Quantifying the Coherence of Pedestrian Groups

Adam W. Kiefer (adam_kiefer@brown.edu)
Department of Cognitive, Linguistic & Psychological Sciences
Brown University
190 Thayer Street
Providence, RI 02912 USA

Stéphane Bonneaud (stephane_bonneaud@brown.edu)
Department of Cognitive, Linguistic & Psychological Sciences
Brown University
190 Thayer Street
Providence, RI 02912 USA

Kevin Rio (kevin_rio@brown.edu)
Department of Cognitive, Linguistic & Psychological Sciences
Brown University
190 Thayer Street
Providence, RI 02912 USA

William H. Warren (bill_warren@brown.edu)
Department of Cognitive, Linguistic & Psychological Sciences
Brown University
190 Thayer Street
Providence, RI 02912 USA

Abstract

Coherent collective behavior emerges from local interactions between individuals that generate group dynamics. An outstanding question is how to quantify group coherence in order to understand the nature of these dynamics. We investigate this problem in the context of a small group of pedestrians instructed simply to walk to a goal. To measure the degree of coordination in a group, we employed principal components analysis to estimate dimensional compression, and cross-recurrence quantification analysis to estimate the coupling strength between individuals. The results indicate lower-dimensional behavior and more stable coupling in real groups compared to reshuffled virtual groups. These findings demonstrate spontaneous local coordination in pedestrian groups that gives rise to coherent collective behavior, and offer an approach for investigating group dynamics in more complex contexts.

Keywords: group locomotion; group coordination; cross-recurrence quantification; principal components analysis

Introduction

Group dynamics arise from local interactions between individuals that are governed by a multi-level set of processes. At the most basic level, these interactions depend on a coupling between individuals based on perceptual information, which may further depend on higher-order cognitive and social constraints. To understand the emergence of collective behavior, it is necessary to begin by characterizing both the local informational coupling and the global group behavior. Such an approach requires a complementary set of analysis tools to quantify observable properties, such as the degree and stability of coordination, at both the individual and group levels.

In the context of locomotion, we focus on the coupling between individual pedestrians that yields the formation of a coherent crowd. A recent dynamical model of locomotor behavior (Fajen & Warren, 2003, 2007; Warren & Fajen, 2008) has characterized both individual behavior and pedestrian interactions, including coordination in leader-follower and side-by-side dyads (Rio & Warren, 2011; Page & Warren, 2012), and may be generalized to coordination in groups (Rio, Bonneaud & Warren, 2012). Here we investigate measures of the degree of coordination in small groups, or group coherence.

Relevant behavioral variables to index the locomotor trajectory of an agent include (1) the agent's direction of travel, or heading (ϕ) and (2) the agent's speed (**s**). Each of these variables can be considered a degree of freedom (DoF) of pedestrian locomotion, and thus the DoF of a group of N pedestrians can be operationally defined as a system consisting of N×2 DoF (i.e., ϕ and **s**).

It has been proposed that behavioral coordination between two agents arises from the coupling of DoF via shared information variables (Riley, Richardson, Shockley & Ramenzoni, 2011). Shared information between agents allows the DoF to directly regulate one another. This permits the characterization of interpersonal coordination in terms of the reduction of DoF, or dimensional compression, due to the behavioral reorganization of the newly assembled system. In the context of pedestrian interactions, a follower controls their speed by nulling change in the leader's visual angle, and a pedestrian walking beside a neighbor controls their speed by nulling change in the neighbor's visual direction. Thus, visual information that serves to couple DoFs (i.e. ϕ and **s**) gives rise to pedestrian coordination and, ultimately, coherent crowds (Bonneaud & Warren, 2012; Moussaïd, Helbing & Theraulaz, 2011; Ondřej, Pettré, Olivier, & Donikian, 2010; Rio, Bonneaud & Warren, 2012).

We aim to advance the analysis of collective behavior by developing methods to quantify the degree of coordination among pedestrians in groups. We focus on both the basic coordination mechanism – the local coupling between pairs of neighbors – and the global characteristic of group coherence. The problem then becomes how to quantify coherence as a measure of collective behavior. To that end, we must identify analysis tools that can be used to characterize coordination at multiple scales of measurement.

Principal Components Analysis (PCA) is one way to quantify the overall dimensional compression of an observed system (Riley et al., 2011). An advantage of PCA is that it can take all of the DoF, or variables of interest, in a given system and identify new collective variables (the principal components), based on relations within high-dimensional datasets. It also indexes the load magnitude of the original variables of interest on the identified principal components, and this can help uncover how the behavioral variables are coupled together in the organized system. These characteristics make PCA an important tool for revealing global properties of a system.

However, PCA is a linear analysis and cannot measure the local coordination between agents. That question requires an analysis tool that quantifies patterns of coordination between two behavioral variables. Cross-recurrence quantification (CRQ) is better suited for this purpose. CRQ is a nonlinear analysis that indexes repeating patterns in a pair of time series (Shockley, Butwill, Zbilut, & Webber, 2002; Webber & Zbilut, 1994), and has already demonstrated its utility in interpersonal coordination (e.g., Ramenzoni, Riley, Shockley & Baker, 2012; Richardson, Dale & Shockley, 2008). In particular, when analyzing side-by-side walking, Page and Warren (2012) found CRQ to output a reliable measure of the coupling strength, or degree of coordination, between the walking speed (**s**) of two pedestrians as their behavior evolved over time. In contrast to PCA, CRQ is limited to a pairwise analysis of time series, and thus provides a measure of coupling strength in a dyad rather than the overall coordination of the group. Taken together, PCA and CRQ allow us to characterize coordination and coherence at a local (i.e., dyad) and more global (i.e., group) level of behavior.

To study group coherence, we began with observations of a simple and highly controlled locomotor task: four pedestrians walking to a common goal. While quantitative measures of crowd dynamics should apply to more complex scenarios (see Moussaïd et al., 2012), we believed this approach would reveal essential coordination dynamics as a first pass to understanding crowd behavior. In the present experiment, we instructed groups of four participants to walk toward one of three goals; the group's initial density was varied on each trial (see Figure 1). As described above, we analyzed time series of two behavioral variables for each participant: the heading direction (ϕ) and speed (**s**). This resulted in a total of eight DoF for the four-agent system. We hypothesized that the groups would exhibit dimensional compression in all conditions, compared to virtual groups we constructed by randomly sampling the same participants from different trials (see Method section). We also expected a greater reduction in DoF as density increased, due to larger changes in visual angle and visual direction at smaller distances, as well as to spatial constraints on walking. With regard to CRQ, we hypothesized that the coupling strength would be greater in all conditions compared to virtual groups, and that the leader-follower pairs would exhibit stronger coupling than the side-by-side pairs, as observed in our previous studies of two pedestrians (Rio & Warren, 2011; Page & Warren, 2012).

Participants

Five groups of four participants (N=20; M age 23.57 ± 0.93 years; 12 females, 8 male) from Brown University and the greater region were compensated $15 for their participation. Participants had no history of cognitive deficits, lower extremity injury, or neuromuscular disorders that would inhibit normal locomotor activity. The experiment was approved by the Brown University Institutional Review Board and adhered to guidelines for the ethical treatment of participants.

Materials and Apparatus

The experiment took place in a 12 × 14 m open room. The head position of each participant was tracked with a MicroTrax inertial tracker affixed atop a lightweight bicycle helmet on the head. Each tracker communicated with an IS-900 ultrasonic overhead grid tracking system (InterSense, Billerica MA, USA) and provided 6 DoF position (4 mm RMS error) and orientation (0.1° RMS error) data at 60 Hz. Three cardboard goal poles (approximately 2 m tall and 0.5 m in diameter) were placed at an initial distance of 8 m and angular offsets of 12.53° to the left (pole 1), 0° (pole 2), and 12.53° to the right (pole 3) of the midpoint of the front two participants (see Figure 1). Colored tape was used to mark four possible starting positions in a square configuration, with initial spacing of 0.5, 1.0, 1.5, or 2.5m on a side.

Design & Procedure

Each group completed eight trials in each of 12 conditions, four densities (0.5, 1.0, 1.5, 2.5m spacing) crossed with three goal positions (left, straight, right; see Figure 1). This resulted in a total of 96 trials, presented in a random order, in each experimental session. Goal position was changed only to vary the task between trials, and thus was not included as a factor in the statistical analyses.

At the beginning of each trial the four participants were randomly assigned to the four positions in the square configuration: (1) front right, (2) front left, (3) back right, or (4) back left (Figure 1). Once they were standing in the correct location, an experimenter gave a "go" signal and the group began to walk straight ahead. As the last participant crossed a line 1 m in front of the initial positions of the front participants, the experimenter gave a verbal command to walk to goal 1, 2, or 3. The only instruction given to the participants was to continue walking to the specified goal at a comfortable pace without stopping. Participants were not told to stay together as a group or to maintain the initial configuration. Each trial lasted approximately 6 to 8 s.

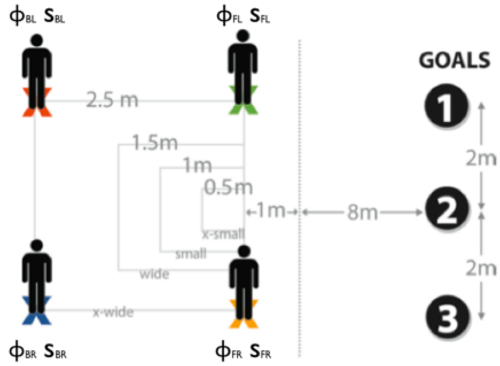

Figure 1: The four possible starting positions for each of the four possible starting densities. From this view, the participants would walk from left to right. Note the dotted line 1 m from the midpoint between the front two participants that represents when the experimenter "goal" command was given. The heading and speed variables (ϕ and **s**, respectively) under each agent indicate the eight DoF of the system (i.e., the eight variables analyzed in the present experiment). FR = Front Right; FL = Front Left; BR = Back Right; BL = Back Left.

Data Reduction and Analysis

The tracking system recorded the head position (*x*- and *z*-coordinates) of each participant at a sampling rate of 60 Hz. The raw (unfiltered) position data were used to compute the participant's speed (**s**) and heading (ϕ) from the displacement between successive samples, according to the following equations:

$$\boldsymbol{s}_i = \frac{((x_i - x_{i-1})^2 + (z_i - z_{i-1})^2)^{.5}}{\Delta t}, \tag{1}$$

$$\phi_i = \tan^{-1}\left(\frac{x_i - x_{i-1}}{z_i - z_{i-1}}\right), \tag{2}$$

where x_i and z_i are the head position on the *i*th frame, in room coordinates. The ϕ and **s** time series were used for all subsequent analyses.

Virtual Group Construction For each real group trial, a paired virtual group trial was constructed by randomly selecting a time series from the same participants in the same group and condition, but from different trials. Thus all task constraints were matched, except that the four participants in the virtual group were not perceptually coupled with each other. The virtual groups were created to ensure that any results that indicated significant coordination between participants were due to the perceptual coupling, not the task constraints (e.g., the common goal, the simultaneous goal command, or similar preferred walking speeds). After random selection of the four time series, they were temporally aligned based on the time the goal command was given by the experimenter. To equate their length (a requirement of both PCA and CRQ analysis), a time series was then potentially cropped at the beginning and/or end. This resulted in four time series of equal length that were aligned by the goal command.

Principal Components Analysis (PCA) PCA identifies linear relationships within multi-dimensional datasets and then maps the original data into a newly defined space. The principal components (i.e., axes of space) represent the dataset's primary dimensions of variation, but do not necessarily map directly onto the original dimensions of the actual measurement. The end result is a representation of potentially new, important variables that best account for the variance within the observed system.

In the context of the present experiment, eight variables of interest representative of the 8 DoFs of the observed system (i.e., ϕ and **s** for each participant) were submitted to a single PCA. The data were normalized using a *z*-score transform prior to analysis. PCA was performed in *Matlab* using the *princomp* function and the results were examined in a similar fashion to Ramenzoni et al. (2012).

First, the number of components that together account for 90% or more of the variance in the data set was determined. To investigate dimensional compression in the real vs. virtual group, a 4×2 mixed-model analysis of variance (ANOVA) was conducted on number of components, with initial density as a within-subjects factor and group (real vs. virtual) as a between-subjects factor, averaged across goal position.

Next, the amount of variance accounted for by the first principal component (PC) in the real vs. virtual group was compared using an identical mixed-model ANOVA. The analysis was limited to the first PC because the subsequent components were dependent on the first PC. Greater variance accounted for by the first PC in the real group indicates dimensional compression, and thus greater coherence, in the visually coupled system.

Finally, the mean correlation coefficient (*r*) for the loading of each behavioral variable on the first PC was examined to investigate which of the eight variables were most influential in characterizing the group's behavior. The *r* values were transformed using a Fisher's z' transform and submitted to a 4×8×2 mixed-model ANOVA with initial density and agent position as within-subjects factors, and group as a between-subjects factor, again averaged across goal position. The aim of this analysis was to examine whether the speed or heading of an agent in a particular position more strongly influenced the group's behavior and whether this influence depended on density.

Cross-recurrence Quantification (CRQ) A nonlinear, two-dimensional cross-recurrence quantification (CRQ) analysis was used to quantify the time-correlated activity between the heading time series of each dyad in the group, and, separately, the speed time series of each dyad (see Figure 2 for the analysis steps). A CRQ analysis is conducted by first embedding the pair of normalized time series in a multidimensional, time-delayed phase space (see Marwan, Romano, Thiel & Kurths, 2007; Ramenzoni et al., 2012; Richardson, Schmidt, & Kay, 2007; Shockley et al., 2002; Webber & Zbilut, 1994). Because not all variables that make up the behavior in a dynamical system are necessarily knowable *a priori*, phase space reconstruction allows for the behavior of these potentially "hidden" variables in the dynamical system to be evaluated via their interaction with, or influence on, the known variable (in this case the **s** time series). Thus, the structure of the reconstructed phase space can reveal the underlying dynamics of the dynamical system as a whole. Specifically, the "neighborliness" of points within some tolerance or radius in phase space can indicate recurrent points in the two time series. These represent states in one time series that closely correspond to previous or future states in the other time series, and can illustrate behavioral patterns of coordination in the observed system. The recurrent points are identified and represented in a cross-recurrence plot (see Figure 2, bottom), from which a suite of measures can then be computed to quantify these patterns (see Shockley et al., 2002 and Marwan et al., 2007 for a review of analysis procedures).

In the present experiment, only cross-maxline (CML) was computed and analyzed: specifically the longest diagonal line of consecutive recurrent points on a cross-recurrence plot. This provides a measure of the longest time interval that the speed (or heading) of two participants was coupled during a given trial. CML is known to be sensitive to the temporal stability of coordination between two time series, associated with the coupling strength between agents (Richardson et al., 2007). A previous CRQ analysis of speed with two pedestrians revealed stronger coupling between leader-follower pairs than side-by-side pairs (Page & Warren, 2012). The parameters used for CRQ were as follows: embedding dimension = 5; delay = 3 data points; radius within which points are counted as recurrent = 1.0% of the actual distance separating points in reconstructed phase space.

Results

PCA

Number of Components The number of components required to account for 90% of the variance was significantly reduced in real groups (M = 3.71 ± 0.12) compared to virtual groups (M = 5.76 ± 0.07), $F(1,8)$ = 233.22, p < .001 (see Figure 3, top). Thus, the visual coupling between agents reduced the DoF of the group significantly more than the external task constraints, indicative of emergent global coherence. Surprisingly, there was no effect of initial density (p > .05), implying that group coherence at low densities was comparable to that at high densities.

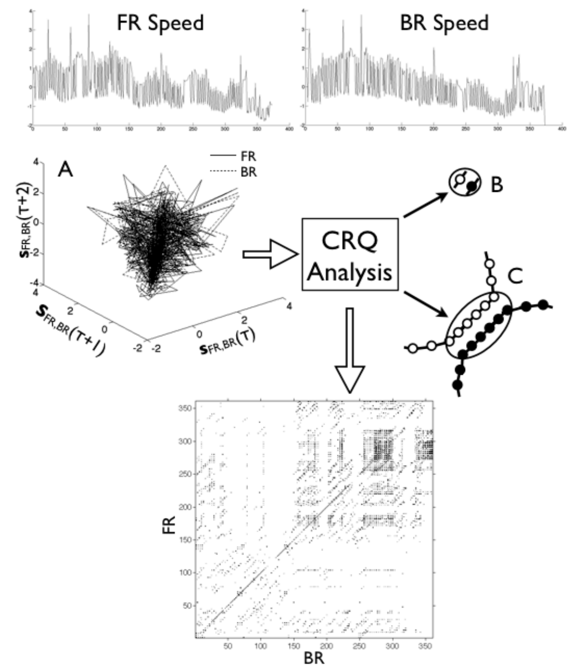

Figure 2: A schematic of the steps in the CRQ analysis. For each trial, the speed time series of the FR agent (top left) and BR agent (top right) are unfolded separately into a shared reconstructed phase space via time-delayed copies of each measured time series, denoted as $\mathbf{s}_{FR,BR}$ (A). Recurrent points within a given radius (B) and strings of recurrent points (C) are identified with respect to each point in phase space and represented in a cross-recurrence plot (bottom) with each axis representative of the $\mathbf{s}_{FR}$ and $\mathbf{s}_{BR}$ time series at each time step. Each pixel indicates a recurrent point, and the diagonal line structures indicate the length of a string of recurrent points, or the co-evolution of the two time series at different time delays. The longest diagonal line, cross-maxline (CML), was computed for each dyad in the group.

PC 1 The first principal component accounted for significantly more variance in real groups (M =52.43% ± 0.79) than in virtual groups (M =39.74% ± 0.45), $F(1,8)$ = 190.42, p < .001. This result confirms dimensional compression in group behavior, due to the visual coupling. There was, again, no effect of initial density (p > .05) on the variance accounted for by PC 1.

Contribution of Variables to PC 1 The composition of the first principal component was further examined to determine the relative contributions of each behavioral variable, by computing the loading (r) of each variable on PC1 (see Figure 3, bottom). A significant agent position × group interaction was observed for r, $F(7,56) = 408.03$, $p < .001$. Follow-up t-tests (Bonferroni corrected $p \leq .01$) indicated that the **s** variable was more strongly correlated with PC1 in the real groups than in the virtual groups (all $p < .001$), whereas the ϕ variable was not (all $p > .01$), for all four agent positions. Within the real groups, the **s** variable was more strongly correlated with PC1 than the ϕ variable ($p < .001$), whereas **s** and ϕ did not significantly differ in the virtual groups (all $p > .01$), for all agent positions. Thus, the behavior of real groups was more coherent than that of virtual groups, primarily due to the coordination of walking speed; thanks to the presence of a common goal, heading was independently aligned in both groups.

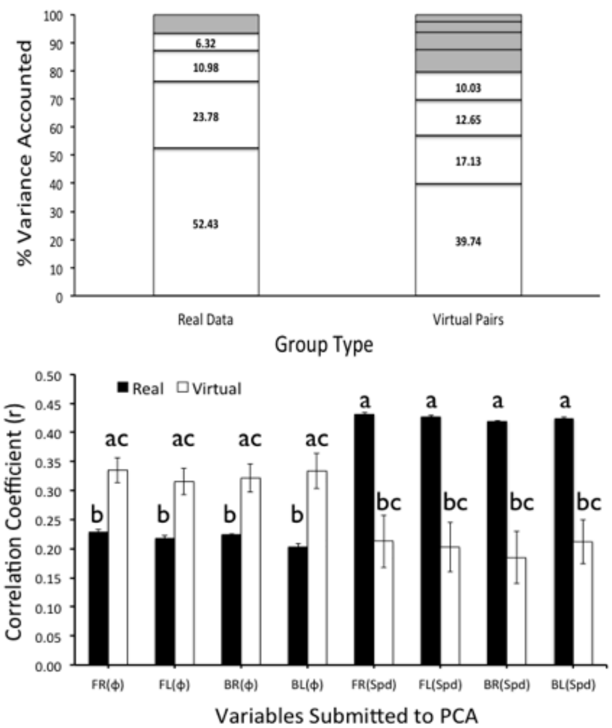

Figure 3: The amount of variance accounted for by each component beginning with PC1 (top), and the loading (r) of behavioral variables onto PC 1 (bottom). Black bars = real groups, white bars = virtual groups, significant differences ($p < .001$) indicated with Duncan Grouping.

CRQ

The results of PCA indicated the importance of speed (more than heading) as a variable of interest in the current dataset. Accordingly, the CRQ analyses focused on the speed time series for all six dyads on each trial. Representative cross-recurrence plots for a real and virtual dyad appear in Figure 4. A significant main effect of group was observed for cross-maxline length (CML), $F(1,8) = 34.83$, $p < .001$. Specifically, the real group exhibited a mean CML ($M = 49.93 \pm 0.03$) more than twice as long as the virtual group ($M = 20.73 \pm 0.02$), irrespective of dyad, goal position, or initial density. Surprisingly, this implies that the coupling is equally stable at high and low densities, and for leader-follower and side-by-side dyads.

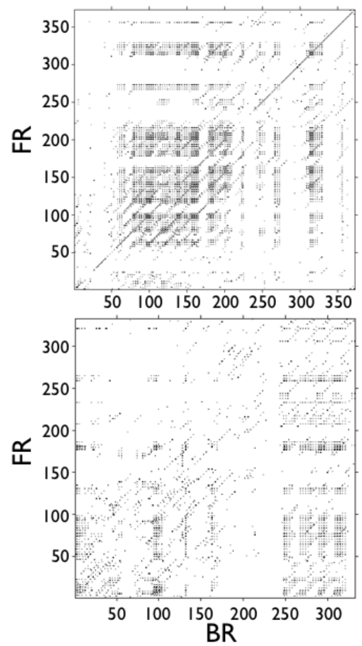

Figure 4: Sample cross-recurrence plots for speed time series from a real (top) and a virtual (bottom) leader-follower dyad. Note the diagonal lines visible in the cross-recurrence plot for the real dyad, indicative of a temporally stable speed coupling between agents.

Discussion

The present experiment attempted to measure the degree of coherence in pedestrian groups, based on analyses of two behavioral variables, heading (ϕ) and speed (**s**), during goal-directed locomotion. We expanded the analysis from interpersonal coordination to the behavior of small groups, as a path to understanding collective crowd dynamics.

The PCA found that visually coupled pedestrian groups exhibited significant dimensional compression across all experimental conditions, compared to virtual groups. The results indicate that the task constraints (e.g. common goal, simultaneous command, preferred walking speed) accounted for a reduction of approximately 2.2 DoF (from 8 to 5.8) in the virtual groups. However, the visual coupling produced a further reduction of approximately 2.1 DoF (from 5.8 to 3.7). This is indicative of a functional reorganization of DoF thanks to the informational coupling of behavioral variables, yielding the emergence of coherent collective behavior.

In addition, PC 1 analysis offers preliminary evidence of a new collective variable that accounts for group coherence in the present case. The loading of behavioral variables on PC1 suggests that agent speed is a primary contributor to the new group dynamics. However, the comparatively weak contribution of the behavioral variable of heading direction is likely due to the external constraint of a common goal in this particular task. Taken together, these findings support the reduction of DoF in interpersonal coordination proposed by Riley et al. (2011; Ramenzoni et al., 2012).

The CRQ analysis provided more specific results about the coupling strength between particular dyads in the group.

The speed variable exhibited a significantly more stable coupling in real groups than virtual groups, with no differences between leader-follower, side-by-side, and diagonal dyads. Taken together, the PCA and CRQ results indicate that the reduction in group DoF in the present task is due in large part to the coordination of speed at the dyad level, resulting from the visual coupling between neighbors.

While the overall results supported the hypothesis of group coordination via local coupling, the analyses diverged from our expectations in two important respects. First, we anticipated that the degree of coordination would increase as group density increased, but we did not observe an effect of density. It is possible that the range of densities tested was too small to observe an effect, or that the task constraints combined with a short walking distance limited the degree of variability in individual trajectories. But consistent with this finding, we previously observed that speed coordination in pairs of pedestrians is also independent of distance over 1-3m (Rio & Warren, 2011). Second, we were surprised that coupling strength did not differ among dyads, given we had previously observed greater speed coordination between leader-follower than side-by-side pairs. Again, it is possible the task constraints may have limited the variability in individual behavior. In subsequent experiments, we are measuring pedestrian groups over longer distances without a common goal or timing signal.

The present work is a starting point for understanding collective behavior in pedestrian groups. We have begun by focusing on the local coupling between agents, on the hypothesis that this generic coordination mechanism will scale up from small groups to large crowds and perhaps to swarms across species. It is likely that other cognitive and social variables also constrain this coupling. For example, cognitive processes such as decision-making and motivation, and social factors such as group membership, dominance relations, and social communication, may influence the selection of goals, neighbors, speeds, and control laws and shape group dynamics. The present experiment suggests an approach to quantifying multi-agent coordination in many of these contexts. Future work will continue to scale up these analyses to larger groups in various pedestrian scenarios, with the aim of understanding the emergence of collective behavior and global patterns in large crowds.

Acknowledgments

This research is funded by NIH 5R01 EY010923-25. The authors would also like to thank Henry Harrison.

References

Bonneaud, S. & Warren, W. H. (2012) A behavioral dynamics approach to modeling realistic pedestrian behavior. *6th International Conference on Pedestrian and Evacuation Dynamics,* Zurich, Switzerland.

Fajen, B. R. & Warren, W. H. (2003). Behavioral dynamics of steering, obstacle avoidance, and route selection. *Journal of Experimental Psychology: Human Perception and Performance*, *29,* 343-362.

Fajen, B. R. & Warren, W. H. (2007). Behavioral dynamics of intercepting a moving target. *Experimental Brain Research, 180,* 303-319

Marwan, N., Romano, M. C., Thiel, M., & Kurths, J. (2007). Recurrence plots for the analysis of complex systems. *Physics Reports*, *438*, 237-329.

Moussaïd, M., Guillot, E. G., Moreau, M., Fehrenback, J., Chabiron, O., et al. (2012). Traffic instabilities in self-organized pedestrian crowds. *PLOS Computational Biology*, *3*, e1002442.
DOI: 10.1371/journal.pcbi.1002442

Moussaïd, M., Helbing, D. & Theraulaz, G. (2011). How simple rules determine pedestrian behavior and crowd disasters. *Proceedings of the National Academy of Sciences of the United States of America*, *17*, 6884-6888.

Ondřej, J., Pettré, J., Olivier, A-H., & Donikian, S. (2010. A synthetic-vision based steering approach for crowd simulation. *ACM Transaction on Graphics 4*, *123*.
DOI: 10.1145/1778765.1778860.

Page, Z. & Warren, W. (2012). Visual control of speed in side-by-side walking. *Journal of Vision*, *12*, 188.

Ramenzoni, V. C., Riley, M. A., Shockley, K. & Baker, A. A. (2012). Interpersonal and intrapersonal coordinative modes for joint and single task performance. *Human Movement Science*, *31*, 1253-1267.

Richardson, M. R., Schmidt, R. C., & Kay, B. A. (2007). Distinguishing the noise and attractor strength of coordinated limb movements using recurrence analysis. *Biological Cybernetics*, *96*, 59-78.

Riley, M. A., Richardson, M. J., Shockley, K. & Ramenzoni, V. (2011). Interpersonal synergies. *Frontiers in Psychology*, *2*, 38.

Rio, K., Bonneaud, S. & Warren, W. H. (2012). Speed coordination in pedestrian groups: Linking individual locomotion with crowd behavior. *Journal of Vision*, *12*, 190.

Rio, K. W. & Warren, W. H. (2011). A speed control law for pedestrian following based on visual angle. *Journal of Vision*, *11*, 899.

Richardson, D., Dale, R., & Shockley, K. (2008). Synchrony and swing in conversation: Coordination, temporal dynamics, and communication. In G. Knoblich (Ed.), *Embodied Communication* (pp. 75-94). New York City, NY: Oxford University Press.

Shockley, K., Butwill, M., Zbilut, J. P., & Webber, C. L. Jr. (2002). Cross recurrence quantification of coupled oscillators. *Physics Letters A*, *305*, 59-69.

Warren, W. H. & Fajen, B. R. (2008) Behavioral dynamics of visually-guided locomotion. In A. Fuchs & V. Jirsa (Eds.), *Coordination: Neural, behavioral, and social dynamics.* Heidelberg: Springer.

Webber, C. L. Jr., & Zbilut, J. P. (1994). Dynamical assessment of physiological systems and states using recurrence plot strategies. *Journal of Applied Physiology*, *76*, 965-973.