by Mr. Martin's co-teacher. Each table had a task to solve that was similar to a task on the pretest. The students were to work together to solve the problem, and the coaches' responsibility was to make sure the students in their group knew how to solve each task. The teachers walked around the room to check in on each group. Once all groups solved the tasks, the groups rotated to the next table and solved a new task together. Each group member had a sheet on which to record their work at each table. While watching the group work, I had the opportunity to look around the room and noticed Mr. Martin's notes on the front whiteboard (see Figure 5.5). This was the first indication I had that he and Mr. Hepner may have some unique ways of communicating mathematics to their students – note step two in the upper right – put on magic parenthesis.

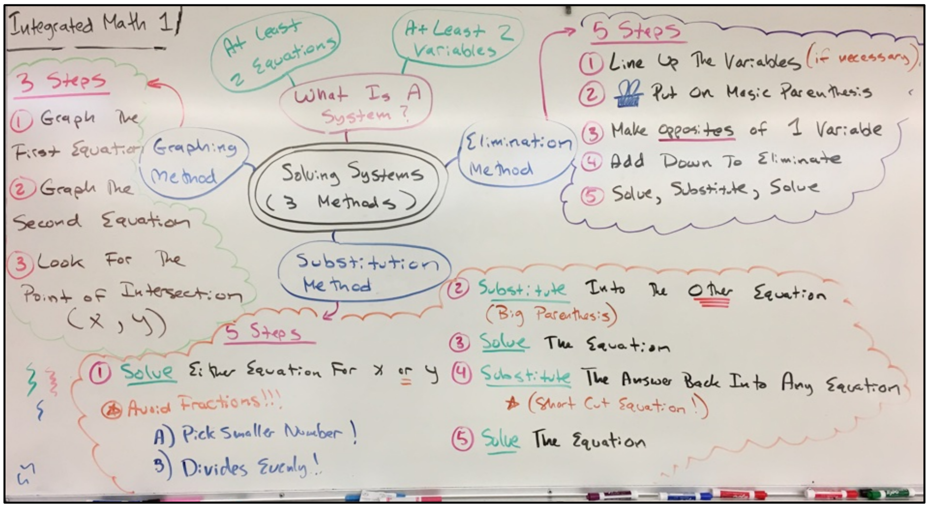


Figure 5.5. Mr. Martin's concept map for systems of equations.

For the second half of the period, I visited Mr. Hepner's room. The students were engaged in the same type of activity, working in groups with student coaches who had scored well on the pretest. I discovered the magic parenthesis step on Mr. Hepner's board as well. In addition, Mr. Hepner had displayed a slide of "special systems" of equations (see Figure 5.6). After the group review concluded, Mr. Hepner handed back the students' pretests and started going over some of the problems with them. He announced about the after school "fixup" session that was happening that day in which students could attend to earn points back on their pretest.

I returned to both teacher's classes after school for the fixup session that ran from 2:15 to 5:00pm. There were approximately 50 students in each teacher's classroom. The teachers had created assignments for the tutoring session and students had to complete some to all of the worksheets based on their pretest score. Mr. Hepner also had student volunteers come in as graders and tutors during this very packed review session. Students were encouraged to help their classmates, but not to copy each other's work since the goal is to master the material and prepare for the unit exam. I also had the opportunity to help students in both classrooms as they completed these assignments, doing my best to mimic some of the questioning tactics Mr.

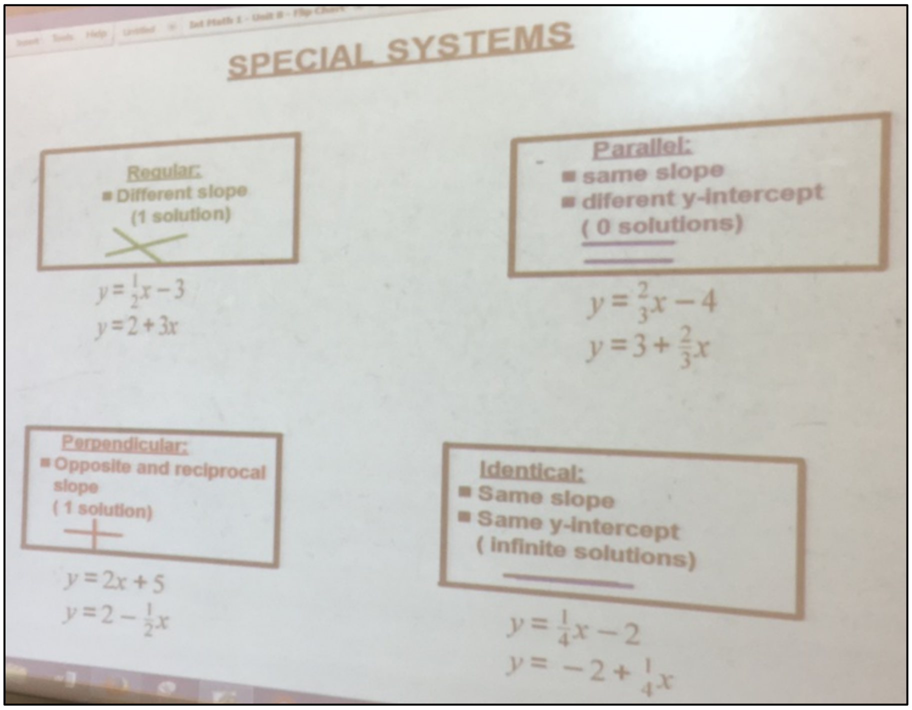


Figure 5.6. Mr. Hepner's display of special systems of linear equations.

Hepner had used during the review earlier that morning to help the students identify how their notes could be used as a resource for solving the tasks on the pretest.

On the day I observed Mr. Martin's class, he reviewed the process they had learned in the previous class session to determine whether a given point is a solution to a given two-variable linear inequality. He completed the first problem on the worksheet with the students, asking questions about the next steps and expecting students to participate by choral responses throughout the problem. If it seemed as though only a few students were answering, he informed the students that wasn't the level of participation for which he was looking. The students were then given time to complete the rest of the worksheet. After noticing he was getting the same question from several students, Mr. Martin went over another problem with the class to clear up the confusion about whether the line should be solid or dotted, referring the students back to the concept map on the front board to find their answers. After reviewing this problem, he asked the students to put the worksheet away.

Next, Mr. Martin asked the students to take out their notes from the previous class so they could finish writing the last set of steps for graphing two-variable inequalities. He informed the students that there's more than one way to teach this concept, but that his way helps students make fewer mistakes. He asked the students to turn their concept map over and start the example on the back side of the page. Mr. Martin then had them return to the concept map to add step one, to change the inequality symbol to an equal sign, which he validated by telling the students this step meant they won't have to worry about flipping the inequality symbol if they divide by a negative. As at other times, he indicated to the students that this was something that was difficult for students to remember, so that's why he recommended his method. As I observed, I wondered if he always addressed the students this way or if this information was for my benefit to explain why he may be doing something that could be perceived as unconventional.

Mr. Martin returned to the example and wrote step one, which also included changing the subtraction to adding the opposite. As he did this, he told the students to do "chop chop" again. Solve for *y* was added as step two in the notes, then Mr. Martin returned to the example, circling the -2*x* and drawing an arrow over the equal sign to a circle with a positive sign inside (see Figure 5.7). The lesson proceeded in this manner, alternating between adding steps to the concept map notes and doing the next step in the example. After graphing the dotted line, Mr. Martin asked the students how many thought the top of the graph should be shaded (about 5 students), the bottom (nobody), and how many weren't sure (a few hands). He then asked the students to come up with a theory on how they could figure out which side should be shaded, and that it's okay to be wrong or make a mistake because they are just theorizing. One student's theory was that if the line is going up (positive slope), you should shade up above the line. Another suggested that if the inequality symbol said greater than, then they should shade above the line. Another student suggested a making a slope triangle but wasn't sure how that would help them decide which way to shade.

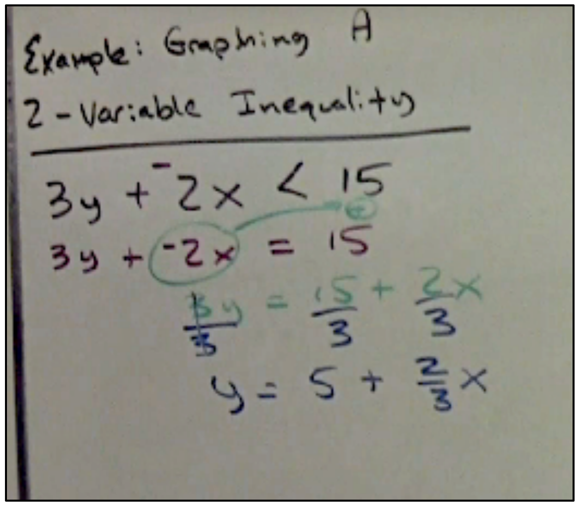


Figure 5.7. Mr. Martin's example of graphing a two-variable inequality.

Mr. Martin then tried to help the students recall what they had just been working on the worksheet at the beginning of class without pointing them directly to it. He asked questions about the shaded area and a student stated that solutions are in the shaded area. Mr. Martin asked the students if a solution could help them figure out where to shade. They replied yes but didn't offer a way to do it. Then Mr. Martin asked someone to give him a point and a student gave him the *y*-intercept of the line, so Mr. Martin more specifically requested a point not on the line. They tested the point by plugging it into the original inequality. Mr. Martin asked the students not to write this down and just follow along as they decided whether this point was a solution or not and which side of the line should be shaded. Mr. Martin then informed the students that this was a reasonable way to get to the solution, but he recommended using the point (0, 0) to simplify the calculations they have to do (provided that the line doesn't pass through the origin). He asked the students to write this portion of the example in their notes. When a student asked if they had to shade their graphs all the way to the edges, Mr. Martin stated that they did because the solution basically goes on forever (also in anticipation of graphing systems of inequalities in the next lesson).

The next portion of class was devoted to guided practice where the students were given whiteboards on which to work out the next example. Mr. Martin and his co-teacher walked around the room checking the students' progress, offering praise and corrections as needed. He then went over the example, indicating that 90% of them successfully graphed the dotted line, but only about half the students got the shading correct on the first try. He then gave the students another problem to try, indicating there would be three more and another worksheet. After two more guided practice examples, Mr. Martin seemed confident that he could move into the next activity. The worksheet was eight more graphing linear inequality problems, each having a point to determine whether it was a solution or not a solution after graphing. After doing the first problem with the students to make sure his expectations were clear, Mr. Martin encouraged the students to complete the worksheet and indicated that he would be coming around to check their work with his answer key.

Because I have adopted a sociocultural perspective of learning, I felt that the students were not given many opportunities to engage in mathematical practices during this lesson, because student participation during the lesson looked like note-taking, doing multiple practice exercises, and providing short choral responses to the teacher. Students were certainly engaged in SMP6, attending to precision, producing correct graphs and calculating accurately and efficiently. Mr. Martin identified SMP5, using appropriate tools strategically, as another mathematical practice in which his students were engaged because they were relying upon their prior knowledge of graphing linear equations and point-testing. However, upon further analysis, one could also say students were engaged in SMP1, make sense of problems and persevere in solving them, because they learned two ways to determine if a point was a solution or not (algebraically and graphically) and Mr. Martin asked the students to make a conjecture about how they would know which side of the line to shade before introducing them to the “(0, 0) test.”

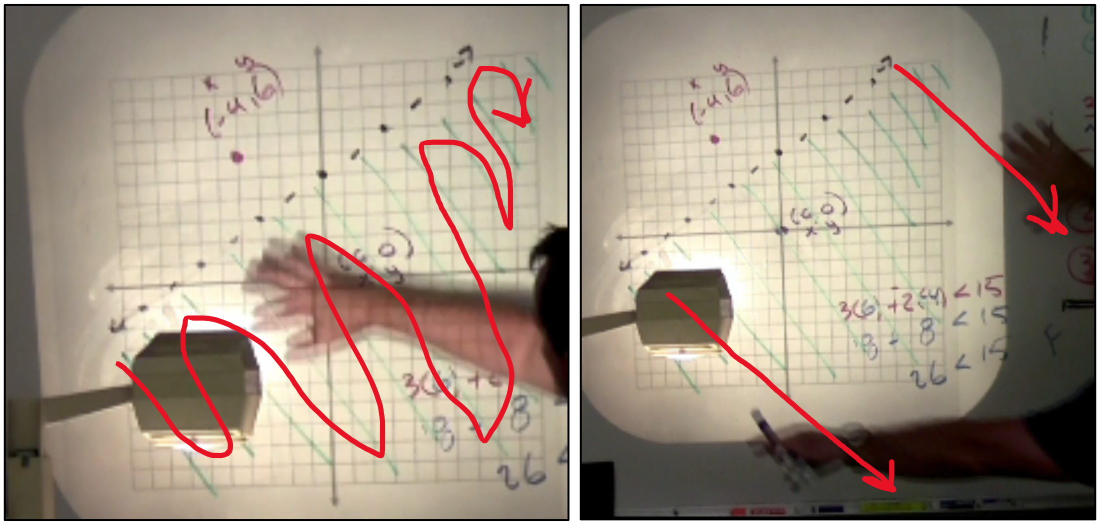


Figure 5.8. Mr. Martin indicating that all points in the shade are solutions to the inequality and demonstrating that the solution set goes on forever.

Mr. Martin used a lot of repetition and structure in his lesson, both linguistically and mathematically, which he claimed is the best way for his student population, especially EB students, to build mathematical knowledge and self-efficacy. His terminology was precise and consistent. He was very methodical in his board work, using concept mapping for notes and color-coding for the steps in his examples. Every example he did on the board followed the exact same format. Mr. Martin also used gestures as he talked about shading the graphs (see Figure 5.8). However, on this day, his students were only given the opportunity to participate in whole class instruction through choral responses, which were at times known answers due to what Mr. Martin was pointing at in the concept map notes. Because all of the tasks in this lesson were decontextualized, there were not as many opportunities to look for the ways in which Mr. Martin would support EB students’ access to context-embedded tasks.

Case Study 3 – Ms. Rainey, School A

Ms. Rainey had been teaching for 40 years at the time of this study, with twelve of those years at School A. As one can imagine, she had taught practically every secondary mathematics course over the course of her teaching career. Currently, she was teaching three sections of honors precalculus. Ms. Rainey earned a bachelor's degree in Spanish, obtained a teaching credential, a TESOL certificate, then later on took the National Teachers Exam (administered by Educational Testing Service (ETS) and later replaced by the Praxis exams) in mathematics to get her math credential, and earned a master's in educational leadership.

Ms. Rainey described her teaching style as finding ways to get students to bump into new mathematical ideas for themselves through carefully sequenced activities. She also explained that she spends a lot of time preparing her slides so that she can talk as little as possible during the class, with the goal of giving students the opportunity to run into those mathematical ideas. Ms. Rainey informed me that she doesn't write out a lesson plan anymore, she creates her slides for class, thinking through the mathematics and what language supports the students may need to access the lesson. She incorporates real-world examples in her notes as often as possible, taking into consideration contexts that are familiar or interesting to her students as well as helping them learn practical life information that relates to mathematics.

Planning. Before planning the lesson I was to observe, Ms. Rainey walked me through her slides from the two lessons that preceded the lesson, describing her teaching style as she went along. She described when she was silent, what questions she asked and when, and how the students had responded to the tasks. Ms. Rainey then shared in general terms what she intended to accomplish next – more practice determining an explicit equation given the terms of a geometric sequence, then finding the sum of a geometric series, and finding a word problem to connect geometric sequences to real life. She then pulled up the slides she had started to create for the lesson and walked me through them.

Ms. Rainey displayed a geometric sequence (2, 4, 8, 16, 32, 64) with the question, “What is the pattern?” She anticipated that she would have some students who would describe the pattern as add two, then add four, then add eight, then add sixteen, so you add twice as much each time, yet they may not see that this is the same as multiplying by two. If that happened, Ms. Rainey planned to ask questions until someone identifies the pattern. Her next slide presented definitions of a geometric sequence and the ratio, which she indicated must be greater than one (she appeared to focus only on geometric growth in this lesson). She explained that she doesn’t have to say anything about this slide – the students would just absorb the information from what is provided. She then displayed five sequences for which the students were to decide if they were geometric. This task was followed by a real-life application presented in a table representing the number of views (in 1000s) each day since the video was posted. She planned to ask questions about the values in the table, such as the number of views on day two. On the next slide she added sequence notation to each column to connect the values in the table to geometric sequences, then would ask the students to find the constant ratio and the tenth term of the sequence. The next slide shows the equation for a geometric sequence. At this point, Ms. Rainey ran out of the slides she had prepared and returned to the beginning to reflect on whether the slides were doing what she intended.

As she clicked back through the slides, she noticed that the table included days since the video was posted, but she hadn’t introduced a situation yet. She inserted a new slide and created a scenario in which you made a video in another class that you were proud of, so you posted it to YouTube. Ms. Rainey then decided it may be overwhelming to see both columns of the table at the same time, so she hid the second column to give students the opportunity to focus on the given information a little bit at a time. Ms. Rainey said she wanted smooth transitions that will be clear to everybody: English learners and people who read slowly. (My interpretation of Ms. Rainey's comment here is that she was listing two groups of people that she was independently concerned about, not equating these two groups.) After the explicit equation, she intended to plan another real-life example, then some guided practice tasks, and finally some independent practice tasks.

It sounded as though Ms. Rainey had concluded her planning, so I asked her if she typically used any curriculum or other resources as she planned. She then described several textbooks that she likes to use as references for real life examples she can incorporate in her lessons. As she was scanning the materials, she found the formal textbook definition of the n^{th} term of a geometric sequence and described that she sometimes shows the formal definitions to her students after they have learned about the topic to build the students' confidence that they can read a textbook definition (now that some of it should look familiar). Ms. Rainey then noticed a problem about the value of an automobile depreciating and expressed that the students would have to learn what depreciate means, but she can refer to common knowledge that the value of a car drops as soon as you drive it off the lot. She planned to tell the students that she's never owned a new car since a two-year-old car is likely to be half the price, expressing that she likes to throw in some life lessons while she teaches math.

As Ms. Rainey continued to skim the textbook, she indicated that there were a lot of viable practice problems in the book, but that the writing is too demanding for their students. She indicated that many high school students at School A read at a third-grade level, so she would need to reword the problem and add a diagram to help communicate the context of the task. Ms.

Rainey indicated that she would consider the sequence of tasks that would lead up to that problem as well as the career in which the mathematics may be used and why the concept is important.

Enactment. Ms. Rainey's lesson proceeded much the same way as she had described in the planning session, with a few additions. There was a new opening problem about withdrawing money from a bank account in which you withdrew a higher amount of money each day until the account was empty. The students were asked to write this situation in summation notation. (Note that multiple solutions are possible in this situation.) As planned, they next identified a geometric pattern, defined a geometric sequence, identified which sequences were geometric, did the YouTube video views example, and defined the explicit equation of a geometric sequence. Ms. Rainey added the formal textbook definition of the n^{th} term of a geometric sequence and asked the students to find the equation of the geometric sequence in the definition and notice what is the same and what is different in the two equations. Ms. Rainey then developed a story about housing prices appreciating over time, related to housing costs in Fresno, CA before the bullet train to the Silicon Valley is built versus housing costs in Fresno after the train is completed. Her story was accompanied by visual images depicting a house with two dollar signs below it next to sapling in 2018 and the same house with five dollar signs below it next to a mature tree in 2030, a map of California, and a picture of a high speed train. The punchline of the story was that housing prices in Fresno will go up, and the students were asked to find the value of the home in the year 2030. In her debriefing interview, Ms. Rainey explained that her students can't imagine owning a home, so she wanted to convey to them that if they get a college degree and a good job, there are other places in which they can live and afford to buy a home. The class ended with a few practice exercises in finding geometric means.

Considering the mathematical practices that students had the opportunity to engage in during this lesson, the two with the highest intensity were SMP1 and SMP7. In the bank account problem, the students needed to analyze givens and constraints, make conjectures about the form of the solution in summation notation, and consider the approaches of others. Throughout the lesson, students were engaged in finding patterns. The lesson also touched on other mathematical practices, but to a much lesser intensity than SMP1 and SMP7. Students needed to consider the applicable domain of the bank account problem (SMP3), they applied math to solve everyday situations (SMP4), they used calculators (SMP5), and calculated accurately and efficiently (SMP6).

Supports for EB students in this lesson included the use of visual images, defining vocabulary such as depreciate and appreciate, and annotating the tasks to draw attention to the salient features such as how many times they would need to multiply by the common ratio to get to the desired term or adding sequence notation to each term in the table to help the students recognize they were looking for a constant ratio. Ms. Rainey also asked a series of questions that clarified vocabulary (e.g., constant, consistent, without change). In general, the students only spoke when giving choral answers to Ms. Rainey's questions. At one point in the lesson, Ms. Rainey asked the students to turn to their partner when she rings the desk bell and finish the following sentence: the pattern is ____. She then called on a student to share what her partner had said. Several times during the lesson students were called up to the board to write their answers to the tasks, but generally were not asked to explain what they wrote.

Case Study 4 – Mr. Estrada, School B

Mr. Estrada was an alumnus of School B and was in his second year of teaching there. Mr. Estrada earned a bachelor's degree in mathematics, held a bilingual teaching credential, and was enrolled in a master's in mathematics education program. Like Mr. Martin and Mr. Hepner, Mr. Estrada started in a different career – computer engineering. After having the opportunity to run some workshops and later teach some courses at a university, Mr. Estrada decided he wanted to teach and started out substitute teaching at a middle school in District B before getting a job at School B. He had about nine years teaching experience total between his university level teaching, substitute teaching, and his two years teaching at School B. At the time of the study, Mr. Estrada was teaching four courses: Integrated Mathematics 3, Integrated Mathematics 3 Bilingual, Integrated Mathematics 1 Bilingual, and AP Calculus BC.

In his Integrated Mathematics 1 Bilingual course, Mr. Estrada had elected not to use the student work packets that the rest of the Integrated Mathematics 1 team used, citing that he felt that work packets are boring. He described his class as working in groups, "Common Core style." Mr. Estrada said a typical day in his class would begin with him giving a lesson and providing the background knowledge that they need for the day, then have the students try some problems together, and then he will give more problems for the students to work on independently or in their groups. If the students are understanding, he describes the rest of the class as more student-driven, which meant giving the students the opportunity to work on the assigned material at their own pace. Mr. Estrada primarily taught his bilingual class in Spanish, but was confident that with some topics, such as graphing linear equations, his students would also know how to communicate about the mathematics in English. He contrasted this lesson with the chapter on exponential functions, which had a lot more specialized vocabulary.

Planning. In his planning interview, Mr. Estrada shared that he would be doing a review lesson of graphing linear equations and inequalities and systems of linear equations and inequalities. His general plan was to have the students work in groups to prepare and present posters in which they focused on slope, y-intercept, points of intersection, and shading solutions. Mr. Estrada then added that he will do a little bit of review at the beginning, like a warmup, and then a couple more problems to get the students started on the lesson. He planned to break the class up into their groups and give each student a worksheet with the problems they needed to graph. Mr. Estrada indicated that he would want the students to show their work on a scratch paper before transferring it to the poster to ensure accuracy. He planned to walk around while the students were working to answer their questions and monitor their progress.

Mr. Estrada expressed that he doesn’t have his own classroom and that this has had an impact on his ability to teach the class. He stated that he doesn’t have access to all of the technology in this room and is limited to using only certain capabilities of the rooms in which he teaches. Mr. Estrada preferred to project the textbook (in English or Spanish, depending on the academic vocabulary needed to discuss the content of the lesson) and help the students understand each part of the tasks as they progressed through the curriculum. He liked to display the Math Notes sections from the CPM textbook for the students and have them copy them down into their notes while he highlights important vocabulary for them. Mr. Estrada explained that even though the CPM textbook has a Spanish translation and he runs the class in Spanish, the students still need a lot of scaffolding to be able to understand the academic language in mathematics in Spanish. Much like Ms. Ryan, he felt that he needed to break down the tasks into smaller manageable chunks and alternate between explaining part of the task, having the students work on that piece, then explaining the next part, having the students complete it, and so on. Mr. Estrada indicated that his way of teaching more closely models the “Mexico way of doing” lessons in which the students are taking notes prior to working on tasks as opposed to being given tasks and asked to work on them without prior instruction.

When asked about mathematical practices, Mr. Estrada expressed that he wants his students to attend to precision. He described this as being precise with their work and doing it correctly. However, he also wanted his students to be speaking about mathematics because he felt that they retain the knowledge better when they have the opportunity to talk about math and ask questions than when they simply take notes for a whole class period. While precise communication is also part of SMP6, Mr. Estrada did not appear to connect this idea of communicating about mathematics with the mathematical practice of precision.

Mr. Estrada shared that the CPM textbook presentation of material is very complex and needs to be broken down for the students, and not just for his bilingual students. He described helping the students identify the key information in the text. Mr. Estrada shared that using sentence starters, providing graphic organizers, altering fonts for emphasis, and considering the placement of items on worksheets were all ways in which he could help his students access the material. He expressed that their population of students won't really complete homework unless the teacher is really on top of things, so he believed that the CPM curriculum needs more practice exercises than it offers so that his students can solve more problems during class since they aren't getting the additional practice at home. In his bilingual course, Mr. Estrada felt free to teach the course as he saw fit, meaning he could give the students written materials in either English or Spanish and teach the lessons in either language, sometimes choosing to use both languages in the same lesson. He wanted his students to understand that it is important to learn both mathematics and English, especially if they plan to go to college in the US.

Enactment. Before reporting on Mr. Estrada's lesson, I should explain that I am not fluent in Spanish. I can understand much more than I can speak and have also realized that I am much better at understanding formal, academic Spanish than conversational Spanish. Because

Mr. Estrada spoke slowly and clearly to his students, wrote information down, had told me his lesson plan ahead of time, and because I knew the mathematics, I felt fairly confident about my ability to understand what was happening in the classroom – until a student asked me what I was going to do with the video, and I had no idea what he was saying to me. For this reason, I passed along the video and my fieldnotes to a willing colleague who enhanced (and corrected) my notes to provide a detailed summary of what was taking place throughout the class. We also discussed the class so I could feel confident about my interpretations of our combined summary and what I had experienced, as well as to double-check that I was accurately recounting what happened in Mr. Estrada's class. While I am confident that this is a reasonable interpretation of the class observation, I must acknowledge that it's possible that I have missed important interactions due to my inability to understand or by having to rely on a colleague to notice those interactions which were relevant to my research questions.

Mr. Estrada greeted his students at the door. Students entered speaking to Mr. Estrada and each other in Spanish. The first eighteen minutes of class were spent on collecting permission slips for my study, changing seats, making announcements about the schedule for their upcoming unit test and final exams, and answering student questions. Mr. Estrada then wrote the warmup on a piece of paper displayed on the document camera and informed the students that they would be practicing graphing linear equations and inequalities and creating a poster by the end of the period.

The warmup was to find the slope and *y*-intercept of a line given in slope-intercept form and to sketch the graph. Mr. Estrada circulated around the room to check student progress on the first question. After the students had completed the warmup, Mr. Estrada returned to the document camera and wrote down the next exercise for the students to complete, which was graphing a system of linear equations given in slope-intercept form. Note that Mr. Estrada did not present a solution to the warmup before moving on. As Mr. Estrada circulated around the room, he initialed student work that he had checked. In the group of students near me, the student who finished the problem first took on the role of checking the work of the other students in her group. Mr. Estrada gave a final task about graphing a system of linear inequalities. After a few minutes, he returned to the document camera and explained to the students when they should use a solid or dashed line based on the inequality symbol. He encouraged the students to differentiate between their shading of the overlapping region and those that aren't overlapping by shading lighter in the non-overlapping regions.

Mr. Estrada then introduced the poster activity. He handed out all of the supplies to each table, and then assigned eight out of the eighteen problems on the worksheets he had just handed out to the class (it took almost a full eight minutes to introduce the activity, handout supplies, and assign the problems they needed to do). Most of the students got right to work on the assigned problems. The students in the group nearest the camera worked individually on the problems and checked their answers with the same student who had checked their work in the opening examples. This same student also started creating the group's poster as the other students completed their graphs. The rest of the period was spent on making posters, with Mr. Estrada circulating to answer questions and checking the accuracy of their graphs. The students nearest the camera continued to work independently on their worksheets and poster, checking in with each other as they completed each graph. However, if they had a question, they tended to ask Mr. Estrada instead of their group members (which he answered).

One of the students asked Mr. Estrada how to graph a line with a slope of four. Mr. Estrada reminded him that he needed to write four as four over one. Later in this conversation he used his arms to distinguish between positive and negative slopes as he talked to the student. In Figure 5.9, Mr. Estrada was talking about a positive slope while making this gesture. Note that from his perspective, he is looking at a line with a positive slope, but the student's perspective is the same as that of the camera's in which his arm resembles a line with a negative slope. The student nodded that he understood the interaction, but it would have been interesting to know how the student interpreted this gesture. Students continued to work on their posters until the end of the period. Unfortunately, there was not enough time for the students to present their posters during this class session.

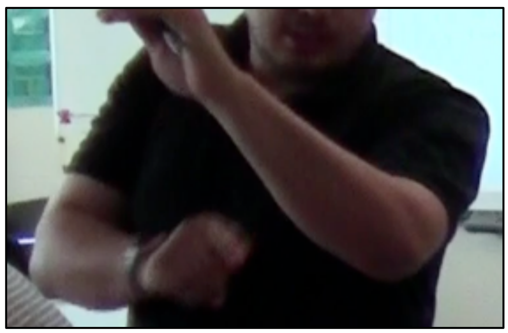


Figure 5.9. Mr. Estrada using his arm to indicate the slope of a line.

While the students were given ample time to work in groups as they created their posters of graphing linear equations and inequalities, many of the students did not use this time to have mathematical discussions. The students tended to work individually on their worksheets, then transfer their graph to the poster. Some students would check in with a groupmate to see if they had the correct answer, but this did not produce extended conversations between the students. Many students checked their answers with Mr. Estrada and asked him questions instead of their groupmates. The students were engaged in SMP6, attending to precision, as they produced accurate graphs. One group of students decided to check their work with Desmos, so these students were also engaged in SMP5, using appropriate tools strategically. Mr. Estrada's goals for the lesson included communicating about the mathematics, both during the creation of the posters and the presentation of the posters at the end of class, but the students' decisions to work individually rather than collaboratively limited their opportunity to engage with others. It seems likely that if there had been time to present the posters, I would have seen more collaboration among the students as they prepared what they would present to the class.

With the entire class conducted in Spanish (which was certainly one way to support students' engagement in mathematics), it was difficult to elaborate the ways in which the EB students were supported in their participation in mathematical practices in the same way as I did in a class that was conducted entirely in English. However, Mr. Estrada did provide opportunities to clarify mathematical content through the use of gestures. He also wrote out the examples he wanted the students to complete at the beginning of the period, structuring the problems to include answer blanks and coordinate axes to signal to the students what he expected the students to do (see Figure 5.10). In his debriefing interview, Mr. Estrada expressed that he was confident his students could have done this lesson in English or Spanish since there wasn't a lot of vocabulary involved in this lesson beyond slope, *y*-intercept, and point of intersection.

Reflecting on Mr. Estrada's statement during his lesson planning interview that his students could do some lessons, like graphing linear equations and inequalities, in English or Spanish, but other topics would be more difficult, I wondered how he knew this to be true. Unfortunately, it did not occur to me at the time to ask why he was so confident about that. During my observation, students spoke Spanish throughout the class and the handouts were in English, but only included a topic and a brief sentence about sketching a graph. I did have a brief conversation with two young men sitting near me who were curious about what I was going to do with the video. One spoke only in Spanish, and the other was able to translate his question for me when I didn't understand all of his words, as well as communicate my answer back to the

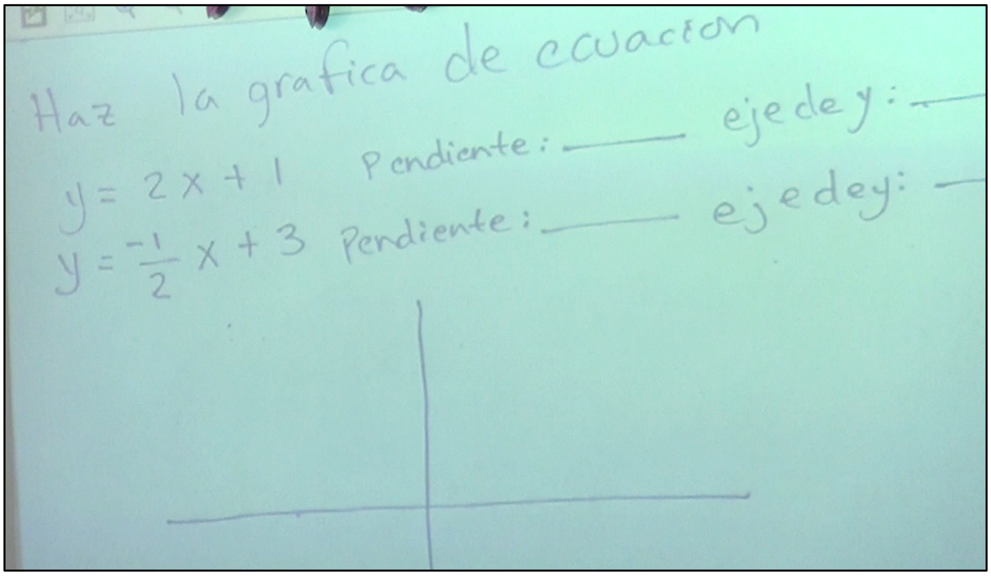


Figure 5.10. Mr. Estrada's second example, complete with answer blanks and coordinate axes.

student. Additionally, I took pictures of five of the posters the students created, and only one group labeled their poster “Graphing Lines” in English, while the rest were in Spanish. (This was also the title of one of the handouts the students were given, so I can’t really say for sure whether the students were using language they knew.) I also noted that after Mr. Estrada introduced the group poster activity, one of the students asked, “¿Cuál es el título?” (What is the title?) and Mr. Estrada responded, “Gráfica de ecuaciones lineales” (Graphs of linear equations). One could cite this is evidence that at least one of the students doesn’t know both English and Spanish for this lesson, but I wouldn’t wish to assume this isn’t just a student asking what the teacher’s expectations were for the lesson.

Case Study 5 – Ms. Ochoa, School B

Ms. Ochoa has worked in District B for 18 years, the last six of those years at School B, her alma mater. She described herself as a struggling math student when she was in high school, having failed two years of math in a row at her previous high school. She then transferred to School B her junior year and found a role model, a Latina counselor who taught one period of precalculus, then she fell in love with mathematics and went on to earn a bachelor's degree in mathematics, a bilingual teaching credential and a master's degree in Curriculum and Instruction. Knowing the impact that her high school counselor and math teacher had on her own life, she wanted to give back and perhaps become that role model for other students like herself. Ms. Ochoa described coming to work at School B as an opportunity to come home.

Over her years of teaching, Ms. Ochoa had taught all levels of mathematics except for Advanced Placement courses, and she was serving part-time as a Teacher on Special Assignment (TOSA) for the district as a curriculum specialist at the time of this study. In School B, she was also serving as the leader of the Integrated Mathematics 1 teachers, working alongside her peers to support the continued implementation of the CPM curriculum. At the time of the study, Ms. Ochoa was also supervising a student teacher, Ms. Carter. Additionally, Ms. Ochoa was finishing her final year of participation in a five-year NSF-supported professional development (PD) project at a local university. The goal of this PD project was not only to improve her own teaching practice but also to help Ms. Ochoa become a more effective teacher leader in her district.

Planning. At School B, the Integrated Math 1 professional learning community (PLC) divided up the work of lesson planning across the team. For every chapter, each teacher was assigned one lesson from the CPM textbook to plan on their own prior to meeting with the PLC. I joined the PLC team for two five-hour days during their spring break as they met to go over the individually planned sections for chapter eight, offering feedback and making adjustments to each lesson before it was finalized and placed in the unit packet created for the students. Day one of the planning sessions began with looking at the district final for their course, counting how many of each type of question appeared (e.g., ten questions related to exponential functions, eight systems of equations, etc.) and whether they have covered that topic or not. Next, the team discussed topics their students were still struggling with on the previous exam and what had been mastered. The PLC team adhered to the recommendations of the CPM authors regarding building fluency and mastery over time, as well as the ratio of current and previous topics on summative assessments. The authors of CPM recommend fewer questions for the current chapter on exams and more questions from earlier chapters that the students are still working toward mastering.

Ms. Ochoa then presented the PLC team with a packet of questions containing Preliminary SAT/National Merit Scholarship Qualifying Test (PSAT/NMSQT) and Smarter Balanced Assessment Consortium (SBAC) released items, asking them to compare to what they are presenting to the students from the CPM curriculum and to consider how well their students may be prepared for standardized tests. The teachers discussed the wording of the questions and the format of the information given, such as using a table with decimal outputs or a table with non-consecutive output values. They selected questions for the chapter eight exam, then kept these questions in mind as they reviewed the lessons they were planning. The rest of the day and the next morning was spent reviewing the nine lessons in the chapter. The teacher who planned the lesson, if present, walked the rest of the team through the lesson, pausing for questions as they went along. The PLC team discussed the mathematics in the lessons (e.g., why can't you do 0^0 and whether they need to discuss why the b in $y = b^x$ can't be negative), the formatting of the problems (e.g., enough space for answers or the ordering of the problems), and the ordering of the lessons. They created a poster highlighting the main mathematical goals of the lesson and which homework problems they would assign for each lesson. Throughout the planning days, it became clear that the teachers had adopted the CPM terminology for the concepts (e.g., initial value, multiplier, equal values method, multiple representations web) as well as activity structures (e.g., Gallery Walks, Mini Posters). While lengthy texts were broken up into smaller pieces and sentence starters were included in lessons, no explicit mention was made about planning supports for engaging EB students in mathematical practices. However, while discussing a lesson on linear and exponential regression using Desmos, SMP5 using appropriate tools strategically was mentioned as well as the idea of using this lesson as to differentiate instruction for their students who need extra challenge.

In her lesson planning interview, Ms. Ochoa described that on a typical day in her classroom I would see her focusing on student thinking and reasoning and providing the students with immediate feedback, yet still allowing for productive struggle that leads to “aha” moments. She discussed using a variety of teaching strategies and structured student interactions. While there are daily differences in the overall structure of her lessons, Ms. Ochoa emphasized that every lesson begins with either a warmup or homework review and ends with a closure activity, but the middle can look very different depending on the day. She then reviewed the lesson that had been planned by the PLC team during spring break. Originally, Ms. Carter, Ms. Ochoa’s student teacher, was scheduled to teach this lesson. However, Ms. Carter had requested that Ms. Ochoa teach a demonstration lesson for her and another prospective teacher from her teacher education course. This request slightly altered my data collection process for Ms. Ochoa in that she was planning for a lesson she was teaching during the next period instead of a day in the near future like the other participating teachers. Unfortunately, this meant a delay in between the class observation and the debriefing interview as Ms. Ochoa was not available to meet a second time that day. However, this also opened up an opportunity to observe her teaching on another day. (Ms. Carter was also a participant. On the day I observed her, she and Ms. Ochoa planned to demonstrate two different solution methods for solving a system of exponential equations, each teaching their preferred method.)

Ms. Carter and Ms. Ochoa spent a short amount of time discussing Ms. Carter's plan for the lesson that was not included in the student packet. They planned to add a "whiteboard drill" to build procedural fluency with exponential functions based on their observations from the previous class session that the students needed a little bit more practice understanding that exponential functions have an input and an output. Ms. Carter had also assigned homework problems, so Ms. Ochoa planned to project the answers to the homework problems and go over any questions with which the students had difficulty. Ms. Ochoa then did an unprompted think-aloud as she read through the printed lesson. This was quite beneficial for me to hear because it quickly became evident that she was considering multiple elements of the lesson simultaneously – the stated mathematical goal, the types of tasks in the lesson, what mathematics should be emphasized in each task to prepare the students for the next task, the participation structures she would use and when, how much time she should spend on each section, how she can assess what the students are understanding, and how to hold the teams accountable for collaborating and ensuring that all teammates understand the material. She did not work out any of the tasks during this planning time, perhaps because the PLC team also created teacher notes for the student packet. The teacher notes typically consisted of the solutions to all of the tasks, suggestions for activity structures, and notes about important mathematical concepts to emphasize.

Enactment. With the planning interview and enactment of the lesson happening so close together, it may not be surprising to report that Ms. Ochoa essentially taught the lesson exactly as planned. The only real difference I could identify was the amount of time spent on the activities. For example, she planned to spend a half an hour on the final teamwork tasks and accountability quiz, but only had about 13 minutes remaining in the class period for this activity (which turned out to be ample time). My overall impression of Ms. Ochoa's teaching was that she has a very positive rapport with her students, always speaking to them in a calm and friendly tone and offering encouragement and compliments about the mathematics and mathematical practices in which the students were involved.

The lesson began with homework review. Ms. Ochoa displayed the answers to the homework and answered student questions on the first two problems. She then displayed the other homework solutions and reminded the students that several math classrooms are open after school for math tutoring if they need more help on the homework. Next, Ms. Ochoa did the whiteboard drill in which one partner wrote down the exponential equation for a specific value of *b*, then handed off the whiteboard to his partner to evaluate the equation at a given *x*-value. As the students finish their problems and hold up their whiteboards, Ms. Ochoa offers immediate feedback (e.g., beautiful, very nice, that's it, try again, you're close). After a few rounds of this type of drill, Ms. Ochoa moved on to the warmup problem in the student packet, which involved writing an exponential function from a given table of values. A student volunteered to read the daily learning target (DLT) to the class, then Ms. Ochoa wrote it on the board on the displayed student packet and elaborated on the plan for the day. She then joked with the students that she was sure that the first thing they thought of when they woke up that morning was that they hoped they would get to learn about exponential equation, so she was there to make all their dreams come true. The students laughed.

Ms. Ochoa worked out the first warmup problem with the students, then asked them to work on the other two, first individually and then check in with their partner and compare answers. While they were working, Ms. Ochoa approached a student who raised his hand. He asked if his answer was correct and Ms. Ochoa encouraged him to ask what his partner thought about his answer. Before walking away, she also encouraged him to ask how his partner thought about the problem and to make sure he told his partner how he thought about the problem as well. I interpreted this move as Ms. Ochoa was simultaneously supporting productive struggle and encouraging mathematical communication. When the students had finished the warmup, Ms. Ochoa asked for a volunteer to share their partner's answer. The volunteer provided a multiplier of negative five instead of positive five. Ms. Ochoa wrote the answers as given and asked the class if they needed to make any adjustments. Another student quickly shared that the multiplier should be positive five and that she knew that because she had made the same mistake, but then realized that if she multiplied a negative three by a negative five, she would get a positive fifteen instead of negative fifteen. Ms. Ochoa complimented the students for their work and for the adjustments, reminding them that "we don't make mistakes, we make adjustments."

As they reviewed the final problem, Ms. Ochoa took the opportunity to discuss consecutive inputs, finding the multiplier by dividing an output by the previous output, and how to find the multiplier if you have non-consecutive inputs. During this discussion, Ms. Ochoa provided opportunities for students to check in with their partners. In one instance, she asked the students to take turns completing the following sentence: to find the multiplier using the outputs, ____. At other times, she told the students if they understood what she just said, then turn to your partner and ask your partner where did Ms. Ochoa confuse you? As they finish the warmup, Ms. Ochoa took another opportunity to praise the students for how well they are doing on the lesson and told them they were ready for the next task.

The next activity was set up as a team task with four parts. Ms. Ochoa reminded the students of the four team roles (task manager, resource manager, facilitator, and recorder/reporter) and their respective responsibilities. She also had a laminated team role sheet to give to each team. Each part of this team task has a situation and students are to make connections between different representations of exponential functions (situations, tables, and equations) to answer questions about the situation. The PLC team had added an activity structure to the tasks in which the teams were allowed to ask for three hints but would be awarded bonus points for completing the tasks without help. Additionally, there were stop signs after each of the four tasks indicating that the team needed to wait for the teacher's approval before going on to the next problem. Ms. Ochoa informed the students that for each situation they would be identifying the multiplier and the initial value and, in some cases, writing the equation. She then wrote $y = a(b)^x$ on the board for reference. About five minutes into the task, Ms. Ochoa called the resource managers to the side of the room for a huddle to discuss the meaning of the phrase *rebound ratio*. Armed with knowledge about the phrase and the numerical value of the ratio, the resource managers returned to their groups to share the information. As students continued to work on the team task, Ms. Ochoa monitored student progress and looked for ways she could compliment the students on their progress. When the first group (a pair) finished, the students were asked to serve as tutors to help Ms. Ochoa answer questions and check answers.

The next portion of the lesson was devoted to graphing three exponential equations and identifying the relationship between the initial value and the multiplier in the equations and where they can be found on the graph. Ms. Ochoa included a table representation in addition to the equation and graph and encouraged the students to make connections between all three. As soon as students completed this task, they were encouraged to move on to the closure task which involved identifying the initial value and the multiplier in all four representations in the multiple representations web (see Figure 5.11). This task would be the source of questions for the team accountability quiz to close the lesson. Teammates were responsible for making sure everyone in the team knew what *a* and *b* were in each representation and could explain how to find it. Ms. Ochoa used the team roles to randomly select two students in each team, asking them two questions each. The students were allowed to decide when their team was ready to take their quiz. If a student could not answer their question, it was up to the team to explain everything to that student and then Ms. Ochoa would return to ask the same student different questions.

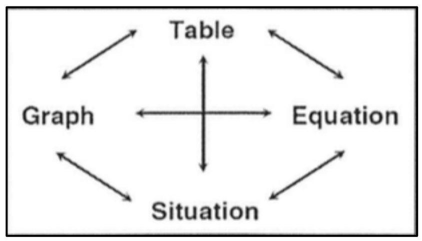


Figure 5.11. CPM's Multiple Representations Web (Dietiker et al., 2014, p. 437).

The students had the opportunity to engage in several mathematical practices throughout this lesson, including SMP1, SMP3, SMP5, SMP6, SMP7, and SMP8. The team task with four situations involving rebound ratios, computer viruses, and a rapidly growing technology company was an opportunity for the students to make sense of problems and persevere in solving them. Recall that Ms. Ochoa informed the students that they would need to find the initial value and the multiplier for each situation and, in some cases, write the equation and provided the general form of the equation on the board. This may have reduced the demands of the team task for the students since they now had information about how they needed to solve the tasks. Yet some of the students still struggled with the task due to the unfamiliar phrase *rebound ratio* as well as having a multiplier that was less than one (i.e., decreasing rebound heights). Students were searching for patterns (SMP7), generalizing to equations (SMP8), and communicating about the mathematics throughout the lesson (SMP6). Students used calculators (SMP5) and there was attention to accuracy (SMP6). Students were also given the opportunity to present solutions, critique the reasoning of others, and justify how they arrived at their answers during the accountability quiz (SMP3).

Ms. Ochoa provided many opportunities for the students to communicate throughout this lesson in a variety of structures (i.e., whole class, pairs, and small groups). While much of the communication among the students and teacher took place in English, the students were not discouraged from using their home language. For those students whose first language was Spanish, Ms. Ochoa was capable of and willing to help the students in Spanish as needed. Comparing the teacher-created work packet to the corresponding lesson in the CPM textbook showed that the PLC team had added the warmup activity to help the students recall what they had learned in a previous chapter about geometric sequences. The team task was taken directly from the CPM textbook, word for word, but the PLC team broke up the larger block of text into separate parts. For example, in the CPM textbook, the situation was presented immediately followed by two or three questions, all in the same paragraph. The PLC team separated the stem of the question (i.e., the description of the situation) from each of the questions and listed each of the questions separately, with their own number, and with blank space between each question to have room to write an answer.

The closure activity was significantly modified by the PLC team from its original form. In the CPM textbook, students were to reflect on a blank (no arrows) multiple representations web and decide what connections they have already learned, haven't discussed but think they know how they are connected, and what they still need to learn. The teams were encouraged to reflect on tasks they have completed in this chapter (and chapter five) and write those task numbers on the arrows they have drawn as evidence of having made the connection between the representations. Instead, the PLC team provided an example of each of the four representations and asked the students how to find the *a* and the *b* in each representation. This modification not only reduced the cognitive demand of the task, but also changed the nature of the task itself and the type of interactions the students could have. This task was no longer about self- or team-reflection about their learning, expressing their ideas in their own words, or considering what they may now be able to do even if it has not been taught yet.

Case Study 6 – Ms. Montez, School B

After completing her student teaching at School B, Ms. Montez was offered a job and she had been there ever since. At the time of the interview, she had taught for 17 years. Over those years, Ms. Montez had the opportunity to teach several versions of algebra courses (Extended Algebras, Algebra T, Algebra One), Algebra Two, Math 12, Precalculus, Finite Math, Integrated One, and Integrated Three. At the time of data collection, she was teaching Integrated Mathematics Three (IM3) and Discrete Mathematics. Ms. Montez earned a bachelor's degree in Mathematics, a master's in Educational Technology, and held a bilingual teaching credential.

When asked what a typical day in her class would look like, Ms. Montez described that her students would be working in groups because of the way the CPM curriculum is organized around team collaboration. She shared that she sometimes does direct instruction, but that most of the time the students "have their own time to discover and figure out the problems" before she does the direct instruction. I asked if she had any regular routines that happened every day, such as a warmup or and exit ticket. Ms. Montez responded that she didn't have regular routines and, instead, thought of her class as always having something new to learn each day unless they have an assessment.

Planning. Like the IM1 teachers, the IM3 teachers had also compiled a unit packet for the students. Ms. Montez explained that four or five years before the district adopted the CPM curriculum, she and the other IM3 teachers (Ms. Ochoa was one of them) co-created the lessons in this packet drawing from a variety of resources (she did not recall what they had used). She shared that their goal was to create activities that would develop the students' conceptual understanding of what a radian is and how to help students understand why the unit circle is labeled the way it is rather than just giving the students a conversion formula or handing the students a completed unit circle to memorize like they had seen in some of the curriculum resources. Ms. Montez shared that their current versions of the packets drew from both the CPM curriculum and the packets the IM3 team had created. As she flipped through the student packet, she indicated that most of it was the curriculum the IM3 team put together, but did point out a ferris wheel problem that was from the CPM textbook.

Ms. Montez first shared the overall goal for the lesson, which was to discover what a radian is and then finish building the unit circle, and then walked me through the rest of the lesson as she referenced the student packet. First, the students would complete a warmup activity to help students review pi and the circumference of a circle. Second, they would discover what a radian is through a measurement activity. Third, they would build up a unit circle by labeling the radian measurements piece by piece. They would start with the quadrantals (nineties), then the forty-fives, then the thirties and sixties. Afterwards, they would compile everything they had done on a single unit circle, all the radian measurements together with the degree measurements and the coordinate points. Finally, they would do some practice problems involving measurement conversions between degrees and radians and using the unit circle to look up values of trigonometric functions.

Next, I asked Ms. Montez to share a little bit more about her role and the students' roles while doing these activities. For the warmup, she planned to have the students work in their teams. She would be available to answer any questions they may have. She shared that she usually has the students come up to the board and answer the questions, and she would intervene only if there was a misunderstanding. For the first part of the radian activity, she would demonstrate the measurements for the students and have them do the activity with her first and then let them repeat the activity in their groups on a differently sized circle. For building the unit circle, she planned to do this section with the students because it was important for the radian labels to be correct. Ms. Montez wanted to avoid having the students make a mistake on this the first time they see it because she was afraid that such a mistake could persist. When they were ready to compile everything onto one unit circle, the students would still be sitting in their groups, but each student needs to have their own complete unit circle, so this may look more like individual work. The students would be allowed to work together in their groups on the practice problems.

Enactment. Like the other teachers who have walked me through their previously created (and in some cases, previously taught) lessons, Ms. Montez enacted her lesson almost exactly as she described to me in the planning interview. She asked the students to get started on the warmup and wrote the DLT (By the end of the lesson, I will be able to know equivalent degree and radian measures on a unit circle as evidenced by completing the investigation pg. 25-26.) on the board for the students to copy down in their packets. The warmup problems involve finding the circumference of circles, so Ms. Montez makes sure the students remember where the pi key is on their calculators. She then clarified the difference between an exact answer (when you write pi) and an approximation (when you convert to decimal). There were at least five more instances during the warmup activity that she explained the difference between exact and approximate. Ms. Montez had the students write their answers to the warmup problems on the board, then asked the students if there were any questions. Nobody asked questions, but Ms. Montez went over most of the warmup with the students in case there was any confusion.

Ms. Montez transitioned to the “What is a Radian?” hands-on activity, sharing what they would be learning about as she handed out strips of paper to each student for their measuring devices. (She also joked about this measuring device being high tech as she distributed them to the students.) Next, she demonstrated how to use the strip of paper to measure the length of the radius and then measure out the radians on the circle (see Figure 5.12). They discussed how many radians were in a semicircle (three and a little more, or π), how many in the whole circle (six and a little more, or 2π). Then the students repeated this activity with a larger circle in order to emphasize that the size of the circle does not alter the number of radians in a circle.

The remainder of the class period was spent determining the radian measurements on the unit circle, building up bit by bit, and then filling out a complete unit circle from all the pieces. Ms. Montez started with the quadrantal angles, referring back to the opening activity to help the students see where π and 2π should be labeled, and then moved on to 45-degree angles, then 30- and 60-degree angles. Ms. Montez lead the students through labeling each piece, asking questions about how to count and label the radians, and the students provided choral responses to her questions. Once they had filled in the radian measurements for the three separate circles, Ms. Montez asked them to fill out the unit circle on the next page, complete with degree and radian measurements and the coordinate points. After about six minutes, Ms. Montez projected her unit circle so the students could check to see if they had filled out their unit circle correctly because it

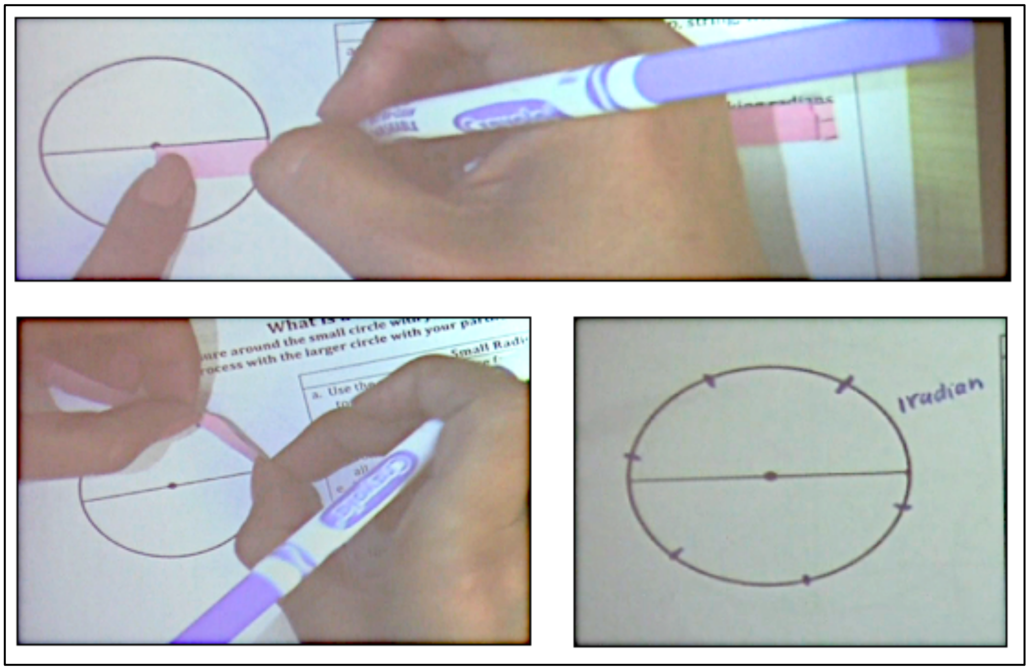


Figure 5.12. Ms. Montez demonstrating how to measure radians.

would be their reference for use on assignments and assessments. (The unit circle that was projected did not have the radian measurements labeled.) For the last ten minutes of class, the students worked on converting between radian and degree measurements and using the unit circle to find the value of trigonometric functions.

During this lesson, students were given the opportunity to engage in four of the mathematical practices. Most prominently, students were attending to precision (SMP6). In building the unit circle and then using it to solve practice exercises, students needed to specify units, label the angle measurements accurately, and calculate efficiently and accurately. Additionally, there were questions in the packet that encouraged the students to express in their own words what a radian measure is and what was the difference between degree and radian measures, which gave the students the opportunity to communicate precisely. In the hands-on radian measurement activity, students were engaged in SMP5, using appropriate tools strategically, (the strips of paper used to measure the radians) and SMP8 by repeating the measurement activity with a larger circle and comparing the two outcomes to notice that the number of radians in a circle remained constant. In the warmup exercises, students used calculators to give approximate answers (SMP5) and reasoned abstractly and quantitatively (SMP2) when solving exercises such as the one in which they figured out how far a piece of gum stuck to a tire travels if the tire rotated one-third of a rotation. Problems of this type also engaged students in SMP1, make sense of problems and persevere in solving them.

There were several instances during the lesson in which the planned activities and Ms. Montez's enactment of the activities provided additional supports for EB students. First, Ms. Montez highlighted (multiple times) the difference between an exact and approximate solution. She mostly explained these terms verbally, but she also pointed to exact and approximate answers on the board as she spoke about them. Second, students were engaged in a hands-on activity to measure radians. Students had the opportunity to build an intuitive sense of what a radian measurement is through physically measuring out the radians on two separate circles. Third, Ms. Montez gestured as she spoke to the students throughout the lesson, like other teachers have also done. For example, when she talked about the circumference of a circle, she would draw a circle in the air with her finger and talk about the distance around the circle. Finally, Ms. Montez was able to translate information into Spanish if anyone needed additional assistance. During the planning interview, she emphasized that she doesn't speak in Spanish for the whole class but will speak Spanish when addressing individual questions. I wish I had asked her to elaborate on this distinction, but I can offer an educated guessed based on the context of the conversation. Ms. Montez was answering a question about what accommodations she makes for EB students in her class, so she specified that for Spanish speakers, she can translate for them, but she's unable to do that for students who speak other languages. It seemed that she emphasized this due to her recognition that her students aren't all Spanish-speaking English learners, so this wasn't an accommodation that would be helpful as a whole class strategy. However, one may wonder if there were other, perhaps politically-motivated, reasons behind her statement, given the history of bilingual education in the state of California.

Cross-case Synthesis

In this next section, I will provide a cross-case synthesis of the curriculum and teacher supports for engaging EB students in mathematical practices. Starting with supports for mathematical practices, I will elaborate on the teachers' interpretations and enactment of mathematical practices. Then I will provide a detailed illustration of SMP6 and the variety of ways the participating teachers interpreted *attend to precision.* Next, I will focus on the ways in which EB students were provided additional supports and provide a comparison among the six case study teachers' enactments of supports for their EB students.

Mathematical Practices

Curriculum and Teacher Support for Mathematical Practices. In Chapter 4, I discussed the potential for the four published curriculum materials and the teacher-created materials to provide supports for engaging students in mathematical practices. To summarize, two curriculums (CME Project and TTA) focused on developing mathematical Habits of Mind (which they connected to the SMPs) over time and the other two (CPM and MVP) directly connected each lesson to the specific practice standards in which they intended students to engage. All of the teacher-created materials emphasized SMP6, attend to precision, through producing accurate graphs and representations, calculating accurately, specifying units, or (in a few lessons) communicating precisely. While some of the teacher-created materials provided opportunities to engage in other practices (SMP2, SMP5, SMP7, SMP8), none of these were common across all of the materials and most of the potential for these practices appeared in only one lesson.

However, neither the published curriculums nor the teacher created materials adequately reflect how these lessons were interpreted or enacted by the teacher. Recall in Remillard (2005), that the enacted curriculum reflects what actually occurs in the classroom when the teacher's plans are used with his students. Using the same coding scheme for SMPs that I refined with a second coder for the curriculum analysis, I coded the video content logs (detailed summaries of the classroom activities, broken into episodes by task then participation structure, that also included selected transcriptions of moments in which teachers provided opportunities to engage students in SMPs or strategies to support EB students' access to or engagement in the lessons) for evidence of SMPs. I then analyzed the frequency (how many different times in the lesson an SMP was coded) and the intensity (e.g., use a calculator to compute a result vs. discuss what one can learn by analyzing the graph or knowing that one needs to attend to the order of operations and use parenthesis to get the correct answer) of the potential engagement in SMPs.

Figure 5.13 represents the results of coding the teachers' enactment of mathematical practices on the day I observed their class. Note that this is not a reflection of student participation in the mathematical practices, as student participation was not the focus of my research questions. In the diagram, red not only indicates that the teacher provided many opportunities throughout the lesson to engage in that practice, but also that the enactment deeply reflected the elaborations provided in the Common Core Standards (NGA & CCSSO, 2010). Orange represents there was a moderate level of opportunity to engage in the practice. Yellow indicates a low level of intensity in the enactment of the standard, such as emphasizing correct answers and specifying units in SMP6 (without attention to precise communication, use of definitions, etc.). Grey indicates there was no evidence of the standard happening during this lesson. Like the analysis of the lessons in the curriculum, it is important to emphasize that this data reflects a single snapshot of one lesson on one day in the school year for each teacher. Not only is this data tied to a single observation, the set of mathematical practices enacted on that particular day are likely to be uniquely tied to the mathematical content in that particular lesson.

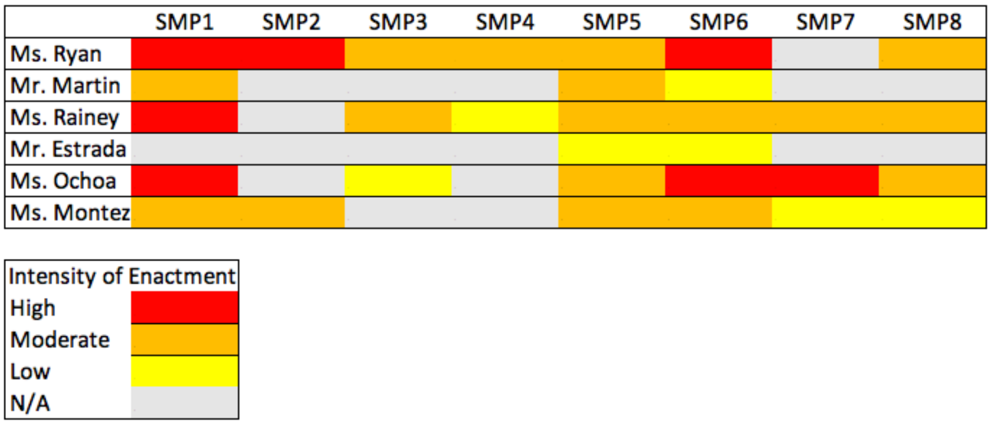


Figure 5.13. Coding of teacher enactment of mathematical practices by intensity.

Recall that I have adopted a situated sociocultural theory of learning for this study. As such, my interpretation of the SMPs necessarily involves student engagement in mathematical communication, preferably with other students. This certainly influenced my interpretation and coding of the enactment of the lessons and it's very likely that others would have different interpretations of the practice standards and what they may look like in a classroom. In fact, this relates to one of my earliest observations that occurred to me as I collected my data; the participating teachers were talking about the same eight mathematical practices, but their interpretations of these standards, the ways they talked about engaging students in those standards, varied from teacher to teacher (and didn't always match the intent of the CCSM elaborations of those standards). I will provide a specific illustration of different interpretations of SMP6 in the next section.

As shown in Figure 5.13, SMP5, use appropriate tools strategically, and SMP6, attend to precision, were represented (to some degree) in every lesson taught by the case study teachers. The elaboration for SMP5 not only mentions the use of tools (e.g., calculators, rulers, protractors, paper and pencil, software, etc.), but also knowing when the tools should be used and the affordances and limitations of using those tools. Perhaps too generously, I coded a moderate level of intensity if students were instructed to use tools during the lesson, reserving a high level of intensity for instances of thinking about the utility of those tools. (Ms. Montez came the closest to having a high level of intensity because she asked the students to explain why the calculator gave different results for $540/2\pi$ versus $540/(2\pi)$, leading to a short reminder of the order of operations.) There are two other instances of SMP5 that stand out to me. In Mr. Estrada's class, one student decided to take out his laptop to use Desmos to check his group's graphs before transferring them to their poster. As this was not part of the planned lesson, nor did the teacher initiate this idea, nor did any other student used this tool, I coded it as low intensity engagement in this practice. The other case I wish to highlight is that of Mr. Martin. When he and Mr. Hepner talked about SMP5 and using tools, they referenced the students' prior mathematical knowledge and a tool that can be used in a new context. For example, students use their knowledge of graphing a linear equation when they graph a two-variable inequality (see Figure 5.14).

Working with a second coder while analyzing the four lessons discussed in Chapter 4 and using the constant comparison method (Strauss & Corbin, 1994) to verify I was applying the SMP codes consistently, reinforced my impression that individuals may hold varied interpretations of the SMPs. As I refined my code book, one of the later additions was that a task must have a context in order to apply SMP2, reason abstractly and quantitatively, and SMP4,

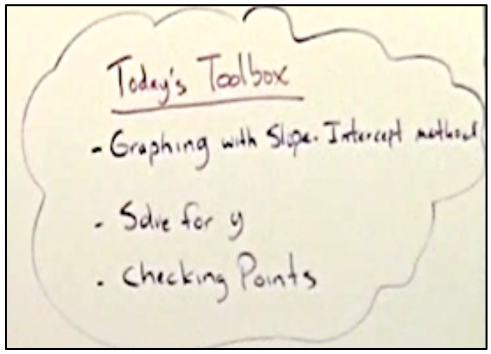


Figure 5.14. The set of tools used in Mr. Hepner's graphing two-variable inequalities lesson.

model with mathematics. By a context, I was referring to a task in which there was a situation or real-world application to consider, not simply answering questions about an equation or graph. SMP1, make sense of problems and persevere in solving them, was briefly included as needing a context then later removed as we found evidence of considering similar problems, making conjectures, and using a different method to check one's answer in tasks that didn't provide a realistic situation to accompany the mathematics. With this in mind, I return to my analysis of the enacted lessons. Note the low occurrence and intensity of codes in SMP2 and SMP4 in Figure 5.13. Two of the teachers, Mr. Martin and Mr. Estrada, had lessons on graphing linear inequalities and both expressed that their goal was to build procedural fluency. Both teachers chose to use practice exercises without contexts, which is one explanation for coding a lower incidence of engagement in mathematical practices than in the other case studies. It is unlikely that we would see this limited engagement in SMPs if I had observed their classes over a longer period of time, teaching other IM1 concepts.

One can see from Figure 5.13 that Ms. Ryan's lesson on graphing related quantities incorporated the most mathematical practices. Recall that Ms. Ryan taught a combination IM1 / SIFE class in which all students were classified as ELs, most of whom had only been in the US for less than a year. Her mathematical goals for this lesson were to build conceptual understanding of graphing quantities over time while developing intuition about graphs of functions. Additionally, Ms. Ryan incorporated general language goals (not necessarily content-specific) in her class every day. She emphasized the importance of having her students listen to, read, write, and speak the language of instruction to build their language proficiency and to prepare the SIFE students for their next mathematics class. Her lesson included sharing student solutions to tasks and discussing the features of the graphs they created, comparing solutions, evaluating and correcting misunderstandings, and building academic vocabulary to talk about features of graphs. While her class represented the most diverse population with the highest proportion of students classified as ELs (100%) and perhaps the most linguistically diverse (most languages) of all the case studies, the lesson she taught on her observation day most closely resembled my vision of engagement in mathematical practices, having taken on a sociocultural lens on learning mathematics.

What is not captured in this analysis of the mathematical practices is the fact that the majority of the participating teachers (82%) talked more generally about students' participation in mathematics rather than citing specific SMPs when asked about practices during their interview. They emphasized interacting, collaborating, working effectively in groups, and discussing mathematics. Some teachers referred to mathematical discourse as the "Common Core way" or the "CPM way" when they shared about having students work together. Considering the enactment of the lessons I observed, mathematical communication is also widely interpreted. While I may have imagined student-student communication, some teachers appeared to consider teacher-student communication and choral responses in whole class discussions as satisfying the perceived call for mathematical discourse. Others who expressed a commitment to student-student communication did not provide students with groupworthy or discussion-worthy tasks (Cohen, 2002), instead they completed practice exercises (Schoenfeld, 1992) together.

Another observation is that coding for enactment of mathematical practices does not capture the additional ways in which the teacher supports her students to engage in the SMPs beyond providing appropriate tasks and participation structures. While observing Ms. Ochoa and again while analyzing her teaching, I noticed her positive interactions with the students and how consistently she looked for ways to encourage and praise students for their diligence. She made sure that there were multiple opportunities throughout the lesson to attend to student reasoning and to offer them feedback. Other teachers (e.g., Mr. Martin, Ms. Ryan, Ms. Rainey, Mr. Estrada) also expressed a desire to give students immediate and ongoing feedback during their lesson planning interviews and did so during the classes I observed. Coupled with giving feedback was an expressed desire to build students' confidence and self-efficacy in mathematics. Sometimes the teacher feedback to students was strictly evaluative (e.g., nice job, that's it, good work, almost there, try again), while other times students were complimented on facets of their individual or work (e.g., your graph looks amazing, I like how you decided to organize the information in a table, I like how you guys are struggling through this together). This constant reassurance cultivated a positive classroom environment and seemed to motivate the students to persevere in completing the assigned tasks.

Finally, teachers that effectively facilitated group work and utilized team roles had a higher incidence and intensity of potential student engagement in mathematical practices. While this observation is certainly influenced by my theory of learning mathematics and my criteria for coding mathematical practices, I found similar results in my modified ELSF coding of the lesson enactments that I will discuss later in this chapter. Assigning (and utilizing) group roles signaled to students that they were expected to work together, whereas having students sitting at a grouping of four desks did not – even when students were assigned to create a group poster. Reflecting again on Ms. Ochoa's class, her use of team roles supported the students' engagement in mathematical practices. For example, when she saw that many of the teams were struggling with the phrase "rebound ratio," she called for a huddle with the resource managers from each team, discussed the phrase, and sent them back to their teams as the expert. This action supported both perseverance (SMP1) and precise communication (SMP6).

SMP6 and the Lack of Precision on Precision! As mentioned previously, the participating teachers' interpretations of the SMPs varied, both from each other's descriptions and from the elaboration of this practice standard in the CCSSM (NGA & CCSSO, 2010). For this illustration, I will include all eleven participating teachers in my analysis, not just the six case study teachers discussed in this chapter, providing an in depth look at SMP6*, attend to precision*. The following description is the elaboration of this SMP in the Common Core State Standards for Mathematics (CCSSM):

> Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. (NGA & CCSSO, 2010, emphasis added)

This description indicates SMP6 is about far more than getting a correct or accurate answer to a given problem, a more common interpretation of the phrase *attend to precision*. This description from the CCSSM highlights that clear communication, both in words and mathematical symbols, appears to be the ultimate goal of this practice standard.

All eleven teachers indicated that they were familiar with the eight SMPs and at least eight had a poster listing the SMPs in their classroom. However, there were two teachers who said they were familiar with the SMPs but could not list any off the top of their heads. One of these teachers (who had twelve years of teaching experience) discussed pedagogical strategies and listed the mathematical skills the students were doing during the lesson. The other teacher (who had forty years of experience) discussed her strategy of getting students to “bump into” a mathematical idea instead of handing it to them, credited the CPM curricular materials for helping her facilitate more effective group work, and talked about allowing students to struggle and develop tenacity.

Of the remaining nine teachers, eight specifically mentioned *attending to precision* in their lesson planning interview, debriefing interview, or both. The teacher who did not talk about attending to precision was an intern teacher who was focused on having the students *justify* why what they were doing made sense. While he didn’t specify attending to precision as a mathematical practice that he emphasized, he did talk about encouraging his students to communicate the math clearly and precisely. Perhaps foreshadowing what is to come, while communicating precisely is part of the elaboration of attending to precision in the CCSSM, this didn’t seem to be part of this teachers’ understanding of this SMP, and he did not connect his emphasis on precise language to SMP6.

For the eight teachers who did mention SMP6, *attend to precision*, there was a striking variety in how they described it. Some of this variety can be attributed to the topic of their lesson on the observation day (see Table 3.2 for topics), while other aspects of this variation are likely due to differing teacher interpretations of the SMP. Referring back to the italicized parts of the elaboration of *attend to precision*, (a) one teacher referred to communicating precisely – “So they need to just be able to use the language properly so that they could communicate what they're thinking to other people" (Mr. Leong, debriefing interview); (b) three teachers talked about accurately labeling, scaling, and sketching graphs – "paying attention to detail as far as the graphing and the shading of the region and just kind of that fine-tuned precision" (Mr. Hepner, debriefing interview); and (c) one teacher talked about calculating accurately and efficiently as he described that his students were checking their answers with each other or on Desmos instead of asking him – "attentive to checking themselves that they were doing the problem correctly" (Mr. Estrada, debriefing interview). Of the remaining three teachers, two described *attend to precision* as being correct all the time or making no mistakes – "attend to precision all the time, you know, be correct" (Ms. Montez, lesson planning interview). The final teacher referred to *attend to precision* in reference to her graph and situation matching activity - "they had to attend to precision, I mean you gotta really pay close attention for that matching, like you really had to look closely and read and understand what it's saying" (Ms. Ryan, debriefing interview). While there is a common thread of correctness in each of the teacher's descriptions, one might consider these statements as fitting on an "attend to precision continuum" of sorts, with making no mistakes at the low end, paying attention to details somewhere in the middle, and reflecting on and communicating about details at the high end.

In two classes, Ms. Montez and Mr. Turner were both conducting lessons on circles and the tasks involved computing with pi. Students were asked to give both exact and approximate solutions, as shared in Ms. Montez's case study. This confusion between exact and approximate answers also came up in Mr. Turner's Integrated Mathematics Two class I observed at School B (same school as Ms. Montez). In that case, early in the lesson the teacher encouraged the students to use the pi button on the calculator instead of just multiplying by 3.14 because they would get a more "exact" answer. I had wondered if this contributed to the students' confusion in his class, but Ms. Montez's students having the same difficulty lead me to wonder about why this distinction is so complicated for students, yet it also illustrates why it is important for us to communicate precisely about mathematics.

While I focused on *attend to precision* for this illustration, similar results could be found with other SMPs. For example, *use appropriate tools strategically* seemed only to mean use of calculators, rulers, or other physical objects for some teachers, while others considered prior knowledge of mathematical procedures, such as using the slope-intercept form of the line to graph the equation of the line, as the tool needed for the lesson at hand. Hearing the different interpretations that teachers have of the SMPs, one possible takeaway from this work is the importance of not assuming that we are all talking about the same idea when we use phrases from the standards such as *attend to precision*.

For EB students, precision with respect to communication is paramount for their opportunity to develop academic literacy in mathematics (Moschkovich, 2015). It is important that we expand our notion of *attend to precision* to include more than correct answers, no mistakes, or carefully labeled axes. As Moschkovich (2015) argued, it is essential for teachers of EB students to adopt a complex view of mathematical discourse, which goes beyond spoken and written words. Gestures, diagrams, tables, graphs, physical objects, and informal everyday language are all a vital part of the mathematical discourse that is situated in a classroom, and these can be resources for EB students and their teachers (Dominguez, 2016; Shein, 2012; E. Turner et al., 2013). Perhaps most importantly, the prevalence of the teachers' invocation of *attend to precision* and their divergent ideas of what this SMP means raises the issue of whether linguistically diverse students have the opportunity to develop this practice.

Emergent Bilingual Students

Curriculum and Teacher Support for Emergent Bilingual Students. In Chapter 4, I presented my analysis of the curriculum and its potential for supporting emergent bilingual students, based upon using the English Learners Success Forum's *Guidelines for Improving Math Materials for English Learners* as a coding system. For my analysis of the teacher provided supports for emergent bilingual students, I modified the ELSF Specifications (see Appendix I) to reflect teacher actions (and eliminated five Specifications, four that referenced building supports over time and one about summative assessment), then coded the lessons holistically, looking for whether the teacher actions occurred never, once, or more than once during the lesson.

A second coder analyzed half of two separate lessons with this coding scheme, and we had 87.8% agreement on one lesson and 97.5% agreement on the other. Because we coded holistically, the discrepancies on the first coding was due to my difficulty with considering only the portion of class the coder observed (one continuous hour of a two-hour class) when I had observed and analyzed approximately 4.5 hours of this teacher's classes. For the second class, I asked him to watch and analyze one hour of the 90-minute class, but this time broke the hour up into three discrete chunks so he would view each activity that happened during the class. The results are displayed in Figure 5.15. In this diagram, green means the teacher enacted the ELSF Specification more than once, yellow means once, and red means not at all. Grey indicates there are no additional Specifications for that Guideline (i.e., only Guideline 4 has four Specifications). From this visual representation, one can not only see the different profiles of the participating teachers selected for the case studies and the degree to which they provided supports (green and yellow) for emergent bilingual students, but we can also look across each row of the charts to quickly assess which ELSF Specifications were not provided (red) by any teacher on the day they were observed.

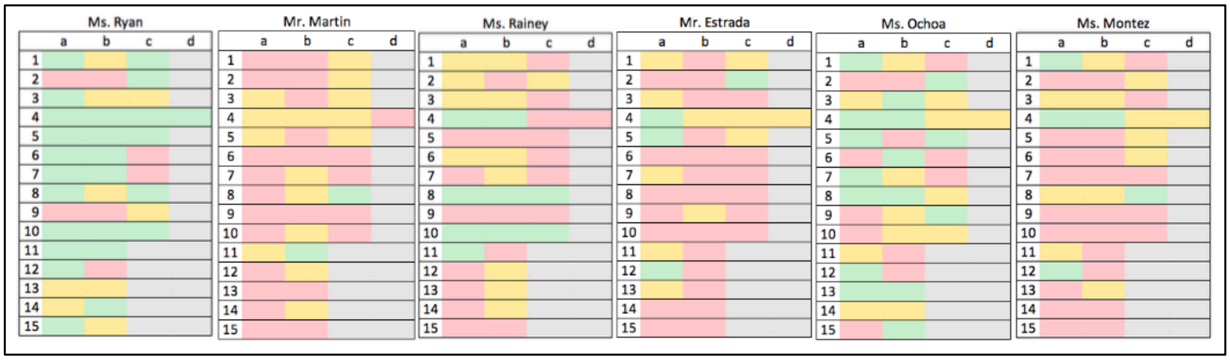


Figure 5.15. ELSF teacher enactment coding analysis.

As with the coding of the curriculum, one must keep in mind that this is a single snapshot, only one lesson, taught by each teacher. Like the curriculum analysis, I interpret these results in some ways as the minimum level of supports provided by these teachers, acknowledging that on other days with different mathematical content, the teachers would likely have provided a different set of supports – perhaps more, or perhaps fewer. The need for additional supports for EB students widely varies across teachers due to the nature of the content of the lesson, how many EB students are in a class, and the levels of language proficiency represented.

One can clearly see the most supports (green and yellow) in Ms. Ryan's graph. Recall that Ms. Ryan's class was composed entirely of newcomers – every single student was classified as an EL – and the class was a combination IM1 / SIFE class. The students on the SIFE (Students with Interrupted Formal Education) roster needed support in learning the language of instruction as well as building foundational mathematical knowledge in order to take a math course the following school year. With this explicit focus of simultaneously developing language and mathematics skills, Ms. Ryan and her co-teacher met far more of the modified ELSF

Specifications than any other participating teacher. In contrast, recall that Mr. Estrada also had a class in which all of the students were classified as ELs. However, his class was a bilingual IM1 class and his EB students all spoke Spanish, as did he. Note the preponderance of red in the profile of this day in his class. The nature of the lesson (a review assignment on graphing linear equations and inequalities) and the fact that the class was conducted in the students' first language greatly reduced the number of ELSF Specifications that could be coded in this lesson. The students didn't need language supports to make sense of unfamiliar contexts or vocabulary or to engage in mathematical practices (recall there were fewer opportunities to engage in mathematical practices in this lesson). With the intent of the ELSF Specifications focused on improving supports for English learners to engage in mathematical practices, coding this bilingual class in which there were no instances of learning or doing mathematics in English on this day resulted in very few opportunities to identify the ways in which the students may have been supported.

Ms. Ochoa's profile shows that she met approximately 70% of the recommended supports for ELs. Her utilization of team tasks, group roles, and structured interactions provided more opportunities to and more supports for students to engage in mathematical practices. She encouraged student participation, asked students to explain things to each other in their own words, and planned for a variety of interactions and activities that incorporated discourse. Contrasting her profile with Mr. Martin's (about 41% of the recommended supports), the differences in their classes are primarily attributable to their use of contextualized versus decontextualized tasks, group work versus whole class choral responses, and the number of opportunities for students to share their thinking with their peers (many times versus once).

Supports that are notably absent from all of the teachers' enactments of their lessons are ELSF Specifications 2a, 2b, 7c, 9a, and 15a. Specifications 2a and 2b call for objectives to include not only mathematical content, but also mathematical practices and language development. While almost all of the teachers had a written objective for their lesson, the majority of these objectives focused only on content standards. In Specification 7c, students are given the opportunity to revise mathematical writing – their own, a peer's, or a fictitious student's – to develop mathematical rigor through language. In some classes, students were given the opportunity to write something in their own words, but there was not an emphasis of discussing what was written for the purpose of improving the clarity of communication in written work. It is possible that Specification 9a, student-student discussions that build on each other's ideas, naturally occurred while students worked together on tasks, but the specification calls for teacher cultivation and facilitation of these types of discourses. During whole class discussions, there weren't instances of students building on another student's ideas. The closest type of interaction would be when the students made corrections to another student's work, but this conversation was typically negotiated through the teacher, not student to student. Finally, the ability to code for Specification 15a, teachers do not conflate language proficiency and mathematical proficiency, was not captured by my interview and observation protocols. While some teachers made statements from which I may be able to infer that some do interpret lower level language proficiency as lower level mathematics proficiency, I chose not to make such inferences given the limited data I had available.

Explicit Use of Strategies for Supporting Emergent Bilingual Students. During the lesson planning and debriefing interviews, I asked the teachers about the supports they provided for ELs and how their curriculum resources helped them make accommodations for ELs. Many of the teachers expressed that their written curriculum was very wordy or had a lot of reading, written at a level that is often beyond their students' reading level, and that the suggested accommodations are not enough for their students. To make the curriculum more accessible to their students, the teachers broke up longer narratives into shorter pieces, perhaps by separating the problem stem from the questions (see Figure 5.16). Others added diagrams, pictures, or other visual representations to support students' access to the content. Several teachers emphasized that they spend a lot of time launching the tasks with their students to check for understanding of the problem situation before asking them to work on solving the task (this reflects some teachers' interpretations of SMP1, make sense of problems and persevere in solving them).

c. A computer virus is affecting the technology center in such a way that each day, a certain portion of virus-free computers is infected. The number of virus-free computers is recorded in the table at right. How many computers are in the technology center? What portion of virus-free computers is infected each day? How many computers will remain virus-free at the end of the third day? Justify your answer.

Days Since Virus Started	Uninfected Computers
0	27
1	18
2	12

d. A computer virus is affecting the technology center in such a way that each day, a certain portion of virus-free computers is infected. The number of virus-free computers is recorded in the table at right.

i. How many computers are in the technology center?

Days Since Virus Started	Uninfected Computers
0	27
1	18
2	12

ii. What portion of virus-free computers is infected each day?

iii. How many computers will remain virus-free at the end of the third day? **Justify your answer.**

Figure 5.16. (Top) CPM textbook presentation of computer virus task. (Bottom) Teacher modifications of this task. (Dietiker et al., 2014, p. 437)

Several teachers spoke about building or emphasizing vocabulary – breaking down longer words into parts or root words, relating words to synonyms of the word, or comparing the everyday use of a word like *reflection* to its mathematical meaning. Some teachers discussed adding sentence frames and word lists to their lessons. Others encourage partner or group talk so that students have the opportunity to talk about mathematics without the intimidation of speaking to the whole class (or the teacher). Teachers who reported being bilingual translated the information for a student who is struggling to understand the math in English. If they don't speak the same language, many teachers turned to translation software for assistance. Teachers also relied upon other students to assist EB students. Teachers described carefully assigning students to groups based upon their prior mathematical achievement and/or language abilities, some high and some low in each group. Others relied on activity structures such as appointment books to give the students the opportunity to work with someone who speaks the same language as they do (when possible). In Ms. Ryan's diverse class of newcomers, she chose to group students with different language backgrounds to facilitate the learning and use of English to communicate with each other rather than relying on speaking their home languages.

Communicating or speaking in mathematics was valued by all of the teachers, yet the teachers' facilitation of communication was enacted in various ways. Some teachers primarily used short choral responses to questions asked while solving tasks, while others encouraged partner talk or group interactions. Others limited the amount of spoken and/or written words used in lessons and assignments. Some teachers built procedural fluency before introducing contextualized problems. Others preferred contextualized, realistic tasks. Some chose to challenge their students on assignments to see how much the students could handle (both mathematically and linguistically). Others felt it was important to focus on fewer pieces of information at a time with more repetition and less language demand. Similar to the varying interpretations of the SMPs, the quality and quantity of mathematical discourse reflected in these descriptions and enactments of student communication offer different levels of access to the students' development of mathematical and language skills. According to Vygotsky (1987) language is developed through social interaction, language and thought are reflexive, and higher thought processes come from mediated activity, so the more opportunities we provide to students to engage with mathematics and their peers in a variety of ways would seem to increase the likelihood that they will learn both language and mathematics.

To analyze the classroom observation videos, I created a coding scheme based on the recommendations from research compiled by Chval and colleagues (2015). My a priori coding scheme (see Table 5.1) was based on these recommendations. As I analyzed my field notes and video content logs, I discovered items that felt like supports for EB students that weren't captured in this list. I temporarily coded them as Other, adding a comment about what was happening to support EB students. Two additional codes emerged from this data – Partner/Group Work and Feedback/Encouragement/Motivation.

The first additional code for partner/group work emerged while analyzing Ms. Ochoa's class and recognizing that calling a huddle of all the resource managers from each team was a means for supporting student engagement in SMPs. Giving students the opportunity to work with a partner or a group opened up additional spaces in which students, particularly EB students, could communicate mathematically with others. This may have helped students who are reluctant to participate in whole class discussions have an opportunity to express their mathematical thinking. Additionally, students may have felt free to speak more informally, ask more questions, or use their home language in these partner or group conversations. However, the degree to which students were given these opportunities varied from class to class. For example, Ms. Ochoa assigned and utilized team roles throughout her lesson, including administering a team accountability quiz at the end of the lesson. After displaying a geometric

Table 5.1. A priori Codes for Supporting EB Students.

Code Book for Enacted Supports for Emergent Bilingual Students

Definitions	Codes
Connect mathematics with students' life experiences and existing knowledge	Connect math to life / prior knowledge
Create classroom environments that are rich in language and mathematics content	Math- & Language-rich
Emphasize meaning and the multiple meanings of words – students may need to communicate through using gestures, drawings, or their first language while they develop command of the target language and mathematics	Word meanings
Use visual supports such as concrete objects, videos, illustrations, and gestures in classroom conversations	Visual supports in conversations
Connect language with mathematical representations (e.g., pictures, tables, graphs, and equations)	Connect language with math
Write essential ideas, concepts, representations, and words on the board without erasing so that students can refer back to it throughout the lesson	Visual references
Discuss examples of students' mathematical writing and provide opportunities to revise their writing	Discuss and revise mathematical writing

sequence, Ms. Rainey asked the students to turn to a partner and finish the following sentence, "The pattern is _____." Mr. Martin assigned students to work in pairs or small groups during the independent practice portion of class, pairing up students he noticed having struggled with the guided practice with a more capable peer (a group of students was also assigned to work with the co-teacher). Both the duration of participation in and the potential for quality mathematical discourse varied widely in these three examples of partner or group work.

The second code, *Feedback/Encouragement/Motivation*, emerged from the data as I found several instances (at least 23 times that were reflected in my field notes and video content logs across six teachers) throughout the 90- and 120-minute classes in which the teachers were either evaluating student work, praising students for their productive participation, or encouraging students to keep going. Four of the six case study teachers had expressed how important it was to give their students immediate feedback, suggesting that when the students know how they are doing, they feel more confident and are more motivated to keep going.

Another change I made to the a priori coding scheme was to separate *connect mathematics with students' life experiences and existing knowledge* into two codes, one for connecting to life experience and the other to prior knowledge. I had coded several instances of teachers reminding their students about mathematics they had previously done in class and fewer instances of connecting math to students' life experiences. Recalling previously learned mathematical facts (or perhaps re-teaching facts that students were presumed to have learned), in comparison to drawing upon and connecting to mathematics in students' lived experiences, felt fundamentally different – knowing through having personal experience with versus knowing because you've been told something before.

The final coding scheme and counts from the enacted lessons appear in Table 5.2. The remaining code for *Other* reflects items that were unique to a single teacher, such as Mr. Martin's repeated use of invented phrases (e.g., "chop chop" and "a solution in the shade") that communicate something mathematical in his class (but perhaps not to the mathematical community) or Ms. Ochoa's voluntary collection of cell phones (in exchange for a homework ticket) at the beginning of class to promote a distraction-free environment, which helped the students focus on their team tasks. Though in quite different ways, both of these examples appeared to have an effect on students' ability to engage in mathematical practices.

To compare the use of supports for EB students across teachers, I created a Document Portrait in MaxQDA. This representation provided a visualization of the proportion of the classroom observation data that was coded for a support for EB students in relation to the whole document. (Each code in Table 5.2 was assigned a unique color in MaxQDA.) In Figure 5.17, I

Table 5.2. Results of Coding Supports for EB Students for the Six Case Study Teachers.

Counts of Supports for EB Students in the Enacted Lessons

	Teachers					
Codes	Ms. Ryan	Mr. Martin	Ms. Rainey	Mr. Estrada	Ms. Ochoa	Ms. Montez
Connect Math to Life	1	0	2	0	1	2
Connect Math to Prior Knowledge	8	3	3	0	1	3
Math- & Language-Rich	9	1	1	0	4	1
Word Meanings	7	2	6	0	1	7
Visual Supports in Conversations	11	6	6	1	4	11
Connect Language with Math	12	2	1	1	3	4
Visual References	1	4	1	0	2	1
Discuss and Revise Mathematical Writing	1	0	0	0	0	2
Partner/Group	1	2	2	2	8	0
Feedback/Encouragement/Motivation	5	7	0	0	10	1
Other	0	8	1	1	2	2

provide three of these document portraits to illustrate the differences between classes with low, moderate, and high levels of support for EB students. Perhaps not surprising, the high level of enactment of supports for EB students was in Ms. Ryan's IM1/SIFE class in which all students were recent newcomers to the US. Out of necessity and a strong commitment to prepare the students for future academic work, her class had an explicit focus on developing language and mathematics simultaneously. What is notable, however, is that she was developing rigorous, conceptual, grade-level mathematical knowledge throughout this lesson, developing an intuitive understanding of functions through graphing everyday situations that were accessible and relatable to her students.

In contrast, the document portrait with low incidence of supports for EB students was in Mr. Estrada's bilingual IM1 class. Because the class was conducted entirely in Spanish (which, arguably, is a means of support for the EB students in this class), there were far fewer opportunities to code ways in which students' language development were supported, as my research questions and coding scheme were written with the assumption that EB students would

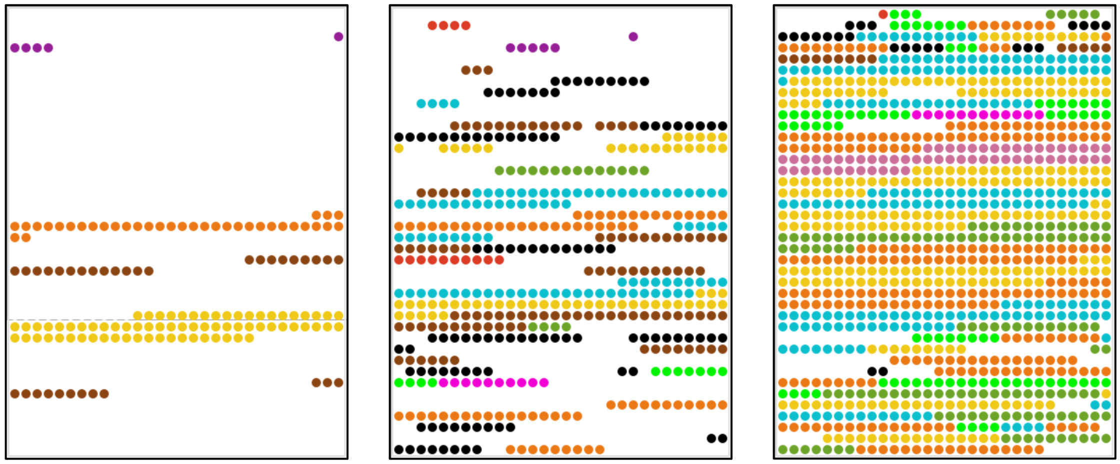


Figure 5.17. Coding of enacted supports for EB students.

be learning the language of instruction alongside learning mathematics. This lesson was also a review of graphing linear equations and inequalities, and there were no contexts associated with these exercises. Thus, another interpretation of this document portrait for Mr. Estrada is the need for language supports was greatly diminished compared to other classes that had contextualized tasks that related to real-life experiences. The students were given the opportunity to work in groups, but many of the students chose to work individually. This also reduced their opportunities to use language as they worked on their posters. The intent was to present the posters at the end of class, but they ran out of time. Had they not run out of time, it's likely that I would have seen more ways in which Mr. Estrada supported the students as they talked about mathematics. Perhaps he may have encouraged the students to present their posters in English. The poster presentations may have given students the opportunity to make comparisons across graphs of the same line that may look different based on the scale they choose for their axes, or students may have shared different strategies for deciding where to shade the solution set for an inequality, thereby creating a space in which to have rich mathematical discussions around what one may have considered purely routine exercises.

The middle document portrait shows a moderate amount of supports for EB students, and the remaining three teachers have profiles that fall between the low and moderate levels, all closer to moderate than low. This portrait is a representation of Ms. Ochoa's class. Note the proportion of black (feedback/encouragement/motivation) and brown (partner/group) dots in this portrait, interwoven throughout the class period. While other classes, such as Mr. Estrada's, also had the opportunity to work in groups for extended periods of time, I only coded instances when the teachers made explicit reference to do something with a partner or group, utilized group roles, or encouraged student-student interactions, so there are fewer instances of the partner/group code in other classes. Looking at the color distributions in the document portraits, the more colors represented in the portrait indicates a wider range of enacted supports for EB students. For example, in Mr. Estrada's graph there are only four colors represented, two brown for explicit mentions of group tasks, orange for connecting language with mathematics in distinguishing between graphing inequalities with solid (*solida*) or dotted (*tachada*) lines, yellow for providing visual supports in conversations (e.g., using gestures such as angling his arm to indicate positive slope), and the purple dots represent his request for the students to put cell phones away at the beginning of the period (similar to Ms. Ochoa).

Tacit Use of Strategies to Support Emergent Bilingual Students. Berliner (2004) asserted that experienced or expert teachers may have a more difficult time communicating pedagogical decisions or describing their teaching practices due to the situatedness of their practical knowledge. Practical knowledge is action-oriented knowledge that is developed over time through personal experience, and this practical knowledge is often tacit (unspoken or unstated). As teachers gain more years of experience and have many opportunities to reflect upon both positive and negative classroom experiences, they develop the ability to recognize patterns of interaction or student behavior that may not be productive and build a system of strategies to change the course of the interaction efficiently, without the need to give this process much thought. As I planned this study, I had anticipated that I may find teachers who don't necessarily recognize all the supports they may provide to their EB students and may do more than they report in their interviews.

Ms. Montez provided an exemplar of an experienced teacher (seventeen years teaching experience all at School B) who provided many more supports for her EB students than she could name. During her lesson planning interview, she indicated that she had helped create the lesson packet they were using four or five years before the district adopted the CPM curriculum. She explained that the textbooks they had at that time simply presented the students with a unit circle, so she and the other teachers wanted to find a more conceptual way for their students to learn about the unit circle and radians than just memorization. When I asked her in what ways she and her team planned supports for EB students, Ms. Montez said that because she is bilingual she can help her students individually in Spanish if they are having trouble with the English, but can only do that for Spanish speakers, not other languages. As for the CPM curriculum, she described that it includes a lot of reading, which is both good and bad for the students classified as ELs - good for helping them develop their English skills and acquire new vocabulary, but difficult because the teachers were spending so much time translating, rewriting, or explaining the text. Ms. Montez also described that "CPM is asking for the students to do it mostly alone," which isn't possible if the students don't understand what they are reading. I then returned to the question about the student packets she and her team had created and asked what she thought was good about them for her EB students. Ms. Montez indicated that there isn't a lot of reading, but they still have critical thinking built in. For example, they included word problems, but they were one sentence long instead of four. Instead of just giving students “drill and kill” simple exercises (e.g., providing a picture of a circle with radius 32 and asking for the circumference), they added situations (e.g., there’s a piece of gum stuck to a tire, how far does the gum travel in one revolution) to the problems to make the students think about what they know and need to do to solve the problem. Finally, I asked Ms. Montez to describe how she addresses language demands when teaching mathematics besides shortening the situation and translating for her Spanish-speaking students. She added that she will draw diagrams or provide pictures of real-life objects to help them understand the context (e.g., for a problem about a ferris wheel, she showed her students the London Eye).

In the debriefing interview, Ms. Montez described grouping her students according to how they are performing in class, “I always mix the teams with like high achievers and then the not, you know, so good students together so that they can help each other.” In addition, she mentioned that she had two students in her class that are newcomers to the US whose English isn’t strong, so she always makes sure there is at least one other person on the team that will be able to translate for them. The following is an excerpt from Ms. Montez’s classroom observation debrief in which I was trying to find out what types of supports she used to engage her EB students in the lesson.

1	LW:	Did you modify or differentiate the lesson for special ed or other groups of students?
2	Ms. M:	No.
3	LW:	And were any of the materials specifically chosen with English learners in mind?
4	Ms. M:	No.
5	LW:	How might you change this lesson if every student in the class was classified as an English learner?
6	Ms. M:	Well, I probably would translate the lesson if everybody needed that. Or I could, not necessarily translate, I could also either translate the writing parts for them to read it in their language, or as we go, if I would translate verbally, like this means this, or

make some definitions like in cards or something for them to see for example what's a revolution, what does degree mean, key words.

7 LW: What if they were all newcomers and spoke very little English?

8 Ms. M: Yes, and then in that case I would have to give them the Spanish, for if it was Spanish, the Spanish version.

9 LW: Mmhmm. What if they didn't speak Spanish? Some other language.

10 Ms. M: Oh my gosh, yeah. So in that case, I don't know. (laughs) I never, yeah, I had that case long, long ago, I had a student from, she spoke Arabic or something like that, but, and she was placed in my bilingual, back then, but I, there was no way that I could help her. So she, she learned Spanish very quickly! (laughs) Because of the students, you know, it was a bilingual class, I think it was like ELD one and two and, and she developed very quickly the Spanish and the English at the same time.

From this excerpt and her lesson planning interview, one might conclude that Ms. Montez didn't have many strategies for supporting her EB students beyond translating, providing diagrams, or using tasks with fewer words to read. However, this is far from accurate. A quick glance at the document portrait created in MaxQDA (see Figure 5.18), the count of enacted supports in Table 5.2, or my modified ELSF Specifications teacher enactment coding in Figure 5.15, provide

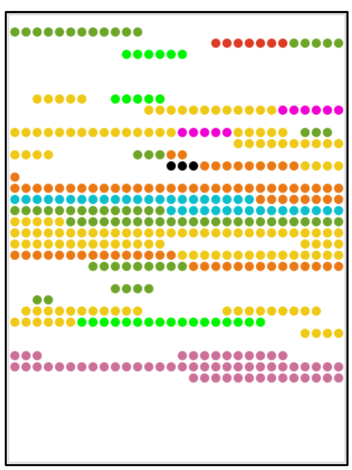


Figure 5.18. Ms. Montez's enacted supports for EB students.

evidence that Ms. Montez was enacting more supports than she accounted for in her interviews. Anecdotally, during Ms. Montez's debriefing interview, Ms. Ochoa happened to be in the work room. After Ms. Montez left, Ms. Ochoa expressed surprise about how Ms. Montez had answered these questions, having both seen Ms. Montez teach (they shared a classroom) and having been part of the team that created the lessons with Ms. Montez. Ms. Ochoa shared that everything that they created was made with EB students in mind, and that's why they included more hands-on activities and investigations to help students develop conceptual understanding and the structured interactions and participation structures that they built into the lessons were all for supporting linguistically diverse students.

Discussion: Tacit versus Explicit Support. In reflecting on the degree to which teachers articulated what they do to support EB students' access to and participation in the mathematical practices of their classes compared to the supports that happened during the enactment of the lessons I observed, several questions came to mind. What is the role of experience? Are there differences between teachers who are monolingual and those who are bilingual who may have experienced being classified as an EL as a student? Does participation in coursework or professional development that is related to teaching EB students play a role in how teachers plan for and enact supports for their EB students? In this section, I will discuss these questions with the participating teachers in mind.

The participating teachers in this study ranged from student teachers (or intern teachers) to a teacher with 40 years of teaching experience, with an average of 12 years teaching experience across all eleven teachers. Four were new teachers (one- or two-years teaching experience), and the rest were experienced teachers with a minimum of nine years teaching experience. In general, and not necessarily surprising, observing teachers with more years of experience left me with the impression that they were confident in their practice and that they had developed ways of interacting with their students that made them comfortable in their classrooms. Considering the document portraits of their enacted lessons and the modified ELSF Specifications, the experienced teachers provided low to moderate levels of support for their EB students.

In contrast, when I conducted my observations in the spring near the end of the school year, it seemed that the two intern teachers were still developing classroom routines with their students and figuring out what style(s) of presenting mathematical content to their students worked well and what didn't. One was very concerned about providing supports for EB students and planned to use sentence frames, a vocabulary list for the chapter, and partner work during his lesson. The other felt that he didn't have any issues with his students not understanding English, so he didn't plan for or enact supports for his EB students (note that 18% of the students on his roster were classified as ELs), citing that they were able to do their work and get the grades they needed to earn. On the other hand, Ms. Ochoa's student teacher provided moderate levels of supports for her EB students, and Ms. Ryan (first full year teaching after a one-year internship) provided the most supports for EB students out of all of the teachers.

Based upon my analyses of the participating teachers in my study, years of experience is not necessarily an indicator of how well teachers support the EB students in their classrooms. Does the teachers' first language (L1) and whether or not they are bilingual (or multilingual) influence the ways they interact with EB students in their classrooms? Four of the participating teachers reported being bilingual and having Spanish as their L1. Three of these teachers are at School B, in which the majority of their students who are classified as ELs or former ELs are Spanish-speakers. In that respect, their ability to translate information for their students when needed alleviated the need to plan for and enact specific strategies for supporting EB students. Additionally, the school offered bilingual mathematics courses, so Spanish-speaking students who were classified as beginners or short-term ELs would likely have been placed in the bilingual course, reducing the number of EB students in the monolingual English language courses. However, it did appear that having been an EL helped the teachers empathize with their students in ways that monolingual speakers could not. Ms. Montez shared the following in response to how she helps her students feel comfortable speaking English in her class.

> Well, I think I'm a very good model because I, you know, what bothers the students is the accent, all the mistakes that we do when we speak, and then (laughs) they see me and it's like ok! Right? I think that gives them a little comfort, like they don't feel, cause I have a student like that and she comes, when she has to go to the front and I ask questions, she will do it, no problem, and the students don't make fun of her. They understand because most of them, maybe some time or another, they were English learners as well and they struggled with that too. (Ms. Montez, debriefing interview)

As one may have deduced from her statement, Ms. Montez had a strong accent and it is interesting to see how she viewed this as a resource for encouraging her EB students to participate verbally in class.

In contrast, Ms. Ryan was a monolingual English speaker who reported knowing a limited amount of French, joking that she could at least count and ask where the restroom is located. Again, her class reflected the highest amount of language supports for EB students, so it does not appear that bilingual teachers have an easier time supporting EB students in their classes. If years of experience or bilingualism doesn’t necessarily indicate how well teachers may support EB students, what about coursework or professional development opportunities? Note that in California, teachers are required to take a course on teaching English learners in the content areas. Overwhelmingly, the experienced participating teachers indicated that they didn’t really recall their teacher education courses. Some knew that they had taken courses on English language development or that they had discussed teaching English learners, but couldn’t really give specifics about what they had learned in these courses. The newer teachers could recall their coursework, but also did not report many specifics about what they had learned. While this may have been a shortcoming in my interview protocol or that I didn't press for enough detail, it left me with the impression that teachers weren't able to readily draw on course work to name supports for how to help EB students learn mathematics.

What, then, did matter? What caused some teachers to focus more on language supports than others? First, perhaps, was sheer necessity. Ms. Ryan's class was full of newcomers. It was a combination IM1/SIFE course in which she was expected to help the students simultaneously develop language and mathematical understanding. With so many different languages (Ms. Ryan listed a minimum of seven in her interview) represented in one class, English had to be spoken as a common language (and with Ms. Ryan being a monolingual English speaker) for whole class communication, while students with the same L1 were allowed to speak in their L1 as needed. In School B, some teachers didn't seem to focus on the presence of EB students in their classrooms, perhaps because they are predominantly Spanish speaking rather than a wide variety of languages and most teachers could support the students in Spanish as needed. Second, teachers who had a vision of mathematics teaching and learning that involved mathematical discourse were more focused on language supports and providing activities that engaged students in mathematical practices. This was clearly evident, for example, in Ms. Ochoa's class. Third, the teachers who had high expectations for their students and engaged them in rigorous, grade-level mathematics with a focus on developing conceptual understanding alongside procedural fluency had more opportunities to support EB students engagement in mathematical practices than those who were focused on developing procedural fluency in one or two topics during the lesson. For example, Ms. Montez's lesson on developing a meaning for radian measure and generalizing across different size circles. Finally, at least half of the teachers spoke in some manner about developing a classroom environment that builds students' mathematical confidence through ongoing feedback and encouragement. For those that were successful in building these environments, their students were more willing to participate in class knowing that the teachers were there to provide mathematical and linguistic support as needed.

Considering these observations, weighing the differences between tacit and explicit supports for EB students become almost negligible. When teachers did explicitly talk about supports, it was primarily around simplifying or clarifying language, providing sentence frames, identifying key words, or using diagrams and pictures. Very few teachers talked about all the ways they use gestures, anchor charts or other visual representations, or encourage students to describe mathematical terms or procedures in their own words, yet each of these happened in almost every class I observed. In that respect, the experienced participating teachers certainly seem to have a great deal of tacit knowledge that they did not share during their interviews.

Conclusion

In this chapter I have presented six case studies of practicing teachers and have summarized the manner in which EB students were supported in engaging in mathematical practices in the participating teachers' linguistically diverse classrooms. Generally speaking, the participating teachers described engaging in mathematical practices as communicating about mathematics and working in groups. When the Standards for Mathematical Practice (NGA & CCSSO, 2010) were referenced by the teachers, it was most common to hear about SMP1, *make sense of problems and persevere in solving them*, SMP5, *use appropriate tools strategically*, and SMP6, *attend to precision.* Yet the teachers' interpretations of these and other practices widely varied, which naturally led to varied enactments of the SMPs. As shown in Figure 5.13, the enactment of the SMPs varied in both opportunities to engage in the practices as well as the intensity to which the mathematical practice was enacted. Classes in which the focus of the lesson was developing procedural fluency, such as graphing linear inequalities, limited students' opportunities to engage in mathematical practices. Lessons that connected mathematics to contexts or situations, focused on developing conceptual (or both conceptual and procedural) knowledge, and utilized a variety of interactional structures (e.g., whole class, partners, groups) incorporated many more of the SMPs.

Similarly, the ways in which the participating teachers supported their EB students' engagement in the mathematical practices during each lesson varied. These variations appeared to depend on multiple facets, including the proportion of EB students in the class, the mathematical content of the lesson, the teachers' vision of mathematics instruction, and whether assigned tasks were intended to build procedural or conceptual fluency (or both). My analyses pointed to the fact that Ms. Ryan was providing the most language supports to her students, yet this does not seem to be surprising due to the fact that her students absolutely needed language support in order to participate in class. In other classes that I found a moderate to high level of language supports, such as Ms. Ochoa's class, the teacher preferred to use tasks that were contextualized, focused on building conceptual understanding as well as procedural fluency, and emphasized connections between mathematical representations.

One thing was certain, these teachers of linguistically diverse students don't all have the same philosophies for teaching mathematics and don't use the same strategies to support their EB students. In many cases, their strategies are contradictory. However, there was a common thread among the participating teachers in terms of supporting EB students: frequent evaluative feedback and encouragement. The teachers expressed that their students need to know if they are on the right track or not to give them the confidence to persevere. Taking another step back in my reflections, as each of these participating teachers shared with me about their students and the way they teach their students, I was completely convinced that each one of them deeply cared about the success of their students and sincerely believed in their curricular and pedagogical choices.

In Chapter 6, I provide a summary of my findings for each research question and discuss some limitations of my research. I return to Remillard's (2005) teacher-curriculum interaction framework to discuss how my findings relate to and may contribute to the development of this framework. In addition, I address implications for research and teaching. Finally, I offer directions for future research that have emerged during this study.

Chapter 6: Conclusion

The goal of this study has been to investigate the ways in which emergent bilingual high school students' engagement in mathematical practices is supported in linguistically diverse classrooms. Drawing on a situated sociocultural theory of learning, I designed a qualitative study to examine how mathematics teachers in two linguistically diverse schools used their adopted curricula materials to develop engagement in mathematical practices among their students and support the mathematics learning of EB students classified as English Learners. The study used a contrastive design, examining curriculum used by teachers in schools that use different Common Core-aligned curricula that provide different types of guidance for teachers to support EB students. Remillard's (2005, 2009) framework for the teacher-curriculum interaction provided a conceptual framework for this study, recognizing that the curriculum developers are communicating their perception of what it means to do mathematics through their materials while the teachers who use the curriculum also have their own perspectives of what it means to do mathematics in their classrooms, influenced by their students and local communities. This study investigated this interaction as teachers plan lessons for their students, particularly attending to the ways in which the text, planned curriculum, and enacted curriculum support EB students.

In this chapter, I revisit my research questions and present a brief review of my findings. Next, I return to Remillard's (2005) teacher-curriculum interaction framework and reflect on my findings in light of the components of her framework. Third, I discuss observations that have surfaced during this study and suggest implications for research and teaching. Finally, I acknowledge the limitations of this study and propose future directions of research.

Brief Summary of Findings

In this section, I return to my research questions and provide a brief summary of my findings for each question. As a reminder, my research questions are:

1. In what ways do the locally adopted, CCSSM aligned, curricula support secondary mathematics teachers to engage their emergent bilingual students in mathematical practices?
    a. What supports are provided explicitly to help teachers engage their emergent bilingual students in these practices?
    b. How do the curriculum materials position emergent bilingual students as learners of mathematics?
2. How do teachers use textbooks, the accompanying teacher-facing resources, and/or other materials to plan and enact lessons that support emergent bilingual students' engagement in mathematical practices?

In Chapter 4, I examined not only the locally adopted curricula, but all of the published or teacher-created materials that the participating teachers used during the classroom observations. The four commercially-available curriculum programs had varying levels of support for both mathematical practices and suggestions for meeting the needs of EB students. Two of these curriculums, CME Project and TTA, focus on developing mathematical habits of mind, while MVP and CPM focus on developing the CCSSM SMPs (recall that the authors of CME and TTA make a correspondence between HoMs and the SMPs). Moreover, the TTA and CME Project curriculum resources had very few (or no) references to EB students. Both the MVP and CPM publishers offer a Spanish edition of their student textbooks. In addition, the CPM authors have included tips for supporting EB students in each section of their textbooks. The MVP authors have also included language supports in the for-purchase teacher resources, but do not include them in the free online edition. My analysis of the voice of each curriculums, operationalized as the occurrence of imperatives and personal pronoun use in the lessons revealed that the authors of these CCSSM-aligned materials have (though likely unintentionally) continued to promote the dominant discourse of mathematics that distances human presence in mathematics and positions the readers as scribblers rather than thinkers of mathematics. This predominant positioning was also common in the teacher-created materials.

In Chapter 5, I presented six case studies of participating teachers, comparing the occurrence of student engagement in mathematical practices and supports for EB students during their lessons. The lesson planning interviews revealed that none of the teachers intended to use the published curriculum resources in their existing form during their enacted lessons (though in practice, one teacher did use the MVP lesson without the modifications he had proposed in the interview). Common teacher modifications to the printed curriculum resources were creating student handouts that included space to work out the solutions to the tasks, breaking longer task statements into shorter pieces or separating a single, multipart set of questions into separate parts, and modifying task situations that the teachers perceived to be unfamiliar to their students. While one may wonder if the teachers modified the curriculum because that's what they thought I was there to see, just over half of the teachers brought in a lesson that they had planned prior to the lesson planning interview.

Opportunities for student engagement in mathematical practices appeared to be linked to the mathematical content of the lesson, the teachers' enactments of partner and/or group work, and whether the students were assigned routine practice exercises or tasks based in realistic contexts. Strategies for supporting EB students' engagement in mathematical practices also varied from teacher to teacher, and the frequency with which different strategies were utilized appeared to be connected not only to the proportion of EB students in the classroom, but also the teachers' perceptions of their students' levels of proficiency in the language of instruction. One teacher shared that he really didn't have any issues with his students not understanding the mathematics due to language. On the day I observed his class, students responded "chorally" to his questions, but a closer analysis would likely reveal that fewer than half of the students spoke during class. Teacher use of supports for EB students also depended on the activity structures implemented and the type and amount of academic language used in the tasks.

The Teacher-Curriculum Interaction

The results of this study can be interpreted using Remillard's (2005) framework for the teacher-curriculum interaction. Recall that the framework has four main components: the teacher, the curriculum, the participatory relationship between the teacher and the curriculum, and the planned and enacted lessons (see Figure 6.1). In this section, I discuss the results of this study in relation to this framework.

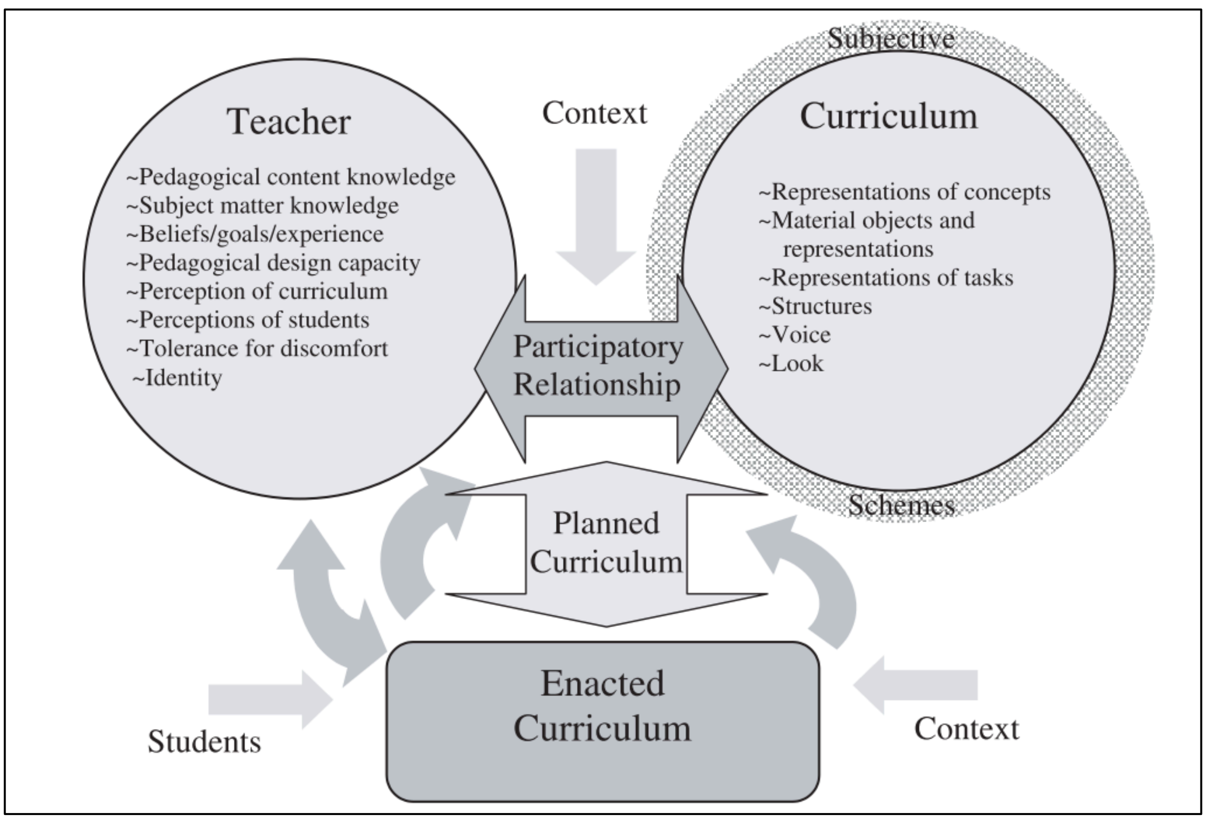


Figure 6.1. Remillard's Teacher-Curriculum Interaction (Remillard, 2005, p. 235).

Reflecting upon the data that I collected and analyzed in the context of linguistically diverse classrooms, it appears that the experienced teachers displayed confidence in their subject matter knowledge and pedagogical content knowledge, while some of the new teachers tended to hedge their descriptions and answer questions with less certainty. To illustrate the two extremes, compare these reflections on the lessons they had just taught. Mr. Martin confidently asserted that he would not change anything about the lesson he just taught, because he had already perfected it over the years and felt that “the beats were pretty much on point all the way through my lesson”. In contrast, one of the intern teachers appeared to second guess many pieces of the lesson: whether he spent too much time at the board guiding the students, if the quartic polynomial exercise was too time-consuming for too little payoff, and lamenting that he frequently utilized direct instruction as a means for keeping students on task and avoiding discipline issues.

Two of the characteristics Remillard (2005) identified in the teacher circle, perceptions of students and perceptions of curriculum, were particularly relevant to the results in this study. At School A, the three experienced teachers shared that the students in their school come into their classes below grade level in both mathematics and reading ability (they explicitly stated that their high school age students tend to have a third-grade reading level). In light of this perception of student ability, they expressed that grade-level mathematics textbooks will always be too difficult for their students to read, and it is necessary for the teachers to develop their own materials that are suitable for their student population. It would appear then that the rejection of the CME Project curriculum for use in School A may not have had anything to do with the curriculum itself, but perhaps the teachers’ perceptions of the suitability of published grade-level curriculum materials for their students in general (represented by the subjective schemes ring on the curriculum component).

At School B, Ms. Ochoa had served on the district textbook adoption committee and shared that the committee had narrowed the curriculum options down to two choices, then each school in the district voted for their preferred curriculum. The mathematics teachers at School B had actually voted for the other curriculum, but CPM received the most votes districtwide. Ms. Ochoa expressed that she was proud of her department for their efforts to embrace the curriculum despite it not being their first choice. However, once the district stopped offering CPM training and supports in the third year of adoption, three of the five participating teachers at School B started supplementing or replacing the CPM curriculum with the materials they had created prior to the adoption of CPM. While not all teachers were consistently using CPM at the time of my study, I did get the sense that all of the teachers were familiar with and had implemented CPM textbooks. It is also noteworthy that District B offered curriculum-aligned professional development, while District A did not provide any additional supports to the teachers to learn about the CME Project curriculum prior to its implementation. These context-related factors influenced the use or lack of use of the curriculum (enhanced or weakened the participatory relationship). Additionally, all of the participating teachers from School B expressed that the reading level and the number of words in the CPM tasks presented a challenge for all of their students, but particularly for the EB students, and that they had to spend a lot of class time making sure that students understand the tasks before they work on them. In School B, district authority over curriculum use and teachers' beliefs about the curriculums' accessibility to their students have both affected the participatory relationship between teachers and curriculum.

Remillard (2005) asserted that focusing on the participatory relationship between teachers and curriculum highlights the influence of the context in which the teachers are situated. This agrees with what I've reported above, that the teachers modify their curriculum materials with their students' perceived abilities and needs in mind. Remillard also posed questions for future research of this interactive relationship that included considering when teachers interact with curriculum resources and how the participatory relationship changes as a result of extended use of the same curriculum. All of the four new teachers in this study actively referred to the student editions of the curriculum materials as they planned their lessons during the interview, only one also consulted a teacher guide (but this was a teacher-created guide that went with the modified CPM materials in the student packet). Recall that the IM1 teachers at School B had divided up the chapter and planned one lesson each prior to meeting with the team to finalize the lessons. During this planning session, there was at least one occasion in which the CPM teacher guide was consulted to determine the mathematical goal for the lesson. With the exception of Ms. Rainey, all of the experienced teachers walked me through a lesson they had either previously planned or taught.

After analyzing the data in this study, I was left with the impression that curriculum developers may have very limited opportunities to convey their intentions for their curriculum to teachers as the materials may only be skimmed or consulted during the first years of implementation. While my sample size was quite small (N=11) and by no means a random or representative sampling of teachers, one can't help but wonder how many high school teachers use curriculum and of those who do, how many utilize the teacher resources that accompany the student textbook? Is it possible that curriculum use is more prominent at the K-6 level than at the high school level? If curriculum developers essentially have only one shot (one year?) at convincing teachers that their textbooks and accompanying resources are valuable, what can be done to improve the chances of long-term use? Offering curriculum-aligned training, having school- or district-wide support, and ongoing curriculum-aligned professional development seem to be headed in the right direction, but may not be enough for prolonged, widespread adoption. What else can be done? Teachers in School B lamented that they were no longer provided time to do collaborative lesson planning and that the district no longer paid for them to attend the CPM conference.

Considering the planned and enacted curriculum component of Remillard's framework, I was surprised by the extent to which the participating teachers' planned lessons closely resembled the enacted lessons, regardless of teachers' years of experience. Since I had anticipated that there would be differences, or even perhaps that a different lesson entirely may be taught due to unforeseen circumstances, I reflected on why this may not have occurred. First, I must acknowledge the influence of my study on what happened in the classroom. My interview protocol clearly indicated that I wanted to observe the teachers teach the lesson they planned with me, so the teachers may have felt obligated to teach that particular lesson on the designated classroom observation day. Second, some (four) of the teachers had previously taught the lesson they "planned" with me and intended to implement the lesson without changes from the previous enactment. Third, the observations took place in the second half of the spring semester, so by this time of the school year, the teachers may have been able to anticipate the needs of their students and how long activities should take. Finally, an interpretation that would be difficult to support with my data but is a possible explanation, the teachers planned and enacted lessons that they believed were adequate for supporting student understanding of the lesson, and did not see a need to modify the lesson based on the students' responses during classes that occurred between the lesson planning interview and teaching event, or during the live class that I observed.

Discussion

In this section, I briefly discuss some of the observations that I have made as I considered the data in light of supporting EB students' engagement in mathematical practices. First, I reflect upon my use of the SMPs and the ELSF Guidelines as coding schemes in my analyses of this data. Second, I consider the uses of curriculum (or lack thereof) observed in this study of secondary mathematics teachers working in linguistically diverse settings and the implications for students. Third, I return to an idea that emerged in this study – the idea of mastery learning and how this term was used in two very different ways by two of my participants. Finally, I contemplate my impression that almost all of the teachers wholeheartedly believe they are providing the mathematical education that their students need.

SMPs and ELSF Guidelines

I utilized the SMPs from the Common Core State Standards (NGA & CCSSO, 2010) and the published ELSF Guidelines (ELSF, n.d.) in my analyses of both the curriculum materials and the enacted lessons. Paralleling the teachers' varied interpretations of the SMPs, I found that it was challenging as a researcher to use the Common Core's SMPs as a coding scheme and to clearly communicate my interpretations of the practices to a second coder (where both of us were deeply knowledgeable of secondary mathematics and standards). Our first attempt at coding a lesson in the CPM textbook for the potential of engaging students in SMPs achieved only about a 44% agreement, even though both coders had been secondary mathematics teachers and were familiar with the Common Core practices. As we discussed the practices, we realized we had different interpretations of the practices as well, and we worked to solidify our mutual understanding of the SMPs. The coding was further complicated by my decision to allow for a task or text to be coded for multiple SMPs, so each codable portion of the lesson could potentially receive zero to eight codes. Also due to the overlapping nature of some of the SMPs, our early coding comparisons lacked agreement. However, after discussing each codable unit in that first lesson, we reached consensus coding for that lesson and repeated the process a few more times, each time refining our interpretations of the SMPs. In the end, we achieved about 91% agreement in our coding of the curriculum. While an underdeveloped coding scheme was partially to blame for these initial difficulties, more notable was our varied interpretations of what counts as a mathematical practice even after we had both studied the elaborations provided by the authors of the CCSSM SMPs. One consideration for mathematics teacher educators and professional developers is if teachers are only reading the short descriptions of the SMPs and not the elaborations (think of the posters I saw in most classrooms listing the eight practices), it is not surprising that the teachers' interpretations widely vary and that enactments of the standards may only appear to be surface level implementations of the standards.

Though the ELSF Guidelines were not designed to be used as a coding scheme, they worked fairly well to focus my analysis of the curriculum resources. Additionally, I was able to modify the ELSF Specifications to code the teacher enactment of lessons that supported EB students. In contrast to the SMPs, using the modified ELSF Specifications to holistically code the enacted lessons yielded a high level of agreement with a second coder, without a need for extended discussions of the code book. (One possible reason the ELSF guidelines were more reliable is because they included a great deal more specificity in description than in the CCSS practice descriptions). Overall, as I reflect on the potential for the ELSF Guidelines to improve curriculum resources that support EB students and consider the ways the participating teachers in this study interacted with curriculum resources, I wonder about the format in which these supports are communicated to the teachers and how they may be taken up by teachers. The ELSF

Specifications include recommendations for supports to appear in both student materials and teacher materials, with approximately 54% of these recommendations specifically referenced as teacher materials. Yet the teachers in this study rarely accessed the teacher materials as they planned or discussed their lessons. Was this simply a reflection of their actions in this artificial lesson planning space, or is it uncommon for teachers to utilize the teacher guides? On the other hand, if the teacher guides provided valuable examples of student learning at varying levels of language proficiency or offered sample transcripts of mathematical discussions with EB students related to the lesson topic, would teachers of linguistically diverse classrooms utilize the teacher guide more frequently?

Curriculum Use

As mentioned in Chapter 5, I didn't see any students with mathematics textbooks in any of the classes I observed. In each case, the teachers had provided worksheets or packets in place of the textbooks. (Recall that some of these materials were modifications of the content in the curriculum. Note also that Ms. Ryan's class typically used the TTA worktexts, but she elected to create more accessible scenarios rather than using the written worktext on the day of my observation.) While both districts have adopted a mathematics curriculum, only two of the eleven participating teachers in this study used the district-adopted curriculum on a regular basis. The CPM curriculum offers student eBooks and District B has a one-to-one laptop program, so the students at School B should all have access to their textbook if desired. Yet this only supports the students whose teachers were still regularly using the district-adopted curriculum in its third year of implementation (only two of five participating teachers at School B).

The district mathematics coach assigned to School A supported the use of the TTA and MVP curriculums alongside or in place of the CME Project curriculum. However, the mathematics department at School A had not specified a particular curriculum to utilize throughout integrated mathematics one, two, and three. Without a cohesive vision for mathematics content coverage, the three participating teachers who taught IM2, IM3, and precalculus expressed that students were not prepared to learn the grade-level mathematics content and that they had to fill in gaps in student knowledge as they went along. Notably, the teachers using the MVP curriculum modules for IM2 and IM3 were teaching lessons from the third module of nine, with approximately one month left in the school year. This means that the classes were presumably one-third of the way through the curriculum if they taught the modules in order, raising questions about student opportunity to learn.

Additionally, the prevailing usage of worksheets and unit packets led me to wonder what resources the students have access to throughout the school year to support their mathematical learning. Even if all student worksheets or unit packets are returned to the students in a timely manner, how many students would keep their returned work for later reference? Of those that do keep their returned work, do the students organize it in a manner that is easily accessible? Without the typical organizational features of a textbook (i.e., table of contents, index, glossary), how do the students know where to look for the help they need in the moment? CPM's eBooks help with this issue, but what about the teacher-created materials that are not used in conjunction with a printed curriculum?

Mastery Learning

Mastery learning is currently a common model in schools. The idea of mastery learning appeared in two very different ways in my data analysis. Mr. Martin and Mr. Hepner referenced mastery-based teaching during their lesson planning interview. The CPM authors referred to mastery in their curriculum resources, suggesting that content is to be mastered over time rather than in a single lesson or chapter. While the word "mastery" was used in both approaches, the meaning of the term and the implementation of "mastery learning" was very different – in fact nearly opposite interpretations of what this means for students and teachers.

For Mr. Martin and Mr. Hepner, mastery-based teaching meant that students should be given ample opportunities to practice a standard before moving on to the next standard. Each module or topic was a stand-alone unit. In the curriculum they authored, the content standards were condensed into what Mr. Martin and Mr. Hepner considered to be a manageable number of key standards to master in ninth grade mathematics. They built up their curriculum around these key standards and provided students numerous opportunities to develop each standard through focused direct instruction, guided practice, and independent practice. This definition of mastery appeared to be closely linked to procedural fluency. Mr. Martin and Mr. Hepner also felt an important element of mastery-based teaching was providing immediate and constant feedback on student work to help the students develop confidence in their mathematical learning. This view of mastery learning may lead students to believe that mathematics is incremental in nature, each topic building upon the previous one. One concern would be that a student who is struggling with a current topic may come to believe that he has reached the limit of his mathematical knowledge and that success in future mathematics is unattainable.

In the CPM curriculum, the homework exercises (called *Review & Preview*) offer mixed, spaced practice. Mixed practice is when students are asked to complete several different kinds of problems in a single assignment (contrast this with massed practice, many similar problems in a single assignment). Spaced practice refers to spreading the learning of a concept over time through repeated exposure to the concept rather than practicing the skill repeatedly in a short amount of time. Because they believe mastery is developed over time, the CPM authors also recommend that assessments should be comprised of 35-50% current material and 50-65% review material. Thus, one should assess basic understanding of new material and intermediate or advanced understanding of review material. This format also allows for the use of assessment questions that span chapters and topics, thereby emphasizing the mathematical connections between different topics. This view of mastery learning emphasizes that learning mathematics takes time and may require multiple exposures to concepts before they are understood. From this standpoint, when mastery is not expected right away, students may feel freer to take risks and attempt problems they feel uncertain about, knowing that their teacher doesn't expect perfection at all times.

Teacher Sincerity

Perhaps one of the most surprising and unexpected observations I had when analyzing the data in this study was my realization that almost every experienced teacher described their practice in a way that convinced me they sincerely believed that how they were teaching mathematics to their students was precisely what their students needed to be successful. Given that there was wide variation in teaching styles and approaches to learning mathematics, some far more aligned with the content standards and the SMPs than others, it was overwhelming to consider what might need to happen in order to convince a teacher that they need to change their practice – or perhaps to consider that our (researchers) assumptions are wrong. If, as a teacher, my perception is that my method is the best method for teaching my students, and my students are successfully learning mathematics by my standards and perhaps even as evidenced by standardized testing, why would I wish to consider trying something completely different?

Rather than simply attempting to describe this phenomenon, the following is an illustration from Mr. Martin's and Mr. Hepner's lesson planning interview (emphasis added) in which they were describing how they support their students' engagement in mathematics in general, but also in SMP1, make sense of problems and persevere in solving them.

1 Mr. M: One of the things that seems to work really well with this demographic is, is not like, showing them eight different things and trying to learn eight different things all at the same time. Focusing in on one or two things at a time, seems to give them this sort of ability to focus on something, practice it and get good at it, and that's where the magic happens, basically. When you start taking kids who have never been good at math, who don't have a lot of self-confidence in math, and you **put 'em in a situation where almost daily they're doing something and doing it well and getting positive feedback, it completely changes the paradigm of how they feel about themselves, and the type of motivation they bring to the class. So that's kind of like where we're always trying to work**. We're trying to work in that space.

2 Mr. H: Especially with the English language learners, you know, who will suffer from also that lack of confidence in language and, and mathematics, you know. So, really just kind of **building up that positive feedback and self-belief that, you know, we're going to be here to support you and this is something that you're going to master.**

It may be difficult to detect their concern for their students from the written transcript, but in person and on video, I'm equally convinced that this pair of teachers fully believed in their practice, based on their confidence, their sharing of earlier successes with students achieving proficiency on standardized tests (before CCSSM), and other things they said in their interviews.

While this doesn't speak to my discussion point about teachers' sincere beliefs in their practice, I would be remiss not to point out that this transcript excerpt also reflects some of the dominant discourses about EB students, students of color, and students with lower socioeconomic status. Despite Mr. Martin's and Mr. Hepner's obvious concern for their students, their statements could also be read as re-inscribing deficit beliefs about their students.

1 Mr. M: One of the things that seems to work really well *with this demographic* is, is not like, showing them eight different things and trying to learn eight different things all at the same time.

Focusing in on one or two things at a time, seems to give them this sort of ability to focus on something, practice it and get good at it, and that's where the magic happens, basically. When you start taking kids who have *never been good at math*, who *don't have a lot of self-confidence in math*, and you **put 'em in a situation where almost daily they're doing something and doing it well and getting positive feedback, it completely changes the paradigm of how they feel about themselves, and the type of motivation they bring to the class. So that's kind of like where we're always trying to work**. We're trying to work in that space.

2 Mr. H: Especially with the *English language learners*, you know, *who will suffer from also that lack of confidence in language and, and mathematics*, you know. So, really just kind of **building up that positive feedback and self-belief that, you know, we're going to be here to support you and this is something that you're going to master.**

This observation broadens my concern from solely convincing a teacher to change their practice, but also to include how we can change the prevailing perceptions of EB students from students who lack something to students who bring valuable and different resources to our classrooms who are capable of learning grade-level mathematics given appropriate supports and opportunities to engage in mathematical learning.

While it is known that these types of deficit discourses about EB students persist and that EB students are often not provided access to grade-level mathematics and high-quality teachers, I wish to end this section with excerpts from two of the participating teachers' debriefing interviews. Knowing that research has found that teachers often shy away from engaging EB students in mathematical practices or mathematical discussions, my final question to the teachers was, "What do you think about engaging English learners in mathematical practices?"

> Well it's, it's important for all students to engage in the mathematical practices. I remember when they first came out with this idea of mathematical practices and how, from K through, you know, graduation, it's the same standards. And the idea appealed to me **because I felt like I had hit a wall**, and this is **before [participating in a 5-year PD program]** also, I had hit a wall. I was **doing my Structured Interactions**, but I was still getting **kids who were focusing only on**

> **the procedural.** And when they started, and when they would work on the same skill to apply it in a different situation, **they could never process or persevere through that**. So **having been a part of [the 5-year PD program]** and **having all this experience now with mathematical practice**, I see **the benefit of them**, not just having structured interactions, but **giving them tasks that give me a window for me to see their thought process** and **what they're understanding** and **how they're formalizing the information**. So those mathematical practices have definitely helped me accomplish and **go past that hurdle of kids focusing only on the procedural** and actually **understanding why they're applying those procedures** and **how can this skill be applied in a new situation**. Which **I'm still learning how to do!** (Ms. Ochoa, debriefing interview, emphasis added)

> I think one is **giving students the confidence to think** about a problem in a way **that's not simply computational** and **more abstract reasoning**…and **working collaboratively**. I mean, **every standard in there, essentially, you're not doing them individually**. This is something that you're mastering: **communicating with your peers like a mathematician**. And so, especially for, for all students, but especially English language learners I think that the **standards are crucial to understand math in a way that's beneficial and in a way that they can grasp it**. So again, it just goes back to having those **models and visuals and representations for English language learners** is the best strategy for them to learn mathematics. (Ms. Carter, debriefing interview, emphasis added)

These excerpts are encouraging to me because they reflect alternative discourses about both mathematical practices and EB students.

Limitations, Implications, and Future Directions

In closing, I would like to acknowledge some limitations to my study and propose potential future directions for this research. With a small sample size of eleven teachers, it would be inappropriate to generalize my findings to the larger population of all secondary mathematics teachers in linguistically diverse classrooms. However, thinking about the wide variety of interpretations of the SMPs represented in this small group of teachers undoubtedly raises one's attentiveness in considering whether this is a broader phenomenon than we may have expected. In addition to differing interpretations, the surface level enactments of the SMPs (e.g., citing engagement in SMP5 when a calculator is used or that SMP6 means accurate calculations) might cause one to wonder how well the short titles of the eight SMPs really communicate the intent of that practice. It seems wise to caution the research community and policy developers to consider how well we are communicating the meaning and intent of our work.

Another limitation of this study is that it only captures a one-lesson snapshot of a teacher's entire school year. Undoubtedly, the SMPs and supports for EB students that were captured on this day were tied specifically to the mathematical content of the lesson. In some classes, data was collected closer to the end of the school year, and the observed lessons were intended as review lessons in preparation for final exams rather than introduction of new material, thus I may not have captured a typical day for these teachers. Although my data represented a single snapshot of each teacher, I have shown that coding enacted lessons for supports for EB students with both my modified version of the ELSF Specifications and my coding scheme based on research provided similar portraits of the teachers' enactments of supports for EB students. Thus, one potential use of this study is as a proof of concept of the coding schemes for capturing important components of teacher enactment of lessons designed to meet the needs of EB students.

When the teachers were recruited for this study, they were informed that the purpose of my study was to learn about how different teachers plan lessons and teach integrated mathematics in classes where some students are learning English. My dissertation title also appeared on the consent form, which included a reference to mathematical practices. In addition, the questions in my lesson planning interview protocol for after the lesson planning portion included questions about the SMPs and supports for EB students. The teachers' awareness of the focus of my study could be a potential validity threat, as teachers may have deviated from their typical style of teaching or included more SMPs in their lesson due to my presence in the classroom. While this limitation is real, it does not diminish from the finding that among these eleven teachers there was a wide variety of interpretations of the meaning of the mathematical practices.

Finally, I present some possible future directions for this research. After conducting my analysis of the voice of the student textbooks (imperatives and personal pronouns), it was difficult not to notice the presence of voice in other parts of my data. Two additional analyses became immediately apparent: (a) analyzing the voice in the teacher guides, and (b) producing a complete transcript of the teacher talk for each classroom observation video and analyzing the voice the teachers use while communicating with their students. These ideas grew out of informally noticing that the MVP teacher notes used a lot of exclusive imperatives in their vision of leading a mathematical discussion about the growing dot pattern and that Ms. Ochoa used inclusive language while telling her students, “it’s important that we understand this.” Other ideas for future research opportunities include gathering student data such as how the students participate in mathematical practices, how EB students feel supported to engage in mathematical practices, and how students interpret the “real life” situations their teachers chose to use in mathematical tasks (e.g., do they actually perceive the situations as a resemblance of their reality?, or when teachers talk about buying a car or a house, do students relate, or are these only real-life applications for the teacher but not the students?)

Appendix A: Lesson Planning Interview Protocol

Before we start, I would like to ask your permission to record this interview. The recordings will primarily be used by me to recall what we discussed. Otherwise, members of my dissertation committee may ask to see the recordings. I will not post these recordings on the internet, or in any way make the video clips available to the general public. I will also use a pseudonym for both you and the school in any publications or presentations. Do you have any questions for me? Do you mind if I record this interview?

(START RECORDING!)

Thank you for agreeing to participate in my dissertation study. My name is Lynda Wynn and I am a graduate student in the Mathematics and Science Education joint doctoral program at San Diego State University and UC San Diego. Dr. Bill Zahner is my graduate advisor at SDSU. You are welcome to contact him should you have any questions or concerns about this study. Both of our contact information is on the assent form.

During this interview, I will ask you some questions about your teaching experience and then I will have you plan a lesson about _________. After you are done planning the lesson, I will ask you additional questions about your lesson planning process, your classroom environment, and educational background.

1. How did you end up teaching here at _________?
2. How long have you been teaching?
3. How long have you taught in this school district?
4. How long have you taught in this school?
5. What mathematics courses have you taught?
6. Which mathematics courses are you currently teaching?
7. If I were to visit your classroom on a typical day, what would I see?

Lesson Planning

For the next 45 minutes, we will talk about how you would create a lesson plan for Section _____. Imagine that you will be teaching this lesson tomorrow, so please sketch out all materials you would want to have for class. Try to include as much detail as possible, such as what activities or problems you would have the students do and how, whether you will use small group or whole class discussions for each activity, etc. I'll check in with you after 30 minutes to reflect on what you have so far while the experience is still fresh in your mind. Don't worry if you aren't completely finished in that time.

Lesson Planning Debrief

1. Can you describe your plan for the lesson? Include details of what activities you will use and how you plan to do the activities, your role and the students' roles in the lesson.
    a. Some teachers also use the internet, other books, or shared materials from another teacher. Did you use any of these while you were putting together this lesson?
    b. Do you use the internet or other books in your daily lesson plans?

2. As a graduate student, I've been taught a lot about the Common Core Standards and the Practice Standards. Are you familiar with the 8 practice standards? In what ways do the practice standards influence your planning and/or teaching? Does your textbook help you incorporate the practice standards in your planning/teaching?
3. As a research assistant and in my own reading, I've learned about making accommodations for groups of students such as ELs and Special Ed students. Does the book help you make any accommodations for certain groups of students? If so, in what ways?

Additional Background Information

1. What degrees have you received? What institution(s) granted the degrees?
2. What type of teacher preparation did you complete? Which institution?
    a. Undergraduate mathematics education preparation
    b. Undergraduate degree outside education; masters' mathematics education
    c. Undergraduate degree outside education; alternative preparation program
    d. Other (Please describe)
3. In what areas are you certified to teach?
4. What do you know about the language background of your students?
5. Tell me about the students in this class: how many students, how many are classified as ELs or former ELs, etc.
    a. How did you find out?
    b. Do you know how to find out?
6. Do you hold a CLAD or BCLAD endorsement? Can you give a brief overview of the types of courses/activities that were required for this endorsement?
7. How were you introduced to the curriculum adopted by your school/district?
8. What challenges and resources does your adopted curriculum present to EL students?
9. How do you address language demands when teaching mathematics? Do you modify materials for your EL students? If so, how?
    a. I'm always looking for new resources. Have you seen any curriculum materials you really like? Do you have resources you like best for your ELs?
10. Do you speak any languages other than English? If so, please state the language(s) and your fluency level. What is your first language?

    If time:
11. Have you attended any types of professional development since you started teaching? What types of PD have been the most influential on your teaching?

Appendix B: Observation Debrief Protocol

Prior to observation: Is there a way that I can offer to help them while I'm observing?

Observation Debrief

1. Let's talk about your lesson on ________. How do you think the lesson went? How would you change the lesson if you were to teach it again?
    a. What were your goals for the lesson?
        i. Mathematical?
        ii. Other goals?
    b. How did the composition of your class shape your goals while planning this lesson?
    c. Can you tell me about some of the discussions/questions the students had? What are some memorable things that students said or did?
    d. What would you say was the most successful part of the lesson?
2. What do you think the students got out of the lesson?
    a. What content did they learn?
    b. What mathematical practices, such as explaining, justifying, or modeling, were the students engaged in during the lesson?
3. Did you modify or differentiate the lesson for SPED or other groups of students?
    a. Were any of the materials specifically chosen with ELs in mind?
4. How might you change the lesson if every student in your class was classified as an EL? What if they were all newcomers and spoke very little English?
5. As a graduate student, I've been taught about mathematical practices. What do you think about engaging ELs in MPs? How do you do it?

Do you have any questions for me?

Appendix C: Cross-text Comparison

Graphs of Exponential Functions

Mathematical Features

Section 8.1.1 *What do Exponential Graphs Look Like?* of the CPM text is designed to be a two-day team investigation of the function $y = b^x$. The students use graphing calculators to produce graphs of the function $y = b^x$ with varying values of *b,* noting similarities and differences in their graphs. This activity is followed by the use of a Desmos eTool to explore the effect of changing the value of b on the graph of the function $y = b^x$. Students are reminded that in a previous chapter (5) they have graphed some exponential functions, recalling graphs of rabbit populations (increasing) and rebound heights of a bouncing ball (decreasing). Guiding questions, called *Discussion Points* are provided in the text for students to discuss in their teams. At the end of the activity, the teams each produce a poster highlighting the features of two types of exponential graphs. Similarly, CME Section 5.17 *Graphs of Exponential Functions*, suggests students explore $y = a \cdot b^x$ to discover which values of *b* produce an exponential growth function and which produce exponential decay. Additionally, the teacher notes encourage the use of tables and expanded-form calculations in order to make patterns visible. Although only one day is allotted for this section, the margin note indicates that this exploration and discussion could take a whole class period. The CME text does not provide suggestions for guiding questions, what students should produce in what form, or suggestions about pacing and wrapping up the lesson (other than assigning some problems). The presentation of these sections gives me the impression that CPM encourages student-centered learning (a little bit of "the sage on the stage") from exploration to presentation of findings, while CME tends to favor teacher-centered learning with some group exploration and the manner in which these exploration findings are summarized seems to be left to the teachers' discretion.

The homework sections in the CME and CPM texts look very different. In CME, all of the problems (ten total) are directly related to graphing $y = b^x$ or $y = a \cdot b^x$. The CPM text has spiral review problems built in to the text, so only three of the fourteen exercises provided in this section are related to graphing exponential functions. CPM also provides a standardized test preparation question, while CME does not have any multiple-choice questions in this section.

Language Access and Production

The CME curriculum does not appear to offer any support for ELs. Language access concerns are not addressed and there is a fair amount of text appearing throughout the section. While the tone seems to be conversational (the student text reads as though it is talking to the student), there are still some fairly long sentences as well as sentences with complicated syntax. For example, the wording of some of the **For Discussion** questions seem like they would be difficult to parse for ELs:

3. How much more money than Berta does Alicia have after 5 years?
4. How can you find how many years it will take for Alicia's savings to double? For Berta's savings to double?

Additionally, there does not appear to be any language production supports in the curriculum to assist ELs.

In contrast, CPM draws attention to the presence and needs of ELs by providing Spanish translations of the student text and handouts and by including a section in the teacher notes for each section of the book entitled **Universal Access**. For this section, teachers are reminded that the language demands of the curriculum are continuing to increase. A sentence frame is suggested to provide access to and promote the production of the mathematical vocabulary in this section:

When the value of b is ____, ($b > 1$, $0 < b < 1$, $b < 0$) the exponential graph is _______ (increasing/decreasing/discrete). Modify as needed.

The teachers are also reminded of a poster that was provided in Chapter 1 about "completely describing graphs" that may be a useful reference for students as they work on the tasks in this section. Additionally, CME provides a "Universal Access Guidebook" which includes a two-page guide entitled "Sequential or Simultaneous Instruction for English Learners."

Appendix D: ELSF Guidelines for Improving Math Materials for ELs (ELSF, n.d.)

ELSF Specifications

Area of Focus I: Interdependence of Mathematical Content, Practices, and Language

1. Strategic opportunities to use and refine both language and mathematics over time
    a. Materials highlight, define, illustrate, and show the purpose for mathematical language within the context of the lesson (not in isolation).
    b. Materials guide teachers to encourage students to build their own understanding of mathematics actively, using language, through sustained activities and experiences.
    c. Materials provide strategies to help students make connections between current language, new language, and mathematical concepts.
    d. Units offer repeated opportunities to develop, refine, and extend language for mathematical purposes over time.
2. Explicit mathematics and language learning goals and pathways
    a. Teacher materials state clear and specific language objectives both for math practices as well as for academic purposes that cut across disciplines.
    b. Student materials contain mathematics and language learning objectives.
    c. Teacher materials articulate a pathway or progression of objectives for content, practices, and language throughout units.
    d. Materials present opportunities for students to use language at different stages within a unit, such as speculating or predicting about a new topic, exploring and reflecting during an experience, presenting afterwards, etc.
3. Regular and varying opportunities to learn, reflect upon, and demonstrate learning of mathematics using a variety of modes and forms
    a. Activities deepen and extend learning through the various modes of communication: speaking, listening to deepen and extend learning through varied modes: speaking, listening, reading, and writing.
    b. Materials include prompts for students to reflect on their own thought processes, language use, methods, and learning of mathematical content.
    c. Materials encourage students to utilize interdisciplinary words and phrases as well as math-specific words and phrases.

Area of Focus II: Scaffolding and Supports for Simultaneous Development

4. Opportunities for students to interact with and produce a variety of methods and representations

   a. Learning activities provide ways for students to generate and interpret a range of mathematical methods and representations (symbols, manipulatives, graphs, tables, words, etc.) and methods.
   b. Teacher materials provide guidance to encourage students to draw comparisons and connections across different methods and representations.
   c. Unit of study includes multiple sensory modalities for student interaction.
   d. Teacher materials provide supports for teacher modeling of reading, writing, listening, speaking, and thinking aloud.
5. Directions for providing specialized individual and small group instruction to ELs
   a. Teacher materials point to strategic opportunities for teachers to meet directly with EL students individually and in small groups.
   b. Teacher materials give guidance on what to look for, listen for, questions to ask, and/or feedback to give when meeting with EL students.
   c. Materials present a balance of opportunities for independent, paired, small-group, and whole-class activities.
6. Guidance for anticipating potential language demands and opportunities in student activities
   a. Teacher materials make suggestions for addressing possible language issues that may interfere with engagement of math content.
   b. Materials demonstrate activities and ways to help students make meaning of typical mathematical texts such as word problems, graphs, tables, etc.
   c. Materials provide activities to help distinguish between common everyday meanings of language and mathematical meanings (table, round, product, origin, similar, etc.) as they emerge in the materials.
   d. Unit amplifies rather than simplifies English language structures and forms that are often used in mathematics.

Area of Focus III: Mathematical Rigor Through Language

7. Explicit guidance for teachers to engage students in using mathematical practices
   a. Materials have targeted opportunities for students to use and develop language functions while engaging in mathematical practices.
   b. Teacher materials point out opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.
   c. Teacher materials provide opportunities for students to revise their own, peers', and/or fictitious mathematical writing.

8. Maintain appropriate challenge and high expectations of mathematics learning for EL students
    a. Materials consistently provide access to cognitively-demanding tasks.
    b. Teacher materials demonstrate when and how to support productive struggle before intervening.
    c. Materials guide the implementation of anchor charts, visual aids, models, and other resources for students to use as a reference.
9. Guidance for facilitating mathematical discussion and co-construction of meaning
    a. Materials include prompts for teachers to cultivate and facilitate back-and-forth mathematical discussions between students that refer to and build on each other's ideas.
    b. Materials provide explicit purposes for communication between students.
    c. Materials allow for equitable participation and risk-taking in conversations.

Area of Focus IV: Leveraging Students' Assets

10. Opportunities to draw on and incorporate students' cultural background and lived experiences in mathematics learning
    a. Teacher materials include relevant and practical suggestions for connecting mathematics content and practices to students' lives.
    b. Materials encourage students to draw on prior knowledge, culture, and experiences.
    c. Materials offer opportunities for clarifying potentially unfamiliar contexts.
11. Suggestions for incorporating and valuing ELs' written and spoken contributions
    a. Teacher materials contain examples (and non-examples) of evidence of students with various language strengths and needs in mathematical practices.
    b. Teacher materials contain explicit guidance for teachers to examine their own values and beliefs about language, ELs, and ways in which that might impact their teaching.
12. Encouragement for ELs to use and build on existing language resources
    a. Activities permit appropriate opportunities for ELs to use and integrate first language (L1) and everyday English in communicating mathematical thinking.
    b. Activities and materials present opportunities for students to ask and pursue their own questions and interests, using their own methods in their chosen contexts.

Area of Focus V: Assessment of Mathematical Content, Practices, and Language

13. Descriptions, illustrations, and examples of quality work and mathematical practices with varying levels of language proficiency

    a. Teacher materials should provide examples of teacher-student and student-student interactions that model and reflect the intent of mathematical practices.
    b. Teacher materials present examples in a way that highlight student potential for English proficiency, not deficit-based.
14. Assessments able to capture and measure students' mathematics and language progress over time
    a. Assessments prompt students to use math practices through language (including but not limited to vocabulary).
    b. Rubrics specifically identify and describe typical mathematical content, practice, and language achievements.
    c. Teacher materials suggest ways to capture students' progress from everyday language to language for more formal academic and mathematical purposes.
15. Guidance for recognizing and attending to student language produced to inform instructional decisions
    a. Teacher materials instruct teachers to avoid interpreting lower level language proficiency as lower level mathematics proficiency.
    b. Unit includes a range of assessments for formative purposes that enable students to draw on and make use of their existing language resources.
    c. Summative assessment tools specifically identify, describe, and measure mathematical and language successes, errors, and misconceptions and guide teachers to score them accordingly.

Appendix E: The CME Project Lesson 4.02

(Cuoco & Kerins, 2016, pp. 279 - 283)

4.02 Equations of Lines

The Assumption in Lesson 3.13 is essential in developing a way to write the equation of a line. From that assumption comes the following theorem.

Theorem 4.1

The slope between any two points on a line is constant.

In other words, if the slope between two points on a line is m, then the slope between any two points on that line must be m.

For Discussion

1. Prove Theorem 4.1.

Remember... By definition, all points on a line are collinear.

In the previous lessons, you defined slope as a measurement between two points. With Theorem 4.1, you can now describe the slope of a line.

Definition

The **slope** of a line is the slope between any two points on the line.

To find the equation of a line, you only need to know one point on the line and the slope of the line. Once you know these, the equation of the line is just a point-tester. You can verify that the slope between some arbitrary point on the line and the fixed point matches the slope of the line.

Minds in Action

Sasha and Tony are trying to find the equation of the line ℓ that goes through points $R(-2, 4)$ and $S(6, 2)$.

Sasha To use a point-tester, we first need to find the slope between R and S.

Tony goes to the board and writes

$$m(R, S) = \frac{2-4}{6-(-2)} = \frac{-2}{8} = -\frac{1}{4}.$$

Tony It's $-\frac{1}{4}$.

Sasha Okay. Now, we want to test some point, say P. We want to see whether the slope between that point and one of the first two, say R, is equal to $-\frac{1}{4}$. If it is, that point is on ℓ. So our test is $m(P, R) \stackrel{?}{=} -\frac{1}{4}$.

It doesn't matter which point you choose as the base point. Either point R or point S will work.

For Discussion

1. Let points A, B, C, and D be on the line ℓ. By the Assumption in Lesson 4.4, $m(A, B) = m(B, C)$ and $m(B, C) = m(C, D)$; so $m(A, B) = m(C, D)$.

Lesson Overview

GOALS

- Write linear equations.
- Determine the slope of a line from its equation.

This lesson presents the central idea of the investigation—an equation is a way of testing to see if a point is on a line. The lesson uses the point-tester concept to provides motivation for writing equations of a line.

The student text purposefully does not stress specific forms of equations. In this lesson, students primarily deal with the form $y - b = m(x - a)$, or the point-slope form. Understanding the point-tester idea enables students to generate and understand more advanced equations, such as conics.

In Lesson 4.03, students experience ways to write a linear equation. If your curriculum requires teaching the specific forms, you can do so in that lesson. Keep the focus of this lesson of generating an equation and what it means to have an arbitrary point, (x, y).

CHECK YOUR UNDERSTANDING

- Core: 1, 3, 5, 6
- Optional: 2, 4

MATERIALS

- graph paper

HOMEWORK

- Core: 7, 9, 11, 12, 13, 16
- Optional: 8, 10, 17, 18
- Extension: 14, 15

VOCABULARY

- parallel lines
- slope of a line

Launch

Review Exercises 10 and 11 from Lesson 4.01. By now, students should know that the point-tester will find infinitely many points collinear with two given points.

Explore

Minds in Action

Sasha and Tony begin their dialog trying to abstract an equation from the point-tester. In Chapter 3, students used the guess-check-generalize method to build one-variable equations based on a single missing quantity. The point-tester ultimately extends the guess-check-generalize method to arbitrary points, (x, y), which ultimately gives students two variables.

This dialog should only serve to solidify the work that students have been doing since the end of Investigation 4A. In addition, the exercises from the previous lesson provided students with several opportunities for them to work through examples such as the example Tony and Sasha work through here. Use this dialog to make sure that students have fully understood how the point-tester works.

4.02 Equations of Lines 279

Example 1

The point-tester in its native form will work on all points but one, the chosen base point. It makes sense, though, because if you try to test the base point in the point-tester, you are essentially calculating $m(R, R)$, the slope of a single point. While $\frac{y-4}{x+2} = -\frac{1}{4}$ is an equation, it is not the equation of a line. It is, in fact, the equation of a line with a singularity, or "hole," at the point $(-2, 4)$.

Example 2

Astute students may notice that the slope of an equation in the form $ax + by = c$ is $-\frac{a}{b}$. Exercises in this investigation will help students discover this fact, so it is not necessary to teach it here.

Wrap Up

Give students two new points. Have them use both methods from the Minds in Action dialog to develop point-tester equations. Then provide them with some new points to test.

Assessment Resources

See Lesson Quiz 4.02 at www.successnetplus.com.

Tony Let's guess and check a point first, like $P(7, 2)$. Tell me everything you do so I can keep track of the steps.

Sasha Well, the slope between $P(7, 2)$ and $R(-2, 4)$ is $m(P, R) = \frac{2-4}{7-(-2)} = \frac{-2}{9} = -\frac{2}{9}$. This slope is different, so P isn't on ℓ. Maybe we should use a variable point.

Tony How do we do that?

Sasha A point has two coordinates, right? So use two variables. Say P is (x, y).

Tony Then the slope from P to R is $m(P, R) = \frac{y-4}{x-(-2)} = \frac{y-4}{x+2}$. The test is $\frac{y-4}{x+2} = -\frac{1}{4}$.

So, that must be the equation of the line ℓ.

Notice how Sasha switches to letters. She uses x for point P's x-coordinate. She uses y for point P's y-coordinate.

Example 1

Problem **What's Wrong Here?** Tony and Sasha go back to check their work. They want to make sure both R and S work in their point-tester equation.

Sasha says, "Okay, let's try R."

Sasha writes the following on the board.

$$\frac{y-4}{x+2} = -\frac{1}{4}$$
$$\frac{4-4}{-2+2} \stackrel{?}{=} -\frac{1}{4}$$
$$\frac{0}{0} \stackrel{?}{=} -\frac{1}{4}$$

Tony states, "We can't divide by 0. Now what?" Why doesn't the point tester equation work for the point R?

Solution Sasha knows she has a problem, because she cannot divide by 0. The easiest way to eliminate that problem is to multiply the variable x out of the denominator. You can do this by multiplying each side by the denominator, $(x + 2)$.

$$\frac{y-4}{x+2} \cdot (x+2) = -\frac{1}{4} \cdot (x+2)$$
$$y - 4 = -\frac{1}{4}(x+2)$$

Multiplying each side of an equation by an expression containing a variable is *not* a basic move. The resulting equation may have more or fewer solutions than the original equation. Here, the resulting equation gains a solution, $(-2, 4)$. This is exactly the solution you were missing.

For You to Do

2. Test whether the points R and S satisfy the equation $y - 4 = -\frac{1}{4}(x + 2)$.

Answers

For You To Do

2. Both R and S satisfy the equation.

You can simplify and change the equation $y - 4 = -\frac{1}{4}(x + 2)$ with basic rules and moves. The different forms the equation can take highlight different information about the line. You will work with the different forms throughout the rest of this investigation.

You can use the basic rules and moves to change any form of an equation of a line to the form $ax + by = c$, where a, b, and c are constants.

Any such equation is a linear equation.

It works the other way, too. The graph of any equation in the form $ax + by = c$ is a line.

Example 2

Problem Find the slope of the line with the equation $2x + y = 5$.

Solution Find two points on the line.

If $x = 0$, then $2(0) + y = 5$. So $y = 5$, and $(0, 5)$ is on the line.

If $x = 1$, then $2(1) + y = 5$. So $y = 3$, and $(1, 3)$ is on the line.

The slope between the points $(0, 5)$ and $(1, 3)$ is $\frac{5-3}{0-1} = \frac{2}{-1} = -2$.
The slope of the line is -2.

Exercises *Practicing Habits of Mind*

Check Your Understanding

1. Use Sasha and Tony's method to find an equation for line ℓ. This time, use point S as the base point. Do you get the same equation? Explain.
2. Try Sasha and Tony's method with two points of your own. Use your equation. Test some points to see if they are on your line.

For Exercises 3 and 4, write an equation for each line.

3. a. through $A(6, 7)$ and $B(12, 1)$ b. through $(5, 4)$ and $(-3, -4)$
   c. through the origin and $(9, 3)$
   d. through $(0, 10)$ and parallel to the line in part (c)
   e. through the origin and parallel to the line in part (a)

Two lines in the same plane are **parallel** if they do not intersect.

Exercises

1. $y - 2 = -\frac{1}{4}(x - 6)$; yes; both equations simplify to $y = -\frac{1}{4}x + \frac{7}{2}$.
2. Answers may vary. Sample: An equation of the line through the points $(5, 6)$ and $(3, 2)$ is $y - 2 = 2(x - 3)$; $(1, -2)$ is on the line because $-2 - 2 = 2(1 - 3)$ simplifies to the true statement $-4 = -4$, and $(6, 6)$ is not on the line because $6 - 2 = 2(6 - 3)$ simplifies to the false statement $4 = 6$.
3. a. $y - 7 = -1(x - 6)$
   b. $y - 4 = x - 5$
   c. $y = \frac{1}{3}x$
   d. $y = \frac{1}{3}x + 10$
   e. $y = -x$

Exercises

HOMEWORK

- Core: 7, 9, 11, 12, 13, 16
- Optional: 8, 10, 17, 18
- Extension: 14, 15

Check Your Understanding

Many exercises in the lesson rely on the fact that parallel lines have the same slope. Investigation 4B formally states this idea.

EXERCISE 1 serves two purposes. First, it gives students practice with manipulating equations and writing them in different forms. Second, and perhaps more importantly, it shows that two equations that appear to be very different, do describe the same equation.

Encourage students to find their own method for proving that the two equations are equivalent. Do not force them into transforming each equation into a specific form. You can suggest that students test that the two equations are equivalent by solving both equations for y and simplifying, but students should not need to choose a particular form to show the equations are equivalent. They should only need to show that you can write both equations in that form.

For instance, an alternate (and equally valid) way to show they are equivalent would be to start with the first equation and end with the second equation. If you have students compare the left side of each equation, they may notice that by subtracting 2 from $y - 2$ they get $y - 4$, so if they subtract 2 from both sides of the first equation, they should end up with the second equation:

$$y - 2 = -\frac{1}{4}(x - 6)$$
$$y - 2 - 2 = -\frac{1}{4}(x - 6) - 2$$
$$y - 4 = -\frac{1}{4}(x - 6) - \frac{8}{4}$$
$$y - 4 = -\frac{1}{4}(x - 6 + 8)$$
$$y - 4 = -\frac{1}{4}(x + 2)$$

EXERCISE 3 Parts (a), (b), and (c) are routine practice. Students may struggle with parts (d) and (e) for two reasons:

1. They may not know students parallel lines have the same slope.
2. Unlike in parts (a), (b), and (c), students no longer start with two points.

ERROR PREVENTION You can help students by structuring the problem. They know a point and the slope. However, some may prefer to use the slope to find a second point, and then follow the process from start to finish. Although the process is not the most efficient (or comprehensive), it may be helpful for some students.

EXERCISE 4 In part (a), students can use a point-tester, as the slope is 0. In part (b), however, students cannot find the slope because it is undefined. For both of these examples, encourage students to make tables of values for points that lie on the lines (they may need to make the graphs first and be reminded of the definitions of *vertical* and *horizontal*). They should see patterns. In a horizontal line, the y-value does not change, so the point-tester should be a $y = c$ equation. In a vertical line, the x-value does not change, so the point-tester should be an $x = c$ line.

EXERCISE 5 You may want to introduce the vocabulary *x-intercept* and *y-intercept*. Discuss how to locate the intercepts (by substituting 0 for x or for y).

LEADING QUESTIONS

- Which lines are parallel? (Answer: The lines in parts (f) and (i) are parallel. The lines in parts (a), (c), and (e) are all parallel to the line in part (h).)
- How do you find the slope of a line with an equation similar to the equation in part (a)? (Answer: The number in front of the parentheses is the slope $\left(\frac{3}{4}\text{ in this case}\right)$.
- Is $\frac{y-3}{3} = \frac{x-4}{4}$ linear? (Answer: yes)
- Is $\frac{y-3}{3} = \frac{4}{x-4}$ linear? (Answer: no)

EXERCISE 6 uses algebra to generalize a conjecture.

ERROR PREVENTION Students may get confused by several different variables. Remind them that the basic rules and moves still apply.

On Your Own

EXERCISE 8 previews $y = ax + b$ form, which is more formally introduced in the next lesson. Students need to use basic moves to solve for y and simplify each point-tester equation. You can extend this problem by asking students where the slope appears in this form (a). You might also ask them to graph the lines to see what the value of b represents (the y-intercept).

EXERCISE 9 asks students to work backward by findings the points and slope from the equation. For most exercises, students find 2 points on the line and then find the slope between points. Ask students if there are any shortcuts for finding slope without actually finding points. Here are some shortcuts:

- For $y = ax + b$ the slope is a.
- For $ax + by = c$, the slope is $-\frac{a}{b}$.
- For point-tester form, one side of the equation states the slope.

EXERCISE 10 Similiar to Exercise 1, this exercise serves two purposes. First, students practice manipulating equations and writing them in different forms. Second, students learn that two equations that look different can describe the same graph.

4. a. a horizontal line that passes through (6, 7)
   b. through (5, 4) and (5, −4)
   c. through the origin that splits the first and third quadrants in half
   d. line through the origin and parallel to the line in part (a)

You can use these hints to help you.
- Test a few points.
- Keep track of your steps!
- Develop a point-tester.
- Translate your point-tester into an equation.

5. For each part:
   - Decide whether the graph is a line. Explain.
   - If the equation represents a line, find the slope and the points where the line crosses the x- and y-axes.

   a. $y - 3 = \frac{3}{4}(x - 8)$ b. $xy = 12$ c. $y = \frac{3}{4}x - 3$
   d. $y = 5x - 7$ e. $3x - 4y = 12$ f. $y = -\frac{5}{3}x - 9$
   g. $x^2 + y^2 = 1$ h. $3x - 4y = 9$ i. $y = -\frac{5}{3}x + 8$

6. Prove that the slope between any two points on the line $3x - y = 7$ is 3.

On Your Own

7. **Write About It** Explain Sasha and Tony's method for finding the equation of a line as an algorithm. Be sure to include precise instructions and steps.
8. Write the equation of the line that includes each pair of points or matches each description. Write the equation in $y = ax + b$ form.
   a. (5, 2) and (−3, −4) b. (5, 4) and the origin
   c. (5, 5) and (7, 7) d. includes (5, 4) and has slope $\frac{2}{3}$
   e. passes through the origin and is parallel to the line in part (a)
   f. passes through the origin and is parallel to the line in part (d)
9. What is the slope of each line that has the description or equation given?
   a. through (−5, 6) and (−3, −4) b. $y = \frac{3}{4}x + 7$
   c. $2x + 4y = 8$ d. $2x + 4y = 15$
   e. $5.1476x + 5.1476y = 15$ f. $y - 3 = \frac{7}{13}(x - 4)$
   g. $y - 7.3591 = \frac{7}{13}(x - 4.0856)$
   h. through the origin and parallel to the line in part (d)

What do the lines in parts (c) and (d) have in common?

10. a. What is the equation for the line containing points $J(3, -6)$ and $K(8, 4)$? Use J as the base point.
    b. What is the equation through $J(3, -6)$ and $K(8, 4)$ if you use K as the base point?
    c. Prove that the two equations in parts (a) and (b) describe the same graph. Use basic rules and moves.

Answers

4. a. $y = 7$ b. $x = 5$
   c. $y = x$ d. $y = 0$
5. See back of book.
6. Choose two points A and B on the line with x-coordinates a and b, respectively (where $a \neq b$). Then $A = (a, 3a - 7)$, $B = (b, 3b - 7)$, and $m(A, B) = \frac{(3b - 7) - (3a - 7)}{b - a} = \frac{3b - 3a}{b - a} = \frac{3(b - a)}{b - a} = 3$.
7. Answers may vary. Sample:
   1. Test a few points.
   2. Keep track of your steps.
   3. Develop a point-tester.
   4. Translate your point-tester into an equation.
8. See back of book.
9. a. −5 b. $\frac{3}{4}$ c. $-\frac{1}{2}$ d. $-\frac{1}{2}$
   e. −1 f. $\frac{7}{13}$ g. $\frac{7}{13}$ h. $-\frac{1}{2}$
10. See back of book.

11. Alejandra is selling candy bars for \$.85 each.
   a. If she sells 20 candy bars, how much does she make?
   b. If she sells 50 candy bars, how much does she make?
   c. Let c be the number of candy bars Alejandra sells. Let s be her sales in dollars. Which equation represents the relationship between c and s?
   A. $c = 0.85(s)$ B. $s = 0.85(c)$
   C. $s = c + 0.85$ D. $s = c + 85$

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12. Standardized Test Prep What is the slope of the line with equation $5x + 2y = 10$?
   A. $-\frac{2}{5}$ B. $-\frac{5}{2}$ C. $\frac{5}{2}$ D. $\frac{2}{5}$

13. Write About It Find an equation of the line that contains (5, 3) and has a slope of −4. Can you find a point that satisfies your equation but is not on the line? Explain.

How many lines pass through (5, 3) and have a slope of −4?

14. Take It Further Explain why the slope between any two points on the line $ax + y = b$ is constant.

15. Take It Further Given $A(5, 2)$, $B(3, 7)$, and $C(10, 4)$, find equations for the lines that form the sides of $\triangle ABC$.

Maintain Your Skills

16. Sketch each pair of equations. Do the lines intersect?
   a. $y - 2 = 2(x - 7)$ and $y - 2 = 5(x - 7)$
   b. $y + 4 = \frac{2}{3}(x - 1)$ and $y + 4 = \frac{3}{2}(x - 1)$
   c. $y - \frac{14}{23} = \frac{1}{7}\left(x + \frac{13}{23}\right)$ and $y - \frac{14}{23} = \frac{1}{6}\left(x + \frac{13}{23}\right)$
   d. $y - 5 = x - 10$ and $y - 5 = 3(x - 10)$
   e. What pattern do you notice in the line intersections?

17. a. Graph the family of equations.
   $2x - 3y = 0$
   $2x - 3y = 1$ $2x - 3y = -1$
   $2x - 3y = 2$ $2x - 3y = -2$
   b. What pattern do you notice among the graphs?

You can use the term family to describe a collection, or a set. This family is the set of equations in the form $2x - 3y = k$, where k is any number.

18. a. Find equations for five distinct lines that include the point (5, 1).
   b. Is there a pattern among the equations? Explain.

11. a. \$17
   b. \$42.50
   c. B
12. B
13. $y - 3 = -4(x - 5)$; no; if a point satisfies a point-tester, that means that the point lies on the line.
14. See back of book.
15. $y - 2 = -\frac{5}{2}(x - 5)$,
   $y - 2 = \frac{2}{5}(x - 5)$,
   $y - 7 = -\frac{3}{7}(x - 3)$
16. See back of book.
17. See back of book.
18. a. Answers may vary. Sample:
   $(y - 1) = (x - 5)$,
   $(y - 1) = -(x - 5)$,
   $(y - 1) = 2(x - 5)$,
   $-2(y - 1) = (x - 5)$,
   $3(y - 1) = (x - 5)$
   b. Each equation is of the form $y - 1 = m(x - 5)$.

EXERCISE 11 gives students practice moving between equations and the situations they represent.

EXERCISE 14 This exercise uses abstract constructs. Since the solution relies on strong algebraic manipulation, the exercise is for more advanced students.

Maintain Your Skills

EXERCISE 17 introduces students to families of lines. The purpose of the exercise is to make conjectures that they can support about this family.

EXERCISE 18 asks students to look at the family of equations passing through the point (5, 1). Students may struggle because they need to choose the slopes. Encourage students to graph a few lines passing through the point. Ask them how to find the equations of the lines. You may also encourage students to write equations in $y = mx + b$ form so they can make some conjectures about m and b.

GOING FURTHER Ask students to experiment with other families by choosing a different point for all their lines to pass through. For comparison, they should use the same slopes as they did for the lines containing the point (5, 1). Ask, "How do your conjectures change when the point changes?"

Additional Resources

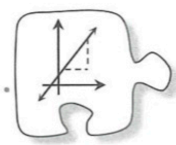
- Solution Manual
- Practice Workbook
- Assessment Resources
- Teaching Resources
- Interactive Textbook
- www.successnetplus.com

Additional Practice: For Lesson 4.02, assign Exercises 1–7.

Appendix F: CPM Lesson 2.1 Student Textbook

(Dietiker et al., 2014, pp. 67 - 71)

2.1.4 What information determines a line?



$y = mx + b$ and More on Slope

In previous lessons, you found the slope and y-intercept of linear functions. You connected slope (growth) and y-intercept (starting value) to their representations in patterns, tables, equations, and graphs. Today you will complete your focus on determining slope and learn how to use slope and the y-intercept to write the equation of a line. During this lesson, keep the following questions in mind:

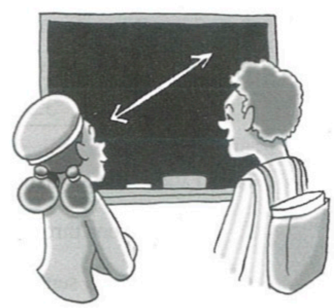
How can we determine the growth? How can we determine the starting value?

Is there enough information to graph the line?

How can we calculate the slope of a line without graphing it?

2-35. Equations for linear patterns can all be written in the form $y = mx + b$.

a. x and y represent **variables**. When you wrote equations relating the figure number to the number of tiles, what did x represent? What did y represent?



b. m and b are **parameters**—they do not change within a given linear situation.

m is also called a **coefficient** since it multiplies a variable (x), and b is a **constant term** since it does not multiply a variable.

What do m and b represent in a linear situation like the tile patterns?

c. What effect does m have on a graph of the line? What effect does b have?

2-36. THE LINE FACTORY

You are an engineer at the city's premiere Line Factory. Your job is to process customers' orders for lines.

Analyze the recent orders below. If the customer has provided enough information to produce one (and only one) line, then pass it on to your production department with an equation and a graph.

However, if you do not have enough information to draw one specific line, draw at least two lines that fit the order and send it back to the customer, so that the customer can be more specific.

The Line Factory uses the Lesson 2.1.4 Resource Page to design orders.

a. Line A goes through the point (2, 5).

b. Line B has a slope of –3 and goes through the origin.

c. Line C goes through points (–3, –2) and (3, 10).

d. Line D has the following table:

x	2	3	4	5
y	1	3.5	6	8.5

e. Line E has a slope of 4.

f. Line F goes through the point (8, –1) and has a slope of $-\frac{3}{4}$.

g. Customer G sent the following table:

x	0	1	2	3	4
y	1	2	4	8	16

2-37. CALCULATING THE SLOPE OF A LINE WITHOUT GRAPHING

While determining the slope of a line that goes through the points (6, 5) and (3, 7), Gloria figured out that $\Delta y = -2$ and $\Delta x = 3$ without graphing.

a. Explain how Gloria could figure out the horizontal and vertical distances of the slope triangle without graphing. Draw a sketch of the line and validate her method.

b. What is the slope of the line?

c. Use Gloria's method (without graphing) to calculate the slope of the line that goes through the points (4, 15) and (2, 11).

d. Use Gloria's method to calculate the slope of the line that goes through the points (28, 86) and (34, 83).

e. Another student found the slope from part (d) to be 2. What error or errors did that student make?

2-38. STEEPEST SLOPE?

What is the steepest line possible? What is its slope? Be ready to justify your statements.

2-39. LEARNING LOG

Consider the equation for a line, $y = mx + b$. What does the m represent? What does the b represent? Now consider the four representations of a linear function: situation (for example, a tile pattern), table, equation, and graph. Where in each of these representations would you look if you wanted to determine the slope? The y-intercept? Title this Learning Log entry "$y = mx + b$", and include today's date.

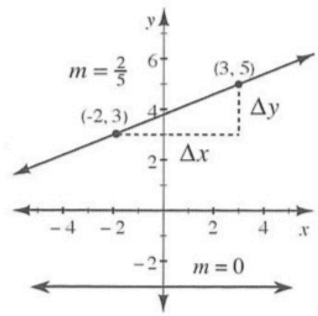
METHODS AND MEANINGS

MATH NOTES

The Slope of a Line

The **slope** of a line is the ratio of the vertical distance to the horizontal distance of a slope triangle formed by two points on a line. The vertical part of the triangle is called **Δy** (read "change in y"), while the horizontal part of the triangle is called **Δx** (read "change in x"). The slope indicates both how steep the line is and its direction, upward or downward, left to right.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$$



Note that "Δ" is the Greek letter "delta" that is often used to represent a difference or a change.

Note that lines slanting upward from left to right have positive slope, while lines slanting downward from left to right have negative slope. A horizontal line has zero slope, while a vertical line has undefined slope.

To calculate the slope of a line, choose two points on the line, draw a slope triangle (as shown in the example above), determine Δy and Δx, and then write the slope ratio. You can verify that your slope correctly resulted in a negative or positive value based on the direction of the line. In the example above, $\Delta y = 2$ and $\Delta x = 5$, so the slope is $\frac{2}{5}$.

Review & Preview

2-40. State the slope and y-intercept of each line.

a. $y = \frac{5}{3}x - 4$

b. $y = -\frac{4}{7}x + 3$

c. $y = -5$

2-41. Without graphing, calculate the slope of each line described below.

a. A line that goes through the points (4, 1) and (2, 5).

b. A line that goes through the origin and the point (10, 5).

c. A vertical line (one that is "up and down") that goes through the point (6, –5).

d. A line that goes through the points (1, 6) and (10, 6).

2-42. Which of the numbers below are correctly written in scientific notation? For each that is not, rewrite it correctly.

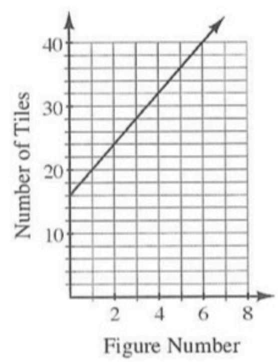
a. 4.51×10^{-2}

b. 0.789×10^{5}

c. 31.5×10^{2}

d. 3.008×10^{-8}

2-43. The graph at right models the number of tiles in a tile pattern.



a. Based on the information in the graph, how many tiles are being added each time (that is, what is the slope of the line)? Pay close attention to the scale of the axes.

b. How many tiles are in Figure 0?

c. Write the equation for the tile pattern.

d. How would the line change if the pattern grew by 12 tiles each time instead?

2-44. Zaheed enjoys walking his dog, Basil, each afternoon. Zaheed typically walks 12 blocks around the neighborhood, which takes about 45 minutes. Today, on his walk around the neighborhood with Basil, Zaheed noted that Basil barked at 17 other dogs. Zaheed wonders if he can predict how many dogs Basil will bark at tomorrow on the way to the dog park, which is 26 blocks away.

a. What assumptions would Zaheed have to make in order to use a proportional equation to make his prediction?

b. Write a proportional equation for Zaheed's situation and solve. Make sure that the precision of your answer is reasonable.

c. What is the unit rate in dogs per block for Zaheed's walks?

CPM Lesson 2.1.4 Teacher Notes

(Dietiker et al., 2014, pp. 164 - 168)

Student lesson pages 67 – 71.

CCSS: A-SSE.1a, A-SSE.1b, A-REI.10, F-IF.4, F-IF.6, F-IF.7a, F-BF.1a, F-LE.1a, F-LE.2, F-LE.5

Lesson 2.1.4 What information determines a line?

$y = mx + b$ and More on Slope

Lesson Objective: Students will formalize $y = mx + b$ form and will explore what information is needed to determine a line. They will continue to write equations of lines given various pieces of information about the slope and y-intercept. They will develop an algorithm for determining the slope of a line through two points without graphing. Finally, students will investigate the slope of vertical lines.

Mathematical Practices: Make sense of problems and persevere in solving them, reason abstractly and quantitatively, model with mathematics, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning

Length of Activity: One day (approximately 50 minutes)

Core Problems: Problems 2-35 through 2-39

Materials: Lesson 2.1.4 Resource Page, one per student

Suggested Lesson Activity: Ask a volunteer to read the lesson introduction. Then start teams on problem 2-35, in which students formalize what m and b represent in $y = mx + b$. Do this as a Think-Pair-Share for each part of the problem. After teams have had a few minutes to discuss their ideas, have them share their ideas with the whole class. Write down the statements they make on the board or overhead. After you have collected statements, ask if anyone disagrees with any of the statements or has questions. Lead a discussion allowing students to clarify their ideas. For inquiring students, more about the vocabulary of equations and expressions can be found in the Lesson 3.2.1 Math Note, in the Appendix in the Lesson A.1.1 Math Note, and of course in the glossary.

Problem 2-36 ("The Line Factory") is a challenging problem that can be done either in teams or as a whole class. Start this as a Teammates Consult (also known as Pencils in the Middle). This will allow students a chance to discuss the directions before they start the problem. Students may not be accustomed to deciding whether or not they have enough information. They may be satisfied with just drawing one example and assuming it is a unique solution. Challenge this idea by asking, "*Is that the only line that fits this criteria?*"

In problem 2-36, students are expected to write equations of lines by identifying the slope and y-intercept graphically. The algebraic method for this is developed in problem 2-37, and further in Section 2.3. Do not rush students to use an algebraic method.

If teams are having trouble with parts (c) and (d) of problem 2-36, suggest that they draw a graph. From the graph, they should be able to determine the slope and y-intercept. You can also call a Huddle or use an I Spy (see the Team Strategies section).

In part (f) of problem 2-36, many students will find it difficult to draw lines with the given slopes through a point other than the y-intercept, even though they have drawn lines with given slopes. Help them by asking, "*If the line started at (0, 2) and had a slope of* $-\frac{3}{4}$, *how would you graph it? What if it passed through (8, –1) instead?*"

Once teams finish parts (a) through (g) of problem 2-36, prompt students to summarize their results with, "*What kind of information is needed to write an equation for a line?*" You may choose to discuss this briefly with the class. Be sure students recognize that they can write an equation of a line given two points, or a point and a slope. Sometimes students do not realize that there is more than one way to do this.

Then have teams move on to problem 2-37, which asks students to devise a method to calculate the slope between two points without graphing. Lead a brief discussion summarizing teams' conclusions from problem 2-37. Have a team explain how to calculate the slope without graphing. We purposely do not provide the formula for slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$, as we want students to rely on the meaning of slope rather than a formula.

Then have students individually answer problem 2-38 before discussing this in teams or with the class. While debriefing the class, ask students to model their lines with their arms or by drawing. Any use of movement in the classroom helps the brain make connections. You may refer back to the stair-step model and ask them what the staircase they are describing would look like.

Students should arrive at the idea that the steepest line is vertical. Many will be tempted to give this a slope of zero, because the Δx of the slope triangle would be zero. This is a good time to raise the idea that slope predicts the slant of the line and that if two lines have the same slant, they must have the same slope. Horizontal lines have already been identified as having a slope of zero. One way to address this is to have students choose a Δy and divide it by zero (Δx) on their calculators. They should get some sort of error message, indicating that zero is not the answer. Some calculators will even display "undefined". Discuss why it is not possible to divide by zero. (One explanation: It is not possible to divide one quantity into zero parts.) Remind students that in the previous lesson, in problem 2-27, they determined that a vertical line is not a function.

Closure: (10 minutes)

Allow students time to complete the Learning Log in problem 2-39. You may choose to have some students share their answers to the second prompt in the Learning Log, in which students consider the multiple representations of a linear function.

Universal Access: Throughout this lesson and especially in problem 2-36, continue to take advantage of visuals. Either while teams are working on problem 2-36 or during your discussion, show the lines formed. Use them as a reference. Reinforce the concept of starting value or the y-intercept by pointing it out and relating it to the Line Factory scenario. Demonstrate slope by drawing and labeling the slope triangles.

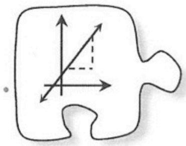
The vocabulary is very important here. If you have your students keep a glossary, Toolkit, or word wall, add at least the words "variable" and "coefficient". Place a labeled diagram similar to the one in the Math Notes box on your Process Word Wall and have students draw it in their notes. When discussing problem 2-35 and defining "m" and "b", students will often ask, *"Why use those letters?"* The answer is, no one really knows. Try asking the students to come up with reasons why "m" and "b" are used. For example, "m" could mean "more" as in, "how many *more* tiles are being added." In Spanish, it could translate to "más", which means "more" or "add". You could justify "b" by saying it is where the pattern "begins".

Team Strategies: Remember you can use a Huddle or an I Spy any time you feel that teams (or one team) might need a little help. For a Huddle you could call one person (Resource Manager) from each team together and give them a hint, ask them a question, or have them quickly share what they are doing. Then they return to their teams to share the information. An I Spy can be used at any time when a team is stuck. One person (Recorder/ Reporter) from a team can quietly spy on another team. That team should not know that the spy is even there. Then the spy returns to his or her team to report back. Sometimes it is a good idea to let a student who has a perceived "low status" be the person chosen to go to the Huddle or be the Spy. That way they go back to their team with important information that the rest of the team needs.

Homework: Problems 2-40 through 2-44

Notes to Self:

2.1.4 What information determines a line?



$y = mx + b$ and More on Slope

In previous lessons, you found the slope and y-intercept of linear functions. You connected slope (growth) and y-intercept (starting value) to their representations in patterns, tables, equations, and graphs. Today you will complete your focus on determining slope and learn how to use slope and the y-intercept to write the equation of a line. During this lesson, keep the following questions in mind:

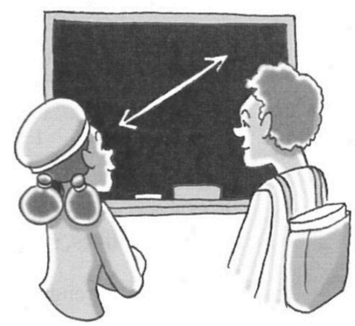
How can we determine the growth? How can we determine the starting value?

Is there enough information to graph the line?

How can we calculate the slope of a line without graphing it?

2-35. Equations for linear patterns can all be written in the form $y = mx + b$.

a. x and y represent **variables**. When you wrote equations relating the figure number to the number of tiles, what did x represent? What did y represent? **[x was the figure number and y was the number of tiles.]**



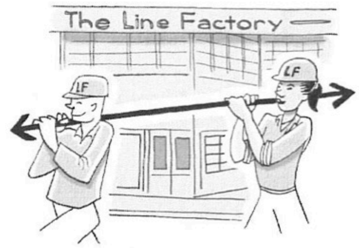
b. m and b are **parameters**—they do not change within a given linear situation.

m is also called a **coefficient** since it multiplies a variable (x), and b is a **constant term** since it does not multiply a variable.

What do m and b represent in a linear situation like the tile patterns? **[m is the slope of the line used to model the growth of a tile pattern and b is the starting value (number of tiles in Figure 0) of the tile pattern.]**

c. What effect does m have on a graph of the line? What effect does b have? **[m changes the steepness; the more positive or the more negative m is, the steeper the line. b changes the starting value (the y-intercept) without changing the steepness.]**

2-36. THE LINE FACTORY



You are an engineer at the city's premiere Line Factory. Your job is to process customers' orders for lines.

Analyze the recent orders below. If the customer has provided enough information to produce one (and only one) line, then pass it on to your production department with an equation and a graph.

However, if you do not have enough information to draw one specific line, draw at least two lines that fit the order and send it back to the customer, so that the customer can be more specific.

The Line Factory uses the Lesson 2.1.4 Resource Page to design orders.

a. Line A goes through the point (2, 5). **[This is not enough information. An infinite number of lines pass through (2, 5).]**

b. Line B has a slope of –3 and goes through the origin. **[$y = -3x$]**

c. Line C goes through points (–3, –2) and (3, 10). **[$y = 2x + 4$]**

d. Line D has the following table: **[$y = 2.5x - 4$]**

x	2	3	4	5
y	1	3.5	6	8.5

e. Line E has a slope of 4. **[This is not enough information. An infinite number of lines, all parallel to each other, have a slope of 4.]**

f. Line F goes through the point (8, –1) and has a slope of $-\frac{3}{4}$. **[$y = -\frac{3}{4}x + 5$]**

g. Customer G sent the following table:

x	0	1	2	3	4
y	1	2	4	8	16

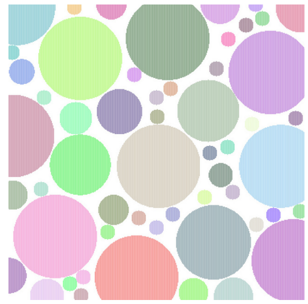
[This is not a line. The slope triangles do not result in the same slope, so the growth is not constant.]

Appendix G: MVP Lesson 1.2 Student Handouts

(Hendrickson et al., 2016a, pp. 6 - 10)

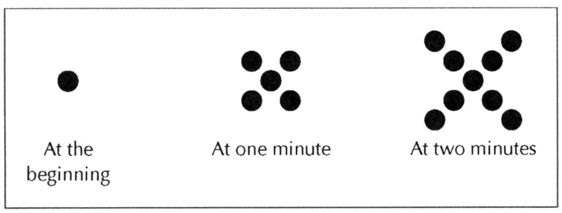
1.2 Growing Dots

A Develop Understanding Task



CC BY Davide Della Casa

https://flic.kr/p/d9Xi2T



1. Describe the pattern that you see in the sequence of figures above.

2. Assuming the pattern continues in the same way, how many dots are there at 3 minutes?

3. How many dots are there at 100 minutes?

4. How many dots are there at t minutes? Solve the problems by your preferred method. Your solution should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

READY, SET, GO! Name Period Date

READY

Topic: Using function notation

To **evaluate** an equation such as $y = 5x + 1$ when given a specific value for x, replace the variable x with the given value and work the problem to find the value of y.
Example: Find y when x = 2. Replace x with 2. $y = 5(2) + 1 = 10 + 1 = 11$.
Therefore, y = 11 when x = 2. The point (2, 11) is one solution to the equation $y = 5x + 1$. Instead of using x *and* y in an equation, mathematicians often write $f(n) = 5n + 1$ because it can give more information. With this notation, the direction to find $f(2)$, means to replace the value of n with 2 and work the problem to find $f(n)$. The point $(n, f(n))$ is in the same location on the graph as (x, y), where n describes the location along the x–axis, and $f(n)$ is the height of the graph.

Given that $f(n) = 8n - 3$ and $g(n) = 3n - 10$, evaluate the following functions with the indicated values.

1. $f(5) =$
2. $g(5) =$
3. $f(-4) =$
4. $g(-4) =$
5. $f(0) =$
6. $g(0) =$
7. $f(1) =$
8. $g(1) =$

Topic: Looking for patterns of change
Complete each table by looking for the pattern.

9.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32			

10.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18				

11.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20				

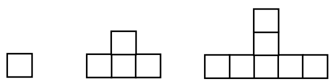
12.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12				

SET

Topic: Use variables to create equations that connect with visual patterns.

In the pictures below, each square represents one tile.



Step 1 Step 2 Step 3 Step 4 Step 5

13. Draw Step 4 and Step 5.

The students in a class were asked to find the number of tiles in a figure by describing how they saw the pattern of tiles changing at each step. Match each student's way of describing the pattern with the appropriate equation below. Note that *"s"* represents the step number and "*n*" represents the number of tiles.

(a) $\boldsymbol{n = (2s - 1) + (s - 1)}$ **(b)** $\boldsymbol{n = 3s - 2}$ **(c)** $\boldsymbol{n = s + 2(s - 1)}$

14. ____Dan explained that the middle "tower" is always the same as the step number. He also pointed out that the 2 arms on each side of the "tower" contain one less block than the step number.

15. ____ Sally counted the number of tiles at each step and made a table. She explained that the number of tiles in each figure was always 3 times the step number minus 2.

step number	1	2	3	4	5	6
number of tiles	1	4	7	10	13	16

16. ____ Nancy focused on the number of blocks in the base compared to the number of blocks above the base. She said the number of base blocks were the odd numbers starting at 1. And the number of tiles above the base followed the pattern 0, 1, 2, 3, 4. She organized her work in the table at the right.

Step number	# in base + #on top
1	1 + 0
2	3 + 1
3	5 + 2
4	7 + 3
5	9 + 4

GO

Topic: The meaning of an exponent

Write each expression using an exponent.

17. $6\times6\times6\times6\times6$

18. $4\times4\times4$

19. $15\times15\times15\times15$

20. $\frac{1}{3}\times\frac{1}{3}$

A) Write each expression in expanded form. B) Then calculate the value of the expression.

21. 7^1

22. 3^2

23. 5^3

24. 10^4

25. $7(2)^3$

26. $10(8^2)$

27. $3(5)^4$

28. $16\left(\frac{1}{2}\right)^3$

MVP Lesson 1.2 Enhanced Teacher Notes

(Hendrickson et al., 2016b)

1.2 Growing Dots– Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to develop representations for arithmetic sequences that students can draw upon throughout the module. The visual representation in the task should evoke lists of numbers, tables, graphs, and equations. Various student methods for counting and considering the growth of the dots will be represented by equivalent expressions that can be directly connected to the visual representation.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus on linear and exponential only)

Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

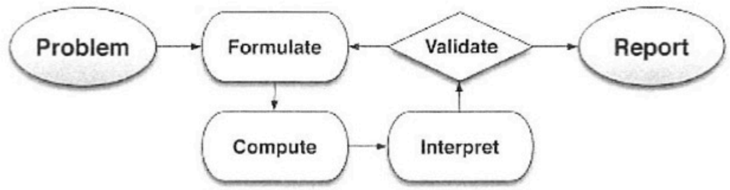
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice:

SMP1: Make sense of problems and persevere in solving them.

This task introduces a growing pattern and asks students to make sense of the pattern using various representations. They will not be able to simply draw the pattern at 100 minutes, so they will need to find ways to predict the result. Students may take multiple approaches to solving the problem, including using number patterns, noticing a visual pattern in the growth of the design or writing equations. Since this is the first pattern they have seen of this type, they may need to try different approaches before they are able to generalize the pattern.

SMP7: Look for and make use of structure.

As students try to write a recursive formula or equation to find the number of dots at t minutes, they may rely on the diagram and the way that they see the pattern changing. Some students may see 4 groups of size *t*, and other students may see *t* groups of size 4.

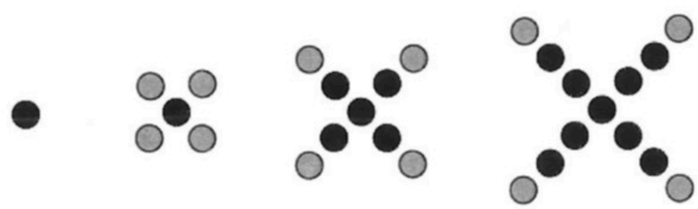
Some students may focus on the y values (the number of dots at a given time, t) in a table or graph, seeing that the number of dots is increasing by 4 each minute. Some may look across a row of the table to see a relationship between the t value and the number of dots at a given time. Different ways of the seeing the structure in the diagram or table may lead to different equations, some recursive and some explicit.

The Teaching Cycle:

Essential question for students: *How can we use mathematical representations to model a pattern?*

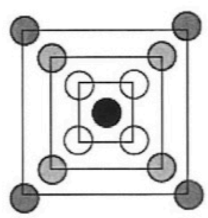
Launch (Whole Class): Start the discussion with the pattern on growing dots drawn on the board or projected for the entire class. Ask students to describe the pattern that they see in the dots (Question #1). Students may describe four dots being added each time in various ways, depending on how they see the growth occurring. This will be explored later in the discussion as students write equations, so there should not be any emphasis placed upon a particular way of seeing the growth. Ask students individually to consider and draw the figure that they would see at 3 minutes (Question #2). Then, ask one student to draw it on the board to give other students a chance to check that they are seeing the pattern.

Explore (Small Group or Pairs): Ask students to complete the task. Monitor students as they work, observing their strategies for counting the dots and thinking about the growth of the figures. Some students may think about the figures recursively, describing the growth by saying that the next figure is obtained by placing four dots onto the previous figure as shown:



Some may think of the figure as four arms of length *t*. with a dot in the middle.

Others may use a "squares" strategy, noticing that a new square is added each minute, as shown:



As students work to find the number of dots at 100 minutes, they may look for patterns in the numbers, writing simply 1, 5, 9, . . . If students are unable to see a pattern, you may encourage them to make a table or graph to connect the number of dots with the time:

Time (Minutes)	Number of Dots
0	1
1	5
2	9
3	13
t	

Watch for students that have used a graph to show the number of dots at a given time and to help write an equation. Encourage students to connect their counting strategy to the equation that they write.

For the discussion, select a student for each of the three counting strategies shown, a table, a graph, a recursive equation, and at least one form of an explicit equation.

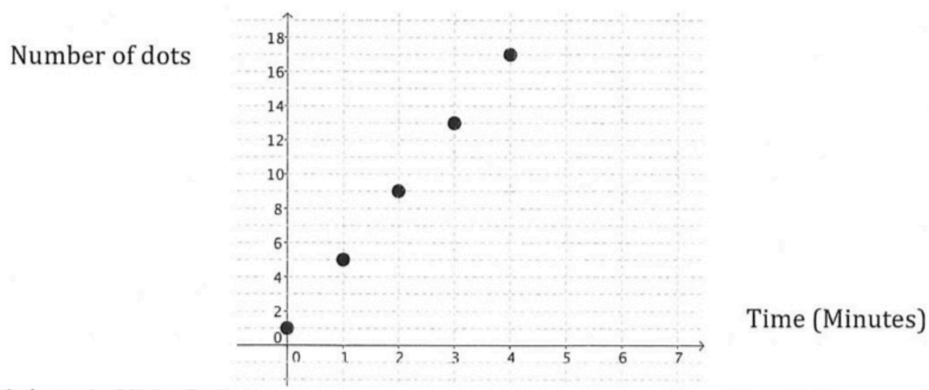
Discuss (Whole Group): Begin the discussion by asking students how many dots that there will be at 100 minutes. There may be some disagreement, typically between 100 and 101. Ask a student that said 101 to explain how they got their answer. If there is general agreement, move on to the discussion of the number of dots at time *t*.

Start by asking a group to chart and explain their table. Ask students what patterns they see in the table. When they describe that the number of dots is growing by 4 each time, add a difference column to the table, as shown.

Time (Minutes)	Number of Dots	Difference
0	1	
		> 4
1	5	
		> 4
2	9	
		> 4
3	13	
...	...	
t		

Ask students where they see the difference of 4 occurring in the figures. Note that the difference between terms is constant each time.

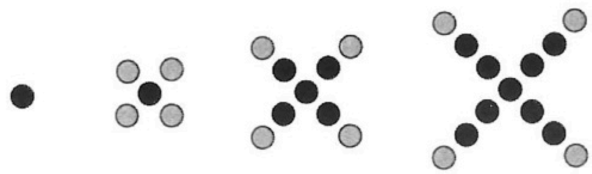
Continue the discussion by asking a group to show their graph. Be sure that it is properly labeled, as shown.



Ask students how they see the constant difference of 4 on the graph. They should recognize that the y-value increases by 4 each time, making a line with a slope of 4.

Now, move the discussion to consider the number of dots at time *t*, as represented by an equation. Start with a group that considered the growth as a recursive pattern, recognizing that the next term is 4 plus the previous term. They may represent the idea as: $X + 4$, with X representing the previous term. This may cause some controversy with students that wrote a different formula. Ask the group to explain their work using the figures. It may be useful to rewrite their formula with words, like:

The number of dots in the current figure = the number of dots in the previous figure + 4

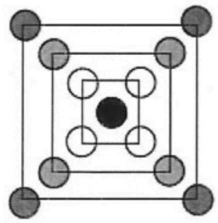


Or simply, *Current = previous + 4*

This may be written in function notation as: $f(t) = f(t-1) + 4$. (Although students have some exposure to function notation in grade 8, they have not seen it used to write recursive formulas. You may choose to introduce this notation in later lessons, simply focusing on writing the recursive idea in words as shown above.)

Next ask a group that has used the "four arms strategy" to write and explain their equation. Their equation should be: $f(t) = 4t + 1$. Ask students to connect their equation to the figure. They should articulate that there is 1 dot in the middle and 4 arms, each with t dots. The 4 in the equation shows 4 groups of size *t*.

Next, ask a group that used the "squares" strategy to describe their equation. They may have written the same equation as the "four arms" group, but ask them to relate each of the numbers in the equation to the figures anyway. In this way of thinking about the figures, there are t groups of 4 dots, plus 1 dot in the middle. Although it is not typically written this way, this counting method would generate the equation $f(t) = t \cdot 4 + 1$.



Now ask students to connect the equations with the table and graphs. Ask them to show what the 4 and the 1 represent in the graph. Ask how they see $4t + 1$ in the table. It may be useful to show this pattern to help see the pattern between the time and the number of dots:

Time (Minutes)	Number of Dots		Difference
0	1	1	
			> 4
1	5	1+4	
			> 4
2	9	1+4+4	
			> 4
3	13	1+4+4+4	
...	...		
t		$1+4t$	

You may also point out that when the table is used to write a recursive equation like $f(t) = f(t-1) + 4$, you may simply look down the table from one output to the next. When writing an explicit formula like $f(t) = 4t + 1$, it is necessary to look across the rows of the table to connect the input with the output.

Finalize the discussion by explaining that this set of figures, equations, table, and graph represent an arithmetic sequence. An arithmetic sequence can be identified by the constant difference between consecutive terms. Tell students that they will be working with other sequences of numbers that may not fit this pattern, but tables, graphs and equations will be useful tools to represent and discuss the sequences.

Exit Ticket: *What representation (table, graph, equation, recursive formula, etc.) did you find most helpful in understanding this problem? Explain your reasons.*

What representation is new or confusing? How might it be helpful in working with problems of this type?

Instructional Supports:

Visual: The task is based upon using a diagram that shows a visual pattern. The task is open to many different student strategies for counting the dots and recognizing a pattern that can be represented in many ways. It is not necessary for all students to have every representation. The class discussion should include a table, a graph, and equations, so that all students have access to these representations, even if they did not include them in their own initial work.

Scaffolding: The task provides scaffolding by asking students to begin by describing the pattern they see. This is to support recognizing their counting strategy, as they did in "Checkerboard Borders". After describing the pattern, students are asked to find the number of dots in specific cases, before generalizing to an expression containing a variable. Finding the number of dots in specific cases encourages the use of a table or some strategy that brings out patterns in the numbers that can be connected to the visual pattern.

Instructional Adaptations:

Academic Language: For many students the academic language used in the essential question may be unfamiliar. The terms "mathematical representation" and "model" are very important in this module and worth explicitly defining for this context. Students should keep a notebook that contains their record of class discussions as well as a vocabulary section. Before launching the task, define the word "mathematical representation" as a mathematical way to show a relationship.

Representation (noun): A mathematical way to show a relationship such as a table, graph, equation, diagram, or story context.

Example: In *Checkerboard Borders*, the representation that I used to show the number of colored tiles was an expression with a variable.

Represent (verb): The act of showing a mathematical relationship with a table, graph, etc.

Example: I chose to represent the survey data with a graph.

Ask students to answer the question: *What is a mathematical representation that you have used?*

Use this sentence starter to respond with the verb "represent":

I ________________________ a function with a ______________________ to show ____________________.

Verb: Represent   Noun: table, graph, equation, etc.

Possible answer: I represented a function with a graph to show how it grows over time.

Challenge Activity:

Challenge students to create all of the representations and to be able to connect each representation to the way that they viewed the diagram. For example, a student that viewed the diagram using the "four arms" strategy could describe the increase of 4 each minute on the table as one dot being added to each of 4 groups.

Aligned Ready, Set, Go Homework: Sequences 1.2

1.2 Growing Dots – Answer Key

1. Refer to the illustrations in the teacher notes for different ways of seeing the pattern. The descriptions fall into two basic categories:

   a. there are 4 groups of size t

   b. there are t groups of size 4

2. Based upon the table below, there will be 13 dots at 3 minutes.

Time (Minutes)	Number of Dots
0	1
1	5
2	9
3	13
...	...
t	

3. At a 100 minutes there will be $4 \cdot 100 + 1 = 401$ dots

4. The number of dots at t minutes can be given in either recursive or explicit form. Students have not yet used recursive notation. Refer to the teacher notes for ideas about how students might write their recursive thinking.

Recursive formula: $f(0) = 1, f(t) = f(t-1) + 4$ or Current = Previous + 4

Explicit formula: $f(t) = 4t + 1$

READY, SET, GO! Name Period Date

READY

Topic: Using function notation

To **evaluate** an equation such as $y = 5x + 1$ when given a specific value for x, replace the variable x with the given value and work the problem to find the value of y.

Example: Find y when x = 2. Replace x with 2. $y = 5(2) + 1 = 10 + 1 = 11$.

Therefore, y = 11 when x = 2. The point (2, 11) is one solution to the equation $y = 5x + 1$. Instead of using x *and* y in an equation, mathematicians often write $f(n) = 5n + 1$ because it can give more information. With this notation, the direction to find $f(2)$, means to replace the value of n with 2 and work the problem to find $f(n)$. The point $(n, f(n))$ is in the same location on the graph as (x, y), where n describes the location along the x–axis, and $f(n)$ is the height of the graph.

Given that $f(n) = 8n - 3$ and $g(n) = 3n - 10$, evaluate the following functions with the indicated values.

1. $f(5) =$ **Answer: $f(5) = 37$**
2. $g(5) =$ **Answer: $g(5) = 5$**
3. $f(-4) =$ **Answer: $f(-4) = -35$**
4. $g(-4) =$ **Answer: $g(-4) = -22$**
5. $f(0) =$ **Answer: $f(0) = -3$**
6. $g(0) =$ **Answer: $g(0) = -10$**
7. $f(1) =$ **Answer: $f(1) = 5$**
8. $g(1) =$ **Answer: $g(1) = -7$**

Topic: Looking for patterns of change

Complete each table by looking for the pattern.

9.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32	**64**	**128**	**256**

10.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18	**2**	**-14**	**-30**	**-46**

11.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20	**10**	**5**	**2.5**	**1.25**

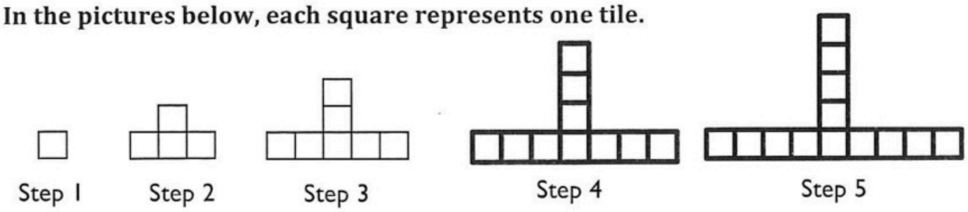
12.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12	**19**	**26**	**33**	**40**

SET

Topic: Use variables to create equations that connect with visual patterns.

In the pictures below, each square represents one tile.



13. Draw Step 4 and Step 5.

The students in a class were asked to find the number of tiles in a figure by describing how they saw the pattern of tiles changing at each step. Match each student's way of describing the pattern with the appropriate equation below. Note that *"s"* represents the step number and *"n"* represents the number of tiles.

(a) $n = (2s - 1) + (s - 1)$ **(b)** $n = 3s - 2$ **(c)** $n = s + 2(s - 1)$

14. __c__ Dan explained that the middle "tower" is always the same as the step number. He also pointed out that the 2 arms on each side of the "tower" contain one less block than the step number.

15. __b__ Sally counted the number of tiles at each step and made a table. She explained that the number of tiles in each figure was always 3 times the step number minus 2.

step number	1	2	3	4	5	6
number of tiles	1	4	7	10	13	16

16. __a__ Nancy focused on the number of blocks in the base compared to the number of blocks above the base. She said the number of base blocks were the odd numbers starting at 1. And the number of tiles above the base followed the pattern 0, 1, 2, 3, 4. She organized her work in the table at the right.

Step number	# in base + #on top
1	1 + 0
2	3 + 1
3	5 + 2
4	7 + 3
5	9 + 4

GO

Topic: The meaning of an exponent

Write each expression using an exponent.

17. $6\times6\times6\times6\times6$

Answer: 6^5

18. $4\times4\times4$

Answer: 4^3

19. $15\times15\times15\times15$

Answer: 15^4

20. $\frac{1}{3}\times\frac{1}{3}$

Answer: $(\frac{1}{3})^2$

A) Write each expression in expanded form. B) Then calculate the value of the expression.

21. 7^1

Answer: $7 = 7$

22. 3^2

Answer: $3\cdot3 = 9$

23. 5^3

Answer: $5\cdot5\cdot5 = 125$

24. 10^4

Answer: $10\cdot10\cdot10\cdot10 = 10000$

25. $7(2)^3$

Answer: $7\cdot2\cdot2\cdot2 = 56$

26. $10(8^2)$

Answer: $10\cdot8\cdot8 = 640$

27. $3(5)^4$

Answer: $3\cdot5\cdot5\cdot5\cdot5 = 1875$

28. $16\left(\frac{1}{2}\right)^3$

Answer: $16\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2$

Appendix H: TTA Unit 6 Lesson 6 Student Worktext

(Mark, Goldenberg, Fries, Kang, & Cordner, 2014, pp. 27 - 31)

Lesson 6: Solutions and Point Testing

IMPORTANT STUFF

① I am point A.
- My y-coordinate satisfies $y = x + -5$.
- My x-coordinate is 4.

Where Am I? (,)

② I am point B.
- $y = x + -5$
- $x = -3$

Where Am I? (,)

③ I am point C.
- $y = x + -5$
- x is 12.

Where Am I? (,)

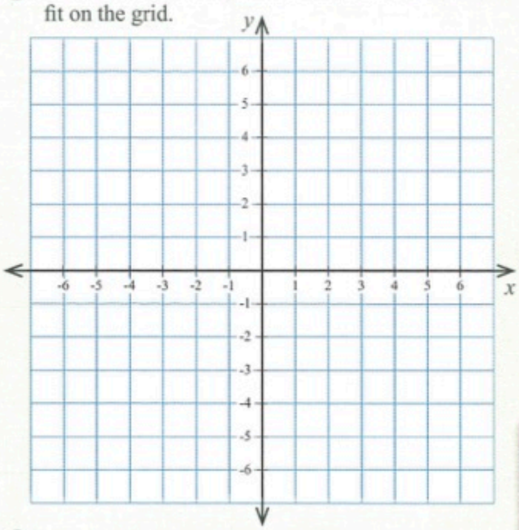
These are *solution points* for the equation $y = x + -5$ because each point gives a pair of numbers (x and y) that, when used together in that equation, make it true.

④ Which of the following points are *also* solution points for $y = x + -5$?
Circle the solution points and cross out the non-solution points.

(45, 40)	(1, -4)	$(\frac{1}{2}, -4\frac{1}{2})$	(-6, 1)	(2, -3)
(3, -2)	$(5\frac{1}{3}, \frac{1}{3})$	(-15, -10)	(10, 5)	(5, 0)

A solution point for $y = x + -5$ is *any* point where the y-value is five less than the x-value.

⑤ Plot all the solution points from problems 1-4 that fit on the grid.



⑥ There are infinitely many more solution points for $y = x + -5$. Find six more, record them in the table, and plot them, too, if they fit.

(x, y)
(0,)

Connecting your solution points will suggest other solution points between the ones you found.

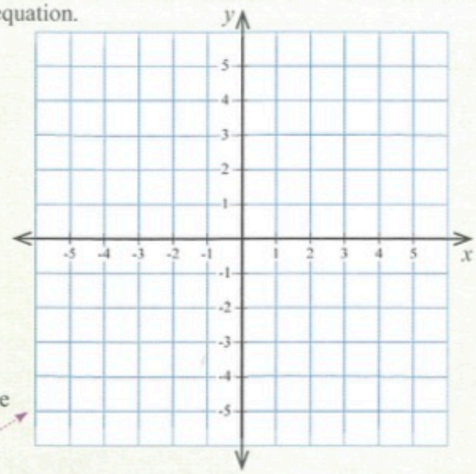
⑦ Just from looking at the graph, these points *look* like they might be solution points. Are they?

ⓐ (1.2, -3.7) ⓑ (-0.5, -5.5)

Discuss & Write What You Think

⑧ The graph of $y = x + -5$ is a line connecting all the solution points. Explain why it makes sense to connect the points when graphing the relationship.

⑨ (x, y) pairs that make $x + y = 2$ true are *solution points* for that equation.



ⓐ Find a solution point if $x = 1$.

(1,)

ⓑ Find a solution point if $x = -3$.

ⓒ Find a solution point if $y = 0$.

ⓓ Find a solution point if $x = 100$.

ⓔ Find three more solution points.

ⓕ You now have seven solution points for this equation. Some of them don't fit on the graph paper shown here. Plot the ones that do and use them to draw the graph of $x + y = 2$.

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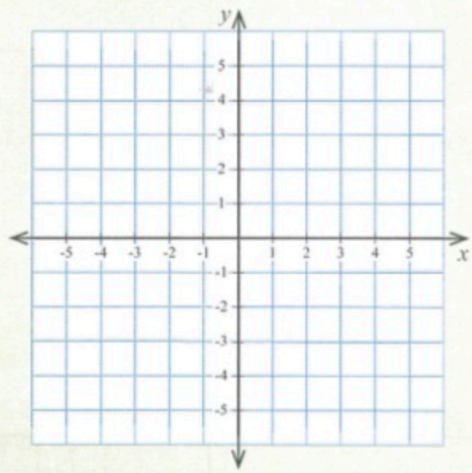
⑩ $y = -2x + 3$



ⓐ Find a solution point if $x = 0$.

ⓑ Circle the solution points; cross out non-solution points.

(4, -5)	(2, 0)	(-2, 7)
(10, -17)	(-1, 5)	(-10, 17)

ⓒ Use the solution points to draw the graph of $y = -2x + 3$. Check more solution points, if necessary.

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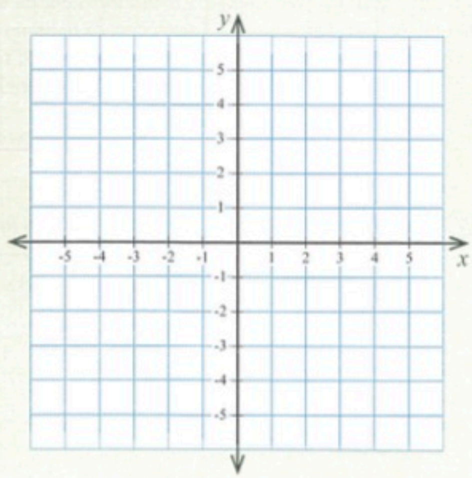
⑪ The graph of $y = x^2 - 5$ is not a straight line.



ⓐ Find a solution point if $x = 0$.

ⓑ Circle the solution points; cross out non-solution points.

(-2, -1)	(3, 4)	(4, 9)	(1, -4)
(2, -1)	(5, -20)	(2, 0)	(-1, -4)
(-6, 31)	(-5, 20)	(-3, 4)	(10, 95)

ⓒ Use the solution points to draw the graph of $y = x^2 - 5$. Check more solution points, if necessary.

STUFF TO MAKE YOU THINK

(12) Solve for x.

(a) $2x + 5 = 7$

If [hand] + 5 = 7, what is [hand]?

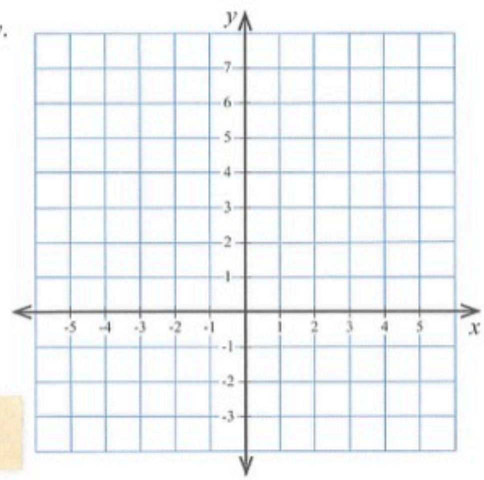
(b) $2x + 5 = 1$

(c) $2x + 5 = -3$

(d) These equations are all specific cases of $2x + 5 = y$.
By solving the equations above, you have found 3 solution points. What are they?

(, 7)   (, 1)   (,)

(e) Use these points (and more, if you want) to graph $2x + 5 = y$.



(f) Use the graph to estimate the solution point if $y = -1$.

(,)

(g) Solve the equation $2x + 5 = -1$ to test your estimate.

(h) What point can help you solve $2x + 5 = 5$? (,)

(i) Find x: $2x + 5 = 5$

This graph summarizes solution points for this equation.
Graphs can be a tool for helping to solve equations.

(13) Solve for x.

(a) $x + 2 = -3$

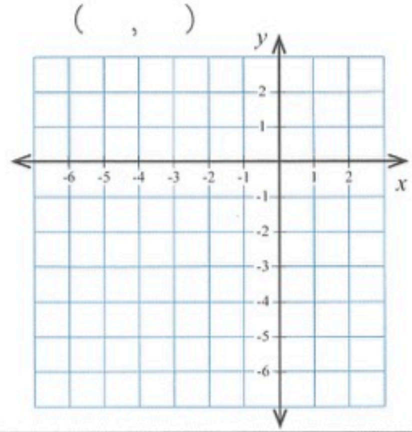
(b) $x + -5 = -3$

(c) $x + -2 = -3$

(d) These equations are all specific cases of $x + y = -3$. What solution points did you just find?

(, 2)   (, -5)   (,)

(e) Use these points (and more, if you want) to graph $x + y = -3$.

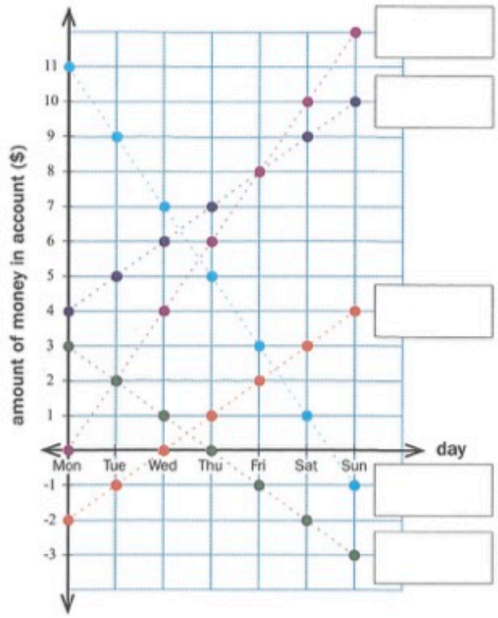


Now use your graph to help you solve equations.

(f) Solve $x + -4.5 = -3$ by estimating a point on the graph and testing it in the equation.

(g) Solve $x + 0.5 = -3$ the same way.

⑭ Five students meet every night to work on the production of a play. They set up individual accounts to keep track of the bags of pretzels they eat as snacks. Any time they bring in pretzels, they add $1 per bag to their own account. Whenever they eat pretzels, they subtract $1 per bag from their account.



By the way, the light dotted lines are only there to help show which points go together. It actually doesn't make sense for the points to be connected. (Why not?)

The clues below will let you figure out which student goes with which graph. Write the correct name by each graph.

Clue 1: Asher, Carla, and Eva all started out with money in their accounts.

Clue 2: Ben, Damian, and Eva all bring more pretzels than they eat.

Clue 3: Carla didn't bring in any bags of pretzels all week but ate two bags every day.

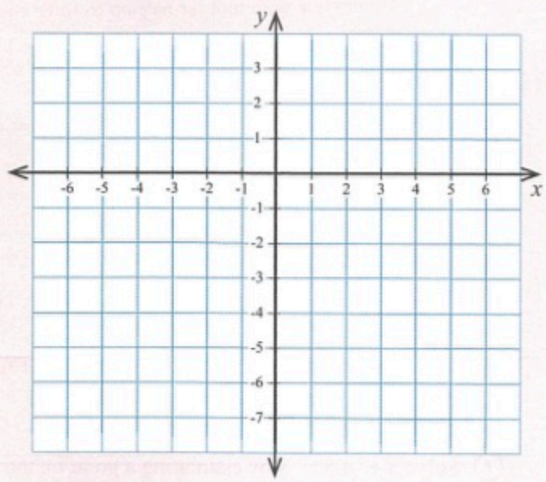
Clue 4: Every day, Asher brought in 3 bags and ate 4 bags.

Clue 5: Ben didn't eat any pretzels all week, but brought in 2 bags per day.

Clue 6: On Thursday, the person with the most money in her account was Eva; Asher had the least.

TOUGH STUFF

Fill in the coordinates, and draw and label each point.



⑮ I am point A.

» My x-coordinate is the solution to $2(x + 4) - 5 = 15$.

» My y-coordinate is half my x-coordinate.

Where Am I? (,)

⑯ I am point B.

» The sum of my x- and y-coordinates is -4.

» Solve $4 = x + 7$ to get my x-coordinate.

Where Am I? (,)

⑰ I am point C.

» $x + y = -5$

» $y > x$

» $x < 0$

» $xy = 0$

Where Am I? (,)

⑱ I am point D.

» $x^2 = 16$

» $y^2 = 9$

» There are *four* possible places I could be. Mark them all.

Where could I be?

(,) (,) (,) (,)

Additional Practice

Fill in the coordinates, and draw and label each point.

Ⓐ I am point A.
» My x-coordinate is 0.
» $y = x - 3$

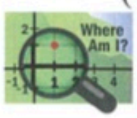
Where Am I? (,)

Ⓑ I am point B.
» My x-coordinate is -1.
» $y = x - 3$

Where Am I? (,)

Ⓒ I am point C.
» My y-coordinate is 0.
» $y = x - 3$

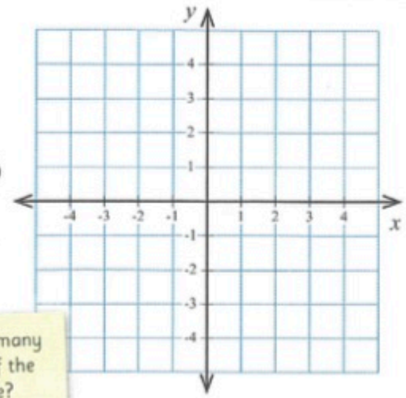
Where Am I? (,)



Ⓓ I am point D.
» $y = x - 3$

Where could I be?

The equation $y = x - 3$ has infinitely many solutions. What does a graph of *all* of the possible locations for point D look like?



Ⓔ Find (x, y) pairs that work for $y = -x + 1$. These are *solution points*.

ⓘ Find a solution point if $x = 0$.

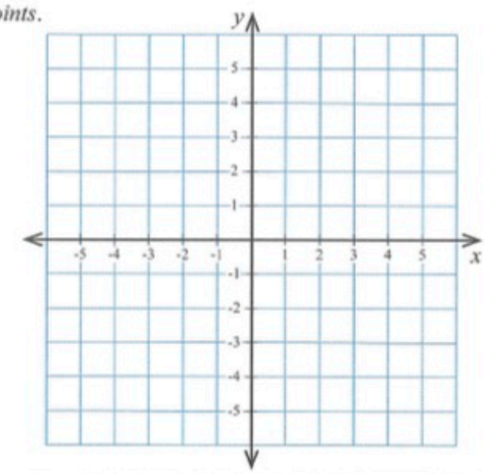
ⓘⓘ Find a solution point if $x = -4$.

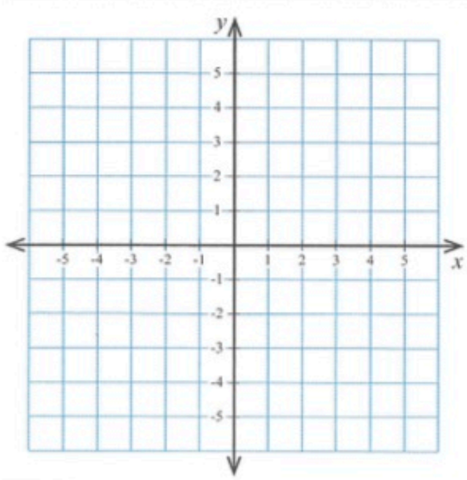
ⓘⓘⓘ Find a solution point if $y = 0$.

ⓘⓥ Find a solution point if $x = -20$.

ⓥ Find three more solution points.

ⓥⓘ Plot the solution points that fit on the graph paper and use them to draw the graph of $y = -x + 1$.





Ⓕ $x + 2y = 8$

ⓘ Find a solution point if $x = 0$.

ⓘⓘ Circle the solution points; cross out non-solution points.

(6, 1)	(10, -1)	(-10, 9)
(20, 6)	(-2, 5)	(2, 4)

ⓘⓘⓘ Use the solution points to draw the graph of $x + 2y = 8$. Check more solution points, if necessary.

TTA Unit 6 Lesson 6 Teaching Guide

(Mark, Goldenberg, Fries, Kang, & Cordner, 2014a, pp. T23 - T26)

Lesson 6:
Solutions and Point Testing

PURPOSE

This lesson introduces students to the idea of a solution point. Students explore the idea that given an equation, *any* point on the coordinate plane either *is* or *is not* a solution point. An equation then serves as a "point-tester": any point can be put into the equation, and the equation will show whether the point is or is not on the graph of that equation. With this understanding of solution points, students develop a greater sense of what the graph of an equation represents: a graph is not just a picture; it is a representation of the *solutions* of the equation, the collection of *all* points that follow the pattern described by the equation.

The fact that for linear equations, the infinite collection of solution points happens to take the shape of a *line* when plotted together is special. This lesson helps students establish the relationship between solutions to an equation and points on a graph. They develop the idea that an equation represents a pattern of calculation that shows up as a visual pattern on the coordinate plane.

Understanding equations as point-testers can help students identify an equation by looking at a graph and make sense of non-linear graphs. The equation of a graph makes a true statement for *every* point on the graph and *only* for that collection of points. Writing an equation means describing a relationship between x and y for those points. This understanding extends to non-linear graphs. For students who understand a graph as a collection of an infinite number of points, the concept of a parabola extends from their understanding of a line; a parabola also marks out a specific collection of solution points to an equation on the coordinate plane.

Students will revisit these ideas in Unit 9: *Points, Slopes, and Lines* when they use point testing to generate linear equations using the knowledge that points on a line follow a pattern of constant rate of change and examine graphs of inequalities where solution points encompass *regions* of the plane rather than a line or curve.

Mental Mathematics *Begin each day with five minutes of Mental Mathematics (pages T51–T64). These activities help students build working memory so that they can keep multiple pieces of mathematical information in mind at once.*

Lesson at a Glance

Preparation

- Prepare to display the "Point Testing" grid on T42 for the Launch.
- *(optional)* The Launch activity suggests using four colors to mark points.

Mental Mathematics (5 min)

Launch: Point Testing (15 min)

- Place students in four groups, each with its own equation to use to test points.
- Discuss "point testing" as a way of investigating the graph of an equation; *any* point can be tested to see if it is a solution point, but once patterns start to emerge, making conjectures and testing specific points can further confirm a suspected pattern.

Student Problem Solving and Discussion (25 min)

- Give students time to work through the Important Stuff and additional problems.
- Ask students to share their responses from the *Discuss & Write* problem, which asks about the logic of connecting the solution points of an equation.
- Discuss what it means to be a solution point and what a graph really represents.

Unit 6 Related Game: Battleshape (See page T37 and Student Worktext page 43.)

These equations and the ones in the Student Worktext are deliberately presented in various forms. The message for students is that all of these equations—regardless of their form—represent a relationship between *x* and *y* and that we can start to make sense of this relationship by examining what happens at specific points.

You may find it helpful to write the equations on the board with each group's associated color.

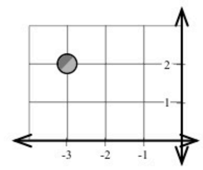
Launch: Point Testing

Split the class into groups or let them work individually. Assign each group or individual one of the equations below:

- Group A will get the equation $\mathbf{y = x + 5}$.
- Group B will get the equation $\mathbf{x + y = {}^{-}1}$.
- Group C will get the equation $\mathbf{2x + y = 2}$.
- Group D will get the equation $\mathbf{y = 2x + 14}$.

Display the "Point Testing" grid on page T42 .

Circle the point (⁻3, 2) and ask students to test that point in their equation. "Testing" a point means using its *x*- and *y*-coordinates as values in the equation and seeing if the result is a true statement. Students in Groups A and B will find that (⁻3, 2) works for their equation. Have students show their evidence (e.g. 2 = ⁻3 + 5 for Group A's equation). Assign each group a color and mark the point with the color(s) of the related equations.

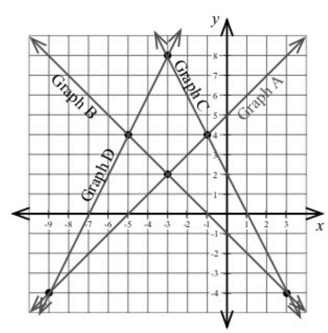


Present the following six points one a time to the whole class, spending about a minute on each point for students to test the point and for the class to verify the results:

(⁻3, 2)	Satisfies equations A and B.
(⁻1, 4)	Satisfies equations A and C.
(⁻5, 4)	Satisfies equations B and D.
(3, ⁻4)	Satisfies equations B and C.
(⁻3, 8)	Satisfies equations C and D.
(⁻9, ⁻4)	Satisfies equations A and D.

Make sure students understand that this process could continue forever, because *any* point on the coordinate plane can be tested. As we test many points, we see that the solution points for a particular equation start to follow a visible pattern.

Mention that the four equations in this activity are special: not all equations have straight line graphs. But finding and testing solution points can always be used as a starting point for graphing equations. As students start to see a pattern emerging in the solution points for an equation, they can predict a solution point and test it. This is a way of gathering evidence to support or revise their hypothesis. Part of mathematics is then going further to understand why particular patterns emerge from particular equations.



Here are some other ways to further the conversation:

- Ask students to predict another possible solution point for their equation and then to test it.
- Ask about the point (-7, -1). It *looks* like it may be on one of the lines, but point testing will confirm that it isn't.
- Ask about the points $(\frac{1}{2}, 5\frac{1}{2})$ and $(\frac{1}{2}, 5\frac{1}{4})$. Solution points don't have to have integer coordinates. Furthermore, *every* point is either on the graph (is a solution point) or it is not. Group A will find that their graph passes through $(\frac{1}{2}, 5\frac{1}{2})$ but not through $(\frac{1}{2}, 5\frac{1}{4})$. Students should get the message that graphing is not just about drawing a "straight line" and should understand that *every* point on that line is a solution for the equation.
- Ask students to predict and test other solution points (including points with non-integer coordinates) until they have found eight solution points for their equation. For these equations, but not for all equations, the graph is a line. Have students draw the line and choose one more point that is off the paper to test, with the equation, if it is on that line.

Student Problem Solving and Discussion

Allow time for students to work on the Important Stuff and additional problems.

Most of the problems are about testing or finding solution points. Help students see an equation as a statement that encodes a pattern of calculations. They have already approached equations in this way in the Think of a Number tricks from Units 1 and 5.

In this lesson, students start with an equation and use it to generate and test points. The repetition that students experience in this lesson is intentional and designed to build the idea that *every* point on a graph is related by the *same* pattern of calculation encoded in the equation. These points, taken as a collection, produce a graph.

Ask students to share written responses for **PROBLEM 8** in the *Discuss & Write* box. The problem is about whether it makes sense to "connect the dots" in the graph of an equation. Students have been thinking about when to connect the dots throughout Unit 6—most notably in Lessons 2 and 5. Listen for students who can explain that the line is just a way of indicating *every* point that follows the pattern of the solution points.

Use these prompts to support good thinking and discussion about graphs.

» **How do you know if a point is on the graph of an equation?** Students may think of an equation as a point-tester. For an equation that describes a relationship involving x and y, the x- and y-coordinates of any point can be plugged into the equation. The point is on the graph if the equation results in a true statement.

Algebraic Habits of Mind

Using Tools Strategically
Help students see that one way that a graph can be used as a tool is to replace a huge table of calculations with a compact picture that summarizes them all. Students who are using tools strategically will be able to identify when graphing can support their mathematical work and reason about a problem by looking at a related graph.

What if . . .

What if students labor over each point in a sequence of problems?

Remind them that the calculations they are performing are actually the same each time because they are using the same equation. Help those students switch from focusing on "what to plug in where" to seeing $y = x + {}^-5$ as an equation and thinking "in each point, y is five less than x" as a way to understand the pattern of calculation. It may help some students to think of this like a Mental Mathematics activity where they are given a prompt and have to subtract 5.

Make clear by your use of language that "solution points for an equation" and "points on the graph of the equation" are two names for the same set of points. The "graph of an equation" is not just what we see but a visual representation of the entire set of solution points for that equation. The "solution points" are not just the entries in the table but *all* the points that make the equation true. They therefore include any point that *could* be in the table.

» **Describe the relationship between the solution points of an equation and the graph of an equation. What about the non-solution points?** Listen for evidence indicating understanding that any point on the coordinate plane can be tested. Graphs of equations represent the set of solution points; furthermore, they include *all* solution points—even the ones that don't fit on the page. *Any* point that is *not* on the graph is a non-solution point, and any point that is not a solution point is *not* on the graph.

» **In problems 1–8, you worked with the equation $y = x + {}^{-}5$. How does that compare with $x - y = 5$?** See if anyone can write the equations in a way that makes the comparison easier. Both can be converted, using legitimate algebraic steps, into the same form. By making the form of the equations as similar as possible, students can isolate specific features that might account for any differences between two graphs. In this case, the two equations represent the same relationship and so have the same solution points and therefore the same graph.

» **What if you were just given an equation (like $2x + y = 20$) and no possible solution points? What would you do to start figuring out what the graph looks like?** In this lesson, students were generally given solution points to test. But students should also realize that they can (and did) generate their own solution points. More of this will be addressed in Lesson 7. Students may start by testing arbitrary points, but they can be more strategic. They can think, for example, "What if x were 0?" and use this as an entry to generate solution points. Students should see that choosing "easy" values for x like 0, ±1, ±2, or ±3 (depending on the equation) is a useful strategy, but not the only correct approach. Later, they will learn that part of a strategy for graphing also includes recognizing features of an equation that make it possible to make predictions about whether the graph will be a line, parabola, circle, V-shape, or other shape.

Appendix I: Modified ELSF Guidelines for Teacher Enactment

Area of Focus I: Interdependence of Mathematical Content, Practices, and Language	
1. Strategic opportunities to use and refine both language and mathematics over time	
e. Teacher highlights, defines, illustrates, and shows the purpose for mathematical language within the context of the lesson (not in isolation).	
f. Teacher encourages students to build their own understanding of mathematics actively, using language, through sustained activities and experiences.	
g. Teacher provides strategies to help students make connections between current language, new language, and mathematical concepts.	
2. Explicit mathematics and language learning goals and pathways	
e. Teacher states clear and specific language objectives both for math practices as well as for academic purposes that cut across disciplines.	
f. Teacher gives students specific mathematics and language learning objectives.	
g. Teacher presents opportunities for students to use language at different stages within the lesson.	
3. Regular and varying opportunities to learn, reflect upon, and demonstrate learning of mathematics using a variety of modes and forms	
d. Teacher provides activities that deepen and extend learning through the various modes of communication: speaking, listening, reading, and writing.	
e. Teacher prompts students to reflect on their own thought processes, language use, methods, and learning of mathematical content.	
f. Teacher encourages students to utilize interdisciplinary words and phrases as well as math-specific words and phrases.	
Area of Focus II: Scaffolding and Supports for Simultaneous Development	
4. Opportunities for students to interact with and produce a variety of methods and representations	
e. Learning activities provide ways for students to generate and interpret a range of mathematical methods and representations (symbols, manipulatives, graphs, tables, words, etc.) and methods.	
f. Teacher encourages students to draw comparisons and connections across different methods and representations.	
g. Lesson includes multiple sensory modalities for student interaction.	
h. Teacher models reading, writing, listening, speaking, and thinking aloud.	

5. Directions for providing specialized individual and small group instruction to ELs	
d. Teacher meets directly with EL students individually and in small groups.	
e. Teacher plans for what to look for, listen for, questions to ask, and/or feedback to give when meeting with EL students.	
f. Lesson provides a balance of opportunities for independent, paired, small-group, and whole-class activities.	
6. Guidance for anticipating potential language demands and opportunities in student activities	
e. Teacher plans for addressing possible language issues that may interfere with engagement of math content.	
f. Teacher plans to or helps students make meaning of typical mathematical texts such as word problems, graphs, tables, etc.	
g. Teacher distinguishes between common everyday meanings of language and mathematical meanings (table, round, product, origin, similar, etc.) as they emerge in the lesson.	
Area of Focus III: Mathematical Rigor Through Language	
7. Explicit guidance for teachers to engage students in using mathematical practices	
d. Teacher plans to have targeted opportunities for students to use and develop language functions while engaging in mathematical practices.	
e. Teacher provides opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.	
f. Teacher provides opportunities for students to revise their own, peers', and/or fictitious mathematical writing.	
8. Maintain appropriate challenge and high expectations of mathematics learning for EL students	
d. Teacher provides access to cognitively-demanding tasks.	
e. Teacher allows students to engage in productive struggle before intervening.	
f. Teacher uses anchor charts, visual aids, models, and other resources for students to use as a reference.	
9. Guidance for facilitating mathematical discussion and co-construction of meaning	
d. Teacher cultivates and facilitates back-and-forth mathematical discussions between students that refer to and build on each other's ideas.	
e. Teacher plans explicit purposes for communication between students.	
f. Teacher allows for equitable participation and risk-taking in conversations.	

Area of Focus IV: Leveraging Students' Assets	
10. Opportunities to draw on and incorporate students' cultural background and lived experiences in mathematics learning	
d. Teacher connects mathematics content and practices to students' lives.	
e. Teacher encourages students to draw on prior knowledge, culture, and experiences.	
f. Teacher clarifies potentially unfamiliar contexts.	
11. Suggestions for incorporating and valuing ELs' written and spoken contributions	
c. Teacher supports contributions of students with various language strengths and needs in mathematical practices.	
d. Teacher reflects on their own values and beliefs about language, ELs, and ways in which that might impact their teaching.	
12. Encouragement for ELs to use and build on existing language resources	
c. Teacher allows ELs to use and integrate first language (L1) and everyday English in communicating mathematical thinking.	
d. Teacher presents opportunities for students to ask and pursue their own questions and interests, using their own methods in their chosen contexts.	
Area of Focus V: Assessment of Mathematical Content, Practices, and Language*	
13. Descriptions, illustrations, and examples of quality work and mathematical practices with varying levels of language proficiency	
a. Teacher encourages teacher-student and student-student interactions that model and reflect the intent of mathematical practices.	
b. Teacher encourages students to express mathematical ideas in their own words.	
14. Assessments able to capture and measure students' mathematics and language progress over time	
a. Teacher prompts students to use math practices through language (including but not limited to vocabulary).	
b. Teacher provides opportunities to move from everyday language to language for more formal academic and mathematical purposes.	
15. Guidance for recognizing and attending to student language produced to inform instructional decisions	
a. Teacher does not interpret lower level language proficiency as lower level mathematics proficiency.	
b. Teacher includes a range of assessments for formative purposes that enable students to draw on and make use of their existing language resources.	

* Consider Assessment as *formative assessment*: any task or activity in which the teacher is gathering information about what the students know throughout the lesson

REFERENCES

Banilower, E. R., Smith, P. S., Weiss, I. R., Malzahn, K. M., Campbell, K. M., & Weiss, A. M. (2013). Report of the 2012 National Survey of Science and Mathematics Education, 1–309. Retrieved from http://www.horizon-research.com/2012nssme/research-products/reports/technical-report/

Barwell, R. (2003). Patterns of attention in the interaction of a primary school mathematics student with English as an additional language. *Educational Studies in Mathematics*, *53*(1), 35–59.

Barwell, R. (2005). Critical issues for language and content in mainstream classrooms: Introduction. *Linguistics and Education*, *16*(2), 143–150. https://doi.org/10.1016/j.linged.2006.01.003

Berger, M. (2005). Vygotsky's theory of concept formation and mathematics education. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (pp. 153–160). Melbourne: PME.

Berliner, D. C. (2004). Describing the behavior and documenting the accomplishments of expert teachers. *Bulletin of Science, Technology and Society*, *24*(3), 200–212. https://doi.org/10.1177/0270467604265535

Blitzer, R. F. (2010). *Precalculus* (4th ed.). Upper Saddle River, NJ: Prentice Hall.

Brown, J., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, *18*(1), 32–41.

Brown, M. W. (2009). The teacher-tool relationship. In *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36).

Bush, B. (2011). CCSSM Curriculum Analysis Tool. Retrieved from http://www.mathedleadership.org/ccss/materials.html

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology*, *3*, 21–29.

Chval, K. B., Pinnow, R. J., & Thomas, A. (2015). Learning how to focus on language while teaching mathematics to English language learners: a case study of Courtney. *Mathematics Education Research Journal*, *27*(1), 103–127.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education*, *28*(3), 258–277.

Cobb, P., & Yackel, E. (1996). Constructivist, emergence, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, *31*(3/4), 175–190.

Cohen, E. (2002). *Designing Groupwork: Strategies for the Heterogeneous Classoom* (2nd ed.).

New York: Teachers College Press.

Confrey, J. (1995). A theory of intellectual development. *For the Learning of Mathematics*, *15*(1), 38–48.

Confrey, Jere, & Smith, E. (1995). Splitting, covariation, and their role in the development of exponential functions. *Journal for Research in Mathematics Education*, *26*(1), 66–86.

Cuoco, A. (2008). CME Project: Implementing and Teaching Guide.

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior*, *15*(4), 375–402. https://doi.org/10.1016/S0732-3123(96)90023-1

Cuoco, A., & Kerins, B. (2016). *Integrated Mathematics 1*. Boston, MA: Pearson.

de Araujo, Z. (2017). Connections between secondary mathematics teachers' beliefs and their selection of tasks for English language learners. *Curriculum Inquiry*, *6784*(September), 1–27. https://doi.org/10.1080/03626784.2017.1368351

de Araujo, Z., Smith, E., & Dwiggins, A. (2018). Examining storylines of emergent bilinguals in algebra textbooks. In *Cambio Center eBrief*. Columbia, Missouri: University of Missouri.

Dietiker, L, Amador, J. M., Earnest, D., Males, L. M., & Stohlmann, M. (2014). Fostering K-12 prospective teachers' curricular noticing. Research symposium at the National Council of Teachers of Mathematics Research Conference, New Orleans, LA.

Dietiker, Leslie, Baldinger, E., & Kassarjian, M. (2014). *Core Connections Integrated I*. Elk Grove, CA: CPM Educational Program.

Dominguez, H. (2016). Mirrors and windows into student noticing. *Teaching Children Mathematics*, *22*(6), 358–365.

EdReports. (n.d.-a). CPM Integrated (2015). Retrieved from https://www.edreports.org/reports/overview/cpm-integrated-2015

EdReports. (n.d.-b). Mathematics Vision Project (MVP) Integrated (2016). Retrieved from https://www.edreports.org/reports/overview/mathematics-vision-project-mvp-integrated-2016

Ellis, A. B., Ozgur, Z., Kulow, T., Williams, C., & Amidon, J. (2012). Quantifying exponential growth: The case of the jactus. *WISDOMe Monographs*, *2*, 93–112.

Ellis, Amy B., Ozgur, Z., Kulow, T., Dogan, M. F., & Amidon, J. (2016). An exponential growth learning trajectory: Students' emerging understanding of exponential growth through covariation. *Mathematical Thinking and Learning*, *18*(3), 151–181. https://doi.org/10.1080/10986065.2016.1183090

ELSF. (n.d.). *Guidelines for Improving Math Materials for English Learners*. English Learner Success Forum. Retrieved from https://www.elsuccessforum.org/resources

Forman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 333–352). Reston, VA: National Council of Teachers of Mathematics.

Freeman, S. (2013). *Algebra II Reproducibles*. (F. Lesser, Ed.). Dayton, OH: Milliken Publishing Company.

Friesen, N. (2012). Doing with icons makes symbols; or, jailbreaking the perfect user interface. *Theory Beyond the Codes*.

Goodwin, C. (1994). Professional vision. *American Anthropologist*, *96*(3), 606–633.

Grossman, P. (1990). *The making of a teacher: Teacher knowledge and teacher education*. Teachers College Press.

Gutiérrez, R. (2002). Beyond essentialism: The complexity of language in teaching mathematics to Latina/o students. *American Educational Research Journal Winter*, *39*(4), 1047–1088. https://doi.org/10.3102/000283120390041047

Hendrickson, S., Hilton, S. C., & Bahr, D. (2008). The Comprehensive Mathematics Instruction (CMI) Framework: A new lens for examining teaching and learning in the mathematics classroom. *Utah Mathematics Teacher*, *Fall*, 44–52.

Hendrickson, S., Honey, J., Kuehl, B., Lemon, T., & Sutorius, J. (2016a). Mathematics vision project. Retrieved from https://www.mathematicsvisionproject.org/curriculum.html

Hendrickson, S., Honey, J., Kuehl, B., Lemon, T., & Sutorius, J. (2016b). *Secondary Math One An Integrated Approach Enhanced Teacher's Notes*. Mathematics Vision Project.

Herbel-Eisenmann, B. (2007). From intended curriculum to written curriculum: examining the "voice" of a mathematics textbook. *Journal for Research in Mathematics Education*, *38*(4), 344–369.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. *Source: Journal for Research in Mathematics Education Journal for Research in Mathematics Education*, *41*(2), 169–202. https://doi.org/10.2307/20720130

Kazemi, E., & Stipek, D. (2001a). Promoting conceptual thinking in four upper-elementary mathematics classes. *The Elementary School Journal*, *102*(1), 59–80.

Kazemi, E., & Stipek, D. (2001b). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, *102*(1), 59–80.

Khisty, L. L., & Chval, K. B. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal*, *14*(3), 154–168. https://doi.org/10.1007/BF03217360

KUTA Software. (n.d.). Retrieved from https://www.kutasoftware.com/index.html

Lappan, G., Fey, T., Fitzgerald, W. M., Friel, S., & Phillips, E. D. (2006). *Connected Mathematics 2: Implenting and Teaching Guide*. Boston, MA: Pearson, Prentice Hall.

Lobato, J., Ellis, A. B., & Muñoz, R. (2003). How "focusing phenomena" in the instructional environment support individual students' generalizations. *Mathematical Thinking and Learning*, *5*(1), 1–36.

Lobato, J., Hohensee, C., & Rhodehamel, B. (2013). Students' mathematical noticing. *Journal for Research in Mathematics Education*, *44*(5), 809–850. https://doi.org/10.5951/jresematheduc.44.5.0809

Lobato, J., & Walker, C. (2019). How Viewers Orient Toward Student Dialogue in Online Math Videos. *Journal of Computers in Mathematics and Science Teaching*, *38*(2), 177–200.

Lobato, J., & Walters, C. D. (2017). A taxonomy of approaches to learning trajectories and progressions. In J. Cai (Ed.), *Compendium for Research in Mathematics Education* (pp. 74–101). Reston, VA: National Council of Teachers of Mathematics.

Males, L. M., Earnest, D., Dietiker, L., & Amador, J. M. (2015). Examining K-12 prospective teachers' curricular noticing. In *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 88–95). East Lansing: Michigan State University.

Mark, J., Goldenberg, E. P., Fries, M., Kang, J. M., & Cordner, T. (2014a). *Series Overview*. Portsmouth, NH: Heinemann.

Mark, J., Goldenberg, E. P., Fries, M., Kang, J. M., & Cordner, T. (2014b). *Transition to Algebra*. Portsmouth, NH: Heinemann.

Mark, J., Goldenberg, E. P., Fries, M., Kang, J. M., & Cordner, T. (2014c). *Transition to Algebra Unit 1 Teaching Guide*. Portsmouth, NH: Heinemann.

Mark, J., Goldenberg, E. P., Fries, M., Kang, J. M., & Cordner, T. (2014d). *Transition to Algebra Unit 6 Student Worktext*. Portsmouth, NH: Heinemann.

Maxwell, J. A. (2013). *Qualitatitve research design: An interactive approach*. SAGE Publications, Inc.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook*. Thousand Oaks, CA: SAGE Publications, Inc.

Moschkovich, J. (1996). Moving up and getting steeper: Negotiating shared descriptions of linear graphs. *The Journal of the Learning Sciences*, *5*(3), 239–277.

Moschkovich, J. (1998). Resources for refining mathematical conceptions: Case studies in learning about linear functions. *Journal of the Learning Sciences*, *7*(2), 209–237. https://doi.org/10.1207/s15327809jls0702_3

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2&3), 189–212. https://doi.org/10.1207/S15327833MTL04023

Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education*, *6*(1), 45–57. Retrieved from http://ed-osprey.gsu.edu/ojs/index.php/JUME/article/viewArticle/204

Moschkovich, J. (2015). Academic literacy in mathematics for English Learners. *Journal of Mathematical Behavior*, *40*, 43–62. https://doi.org/10.1016/j.jmathb.2015.01.005

Mosqueda, E., & Maldonado, S. I. (2013). The effects of English language proficiency and curricular pathways: Latina/os' mathematics achievement in secondary schools. *Equity & Excellence in Education*, *46*(2), 202–219. https://doi.org/10.1080/10665684.2013.780647

National Council of Teachers of Mathematics. (2000). *Principles and Standards of School Mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors.

National Research Council. (2001). *Adding in up: Helping children learn mathematics*. (J. Kilpatrick, J. Swafford, & B. Findell, Eds.).

NCTM. (1989). *Curriculum and Evaluation Standards for School Mathematics*. *Curriculum and Evaluation Standards Report*. Reston, VA: NCTM.

O'Connor, M. C. (1998). Language socialization in the mathematics classroom: Discourse practices and mathematical thinking. *Talking Mathematics: Studies of Teaching and Learning in School*, 17–55.

Patton, M. Q. (2002). *Qualitative research & evaluation methods*. New York, New Delhi, London: Thousand Oaks Sage Publications.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, *75*(2), 211–246.

Remillard, J. T. (2009). Considering what we know about the relationship between teachers and curriculum materials. In *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 85–92).

Rentner, D. S., & Kober, N. (2014). *Common Core State Standards in 2014: Curriculum and Professional Development at the District Level*. Washington, DC. Retrieved from http://www.cep-dc.org/displayDocument.cfm?DocumentID=441

Ross, K. E. L. (2014). Professional development for practicing mathematics teachers: A critical connection to English language learner students in mainstream USA classrooms. *Journal of Mathematics Teacher Education*, *17*(1), 85–100. https://doi.org/10.1007/s10857-013-9250-7

Roth McDuffie, A., Choppin, J., Drake, C., Davis, J. D., & Brown, J. (2018). Middle school teachers' differing perceptions and use of curriculum materials and the common core. *Journal of Mathematics Teacher Education*, *21*(6), 545–577. https://doi.org/10.1007/s10857-017-9368-0

Säljö, R., & Wyndhamn, J. (1996). Solving everyday problems in the formal setting: An empirical study of the school as context for thought. In S. Chaiklin & J. Lave (Eds.), *Understanding practice* (pp. 327–342). New York: Cambridge University Press.

Schoenfeld, A. H. (1992). *Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics*. (D. Grouws, Ed.), *Handbook for Research on Mathematics Teaching and Learning*. New York: Macmillan. https://doi.org/10.1136/bmj.1.6053.66

Schütz, R. (2004). *Vygotsky and language acquisition*.

Secada, W. (1992). Evaluating the mathematics education of limited English proficient students in a time of educational change. In *Focus on Evaluation and Measurement Series*. Retrieved from ERIC Document No. ED 349 828

Selling, S. K. (2016). Making mathematical practices explicit in urban middle and high school mathematics classrooms. *Journal for Research in Mathematics Education*, *47*(5), 505–551. https://doi.org/10.1007/s13398-014-0173-7.2

Shein, P. P. (2012). Seeing with two eyes: A teacher's use of gestures in questioning and revoicing to engage English language learners in the repair of mathematical errors. *Journal for Research in Mathematics Education*, *43*(2), 182–222. https://doi.org/10.5951/jresematheduc.43.2.0182

Shroyer, J. (1984). The LES Instructional Model: Launch-Explore-Summarize. In *Honors Teachers Workshop of Middle Grade Mathematics Proceedings* (pp. 101–114).

Smith, J. P., & Thompson, P. W. (2008). Quantitative reasoning and the development of algebraic reasoning. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (Vol. 15, pp. 95–132). New York, NY: Lawrence Erlbaum Associates. Retrieved from http://books.google.com/books?id=Ve7uAAAAMAAJ

State of California Commission on Teacher Credentialing. (2016). Single Subject Teaching Credential Requirements for Teachers Prepared in California. Retrieved November 15, 2017, from https://www.ctc.ca.gov/docs/default-source/leaflets/cl560c.pdf?sfvrsn=8db75dfc_0

Stein, M. K., Remillard, J. T., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 319–370). Greenwich, CT: Information Age Publishing.

Stephan, M., & Akyuz, D. (2012). A proposed instructional theory for integer addition and subtraction. *Journal for Research in Mathematics Education*, *43*(4), 428–464.

Strauss, A. L., & Corbin, J. (1994). Grounded theory methodology: An overview. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 273–285). Thousand Oaks: Sage Publications.

Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 179–234). Albany, NY: SUNY Press.

Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English Learners' Participation in Mathematical Discussion : Shifting Positionings and Dynamic Identities. *Journal for Research in Mathematics Education*, *44*(1), 199–234. https://doi.org/10.5951/jresematheduc.44.1.0199

Turner, E. E., Dominguez, H., Empson, S., & Maldonado, L. A. (2013). Latino/a bilinguals and their teachers developing a shared communicative space. *Educational Studies in Mathematics*, *84*(3), 349–370. https://doi.org/10.1007/s10649-013-9486-2

US Department of Education. (2013). *The Biennial Report to Congress on the Implementation of the Title III State Formula Grant Program*. Retrieved from http://www.ncela.us/files/uploads/3/Biennial_Report_0810.pdf

Vygotsky, L. (1987). *Thought and Language*. Cambridge, MA: MIT Press.

Yopp, H. K., Spycher, P., & Brynelson, N. (2016). California's vision of ELA/ELD instruction. *The California Reader*, *49*(3), 8–20.

Zahner, W. (2015). The rise and run of a computational understanding of slope in a conceptually focused bilingual algebra class. *Educational Studies in Mathematics*, *88*(1), 19–41. https://doi.org/10.1007/s10649-014-9575-x

Zahner, W., Velazquez, G., Moschkovich, J., Vahey, P., & Lara-Meloy, T. (2012). Mathematics teaching practices with technology that support conceptual understanding for Latino/a students. *The Journal of Mathematical Behavior*, *31*(4), 431–446. https://doi.org/10.1016/j.jmathb.2012.06.002