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Masters of Paradox: Paradoxes of Self-Reference in the Eastern Islamic World

By

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Abstract

Masters of Paradox: Paradoxes of Self-Reference in the Eastern Islamic World

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Widely studied throughout the eastern Islamic world from the 13th century to the 19th were three paradoxes of self-reference: the paradox of the absolute unknown (*shubhat al-majhūl al-muṭlaq/al-jidhr al-abkam*), the Liar paradox (*shubhat al-jidhr al-aṣamm*), and the paradox of the totality of all relations (*shubhat majmū‘ al-nisab*). Over six hundred years’ worth of debate is here clearly organized and summarized, its philosophical significance carefully analyzed, and its historical trajectory documented. The broader historical results bear on questions that have occupied the minds of specialists in the field for decades, including the question of intellectual decline in the post-classical Islamic world.

Introduction

Studied here are three paradoxes of self-reference extensively investigated by philosophers and logicians of the eastern Islamic world from the 13th century to the 19th. These are (1) the paradox of the absolute unknown (*shubhat al-majhūl al-muṭlaq/al-jidhr al-abkam*), (2) the Liar paradox (*shubhat al-jidhr al-aṣamm*), and (3) the totality of all relations paradox (*shubhat majmū' al-nisab*). Both philosophically and historically, these three paradoxes are sensibly grouped together.

In order to understand how the three paradoxes hang together philosophically, we must first understand the paradoxes themselves. The paradox of the absolute unknown is born of the standard Arabic logical doctrine that representations (*taṣawwūrāt*) are prior to judgements (*taṣdīqāt*). In order to pass a judgement, one must possess the relevant representations, namely the subject-term (*mawḍū'*), predicate-term (*maḥmūl*), and nexus (*nisba*). Hence, (I) “Being a subject of predication requires being known somehow” (*al-ḥukm 'alā shay' yastadī taṣawwur al-maḥkūm 'alayhi bi-wajh mā*) is true. But if (I) is true, so too must be its contrapositive, (II) “Absolute unknowns cannot be subjects of predication” (*kull majlūl muṭlaqan yamtani' al-ḥukm 'alayhi*), yet (II) would appear to lead to paradox.

“Absolute unknowns” must be either (a) absolutely unknown or (b) known in some way. If they are (a) absolutely unknown, it would seem that (II) contradicts itself, since it takes something absolutely unknown, namely “absolute unknowns,” as a subject of predication, which is precisely what it says cannot be done. If, on the other hand, “absolute unknowns” are (b) known in some way, another problem results: if something is known, as we have just granted “absolute unknowns” are, it ought to be capable of being a subject of predication. Yet that is exactly what (II) says cannot be. Whether “absolute unknowns” are considered to be absolutely unknown or known in some way, paradox ensues.

Although the Liar paradox was formulated in numerous ways over the course of its study in the Islamic world, one formulation was (L) “This sentence is false.” Is (L) true or false? Presumably it must be only one or the other. If it is in fact true, then its content corresponds with the fact of the matter, which is to say that it is false, since its content is that it is false. Likewise, if it is false, then its content does not correspond with the fact of the matter, which is to say that it must be true. Either way, we encounter contradiction.

The paradox of the totality of all relations asks us to consider the totality of all relations. Included would be innumerable spatial, causal, and dependence relations alongside many others. Crucially figuring among the relations are the containment relations that take each part of the totality and the totality itself as its two relata. It is self-evident (*darūrī/badīhī*) that a part is prior to the whole. That is, each part of the totality of all relations must be prior to the totality itself. It is also self-evident, however, that every relation is posterior to its relata, since a relation can only exist by virtue of its relata. That is, each part of the totality must be posterior to the totality itself. But this is a contradiction: something cannot be *both* prior and posterior to another.

To link all three of these paradoxes together merely by saying that they are all self-referential would not be saying enough. Obviously, all share in self-reference: (II) “Absolute unknowns cannot be subjects of predication” refers to itself insofar as its own subject-term—

“absolute unknowns”—is a subject of predication, just as “this” in the Liar sentence, “This sentence is false,” refers to itself, and the totality of all relations contains its constitutive containment relations which take it as one of their relata.

There may be another, perhaps more enlightening way, to put the same point. One of the underlying difficulties that surfaces in the literature of the paradox of the absolute unknown is that merely knowing that “absolute unknown” has an instance seems to disqualify the instance from indeed being an instance of “absolute unknown,” since by virtue of being an instance it would be *known* to be an absolute unknown and therefore could hardly be an absolute unknown. Hence, the extension of the term “absolute unknown” appears itself to be problematic. Insofar as it is legitimate to recast the Liar paradox as asking whether the term “not true of itself” figures among its own instances, it too can be framed as a problem of extension. Likewise, the paradox of the totality of all relations clearly indicates that the extension of the term “relation” is problematic: if we consider the totality of all relations (the *extension* of “relation”), we find upon analysis that it leads to contradiction.

Historically, all three problems are either explicitly or implicitly linked in the Islamic world. At least one solution deployed for the Liar or the paradox of the absolute unknown is deployed (sometimes in a modified form) to solve at least one of the other two besides it. For example, the solution to the paradox of the absolute unknown devised by Mullā Ṣadrā is deployed to solve the Liar paradox by the commentator of the *Sullam al-‘ulūm*, al-Mubārakī.¹ Mīr Dāmād thinks his solution to the Liar paradox can also be used to solve the totality of all relations paradox.²

But what is a “paradox,” and did philosophers of the Islamic world consider the three problems here studied to fall under that concept? A concept commonly denoted by the contemporary English word “paradox” is some argument which appears to show that some accepted truth is false. Of course, it is not any old truth which the paradox appears to disprove, but a universal truth (i.e., a true proposition with a universal, as opposed to a particular, subject-term). Some paradoxes are easy to resolve: they turn on the unjustified denial of some other accepted truth, and a solution lies merely in spotting it. Others are insoluble and must be accommodated. The “truths” they undermine must be revised, not rescued.

This latter, insoluble kind of paradox Quine calls an “antinomy.” The doctrines such antinomies undermine are so tightly nestled in the network of our doxastic commitments that, to “solve” them, we are forced to revise “part of our conceptual heritage.”³ The beliefs undermined connect to virtually all other nodes in our network of belief. When they are no longer acceptable, neither are a vast swath of the nodes they used to touch.

A profitable metaphor may be to compare such core beliefs—those which touch almost every other node in a network of belief—to the sun. They shine their rays on all the eye can see, illuminating our sight. Yet even though the sun is what enables us to see, if we glance at it for a second, our sight is snatched away, and we are plunged into darkness. Similarly, so long as we

¹ See solution (13) and the “Historical Development” section in the Liar chapter.

² Muḥammad Bāqir b. Muḥammad Mīr Dāmād, *Muṣannafāt-i Mīr Dāmād*, ed. ‘Abd Allāh Nūrānī, vol. 2 (Tehran: Anjuman-i āthār-o mafākher-i farhangī, 2003), 686.21.

³ W. V. Quine, “The Ways of Paradox,” in *The Ways of Paradox, and Other Essays* (Harvard University Press, 1976), 9.

keep our eyes only on non-core beliefs, we see well, and can conveniently navigate the intellectual world. However, as soon as we turn to the core beliefs themselves—focus our eyes directly on the sun—we become intellectually blind and lose the ability to understand. The sun is not good for seeing the sun, but it works quite well for seeing other things. The beliefs which undergird some antinomies are quite good for understanding many other things, but fall apart when applied self-referentially and produce antinomies.

This breakdown upon self-reference is, I think, a theme in philosophy, and self-reference is the source of much significant thought. Reflexive thinking is a fundamental research method in philosophy: ask questions of the thing itself, and you are often led to insight. Indeed, that knowledge for which philosophy is most often construed as the quest, that knowledge frequently identified as the root of philosophy itself—self-knowledge—has as its principle (*mabdaʿ*) reflexivity. Muslim philosophers and other seekers of wisdom (e.g., Sufis) recognized this method implicitly in their acknowledgment that knowledge of the universe itself (*al-insān al-kabīr*) could be known from self-reflection (*al-ʿālam al-ṣaghīr*). And, as is well known, the Delphic dictum, “Know thyself” was enshrined in the words of the Prophet, “Whoever knows himself knows his Lord.” Perhaps most fundamentally, the very being of the entire cosmos is, according to some Sufis, merely a self-referential thought: everything is merely a manifestation of God contemplating Himself, since, after all, (they would say) there is nothing besides Him. To think deeply about self-reference and paradoxes of self-reference is, therefore, to closely examine the fulcrum on which philosophy teeters (and sometimes appears to slip off).

Did philosophers of the Islamic world have the concept “paradox”? What about the more specific concept “antinomy”? It is hard to doubt that they had the concept “paradox,” although it is not clear that they had a single word with the exact same denotation. As I have explained it, “paradox” is contained in their concept of *mughālaṭa* (sophism, fallacy): “According to logicians, a *mughālaṭa* is a syllogism invalid in either its form, matter, or both.”⁴ As logicians of the Islamic world knew well, some invalid syllogisms nonetheless produce true conclusions, which amounts to the main difference between *mughālaṭa* and “paradox.” The latter must produce a conclusion that contradicts accepted truth, while the former need not. In spite of this distinction, I know of no example of a *mughālaṭa* given in that section of an Arabic logic textbook which produces a conclusion considered true.

There is some evidence that philosophers of the Islamic world had the concept “antinomy” (here, merely an insoluble “paradox,” ignoring all talk of Quinean “webs of belief”) and that they thought some existed. Many authors introduce their solutions to the three problems of this study with declarations of difficulty, these becoming increasingly grandiose with and after Taftāzānī (d. 1390). Before then, however, they are rather succinct:

⁴ M. A. Tahānawī, *Kashshāf iṣṭilāḥāt al-funūn wa-l-ʿulūm*, ed. R. ʿAjam, vol. 2 (Maktabat Lubnān Nāshirūn, 1996), 1602rhs.17-19.

“One of the sophisticated (*latīf*) sophisms...” (Abharī, Liar);⁵ “One of the difficult (*ṣa‘b*) sophisms...” (Abharī, Liar);⁶ “Here there is raised a difficult question...” (Ḥillī, absolute unknown);⁷ “This is a well-known sophism, and all scholars have been bewildered as to how to solve it. They have blackened page after page, but none of it has been convincing (*muqni‘*).” (Samarqandī, Liar)⁸

Indeed, many of these pre-Taftāzānī philosophers simply posed a solution to the given problem without any comment on its difficulty. Rāzī (d. 1210), Ṭūsī (d. 1274), Kātibī (*Sharḥ al-Kashf*) (d. 1277), Ibn Kammūna (*al-Jadīd*) (d. 1284), and Ḥillī (d. 1325) pose solutions to, e.g., the Liar without any talk of difficulty. Not so Taftāzānī and other later philosophers. Many break into rhymed prose:

“If in studying the problem of the absolute unknown you’ve demanded it be made this clear, it’s right to say to you, ‘Put out the candle of night austere—morning is here!’” (Jurjānī, absolute unknown);⁹ “This is a sophism (*mughliṭa*) which has bewildered the greatest minds and the virtuosos of the cognoscenti (*‘uqūl al-‘uqalā’ wa-fuḥūl al-adhkiyā’*)... I have reviewed all of the positions, though none has slaked my thirst. Though I have contemplated considerably, less than little has come to me.” (Taftāzānī, Liar);¹⁰ “How long the views of the learned *imāms* have quarreled over [the Liar], how long the opinions of the brilliant and those possessed of understanding have conflicted!” (Dawānī, Liar);¹¹ “I was asked about the well-known sophism which has bewildered the greatest minds (*‘uqūl al-‘uqalā’*) and which even the virtuosos of the cognoscenti (*fuḥūl al-adhkiyā’*) have not been able to penetrate; the one that has defied the scholars of all ages (*a‘yat ‘ulamā’ al-a‘ṣār*) and thwarted the learned of all places (*a‘jazat fuḍalā’ al-amṣār*).” (Dashtakī, Liar);¹² “Another matter belonging here is the enigma (*mu‘ḍila*) called ‘the imperfect root’ (*jidhr aṣamm*). It is the incurable disease (*dā’ ‘uḍāl*) that has defied (*a‘yat*) former and later generations, those who are the spiritual fathers and doctors in care of the logical arts and demonstrative sciences... [Scholars] have expressed [the Liar] in various ways and formulations (*wujūh was-taqrīrāt*), and they have, by way of escaping the enigma (*i‘ḍāl*), various stratagems and considerations (*ḥiyal wa-tadbīrāt*). How can I describe to

⁵ Athīr al-Dīn Abharī, *Kashf al-ḥaqā’iq fī tahrīr al-daqa’iq*, ed. Hüseyin Sarioğlu (Istanbul, 1998), 216.16; Athīr al-Dīn Abharī, *Khulāṣat al-afkār wa-naqāwat al-asrār*, ed. Mahdī ‘Azīmī and Hāshim Qurbānī, 2017, 382. antepenultimate.

⁶ Naṣīr al-Dīn Muḥammad b. Muḥammad Ṭūsī and Athīr al-Dīn Abharī, “Ta‘dīl al-mi‘yār fī naqd Tanzīl al-afkār,” in *Manṭiq-o mabāhiṭh-i alfāz*, ed. Toshihiko Izutsu and Mehdi Mohaghegh (Tehran: Dāneshgāh-i Tehrān, 1991), 235.15.

⁷ al-Ḥasan b. Yūsuf b. al-Muṭahhar Ḥillī, *al-Asrār al-khaṭiyya fī l-‘ulūm al-‘aqliyya* (Qum: Markaz al-Abḥāth wa-l-Dirāsāt al-Islāmiyya, Qism Iḥyā’ al-Turāth al-Islāmī, 2000), 12.18.

⁸ Shams al-Dīn Samarqandī, *Kıstāsu’l-eskār: Düşüncenin kıstası*, ed. Âtîf Efendi Kütüphanesi, trans. Necmettin Pehlivan (Istanbul: Türkiye Yazma Eserler Kurumu Başkanlığı, 2014), 551.18-19.

⁹ Quṭb al-Dīn Muḥammad b. Muḥammad Taḥtānī and ‘Alī b. Muḥammad Jurjānī, *Lawāmi‘ al-asrār Sharḥ Maṭālī‘ al-anwār fī l-manṭiq* (Istanbul: Dār al-ṭibā‘a al-āmira, 1861), 61.23-5.

¹⁰ Sa‘d al-Dīn Taftāzānī, *Sharḥ al-Maqāṣid*, 2nd ed., vol. 4 (Beirut: ‘Ālam al-kutub, n.d.), 287.4-6.

¹¹ Muḥammad b. As‘ad Dawānī, “Nihāyat al-kalām fī shubhat kull kalāmī kādhīb,” in *Davāzdah risāla dar pāradūks-i durūghū*, ed. Aḥad Farāmarz Qarāmalekī and Ṭayyibah ‘Ārifniyā (Tehran: Mu‘assasa-yi Pizhūheshī-yi Ḥikmat va Falsafah-yi Īrān, 2007), 101.9-10.

¹² Ṣadr al-Dīn Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” in *Davāzdah risāla dar pāradūks-i durūghū*, ed. Aḥad Farāmarz Qarāmalekī and Ṭayyibah ‘Ārifniyā (Tehran: Mu‘assasa-yi Pizhūheshī-yi Ḥikmat va Falsafah-yi Īrān, 2007), 27.5-7.

you how many a person's foot has slipped here (*kamm zallat aqdām aqwām*), whether an early verifier or a verifier of earlier generations (*mutaqaddimat al-muḥaqqiqīn wa-muḥaqqiqat al-mutaqaddimīn*), whether a fool of a philophastr or a philophastr of later generations (*muta'assifat al-mutafalsifīn wa-mutafalsifat al-muta'akhhirīn*)? Those who have most closely approached the wide-open spaces of the correct view and gotten nearest to hitting on the truth are those among them who have confessed to being incapable... With a similar solution, another enigmatic knot (*'aqd al-i'ḍāl*) considered a difficult doubt (*ustuṣ'iba min al-shukūk*) is undone (*yanḥall*)..." (Mīr Dāmād, Liar; totality of all relations);¹³ "With this, the well-known doubt (*shubha mashhūra*) is blocked..." (Mullā Ṣadrā, absolute unknown)¹⁴

Other scholars besides these make similar rhetorical remarks of varying originality (Khafrī, e.g.).

While these remarks undoubtedly indicate that philosophers of the Islamic world considered some of the paradoxes that they studied to be difficult to solve, they also indicate that they at least entertained the idea that there may be insoluble paradoxes, that there could be "antinomies." Their language smacks of insolubility. The first point to make is that the quotes above clearly imply that all solutions of which the authors were aware were, according to their lights, incorrect. In their view, no one, up to their time, had been able to solve these problems.

Additionally, the wording in which they couched their remarks suggests that the paradoxes were insoluble. Although some of the same can be found in earlier accounts, Mīr Dāmād's is particularly rife with locutions indicative of insolubility. Two roots of importance that were sometimes paired were *'ḍ-l* and *'y-y*. Both have medical connotations. The root *'ḍ-l* in the forms *'uḍāl* (adjective), *mu'ḍil/mu'ḍila* (adjective & noun), and *a'ḍala* (verb) denoted incomprehensibility, difficulty, and inexorability:

Something *mu'ḍil* is that which one cannot figure out (*la yuhtadā li-wajhihi*). *Mu'ḍilāt* are extremely difficult things (*shadā'id*). It has been narrated from 'Umar, may God be pleased with him, that he said, "The people of Kūfa *a'ḍala* me. They were displeased with every leader and no leader was pleased with them." Al-Umawī says the following regarding "...*a'ḍala* me...": "The verb is related to the word *'uḍāl*, which is something extremely difficult under whose weight a person cannot stand. ['Umar] meant [by '*a'ḍala* me'] 'All stratagems [regarding the Kufans] were closed off to me; Remaining on good terms (*mudārāt*) with them was difficult (*ṣa'b*) for me.'" ¹⁵

Ibn al-Athīr said, "An *'uḍāl* illness is that sickness which defies (*yu'jizu*) doctors and thus has no cure."¹⁶

The root *'y-y* in the forms *'ayya/ya'yā* (verb, not used in rhetoric of difficulty cited above), *a'yā* (verb, used), and *'ayāyā* (adjective & noun, not used) denoted incapability, ignorance, and impotence:

¹³ Mīr Dāmād, *Muṣannafāt-i Mīr Dāmād*, 2:685.10-12, 685.20-23, 686.21.

¹⁴ Ṣadr al-Dīn Muḥammad b. Ibrāhīm al-Shīrazī, *al-Hikma al-muta'aliya fī l-asfār al-'aqliyya al-arba'a*, 4th ed., vol. 1 (Beirut: Dār iḥyā' al-turāth al-'arabī, 1990), 239.10.

¹⁵ Abū l-Faḍl Jamāl al-Dīn Muḥammad b. Mukarram Ibn Manẓūr, *Lisān al-'arab*, vol. 11 (Qom: Nashr adab al-ḥawza, 1405h), 452lhs.16-22.

¹⁶ Ibn Manẓūr, 11:452lhs.1-2.

A man who takes up some work *ya 'yā* it when he can't figure out how to do it (*idhā lam yahtadi li-wajhi 'amalihi*).¹⁷

So-and-so *a 'yā* me: that is, “I did not know (*jahiltu*) so-and-so.”¹⁸

So-and-so *'ayya* of talking: that is, he has dyslogia. He *a 'yā* the animal: that is, it was exhausted. The journey *a 'yā* the camel, etc.: that is, “The journey exhausted and incapacitated (*tallahahu*) it.”¹⁹

The *ḥadīth* of Umm Zar': The sixth woman (*al-mar'a al-sādisa*) said, “My husband is *'ayāyā* and stupid (*tabāqā*)—he has every illness (*kull dā' lahu dā'*).” Abū 'Ubayd said [commenting on the *ḥadīth*], “A camel that is *'ayāyā* is one which neither impregnates nor has sex, and the word has the same meaning for men.” Ibn al-Athīr explained the word as follows: “Someone *'ayāyā* is impotent and cannot penetrate (*mubāḍa'a*) women.” Jawharī said, “A man is *'ayāyā* if he *'ayya* of something or of talking.”²⁰

While this linguistic information supports the conclusion that some philosophers of the Islamic world thought of these paradoxes as antinomies, there remains room for doubt. The insolubility of an antinomy lies not in the incapability of any given philosopher but in the antinomy itself: it is not an antinomy by virtue of the fact that no one has yet been able to pinpoint the error in the apparently correct reasoning but by virtue of the fact that there is no error. To “solve” the antinomy we must give up the view it undermines, the view we once thought to be true. In other words, an antinomy is, properly speaking, a *reductio ad absurdum* of a previously accepted truth—it is just not yet widely recognized as such.

The denotation and connotation of all six words derived from both roots do not necessarily support an exact comparison with antinomies. However much philosophers of the Islamic world recognized that doctors were incapable of curing certain *'uḍāl* illnesses, it does not therefrom follow that they thought that those illnesses could never be cured using current medical theory. Perhaps they thought that no one had yet understood how to apply established medical theory appropriately so as to cure them. Of course, the alternative explanation—that doctors would have to radically revise medical theory in order to develop novel and effective cures for *'uḍāl* illnesses—remains plausible.

In spite of all the rhetorical flourishes and declarations of insuperable difficulty, almost all authors proffered a solution to each of the three paradoxes here studied. Yet even this does not decide the case of whether philosophers of the Islamic tradition had the concept “antinomy” or thought some *shubah* (doubts) were antinomies: that most offered a solution does not indicate that they thought the paradox was, in every sense, soluble. Indeed, some authors gave in to a paradox by revising the basic assumptions it undermines. For example, to “solve” the Liar paradox, Ibn Kammūna (d. 13th c.)—as well as later authors inspired by his solution²¹—abandons the commonly accepted fundamental doctrine that propositions *must* be either true or false for the doctrine that

¹⁷ Abū l-Faḍl Jamāl al-Dīn Muḥammad b. Mukarram Ibn Manẓūr, *Lisān al-'arab*, vol. 15 (Qom: Nashr adab al-ḥawza, 1405h), 111lhs.ultimate-112rhs.1.

¹⁸ Ibn Manẓūr, 15:112.20rhs.

¹⁹ Ibn Manẓūr, 15:112rhs.25-ultimate.

²⁰ Ibn Manẓūr, 15:112lhs.14-20.

²¹ For example, Fakhr al-Dīn al-Sammākī (d. 1577). See Khaled El-Rouayheb, *The Development of Arabic Logic (1200-1800)* (Basel: Schwabe Verlag, 2019), 118.

propositions need only be capable of being true or false.²² Kātibī introduces the totality of all relations paradox precisely as a *reductio* proof against the thesis it undermines.²³

Complicating matters further still, one or two philosophers appear even to admit that certain paradoxes *cannot* be solved. Regarding the Liar, Taftāzānī (d. 14th c.) writes, “But, in my opinion, the correct position on this proposition [i.e. the Liar paradox] is to abandon (*tark*) trying to answer it and to admit to being incapable (*‘ajz*) of solving it.”²⁴ Regarding the thought-experiment version of the paradox of the absolute unknown, Mullā Ḥasan (d. 18th c.) says, “Yes, at this juncture, there is another powerful (*qawiyya*) doubt (*shubha*), which cannot be undone (*lā tanhall*) by the [fumbling] fingers of reason (*anāmil al-anzār*)—except with the help of God (*illā bi-ta’yīd allāh*), exalted is He.”²⁵ Indeed, Mullā Ḥasan suggests no solution to this version of the paradox but only cites others in order to reject them.

Of course, both quotes could be interpreted such that they mean that the incapacity is exclusively in the subject (philosophers to date) rather than the object (the paradox itself), but the latter interpretation strikes me as more convincing. Presumably, Taftāzānī would not recommend “abandoning trying to solve it” if one day someone sufficiently capable could solve it. Frankly, earlier in the passage quoted, Taftāzānī uses a variant on the Liar to disprove a rival theological school’s position on the question of whether acts are intrinsically moral or immoral, and, hence, he would have had a theological motive to endorse the idea that the Liar cannot be solved—not because philosophers up to his time have lacked ingenuity, but because it itself is insoluble. Regarding the paradox of the absolute unknown, a student of Mullā Ḥasan, Mullā Mubīn (d. 1810), may be interpreted to confirm my interpretation: “[Mullā Ḥasan] held that no response is possible (*qāla bi-ta’adhdhur al-jawāb ‘anhu*).”²⁶

Aside from philosophical importance, the three paradoxes of self-reference here studied also have great historical and academic importance. Based on wide-spread manuscript evidence, it is clear that many philosophers and logicians in the Islamic world thought that these problems deserved concerted effort. If the history of philosophy in the Islamic world is worth writing at all, surely this chapter on paradoxes should not be absent. Scholars of the tradition were particularly interested in these specific problems. While other paradoxes were certainly studied, it is a belief borne out by the historical record that paradoxes of self-reference were of special interest.

One such historical indicator is that many independent treatises were dedicated to both the Liar paradox and the paradox of the absolute unknown.²⁷ Instead of being mixed into a larger

²² See solution (4) in the chapter on the Liar paradox.

²³ Shams al-Dīn Ibn Mubārakshāh et al., *Sharḥ Ḥikmat al-‘ayn ma’a ḥawāshī* (Kazan: al-Maṭba‘a al-Mīriyyah, 1321), 292–93, <https://catalog.hathitrust.org/Record/100329640>.

²⁴ Taftāzānī, *Sharḥ al-Maqāṣid*, 4:287.18–19.

²⁵ Muḥammad Ḥasan b. Ghulām Muṣṭafā Sihālawī et al., *Mullā Ḥasan ‘alā matn Sullam al-‘ulūm* (Dehli: Kutubkhānah-yi rashīdiyya, 1963), 46.11–12.

²⁶ Mullā Muḥammad Mubīn, *Kitāb Mir’āt al-shurūḥ*, vol. 1 (Egypt: Ṭab‘at al-sa’āda, 1321ah), 53.21. For the meaning of *ta’adhdhur*, see Abū l-Faḍl Jamāl al-Dīn Muḥammad b. Mukarram Ibn Manzūr, *Lisān al-‘arab*, vol. 4 (Qom: Nashr adab al-ḥawza, 1405h), 549rhs.10–11: “Something *ta’adhdhara* for someone when it is difficult (*ṣa’uba wa-ta’assara*).” Hans Wehr (fourth edition) gives “difficulty, impossibility, impracticability, unfeasibility” for *ta’adhdhur*.

²⁷ The totality of all relations paradox is an exception. Although it has roughly as long of a history of study in the Islamic world as either of the other self-referential paradoxes, it never garnered the same amount of attention.

textbook or study of logic, the two problems were singled out for separate studies. Indeed, sometimes the two were studied together in a single treatise.²⁸ By contrast, few other aporia commonly raised in logic textbooks garnered quite as many independent treatises. For example, as far as I am aware, no independent treatises were dedicated to Meno's paradox, although it was discussed in works of logic.

There are some apparent exceptions to this trend. Consider the logical aporia *shubhat al-istilzām* or *shubhat 'āmmat al-wurūd*, both of which originated in the fecund 13th century. Each merited several independent treatises, the former more than the latter. Nonetheless, my overwhelming suspicion is that, were manuscript libraries carefully examined, and the number of treatises on each paradox counted, there would be significantly more on self-referential paradoxes, the paradox of the absolute unknown and the Liar. I further suspect that an analysis of the solutions proffered for both *shubhat al-istilzām* and *'āmmat al-wurūd* would clearly differentiate them from self-referential paradoxes: the latter would have more independent solutions than either *shubhat al-istilzām* or *'āmmat al-wurūd*.

The philosophical evidence shows that scholars seemed to reach a consensus on some aporia but not on these three paradoxes of self-reference. To take Meno's paradox as an example once again, philosophers appear to have satisfied themselves with a modified version of Avicenna's solution to the problem—after, of course, numerous additions and qualifications. Although the matter is subject to further research, it appears that there was little further development of the literature on that problem after the 15th century. My suspicion is that further inquiry will yield a similar conclusion for *shubhat al-istilzām* and *shubhat 'āmmat al-wurūd*, namely that far greater consensus on a satisfactory solution was reached for those problems, in contrast to solutions for any of the three paradoxes of self-reference I study here.

Indeed, there is a complete absence of consensus on how to solve these problems properly. Few solutions went unscathed by criticism, and many of the brightest stars of Islamic intellectual history refused to follow in their predecessors' footsteps and instead developed their own unique solutions. Therefore, my hypothesis would lead me to conjecture further that far more independent solutions were proposed for these three self-referential paradoxes than were for *shubhat al-istilzām*, *shubhat 'āmmat al-wurūd*, or almost any other problem in logic then open to debate. But all of this awaits empirical confirmation from further investigation of the sources.

A comment about the study's scope is here in order. I have advertised this work as a study of paradoxes in the eastern Islamic world. I mean by "eastern Islamic world" those regions between Iraq and Bangladesh, including Iran, Central Asia, and India, among other areas. Virtually all of the authors here surveyed either hailed from or lived in these lands. Naturally, I have not had the success to examine everything ever written in the eastern Islamic world about these three paradoxes. If it is even possible in principle—some works may be lost, or perhaps some philosophers had ideas that they never committed to writing—it would probably take decades.

I have instead pursued a much more modest (and, in my mind, desirable) goal. I have tried to check the most famous authors and those who enjoyed less acclaim but were nonetheless easily available to me. Temporally, I focus on the period after the 13th century, mostly because two of

²⁸ For example, Muḥammad Shafī' al-Bihārī Dihlawī, "al-Fawā'id fī l-mantiq" (n.d.), MS Rampur 3447.

the three paradoxes have yet to be traced any earlier, and in the case of the third (the Liar), the amount of research conducted before the 13th century pales in comparison to the amount conducted after. My expertise was another limitation: it is the eastern Islamic world's intellectual output that I know best. Contemporary research suggests this limitation may be fortuitously unobjectionable. The western parts of the Islamic world appear to have had less interest in such logical paradoxes.²⁹

I must also explain the (what may appear to some specialists) strange format of the study. Each chapter begins with a statement of the paradox as it was formulated by members of the tradition. This will entail giving numerous formulations, since all three paradoxes are perhaps better called *families* of paradoxes. Once the problem space is carefully laid out (*tahrīr maḥall al-nizā'*), the various independent solutions, their attendant objections, and the resolutions of those objections are presented and analyzed. Each chapter is sealed with a historical summary of the research on the paradox. An appendix includes translations of representative solutions, and, in the case of the totality of all relations paradox, of representative formulations.

There are several reasons for adopting this relatively unconventional format:

1. **Textual fidelity:** The organization of the philosophical portion of each chapter parallels philosophical research in the Islamic world in that the discussion there was frequently structured around the formulation of the problem, solutions to the problem, objections to the solutions, and resolutions of the objections.
2. **Concision:** Philosophy in the Islamic world was highly cumulative in nature. Most authors had little original content to add to the discussion of any given theorem (*mas'ala*)—in general one new solution, one new objection, or one new rejoinder. Few indeed are those who did any more. Most of what any given author had to say was reproduced from the work of other scholars. Hence, an informative approach (philosophically) is to cut out repetition and get an overview of all the original contributions the tradition made.
3. **Scope and organization:** The formulations, solutions, objections, and resolutions here given are culled from many works and presented in immediate proximity to one another, which is, in itself, a great aide to understanding a given philosophical problem in its entirety, as it was explored in the Islamic world. Rarely did one author's philosophical work summarize all prior insights into and contributions to an issue. Hence, seldom will a primary source offer the philosophical depth and breadth that a study of this kind could.
4. **Accessibility to practitioners:** Some histories are read by those who fall under the description of whatever the history is about. For example, many politicians (and political scientists) read political and social histories; many philosophers read the history of philosophy; many artists read the history of art; etc. (Few doctors or scientists read the history of medicine or science). As the history of philosophy in the Islamic world is currently written, it is rarely accessible to or meant to engage those who have a primary interest in philosophy *simpliciter*.

If a philosopher picks up a book on philosophy in the Islamic world, he is interested to know about the philosophy, not necessarily about all of its historical, social, political, and economic accoutrements. Yet few indeed are the publications that can claim to serve such a role. While the specialist-facing side of the discipline is thriving, the practitioner-

²⁹ El-Rouayheb, *The Development of Arabic Logic (1200-1800)*, 202.

facing side is near to non-existent. Making the history of philosophy in the Islamic world more accessible to practitioners would help move it further into the mainstream of academia and into dialogue with the history of philosophy more generally and with philosophy proper. It would also help enable a fair consideration of the question of whether or not Western philosophy departments need to “decolonize,” as some pundits have put it: that is, whether or not philosophy departments should incorporate in their teaching curricula material on the thought of non-European philosophers.

The philosophical content here is stripped of all extraneous information, instead of being mixed in with philological interpretation or historical details, making it easier to identify and forcing greater clarity of presentation. Readers with a primarily philosophical interest do not need to weed through biographical or political details in order to get at the philosophical root of the matter. Moreover, when philosophical and historical information are so carefully separated and organized, vague and excessively concise paraphrasing of philosophical content that ought to be more fully explained becomes obvious.

5. **Facilitation of historical understanding:** By giving historical information its own section, historians will more easily find what they are looking for. The historical progression of the study of a given issue and the network of scholars involved becomes all the more obvious when extraneous philosophical details are excluded.
6. **Emphasis of new material:** Dispensing with introductory material—whether of a historical or philosophical nature—that is easily found in other published works allows those familiar with the existing secondary literature to quickly access new additions.

*Wa-idhā taraqqayta fī mabāḥith al-majhūl al-muṭlaq ilā hādhihi l-daraja min al-istīdāḥ
ḥaqqā laka an yuqāl afī 'al-miṣbāḥ
fa-qad ṭala 'a l-ṣabāḥ*

*If in studying the problem of the absolute unknown you've demanded it be made this clear,
It's right to say to you, "Put out the candle of night austere—
Morning is here!"*

al-Sayyid al-Sharīf al-Jurjānī

There are known knowns—there are things we know we know. We also know there are known unknowns. That is to say, we know there are some things we do not know. But there are also unknown unknowns: the ones we don't know we don't know.

Donald Rumsfeld, Former US Secretary of Defense

The Paradox of the Absolute Unknown

Formulations of the Problem

There are at least two clearly distinguishable formulations of the paradox of the absolute unknown in the texts, other variations being linguistic in nature. The first is a logical formulation, and the second metaphysical.

The Logical Formulation

It is reasonable to assume that we cannot pass a judgement on something without already knowing it in some way.³⁰ If we are going to affirm or deny some predicate of some subject-term, we need at least to know something about the latter. After all, we must have some understanding of what it is we are making the affirmation or denial about—otherwise, our statement could hardly make sense. In sum, (I) “Being a subject of predication requires being known somehow” (*al-ḥukm 'alā shay' yastad 'ī taṣawwur al-maḥkūm 'alayhi bi-wajh mā*) seems obviously true.

If (I) is true, however, its contrapositive (II) “Absolute unknowns cannot be subjects of predication” (*kull majūl muṭlaqan yamtani 'al-ḥukm 'alayhi*)³¹ must be as well, yet it seems that it could not be. “Absolute unknowns” must be (a) absolutely unknown or else (b) known in some way. If they are (a) absolutely unknown, then (II) seems to contradict itself, since its content is that absolute unknowns cannot be subjects of predication, yet it itself is an instance of just such

³⁰ ‘In some way’ or ‘somehow’ (*bi-wajh mā*) means ‘as falling under any description, essential or otherwise.’ To know something ‘in some way’ or ‘somehow’ would be to know it insofar as it falls under some description (e.g. insofar as it falls under the description “thing”). If something was unknown such that it was not known as falling under any description, it would be ‘absolutely’ (*muṭlaqan*) unknown. “The meaning of [something being an ‘absolute unknown’] is that no individual at all (*shakḥ min al-ashkhāṣ*) knows (*taṣawwur*) it in any way whatsoever (*bi-wajh min al-wujūh*).” Taḥṭānī and Jurjānī, *Lawāmi' al-asrār*, 61.20-21. “An absolute unknown—i.e., that which cannot be known (*mā min sha'nihi an lā yu'lam*)...” ‘Abd Allāh Muḥammad al-Sā'infūrī, “Sharḥ Sullam Al-'ulūm” (n.d.), 12a.17, MS Rampur 3399.

³¹ Some logicians of the tradition object that (II) is not, strictly speaking, the contrapositive of (I), and then proceed to rephrase (II). Most, however, do not bother with this technicality.

predication: “absolute unknowns” are absolutely unknown, but they are subjects of predication, namely, of the predicate “cannot be a subject of predication.”

If, on the other hand, “absolute unknowns” are (b) known in some way, a similar problem arises. It seems obviously true that if something is known, it can be a subject of predication—at the very least, one could say of it that it is known. Therefore, if “absolute unknowns” are (b) known in some way, they should be capable of being subjects of predication, but that would be directly contrary to what (II) says, namely that “absolute unknowns” *cannot* be subjects of predication. Although (I) is perfectly plausible, and therefore (II), its contrapositive, ought to be equally plausible, the latter nonetheless leads to a dilemma.

There are a number of tasks any successful solution will have to accomplish. Perhaps most important is to preserve the truth of (II) and explain away the contradictions to which it apparently leads. In so doing, a constant obstacle will present itself in simultaneously accommodating the following two obvious truths: (A) “A subject of predication is somehow known” and (B) “Something that is somehow known can be a subject of predication.”

The Metaphysical Formulation

Although the metaphysical formulation doubtless traces its origins back to its logical counterpart and is seldom found independent of it in the literature, its force as a philosophical problem is not intimately bound up with it.³² It takes the form of a thought experiment: suppose there were a person who had no other concept besides “absolute unknown.” Now, things can be either (a) known in some way to this person or (b) absolutely unknown to him.

If something is (a) in some way known to him, it must be the case that this thing falls under the concept “absolute unknown,” since, as we assumed, he has no other concept, and so no other way to know anything. If that is the case, “absolute unknown” must be true of it, yet it is precisely this “absolute unknown” that is thereby known. This is a contradiction.

If instead something is (b) absolutely unknown to him, it follows that it falls under his concept of “absolute unknown,” by virtue of the fact that it is absolutely unknown. The fact that this thing falls under his concept of “absolute unknown,” however, is a way in which that thing is known, and hence, it is known in some way. But this implies a contradiction: that it is both known in some way and absolutely unknown.

Attempted Solutions and Objections Thereto

The Logical Formulation

In the sources I have examined, there are at least eight distinct solutions proposed for the logical formulation. Five of the six most significant of these follow the same model: they deploy a distinction to dissolve the apparent contradiction the paradox produces. Only the third solution below (“accidents of obtaining in the mind”) is an exception.

³² For further remarks of a historical nature, see below.

1. The Description/Referent (*waṣfī/dhātī*) Distinction

A proposition can be interpreted in two ways. The proposition's subject of predication can be taken (a) as the referent of the subject-term insofar as that referent falls under the subject-term's description (*waṣf*) or (b) as merely its referent (*dhāt*), irrespective of the fact that it may fall under that description. Depending on the interpretation, the proposition's truth-value may change. Consider the example "Someone standing is necessarily standing." If the subject of predication is the subject-term's referent insofar as it falls under the description of the subject-term—namely, an instance of the concept of 'someone standing'—then the proposition is true. After all, essential to (constitutive of) the concept 'someone standing' is 'standing,' and therefore, 'standing' is necessarily true of any instance of 'someone standing.' The same is not so of the bare referents of 'someone standing,' human-beings. Human-beings are not necessarily standing, no matter the interpretation of necessity. Indeed, sometimes they sit, lie down, crouch, etc. In sum, the truth of a proposition is a function of the description/referent interpretation of its subject-term, among other variables.

Let us bring this distinction to bear on the problem at hand. If (II)'s subject-term is interpreted merely as referent (*dhātī*), then it really is the case that—as (II) expresses—it cannot be a subject of predication, since the referent is utterly unknown: indeed, considered as mere referent, it is not even known insofar as it is known to be unknown. Therefore, (II) taken as a *dhātī* proposition could not be true, since that would imply that the bare referent could be a subject of predication, which would imply that it is somehow known. On the other hand, if (II)'s subject-term is interpreted as the referent under the description of the subject-term (*waṣfī*), then the subject of predication of (II) will in fact be known: for something to be described as being unknowable is for it to be known—at least insofar as it is known to be unknowable.³³ Hence, (II) would be true under this interpretation of the subject-term.

The force of this solution seems just to be that the horns of the dilemma—whether (II)'s subject-term is absolutely unknown or known in some way—can be grappled with by exposing an ambiguity of interpretation. Depending on which interpretation of the subject-term is identified as the actual subject of predication, (II) will be true or false. That is, we merely need to keep this ambiguity in mind, and we will see that it is absurd neither for (II)'s subject-term to be absolutely unknown nor for it to be known in some way: if the actual subject of predication in (II) is the bare referent, then it is, strictly speaking, absolutely unknown and really couldn't be a subject of predication;³⁴ if the actual subject of predication is the referent insofar as it falls under the description "absolute unknown," then it is fine to say of it that it could not be a subject of predication, insofar as it is known to be unknowable. Therefore, since the actual subject of predication differs according to either interpretation, it is no contradiction to hold that "absolute unknowns" both can and cannot be subjects of predication.³⁵

³³ In Rumsfeldian terms, every "unknown unknown" is, in light of the fact that it is known to be an unknown unknown, a "known unknown."

³⁴ It is easy to see how someone could here object, saying, "Ah! But you just said it! '(II)'s subject of predication cannot be a subject of predication'!" If one were to respond that nothing can be said of an absolute unknown considered as bare referent, the objection would simply resurface: "But didn't you just say something about it? 'Nothing can be said of it'?" Perhaps this line of objection ultimately reduces to the first objection given below and is therefore amenable to the same resolution.

³⁵ Tūsī and Athīr al-Dīn Abharī, "Ta'dīl al-mi'yār fī naqd Tanzīl al-afkār," 143.

Objection: Something known can be a subject of predication

The description/referent solution tells us that, granting that one interprets the subject-term as the referent under the description of “absolute unknown,” (II)’s subject-term is known in some way, and, therefore, (II) is, on this interpretation, true. It is obvious, however, that if something is known, it follows that one can predicate of it that it can be a subject of predication. But that (II)’s subject-term—namely “absolute unknowns”—can be a subject of predication apparently contradicts what (II) itself says: “Absolute unknowns cannot be subjects of predication.”³⁶

Resolution: The predication involved is not the one described

It seems this objection can easily be answered. The statements

(III) “(II)’s subject-term can be a subject of predication” and

(IV) “(II)’s subject-term cannot be a subject of predication”

are true or false depending on what “(II)’s subject-term” refers to. If it refers to the bare referent of “absolute unknowns,” then (III) is false and (IV) true. If it instead refers to the referent of “absolute unknowns” insofar as it is known to fall under the description “absolute unknowns,” then (IV) is false and (III) true. Since the subject-terms of (III) and (IV) differ in the two cases, there is no contradiction.

When (II)’s subject-term is interpreted as the referent insofar as it falls under the description “absolute unknown,” and (II) is therefore true, the impossibility of predication described in (II) is not the very same predication it contains but rather the predication to which the bare referent would (counter-factually) be subject. Hence, there is no contradiction entailed by (II)’s being true in the case that its subject-term is interpreted as the referent insofar as it falls under the description “absolute unknowns.”³⁷

Objection: The strengthened absolute unknown

If (I) “Being a subject of predication requires being known somehow” is true, then not only is its contrapositive (II) “Absolute unknowns cannot be subjects of predication” true, but so is (II’s) “Perpetual, absolute unknowns could never be subjects of predication.” The subject-term of (II’s), therefore, could never be known in any way, since that would be a contradiction in terms (it is, after all, *perpetually* absolutely unknown). That the subject-term *could* in some way be known, however, is essential to the description/referent solution, and, hence, that solution is no solution at all in the face of this formulation.³⁸

³⁶ This is how the objection is formulated by Ḥillī (*al-Asrār al-khaḍīyya*, 13.9-11.). Taḥṭānī and, following him, Jurjānī, phrase it slightly differently, though the content remains the same (*Lawāmi‘ al-asrār*, 19.8-11 and 59.1-5.).

³⁷ Taḥṭānī and Jurjānī, *Lawāmi‘ al-asrār*, 58.14-59.5.

³⁸ Shams al-Dīn Samarqandī, *Kistāsu‘l-efkār*, 83.13-16. A similar strengthening of the problem is formulated by Khūnajī, to which Samarqandī’s solution would seem to be equally applicable: “Anything that is unknown to us in all ways is impossible for us to make a subject of predication.” Muḥammad b. Nāmāwar Khūnajī, *Kashf al-asrār ‘an*

Resolution: Necessary facts and contradictory qualifications

It is simply a necessary fact that something unknown is known insofar as it is inevitably known to be unknown. Therefore, to qualify something that is otherwise absolutely unknown as “perpetually” absolutely unknown or “necessarily” absolutely unknown in order to disqualify it even from being known merely insofar as it is known to be unknown is tantamount to qualifying something with its contradictory.³⁹ That is, such a qualification would be akin to qualifying, e.g., “even” as “odd.” The upshot seems to be that every object of the mind in itself implies that it can be known, and to deny that would be to deny a necessary fact about all such objects.

2. Existential Propositions (*qaḍāyā khārijīyya*) and Essential Propositions (*qaḍāyā haqīqīyya*)

In order for some propositions to be true, their subject-terms must exist concretely. These are called existential propositions (*qaḍāyā khārijīyya*) in the tradition. In contrast, there is another class of proposition, the essential proposition (*qaḍīyya haqīqīyya*), for which no such condition holds: these propositions are true or false irrespective of the concrete existence of their subject-terms. Ultimately, this distinction goes back to an ambiguity in interpretation. Does the subject-term refer only to those things that are actually qualified by it (existential), or to anything which may be qualified by it, even if only hypothetically (essential)?

To complete the explanation, let me give an example of each kind of proposition. A standard example of the essential proposition is “The sum of the internal angles of a triangle is equal to two right angles.” Interpreted such that the subject-term is not limited to actually existing triangles, the statement is true even if there never had been and never could be a triangle. The proposition is true in virtue of the essence (*haqīqa*) of the subject-term (‘the sum of the internal angles of a triangle’) and predicate (‘equal to two right angles’). The existential proposition can be illustrated using the following scenario: suppose that it happened that the only color remaining was the color black. In this case, the statement “Every color is black,” interpreted as an existential proposition, would be true, since the instances of the subject-term, “every color,” would be limited to those actually existing, though the same statement would be false interpreted as an essential one.⁴⁰ Hence, the truth-value of a statement depends on, among other things, whether it is taken to express an existential or essential proposition.

Logicians of the tradition deploy this distinction to overcome the paradox of the absolute unknown. The sentence, (II) “Absolute unknowns cannot be subjects of predication,” is only paradoxical as long as we are fooled by a sleight of hand made under cover of ambiguity. That cover will quickly lift, however, if we read (II) keeping the existential-essential proposition distinction firmly before our eyes. Taken as an existential proposition, (II) will be false—these logicians hold—since its subject-term could not refer, and any existential, affirmative

ghawāmiḍ al-afkār, ed. Khaled El-Rouayheb (Tehran: Mu’assaseh-yi pizhūheshī-yi ḥikmat-o falsafeh-yi īrān, Mu’assaseh-yi muṭāla‘āt-i islāmī-yi dāneshgāh-i āzād-i birlīn, 2010), 9.17-18.

³⁹ Shams al-Dīn Samarqandī, *Kistāsu’l-efkār*, 83.17-19.

⁴⁰ Khūnājī, *Kashf al-asrār ‘an ghawāmiḍ al-afkār*, 85.5-7.

proposition⁴¹ with a non-referring subject-term is false. “Absolute unknowns” must be non-referring in light of the fact that were any instance of “absolute unknowns” to exist, it would per force be known, at least insofar as it is known to exist,⁴² and therefore there could be no instance of “absolute unknowns.” The horn of the dilemma where (II)’s subject-term is known in some way is thus dodged by claiming that (II) would simply be false.

If (II) is instead taken as an essential proposition, it would be true: were an absolute unknown to exist, it would indeed be impossible for it to be a subject of predication. Note that this counterfactual does not lead to a contradiction with (II). Although (II)’s subject-term would be known in a way, the content of (II) “Absolute unknowns cannot be subjects of predication” nonetheless threatens no contradiction, since the predication it describes, namely “cannot be a subject of predication,” is not predication of a partially known subject-term as in (II) itself but of an absolutely unknown subject-term.⁴³ Therefore, one cannot object that (II) contradicts itself, since the possibility of predication it is denying is of absolute unknowns, whereas (II)’s subject-term is not absolutely unknown.

Objection: Not all formulations are satisfied

The solution based on the existential-essential proposition distinction tells us that (II), when interpreted as an existential proposition, is false, since existential, affirmative propositions with non-referring subject-terms are false, and nothing answers to (II)’s subject-term, “Absolute unknowns.”⁴⁴ But (II)—and therefore the paradox as a whole—can be reformulated such that it is instead a negative proposition, in which case the fact that its subject-term does not refer will not be sufficient to make it false.⁴⁵

Objection: Absolute unknowns can exist

Were it even to be granted that the only formulations of the paradox which need be solved were those that are affirmative propositions, this solution would still fail, since it relies on the fact that the subject-term of any such affirmative formulation does not refer, which is simply false. There is no reason why there couldn’t be “absolute unknowns.” It is not the case, as claimed above, that, were an absolute unknown to exist, it would per force be known, at least insofar as it is known to exist, since it could be that there are absolute unknowns of which no mind is aware.⁴⁶

⁴¹ Unfortunately, my translation of (II) as “Absolute unknowns cannot be subjects of predication” makes this point moot, since it is, in fact, a negative proposition (“cannot”), not an affirmative one like it is in Arabic. A rendering of (II) such that it would remain an affirmative proposition, and thus one that would make the point valid, is “It is impossible for absolute unknowns to be subjects of predication.” See, however, the objection below.

⁴² Cf. Joep Lameer, “غير المعلوم يمتنع الحكم عليه” (Ghayr al-Ma’lūm Yamtani’ al-Ḥukm ‘alayhi) An Exploratory Anthology of a False Paradox in Medieval Islamic Philosophy,” *Oriens* 42, no. 3–4 (2014): 419.

⁴³ Maḥmūd b. ‘Abd al-Raḥmān Iṣfahānī, “Tanwīr al-Maṭālī ‘fī Sharḥ Maṭālī ‘al-anwār” (Princeton University, 1300), 14a.3–4, Special Collections - South East (MSS), <https://catalog.princeton.edu/catalog/4793400>.

⁴⁴ As I note above, for this objection to have any meaning, one must keep in mind that a common rendering of (II) in Arabic makes it an affirmative proposition. To capture this, one might translate it as “It is impossible for absolute unknowns to be subjects of predication” instead of as “Absolute unknowns cannot be subjects of predication.”

⁴⁵ Shams al-Dīn Samarqandī, *Kistāsu ‘l-efkār*, 83.2–3; Iṣfahānī, “Tanwīr al-maṭālī,” 14a.4–6.

⁴⁶ Iṣfahānī, “Tanwīr al-maṭālī,” 14a.6–8; Shams al-Dīn Samarqandī, *Kistāsu ‘l-efkār*, 83.4–6.

Objection: If one is essential, then so too must be the other

The paradox springs from the claim that (I) “Being a subject of predication requires being known somehow.” It is the contrapositive of this claim that yields the problem proposition (II) “Absolute unknowns cannot be subjects of predication.” Since (I) entails (II), we can write both as a conditional proposition (*sharṭiyya muṭṭaṣila*), “If being a subject of predication requires being known somehow, then absolute unknowns cannot be subjects of predication.”

The existential-essential proposition distinction solution consists in claiming that (II) is true only as an essential proposition and explaining away any attendant difficulties. But since (II) is the contrapositive of (I), and therefore must figure in a conditional proposition like the one above, it must be the case that both (I) and (II) are of the same kind of proposition, whether existential or essential, since a single conditional could hardly have an antecedent of one kind and a consequent of another. Yet it is clear that (I) is true construed as an existential proposition, and therefore, its contrapositive must also be true as an existential proposition. This contradicts the solution, however, since it relies on the claim that (II) is false if taken as an existential proposition.⁴⁷

3. Accidents of Obtaining in the Mind (*ma'rūd al-ḥuṣūl fī l-dhihn*)

The contents of the mind have certain accidental properties merely as a result of being in the mind. For example, a universal concept like, “man,” is rendered particular, by virtue of being a particular concept in a particular mind. Likewise, a mental representation will inevitably have the property “known,” since it is present in the mind, and therefore known. Hence, even a concept like “absolute unknown” will, by virtue of coming to be in the mind, be known.

But when we consider such concepts in order to determine if they have instances or in order to make a judgement involving them, we ought not take such accidental properties into consideration. Hence, “Absolute unknowns cannot be subjects of predication” is true without contradiction, since when we pass judgement on “absolute unknowns,” we should not take into account its accidental properties (most importantly, “being known”) resulting merely from its presence in the mind but only the content of the concept itself.⁴⁸

4. Instances (*bi-l-dhāt*) and Supposition (*bi-l-farḍ*)

Like all universal concepts, “absolute unknown” would apply to its instances, and its instances would be absolutely unknown. Although the mind must take further steps beyond simply conceiving of “absolute unknown” to realize it, these instances would be *known* to the mind merely by virtue of the fact that the mind knows “absolute unknown” would apply to them. Until one has this realization, however, the subject-term of “Absolute unknowns cannot be subjects of predication” will be absolutely unknown—not even known insofar as it is known to be unknown—

⁴⁷ Hillī, *al-Asrār al-khaṭiyya*, 13.penultimate.

⁴⁸ Ṣadr al-Sharīʿa al-Maḥbūbī, “Sharḥ Taʿdīl al-ʿulūm” (n.d.), 8a.11-23, Cod. Arab. 045, Universitätsbibliothek Leipzig, https://www.islamic-manuscripts.net/receive/IslamHSBook_islamhs_00006096.

and the proposition (II) will therefore be false, since nothing could be said about absolute unknowns, not even that they “cannot be subjects of predication.” Upon acknowledging that the instances of “absolute unknown” would indeed be known by virtue of the fact that they are known to be absolutely unknown, the mind can assent to “Absolute unknowns cannot be subjects of predication,” by means of supposing the instances merely to be absolutely unknown, ignoring the fact that they are thereby known and therefore *could* be subjects of predication.⁴⁹

5. Primary Predication (*ḥaml awwalī*) and Standard Predication (*ḥaml shā’i*)

Predication that either is an expression of intensional containment, intensional identity, or known to be true merely by conceiving of the subject-term and predicate is called primary (*ḥaml awwalī*) in the tradition.⁵⁰ Examples are “Man is a rational animal,” since ‘rational animal’ is constitutive of the concept ‘man,’ and “Man is universal” since being ‘universal’ is an immediate *per se* accident (*lāzim bayyin*) of ‘man’ insofar as it exists in the mind. Predication that is not tantamount to one concept being constitutive or an immediate *per se* accident of another but to one being co-extensive with another is termed standard (*ḥaml shā’i*).⁵¹ “Man is risible” is an example since anything that ‘man’ is true of, ‘risible’ is true of, and *vice versa*, though the conceptual content of ‘man’ is distinct from that of ‘risible.’ Likewise, “Man is particular” would be true according to a standard predication reading, since the instances of ‘man’ are indeed all among the instances of ‘particular.’⁵²

Further examples can help make the distinction clear. Consider the statement “Particulars are particular.” Read such that it expresses primary predication, it is false: the concept ‘particular’ is in fact universal, considering as how it can apply to many individuals. In contrast, if it is read such that it expresses standard predication, it is true, since all the instances of ‘particulars’ are, in fact, contained in the extension of ‘particular’: Socrates is particular, my computer is particular, etc.⁵³ Take yet another example: “Verbs cannot be subjects of predication.” If it is read as expressing primary predication, then it is false, since the concept ‘verbs’ can in fact be a subject of predication, as this very sentence demonstrates; read as expressing standard predication, however, it will be true: none of the instances of ‘verbs’—e.g., ‘runs,’ ‘swam,’ ‘fought,’ etc.—can take the place of subject-term in a sentence, and so all are included in the extension of ‘cannot be subjects of predication.’

Before discussing how these two kinds of predication were deployed to produce a solution to the paradox, it is illustrative to note that this dichotomy is distinct from the dichotomy between essential and existential propositions. The latter dichotomy turns on whether the subject-term is

⁴⁹ Taḥṭānī and Jurjānī, *Lawāmi‘ al-asrār*, 60.4-18.

⁵⁰ “It has been called ‘primary’ because such propositions are primarily true or false.” M. A. Tahānawī, *Kashshāf iṣṭilāḥāt al-funūn wa-l-‘ulūm*, ed. R. ‘Ajam, vol. 1 (Maktabat Lubnān Nāshirūn, 1996), 717lhs.27-8. That is, all such propositions are of the class of primary propositions (*awwaliyyāt*), those that are known to be true merely upon conceiving of the subject-term and predicate.

⁵¹ “It is the [kind of predication] used in the sciences... It has been called ‘standard’ because of how commonly and standardly it is used.” Tahānawī, 1:716lhs.11-12, 718rhs.15-16. Cf. Lameer, “Ghayr Al-Ma‘lūm,” 438, 438n108.

⁵² Ṣadr al-Dīn Muḥammad b. Ibrāhīm al-Shīrāzī, *Manṭiq-i nuvīn: mushtamil bar al-Lama‘āt al-mashriqiyya fī l-funūn al-manṭiqiyya*, trans. ‘Abd al-Muḥsin Mishkāṭ al-Dīnī (Tehran: Mu’assasah-yi intishārāt-i āgāh, 1981), 7.5-7. For a very clear presentation of the distinction, see the Persian commentary of the translator starting on p. 158.

⁵³ al-Shīrāzī, 20.2-8.

taken to apply only to its actually existing instances or to hypothetical instances as well, while the former dichotomy is grounded in the intensional and extensional relations between the subject-term and predicate. Therefore, whether a statement is interpreted to express primary or standard predication has no bearing on whether it also be interpreted as an essential or existential proposition.

Take “Man is universal” for example. When it is read as expressing primary predication, it can be rephrased as “The concept ‘man’ is universal,” and, as is plain to see, this could be further interpreted either as an essential or existential proposition. When taken as the former, it can be rephrased as, “Were there any such concept as ‘man’ it would be universal,” and, taken as the latter, as “The concept ‘man’ that anyone actually has is universal.” The same can be shown for a statement interpreted as expressing standard predication, like “Man is risible.” Taken as expressing standard predication, it can be formulated as “The instances of the concept ‘man’ are among the instances of the concept ‘risible,’” and this statement is perfectly amenable to being interpreted either as an essential or existential proposition. In the former case, it would read “Were there to be an instance of the concept ‘man,’ it would be among the instances of the concept ‘risible,’” and in the latter, “The actual instances of the concept ‘man,’ are among the instances of the concept ‘risible.’”⁵⁴

The tradition deploys the distinction between primary and standard predication in an attempt to solve the paradox.⁵⁵ The crux of the solution is that some terms can be primarily but not standardly predicated of others. In other words, a given statement of the form “A is B” will be either true or false interpreted as expressing primary predication, yet the very same statement will bear the opposite truth-value when interpreted as expressing standard predication.

This reasoning is applied to a number of *aporetic* statements, including (II) “Absolute unknowns cannot be subjects of predication.” Notice that, interpreted as expressing primary predication, (II) is false: “The concept ‘absolute unknown’ cannot be a subject of predication” is false, since the concept ‘absolute unknown,’ does in fact exist in a person’s mind, is thus known, and so can indeed be a subject of predication. “*Can* be a subject of predication” would appear to be a *per se* accident of any concept, even “absolute unknown.” But interpreted as expressing standard predication—“The instances of ‘absolute unknown’ are among the instances of the concept ‘cannot be a subject of predication’”—it is true, since the individuals that ‘absolute unknown’ is true of could not be subjects of predication, since they could not be known—on pain of no longer being absolutely unknown!

This last point can be further explained by focusing in closer on the nature of subject-terms. A subject-term *x*, when interpreted according to standard predication, can be analyzed as the proposition, “Something is described by *x*” (*‘aqd al-waḍ’*). Some subject-terms, however, immediately yield a contradiction when subjected to such analysis. Take the term ‘indescribable.’ If it figured as a subject-term in a proposition expressing standard predication, we ought to be able to analyze it as any other, yet if we do, we get “Something is described by ‘indescribable,’” a contradiction. That is, some concepts, by virtue of their content, could not actually apply to the instances that ought to fall under them. “Absolute unknown” is one such concept: as soon as one identifies the individuals to which “absolute unknown” applies, those individuals could not have

⁵⁴ Cf. Lameer, “Ghayr Al-Ma’lūm,” 439.4-6.

⁵⁵ al-Shīrazī, *Manṭiq-i nuvīn*, 15.5; al-Shīrazī, *al-Asfār*, 1:239.3-11.

it applied to them. Of course, that is not to deny that some individuals (we know not which) *could* have it applied to them, *were* they known.

To put the same point differently, some terms can be self-predicated when that predication is taken as primary but not when taken as standard.⁵⁶ Hence, “Absolute unknowns are absolutely unknown” is true when taken in the sense of “The concept ‘absolute unknown’ is made up of the content ‘absolute unknown’,” but false when taken in the sense of “The instances of the concept ‘absolute unknown’ are among the instances of the concept ‘absolutely unknown.’” No instance of ‘absolute unknown’ could have the predicate ‘absolutely unknown’ applied to it, insofar as that entails that it would thereby be known to be absolutely unknown. To be a subject-term *x* in a self-predicating proposition taken in the sense of standard predication is just for the proposition “The instances (of *x*) are described by *x*” to be true. For ‘absolute unknown,’ this would mean that, for it to appear as subject-term in a proposition expressing standard predication, the proposition “The instances (of ‘absolute unknown’) are described by ‘absolute unknown’” would have to be true, but that simply could not be.

It is worth here returning to a possible gap in the reasoning I hinted at earlier. Consider the concept ‘number.’ There are many numbers of which I simply have never conceived, and there are even some of which I *cannot* conceive (at least in a certain sense of “conceive of”), since they are so large. Does the fact that I cannot positively identify the specific individuals that ‘number’ would apply to imply that ‘number’ is not true of them actually? The answer seems to be “No, ‘number’ would still apply to them despite the fact that I cannot conceive of them in any way more specific than under the description ‘number too big for me to conceive of’.”

Likewise, although it appears that I cannot conceive of any specific instance of ‘absolute unknown’ without thereby disqualifying it from being one, I seem perfectly well entitled to conceive of the idea that there are some individuals (we know not which) that ‘absolute unknown’ applies to, without simultaneously forfeiting the right to term them as such. That is, there actually seem to be two kinds of standard predication, one where a predicate is true of instances of the subject-term without further knowledge of what identifies those instances, and one where a predicate is true of those instances insofar as they are known as individuals.⁵⁷

This observation, however, does not seem to generate any real difficulty. The solution claims to have undone the contradiction by drawing our attention to a condition on contradiction that has not been met: just like conditions of time and aspect must be met for a contradiction to obtain between propositions, so too must this condition of predication-type.⁵⁸ That is, if two propositions are to be contradictory, they must also express the same kind of predication, primary or standard. If, as the foregoing paragraph implies, there are actually two kinds of standard predication, the solution remains unscathed: so long as there is a distinction, whether it be bi- or tri-partite, there will be no contradiction.

6. Essence (*bi-l-dhāt*) and Accident (*bi-l-‘araḍ*)

⁵⁶ al-Shīrazī, *Manṭiq-i nuvīn*, 15.6; al-Shīrazī, *al-Asfār*, 1:239.11-12.

⁵⁷ Ṣadrā’s teacher, Mīr Dāmād, relies on drawing just this distinction in his attempt to solve the Liar paradox: Mīr Dāmād, *Muṣannaḡāt-i Mīr Dāmād*, 2:670, 685–86.

⁵⁸ Ṣadrā says just this at *Manṭiq-i nuvīn*, 20.5.

A predicate can be true of a subject essentially (*bi-l-dhāt*) or accidentally (*bi-l-‘araḍ*). In the first case, the predicate is constitutive of the essence of the subject. Were we to list out what it is that makes the subject the kind of thing that it is, we would find the predicate among its items. The second case is the opposite: performing the same analysis, we would *not* find the predicate.

This distinction is deployed to solve the paradox, which hinges on the question of whether the subject in (II) “Absolute unknowns cannot be subjects of predication” is known somehow or absolutely unknown. The response is that “absolute unknowns” is, on the one hand, known essentially, and, on the other, absolutely unknown accidentally. It is essentially known because “absolute unknowns” is a concept, and to be known is constitutive of being a concept. Insofar as “absolute unknown” is true of any object, however, that object would be absolutely unknown accidentally: there is no object whose essence is constituted by being absolutely unknown, similar to how there is no object whose essence is constituted by being a “thing” or “existent.”⁵⁹

By drawing this distinction, the contradiction to which either horn of the paradox would lead is prevented. When “absolute unknowns” is interpreted as concept and therefore essentially known, (II) “Absolute unknowns cannot be subjects of predication” is true. When it is interpreted as object and therefore accidentally absolutely unknown, (II) is false.⁶⁰

The Metaphysical Formulation

1. Mere Presence in the Mind Is Not Sufficient for Qualifying as Knowledge

By means of some concept extraneous to a thing itself or by means of the concept of the thing’s essence, we come to know that thing. However, the concept by which we know an object of knowledge cannot be such that it contradicts the possibility of coming to know it at all. Such a concept would be incapable of yielding knowledge of anything due to the very nature of the concept and the nature of what it is to know something. If the content of the concept is that its instances are unknown, then that concept is prohibitive of the process by which something can become known: the concept must be capable of serving as a “mirror” (*mir’āt*) by which the instances are reflected in the mind.

In the scenario posed by the thought experiment, things would indeed be “absolutely unknown” to this mono-conceptual individual, and his lone concept, “absolute unknown,” would therefore apply to them. However, it does not follow that they would thereby be *known* to him. Something is known to someone only when the mediating concept does not contradict the very meaning of “coming to be known.” Of course, that is precisely the case here: “absolute unknown” could never function as the mirror that reflects its instances into the mind, since doing so would contradict its very content. “Absolute unknown” would have to be capable of serving as an intermediary for making its instances known, but by serving as that intermediary and applying to them, they become absolutely *unknown* not known.

⁵⁹ Although nowhere mentioned in the texts, proof would be that “unknown” is a relational predicate—that is, it demands some knower—and no essence could be constituted by its relation to something else (or the absence thereof), since the relatum must be prior to the relation, yet the relation (“unknown”) that is constitutive of the quiddity would have to be prior to its relatum, the quiddity itself.

⁶⁰ ‘Abd al-‘Alī Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm ‘alā Sullam al-‘ulūm*, ed. ‘Abd an-Naṣīr Aḥmad al-Malībārī (Kuwait: Dār al-Ḍiyā’, 2017), 266.9-12.

The solution amounts to throwing out the concept “absolute unknown” as inherently problematic. While it may not be so obviously contradictory as “an even odd” or “a square circle,” upon analysis, “absolute unknown” is contradictory nonetheless, though more in the way that “largest prime number” is. Since a concept is precisely the vehicle for obtaining knowledge about some object, and “absolute unknown” is a concept, its content—“absolute unknown”—contradicts the fact that it is a concept—something used for obtaining knowledge.

Objection: Mere verbal solution

Something is known when the concept of its essence itself or some aspect true of it obtains in the mind, and it is unknown when neither have obtained in the mind. Since the solution grants that something obtains in the hypothetical person’s mind, it will be known. Therefore, the same contradiction as explained in the original problem statement remains. To redefine “known” with an added condition is fine, but the contradiction between the more typical understanding of that word and the more typical understanding of “unknown” has not thereby been undone.⁶¹

Resolution: Partial knowledge of x requires knowledge that it is knowledge of x

One cannot say that something is known merely by virtue of the fact that some aspect of the thing has obtained in the mind. One must further know that this aspect is true of something else, and so in order to know x insofar as it is y , one must know x , y , and the fact that x is y . However, in the thought experiment, the person knows only “absolute unknown” but neither the objects it is true of nor the fact that it is true of those objects. That “absolute unknown” would, independent of the mono-conceptual person’s knowledge, be true of those objects is not sufficient for one to claim that the person knows them by means of that concept, even though he is in possession of the concept. Therefore, things are “absolutely unknown” to him in the sense that, independent of his knowledge, “absolute unknown” applies to them, but he does not know them to be absolutely unknown, because he has no knowledge of the things in themselves of which “absolute unknown” is true nor does he have knowledge that “absolute unknown” applies to those things.⁶²

2. The Thought Experiment is Absurd

(a) The nature of the material stage of the intellect does not accord with the thought experiment

In the first stages of life, the mind has only the most self-evident of self-evident concepts, like “existence” and others. Therefore, it cannot be admitted that “absolute unknown” would be one of these first concepts, and so the whole thought experiment is baseless.⁶³

Objection: The scenario is not impossible

⁶¹ Baḥr al-‘Ulūm, 269.1-4.

⁶² Mubīn, *Mir’āt al-shurūḥ*, 1321ah, 1:54.7-16.

⁶³ Sihālawī et al., *Mullā Ḥasan*, 47n2.

The scenario proposed in the thought experiment is far from impossible. Since it is not impossible, neither should be the consequences, and an explanation is therefore still required.⁶⁴

(b) The very conditions of the thought experiment lead to a contradiction

If someone genuinely understood the concept “absolute unknown,” he would have to know a whole host of other concepts like “known,” “essence,” “accident,” etc. But that would be to contradict what was assumed—that the person in the thought experiment has no other concept besides “absolute unknown.”⁶⁵

Summary and Analysis

Historically speaking, it seems that the least successful solution of those here presented is the one founded on the essential-existential proposition distinction. The objections against it are many, yet a millennium of tradition has produced no resolutions, which is hardly a reflection of a lack of ingenuity: despite the solution’s merits, the objections strike at real weaknesses. It is not obvious, e.g., that there could exist nothing that is absolutely unknown, yet the solution expects us to take it for granted, offering little justification.

The essence and accident solution also suffers from weaknesses. It fumbles from the very beginning by misidentifying the distinction supposed to be its deliverance. While appearing to be grounded in the distinction between essence and accident, in reality, the more fundamental distinction it relies on is that between intension and extension. “Absolute unknowns” is only essentially known because it is taken to be a concept, and it is only accidentally unknown because it is taken as an instance. In this respect, the solution boils down to the primary/standard predication solution.

The essence and accident solution also shares something with the accidents of obtaining in the mind solution. Both make the fundamental insight that to be a concept is to be known, and therefore “absolute unknown” must be known. But the accidents of obtaining in the mind solution develops this insight in a different way. Such accidental properties like “being known” are not to be taken into account when passing judgement on the concept itself. Hence, the paradox sentence “Absolute unknowns cannot be subjects of predication” is not a paradox at all, since only the content of “absolute unknowns” should be considered and not its accident “known.”

The description/referent distinction solution enjoyed great success. Although objections were raised, all of them found some plausible resolution, which shows both the solution’s historical popularity—logicians thought highly enough of it to invest their time in resolving objections—as well as its philosophical strength—not every solution can even in principle be so well defended against objections, let alone in practice.

While its proponents sometimes presented it as an improvement on the essential/existential proposition distinction solution, the instances and supposition solution seems to me to be instead

⁶⁴ Mubīn, *Mir’āt al-shurūḥ*, 1321ah, 1:53.18-20.

⁶⁵ Sharaf al-Dīn b. Muḥyī l-Dīn b. Ṣadr al-Dīn b. Muḥammad al-A‘zamī Laknawī, “Risāla fī l-jidhr al-abkam” (n.d.), 190a.15-18, MS Rampur 3307, 190a-b.

an improvement on the description/referent distinction solution. In the latter, the paradox sentence “Absolute unknowns cannot be subjects of predication” is deemed false when the subject-term is taken as bare referent, since, the reasoning went, such instances would not even be known to be unknown, let alone incapable of being subjects of predication. But this does seem to ignore the fact that the person so judging the statement could only have locked on to the bare referents by means of the concept (since, they are, after all, otherwise “absolute unknowns”), and so at some level must be aware of the fact that those “bare” referents fall under the concept “absolute unknown,” making them less bare than they were supposed to be. The instances and supposition solution rectifies this problem by acknowledging that the mind tracks the referents by means of the concept, adding that it may not realize the consequences—namely, that by knowing the referents fall under the concept, the concept itself may no longer apply to its instances, which explanation better suits lived experience.

Although the instances and supposition solution closes one door left ajar by the description/referent distinction solution, it opens another. There is a plausible argument to be made that a concept can only act as a means of knowing its instances so long as the very content of that concept does not contradict coming to know them. Few concepts more obviously violate that condition than “absolute unknown,” yet the instances and supposition solution relies on the idea that one could use “absolute unknown” as a means to know its instances. To make the solution more convincing, some argument against this condition would need to be added.

The primary/standard predication solution, however, appears to me one of the greatest here examined. It captures virtually all of the strengths of the description/referent distinction solution while adding further insight. It rightly points out that the paradox is fundamentally one of self-predication. The paradox turns on the fact that predication implies knowledge and vice versa. Therefore, “cannot be subjects of predication” implies “cannot be known,” and so the paradox can be more simply stated “Absolute unknowns cannot be known,” or better, “Absolute unknowns are absolutely unknown.” Furthermore, the solution precisely identifies the problem cases of self-predication by deploying the distinction on which it is founded. When a term can be primarily but not standardly self-predicated, it will be a problem case.

Examples additional to those above can be given. “God’s partner is impossible” is just such a problem case, and could be rephrased as “Impossible things are impossible,” to show more explicitly that it is a case of self-predication. Interpreted so that the predication is primary, the statement is unproblematic. “Impossible” is indeed constitutive of the concept “impossible things.” No instance of “impossible things,” however, could fall under “impossible,” since what it is for something to be impossible is just that it does not exist, and “impossible” necessarily has neither concrete nor mental instances.

While there are many advantages to this solution, it suffers from the same problem that the rest do. In every solution but one, the contradiction that the paradox leads to is explained away by giving alternate interpretations such that one interpretation is true and the other false. When the paradox sentence “Absolute unknowns cannot be subjects of predication” is interpreted such that it is false, the grounds given for its being false are that “absolute unknowns” would be absolutely unknown and so could not be subjects of predication. The difficulty is that this seems to be affirming precisely what it is supposed to be denying—that absolute unknowns cannot be subjects of predication. The paradox resurfaces.

Perhaps the root of the difficulty lies in the concept “absolute unknown” itself. What could be known to answer to it? The fact that the concept is possessed by some thinker demands that there be no instance of it known to anyone. If someone *knew* “absolute unknown” had an instance, that instance would hardly be absolutely unknown and so could not after all be known as its instance.

But in order for some term (say, “absolute unknown”) to be true of some object, must someone know that some object falls under the term? Certainly, the object need not be known as a particular. It could be known in no more specific a way than as one individual among many falling under the term in question or any other term. But need it be known at all by anyone? If there were never any minds, never anything that could *know*, would there be any sense in which any object is the instance of any term?

There may be a structural parallel between this paradox and Russell’s. If an instance is in the extension of “absolute unknown,” it should be out of it, since it would thereby be known, and if it is outside of any extension, and hence an absolute unknown, it would *perforce* have to be in the extension of “absolute unknown.” The situation bears a resemblance to how the set of all sets not containing themselves ought to contain itself if it does not contain itself, and not contain itself if it does. Of course, the difference is that an object could fall outside the extension of “absolute unknown” but in the extension of some other term, and there would be no problem.

Still other avenues of solution seem to have been available. One could argue that “subject” (“knower”) is inconceivable without “object” (“known”) and *vice versa*.⁶⁶ Yet to be an “absolute unknown” is exactly to be an object with no subject cognizing it. Therefore, one may conclude, the very concept is absurd. Another way to put the point is as follows: Every “absolute unknown” would have to be an object of knowledge, and, as noted above, “cannot be a subject of predication” implies “cannot be known.” Therefore, “Absolute unknowns cannot be subjects of predication” should have the same truth value as “Some objects of knowledge cannot be known,” which is seems to be a logical contradiction. This analysis would be of the same kind as one given for the metaphysical formulation and one given for the “totality of all relations” paradox, insofar as all claim that the very framing of the paradox relies on a contradiction.

It is informative to consider why this avenue of solution may not have been attractive to most Arabic logicians, even if some did tread similar paths. Virtually all of them would have had theological commitments to maintain, one of which may have been that God knows everything, which is just to say that there are *no* absolute unknowns. Nonetheless, certain logicians go out of their way to countenance the idea that absolute unknowns could, in principle exist, even if they could never be known to exist. One wonders if they thought this implied—as, on an uncontroversial reading, it would—that God could be ignorant of something.

Theologically speaking, then, there was every reason to go along with other logicians who suggested that no “absolute unknown” could ever actually exist, yet many resisted. Perhaps their motivation for resisting, therefore, was a more basic philosophical commitment. The claim that something entirely unknown could exist is a litmus test for realism—the belief that things have the properties they do (*sharṭ māhawī*) and exist at all (*sharṭ wujūdī*) independent of any mind’s

⁶⁶ In Arabic logical terms, the view is that “subject” and “object” are *mutadāyifān* (mutually dependent concepts) like “parent” and “child” are: every instance of “object” is cognized by some instance of “subject,” and every instance of “subject” cognizes some instance of “object.”

knowing them. It is only an idealist—someone who denies either clause of realism—who could go along with such a claim. Since I suspect it is the case that many if not most logicians in the tradition were realists, any in principle denial of the existence of “absolute unknowns” would have been off-limits. In this light, the paradox can be seen as a proxy for discussing realism. The attempts to solve the paradox while maintaining the possibility that some absolute unknown exists can be construed as resisting idealism, while those solutions that declare “absolute unknown” incoherent are perhaps a way of yielding to it.

Historical Development

The first person to have posed the logical formulation of the problem in Arabic appears to have been Fakhr al-Dīn al-Rāzī (d. 1210).⁶⁷ The problem was then taken up by virtually all of the most famous logicians of the following generations: Afḍal al-Dīn al-Khūnajī (d. 1248), Athīr al-Dīn al-Abharī (d. 1265), Naṣīr al-Dīn al-Ṭūsī (d. 1274), Najm al-Dīn al-Kātibī (d. 1277), Sīrāj al-Dīn al-Urmawī (d. 1283), Shams al-Dīn al-Shahrazūrī (fl. 1288), Shams al-Dīn al-Samarqandī (d. 1322), Ibn al-Muṭahhar al-Ḥillī (d. 1325), Ṣadr al-Sharī‘a al-Maḥbūbī (d. 1346), Maḥmūd b. ‘Abd al-Raḥmān al-Iṣfahānī (d. 1348/9), Quṭb al-Dīn al-Taḥṭānī (d. 1365), and al-Sharīf al-Jurjānī (d. 1413). While Rāzī’s response to this problem is not entirely clear—he seems ultimately to rely on the claim of self-evidence to refute it—he unambiguously plants the seeds of what comes to be one of the major attempts at a solution, the *wasfī-dhātī* (description-reference) distinction solution.⁶⁸ This is confirmed by Kātibī’s interpretation in his commentary on Rāzī’s *Mulakhkhaṣṣ*, *al-Munaṣṣaṣ*.

But the first major solution I have seen to be explicitly and clearly presented is the essential-existential proposition distinction solution, at the hands of Khūnajī (d. 1248) in his logic textbook *Kashf al-asrār*.⁶⁹ Khūnajī’s view, though rarely paired with his name,⁷⁰ often figures in discussions of the problem. Some logicians, like Urmawī (d. 1283) and Shahrazūrī (fl. 1288), adopted it.⁷¹ Even if others could not endorse it, almost everyone who later examines the problem critically discusses his solution, and, in the end, the little work done to develop Khūnajī’s solution came in the form of answering the objections of its detractors.

Though some logicians thought the basic thrust of Khūnajī’s attempt hit the mark, equally many seem to have thought otherwise; indeed, they argued it was downright misguided. Kātibī (d. 1277), Samarqandī (d. 1322), Ḥillī (d. 1325), and Iṣfahānī (d. 1348/9) all raised difficult objections. Half of these authors, however, merely reproduced the ideas of their predecessors: Ḥillī repeats Kātibī’s objection, and Iṣfahānī includes, in essence, nothing more than Samarqandī does. In fact, Samarqandī seems to be the outstanding personality of the bunch: his presentation of the discussion in general and of the objections to Khūnajī’s solution in particular is not only exceedingly clear

⁶⁷ Fakhr al-Dīn Muḥammad b. ‘Umar Rāzī, *Manṭiq al-Mulakhkhaṣṣ* (Tehran: Inteshārāt-i Dāneshgāh-i Imām Ṣādiq, 2002), 8.

⁶⁸ Rāzī, 7–8.

⁶⁹ Khūnajī, *Kashf al-asrār ‘an ghawāmiḍ al-afkār*, 9–10.

⁷⁰ An uncommon reference to *ṣāhib al-kashf* is found in Ṣadr al-Sharī‘a al-Maḥbūbī, “Sharḥ Ta‘dīl al-‘ulūm,” 14b.11.

⁷¹ Shams al-Dīn al-Shahrazūrī, *Rasā’il al-shajara al-ilāhiyya fī ‘ulūm al-ḥaqā’iq al-rabbāniyya*, ed. Necip Görgün, vol. 1 (Beirut: Dār Ṣādir, 2007), 48.16.

but also original, insofar as he introduces objections I have been unable to track down in the work of earlier logicians.

Although Ḥillī is not particularly original, there are other factors that make his work on the problem worth further consideration. Ḥillī appears to have been rather non-committal in his stance. He entertains the paradox in both his *Asrār al-khaṭṭiyya* and his commentary *al-Qawā'id al-jaliyya fī sharḥ al-Risāla al-shamsiyya*. In the former, he implies that he has a stronger preference for the description-referent solution than he does for the other two he reports by virtue of the fact that the one is introduced as “Response:... (*al-jawāb*)” while he prefaces the others with “Some have responded... (*ajāba ba'dhum*)”.⁷² That being said, Ḥillī spares none of the three solutions from criticism. Though he poses a problem for each, he makes no attempt to resolve any.

In *al-Qawā'id al-jaliyya*, however, he touts the essential-existential proposition distinction as the solution.⁷³ But the treatment is brief in this work, and the fact that he raises no objections to what he knew was a disputed solution makes me think he was not entirely convinced. It is worth noting that this preference cannot be explained as a bi-product of the constraints of commentary: the problem is not mentioned at all in the base-text, Kātibī's *al-Risāla al-shamsiyya*, such that Ḥillī would be constrained to favor one solution over another simply because it was the one given by the author (*muṣannif/mātin*). Moreover, Kātibī, in his commentary on Rāzī's *Mulakhkhaṣ*, does not seem to have unconditionally supported the essential-existential proposition distinction solution, if he supported it all.⁷⁴

This is significant because Ḥillī is often (rightfully) thought to be a close follower of one of his teachers, Naṣīr al-Dīn al-Ṭūsī.⁷⁵ Here, however, we see that Ḥillī mostly follows his own path, even if it is one of uncertainty: Even when he does express a preference for Ṭūsī's solution, he criticizes it, and in his other treatment of the problem, he makes no mention of it at all.

Abharī (d. 1265) tries to solve the paradox in at least three of his works. He has a variety of things to say, though it is not always clear to me what exactly they mean or how they are supposed to solve the problem. What is clear is that he attempts a Rāzī-inspired description-referent style solution in at least one work.⁷⁶ Ultimately, however, it is Abharī's student, Ṭūsī (d. 1274), who presents a clear and unambiguous description-referent solution, considerably improving on his forbears' work.⁷⁷ Ṭūsī's colleague, Kātibī (d. 1277), fits Rāzī's argument into a description-referent solution in his commentary on Rāzī's *Mulakhkhaṣ*, though it is obvious to me that Ṭūsī has given a better account of the solution, irrespective of who was the first to publish.⁷⁸

⁷² Ḥillī, *al-Asrār al-khaṭṭiyya*, 13.1, 13.12, 14.2.

⁷³ al-Ḥasan b. Yūsuf b. al-Muṭahhar Ḥillī, *al-Qawā'id al-jaliyya fī sharḥ al-Risāla al-shamsiyya*, ed. Fāris Ḥassūn Tabrīziyyān (Qum: Mu'assasat al-Nashr al-Islāmī al-Tābi'a bi-Jamā'at al-Mudarrisīn, 1992), 192.

⁷⁴ Najm al-Dīn Kātibī, “al-Munaṣṣaṣ fī Sharḥ al-Mulakhkhaṣ” (n.d.), 2a.6, Fazil Ahmed Pasha, Süleymaniye Library.

⁷⁵ See, e.g., K. El-Rouayheb's introduction to Khūnajī, *Kashf al-asrār 'an ghawāmiḍ al-afkār*, xxxv: “...this being one of the few occasions on which Ḥillī expressed disagreement with his teacher.”

⁷⁶ Athīr al-Dīn Abharī, *Muntahā l-afkār fī ibānat al-asrār*, ed. Mahdī 'Aẓīmī and Hāshim Qurbānī (Tehran: Intishārāt-i Hikmat, 1395), 200.

⁷⁷ Ṭūsī and Athīr al-Dīn Abharī, “Ta'dīl al-mi'yār fī naqd Tanzīl al-afkār,” 143.13.

⁷⁸ Kātibī, “al-Munaṣṣaṣ,” 1b.23.

While Rāzī may have first planted the seeds of the solution, and Abharī watered and watched over it, it first came to full bloom by Tūsī’s tending.

Many later logicians cite this solution approvingly. In the generations immediately following Tūsī, Samarqandī, Ḥillī, and Iṣfahānī all argued for it.⁷⁹ None of them further develops the solution itself, but Samarqandī and Ḥillī each take an objection into account. The latter leaves his unanswered, while the former has a resolution. Although Taḥṭānī (d. 1365) does not make the solution his own, he poses yet another objection and then defends against it.⁸⁰ Ṣadr al-Sharī‘a al-Maḥbūbī (d. 1346) cites the solution from his Transoxianan forebear Samarqandī and levels a few criticisms against it.⁸¹

Perhaps Ṣadr al-Sharī‘a’s more significant contribution to the literature on the paradox of the absolute unknown, however, is a solution that I have not been able to trace any earlier, the accidents of obtaining in the mind solution. While he does not claim to be the first to have devised the solution, he does give it pride of place by mentioning it first and again (very briefly) last.⁸² It is also the only solution he does not subject to criticism. In addition to this solution, Ṣadr al-Sharī‘a proposes a modified version of Tūsī’s solution, deploying a distinction between “the time of passing the judgement” (*ḥāl al-ḥukm*) and “the time the judgement describes” (*ḥāl i’tibār al-ḥukm*).⁸³

Taḥṭānī may also have been responsible for developing a novel solution. He does not appear to claim credit for it but instead frames it as the “verification” (*taḥqīq*) of the solution that the base-text author, Urmawī, gives, namely the essential-existential proposition distinction solution.⁸⁴ Given how incredibly terse his formulation of it is—a single sentence—my suspicion is that he was not the first to have devised it. It is not until Jurjānī’s (d. 1413) gloss on Taḥṭānī’s *Lawāmi‘ al-asrār* that I have been able to find a thorough elaboration of the *bi-l-dhāt/bi-l-farḍ* (instances and supposition) solution.

The work of Taḥṭānī and Jurjānī is especially influential when seen in the light of the Mughal logical tradition. It is possible that Muḥibb Allāh al-Bihārī (d. 1707)—the author of one of the most commented upon logic text-books of the period,⁸⁵ *Sullam al-‘ulūm*—adopted Jurjānī’s position. Commentators differ on the point. Muḥammad ‘Alī al-Mubārakī al-Jawnfūrī (fl. 1709), Qāḍī Mubārak (d. 1749), and the later Baḥr al-‘Ulūm (d. 1810) read Bihārī such that he takes a solution of his own.⁸⁶ In contrast, ‘Abd Allāh Muḥammad al-Sā’infūrī (fl. 1707, likely the earliest commentator), Fīrūz b. Maḥabba (fl. 1707), Aḥmad ‘Abd al-Ḥaqq b. Muḥammad Sa‘īd b. Quṭb al-Dīn al-Sihālāwī (d. 1754), Mullā Ḥasan (d. 1784), and his student Mullā Mubīn (d. 1810) all

⁷⁹ Shams al-Dīn Samarqandī, *Kistāsu’l-efkār*, 83.8; Ḥillī, *al-Asrār al-khaṣṣiya*, 13.1; Iṣfahānī, “Tanwīr al-maṭālī’,” 14a.11-12.

⁸⁰ Taḥṭānī and Jurjānī, *Lawāmi‘ al-asrār*, 19.8-11.

⁸¹ Ṣadr al-Sharī‘a al-Maḥbūbī, “Sharḥ Ta‘dīl al-‘ulūm,” 15a.7, 15b.10.

⁸² Ṣadr al-Sharī‘a al-Maḥbūbī, 8a.11-23, 15b.22.

⁸³ Ṣadr al-Sharī‘a al-Maḥbūbī, 15a.21.

⁸⁴ Taḥṭānī and Jurjānī, *Lawāmi‘ al-asrār*, 19.27-29, 61.9.

⁸⁵ Asad Q. Ahmed, “The Sullam Al-‘ulūm of Muḥibballāh Al-Bihārī,” in *The Oxford Handbook of Islamic Philosophy*, ed. Khaled El-Rouayheb and Sabine Schmidtke (Oxford, 2016), 489. See also Asad Q. Ahmed, *Palimpsests of Themselves* (University of California Press, 2022).

⁸⁶ Muḥammad ‘Alī Mubārakī, “Mī‘rāj al-fuḥūm fī sharḥ Sullam al-‘ulūm” (n.d.), 15a.10, MS Rampur 3401; Ibn Muḥammad Dā‘im Qāḍī Mubārak, *Kitāb Sullam al-‘ulūm wa-ḥāshiyatuhu l-mashhūra bi-l-qāḍī ma‘a minḥiyātihī* (Kazan: al-Maṭba‘a al-malikiyya, 1887), 51.6; Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm*, 266.11.

interpret the passage in question as propounding Jurjānī's instances and supposition solution, although the latter three acknowledge the possibility of the other reading.⁸⁷ Neither Mubārakī, Qāḍī Mubārak, nor Baḥr al-ʿUlūm, however, mention the alternate interpretation.

The dispute is of particular interest because it is not so much an issue of interpretation as it is one of orthography. What Sāʾ infūrī, Mullā Fīrūz, ʿAbd al-Ḥaqq al-Sihālawī, Mullā Ḥasan, and Mullā Mubīn read as *fard* Mubārakī, Qāḍī Mubārak, and Baḥr al-ʿUlūm read as *ʿaraḍ*, the difference of one letter (*ʿayn* or *fāʾ*) making for the difference in solution. It is hard to be certain which is the reading Bihārī actually intended, but one reason to prefer *ʿaraḍ* (such that the solution is Bihārī's) over *fard* (such that it is Jurjānī's) occurs to me. Virtually all of the commentators note that, in his self-gloss, Bihārī promises further explanation of the problem in a later section. The brief discussion of the problem there seems to me to conform better with the essence and accident solution instead of the instances and supposition one.⁸⁸

Whatever the correct interpretation may be, few of the commentators are enamored of Jurjānī's solution. Qāḍī Mubārak is the most supportive, posing two objections and answering them both in his self-gloss. Based on how Baḥr al-ʿUlūm considers Bihārī's essence and accident solution as "artificial" (*takalluf*) and impotent, one suspects he wouldn't have been much kinder to the instances and supposition solution, though he makes no mention of it.⁸⁹ Both Mullā Ḥasan and Mullā Mubīn elaborate the solution, but neither takes much of a stance on it. They simply present it and then move on without raising any objections. All of the commentators take either a similarly disenchanted attitude to Bihārī's solution or an explicitly negative one.

Commentators of the *Sullam* seem to have been more interested in a related problem. Although the difficulty is nowhere mentioned by Bihārī, some commentators nonetheless raise a thought experiment inspired by the original absolute unknown paradox. If Mullā Ḥasan was not unconditionally the first to have posed it, he was almost certainly the first commentator of the *Sullam* to have done so. Among a couple other related problems he raises, he identifies the thought experiment as particularly resistant to solution. His formulation is somewhat idiosyncratic, involving two persons, instead of the later formulation involving only one, a further reason to suspect that Mullā Ḥasan may have been the problem's originator.⁹⁰

There is little agreement among commentators on a solution. Mullā Ḥasan explicitly rejects the instances and supposition solution, offering no replacement.⁹¹ Baḥr al-ʿUlūm considers two solutions, though he objects to the first, claiming it inadequate, and is non-committal to the second.⁹² Mullā Mubīn advocates the solution Baḥr al-ʿUlūm rejects and defends it against his objection, considerably enlarging on it in the process, and then rejects the solution Baḥr al-ʿUlūm appears ambivalent about, raising an objection.⁹³

⁸⁷ al-Sāʾ infūrī, "Sharḥ Sullam Al-ʿulūm," 12b.7; Muḥammad Fīrūz Ibn Maḥabbat, "Sirāj Wahhāj" (n.d.), 24b.6, MS Rampur 2149; Aḥmad ʿAbd al-Ḥaqq Al-Anṣārī al-Sihālī, "Suddat al-ʿulūm" (n.d.), 23.13, MS Rampur 3402; Sihālawī et al., *Mullā Ḥasan*, 25.4; Mubīn, *Mirʾāt al-shurūḥ*, 1321ah, 1:52.10, 52.18.

⁸⁸ Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, 502–3.

⁸⁹ Baḥr al-ʿUlūm, 267.9–10.

⁹⁰ Sihālawī et al., *Mullā Ḥasan*, 46.10–20. Despite the idiosyncrasy, the content is identical to that of the later formulation.

⁹¹ Sihālawī et al., 47.2–4.

⁹² Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, 268.12–269.

⁹³ Mubīn, *Mirʾāt al-shurūḥ*, 1321ah, 1:53.3, 53.7–9, 53.18–20.

Besides the oft discussed instances and supposition solution, the *Sullam* tradition seems to have mostly ignored earlier work on the problem. There is some mention of the essential/existential proposition distinction solution in Qāḍī Mubārak,⁹⁴ and Mullā Ṣadrā's primary/standard predication solution is quoted verbatim by Mubārakī,⁹⁵ but that is the extent of it. Indeed, the description/referent solution is, so far as I have seen, never raised.

From the first appearance of the paradox of the absolute unknown in Arabic in the 12th century up until the 19th century, there was steady innovation. Unsurprisingly, an explosion of activity ushers the problem into the tradition: within a century (13th), two lasting solutions are put forward (description-referent & essential/existential proposition). The following century sees standardization in the formulation of these solutions and their further development in the form of posing and resolving objections. Around the middle of the 14th century, two more independent solutions are proposed (accidents of obtaining in the mind & instances and supposition), and more objections are devised and resolved. A fifth solution is published three centuries later (primary/standard predication). A century after that, yet another solution is devised (essence and accident), and the thought experiment formulation of the paradox surfaces, yielding two more solutions and attendant objections and resolutions.

It should be no surprise that a third of the six significant solutions to the logical version of the paradox were formulated in the first century of the problem's life. By virtue of sheer novelty, a fresh difficulty is bound to attract more interest than one with a long history. Then there are the advantages of early investment: the solutions nearest to mind are immediately claimed by those given first access to the problem, leaving only those accessible by more difficult paths and therefore in reach only of those with more adventurous minds. Furthermore, a trail already cut is sure to dissuade a traveler from the trouble of making his own. That is, access to ready-made solutions doubtless stymies innovation, and not only insofar as it begets intellectual torpor. The mere fact of already having seen a solution to a problem will bias the mind and inhibit creativity, active though it may otherwise be.

With all of these challenges inherent to a philosophical tradition standing in the way of later logicians, they nonetheless exhibit great invention. Arguably the strongest solutions are those posed centuries after the problem's first appearance. (Here, some of the advantages of arriving late become clear: hindsight and accumulated experience). Only very late into the centuries-long investigation is the novel and rather creative metaphysical formulation of the paradox introduced. Considering that new formulations, solutions, or objections continued to be proposed through the 18th century and into the 19th, there can be little grounds for thinking that the Islamic intellectual tradition had gone entirely stale—at least not when it comes to this particular logical problem. And, indeed, even if this evidence cannot be immediately generalized so as to exculpate other intellectual endeavors, it should at least be cause for hope that they too escaped death by stagnation.

What is to be said by way of fairness is that the progress on the problem was relatively slow. Roughly seven centuries produced only two formulations, eight significant solutions, and slightly more objections. But, of course, speed of progress is relative. To what is it to be compared? I suspect that virtually any philosophical problem of similar complexity pursued over the same

⁹⁴ Qāḍī Mubārak, *Hāshiyat Sullam al-'ulūm*, 50.22.

⁹⁵ Mubārakī, "Mi'rāj al-fuhūm," 91a.21-91b.2.

length of time would allow little more innovation. Moreover, this study is hardly exhaustive: it excludes the vast majority of the western Islamic world.

Regardless, plodding progress has its merits. The fact that progress can be plodding at all demands a long duration of time, and in order for progress to be genuine over that period of time, there has to be continuity in the investigation—logicians must be discussing the same problem with substantively similar tools at their disposal. The alternative to ‘plodding progress,’ therefore, seems frequently to be false progress: a dazzling variety of philosophical work is done over a comparably shorter period of time, yet the investigation conducted by one philosopher is scarcely continuous with that of his predecessor insofar as basic assumptions underlying the investigation are as frequently exchanged for others as solutions are proposed. The plodding progress of the Islamic tradition is, therefore, rather scientific (in the modern sense of the term). Generation by generation, one brick after another is carefully inserted into the edifice of knowledge.

Appendix of Translations

Logical Formulation

Description-Referent Solution – Abharī & Ṭūsī, *Naqd Tanzīl al-afkār*

[Abharī]

Objection: Were (1) “Every object of judgement is known in some way” true, it would be the case that (2) “No absolute unknown is an object of judgement.” But (2) is false, since a contradiction follows if it is true, granting its own object of judgement is absolutely unknown; granting it is known in some way, then it still follows that (2) is false, since anything known in some way can be judged as being known in some way, and therefore (2)’s object of judgement can be judged as being known in some way.

My response: I do not grant that, were it known in some way, (2) would be false. And why, I ask, do you hold that it follows from “Anything known can be judged as being known in some way” that (2) is false? I ask because

(a) (2)’s object of judgment is known in some way

and

(b) Anything known in some way can be judged as being known in some way

produce the conclusion that

(c) (2)’s object of judgement can be judged as being known in some way

but (c) does not entail that (d) “‘No absolute unknown is an object of judgement’ is false,” seeing as how the subject-terms are different.

[Ṭūsī]

The scientific approach to the issue (*wa-l-tahqīq fī dhālik*) is as follows: “absolute unknown” can be taken in either of two ways: (A) only the referent of the utterance or (B) its referent along with its being described as absolutely unknown. According to (B), (2)’s object of judgement is not absolutely unknown, since its being described as unknown is itself something known, and something described by something known must, insofar as it falls under that description, also be known. Hence, the object of judgement in “No absolute unknown is an object of judgement,” insofar as it is impossible to pass a judgement on it, is to be taken in accord with (A). Insofar as (2)’s object of judgement can be judged as being impossible to pass a judgement on, it is to be taken in accord with (B). This is the meaning of ‘the subject-terms being different.’⁹⁶

Description-Referent Solution – Jurjānī, *Hāshiyat Sharḥ al-Maṭāli‘*

The second solution is that “absolute unknown”—i.e. some object falling under the description “unknown”—can be considered from two respects. The first is the object

⁹⁶ Ṭūsī and Athīr al-Dīn Abharī, “Ta’ dīl al-mi’yār fī naqd Tanzīl al-afkār,” 143.

considered from that respect—i.e. from the respect of falling under the description “unknown”—*and the second is the object considered not from that respect. The judgement [in “Absolute unknowns cannot be subjects of predication”] that it is impossible to pass a judgement therefore can also be considered from two respects. The first is that the judgement can be made, and the second is that it cannot.* The fact that the judgement can be made is grounded in considering “absolute unknowns” according to the first respect, and the fact that it cannot be made is grounded in considering it according to the second. *Hence, the subject-term in both*—i.e. in the statements, “No absolute unknown can be a subject of predication” and “Some absolute unknowns can be subjects of predication”—*differ according to the respect and there is therefore no contradiction between them except by way of being contradictories, but in no other way.*⁹⁷

Were one to object, “This solution demands that the fact that an object falls under the description ‘unknown’ is the cause for the fact that the judgement [in “Absolute unknowns cannot be subjects of predication”] can be made rather than the cause for the fact that it cannot be made, yet the reverse is so,” we would respond: The author means that the judgement [“Absolute unknowns cannot be subjects of predication”] can be made and is not impossible when [“absolute unknowns”] is considered from the respect that it is known insofar as it falls under the description “unknown,” and that the judgement cannot be made when [“absolute unknowns”] is not considered from the respect that it is known from that respect. The gist of the solution is that [the judgement in “Absolute unknowns cannot be subjects of predication”] can be made due to the fact that “absolute unknowns” is considered from the respect of being known by falling under the description of “being unknown,” and [the judgement] cannot be made due to the fact that the object falls under that description [namely, “unknown”].

Don’t you see that the author said firstly that “being unknown” is something known about something and then said secondly that, from that first respect, [“absolute unknowns”] is known. The author considered [“absolute unknowns”] to be known insofar as they fall under the description “unknown.” Considering [“absolute unknowns”] from this respect makes the fact that [“absolute unknowns”] fall under [the description “unknown”] the cause for the fact that the judgement [“Absolute unknowns cannot be subjects of predication”] can be made. If one considers [“absolute unknowns”] in abstraction from the fact that they are, by virtue of falling under the description “unknown,” thereby known, then they are absolutely unknown, as the author says explicitly: *Something falling under the description “unknown” can be considered as “known” only by virtue of consideration of that very fact.* It is this “being unknown” [abstracted from being known insofar as it falls under the description “unknown”] that is the cause for the fact that the judgement [“Absolute unknowns cannot be subjects of predication”] cannot be made.⁹⁸

Essential/Existential Proposition Distinction Solution – Khūnajī, *Kashf al-asrār*

⁹⁷ Formally speaking, these two statements are contradictories, since one is a universal negative and the other a particular affirmative.

⁹⁸ Taḥṭānī and Jurjānī, *Lawāmi‘ al-asrār*, 58.14-58.28.

The response is that if the consequent in “If being a subject of predication requires being known somehow, then it is true that absolute unknowns cannot be subjects of predication” were taken as an affirmative existential proposition, it would be false, since no such subject-term exists concretely by virtue of the necessary fact that every existent is known in some respect, if only insofar as it is an “existent” or a “thing.” Therefore, the truth of the conditional would not follow. If the consequent were instead taken as an essential proposition, it would be true without the previously mentioned⁹⁹ contradiction arising, as would be the case were it taken as a negative proposition, whether essential or existential.¹⁰⁰

Accidents of Obtaining in the Mind Solution – Ṣadr al-Sharī‘a, *Sharḥ Ta‘dīl al-‘ulūm*

When one has come to know that if that which obtains in the mind corresponds—in the sense earlier explained—then it is true, one knows that it is that which corresponds that is that which obtains in the mind. That which obtains in the mind is the object of the accidental properties of the [process of something’s] obtaining in the mind: this [process] of obtaining is accidental to that object [in the mind]. But that which corresponds is the object which obtains in the mind, and no consideration should be given to the accidents [resulting from obtaining in the mind] in determining whether or not the object corresponds.

Hence, if we conceive of some universal or of “absolute unknown,” in a way such that it corresponds, then there can be no doubt that the universal will have become an individual due to its coming to be in the mind, and “absolute unknown” will have become known. However, these accidental properties of “being an individual” and “being known” [which are a result of coming to be in the mind] do not negate the fact that the object in the mind is “universal” or “absolutely unknown,” since the universal is that which has obtained in the mind independent of the [process] of coming to obtain in the mind being included in it. Likewise is the case of “absolute unknown”: when one conceives of “absolute unknown,” the [accidental properties resultant from] conceiving of it are not included in it.

Thus, in “Absolute unknowns cannot be subjects of predication,” “cannot be subjects of predication” is predicated of the object [in the mind] and the accidental property of “being known” [that results from the process of obtaining in the mind] does not negate that, since the object [in the mind] obtains in the mind insofar as it is in itself, namely “absolute unknown,” and the accidental property [viz., “being known,” which is a consequence of coming to be in the mind] is not included in that object. Therefore, there is no contradiction.¹⁰¹

Instances and Supposition Solution – Jurjānī, *Hāshiyat Sharḥ al-Maṭāli‘*

As for explaining “‘Perpetual absolute unknowns’ are known by means of its instances and absolutely unknown by means of supposition”: If one asserts, “Perpetual absolute

⁹⁹ I.e., the contradiction the paradox yields.

¹⁰⁰ Khūnajī, *Kashf al-asrār ‘an ghawāmiḍ al-afkār*, 10.1-6.

¹⁰¹ Ṣadr al-Sharī‘a al-Maḥbūbī, “Sharḥ Ta‘dīl al-‘ulūm,” 8a.11-23.

unknowns are such-and-such,” there can be no doubt that, by means of the concept of the subject-term’s description, the mind turns to its instances, making the concept a mirror for observing them in a universal, undifferentiated (*ijmālī*) fashion. Hence, it is certain that, in this fashion, the instances are known. The instances are the referents of “perpetual absolute unknowns,” and so the referents must be known by virtue of the fact that they fall under the description “perpetual absolute unknowns.” The fact that the referents are thereby known is known necessarily. If, however, the subject-term’s referents are known in some way, they must not be perpetually absolutely unknown according to how things are in themselves (*fi nafs al-amr*) but rather only according to the mind’s supposition, by means of which it conceives of them as falling under the concept “perpetual absolute unknown.” The judgment that these referents “cannot be subjects of predication” is true by virtue of the fact that they can be considered known, and the denial of the same judgement is true in virtue of the fact that they can be supposed to fall under the description “perpetual absolute unknowns.”

If you were to object, “If the referents of ‘perpetual absolute unknowns’ are known to the mind, how is it correct to judge of them that it is impossible for them to be subjects of predication, in light of the fact that being known demands being capable of being a subject of predication?” I would respond: Even if the referents of “perpetual absolute unknowns” are known to the mind, the mind does not attend to them insofar as they fall under the description “known” but insofar as they fall under the description “unknown.” The gist of this is that the concept “perpetual absolute unknowns” is a universal concept, and—just as it can for any other universal concept—the mind can make “perpetual absolute unknowns” such that it is perceived in itself, independent of anything else (*li-l-‘aql an yaj‘alahu malhūzan bi-l-dhāt*) or it can make it such that it is a mirror for perceiving its particulars (*an yaj‘alahu mir‘ātan li-mulāḥaẓat al-juz’iyyāt*).

If the mind makes “perpetual absolute unknowns” a mirror for its particulars, it will perceive them insofar as they fall under the description of the concept, which perception is precisely what causes it to be impossible for them to be subjects of predication, and therefore the mind judges it impossible to make the judgement [namely, “Perpetual absolute unknowns cannot be subjects of predication”]. The particulars of “perpetual absolute unknowns” become known in consequence of perceiving them as falling under the concept’s description, but they are not then [in the time of negating the judgement that “Perpetual absolute unknowns cannot be subjects of predication”] yet perceived by the mind as falling under the description “known,” since, in order to be perceived as so “known,” the mind must make a second perception in addition to the first, namely that the particulars of “perpetual absolute unknowns” fall under its description. Hence, once the mind so perceives them—i.e. perceives them as being known—it judges that one can predicate of them that they cannot be subjects of predication, since it would indeed be impossible (*ḥakama ‘alayhā bi-ṣiḥḥat al-ḥukm li-mtinā’ih*).¹⁰²

Primary/Standard Predication Solution – Mullā Ṣadrā, *al-Ḥikma al-muta‘āliya fi l-asfār al-arba‘a*

¹⁰² Taḥtānī and Jurjānī, *Lawāmi‘ al-asrār*, 60.3-10.

I say: the intellect can conceive of anything, even impossibilia, like ‘absolute non-existent,’ ‘absolute unknown,’ ‘the coincidence of contradictories,’ ‘God’s equal,’ etc. insofar as they are concepts. It can then make appropriate judgements about these, forming affirmative essential propositions (*haliyyāt ghayr battiyya*). The subject-terms of such propositions can be validly predicated of insofar as they are concepts in a mind, have some kind of existence, and can be termed ‘things,’ ‘possibilia,’ or even ‘accidents,’ ‘qualities of the soul,’ ‘knowledge,’ and similar things. Insofar as these subject-terms are non-referring descriptions (*‘unwān li-umūr bāṭila*), however, it becomes impossible to make any judgement about them. When these two ways of considering such subject-terms are taken together, one can judge that such subject-terms are not predicated of, not judged, do not exist, etc.

It is this explanation that does away with the famous paradox, “Absolute unknowns cannot be predicated of.” The secret to it is that some concepts are true of themselves by primary predication, yet not true of themselves by standard predication: the concept ‘God’s equal’ or ‘absolute unknown,’ is true of itself according to only one of the two kinds of predication, while its contradictory is true of it according to the other; and this is what makes valid the judgement that such concepts cannot possibly exist.¹⁰³

Essence and Accident Solution – Baḥr al-‘Ulūm, *Sharḥ Sullam al-‘ulūm*

*The solution is that “absolute unknowns” is known essentially, due to its form obtaining in the mind. According to this respect, it can be a subject of predication. And absolutely unknown accidentally, due to its applying to unknown things. Therefore, it can be a subject of predication according to one respect and cannot be according to the other.*¹⁰⁴

Metaphysical Formulation

Presence in the Mind is not Sufficient for Knowledge – Mullā Mubīn, *Mir’āt al-shurūḥ*

There is here another doubt to be raised: Suppose that, e.g., Zayd has the concept “absolute unknown” at the very beginning of the material stage of the intellect’s development, and that his mind is otherwise empty of all concepts. Things would then have to be either (a) known or (b) unknown to him.

If they are (a) known, they could be known by him in no way besides by intermediary of the concept “absolute unknown,” since it was assumed that his mind lacks every other concept besides it. Therefore, if they are known by means of that concept, the concept will be true of them, and whatever “absolute unknown” is true of is unknown—but then things would have to be unknown despite the fact that we assumed them to be known. This is absurd.

¹⁰³ al-Shīrazī, *al-Aṣfār*, 1:239.3-40.2.

¹⁰⁴ Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm*, 266.9-12. Bihārī’s base-text is in italics.

If they are (b) absolutely unknown, then “absolute unknown” will be true of them, and will be one of their aspects.¹⁰⁵ This aspect will then obtain in Zayd’s mind, and so things will become known insofar as they are known by that aspect—despite the fact that they are absolutely unknown. This is absurd.

Some have responded as follows: Something is known when one of its aspects obtains in the mind so long as that aspect is not contrary to the thing being known. Were it contrary to that, how could it serve as the source for making the thing known? In light of this, we hold that things would be absolutely unknown to the person, the concept “absolute unknown” being true of them, and one of their aspects. But it does not follow from the fact that this aspect of theirs obtains in his mind that they become known to him, since the aspect contravenes their becoming known.

The objection, “We call what has obtained in the mind in itself or by means of one of its aspects ‘known,’ and the contradictory ‘absolutely unknown.’ If you like, call these two concepts by other names, but there can be no doubt that the two are contradictory and that they lead to a contradiction according to the explanation given” amounts to nothing. For something to be known is for it to be known in itself or by means of one of its aspects. However, something’s aspect obtaining in the mind with no relation or consideration of the fact that it is that thing’s aspect does not make that thing thereby known. As some verifiers have said in refuting partial knowledge [lit. knowledge of an aspect], such knowledge is not tantamount to knowledge of the thing that possesses the aspect. Something that obtains in the mind in itself or by means of one of its aspects is called “known” only insofar as the aspect belongs to the thing. But when Zayd has the concept “absolute unknown” at the beginning of the material stage of his intellect’s development, he is attending neither to anything else nor to the fact that such a thing is known by means of this aspect (“absolute unknown”). The fact that “absolute unknown” would be, as a matter of fact, an aspect of things, would be insufficient to say Zayd knows them, especially since “absolute unknown” is contrary to their becoming known...¹⁰⁶

¹⁰⁵ “Aspect” is a translation of *wajh*, which, in this context, means “accidental property.”

¹⁰⁶ Mubīn, *Mir’āt al-shurūh*, 1321ah, 1:52.23-53.15.

The Liar Paradox

Formulations of the Problem

There are more ways to formulate the Liar paradox than there are to formulate any other paradox here studied. Several are avenues for insight into the nature of the problem. Others, however, are of less philosophical value, it seems to me. These latter are intended merely as a way to prevent misunderstandings, a way to make the crucial self-referential nature of the paradox sentence undeniably explicit. Of this kind are the “time and space” formulations given below.

Single Sentence with Demonstrative Pronoun

Someone says, “This very sentence is false,” referring to that sentence. It appears to be the case that if the sentence is true, then it must also be false. For it to be true means that its content accurately reflects reality, and its content is that it is false—yet we assumed that it was true in reality, which amounts to a contradiction. If the sentence is instead assumed false, then it must also be true. For a sentence to be false is for its content to lack correspondence with reality, yet the sentence’s content corresponds with what we assumed to be the case, namely that the sentence is false, which is to say that it is, contrary to assumption, true.

Almost all of the following formulations are either explicitly or implicitly paired with this same reasoning for why a paradox results, and hence I do not repeat it.

Time and Space

Someone says, “Everything I say this hour is false,” and then says nothing else that hour.¹⁰⁷

Someone steps into a house and, after rattling off one false sentence, says, “Everything I say in this house is false” and then steps out.¹⁰⁸

A life-long liar says on his death-bed, “Everything I’ve said is false,” and then gives up the ghost without uttering another word.¹⁰⁹

Someone who has never once lied says, “I am a liar.”¹¹⁰

¹⁰⁷ Shams al-Dīn Samarqandī, *Kistāsu’l-efkār*, 551.11-12; Tūsī and Athīr al-Dīn Abharī, “Ta’ dīl al-mi’yār fī naqd Tanzīl al-afkār,” 235.15-16; Abharī, *Khulāṣat al-afkār*, 382; Najm al-Dīn Kātibī and Afḍal al-Dīn Khūnajī, “Sharḥ Kashf al-asrār” (Istanbul, n.d.), 226a.8, Carullah 1418, Süleymaniye Library.

¹⁰⁸ Abharī, *Kashf al-ḥaqā’iq*, 217.4-6; ‘Izz al-Dawla Sa’d Ibn Kammūna, *al-Kāshif: al-Jadīd fī l-ḥikma*, ed. Hāmid Nājī Iṣfahānī (Tehran: Mu’assasa-yi Pizhūheshī-yi Ḥikmat va Falsafah-yi Īrān, n.d.), 74.ultimate-75.1.

¹⁰⁹ Abharī, *Kashf al-ḥaqā’iq*, 216.16-17.

¹¹⁰ Fakhr al-Dīn Muḥammad b. ‘Umar al-Rāzī, *al-Maḥṣūl fī ‘ilm uṣūl al-fiqh*, ed. Tāhā Jābir Fayyād ‘Alwānī, vol. 4 (Beirut: Mu’assasat al-risāla, 1992), 291.7-14; Shams al-Dīn Khafīrī, “Ḥayrat al-fuḍalā’,” in *Davāzdah risāla dar pāradūks-i durūghū*, ed. Aḥad Farāmarz Qarāmalekī and Tayyibah ‘Ārifniyā (Tehran: Mu’assasa-yi Pizhūheshī-yi Ḥikmat va Falsafah-yi Īrān, 2007), 274.7-8; ‘Abd-al-Qāhir b. Tāhir al-Baghdādī, *Uṣūl al-dīn*, ed. Aḥmad Shams al-Dīn (Beirut: Dār al-Kutub al-‘Ilmiyya, 2002), 28.3.

Circular

One day someone says only, “What I will say tomorrow is false,” and then the next day says only, “What I said yesterday is true.”¹¹¹

Someone says at 1 o’clock, “What I will say at 2 o’clock is true.” At 2 o’clock he says, “What I will say at 3 o’clock is true,” and at 3 o’clock he says, “What I said at 1 o’clock is false.”¹¹²

One person says of a second, “What he says is false.” The second says of a third, “What he says is false,” and the third says of the first, “What he says is true.”¹¹³

Infinite

Consider an infinite line of people. Each says of the next, “What he says is false.” Assuming that all of these sentences are neither true nor false, a contradiction will follow. The content of the first sentence, for example, is that the second is false. This content, however, does not correspond with the fact of the matter, since we assumed that all the sentences—including the second—are neither true nor false. Therefore, the first sentence ought to be false instead of being neither true nor false, since its content does not correspond to reality, which is just what it is for a sentence to be false. This is a contradiction.

If we consider the second and third sentences, matters will be worse still. According to reasoning similar to the above, it can be argued that the second sentence is false: it says of the third that it is false, yet the third was assumed to be neither true nor false, and therefore, the second ought to be false, since it incorrectly describes the third as false. If it is granted that the second is indeed false, then the first must be true, contrary to what we just argued. The first says that the second is false, which our most recent line of reasoning shows must be so. Therefore, the first must be true, since it correctly describes the fact that the second sentence is false. In sum, the first sentence is neither true nor false according to assumption, yet apparently sound reasoning shows that it must be both true and false.¹¹⁴

Although a distinct problem, the Truth Teller is worth mentioning here, considering how closely related it is to and often it was discussed alongside the Liar: “This sentence is true.”¹¹⁵ Contrary to the Liar, the Truth Teller entails no contradiction. Rather, the problem is that bivalence seems to be violated: there are no clear grounds on which the Truth Teller should be true or false—yet every statement must be one or the other.

¹¹¹ Taftāzānī, *Sharḥ al-Maqāṣid*, 4:287.1-4.

¹¹² Hassan Rezakhany, “Jalāl ad-Dīn ad-Dawānī’s Solution to the Liar Paradox and Its Reception in Qāḍī Mubārak and Mullā Mubīn,” *Journal of South Asian Intellectual History* 1, no. 2 (2018): 189n28.

¹¹³ Aḥad Farāmarz Qarāmalikī, “Āqā Ḥusayn-i Khwānsārī va-mu‘ammā-yi durūghū,” *Kheradnāmah-yi Šadrā*, no. 10 (Winter 1376): 85.19.

¹¹⁴ Farāmarz Qarāmalikī, 85.8-18.

¹¹⁵ Rezakhany, “Jalāl ad-Dīn ad-Dawānī’s Solution,” 210.

Attempted Solutions and Objections Thereto

While every solution I have encountered is explicitly directed at one or two formulations in particular, a frequent objection was that a proposed solution could not accommodate other formulations.¹¹⁶ This seems to indicate that philosophers understood solutions as intended to solve the paradox generally and not only one formulation of it—even if they weren’t successful. In principle, they expected their solutions to satisfy all formulations.¹¹⁷ Therefore, I have not grouped the solutions according to formulation.

1. *No contradiction obtains if it is false*

A given Liar sentence must be either true or false. If we suppose it is true, it can easily be shown that it must also be false: if “Everything I say this hour is false” is true, and the speaker utters nothing more, then the very same sentence must also be false, since it truly describes itself as false. That is to say, the truth of the sentence is tantamount to the contradictory conjunction of its being true and its being false.

By contrast, no contradiction follows from the assumption that the sentence is false. It is well known that for a sentence to be false is for its truth to be negated. Since the Liar sentence’s truth is tantamount to the conjunction of its truth and falsehood, its falsehood must imply the negation of that same conjunction, namely the exclusive disjunction of its falsehood and truth. Therefore, if the Liar sentence is false, it does not imply that it is also true, since the disjunction could be true with only the false disjunct being true.¹¹⁸

Objection: Fallacious reasoning

“[T]he truth of the [Liar] sentence is tantamount to the conjunction of its being true and its being false” could mean either (1) that its truth is intensionally identical with the conjunction of its being true and its being false or (2) that its truth entails that it is false as well. (1) simply could not be.¹¹⁹ (2), therefore, must be the correct interpretation.

If (2) is the correct interpretation, however, the solution relies on a fallacy. Let p be “The Liar sentence is true” and q be “The Liar sentence is true, and the Liar sentence is false.” According to interpretation (2)—namely that the Liar sentence’s being true *entails* that it is also false—we

¹¹⁶ See, e.g., Muḥammad b. ‘Abd al-Fattāḥ al-Tunkābunī Māzandarānī, “Risāla fī jidhr al-aṣamm” (n.d.), 30a.8-10, MS Rampur 574.

¹¹⁷ Indeed, Khafīrī says just that: “The true solution [to this paradox] will work for all possible formulations, or something similar to it will.” Khafīrī, “Ḥayrat al-fuḍalā’,” 289.3-4.

¹¹⁸ Abharī, *Kashf al-ḥaqā’iq*, 217.13-218.7; Ṭūsī and Athīr al-Dīn Abharī, “Ta’dīl al-mi’yār fī naqd Tanzīl al-afkār,” 236.7-9; Kātībī and Khūnajī, “Sharḥ Kashf al-asrār,” 226a.21-24. Cf. Khaled El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz: The Exchange Between Ṣadr al-Dīn al-Dashtakī and Jalāl al-Dīn al-Dawānī,” *British Journal for the History of Philosophy*, 2019, 4. Cf. Ahmed Alwishah and David Sanson, “The Early Arabic Liar: The Liar Paradox in the Islamic World from the Mid-Ninth to the Mid-Thirteenth Centuries CE,” *Vivarium* 47 (2009): 107–13.

¹¹⁹ Ṭūsī gives no justification for why interpretation (1) could not be what Abharī meant or—granting it is what he meant—why it could not be true. Zarepour’s interpretation of the solution crucially relies on assuming that (1) is what Abharī meant. See the summary and analysis section below for more on this issue of exegesis, in light of which I have tried to keep my presentation open-ended.

assume that $p \rightarrow q$. Furthermore, let us assume that $\sim p$ (i.e., that the Liar sentence is *not* true). Just because $p \rightarrow q$, it does not follow that $\sim p \rightarrow \sim q$, but this is the crucial step of the foregoing solution. It would have us believe the following:

- | | |
|---|-----------|
| 1. $(p \rightarrow q) \rightarrow (\sim p \rightarrow \sim q)$ | assumed |
| 2. $p \rightarrow q$ | assumed |
| 3. $\therefore \sim p \rightarrow \sim q$ | (1) & (2) |
| 4. $\sim p$ | assumed |
| 5. $\therefore \sim q$ (i.e., no contradiction need follow from the Liar's being false) | (3) & (4) |

Yet (1) is far from being obviously true. Indeed, what does follow from $\sim p$ (namely the assumption that the Liar sentence is *not* true), is that it is false, which further entails that the sole instance of the subject-term, “Everything I say this hour,” namely, the Liar sentence itself, is not false, as it claims itself to be, but rather true. This is just to say, however, that if the Liar sentence is assumed false, it follows that it is true, and the problem remains.¹²⁰

Resolution: Re-formulation

Perhaps the solution should not be interpreted as it was in the previous objection. Instead, consider it this way: ‘true’ and ‘false’ are contradictories, such that the application of one follows from the negation of the other. But, in the case of the Liar sentence, the opposite is the case: truth entails falsehood. Therefore, it ought not follow that the Liar sentence will be true from the fact that it is not false. Let p be “The Liar sentence is true,” and r be “The Liar sentence is false.” Hence, we know $p \rightarrow r$ (if the Liar sentence is true, then it is false) and we know $\sim r$ (the Liar sentence is not false). Yet it is a fallacy to conclude therefrom that p (“the Liar sentence is true”), since it does not follow from $p \rightarrow r$ that $\sim r \rightarrow p$. Indeed, all that follows is $\sim p$, since $p \rightarrow r$ implies $\sim r \rightarrow \sim p$, and we assumed $\sim r$. In sum, it is absurd for the Liar sentence to be true, since that entails that it is also false, but it is not absurd for it to be false, since that only entails that it is not true ($\sim p$).¹²¹

Objection: The Liar's being false does entail its being true

Just like the Liar's being true entails its being false, its being false entails its being true. If it is false, then the report it makes about itself that it is false must be true, and the difficulty resurfaces.¹²²

Resolution: From the relevant respect

It is true that the Liar's being true—insofar as it is a proposition—entails its being false—insofar as it is what that proposition represents. But that is beside the point. The point was instead that the

¹²⁰ Ṭūsī and Athīr al-Dīn Abharī, “Ta’ dīl al-mi’yār fī naqd Tanzīl al-afkār,” 236.10-14. The objection is Ṭūsī’s.

¹²¹ Ṭūsī and Athīr al-Dīn Abharī, 236.15-20.

¹²² Ṭūsī and Athīr al-Dīn Abharī, 236.21-3; Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 30.4-7.

Liar's being false—insofar as it is a proposition—does *not* entail that it is true. Rather, it entails that it is *not* false—insofar as it is what the proposition represents—and that it is false—insofar as it is a proposition. We do not grant, however, that denying the Liar is false would entail that it is true in this case.¹²³

Objection: The Liar's being false is also tantamount to the conjunction of its being true and false

If, as the solution proposes, we grant that the Liar sentence's being true is tantamount to its being true and its being false, the same should be the case for the Liar sentence's being false. That is, if the Liar sentence's being true is “tantamount to the conjunction of its being true and its being false,” the Liar sentence's being false should be tantamount to the same thing, in which case the usual absurdity resurfaces: its being false entails its being true and false, and its being true entails its being true and false. Otherwise, it would have to be a statement that is neither true nor false, which is also absurd, since a statement is just that sentence which must be either true or false.¹²⁴

Objection: What it actually is for the Liar to be true

The Liar's being true is simply for its predicate (“false”) to apply to its subject-term (the sentence itself), not “the conjunction of its being true and false.” Therefore, its being false is just the opposite, namely that its predicate does not apply to its subject-term. Hence, the same contradiction as before will resurface.¹²⁵

2. *Self-referential statements are neither true nor false*

‘True’ and ‘false’ apply only to those propositions that are distinct from what they represent. Truth is correspondence, and for correspondence between a proposition and what it represents to be conceivable, there must be duality: the proposition must be distinct from what it represents. Without duality, such a proposition would lack the usual features of a proposition. It would, for example, have no contradictory, since were the contradictory of what the proposition represents actualized, there would be no proposition at all (considering that what a self-referential proposition represents is nothing other than itself)! If the proposition represents itself, and what it represents were non-existent, there could be no proposition.¹²⁶

Objection: Obviously True Self-referential Statements

The fact is that correspondence does not demand duality. Consider the sentences, “This statement is compound,” “Every statement is either true or false,” “Everything I say this hour is in Arabic,” or “This sentence is a performative.” All are self-referential, yet all are obviously either true or

¹²³ Ṭūsī and Athīr al-Dīn Abharī, “Ta’ dīl al-mi’yār fī naqd Tanzīl al-afkār,” 237.1-4.

¹²⁴ Dawānī, “Nihāyat al-kalām fī shubhat kull kalāmī kādhīb,” 107.12-18.

¹²⁵ Khafrī, “Ḥayrat al-fuḍalā’,” 277.6-9.

¹²⁶ Ṭūsī and Athīr al-Dīn Abharī, “Ta’ dīl al-mi’yār fī naqd Tanzīl al-afkār,” 237.5-17; Aḥad Farāmarz Qarāmalikī, “Mu’ammā-yi jidhr-i aşamm nazd-i mantiqdānān-i qarn-i haftom,” *Kheradnāmah-yi Šadrā*, no. 7 (Spring 1376): 74–75; Alwishah and Sanson, “The Early Arabic Liar,” 113–23; Mubīn, *Mir’āt al-shurūḥ*, 1321ah, 1:101.21-25.

false. Therefore, the problem with Liar sentences could not be merely that they are self-referential.¹²⁷

Objection: Self-referential statements can have correspondence

It is not a condition on the kind of correspondence involved in statements for the statement to be non-self-referential.¹²⁸

Objection: Strengthened Liar

If the Liar is neither true nor false, it follows that it is also false, since it describes itself as “false,” which, according to the assumption that the Liar is neither true nor false, is inaccurate. Therefore, it is both (1) neither true *nor* false and (2) false, and the usual contradiction reappears.¹²⁹

Objection: Not all formulations are satisfied

Consider the morning-evening formulation, where someone in the morning says only, “Everything I will say this evening is true,” and in the evening says only, “Everything I said this morning is false.” Each proposition is distinct from what it represents, and therefore, according to the proposed solution, both sentences ought to be unproblematically true or false, yet this is not so.¹³⁰

3. *Self-referential statements are false*

The Liar sentence is self-referential and therefore could not be true. Since (i) truth is correspondence between the statement and what it describes, (ii) the Liar sentence is self-referential and is therefore both statement and what it the statement describes, and (iii) these two must be distinct in order for there to be correspondence between them, the Liar could not be true. Since it could not be true, it must therefore be false.¹³¹

Objection: It follows therefrom that the Liar is also true

It is easy to see that, assuming that the Liar sentence is false, it would also have to be true, which would leave us with the same contradiction as before devising this solution.¹³²

¹²⁷ Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 34.2-7; Khafīrī, “Ḥayrat al-fuḍalā’,” 137. antepenultimate-138.2; Farāmarz Qarāmalikī, “Mu’ammā-yi jidhr-i aṣamm nazd-i mantiqdānān-i qarn-i haftom,” 75. The third sentence, “Everything I say this hour is in Arabic,” is found in Māzandarānī, “Risāla fī jidhr al-aṣamm,” 30a.11-12. The fourth, “This sentence is a performative,” is found in (author unknown) “Risāla fī jidhr al-aṣamm” (n.d.), 116.19, MS Rampur 1782.

¹²⁸ Khafīrī, “Ḥayrat al-fuḍalā’,” 279.4-5.

¹²⁹ Khafīrī, 279.5-7.

¹³⁰ Māzandarānī, “Risāla fī jidhr al-aṣamm,” 30a.8-10.

¹³¹ Ibn Kammūna, *al-Kāshif*, 75.6-8.

¹³² Ibn Kammūna, 75.9.

Resolution: It follows only if correspondence were possible

The Liar is not false in the sense that there is a statement and a distinct object it describes, but they do not correspond. Rather, it is false only in the sense that there is no correspondence whatsoever, because the statement is not distinct from what it describes.¹³³

4. *Statements can be neither true nor false*

The Liar sentence is only problematic because we think it must be either true or false. Although the Liar sentence is indeed a statement, statements need not be either true or false. Some are neither, and it just so happens that the Liar sentence is one such statement that is neither true nor false. Furthermore, one should note that denying that statements must be either true or false in no wise eliminates the distinction between statements and all other sentences: a statement is a sentence which *can* be either true or false, though it *need* not be. No other sentence has such potential.¹³⁴

Objection: Insufficient—no way to distinguish

Since, according to the solution, a sentence need only have the potential for being true or false in order to be a statement, any statement could be neither true nor false. How can we distinguish which are only either true or false, such that the fact that one is true implies that it is not false, and which are neither? What is it that makes the Liar sentence neither true nor false?¹³⁵

5. *Two Sentences, So No Contradiction*

The Liar sentence, (1) “Everything I say this hour is false,” really represents two sentences. One is the sentence itself, and the other is (2) “(1) is false.” If (2) is true, then (1) is false, and if (1) is true, then (2) is false. That the truth or falsehood of either is entailed by the other’s truth or falsehood is no contradiction, however, since there are two statements, and a contradiction occurs only when one statement is both true and false. Moreover, (1) is simply false. It follows that some of the instances of (1)’s subject-term must therefore be true, and it is (2) that is just such an instance that will always be true.¹³⁶

Objection: Only one sentence

¹³³ Ibn Kammūna, 75.9-11.

¹³⁴ Khafrī, “Ḥayrat al-fuḍalā’,” 119–20; Farāmarz Qarāmalikī, “Mu‘ammā-yi jidhr-i ašamm nazd-i mantiqdānān-i qarn-i haftom,” 77.penultimate (lhs)-78.8rhs. For a later author (Fakhr al-Dīn al-Sammākī, d. 1577) who held the same position, see El-Rouayheb, *The Development of Arabic Logic (1200-1800)*, 118.

¹³⁵ Dashtakī, “Risāla fī ḥall shubhat jidhr al-ašamm,” 35–36.

¹³⁶ Shams al-Dīn Samarqandī, *Kistāsu’l-efkār*, 553.1-7; Khafrī, “Ḥayrat al-fuḍalā’,” 112; Farāmarz Qarāmalikī, “Mu‘ammā-yi jidhr-i ašamm nazd-i mantiqdānān-i qarn-i haftom,” 78.5-ultimate (lhs); El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 5.

The solution depends upon the assumption that there are in reality two sentences, which is obviously false. There is only one sentence.¹³⁷

6. *Predicate of subject-term, predicate of proposition*

A truth-value can be predicated of a subject-term or it can be predicated of a proposition. A contradiction occurs only in the case that “true” and “false” are predicates of the same subject-term or predicates of the same proposition. If we assume that “This sentence is false” is false, it only follows that the sentence’s contradictory, namely “This sentence is true” is true. Hence, “false” would be a predicate of a proposition (“This sentence is false”), “true” a predicate of a subject-term (“this sentence” in “This sentence is true”). The outcome would be similar if we assumed “This sentence is false” is true.¹³⁸

7. *Sentences producing what they purport to describe are not statements*

The Liar sentence is not the kind of utterance which is capable of being true or false, since it does not report about anything real. Something is “real” when it subsists independent of being described by a statement. But the state of affairs the Liar sentence purports to describe (that it itself is false) is generated precisely by the attempt at description: it is only by virtue of its own description of itself (that it is false) that the state of affairs obtains or not (that it is false or not). Therefore, the Liar could be neither true nor false.¹³⁹

Objection: Strengthened Liar

For a sentence to be false is for it to inaccurately describe the fact of the matter. The Liar sentence describes itself as false. If, as this solution claims, the Liar sentence is neither true nor false, it follows that it must therefore be false, since it inaccurately describes the fact of the matter. The usual contradiction returns.¹⁴⁰

Resolution: No Implication

The Liar sentence is not capable of being either true or false. It does not satisfy the conditions under which the contraries ‘true’ or ‘false’ could apply to it, namely being a statement. Hence, it does not follow from the fact that it is not false, that it is true, much in the same way that it does not follow from the fact that a boy is not bearded that he is clean-shaven: a boy cannot grow facial hair and hence could not be either bearded or clean-shaven; he is lacking the condition according to which the contraries ‘bearded’ and ‘clean-shaven’ could apply to someone, namely being a mature male. Like ‘bearded’ and ‘clean-shaven,’ ‘true’ and ‘false’ are contraries, and therefore

¹³⁷ Khafrī, “Ḥayrat al-fuḍalā’,” 114.1-6. The objection is also Dashtakī’s.

¹³⁸ Taftāzānī, *Sharḥ al-Maqāṣid*, 4:287.7-18.

¹³⁹ Rezakhany, “Jalāl ad-Dīn ad-Dawānī’s Solution,” 186–207.

¹⁴⁰ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 13. The objection is Dawānī’s.

apply only to subjects that meet certain conditions, and not contradictories, one of which must apply to every subject.¹⁴¹

Objection: Not a condition

It is not a condition on being a statement that the statement correspond or not independent of consideration of the statement itself. Indeed, there are obviously true statements where this is the case. Consider someone who says, “Everything I say today is my speech,” and then nothing else for the rest of the day. The sentence is obviously true, yet it could not be deemed true without itself being considered in the process of deeming it to be true.¹⁴²

8. *The Judgement Makes It So*

For a statement to be false is (1) for what it describes to not correspond with how things are in themselves, though in principle it could correspond with them, and (2) for the mind judging the statement to have no part in making it false by passing that very judgement. The Liar, however, lacks correspondence with things in themselves only by virtue of the very judgement that it lacks correspondence. The Liar cannot be considered false without passing the judgement that it “is false.”

In light of this dual condition on a statement’s being false, the Liar sentence is to be understood as follows: “This sentence is false independent of the fact that the mind passes judgment on it.” But that is false, since it cannot lack correspondence independent of being so judged. That the Liar is indeed in this sense false does not contradict the fact that it is neither true nor false when ‘false’ in “This sentence is false” is understood merely as the lack of correspondence with things in themselves, merely as the first condition. In that case, it would be neither true nor false since it violates the second condition, namely that the mind’s passing the judgement not cause it to be false.¹⁴³ Indeed, without passing judgement that the Liar is false (or negating it) and taking that judgment (or its negation) into account, there would be no grounds on which to say it is either true or false. In this respect, it would be just like the Truth Teller: it also lacks any grounds for determining its truth value.

9. *For each truth-value ascription, there must be a distinct statement*

For each ascription of “true” or “false,” there must be a statement that is thereby described, in order for the ascription to be possible. Hence, “Zayd’s statement is false” or “Zayd’s statement is true,” could be neither true nor false, if Zayd had made no other statement, since there would be nothing for either ‘true’ or ‘false’ to describe. Likewise, the statement “‘Zayd’s statement is false’ is true,” could be neither true nor false, unless there were a statement about Zayd’s statement

¹⁴¹ RezaKhany, “Jalāl ad-Dīn ad-Dawānī’s Solution,” 204–7. The resolution is Dawānī’s.

¹⁴² Khafīrī, “Ḥayrat al-fuḍalā’,” 282.7-12.

¹⁴³ Shams al-Dīn Khafīrī, “Ibrat al-fuḍalā’,” in *Davāzdah risāla dar pāradūks-i durūghgū*, ed. Aḥad Farāmarz Qarāmalekī and Ṭayyibah ‘Ārifniyā (Tehran: Mu’assasa-yi Pizhūheshī-yi Ḥikmat va Falsafah-yi Irān, 2007), 268.18-269.3, 269.7-269.11; Khafīrī, “Ḥayrat al-fuḍalā’,” 308.10-309.7.

(“Zayd’s statement is false”) and a statement made by Zayd. The Liar sentence, however, contains two truth-value ascriptions, yet there is only one statement. The Liar sentence itself, “Everything I say this hour is false,” contains the first as predicate, and it contains it again in the subject-term, “everything I say this hour,” which refers to the same sentence. Therefore, the Liar sentence could be neither true nor false, since there are more truth-value ascriptions than statements.¹⁴⁴

Objection: An existential proposition is false when the subject-term does not refer

Even granting that there are not enough statements for the Liar sentence’s truth-value ascriptions, the Liar sentence would still be false. This is because the Liar sentence is an existential proposition, and existential propositions are false when their subject-terms do not refer.¹⁴⁵ If the claim is that there are not enough statements for the truth-value ascriptions, then the Liar’s subject-term must not refer, and the Liar should simply be false. But if we grant that the Liar sentence is false, the paradox, of course, re-emerges.¹⁴⁶

Resolution: Only when the relevant condition is satisfied

It is true that an existential proposition is false when its subject-term is non-referring—but only under appropriate conditions. One of these is for there to be at least one statement additional to each existential statement making a truth-value ascription. Since this condition is not met in the case of the Liar sentence, it is neither true nor false, not false.¹⁴⁷

Objection: Strengthened Liar

If, as this solution claims, the Liar sentence is neither true nor false, it follows that it must be false, which is a contradiction. For a sentence to be false is for it to inaccurately describe the fact of the matter. According to this solution, the Liar sentence is neither true nor false, yet it describes itself as false. Therefore, the Liar sentence is false.¹⁴⁸

Resolution: The Liar lacks the capacity to be true or false

The solution was meant to show that every Liar sentence is neither true nor false in the sense that it has no capacity to be either. The Liar sentence is lacking one of the conditions on the possibility for it to be either true or false, namely that there be enough statements for the number of truth-value ascriptions. Since ‘true’ and ‘false,’ are contraries, and therefore apply only to those subjects

¹⁴⁴ Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 55–56; Khafī, “Ḥayrat al-fuḍalā’,” 284–85.

¹⁴⁵ For an explanation of the existential/essential proposition distinction, see the existential/essential proposition solution in the chapter on the paradox of the absolute unknown.

¹⁴⁶ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 13; Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 57.3–7. The objection is Dawānī’s. See also Khafī, “Ḥayrat al-fuḍalā’,” 285–87.

¹⁴⁷ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 14; Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 57.15–58.8.

¹⁴⁸ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 13; Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 59.16–60.14, 57.17–58.1. The objection is Dawānī’s.

that meet relevant conditions, neither need apply to the Liar, which lacks one of those conditions. An analogous case is a boy who is neither ‘bearded’ nor ‘clean-shaven,’ since he lacks the condition under which either of those contraries could apply to him, namely the capacity for facial hair. Simply because he is not bearded, it does not follow that he is clean-shaven, and *vice versa*. That would only be the case if he met the condition for either of the contraries to apply in the first place. Likewise, just because the Liar sentence is not true, it does not follow that it is false, and *vice versa*. That would only be the case if it met the condition for either ‘true’ or ‘false’ to apply to it in the first place.¹⁴⁹

Objection: Not all formulations satisfied

Consider the scenario where someone says, “Everything I say this hour is false,” and then makes additional false statements that hour. In this case, the Liar sentence would no longer refer only to itself, and there would be at least one distinct statement for each truth-value ascription, the absence of which condition was supposed to account for why every Liar sentence must be neither true nor false. Yet even though the Liar refers to additional statements, and the ratio between statements and truth-value ascriptions is right, the paradox remains.¹⁵⁰

10. *A statement must ultimately be grounded in a non-statement*

Only a statement which ultimately describes a non-statement can be true or false. Hence, if there were a sequence of circularly referring statements, like if Zayd said, “What ‘Amr says is false,” and ‘Amr said, “What Bakr says is false,” and Bakr said, “What Zayd said is true,” none would be true or false. Likewise, if we supposed an infinite sequence of statements, like if Zayd said, “What ‘Amr says is false,” and ‘Amr said, “What Bakr says is false,” and Bakr said, “What Khālīd says is false,” etc., no sentence in the sequence could be true or false, since none describes anything besides another statement.¹⁵¹

Objection: Infinite strengthened Liars

Consider again an infinite line of people. Each says of the next, “What he says is false.” Assuming that all of these sentences are neither true nor false, a contradiction will follow. The content of the first sentence, for example, is that the second is false. This content, however, does not correspond with the fact of the matter, since we assumed that all the sentences—including the second—are neither true nor false. Therefore, the first sentence ought to be false instead of being neither true

¹⁴⁹ Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 61.4-11.

¹⁵⁰ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 13–14; Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 59.14-16. The objection is Dawānī’s. As El-Rouayheb notes, Dashtakī does not appear to reply to this objection. An equally difficult case for Dashtakī’s solution, I think, would be the “morning-evening” formulation. Here there are again two statements, but it is unclear how one could justify the claim that there are any more than two truth-value ascriptions.

¹⁵¹ Farāmarz Qarāmalikī, “Aqā Ḥusayn-i Khwānsārī va-mu‘ammā-yi durūghū,” 84–85. See a similar “infinite Liars” scenario in Māzandarānī, “Risāla fī jidhr al-aṣamm,” 29a.17-19.

nor false, since its content does not correspond to reality, which is just what it is for a sentence to be false. This is a contradiction.

If we consider the second and third sentences, matters will be worse still. According to reasoning similar to the above, it can be argued that the second sentence is false: it says of the third that it is false, yet the third was assumed to be neither true nor false, and therefore, the second ought to be false, since it incorrectly describes the third as false. If it is granted that the second is indeed false, then the first must be true, contrary to what we just argued. The first says that the second is false, which our most recent line of reasoning shows must be so. Therefore, the first must be true, since it correctly describes the fact that the second sentence is false. In sum, the first sentence is neither true nor false according to assumption, yet apparently sound reasoning shows that it must be both true and false.¹⁵²

Resolution: Three respects

Each of the infinite sentences can be considered in one of three respects: (1) It is the first sentence in the sequence. In this case, it will be neither true nor false, since, as explained, it refers to nothing besides other statements and is never grounded in anything real. (2) It lies outside of the sequence of statements and the sentence it refers to (the second) is the start of the sequence. In this case, since the second sentence (now the first in the sequence) must be neither true nor false according to reasoning given in (1), the sentence in question, lying outside the sequence, must be false, since it describes the second sentence as false, while it is neither true nor false. (3) It lies outside of the sequence and refers to the second sentence which now also lies outside of the sequence and refers to the third sentence which begins the sequence. The third sentence, for the same reason given in (1), is neither true nor false, and the second sentence, according to the reasoning given in (2), is false, so the sentence in question, which refers to the second sentence, must be true, since it accurately describes the second sentence as false.¹⁵³

11. *Undifferentiated and Differentiated (ijmāl wa-taḥṣīl)*

The Liar sentence is not, strictly speaking, self-referential. In order for a statement to be self-referential, the subject-term must be identical with the statement itself. But in the case of the Liar, it is not: while the subject-term and statement are, taken in themselves, identical, they are distinct when considered as undifferentiated or differentiated.

The Liar sentence insofar as it is contained within its own subject-term is undifferentiated: We do not conceive of the sentence's constituents—subject-term, predicate-term, and relation

¹⁵² Farāmarz Qarāmalikī, “Aqā Ḥusayn-i Khwānsārī va-mu‘ammā-yi durūghū,” 85.

¹⁵³ Farāmarz Qarāmalikī, 85–86. There are at least two mysteries about this resolution: (1) Why doesn't Khwānsārī simply insist on the terms of his solution? Instead of answering the infinite strengthened Liars objection by proposing how each statement can be true, false, or neither true nor false according to one of three respects, he should simply insist that none could be either true or false, since they report only about other statements, much as Dawānī does in his response to the strengthened Liar; (2) Doesn't this resolution contravene his solution? How can a sentence which describes only other sentences and is never grounded in anything besides have a truth value? But Khwānsārī seems to be saying that a sentence that describes but falls outside of an infinite, ungrounded sequence of sentences can nonetheless be true or false.

between them—individually, when we conceive of it as subject-term. Considered as sentence, it is differentiated: we first consider the subject-term, then the predicate-term, and finally the relation between them. Insofar as the subject-term and sentence are not identical due to this distinction of respects, the sentence is not self-referential. Moreover, this distinction can be used to show that there is no contradiction. The Liar is true insofar as it is differentiated and false insofar as it is undifferentiated.¹⁵⁴

Objection: Proposition or not?

Is the Liar's subject-term—which is the sentence insofar as it is undifferentiated—a proposition or not? If it is, then it must represent something. It couldn't represent the differentiated Liar, since it is itself a representation of the undifferentiated Liar. A proposition's object of representation must be contained in its subject-term, which means that it must be prior to the proposition, since the subject-term is one of its constituent parts. But the differentiated Liar is in fact posterior to the undifferentiated Liar, by virtue of the fact that the latter is the former's subject-term. Hence, were each the object of representation of the other, it would entail a vicious circle in that each would have to be prior to the other. The only other possibility for an object of representation for the undifferentiated Liar would be that it itself have some undifferentiated object of representation, which would be absurd, since we could always ask about the object of representation of this undifferentiated object of representation, and an infinite regress would ensue.

If the undifferentiated Liar is not a proposition, then how could it be false, as the solution declares? Only propositions are true or false. Furthermore, how could it be anything other than a proposition, considering the fact that it has all the parts of one—subject-term, predicate-term, and relation?¹⁵⁵

Objection: The contradiction remains

The undifferentiated Liar is either true or false. If it is true, then it follows that it must also be false, since a statement is true just when its predicate applies to its subject-term, and the Liar's predicate is "false." Likewise, if it is false, it follows that it must also be true, according to parallel reasoning. Hence, drawing the differentiated-undifferentiated distinction does nothing to solve the paradox.¹⁵⁶

Objection: No proposition is undifferentiated

No proposition can be contained in the subject-term of another, since the relation of predication must be explicitly present in a proposition. Were it undifferentiated, its relation would not be distinguishable, and hence it could not be a proposition, and, by extension, neither true nor false. Therefore, the solution does not work.¹⁵⁷

¹⁵⁴ Mubīn, *Mir'āt al-shurūh*, 1321ah, 1:102; Khafī, "Ḥayrat al-fuḍalā'," 292–95.

¹⁵⁵ Mubīn, *Mir'āt al-shurūh*, 1321ah, 1:102.24–103.3; Sihālawī et al., *Mullā Ḥasan*, 71.23–72.1.

¹⁵⁶ Mubīn, *Mir'āt al-shurūh*, 1321ah, 1:103.3–6.

¹⁵⁷ Mubīn, 1:103.6–9; Sihālawī et al., *Mullā Ḥasan*, 71.13–15.

Response: Even undifferentiated propositions remain propositions

The respect in which a proposition is considered—whether as differentiated or as undifferentiated—does not strip it of its essential nature, namely that it is a proposition and must be either true or false. The subject-term, predicate, and relation remain distinct even in an undifferentiated proposition. The difference between differentiated and undifferentiated proposition is merely one of respect.¹⁵⁸

12. False from One Respect, Neither from Another

The Liar sentence can be considered from two respects: (1) What ‘false’ applies to—the proposition containing ‘false’ as predicate, which proposition contains itself as subject-term and (2) the subject-term ‘false’ is predicated of within the proposition, which subject-term is contained by itself considered as a proposition. The Liar as considered from the first respect is false, and therefore its predicate (‘false’) is negated of its subject-term (the Liar considered from the second respect). The Liar as considered from the second respect is neither true nor false, since it is not a proposition but a representation, a subject-term. There is no contradiction since there is the difference of these two respects.¹⁵⁹

13. Essential Predication, Accidental Predication

The Liar sentence contains within itself the predicate ‘false’ and predicates it of itself. Since it contains the predicate ‘false,’ the predicate must necessarily apply to the subject-term, just as any essential constituent must necessarily apply to the thing it constitutes. The proposition must therefore be true. A true proposition can be either essentially true or accidentally true according to the way in which the predicate applies to the subject-term, essentially or accidentally. Hence, the Liar is essentially false, since the predicate ‘false’ it contains must be predicated of its subject-term essentially, but only accidentally true. Recall that a predicate is essential if it is contained in the concept of the subject-term and accidental if it is not, and only true of the concept’s instances. Here, since the predicate ‘false’ is contained in the very concept of the Liar sentence—no Liar sentence can omit it and remain paradoxical—it must apply essentially to it, and since ‘true’ is not contained in its concept, it could only be accidental. Moreover, it is a condition on contradiction that the mode of predication be identical. Since this is not the case, it is no problem that the Liar’s being (essentially) false entails that it is (accidentally) true.¹⁶⁰

14. Mere Instance, Specific Instance

A concept’s instance can be considered insofar as it is either merely the instance of that concept and therefore in no way distinguished from other instances of the same concept (*sinkh al-fardiyya*) or insofar as it is a specific instance, in which case it must bear some characteristic identifying it

¹⁵⁸ Sihālawī et al., *Mullā Ḥasan*, 71.16-21.

¹⁵⁹ Khafrī, “Ḥayrat al-fuḍalā’,” 302.4-11; Khafrī, “‘Ibrat al-fuḍalā’,” 268.8-14.

¹⁶⁰ Mubārakī, “Mi’rāj al-fuhūm,” 28b.12-21.

from other instances (*khuṣūṣ al-fardiyya*).¹⁶¹ In general, a predicate applies first to the description of the subject-term (‘*unwān*’), and then to the instances of that description. The description applies to the instances insofar as they are merely considered as instances (*sinkh al-fardiyya*), not insofar as they are specific instances (*khuṣūṣ al-fardiyya*). The Liar sentence “Everything I say this hour is false” leads to contradiction only because this rule of predication is being violated: when we predicate “false” of “everything I say this hour,” we are attending to the specific instances of the latter, namely, the sentence itself, “Everything I say this hour is false.” If we attended only to the mere instances of the Liar sentence’s subject-term, we would never be able to derive the opposite from the fact that we assumed that it is true or false: Without considering the specific instances of “everything I say this hour,” we would never know that they contain the predicate “false,” and we could therefore never deduce a contradiction.¹⁶²

Objection: Not all formulations satisfied

The solution cannot accommodate the formulation “This sentence is false,” since it is an individual proposition, that proposition where the subject-term is a particular. Hence, there will be no universal subject-term with instances, as in the sentence, “Everything I say this hour is false,” such that one could draw the distinction between mere and specific instances. Without this distinction, the solution amounts to nothing.¹⁶³

Objection: Predicating of a mere instance entails predicating of a specific instance

Passing a judgement on a subject-term entails passing the same judgement on its specific instances; otherwise, syllogistic reasoning would be invalid, since the major-term could not be transferred from the middle to the minor. For example, “All men are mortal, and Socrates is a man.” If a predicate applies only to the *mere* instances of its subject-term, and that does not entail that they also apply to the *specific* instances, then one could not deduce that “mortal” applies to the specific instance of “man/men,” namely “Socrates,” and could therefore not conclude that “Socrates is mortal.” Indeed, no universal statement, “F is G” would justify claiming that a specific instance of F is also G. But that would be absurd.¹⁶⁴

Objection: Only one sentence

Despite the fact that there are numerous ways to consider the same sentence, there nonetheless remains only one sentence. A sentence, in itself, can only be true or false, not both. The solution does not answer whether the sentence in itself must be true or false.¹⁶⁵

¹⁶¹ Mīr Dāmād, *Muṣannaḥāt-i Mīr Dāmād*, 2:670.

¹⁶² Mīr Dāmād, 2:685.10-686.20.

¹⁶³ Qāḍī Mubārak Ibn Muḥammad Dā’im, *Kitāb Sullam Al-‘ulūm Wa-Hāshiyatuhu l-Mashhūra Bi-l-Qāḍī Ma’a Minhuyātihi* (Kazan: al-Maṭba’a al-malikiyya, 1887), 78.9-10.

¹⁶⁴ Ibn Muḥammad Dā’im, 78.10-15.

¹⁶⁵ Al-Anṣārī al-Sihālī, “Suddat al-‘ulūm,” 91.4-5; Mubīn, *Mir’āt al-shurūh*, 1321ah, 1:103.17-20.

Summary and Analysis

Perhaps the most intractable of this study's paradoxes is the Liar. The great variety of solutions confirms as much. For the same reason, identifying a clearly superior solution is difficult, and criticizing is easy, since many solutions suffer from obvious defects. In spite of these considerations, Dawānī and Khafrī (7& 8) appear to me to have devised the most compelling solutions.

Both identify the quality that arguably sets Liar sentences apart from all others: that a Liar sentence is merely judged or considered makes it have a given truth-value. Without someone considering the fact that the Liar predicates 'false' of itself, the Liar cannot be true or false. Hence, without someone passing judgement on the Liar, it cannot be true or false. Ostensibly, the Liar sentence takes the form of a statement describing a certain state of affairs, namely that it itself is false. In reality, however, it does not describe this state of affairs, but produces it, when it is considered or judged.

Other statements, by contrast, do not produce the state of affairs they purport to describe, are not true or false merely by virtue of the fact that they were judged or considered. Some of these other statements are even self-referential: for example, "This sentence is in English." It refers to itself, but it describes an independent state of affairs: the mere fact that we think or judge it does not make it English. Once judged or thought, no further purely logical manipulations or deductions will allow us to conclude that it is indeed English or not, in contrast to a Liar sentence.

Many of the other solutions suffer from implausibility. Figuring among these appears to be Abharī's solution (1). It is easy to think that it trades on a fallacy, on affirming the consequent. Abharī begins by explaining that "The Liar is true" (p) is *tantamount* (*'ibāra 'an*) to "The Liar is true, and the Liar is false" (q). He next claims that the negation of "The Liar is true" (p) ("The Liar is not true," $\sim p$) must therefore entail the negation of "The Liar is true, and the Liar is false," (q) yielding "Either the Liar is not true or the Liar is not false" ($\sim q$). Only one of the disjuncts must be true for $\sim q$ itself to be true, and it could very well be "The Liar is not true," which would, of course, not contradict what entails it, namely $\sim p$, "The Liar is not true."

If we interpret this "tantamount to" to mean "entails," it follows that Abharī is claiming that $(p \rightarrow q) \rightarrow (\sim p \rightarrow \sim q)$, which would only be true if we assume $q \rightarrow p$ in addition to $p \rightarrow q$. Hence, unless we grant the biconditional implication between p and q , Abharī's solution will trade on the fallacy of affirming the consequent. But even if we grant that the implication is biconditional, there are still difficulties.

While it is clearly the case that $q \rightarrow p$ ("The Liar is true, and the Liar is false" implies "The Liar is true"), it is unclear why one must conclude therefrom, as Abharī does, that the Liar is false only, and why one should not also conclude that the Liar is true. Indeed, the same reasoning Abharī used to show that the Liar is false without contradiction seems to apply perfectly well for showing that the Liar is true without contradiction. If $p \rightarrow q$ and $q \rightarrow p$, why should it not also be the case that "The Liar is false" (r) $\rightarrow q$ and $q \rightarrow r$? Suppose then that $r \rightarrow q$ and $q \rightarrow r$ are also the case. Therefore, $\sim r \rightarrow \sim q$, and the Liar need not also be false, and so the Liar can be true without contradiction, the opposite of Abharī's solution.

In contrast to both Ṭūsī and Qarāmalekī, Zarepour argues that Abhari's solution is not fallacious.¹⁶⁶ We must interpret Abhari, Zarepour says, as holding that the truth of the Liar is "tantamount to" the conjunction of its truth and falsity in the sense that it *biconditionally* implies this conjunction (i.e., we should assume $q \rightarrow p$ in addition to $p \rightarrow q$).¹⁶⁷ More importantly, Zarepour interprets Abhari's solution such that it replaces the standard Tarskian truth-schema with another. The new schema blocks the objection that one could use Abhari's reasoning to establish the opposite conclusion (namely that the Liar is true without contradiction) by making it impossible to derive $r \rightarrow q$. This being said, as Zarepour acknowledges, Abhari's solution so interpreted is only as convincing as the new truth-schema is.¹⁶⁸

Indeed, it's worth pointing out that objections levelled against Abhari's solution by later readers (Dawānī and Khafrī) suggest that they did not understand him to have replaced the truth-schema. Dawānī makes a very closely related objection to the one I have outlined above that relies on insisting that $r \rightarrow q$.¹⁶⁹ Khafrī is even more explicit in his rejection of any revised truth schema: the Liar's being true is not tantamount to the conjunction of its being true and false, but rather its being true is simply its predicate's applying to its subject-term.¹⁷⁰ Presumably, if either thought Abhari's solution was founded on a replacement of the truth-schema, they would have attacked the replacement itself, not the consequence of the replacement. This all being said, it has often happened that later authors willfully or out of ignorance misunderstand the work of another. And whether or not Zarepour's interpretation reflects the actual intention of Abhari, it is a charitable interpretation which makes the solution more plausible and thereby advances philosophical understanding of the problem, which, in my mind, is far more significant.

Ṭūsī's argument is more difficult to salvage from obvious difficulties. He insists that self-referential sentences in general are neither true nor false. The problem, as so many later logicians noted, is that there are numerous self-referential sentences that are obviously either true or false. Worse still, his justification for wholesale rejecting self-referential sentences was also found to be lacking: correspondence does not, as he claimed, require the subject-term to be non-identical with the judgement itself.¹⁷¹

The solution Ibn Kammūna (4) was most famous for is about as unintuitive and as difficult to defend as Ṭūsī's, if not more so. That Liar sentences simply belong to a class of statement that can unproblematically be neither true nor false is not explanatory: Why do these sentences but not others belong to such a class? Furthermore, the solution has no obvious defense against the strengthened Liar objection, by which one shows that if we assume the Liar is neither true nor false it follows that it must also be false.

Samarqandī's solution (5) is likewise implausible. It turns on the assumption that every Liar sentence is, upon analysis, actually two sentences, and no contradiction would therefore arise, since each sentence bears the opposite truth-value. Later philosophers were unyielding to the

¹⁶⁶ Farāmarz Qarāmalikī, "Mu'ammā-yi jidhr-i aşamm nazd-i mantiqdānān-i qarn-i haftom," 74.1-2rhs.

¹⁶⁷ Mohammad Saleh Zarepour, "Abhari's Solution to the Liar Paradox: A Logical Analysis," *History and Philosophy of Logic* 42, no. 1 (January 2, 2021): 9.14-16, <https://doi.org/10.1080/01445340.2020.1793291>.

¹⁶⁸ Find Zarepour's maneuver for avoiding this objection by replacing the standard truth-schema on 9–13 of his article.

¹⁶⁹ Dawānī, "Nihāyat al-kalām fī shubhat kull kalāmī kādhib," 107.10-11.

¹⁷⁰ Khafrī, "Ḥayrat al-fudālā'," 277.6-9.

¹⁷¹ For references, see the objections to solution (2) above.

suggestion that a Liar sentence was, in reality, two sentences. They insisted that there was only one sentence.

Besides this most fundamental objection, there seem to me to be more that were, to my knowledge, never raised. Even if one granted that every Liar sentence could legitimately be analyzed into two sentences, a serious mystery remains: which of these two sentences is true and which false? Samarqandī provides no reason for either being either. In fact, his presentation suggests that either of the two could bear either truth-value.¹⁷² It is quite unusual for a statement to be true or false merely by virtue of the fact that one assumes it to be so, and therefore, an explanation is needed.

Another crushing blow to Samarqandī's solution (5) is that it is insufficiently robust. It is far from clear that the solution could accommodate any formulation of the Liar utilizing more than one sentence. Would each of the sentences in the "morning-evening" scenario be analyzed into two more, yielding a total of four? What of Khwānsārī's infinite line of Liars? How could Samarqandī's solution be applied in that case? And what of the Truth-Teller ("This sentence is true")?

Given that Taftāzānī's solution (6) does not greatly distinguish itself from its competitors, it is all the more surprising how quickly it was dismissed. In terms of ingenuity, it is neither much better nor much worse than other solutions: Taftāzānī introduces a distinction among truth-value predicates and explains that there can only be a contradiction when opposite truth-value predicates of the same kind apply to the same subject. He then proceeds to show that this condition on contradiction is not met in the case of the Liar.

Unfortunately, the solution turns on a fallacy. Taftāzānī says that assuming "This sentence is false" is false only entails that "This sentence is true" is true. Such a claim, at first glance, appears eminently plausible. In general, "A is B" is false entails "A is not B" is true. However, in the case of self-referential sentences, the subject-term will be changed when the predicate is changed. Hence, 'this sentence' will only be the same words, not the same subject-term, in each sentence. Therefore, "'This sentence is false' is false" is false entails "'This sentence is false' is true" is true. Of course, once the fallacy is eliminated, the same old contradiction reappears.

Even though the solution is fallacious—and obviously so—it seems to have at least warranted criticism. Indeed, the solution is far more sophisticated than Jurjānī's, which elicited at least some responses from later philosophers. The reason Taftāzānī was so easily dismissed was not merely the weakness of his solution but that he allowed himself to be dismissed. He openly admitted that he thought the solution was no good and that ultimately there was no solution to the Liar. Apparently, later philosophers saw this admission as sufficient grounds for rejecting the solution without further explanation.¹⁷³

Dashtakī's solution (9) is, in many ways, successful, though it cannot overcome at least one objection. Dashtakī insists that for every truth-value ascription there must be at least two sentences. Other solutions often stumble over the strengthened Liar or the fact that some self-referential sentences are obviously either true or false, but Dashtakī's solution is, in the final

¹⁷² Shams al-Dīn Samarqandī, *Kistāsu'l-efkār*, 553.5-6.

¹⁷³ Dashtakī, "Risāla fī ḥall shubhat jidhr al-aṣamm," 29; Dawānī, "Nihāyat al-kalām fī shubhat kull kalāmī kādhīb," 106.penultimate-ultimate.

analysis, slowed by neither. Unfortunately, however, it seems that his solution cannot accommodate simple reformulations of the paradox, like when someone utters a false sentence in addition to “This speech is false.”¹⁷⁴ In this case, there will be more statements than truth-value ascriptions, which, by Dashtakī’s own admission, ought to prevent paradox, yet paradox remains nonetheless.

While Āqā Ḥusayn Khwānsārī’s (d. 1687) solution (10) is novel, it is not very plausible. He argued that every statement must be grounded in a non-statement, if it is to be true or false. That is, statements that describe only other statements, which themselves describe nothing other than statements, can be neither true nor false. Intensionally, the solution has the merit that it does not impugn self-reference, but, extensionally, it does just that: there is no self-referential sentence that is grounded in a non-sentence. Hence, it appears that Khwānsārī would have to be committed to the counter-intuitive position that sentences like “This sentence is in English” or “This sentence is compound” are neither true nor false.

Mughal commentators did a fine job skewering Bihārī’s solution (11) with objections. Bihārī argued that the Liar is true or false from different respects. If it is considered in an undifferentiated fashion (*bi-l-ijmāl*) such that the subject-term, predicate, and relation between them are conceived of all together, it is false, and if considered in the opposite way, in a differentiated fashion (*bi-l-tafṣīl*), it is true. In addition to the many objections raised in the *Sullam* commentary tradition, the most fundamental seems to be that the solution is non-explanatory: why is the Liar false when considered undifferentiated and true when considered differentiated? Why not instead true when undifferentiated and false when differentiated? Neither Bihārī nor (as far as I have seen) his commentators have a response. Interestingly, Khafrī considers this objection, and attempts a response, though I cannot make sense of it. Be that as it may, Khafrī concedes that the response is weak.¹⁷⁵

Despite all of his characteristic bravado, Mīr Dāmād’s solution (14) is neither highly original nor very defensible. The fundamental approach is common enough: he wishes to draw a distinction of respects to argue that no contradiction is entailed. Whether the Liar is assumed to be true or false, the opposite will not follow, when the identified distinction is kept firmly in mind. Although the general approach is by no means novel, the distinction he identifies had not before been deployed for solving the Liar. Commentators of the *Sullam* cut Mīr Dāmād down to size, pointing out some difficult objections.

Historical Development

How exactly the Liar paradox entered the Islamic world is still unclear.¹⁷⁶ The earliest philosophical giants were silent: Neither Fārābī (d. 950/1) nor Avicenna (d. 1037) had anything to say on the problem.¹⁷⁷ It is first genuinely discussed in extant writing by theologian ‘Abd al-Qāhir

¹⁷⁴ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 13–14; Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 59.14–16.

¹⁷⁵ Khafrī, “Ḥayrat al-fuḍalā’,” 294.14–295.5.

¹⁷⁶ Alwishah and Sanson, “The Early Arabic Liar,” 99.

¹⁷⁷ Fārābī mentions it at two points in his logical writings but only as an example and never as a problem that needs solving. Abū Naṣr Muḥammad b. Muḥammad al-Fārābī, *al-Manṭiqiyyāt li-l-Fārābī*, 2nd ed., vol. 1 (Qom: Manshūrāt Maktabat Āyatallāh al-‘Uzmā al-Mar‘ashī an-Najafī, 2012), 246.9–10; Abū Naṣr Muḥammad b. Muḥammad al-

al-Bāghdādī (d. 1037/8).¹⁷⁸ After a century and a half, it is again found in Fakhr al-Dīn al-Rāzī's (d. 1210) monumental work on legal theory, *al-Maḥṣūl*.¹⁷⁹ By the 13th century, the Liar paradox had found a firm foothold in philosophical discourse which it would maintain with increasing strength for the next six centuries.

Many of the best-known 13th and 14th century philosophers discussed the Liar. The most original were Abharī (d. 1265), Ṭūsī (d. 1274), Ibn Kammūna (d. 1284), and Samarqandī (d. 1322). Each formulates a distinct solution. Kātibī (d. 1277) and Shahrāzūrī (d. 1288) simply repeat Abharī's, as Ḥillī (d. 1325) repeats Ṭūsī's. Neither Kātibī, Shahrāzūrī, nor Ḥillī considers objections. Kātibī alone seems to have contributed something to the literature: he appears to have been the first to give a two sentence (in the style of "morning-evening") formulation of the paradox.¹⁸⁰ Ibn Kammūna (d. 1284) elaborates three solutions in his *al-Jadīd fī l-ḥikma* alone: a modified version of Ṭūsī's, Abharī's, and another grounded in the fact that no paradox follows when the Liar sentence is interpreted as an essential as opposed to an existential proposition. In a letter to Kātibī, he proposes yet another solution (4), the one he was remembered for, holding that a proposition need not be either true or false in actuality but rather need only have the potential to be either.

Although Taftāzānī's (d. 1390) solution was rarely if ever given any serious consideration, he nonetheless made significant contributions to the Liar literature. Dawānī and Dashtakī are among the few scholars who actually name him and quote his solution, and neither deem it worthy of criticism. Both dismiss it on the grounds that it is obviously weak, and, more importantly, that Taftāzānī himself thought it insufficient, which is, ironically, one of Taftāzānī's very contributions: he was the first and one of the few philosophers who openly admitted that there was no solution to the Liar paradox. While it is possible to interpret, "The correct position on this proposition is, in my view, to abandon the attempt at any response and instead to admit to being incapable of solving the difficulty" not as a claim that no solution can or will be found but rather as an admission of current and (perhaps) temporary inability, the more radical interpretation appears to me to be more likely, in light both of Taftāzānī's own framing of the problem and later framings of it as one which has "bewildered the intelligentsia" (*tahayyara fī ḥallihā 'uqūl al-'uqalā' wa-fuḥūl al-adhkiyā*).¹⁸¹

Taftāzānī's other great contribution to the literature was naming the problem, which would set part of the scholarly research agenda for the next several centuries. The name he gave, "*jidhr al-aṣamm*," generated an exegetical task for virtually everyone who would write on the Liar after him: explain the meaning of this name and why the Liar is named by it. By the 19th century, any of four explanations had become common:

Fārābī, *al-Manṭiqiyyāt li-l-Fārābī*, 2nd ed., vol. 2 (Qom: Manshūrāt Maktabat Āyatallāh al-'Uzmā al-Mar'ashī an-Najafī, 2012), 348.9-13. For Avicenna, see Alwishah and Sanson, "The Early Arabic Liar," 99.

¹⁷⁸ Alwishah and Sanson, "The Early Arabic Liar," 100. It is also discussed by al-Qāḍī 'Abd al-Jabbār in his *Mughnī*, according to Qarāmalekī: Aḥad Farāmarz Qarāmalekī and Ṭayyibah 'Ārifniyā, eds., *Davāzdah risāla dar pāradūks-i durūghū* (Tehran: Mu'assasa-yi Pizhūheshī-yi Ḥikmat va Falsafah-yi Irān, 2007), hashtād-o do.

¹⁷⁹ I have not seen that 11th and 12th century philosophers like Bahmanyār (d. 1066), al-Lawkarī (d. 1109), al-Ghazālī (d. 1111), or Suhrawardī (d. 1191) discuss the Liar anywhere in their works.

¹⁸⁰ Qarāmalekī and 'Ārifniyā, *Davāzdah risālah*, sī-o shesh tā sī-haft.

¹⁸¹ Taftāzānī, *Sharḥ al-Maqāṣid*, 4:287. It is worth repeating here that Taftāzānī's remarks on how difficult it is to solve the Liar paradox appear to be the basis of and inspiration for later Timurid, Safavid, and Mughal authors' rhetorical flourishes to the same effect. See the Introduction for further details.

1. Mathematical. *Jidhr al-aṣamm* in mathematical terminology meant “imperfect square.” *Jidhr* simply means “root” or “square” (in the mathematical sense of finding a number whose product with itself is the number given), and *aṣamm* means “non-square” (again, in the mathematical sense of a number that cannot be represented as the product of a whole number with itself). “This problem was called ‘the imperfect square’ because human ability is insufficient for extracting the answer,” just as it is insufficient for extracting the root of a non-square number.¹⁸²

Being impossible to solve is often cited as what the Liar shares with imperfect roots such that they also share a name, but there are hints that there is another similarity. One commentator of the *Sullam al-‘ulūm* writes, “The sophism is thus named due to the similarity [with imperfect squares], as is obvious to the learned.”¹⁸³ Perhaps the additional similarity is that an “imperfect square” is also, in a way, contradictory: it asks us to take a square of something that has no square (a non-square number), like the Liar asks us to determine a single truth-value of a proposition that apparently must be both true and false.

2. Linguistic. Taking *jidhr al-aṣamm* in a non-technical sense, it can mean “deaf problem,” i.e., the problem is deaf to any solution.¹⁸⁴
3. Linguistic. According to another non-technical interpretation of the phrase, *jidhr* means “uprooting” and *aṣamm* means “foundation,” yielding “the uprooting of the foundation.” The Liar would “uproot the foundation” in the sense that it contravenes the law of non-contradiction.¹⁸⁵
4. Personal name. The least favored of these explanations was that *jidhr al-aṣamm* was the name of the deaf (*aṣamm*) originator of the paradox.¹⁸⁶

In the late 15th century, there was an explosion of interest in the Liar among Iranian scholars, due mostly to Jalāl al-Dīn al-Dawānī and Ṣadr al-Dīn al-Dashtakī’s heated and prolonged debate over it. Their approach to the paradox is, in comparison to that of their forebears, highly systematic and far more elaborate. They catalogue and criticize many of the solutions advanced by past scholars. Logicians before them had adduced and criticized alternative solutions but never to the same extent.

In addition to the unprecedented amount of attention each dedicated to the paradox, they also devised original solutions and objections. While Dawānī struggled to make his position clear in his writings and seems to have changed it at a certain point, his most interesting solution amounts to the fact that no Liar sentence can be true or false since none is a statement, and none is a statement because none describes a state of affairs but rather produces it. Dashtakī radically changed his solution over the years.¹⁸⁷ His final and strongest position was grounded in the claim that for every statement with a truth-value predicate, there must be that many additional statements.

¹⁸² al-Sā’infūrī, “Sharḥ Sullam Al-‘ulūm,” 25a.8-16. See also Tahānawī, *Kashshāf*, 1996, 2:1168lhs.5-12.

¹⁸³ Ibn Maḥabbat, “Sirāj Wahhāj,” 41a.11-12.

¹⁸⁴ Rezakhany, “Jalāl ad-Dīn ad-Dawānī’s Solution,” 208.

¹⁸⁵ Kamāl al-Dīn Bukhārī, “al-Muntakhab min Sharḥ Risālat al-mughālaṭāt,” in *Davāzdah risāla dar pāradūks-i durūghgū*, ed. Aḥad Farāmarz Qarāmalekī and Ṭayyibah ‘Arifniyā (Tehran: Mu’assasa-yi Pizhūheshī-yi Hikmat va Falsafah-yi Irān, 2007), 316.

¹⁸⁶ Bukhārī, 316.

¹⁸⁷ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 7.

Since the Liar contains a truth-value predicate (“false”), but there is only one sentence, it is invalid and therefore neither true nor false.

Furthermore, they set a trend in the Safavid world. Extensive treatment of the Liar became the norm. It became common to present and criticize the views of numerous other scholars, supply at least one novel solution, and discuss potential objections over the course of several pages. Both Shams al-Dīn al-Khafrī (d. 1535/6) and Āqā Ḥusayn Khwānsārī (d. 1687) follow the same pattern, to say nothing of Ghiyāth al-Dīn al-Dashtakī (d. 1542). By contrast, previous scholars rarely devoted more than a page to the problem.

Safavid scholars had much to contribute. In his *Ḥayrat al-fuḍalāʾ*, Khafrī formulates five solutions of his own. Perhaps the most compelling of these is closest to my understanding of Dawānī’s. Like Dawānī (7), Khafrī (8) recognizes that judging a Liar sentence itself causes it to be false, and therefore, the mind produces what it purports to describe. Furthermore, Khafrī makes it a condition on a proposition’s being false that the mind’s passing judgement on it plays no part in making it false, much as Dawānī does for a proposition’s being truth-apt at all. Khwānsārī (10) devises a more original solution, demanding that every valid proposition ultimately be grounded in a non-proposition. Perhaps more significant is the “infinite Liars” scenario he proposes, the earliest such scenario I have encountered. Mīr Dāmād (d. 1631/2) gives an elaborate solution from respects that is supposed to block the Liar’s being true (false) entailing that it is also false (true).¹⁸⁸

Another advance made in the Liar paradox literature during this period is the insistence that a solution to the Liar must answer all formulations. Dawānī and Dashtakī are, to my knowledge, the first to use this as a way to disqualify a solution.¹⁸⁹ A generation later, Khafrī is, more than once, extremely explicit on this point:

It should be clear to you that the solutions previously given cannot be applied to all the formulations given, and it is thereby known that that they could not be correct. The true solution will apply to all formulations, or else some version of it will.¹⁹⁰

Mīr Dāmād goes out of his way to point out that his solution to the Liar also solves the totality of all relations paradox.¹⁹¹ Later Mughal logicians would make similar if less explicit demands that a solution accommodate all formulations.¹⁹²

The Safavid era witnessed yet another development in the treatment of the Liar paradox. In the 17th century, treatises dedicated to sophisms (*mughālaṭāt*) or “doubts” (*tashkīkāt*) begin to appear. It was rare for these to exclude the Liar paradox. Unfortunately, however, it was also rare for them to contain much original research. Indeed, they are often rather concise, in comparison to contemporaneous studies dedicated to a single paradox or even to those which merely include discussion of a paradox.

¹⁸⁸ Mīr Dāmād, *Muṣannaḡāt-i Mīr Dāmād*, 2:685–86.

¹⁸⁹ El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz,” 13–14; Dashtakī, “Risāla fī ḥall shubhat jidhr al-aṣamm,” 59.14–16. The objection is Dawānī’s.

¹⁹⁰ Khafrī, “Ḥayrat al-fuḍalāʾ,” 289.2–4. See also 308.10–11.

¹⁹¹ Mīr Dāmād, *Muṣannaḡāt-i Mīr Dāmād*, 2:686.21.

¹⁹² Ibn Muḥammad Dāʾim, *Kitāb Sullam Al-ʿulūm Wa-Ḥāshiyatuhu l-Mashhūra Bi-l-Qāḍī Maʿa Minhuyātihi*, 78.9–10.

In the Mughal world, the *Sullam al-‘ulūm* was a central hub for discussion of the Liar. Every commentary dedicated at least one page to the problem. While there was some original innovation, the scope of previous solutions examined was narrower than in Iran. The vast majority of commentaries discuss only the solutions of Dawānī, Mīr Dāmād, and the author, Bihārī. Mughal treatises on the paradox falling outside of the *Sullam* commentary tradition appear to have been similarly limited in the scope of previous solutions examined.

There were, however, exceptions, especially at the inception of the *Sullam* tradition. One early commentator, Mubārakī, mentions Samarqandī and devises a solution obviously inspired by Mullā Ṣadrā.¹⁹³ Another quotes an early solution of Dashtakī’s, though without attribution.¹⁹⁴ Another appears to quote a later solution of Dashtakī’s as well as Ibn Kammūna’s most famous solution (4), though he mentions neither by name.¹⁹⁵ After these earliest commentaries, I have not found any of these authors’ work discussed in the *Sullam*’s section on the Liar. Although later commentaries regularly spend time on it, only one of the earliest commentaries (al-Anṣārī al-Sihālī’s *Suddat al-‘ulūm*) discusses Mīr Dāmād’s solution.¹⁹⁶ Even some later commentators, however, ignore his work on the paradox, like Mullā Ḥasan (a Shi‘ite) and Baḥr al-‘Ulūm (a commentator of Mīr Dāmād’s *al-Ufuq al-mubīn*).

Dawānī is consistently misinterpreted in both Safavid and Mughal research on the Liar. Khafī comes closest to understanding him correctly, while still falling short. By and large, however, Dawānī’s solution is thought to amount to little more than Ṭūsī’s in work from both empires. This misunderstanding is particularly pronounced in the Mughal world, where “Dawānī’s” solution is so consistently examined in *Sullam* commentaries and other treatises, and it is present even in the earliest works of Mughal logic.¹⁹⁷

The key difference between Dawānī’s solution and Ṭūsī’s is that the latter’s rejects all self-referential statements as incapable of bearing a truth-value. Dawānī went to pains to avoid the same conclusion.¹⁹⁸ Mughal philosophers only slightly modified Ṭūsī’s solution when attributing it to Dawānī. Instead of impugning self-referential statements on the grounds that there must be two things in order for there to be correspondence or the lack thereof between them and hence truth or falsehood, they rejected self-reference as incoherent in itself. Perhaps the clearest statement of this rejection is found in Mullā Ḥasan’s (d. 1784) commentary on the *Sullam*:

Rule: Every compound utterance that contains a subject-term and predicate and that contains itself in its subject-term... is a performative and in no wise a statement. Indeed, it is incoherent to think such a sentence could represent something else, since that would demand an object of representation that is prior to the statement insofar as things are in themselves. Any instance of the subject-term [including the sentence itself] would be prior to the object of representation [and therefore also prior to the sentence itself]. Hence, it would follow from holding that such sentences can represent that something can be prior to itself by stages, which is absurd.

¹⁹³ Mubārakī, “Mi‘rāj al-fuhūm,” 28a.22, 28b.12-21.

¹⁹⁴ al-Sā’infūrī, “Sharḥ Sullam Al-‘ulūm,” 24a.13-24b.2.

¹⁹⁵ Ibn Maḥabbat, “Sirāj Wahhāj,” 40a.3, 40a.13.

¹⁹⁶ Neither Sā’infūrī, Mubārakī, nor Ibn Maḥabbat mentions him.

¹⁹⁷ Mīr Zāhid Harawī, “Mīr Zāhid Mullā Jalāl” (n.d.), 36b.14-15, MS Rampur 3357.

¹⁹⁸ On the Mughal misinterpretation of Dawānī, see Rezakhany, “Jalāl ad-Dīn ad-Dawānī’s Solution.”

Therefore, assuming such sentences can represent and that the Liar is a statement is also absurd.¹⁹⁹

In sum, any self-referential sentence (one that “contains itself in its subject-term”) entails that it be prior to itself, if one assumes that it could be a statement. Hence, all self-referential statements are absurd.

Some Mughal logicians seem also to have misunderstood Mīr Dāmād’s solution (14). They thought that Mīr Dāmād was claiming that the Liar sentence was true according to one respect and false according to another.²⁰⁰ This interpretation does not seem to stand up to textual scrutiny, however. Indeed, Mīr Dāmād seems simply to have held that predicates apply to mere instances alone, not to specific instances. His claim, therefore, does not determine whether the Liar sentence is true or false (a philosophical demerit) but only prevents contradiction by blocking the Liar’s being true (false) entailing its being false (true). But even if we assume that these Mughal logicians’ interpretation is correct, it remains unclear why the Liar sentence should be true according to one respect and false according to the other instead of the reverse.

Despite the limited scope of past solutions and some misunderstandings, Mughal authors nonetheless made original contributions to the Liar paradox literature. Perhaps chief among these is the solution of Mubārakī (c. after 1709). In his commentary on the *Sullam*, he elaborates a Ṣadrian style solution, parallel to Mullā Ṣadrā’s solution to the paradox of the absolutely unknown, which he quotes approvingly in the same commentary.²⁰¹ Besides the solution’s philosophical interest, it is also historically interesting: Mullā Ṣadrā never attempted to solve the Liar paradox,²⁰² but here we have a Mughal author, who was clearly an admirer of sorts of Mullā Ṣadrā, deploying his strategies to solve the problem.

The other candidates for being considered original Mughal solutions suffer from difficulties. Bihārī’s solution appears to find precedent in Khafrī’s work. One of Khafrī’s solutions bears a resemblance to Bihārī’s. While there is no evidence that Bihārī read Khafrī and the similarity is not complete, it does seem sufficient to arouse suspicion. Indeed, Khafrī even uses the terms “differentiated” (*tafṣīlī*) and “undifferentiated” (*ijmālī*), as Bihārī does.²⁰³ Bihārī’s commentators, however, assign a fuller meaning to these terms than Khafrī did. The latter means by them only “insofar as the proposition contains itself” and “insofar as it is contained by itself,” respectively.²⁰⁴ The former explain that *tafṣīlī* means “insofar as the subject-term, predicate, and relation are all considered individually,” and *ijmālī* is the opposite.²⁰⁵ Al-Anṣārī al-Sihālī (d. 1754) presents his own solution, but I cannot quite understand it, unfortunately.²⁰⁶

Original work on the paradox in the Mughal world was most concentrated not in solutions but in objections to solutions and in their resolution. Although there were a few original solutions

¹⁹⁹ Sihālawī et al., *Mullā Ḥasan*, 72.19-73.5.

²⁰⁰ Al-Anṣārī al-Sihālī, “Suddat al-‘ulūm,” 91.4; Mubīn, *Mir’āt al-shurūḥ*, 1321ah, 1:103.17. Note that Qāḍī Mubārak makes no such interpretation.

²⁰¹ Mubārakī, “Mī’rāj al-fuhūm,” 91a.21-91b.2.

²⁰² Aḥad Farāmarz Qarāmalikī, “Mu’ammā-yi jidhr-i aṣamm nazd-i mutikallimān,” *Kheradnāmah-yi Ṣadrā*, no. 5 & 6 (Fall & Winter 1375): 69.23-4rhs.

²⁰³ Khafrī, “Ḥayrat al-fuḍalā’,” 294.10-11.

²⁰⁴ Khafrī, 294.10-12.

²⁰⁵ Mubīn, *Mir’āt al-shurūḥ*, 1321ah, 1:102.9-11.

²⁰⁶ Al-Anṣārī al-Sihālī, “Suddat al-‘ulūm,” 91.9-13.

devised by Mughal logicians, they appear, for the most part, to have contented themselves with (pseudo-) Dawānī's, Mīr Dāmād's, and Bihārī's. These inherited solutions were by no means accepted uncritically. Even when, as was frequently the case, a commentator sided with one of these three logicians, it was only after considering several objections. Moreover, the objections were sometimes original and frequently damning.

Appendix of Translations²⁰⁷

(1) Abharī and Ṭūsī, *Ta‘dīl al-mi‘yār fī naqd Tanzīl al-afkār*

[Abharī]

The solution to [the Liar] is as follows: We do not grant that if [“Everything I say this hour is false”] were false, it would follow that some instances of “Everything I say this hour” would be true. This is because for [“Everything I say this hour is false”] to be true is tantamount to the conjunction of its being true and false [i.e., ““Everything I say this hour is false” is true and ‘Everything I say this hour is false’ is false.”]. Hence, were [the Liar sentence] false, it would follow that the conjunction of its being true and false would be negated, but the negation of the conjunction of its being true and false does not entail that it be true.²⁰⁸

[Ṭūsī]

If Abharī means by, “...for [‘Everything I say this hour is false’] to be true is tantamount to the conjunction of its being true and false” that the intension of [the Liar sentence’s] truth is that conjunction, it cannot be granted. If, instead, he means that [the Liar sentence’s] truth entails its falsehood, we cannot further grant that its falsehood entails the negation of what its truth entails (‘*adam dhālika l-istilzām*). Rather, its falsehood would only entail the negation of falsehood from “Everything I say this hour,” which in turn entails that “Everything I say this hour” is not false, which is all we mean by “true.” Thus, it is clear that [the Liar sentence’s] falsehood entails its truth, and the problem returns.

It would have been better for [Abharī] to have said that the [Liar sentence’s] falsehood entails its truth insofar as things are in themselves—that is, we hold that ‘true’ and ‘false’ are contradictories, such that the one entails the negation of the other. If, however, we assume that the truth [of the Liar sentence] entails its falsehood, then denying that it is false would *not* entail that it is true, since the contradictory of the consequent ($\sim q$) [paired with a conditional statement, “If p , then q ,” in a syllogism] will not produce the antecedent (p), but rather only the negation of [the Liar sentence’s] truth ($\sim p$).²⁰⁹ Hence, while an absurdity is yielded by [the Liar sentence’s] truth, since it entails that it is also false, its being false is not absurd, since that only entails that it is not true.²¹⁰

²⁰⁷ Omitted are translations of Dawānī’s (7) and Dashtakī’s (9) solutions, since both can easily be found in recent publications, and both wrote so extensively on the Liar. See Rezakhany, “Jalāl ad-Dīn ad-Dawānī’s Solution” and El-Rouayheb, “The Liar Paradox in Fifteenth-Century Shiraz.”

²⁰⁸ Ṭūsī and Athīr al-Dīn Abharī, “Ta‘dīl al-mi‘yār fī naqd Tanzīl al-afkār,” 236.7-9.

²⁰⁹ I.e., Ṭūsī is thinking as follows:

If “Everything I say is false” is true, it follows that it is false.

Assume “Everything I say is false” is not false.

Conclusion: “Everything I say is false” is not true.

The syllogism is of the form

If p , then q

$\sim q$

Conclusion: $\sim p$.

²¹⁰ Ṭūsī and Athīr al-Dīn Abharī, “Ta‘dīl al-mi‘yār fī naqd Tanzīl al-afkār,” 236.10-20.

(2) Ṭūsī, *Ta 'dīl al-mi 'yār fī naqḍ Tanzīl al-afkār*

The correct position [on this problem], however, is that ‘true’ and ‘false’ apply only to propositions that are not the same as what they represent, such that it is even conceivable that the proposition and its opposite could occur as contradictories, in the sense that if one corresponded to what it represented, the other would not, and the first would be true and the other false. Moreover, negating one would entail affirming the other, and affirming one would entail negating the other.

If, by contrast, the proposition is identical with what it represents, it is inconceivable for it to be true or false, since correspondence would be inconceivable. Indeed, correspondence is only conceivable between two things. Likewise, contradiction would be inconceivable, since if there is only one thing and it is affirmed, there is nothing to be negated, and if the one thing is negated, nothing remains such that it would be conceivable to affirm it.

If such a [self-referential] proposition reported about itself that it was false, correspondence between two things—in the sense of truth—would be inconceivable. Only the lack of correspondence would be conceivable—though not in the sense of the lack of correspondence that is the opposite of a *habitus*, i.e. falsehood, but rather in the sense of the absence of correspondence, i.e. the lack of truth.

Hence, it could not be true of such a proposition that it is true or false. Judging it to be either entails an absurdity. Rather, it is true of such a proposition that it is neither true nor false. Furthermore, negating one from it does not entail affirming the other of it, and so no absurdity arises.²¹¹

(2) Mubīn, *Mir 'āt al-shurūḥ*²¹²

Verifier Dawānī's response to this [paradox] is that it is not a statement, “*because reporting about itself is inconceivable*.” Yet there is no utterance in [the Liar sentence] about which to make a report besides it itself. Were it a statement, it would be a report about itself, but a statement reporting about itself is inconceivable, on pain of²¹³ something's being prior to itself. The report and what it reports about cannot be identical, and if there is no object of a report, there could be no report. Hence, [the Liar sentence] is not a statement but rather a performative in the form of a statement.²¹⁴

(3) Ibn Kammūna, *al-Jadīd fī l-ḥikma*

Solution: [The Liar] is a report about itself, and therefore the report and the object of reporting are one. Hence, it cannot be true, since the concept of ‘truth’ is ‘correspondence of the report to its object,’ and correspondence cannot be had without some kind of duality,

²¹¹ Ṭūsī and Athīr al-Dīn Abharī, 237.5-17.

²¹² The *matn* (base-text) of Bihārī is in italics.

²¹³ Reading *wa-illā yalzam* instead of *wa-lā yalzam* at 101.23.

²¹⁴ Mubīn, *Mir 'āt al-shurūḥ*, 1321ah, 1:101.21-25.

but there is no such duality here. As a result, [the Liar] must be false, since there is no correspondence. It does not follow from its being false in this sense that it must also be true. That would only follow were there some duality along with no correspondence.²¹⁵

(4) Ibn Kammūna, *Mukātabāt*

We do not grant that “everything I say this hour” must be either false or true. We deny the dichotomy. If it is objected that [the Liar] is a statement, and that every statement must be one or the other, since it is by that characteristic that statements are distinguished from all other sentences, then we respond as follows: we do not grant that statements are distinguished from all other sentences by that characteristic. Rather, they are distinguished therefrom by the fact that they are capable of being true or false. But that would not prevent a statement from being neither true nor false, nor is it a condition that it must be either true or false.

In sum, that a given judgement has the potential to be true or false does not entail that it actually be either.²¹⁶

(5) Samarqandī, *Qisṭās al-afkār*

Solution: What it is that a given statement represents is determined only by the intention of the one making the statement. If by “everything I say” he intends, “everything I say besides this,” there is no contradiction, since, in that case, it would not be the truth or falsehood of that very sentence that would follow from its being false or true but the truth or falsehood of some other sentence. If, instead, he means “everything I say including this,” it would be as if he uttered the [Liar] sentence and then said, “That sentence is false,” thus combining in this single utterance two statements, each of which is linked to the other. That one is true demands that the other is false and that one is false demands that the other is true, which, however, is no contradiction, since ‘true’ and ‘false’ do not apply, in that case, to the same thing.²¹⁷

(6) Taftāzānī, *Sharḥ al-Maqāṣid*

‘True’ and ‘false’ can be predicates of a proposition (*ḥāl ḥukm*)—i.e., of the affirmative or negative relation, the judgement of which as ‘true’ or ‘false’ is a concomitant of all propositions—or they can be predicates of a subject-term (*ḥukm*)—i.e., predicates in a proposition, predicated of something paronymously, as in “This is true” or “That is false.” ‘True’ and ‘false’ will contradict only if they are both predicates of the same proposition or of the same subject-term, but not if one is a predicate of a proposition and the other is a predicate of its subject-term, due either to an obvious difference in their referents, as in

²¹⁵ Ibn Kammūna, *al-Kāshif*, 75.6-10.

²¹⁶ Dawānī, “Nihāyat al-kalām fī shubhat kull kalāmī kādhib,” 119–20.

²¹⁷ Shams al-Dīn Samarqandī, *Kisṭāsu’l-efkār*, 553.1-7.

“‘The sky is below us’ is true” or “‘The sky is below us’ is false,”²¹⁸ or to a hidden difference, as in the individual proposition upon which the paradox turns.

If we assume that [the Liar sentence] is false, nothing follows therefrom besides that its contradictory is true, namely “What I say now is true,”²¹⁹ and therefore ‘true’ is a predicate of a subject-term in “What I say now is true,” and is not [the Liar’s] predicate of proposition. Indeed, [the Liar’s] predicate of proposition is ‘false,’ in accord with what we assumed. ‘True’ is also the predicate of proposition of “What I say now is true,” as well as a predicate of the [Liar’s] subject-term, “what I say now.” Hence, ‘true’ and ‘false’ do not both apply to a single proposition or to a single subject-term. The same would be the case were we to assume the Liar is true.²²⁰

(8) Khafīrī, *Ibrat al-fuḍalā*’

The third solution: The meaning of a sentence’s being “false” is that, independent of the mind’s judging so, it does not correspond to things in themselves but is otherwise capable of such correspondence. Hence, the import of “Everything I say this hour is false” is that the sentence does not correspond to things in themselves, independent of the very judgement that it is false, but is capable of correspondence. Were this condition to fail, the proposition would be false. But “false” applies to the Liar only by virtue of the mind’s judgement that it is false...

In sum, the solution is the following: The judgement expressed by the Liar is that “false” applies to it not by virtue of the mind’s judgement. This judgement is false, since “false not²²¹ by virtue of the mind’s judgement” is negated of it due to the negation of the conjunction, namely “being false and not by virtue of the mind’s judgement.” The Liar’s judgement is false not insofar as “false” is to be negated of it, but insofar as “not by virtue of the mind’s judgement” is negated of it.²²²

(8) Khafīrī, *Ḥayrat al-fuḍalā*’

Let it be known that a sign that these solutions—especially the last [i.e. (8)]—are correct is the fact that they apply to all formulations of the paradox, as is clear. Consider, for example, if the last solution were applied to the fourth formulation: the formulation would be that someone says, “I am a liar.” The meaning of this statement is that, the speaker, ignoring his own judgement about himself that he is a liar, would be a liar—but that is not the case. Indeed, he is only a liar by virtue of that very judgement. Hence, the statement’s being false entails no contradiction, as is clear.

²¹⁸ In “‘The sky is below us’ is true” (or “‘The sky is below us’ is false”), the predicate of the subject-term is ‘true’ (or ‘false’) and the predicate of the proposition is the opposite, since the proposition itself, namely “‘The sky is below us’ is true” (or “‘The sky is below us’ is false”), is false (or true), yet no contradiction follows therefrom.

²¹⁹ The Liar sentence Taftāzānī is working with is “What I say now is not true.”

²²⁰ Taftāzānī, *Sharḥ al-Maqāṣid*, 4:287.7-16.

²²¹ There must be a *laysa* between *lahā* and *bi-‘tibār* at 269.8.

²²² Khafīrī, “‘Ibrat al-fuḍalā’,” 268.18-269.3, 269.7-269.11.

Consider if it were applied to the second formulation: the formulation would be that both the morning and evening sentences would be false by virtue of the judgement in either.²²³ But no conjunction of ‘false’ and ‘not false’ or ‘true’ and ‘false’ in either follows therefrom. The judgement in either is that the other is either true or false independent of considering the judgement that the other is either and independent²²⁴ of considering the judgement that it itself is either. Hence, neither is true or false unless the judgement that the other is true or false is taken into account, since each depends on the other to be true or false. Thus, for both to be false in consideration of both judgements contained in either about the other would not contradict both of them being neither true nor false in abstraction of both judgements. Therefore, neither is a contradiction entailed nor a conjunction of ‘true’ and ‘false’ in either statement. The application of the solution to the rest of the formulations should be clear to you.²²⁵

(10) Khwānsārī, *Hāshiyat ‘alā l-Hāshiya al-qadīma ‘alā l-Tajrīd*

The correct solution is to hold that the [Liar sentence] is a statement, though it is neither true nor false. ‘True’ and ‘false’ apply to a statement insofar as it represents a given reality and is described by either in comparison to it, insofar as it does or does not correspond with it. If a statement represents merely another representation, it will be true or false in comparison to the second representation’s object. Likewise, were some representation never to end in an object of representation, it would be neither true nor false. For example, if Zayd states, “What ‘Amr says is true,” and ‘Amr says, “What Bakr says is true,” and Bakr says, “What Khālīd says is true,” and so on *ad infinitum*, none would be true or false in such a scenario. Since the representation is not grounded in any reality, it is a representation of nothing. This is obvious upon reference to one’s own consciousness. The same is true in the case of circularity—the same would apply as does to the case of an infinite regress, as is obvious.

In sum, if a statement is grounded in something which is not to be considered a representation, then it can be true or false in consideration of its correspondence or lack thereof. If, however, it is grounded in no such thing, then it is neither true nor false, and it is equivalent to a performative. Were it not for some of what the Verifier [Dawānī] has said, as we have pointed out, it would be possible to interpret his view as ours.²²⁶

(11) Mubīn & Bihārī, *Mir’āt al-shurūḥ*

The truth is that this statement with all its parts, namely subject-term, predicate, and relation, are subsumed into the subject-term, i.e. they are made the subject-term, hence the relation in the subject-term is considered in an undifferentiated fashion, so that the subject-term won’t be dependent (li-allā yakūna ghayra mustaqill). Thus it—the relation—is an

²²³ This is just what the Arabic says, but it seems to me that it ought to read, “the formulation would be that both the morning and evening sentences would be both false and true by virtue of the judgement in either.”

²²⁴ Reading *wa-lā* instead of *awwal^{an}* at 309.2.

²²⁵ Khafī, “Ḥayrat al-fuḍalā’,” 308.10-309.7.

²²⁶ Farāmarz Qarāmalikī, “Aqā Ḥusayn-i Khwānsārī va-mu‘ammā-yi durūghū,” 84.25-85.6.

object of representation, i.e. in itself. *With respect to conviction attaching to it*, i.e. to the relation, *it is considered in a differentiated fashion*, and *it*, i.e. the differentiated relation, *is a representation of itself*. Therefore, the difference between the representation and the object of representation is only one of consideration, between being undifferentiated and being differentiated.

The essence of this solution, in accord with the first formulation [of the paradox],²²⁷ is [the Liar sentence] is a statement containing both representation and an object of representation, which are different. It does not entail that it represents itself [i.e. is self-referential], as the Verifier [Dawānī] has understood, since its relation can be considered according to two respects, that of being undifferentiated, such that the subject-term, predicate, and relation are all considered together—according to this consideration the statement is an object of representation—and that of being differentiated, such that the subject-term and predicate are first considered, and only then is the judgement made that the relation between them is actual or not—according to this consideration the statement is a representation. Therefore, the representation and the object of representation are distinct, and it does not follow that [the Liar] represents itself. This solution thereby goes beyond that of the Verifier [Dawānī], as [Bihārī's] auto-gloss indicates: “Just as this solution is a response to the paradox, it is also a response to the Verifier Dawānī's solution, since the reason he denies that [the Liar sentence] is a statement is that it does not represent. If, however, as you have seen, it does represent, his claim is baseless.”²²⁸

(12) Khafīrī, *Ibrat al-fuḍalā'*

In sum, the solution is that the Liar, insofar as it is a statement about itself, is false, but not insofar as it is the statement's subject-term. The Liar contains the judgement that, insofar as the Liar is a subject-term, it is false. But insofar as the judgement is so taken, it is neither true nor false, since, insofar as it is a subject-term, the Liar could be neither true nor false.²²⁹ Rather, the Liar is false insofar as it is a statement. But this entails only that ‘false’ applies to the Liar in the sense that ‘false’ is negated of the instance of its subject-term. From this negation, however, it does not follow that the Liar as a statement has ‘false’ negated from it. Rather, ‘false’ applies to it insofar as it is a statement containing a report about itself, though not insofar as it is the subject-term of that statement.²³⁰

(13) Mubārakī, *Mi'rāj al-fuhūm fī Sharḥ Sullam al-'ulūm*

There are various solutions, mentioning all of which along with their objections would excessively prolong the discussion, but this you can say: it is impossible for the Liar to be false, due to the necessary fact that the proposition is a judgement about itself and that it contains within itself ‘false.’ Hence, ‘false’ applies to the Liar in the same way that

²²⁷ Reading *al-taqrīr al-awwal* for *taqdīr al-awwal* at 102.7.

²²⁸ Mubīn, *Mir'āt al-shurūḥ*, 1321ah, 1:102.2-14.

²²⁹ Since subject-terms are not propositions, strictly speaking, and only propositions can be true or false.

²³⁰ Khafīrī, “‘Ibrat al-fuḍalā’,” 268.8-14.

something's essential constituent applies to it, and something's essential constituent applies to it necessarily in the same way that something applies to itself.

If the Liar is not false, then it must necessarily be true. 'True' means that the predicate applies to the subject-term insofar as things are in themselves in accord with the mode in which it is a predicate. If the predicate is essential, then its being true that it applies to the subject-term is also essential, and if the predicate is accidental, then its being true is also accidental. In the case of the Liar, its being true means that 'false' applies to the Liar proposition essentially, and, therefore, there is no impossibility in the Liar's truth entailing its falsehood, nor in its being both true and false, despite the fact that there is a genuine disjunction between the two, since 'true' is predicated of the Liar accidentally and false essentially. There is no contradiction here since it is a condition on contradiction that the mode of predication be the same.²³¹

(14) Mīr Dāmād, *al-Ufuq al-mubīn*

An instance is a mere instance (*sinkh al-fardiyya*) if it is something's nature merely insofar as it is that something's nature and all its particularities are removed. An instance is a specific instance (*khuṣūṣ al-fardiyya*) if it is a particular of the thing, that is, if it is one particular instance, being unconnected to the nature of an instance, i.e., being an instance.

Let us consider something narrower than a given nature, making it a species of that nature, like "man" with respect to "animal." When we do this, we find that "man" is a species only because it is merely that nature [i.e., "animal"] along with [any] differentia,²³² not because it is specified by some *specific* differentia, like "rational." It is the specificity that makes "man" the specific species it is—the nature of species insofar as it is a species does not...²³³

Haven't I already sufficiently clarified in the section, "Rule for Instances,"²³⁴ such that, if intelligence took you by the hand, you would realize that [the Liar] is an instance of its own subject-term insofar as it is composed of the nature of "everything I say this hour" along with a qualification specifying that nature, not insofar as the subject-term has the specific predicate applied to it. The latter is what it is to be a specific instance (*khuṣūṣ al-fardiyya*), not what it is to be a mere instance (*sinkh al-fardiyya*), as you learned earlier.

I do not think you will now doubt that a judgement is transferred from the subject-term's description only to its mere instances (*sinkh al-fardiyya*). A specific thing being a specific instance is a consideration distinct from its being considered merely an instance...²³⁵ A judgement is transferred from the subject-term's description to an instance by virtue of the fact that [the instance] is considered insofar as it is [merely] an instance, not insofar as it is *this* instance, specifically...²³⁶

²³¹ Mubārakī, "Mī' rāj al-fuhūm," 28b.12-21.

²³² Reading *fāṣl* for *faḍl* at 670.5.

²³³ Mīr Dāmād, *Muṣannaḳāt-i Mīr Dāmād*, 2:670.2-8.

²³⁴ The paragraph translated just above is from that section.

²³⁵ Mīr Dāmād, *Muṣannaḳāt-i Mīr Dāmād*, 2:686.1-8.

²³⁶ Mīr Dāmād, 2:686.10-11.

Hence, this statement itself [the Liar], despite whatever its specific predicate [“false”] may be, falls under the description of its own subject-term [“everything I say this hour”]. The judgement [“is false”] only applies to it [the Liar] from that aspect [namely the aspect that it is a mere instance of the description of the subject-term (*sinkh al-fardiyya*)]. [The Liar’s judgement], from this aspect, is isolated from [whatever specific] predicate [the Liar sentence may contain]. This is because the specific predicate [contained by an instance] is only considered in a specific instance (*khuṣūṣ al-fardiyya*)—but [the predicate of a proposition] is not applied [to specific instances of the subject-term]. The only way the truth [of the Liar] entails its falsehood and *vice versa* is by considering the [subject-term’s specific instances]. [This entailment does not arise when only the mere instances of the subject-term are considered (*sinkh al-fardiyya*)].²³⁷

²³⁷ Mīr Dāmād, 2:686.14-19. For another translation and analysis, see El-Rouayheb, *The Development of Arabic Logic (1200-1800)*, 149–50, 190.

The Totality of All Relations Paradox

Formulations of the Problem

The problem's formulations can, on the surface, easily be divided into two kinds: those phrased solely in terms of relations (*nisab*) and those in terms of concepts (*mafhūmāt*). The most sensible way to classify them, however, is instead in terms of the difficulty each identifies.

Contradiction

Consider the totality (*majmūʿ*) of all relations (*nisab*) (call it R). Some of the relations included in R then are, for example, the spatial relation between my bookcase and my computer, the causal relation between me and these words, and the relation of dependence between 1 and 2. More important than any of these three for the purpose of the problem, however, is the containment relation between R and each of its constituents. R contains the very containment relation between itself and any one of its constituents.

Now consider the following two facts. (1) The parts of any totality are prior to it. That is, in order to have a totality, one must first have the constituents it is made up of. Since the containment relation is a part of the totality R, it must therefore be prior to it. (2) Relations are posterior to the things they relate. In other words, a relation can only obtain between things if they have obtained. But that would mean that the containment relation between R and its constituents must be posterior to R (one of its relata). If these two facts and our inferences therefrom withstand scrutiny, a contradiction arises: the same relation is both prior and posterior to R (at the same time, from the same respect, etc.).²³⁸

The contradiction, however, is not always located in the containment relation's contradictory properties. Instead, it is sometimes identified in the fact that what was assumed a totality of all relations could not be one. The terms of the paradox assume a totality of all relations, yet we learn that the containment relation could not be part of the totality. Therefore, what was assumed to be a totality of all relations is no such totality at all.²³⁹

Absurdities concerning Relations

Instead of locating a contradiction in the fact that the totality R will be both prior and posterior to the part-whole relation between any one of its constituents and itself, the problem found in this formulation is that it is impossible for a relation to take itself as a relatum or for a relation to be prior to its relatum.²⁴⁰ Since the part-whole relation is a constituent of R, it will have to relate itself to R, which means that the relation will take itself as a relatum. On another analysis,

²³⁸ Mīr Dāmād, *Muṣannaḡāt-i Mīr Dāmād*, 2:686.21-687.1; Muḥammad Mahdī b. Abī Dharr Narāqī, *Mushkilāt al-ʿulūm* (Tehran, 1887), 58.1-4.

²³⁹ Amīr Faḍl Allāh Astarābādī, "Ḥall al-mughālaṭāt," 121.5-10, 5-32907/3, Nat'l Library and Archives of I. R. Iran, accessed February 24, 2021, <http://dl.nlai.ir/UI/f1168ed8-d338-42d3-906e-8c35ea4e00a8/LRRView.aspx>.

²⁴⁰ Walī Allāh Laknawī, *Ḥāshiyat al-Ṣadrā*, ed. Muḥammad Iḥsān Allāh Laknawī (Lucknow: n.p., 1885), 102.7. Muḥibb Allāh Bihārī and ʿAbd al-ʿAlī Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm ʿalā Sullam al-ʿulūm*, ed. ʿAbd al-Naṣīr Aḥmad al-Malībārī (Kuwait: Dār al-Ḍiyāʾ, 2017), 581.12-14. Mīr Zāhid Harawī, *al-Ḥāshiyah li-Mīr Zāhid ʿalā Sharḥ al-Mawāqif*, ed. Muḥammad Ṣādiq ʿAlī Laknawī (Lucknow: al-Maṭbaʿ al-rafiʿ al-ʿālī al-muʿzā ilā Munshī Nūl Kishūr, 1876), 75.20.

the containment relation will be prior to its relata: since the totality includes all relations, its containment relation too must be included, and all parts are prior to the whole to which they belong. Therefore, the containment relation must be prior to its relata, which contradicts the well-known fact that relations are posterior to their relata.

Absurdity that a Whole Cannot Contain its Contradictory as Part

What is considered by philosophers in the tradition to be yet another rendering of the same problem phrases things much more broadly.²⁴¹ Now the totality is made up of concepts as opposed to merely relations: Consider the totality of all concepts (*mafhūmāt*), such that none are excluded. The contradictory of this totality is also a concept, and must, therefore, also be included in it. But it follows from this that the totality will have its contradictory as a part, which is absurd.²⁴²

Depending on the formulation, the problem presents us with one of three difficulties: (1) a contradiction (something is both prior and posterior to something else, or else what was assumed a complete totality could not be so); (2) a relation is identical with or prior to its relatum (whereas we know that a relation is necessarily posterior to its relatum); (3) a whole contains its own contradictory as a part (whereas we know this to be absurd).

Attempted Solutions and Objections Thereto

There are at least two kinds of solution pursued in the sources. The first strategy is to reason that there simply could be no such totality, and so there is no problem.²⁴³ The second is instead to explain why no problem would arise even if there were such a totality. I have not organized the solutions according to formulation, since it seems to me that few are in principle restricted to one formulation or another, even if every solution was proposed for only one formulation.

1. There Could Be No Such Totality

(a) The very concept of such a totality is contradictory

This solution contends that there is no such totality as R since R is contradictory in its very concept. We will recall that R, according to one of the formulations, was specified as the totality of all

²⁴¹ Laknawī, *Hāshiyat al-Ṣadrā*, 102.4-7.

²⁴² Bihārī and Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, 581.1-3, 581.8-10.

²⁴³ This may be reminiscent of a common response to the barber-framing of Russell's Paradox: "The proper conclusion to draw is just that there is no such barber. We are confronted with nothing more mysterious than what logicians have been referring to for a couple of thousand years as a *reductio ad absurdum*." Quine, "The Ways of Paradox," 2. Quine misses the point that the problem has nothing *per se* to do with the barber but with the rule the barber has set for himself: I will shave anyone who does not shave himself. The paradox is that what appears to be a rule that can unproblematically apply to any and all instances breaks when applied to the person who follows the rule—will the barber shave himself?

concepts.²⁴⁴ The concept “concept,” however, has an undelimited number of instances, and therefore another instance could always be added to its extension. Yet the specification of R runs contrary to this fact by demanding that *all* concepts be included, implying that a limit could be imposed on the number of concepts.

In consequence, to consider “the totality of all concepts such that none are excluded” is to consider something impossible, since one part of the specification (namely, the nature of “concepts”) dictates that the totality be infinite, while the other part (“such that none are excluded”) dictates that it be finite. Since this totality is impossible, it has no instance in the external world. Thus, it is only an impossible concept invented by the mind, and so it is unproblematic if it entails other impossibilities, like something having its contradictory as a part or a relation being identical to its relatum.²⁴⁵

Objection: Counter-example: the totality of all whole numbers

Consider the totality of all whole numbers such that none are excluded. Although the whole numbers stop at no limit, we can nonetheless make true judgements about the totality of all of them. Therefore, the fact that there are limitless concepts prevents one neither from considering the totality of all concepts nor from passing judgments on it, just as the fact that there are limitless whole numbers prevents one neither from considering the totality of all whole numbers nor from passing judgements on it.²⁴⁶

(b) *There is no such totality*

A much simpler version of (a) consists of the claim that there is no such totality, neither in the external world nor in the mind. That there is no such thing in the external world is simply assumed—I have seen no attempt to substantiate it. There is no such totality in the mind because it is only the concept of such a totality, not the totality itself, that obtains in the mind. Indeed, the human mind could not conceive of all the infinite individual parts composing the totality (*bi-l-tafṣīl*) in order for it to exist in the mind.²⁴⁷

*Objection: Counter-example*²⁴⁸

²⁴⁴ Although this solution is phrased in terms of the “all concepts” formulation, as opposed to the “all relations” one, it should be clear that it can nonetheless be made applicable to the other merely by exchanging “concepts” for “relations.”

²⁴⁵ Ḥamd Allāh Sandīlī, *Sharḥ Sullam Mawlawī Ḥamd Allāh*, 2nd ed. (Kānpūr: n.p., 1862), 148.5-10; Ibn Mubārakshāh et al., *Sharḥ Hikmat al-‘ayn ma‘a ḥawāshī*, 293.6-22. In the latter source, the solution is Mīrzā Jān al-Bāghnawī’s.

²⁴⁶ Ibn Muḥammad Dā‘im, *Kitāb Sullam Al-‘ulūm Wa-Ḥāshiyatuhu l-Mashhūra Bi-l-Qāḍī Ma‘a Minhuyātihi*, 258.7-14.

²⁴⁷ Harawī, *Ḥāshiyat Sharḥ al-Mawāqif*, 75.20-22.

²⁴⁸ Mīrzā Jān al-Bāghnawī considers an analogous objection regarding the totality of things subject to God’s power. He makes some attempts to resolve it but seems to be aware that he gets nowhere. Ibn Mubārakshāh et al., *Sharḥ Hikmat al-‘ayn ma‘a ḥawāshī*, 292.16-27.

Both solutions (a) and (b) are susceptible to a theological objection. Consider the totality of all things known to God. Certainly, this is an extant totality encompassing the totality of all concepts and the totality of all relations. Furthermore, since the negation of this totality is also known to God (on pain of divine ignorance), it must contain its contradictory as a part.²⁴⁹

(c) *No such totality could be a relatum*

To suppose that such a totality could be a relatum and one of its parts another relatum would be to suppose a contradiction. According to assumption, there is no concept (or relation) left out of the totality; therefore, there could be no relation left to relate the totality to any of its parts, since every relation must be external to its relata, but, by assumption, all are internal to the totality. The paradox amounts to asking, “Is the relation between the totality of all concepts (relations) and any of its parts—which relation itself is both internal and external to the totality—internal to the totality or external to it?”²⁵⁰

2. No Problem Arises from Assuming Such a Totality

(a) *The contradictory properties are both true in different respects*

This strategy explains that any contradiction or absurdity is only apparent, since, in fact, the condition of “unity of subject” for a contradiction to obtain has not been met. Each contradictory property is indeed true of the containment relation but in a different respect. The respect in which each contradictory property is true differs according to formulation.

In the case of formulations where the difficulty identified is that a whole contains its contradictory as part, the solution is phrased as follows: it is only insofar as “the negation of the totality of all concepts” is a concept that it figures in the totality as a part, and only insofar as it is a negation of “the totality of all concepts” that it is the totality’s contradictory.²⁵¹ Likewise, for the contradiction formulation, it is only insofar as the part-whole relation is a means for considering how a part of R belongs to R that it is posterior to R, and only insofar as it is considered to be a relation that it is part of the totality and so prior to it.²⁵² Another way to draw the distinction and so another version of the solution is that it is only insofar as the part-whole relation is specified by its relata that it is posterior to the totality, and only insofar as it is a mere relation that it is part of the totality and so prior to it.²⁵³

Objection: Relations are individuated by their relata

²⁴⁹ Bihārī and Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm*, 582.14-19. This objection can easily be modified to apply to the other two formulations of the problem. For a resolution insofar as it applies to one of the formulations, see solution 2b.

²⁵⁰ Narāqī, *Mushkilāt al-‘ulūm*, 57.5-11.

²⁵¹ Mullā Muḥammad Mubīn, *Kitāb Mir’āt al-shurūḥ*, vol. 2 (Egypt, 1909), 124.12-14.

²⁵² Mubīn, 2:124.14-15.

²⁵³ Ibn Mubārakshāh et al., *Sharḥ Hikmat al-‘ayn ma’a ḥawāshī*, 292n1. The solution is Jurjānī’s.

The mere fact that we consider a relation from one respect and then from another cannot be sufficient reason for the fact that it belongs to a totality and so is prior to it or does not belong and so is posterior. Indeed, a relation is individuated by means of its relata, and, therefore, once a relation is specified by particular relata and thus contained in the totality, it could not fall out of it merely by being considered from another respect.²⁵⁴

Objection: The totality of relations specified by their relata

The solution cannot accommodate a simple rephrasing of the problem: instead of making the totality a totality of relations *simpliciter*, make it the totality of relations specified by their relata. Therefore, no ambiguity is left open for deploying a distinction of respects, and the problem remains unsolved.²⁵⁵

(b) *A whole containing its own contradictory as part is not impossible*

This last solution is perhaps the only one of the bunch that cannot obviously be extended to other formulations. It therefore only serves as a solution for those formulations where the paradox is grounded in the intuition that a whole cannot contain its own contradictory as part. The general strategy can be described as one of carefully explaining the relevant terms in order to show that the ostensible problem is rather more prosaic than paradoxical.

The analysis takes the form of a distinction. Only conceptual wholes cannot have their contradictories as a part; external wholes, on the other hand, are constrained by no such limitation. Take, for example, black paint, an external whole. It could very well contain its contradictory, white paint, among the constituent parts that compose it. Likewise, granting that the problematic totalities under consideration could exist, the fact that they would contain their contradictories as parts would be no reason to balk. It would therefore be perfectly fine for the totality of God's knowledge—recalling the aforementioned counter example—to contain its very contradictory as a part.²⁵⁶

Summary & Analysis

Most authors seem to have understood this paradox as one of relations, which, while correct, is also potentially misleading. All members of a totality are *related* in a certain way such that they can be called a totality at all, and totalities are, therefore, entities constituted by relations. In this sense, the paradox is in fact also one about totalities. Consequently, the paradox is most threatening, and therefore most enlightening, when formulated as the totality of all relations (as opposed to concepts). Formulations given solely in terms of relations immediately block solutions built on drawing a distinction between the troublesome containment relation conceived of as a relation on the one hand and as a concept on the other (see, e.g., some variants of 2a).

²⁵⁴ Ibn Mubārakshāh et al., 292.7-13. The objection is Mīrzā Jān al-Bāghnawī's, in his supergloss on Jurjānī's gloss.

²⁵⁵ Laknawī, *Hāshiyat al-Ṣadrā*, 103.20-23.

²⁵⁶ Bihārī and Baḥr al-'Ulūm, *Sharḥ Baḥr al-'Ulūm*, 582.ultimate-583.7.

While most of the solutions are plausible, they nonetheless face difficult objections. Solution 1c is an exception. I could locate no objection to it in the literature. According to 1c, it is no problem if absurdities follow from the paradox scenario, since the scenario itself is absurd. Were there such a totality as R, it would be absurd to suppose that R could be related to one of its parts. Since R is, by definition, the totality of all relations, and since every relation is distinct from its relata, the relation between R and one of its parts could not be external to it, yet it would have to be for R to be its relatum.

Solution 1c enjoys a great advantage over its relative solution 1a. The latter locates R's absurdity in the fact that it purports to be a totality and therefore to be finite, yet relations and concepts have, by their very nature, an undelimited number of instances. Unfortunately, such a solution throws out the baby with the bath water: by ridding ourselves of the paradox in this way, we also get rid of any totality formed of a description with an undelimited number of instances. Hence, we would be eliminating the totality of all numbers, the totality of all possible things, and the totality of all propositions, among others.²⁵⁷ While solution 1c follows the same pattern as 1a in that both hold that the very formulation of the paradox conceals absurdity, it differs from it in locating the absurdity in R's role as relatum and what that entails, as opposed to locating it in the contradiction of an infinite totality.

Although it is not raised in the literature, I think there is at least one plausible objection to solution 1c. If what it is for something to be a totality is for its members to be related to it by a containment relation, then solution 1c amounts to little more than the mere denial that a totality could be formed of all relations. 1c explains that it is absurd for there to be a relation lying outside of R that relates R to its parts, since we assumed at outset that R contains all relations. Yet if R could not be related to its parts by a containment relation, then it could not be a totality in the first place. Hence, the solution does little more than assert that there could be no totality of all relations.

According to this criticism, 1c does not account for our intuition that we can form a totality from any description, including relations. We can unproblematically conceive of the totality of all red things, of all chairs, of all stones, etc.—we should *prima facie* be able to do the same for relations and concepts.²⁵⁸ In addition to explaining the absurdity that follows on supposing a totality of all relations that is related to each of its parts by yet another relation, it should also explain what it is about relations and concepts themselves that makes them different from other objects, such that they suffer from this problem while others do not.

Indeed, not all objects can be used to formulate the paradox. “Red things,” for example, does not work. Take the totality of all red things. There will be a containment relation between each red thing and the totality itself. This containment relation is *not* a red thing, and hence no absurdity arises. The necessary condition for generating the paradox, therefore, is that the description of the object used to form the totality be true of a relation. Descriptions that can be

²⁵⁷ If the wide-ranging significance of this fact is not plain enough as it stands, recall that one of Ibn Sīnā's proofs for the existence of a necessary being crucially relies on the notion of a “totality of all possible things.” If no such totality could be validly formed, the proof would rot at the roots.

²⁵⁸ Although it is not so given in the texts, the paradox can also be formulated using “things,” “existents,” “possibilia,” or “individuals”: Consider the totality of all individuals. There will be a relation between each individual contained and the totality. Each containment relation that stands between them is also an individual and therefore belongs in the totality. All the parts of the totality must be prior to it, by virtue of being parts, but some of the members, namely the relations, must be posterior to it, since it is one of their relata. This is a contradiction.

subsumed under none of the categories—transcendentals—can all be used to generate the paradox (existent, thing, one, possible). Philosophers of the tradition fiercely debated how to categorize knowledge (*ma'lūm*) and hence concepts. Are they qualities? Substances? Or of the same category as whatever they represent?²⁵⁹ Given such debates, it is plausible to argue that a relation would be a concept, and therefore “concept” too can be used to generate the paradox.

Even though not every description can be used to formulate the paradox, the paradox is significant about totalities. It has nothing, essentially, to do with the description used to form the totality. Relations are a persistent feature of all formulations only because there will always be a containment relation if there is a totality. There could be no totality without one. The paradox teaches us that no totality formed of relations or of any predicate true of a relation will be sound.

The totality of all relations (or the totality of any predicate true of a relation) distinguishes itself from other totalities in yet another way: while such totalities produce contradictions if the description is taken absolutely, without restriction, they can nonetheless be legitimately formed when their objects are limited to those which exist or existed before the time the totality was formed. That is, the totality of all those relations existing before the totality causes no problem at all. Other totalities (like the totality of all red things), by contrast, can be taken absolutely—we need not qualify the description, “red things” with “existing before the totality.”

It is interesting to think about how the problem bears on our knowledge of the extension of certain terms, even if this issue is not raised by philosophers of the tradition. A common way of thinking of the extension of a term *x* is as the totality of all objects of which *x* is true, even if only in potentiality. Yet if the extension of a term is to be thought of as the totality of those objects of which it is true, it follows as a consequence of the paradox that certain terms are somehow flawed: their extensions cannot properly be defined. Hence, a further way that the paradox is paradoxical is that it gives us reason to think there is something wrong with the term “relation” and any term true of it (and, by extension, something wrong with totalities).

The next most promising solution is 2a, which argues that the contradictory conclusions that the paradox entails are not genuinely contradictory but only apparently so. They result from a failure to observe the conditions of contradiction. The one contradictory property (“being prior to the totality”) belongs to the containment relation considered according to one respect and the other (“being posterior to the totality”) belongs to it according to another.

There is some surface level implausibility to the solution, unfortunately. It is unclear how the fact that it can be considered from one respect as well as from another negates the fact that the very same relation bears contradictory properties. Even if we grant that the containment relation is part of and prior to *R* only insofar as it is merely a relation and posterior to it only insofar as it is a means for observing how the parts of *R* belong to it, or instead prior insofar as it is a mere relation and posterior insofar as it is individuated by its relata, the fact remains that there is only one relation. How could the same relation be both in and out of the same totality, even if it is in it on account of one description and out of it on account of another? The objection that the very identity of a relation is a function of its relata gets at the same point.

²⁵⁹ For such debates, see Sulaymān, ‘Abbās Muḥammad Ḥasan, “Ishkāliyyat al-wujūd al-dhihnī fī l-falsafa al-islāmiyya,” in *Dirāsāt fī l-falsafa al-islāmiyya* (Alexandria: Dār al-ma‘rifat al-jāmi‘iyya, 2001), 147–90.

The final solution described, 2b, is also rather plausible. Of course, its plausibility comes at the price of limitation: the solution applies only to those formulations of the paradox in which the difficulty to overcome is that a whole contains its contradictory as part. It consists of a partial concession to the paradox: Some wholes *can* contain their contradictories as a part. Only concepts cannot contain the contradictory of their own content as a constitutive conceptual part. External wholes, however, can very well contain a contradictory part.

It is worth noting a connection to the Liar paradox. It can be argued that Liar sentences generate their own truth-value. If a Liar sentence is assumed false (or true), that very assumption generates another truth-value, true (or false). Likewise, the totality of all relations generates further containment relations after it has been formed (i.e., after what was supposed to be all of the containment relations were gathered).

Historical Development of the Problem

Kātibī (d. 1276) is the first I have seen pose the problem. He raises it in the section of *Hikmat al-‘ayn* on whether necessity is positive or privative. Kātibī, who held that it is positive, is led to grant that, were it positive, it would be a relation, a relation between something’s quiddity and its existence. If that were so, however, he would apparently be forced to grant further that the necessity of a necessary being would exist before the necessary being itself does in order for the necessity relation to relate the necessary being’s quiddity to its existence, since were the necessary being to exist before its necessity, it could hardly be a necessary being.²⁶⁰

Further, if necessity (a relation) must exist before the necessary being itself (the relatum), it would imply that a relation need not be posterior to its relata. At this juncture, he raises the set of all relations problem as a counter-example in order to vitiate the objection that a relation cannot need be posterior to its relatum: no impossibility need follow, since the totality of all relations scenario shows that a relation need not be posterior to its relata. Therefore, it is possible for a necessary being (the relatum) to be identical with necessity itself (the relation), and, furthermore, necessity need not exist prior to the necessary being in order to relate its quiddity to existence.²⁶¹

The earliest commentators of *Hikmat al-‘ayn* have little to say about the problem. Hillī (d. 1325), who appears to have been the work’s first *shāriḥ* (commentator),²⁶² dedicates a succinct 2a-style solution to it, though only insofar as it is a counter-example adduced by Kātibī rather than as an independent problem.²⁶³ Ibn Mubārakshāh (al-Mīrak al-Bukhārī, fl. 1354) is even less interested, not really even proffering a solution.²⁶⁴ This is perhaps unsurprising, since the problem itself is by no means integral to the discussion in which it happens to figure.²⁶⁵ It is, after all,

²⁶⁰ Shams al-Dīn Ibn Mubārakshāh et al., *Hāshiyat Sharḥ Hikmat al-‘ayn* (Kazan: al-Maṭba‘a al-Karīmiyya, 1901), 74.8-9.

²⁶¹ Ibn Mubārakshāh et al., *Sharḥ Hikmat al-‘ayn ma‘a ḥawāshī*, 292-93.

²⁶² See the preface to Najm al-Dīn al-Kātibī and Shams al-Dīn Ibn Mubārakshāh, *Sharḥ Hikmat al-‘ayn*, ed. Ja‘far Zāhedī (Mu‘assasah-ye chāp-o enteshārāt-o gerāfik-e dāneshgāh-e ferdowsī, 1974), 13.20: “*Īdāḥ al-maqāṣid* was the first commentary on *Hikmat al-‘ayn*...”

²⁶³ al-Ḥasan b. Yūsuf b. al-Muṭahhar Hillī and ‘Alī b. ‘Umar Kātibī, *Īdāḥ al-maqāṣid min Hikmat ‘ayn al-qawā‘id*, ed. ‘Alī Naqī Munzavī (Chāpkhāneh-yi Dāneshgāh, 1959), 78.

²⁶⁴ Kātibī and Ibn Mubārakshāh, *Sharḥ Hikmat al-‘ayn*, 84.

²⁶⁵ Some academics have pointed out that an identifying characteristic of early commentaries, such as Hillī’s on Ṭūsī’s *Tajrīd al-i‘tiqād*, is that they tend to dispense with commenting the base-text’s introduction, unlike later

merely one way of resolving an objection to a position on the real issue, namely whether necessity is positive or privative. At last, al-Sharīf al-Jurjānī (d. 1413), here Ibn Mubārakshāh's *muḥashshī* (glossator), dedicates a note to the problem, giving a more extensive 2a-type solution.²⁶⁶ Mīrzā Jān al-Bāghnawī (d. 1587) continues Jurjānī's attitude of treating the difficulty as an independent problem in his supergloss. He seems to have been the first to propose solution 1a.

The paradox appears outside of the *Hikmat al-ʿayn* commentary tradition in the work of Mīr Dāmād (d. 1631).²⁶⁷ In his *al-Ufuq al-mubīn*, he treats the problem as one in logic.²⁶⁸ Instead of discussing it in a section on modality, he attempts to solve it as a case parallel to the Liar paradox, in order, apparently, to demonstrate how robust his solution to the Liar is. This is particularly interesting because it seems to indicate that he sees the two problems as closely related. That Russell and some other later philosophers have thought that there is a relationship between Russell's Paradox and the Liar would give further credence to the argument below that the totality of all relations paradox is a mereological relative of Russell's.²⁶⁹

In Mughal India, the totality of all relations paradox is treated in Mīr Zāhid al-Harawī's (d. 1689) gloss on Jurjānī's *Sharḥ al-Mawāqif fī ʿilm al-kalām*. Mīr Zāhid is the first author I have found to raise the issue in that commentary tradition. It is again in the context of modality that the puzzle is raised and again at the suggestion that necessity is a relation between existence and a quiddity. While Mīr Zāhid's solution is rather unimpressive (of the 1b variety), his formulation of the problem is interesting and perhaps even novel. Unlike earlier formulations I have seen, where the totality is made up of relations, Mīr Zāhid's is made up of concepts. Besides this, however, the formulation is identical, again invoking the containment relation—which, of course, is also a concept—to undermine our intuition that a relation is inevitably posterior to its relata and never identical with either of them.

Hence, we have here a kind of hybrid formulation: Mīr Zāhid phrases the problem in terms of concepts, but the difficulty he identifies is not the fact that a totality contains its contradictory as part—as is normally the case for the formulation using concepts—it is instead one of the difficulties usually paired with the relations phrasing. Calling this a 'hybrid' formulation may be a bit too charitable, however. It seems more accurate to say that it is a mixed-up or confused formulation, since substituting 'concepts' for 'relations' adds nothing to the problem. If anything, it takes away from it by making solutions of kind 2a more plausible, thereby deceptively

commentaries, in which there are lengthy and often metaphysical or theological discussions inserted as commentary on it. One may hypothesize that the fact that Ḥillī and Ibn Mubārakshāh ignore problems that are—even if interesting in their own right—in truth, only incidental to the issue at hand, whereas al-Jurjānī takes up every little issue in the text, even if it is obvious that the author never intended it to be taken up for consideration in its own right, is another characteristic distinguishing earlier commentaries from later ones.

²⁶⁶ For a full translation, see the appendix.

²⁶⁷ Mīr Dāmād, *Muṣannaḥāt-i Mīr Dāmād*, 2:687.

²⁶⁸ A contemporary of Dāmād's includes the problem directly after the Liar paradox in a treatise on logical sophisms: Amīr Faḍl Allāh Astarābādī, "Ḥall al-mughālaṭāt," 121. The presence of the problem in this treatise hints that it may have had a more vibrant life in the Safavid world than I have been able here to document.

²⁶⁹ Consensus, however, seems to be on the side of the view that Russell's Paradox and the Liar are not substantively related and are instead distinct problems. That there is a consensus, of course, does not affect the point at hand, namely that the relation between the Liar and Russell's Paradox has been a matter of philosophical discussion.

simplifying the difficulty it would otherwise present were it phrased solely in terms of relations.²⁷⁰ At least one later Indian logician and one Iranian reproduce this “mix-up.”²⁷¹

While Mīr Zāhid may have been the first to introduce this problem into the *Mawāqif* commentary tradition, he was perhaps also the last. Neither subsequent Indian glossators of Jurjānī’s *Sharḥ* nor Mīr Zāhid’s own glossators engage the problem. While the former—e.g. ‘Abd al-Ḥakīm Siyālkūtī (d. 1656/7)—comment without raising the problem, the latter—e.g. ‘Abd al-‘Alī Baḥr al-‘Ulūm or Faḍl-i Ḥaqq al-Khayrābādī—end their commentaries well before the opportunity for discussion presents itself. It seems that later scholars were interested only in the first section (*marṣad*) of Mīr Zāhid’s gloss, which is on the topic of existence and non-existence, and includes extensive discussions of mental existence (*wujūd dhihnī*).

By contrast, the commentary tradition on the *Sullam al-‘ulūm* represents the venue where this puzzle was perhaps most extensively discussed. Unfortunately, I have found no commentator who gives the relations-based formulation that threatens contradiction. Instead, it is only formulations which contravene our intuitions about relations and their relata, and the fact that a whole cannot contain its contradictory as part. Despite this philosophical demerit, the presentation in this tradition is, historically speaking, particularly interesting. The author of the base-text, Muḥibb Allāh al-Bihārī, seems to be the first to have formulated the puzzle such that the difficulty to be resolved is that a whole contains its contradictory as part.²⁷²

Still of greater historical interest is the problem’s radical shift in location. We will recall that, in its earliest days of discussion, the paradox was found in sections on general metaphysics (*umūr ‘amma*), embedded in the question on whether necessity was a positive or privative property. Now, in the *Sullam al-‘ulūm* commentary tradition, it is located in a logic textbook in the section on contradiction, and identified as a “doubt” (*shakk*) or “difficulty” (*ishkāl*) pertaining thereto. What began, therefore, as a problem in metaphysics is, in the later Indian context, chiefly treated as one in logic. The same was true, as we noticed before, in the case of earlier Safavid philosophers Mīr Dāmād—who discusses the problem as an afterthought to the Liar in the predication section of *al-Ufuq al-mubīn*—and Amīr Faḍl Allāh Astarābādī—who includes it in his treatise on logical sophisms.

Commentators of the *Sullam* held almost every solution devised prior to them and appear even to have devised one of their own. Many authors give at least two, one of which is inevitably the “contradictory concept” (1a) solution advocated by Bihārī in the *matn* (base-text). Although there were several original contributions made by the commentators, they were obviously acquainted with prior literature. In his *Suddat al-‘ulūm*, al-Anṣārī al-Sihālī reproduces the relevant section from Mīr Zāhid’s gloss on Jurjānī’s *Sharḥ al-Mawāqif*,²⁷³ and Qāḍī Mubārak’s commentary leans heavily upon Mīr Dāmād’s wording in *al-Ufuq al-mubīn*.²⁷⁴ Baḥr al-‘Ulūm, inspired by his teacher Qāḍī Mubārak, seems to be the first to formulate solution 2b (“A whole

²⁷⁰ Compare the 2a solution by Jurjānī with the one by Mullā Mubīn, both translated in the appendix.

²⁷¹ For example, Mubīn, *Mīr ‘āt al-shurūḥ*, 1321ah, 1:123.22; Narāqī, *Mushkilāt al-‘ulūm*, 58.1-4. Ḥamd Allāh, Baḥr al-‘Ulūm, Faḍl-i Ḥaqq al-Khayrābādī, and Walī Allāh al-Laknawī all give the standard relations formulation, as do the Safavid authors here studied.

²⁷² So Baḥr al-‘Ulūm tells us in a marginal note, quoting Bihārī’s lost self-commentary. See *Sharḥ Baḥr al-‘Ulūm*, 581n1.

²⁷³ Al-Anṣārī al-Sihālī, “Suddat al-‘ulūm,” 347.13-17. Compare with Harawī, *Hāshiyat Sharḥ al-Mawāqif*, 75.19-21.

²⁷⁴ Ibn Muḥammad Dā’im, *Kitāb Sullam Al-‘ulūm Wa-Hāshiyatuhu l-Mashhūra Bi-l-Qāḍī Ma’a Minhuyātihi*, 257.13 & 257.17-21. Compare with Mīr Dāmād, *Muṣannafāt-i Mīr Dāmād*, 2:686.21-687.6.

containing its own contradictory as part is not impossible”),²⁷⁵ and Qāḍī Mubārak raises a crushing and, as far as I know, original objection against 1a-style solutions.

The problem does not remain only in books on logic but even makes its way into Mughal commentaries on physics (*ṭabī‘iyyāt*). Walī Allāh al-Laknawī, one of the many glossators of Mullā Ṣadrā’s (d. 1640) *Sharḥ Hidāyat al-ḥikma*, raises the paradox in the discussion on atomism as one difficulty among several facing those who deny atomism and advocate infinite divisibility. Laknawī’s choice of phrase clarifies the link between the totality of all relations/concepts paradox and atomism: “The upshot of the solution is that the totality of a body’s parts such that none are excluded is finite...”²⁷⁶ This is of course the very same formula used to pose the totality of all relations/concepts paradox (“The totality of all relations such that none are excluded”). Aside from mere verbal similarities, the two problems have much in common conceptually: both deal with totalities (one of material parts, the other of concepts) and infinity. Most significantly, however, Laknawī believes that all the difficulties he raises (including the totality of all relations/concepts paradox) are connected by the same solution.²⁷⁷

The paradox continued to be discussed in the post-Safavid world, though how extensively, it is too soon to say, due to the fact that many post-Safavid logical and philosophical works remain in manuscript and are relatively inaccessible as a result of contemporary political tensions. What can be said for sure, however, is that it does find a place in a potpourri style work by Muḥammad Mahdī b. Abī Dharr al-Narāqī (al-Muḥaqqiq al-Narāqī, d. 1795), which covers difficult questions in exegesis, ḥadīth, philosophy, logic, geometry, medicine, Arabic, etc.²⁷⁸

Narāqī’s contribution is noteworthy for a few reasons. The first is that he proposes a solution I have encountered in no earlier tract (1c). The second is that the paradox is an (extended) after thought to another, the “donkey paradox” (*shubha ḥimāriyya*). It goes as follows: Does something that—whether it exists or not—entails that Zayd is a donkey exist or not? In either case, an impossibility is entailed (Zayd is a donkey).²⁷⁹ Narāqī was not the first to relate these two paradoxes, however. He was preceded on that point by Mīrzā Jān al-Bāghnawī.²⁸⁰ Narāqī and Bāghnawī think that this paradox shares the same solution as the totality of all relations/concepts paradox, much as Mīr Dāmād thought the Liar did, and Walī Allāh al-Laknawī difficulties with anti-atomist positions in physics.²⁸¹

The fact that at least three philosophers link this paradox to another, along with the not entirely unrelated fact that the paradox appears in such a variety of disciplines (metaphysics, physics, logic), suggests that it was particularly slippery. Scholars could not quite figure out how

²⁷⁵ Bihārī and Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm*, 582.ultimate-583.7. Compare with Ibn Muḥammad Dā’im, *Kitāb Sullam Al-‘ulūm Wa-Ḥāshiyatuhu l-Mashhūra Bi-l-Qāḍī Ma‘a Minhuyātihi*, 257.5-13.

²⁷⁶ Laknawī, *Ḥāshiyat al-Ṣadrā*, 102.9.

²⁷⁷ Laknawī, 102.7-8: “Such are multiple formulations of the difficulty (*shubha*). The solution to all, however, is but one...”

²⁷⁸ Narāqī, *Mushkilāt al-‘ulūm*, 1.

²⁷⁹ Narāqī, 56.16-18.

²⁸⁰ Ibn Mubārakshāh et al., *Sharḥ Hikmat al-‘ayn ma‘a ḥawāshī*, 293.16-18.

²⁸¹ A certain Mullā M. N. Shīrāzī connects the problem to both the donkey paradox and anti-atomist difficulties (among others pertaining to infinite totalities) in a one-page entry on the paradox. It is not clear to me if it belongs to some other work and is merely excerpted or if this is simply a free standing note by the author. Mullā M. N. Shīrāzī, “n. t.,” 85, 5-32907/3, Nat’l Library and Archives of I. R. Iran, accessed February 24, 2021, <http://dl.nlai.ir/UI/f1168ed8-d338-42d3-906e-8c35ea4e00a8/LRRView.aspx>.

to categorize it. Unfortunately, no scholar ever explains what it is that unites two such paradoxes. In general, the only explanation given for the fact one paradox is linked to another is that they share a solution.

From the 13th to the 18th century, the Arabic philosophical tradition developed three significant formulations of and five solutions to the totality of all relations/concepts paradox. This activity appears to have been concentrated in later centuries. Until the 14th, the paradox remained in gestation: it was barely formulated as an independent problem and had only received a single cursory solution at the hands of Ḥillī (d. 1325). It is with Jurjānī (d. 1413) that the problem is first treated as an important difficulty in its own right, and he spends a few paragraphs elaborating solution 2a. Around a hundred and fifty years later, Mīrẓā Jān al-Bāghnawī (d. 1587) criticizes Jurjānī's solution and develops his own (1a). From the 17th to the end of the 18th centuries, three more solutions, in addition to two more formulations and a couple of objections to prior solutions, are developed mostly by Mughal philosophers (1b, 2b) but also by Qājār philosophers (1c). These last two centuries appear to be, from my limited survey of the literature, the most dynamic.

The extensive treatment the paradox received is all the more surprising in light of the fact that it was likely considered less important than others. One indicator of its minor status is that there appear to be no independent treatises dedicated to it, while this was exceedingly common for the Liar paradox and the paradox of the absolute unknown, among others. Instead, the totality of all relations/concepts paradox was always mixed into larger works. Another indicator is that it was frequently treated as an afterthought to some related difficulty. Nonetheless, despite its secondary status, the problem warranted a response from the tradition comparable to those of other more popular paradoxes. Output was robust in terms of solutions, respectable in terms of formulations, and lacked only in objections. For whatever reason, there appears to have been less criticism of alternative solutions.

Given the fact that this paradox was not even posed until the beginning of the post-classical period and that it was not taken seriously as a problem in its own right until the 14th century, it could hardly serve as evidence of post-classical decline. Indeed, it is another instance of continued innovation. The later tradition proposed and developed an entirely new problem and then devised and debated potential solutions. What is more, this paradox, like the paradox of the absolute unknown, was apparently indigenous to the Islamic world: there is, as of yet, no evidence to suggest that it was inherited from Greek or any other foreign civilization.

Further, the Islamic world arguably beat Russell to his eponymous paradox—by at least five centuries. While I would not wish here to argue that the two paradoxes are completely identical, there is a strong case to be made that they are identical functionally. By this I mean that both paradoxes serve to upset the same intuition: both show that not every property can be used to form a sound totality (or set). Russell's Paradox does this using the property "being a set that does not contain itself" and the totality of all relations paradox does it using the property "being a relation." Each involves some auxiliary assumptions assisting in creating the paradox. In the case of Russell's Paradox, these include the assumption that self-containing sets are even coherent; in the case of the totality of all relations paradox, they include the idea that the parts of a whole must be prior to it and that a relation must be posterior to its relata.

In light of the research presented in this chapter, the hypothesis of philosophical stagnation in the post-classical Islamic world appears even more untenable. Perhaps a reasonable objection by a defender of that hypothesis would be that the chronology here given is subject to further research. It may be that some of the solutions I report as being discovered very late (for example, 1c or 2b) were in fact discovered a century or two earlier, in the 16th as opposed to the 18th century. While this is possible, it does little to harm the broader conclusion that, between the 12th and 16th centuries, original philosophical research was extensively—even if slowly—conducted in the Islamic world. Furthermore, the review of the literature on this paradox was hardly exhaustive: it does not even account for the western Islamic world.

Appendix of Translations

Formulation: contradiction, al-Muḥaqqiq al-Narāqī, *Mushkilāt al-‘ulūm*

Let us take the totality of all concepts such that not a single concept is excluded. Then we relate this totality—from which no concept is absent—to its part. [Upon so doing,] there is no doubt that a relation will obtain between the totality and its part. This relation will be both internal to the totality, since it was assumed²⁸² that no concept whatsoever was excluded from it, as well as external to it, since relations are external to their relata.²⁸³

Formulation: contradiction (of assumption), Astarābādī, *Ḥall al-mughālaṭāt*

Take the totality of all relations such that none are excluded, all included in it, each a part of it, and the whole bears a relation to each of its parts. It is necessary that this relation is not [contained in] the whole, since a relation is external to its relata, from which it follows that the totality is not, as assumed, a totality.²⁸⁴

Formulation: identity of relation & relata, Baḥr al-‘Ulūm, *Sharḥ Sullam al-‘ulūm*

The formulation: Let us take all relations such that no relation is excluded. This whole has a relation to its parts, among which is the very same relation, and, therefore, the whole bears this relation to the relation itself, from which it follows that a relation can be identical to its relatum, despite what is said about how they must be distinct.²⁸⁵

Formulation: relation prior to relatum, Mīr Zāhid, *Ḥāshiyat Sharḥ al-Mawāqif*

If one considered the totality of all concepts such that no concept was excluded—whether [the concept] be considered under the description [“concept”] or not, and whether it have obtained by the time of the consideration or not—the totality would doubtless bear a relation to each of its parts. Now, this relation could not be external to the totality—else, it wouldn’t be “such that no concept was excluded”—and therefore it follows that a relation can be prior to one of its relata.²⁸⁶

Formulation: contradictory part, Bihārī and Baḥr al-‘Ulūm, *Sharḥ Sullam al-‘ulūm*

If we take the totality of concepts such that none are excluded, which thereby yields a concept, the negation of this totality is its contradictory, and that negation is also a concept

²⁸² Reading *li-farḍihi* for *li-‘ardihi*.

²⁸³ Narāqī, *Mushkilāt al-‘ulūm*, 58.1-4.

²⁸⁴ Amīr Faḍl Allāh Astarābādī, “Ḥall al-mughālaṭāt,” 121.5-10.

²⁸⁵ Bihārī and Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm*, 581.12-14.

²⁸⁶ Harawī, *Ḥāshiyat Sharḥ al-Mawāqif*, 75.17-20.

*and is included in the totality. Therefore, the contradictory of the totality figures as its part, which is absurd.*²⁸⁷

(1a) Ḥamd Allāh, *Sharḥ Sullam al-‘ulūm*

The response, in sum: since concepts are not infinite in actuality but are rather infinite in the way that numbers, the parts of a body, and the things under God’s power are—in the sense that they do not stop increasing at any limit—to consider “the totality of concepts such that none are left out” is to consider two contradictory descriptions. “The totality of concepts” demands that it be possible to augment the totality with more concepts, while “such that none are left out” demands that it be impossible. The description, “The totality of concepts such that none are left out” is akin to the description, “The totality that is both infinite and not infinite.” Hence, this totality is not instantiated such that it could be a whole that contains its contradictory as part. Indeed, there is nothing even in the mind except the invented concept compounded of contradictory descriptions.²⁸⁸

(1b) Walī Allāh al-Laknawī, *Ḥāshiyat al-Ṣadrā*

As for talk of the totality of relations: one may respond to this by saying that the relation can be considered in two ways. The first is that it be considered insofar as it takes particular relata. So considered it is posterior to its relata. The second is that it be considered insofar as it is merely a relation, ignoring its particular relata. Under this consideration it is necessarily included in the totality, due to the fact that it is relations insofar as they are relations that [form the totality], not relations insofar as they have specific relata.

Even though this is the solution chosen by *bāqir al-muḥaqqiqīn* [Mīr Dāmād] in his *Qabasāt*,²⁸⁹ it does not completely uproot the matter of the difficulty. One could object by saying that the totality of particular relations such that none are excluded is itself certainly a totality. Each of the relations is a part of the totality. There is no doubt, furthermore, that this totality has a relation to its parts. Hence, this particular relation considered as being a relation between these two relata is certainly included in the totality, from which follows that a relation can be identical to its relatum, and yet the solution Mīr Dāmād mentioned is no defense here. One can escape this problem, then, using what we have mentioned, namely that the description “totality of relations” as specified answers to nothing such that it could have a relation to its parts and so entail that a relation can be identical to its relatum.²⁹⁰

(1c) Muḥammad Mahdī b. Abī Dharr al-Narāqī, *Mushkilāt al-‘ulūm*

²⁸⁷ Bihārī and Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm*, 581.8-10. Bihārī’s base-text is in italics.

²⁸⁸ Sandīlī, *Sharḥ al-Sullam*, 148.5-10.

²⁸⁹ This is vowelised as *fatshā’ih*, which is obviously wrong, and has the same consonantal structure as *qabasātih*. I was not able to locate any discussion of the problem in that work.

²⁹⁰ Laknawī, *Ḥāshiyat al-Ṣadrā*, 103.18-23.

The assumption that the totality of all concepts such that no possible concept is excluded is not consistent with the assumption that the same totality is related to one of its parts. Indeed, to assume that the totality is as noted and that it is related to one of its parts implies a contradiction: the fact that the totality is such that it excludes no possible concept implies that it could not be related to anything, while the fact that it is related to its part implies the opposite. It is as if someone were asking, “Is the relation between the totality of all concepts and any of its parts, which relation is itself both external and internal to the totality, external to it or not?”

Yes, one can indeed consider the totality of all concepts which have obtained by the time of its consideration, though it is obvious that the relation between it and any of its parts would not be among them, seeing as how [that relation] would not have yet obtained at the time of the assumption. If, however, one considered the totality of all concepts such that it excludes none that could possibly obtain or be supposed to obtain—even if after the totality has been considered—the assumption of this totality could not be consistent with the assumption that it is related to anything.²⁹¹

(2a) Jurjānī, *Hāshiya ‘alā Sharḥ Hikmat al-‘ayn*

There is no doubt that the mind judges self-evidently that the relation between any two things is, by its very nature (*bi-l-dhāt*), posterior to both of them—whether the two things be real or merely objects of the mind, or one of them be real and the other an object of the mind. This universal proposition is, however, violated by the example given by the author. We must, therefore, either reconcile the universal proposition and the counter-example or give one of them up. Hence, we say the following—and God is the one who grants success:

The totality of relations, insofar as it is a mere object of the mind, exists nowhere besides the mind, and therefore, the totality has no relation between itself and another of its constituent relations except in the mind. Moreover, it is obvious that so long as the mind does not attend to the totality, there can be no relation between it and anything else. Thus, the relation is, in the mind, posterior to its relata insofar as it takes two particular relata, and, insofar as it is merely a relation, ignoring its particular relata, it is contained in the totality. When the mind brings about the totality, it attends to its constituents insofar as they are merely relations, not insofar as they have particular relata. Indeed, it is impossible for the mind to do the latter.

If this is clear to you, then the meaning of the aforementioned universal proposition is as follows: “Every relation is posterior to its relata *insofar as it has particular relata*.” Such a proposition does not contradict the fact that a relation could be, from another respect, prior to either of its relata. Know this.²⁹²

(2a) Mullā Mubīn, *Mir’āt al-shurūḥ*

²⁹¹ Narāqī, *Mushkilāt al-‘ulūm*, 57.5-11.

²⁹² Ibn Mubārakshāh et al., *Sharḥ Hikmat al-‘ayn ma’a ḥawāshī*, 292n1.

One could also respond by saying that the concept is a part from one respect and a contradictory from another. The negation of the totality of concepts is included in the totality of concepts *insofar as it is a concept*, and *insofar as it is the negation of the totality*, it is its contradictory. The same can be said of the relation: *insofar as it is a means for attending to the state of the whole and the part*, it is posterior to its relata, and *insofar as it is thought under the description of being a concept*, it is included in the totality. Deliberate on this. Perhaps it is a way of getting at what has been said about how the fact that concepts, considered altogether as a whole (*min ḥayth al-ijmāl*), stop at a limit does not contradict the fact that concepts, considered one by one (*min ḥayth al-tafṣīl*), stop at no limit.²⁹³

(2b) Baḥr al-‘Ulūm, *Sharḥ Sullam al-‘ulūm*

The correct solution is as follows: “the totality of all concepts” is both a mental concept (*mafhūm taṣawwūrī*) and external compound (*murakkab khārijī*). No concept of an external part can be predicated of its whole, nor can it along with the whole be predicated of some third thing. The most that follows from the scenario is that a mental concept is an external part of its contradictory, which we refuse to grant is impossible. What is impossible is for both contradictories to be true of the same subject, but that does not follow here. Indeed, the totality of all concepts and its contradictory cannot both be true of the same thing. And, yes, it is true that it is impossible for one thing to be a mental part (*juz’ ‘aqlī*) of its contradictory, since it entails that contradictories could be true of the same thing, namely their instance, since a thing’s essential constituents must all be true of it (*istiḥālat infikāk al-dhātī*).²⁹⁴

²⁹³ Mubīn, *Mir’āt al-shurūḥ*, 1909, 2:124.12-17.

²⁹⁴ Bihārī and Baḥr al-‘Ulūm, *Sharḥ Baḥr al-‘Ulūm*, 582.ultimate-583.7.

Conclusion

Several non-philosophical conclusions can be drawn from the foregoing chapters. Most are historical or ideological in nature, and all, in some way or another, characterize the philosophical enterprise of the Islamic world. The first bears on the relationship between religion and philosophy in the Islamic world, the second on the research methods there pursued, and the third on the tradition's continued intellectual fecundity.

Sometimes it is thought that philosophy in the Islamic world is closely related to Islam. The thesis has been expressed in many different ways with differing degrees of explicitness. At perhaps the least explicit level, it appears in the way that *falsafa*²⁹⁵ is sometimes referred to in English: "Islamic philosophy." Others take the thesis further. For example, "Islamic philosophers" went to pains to ensure that their notion of God was Islamic, they would say. The tendency reaches its apex with the view that—after a certain time—the entire philosophical enterprise in the Islamic world amounted to little more than religion thinly veiled by the trappings of philosophy.

Perhaps the strongest support for such a view is the observation that the conclusions of *falsafa* conveniently resemble or sometimes completely correspond to Islamic views. Yet the same is true of contemporary Western philosophy, for example. It often conveniently defends contemporary public orthodoxy, in the sense that some popular philosophical positions are a defense of prevailing public views or sentiments, even if in a qualified form. Such "orthodox" views are ones that seldom can be publicly contravened without great risk of social consequence (like ostracization, shaming, loss of wealth or health, etc.). Yet we do not call contemporary philosophers "secular philosophers," "democratic philosophers," "scientific philosophers," or "Western philosophers" (in the sense that they are committed to secular, democratic, scientific, or Western social/ethical views at the expense of proof). There are far-reaching and deep-running parallels between our own *Sitz im Leben* and that of past, foreign civilizations, like the Islamic world's. Islam played more or less the same role in the Islamic world as secularism, democracy, and Western values do in the contemporary West.

Alas, it is not always justification that leads philosophers (especially establishment philosophers, philosophers who are institutionally entrenched and funded by those in possession of great political and economic power) to their conclusions. That the philosophers of any society often find its public dogmas well justified is not a fortuitous coincidence. Philosophers of the Islamic world were no exception.

Yet this fact by no means entails the conclusion that everything philosophers prove or conclude is consciously intended to defend prevailing orthodoxy against what they otherwise know to be the truth. Although there are more than enough such ugly examples of philosophical propaganda, the dynamic is, on the whole, far more subtle. Like all human beings, philosophers are subject to the youthful brainwashing we more commonly call "socialization." They are engrained with certain views and ways of thinking at a highly impressionable age, and, as years pass, these become very hard to efface entirely. Worse, these early tendencies are reinforced and maintained over a lifetime by a multitude of social, economic, and psychological factors.

²⁹⁵ I use the term *falsafa* as a convenient way to refer to the philosophical enterprise conducted in Islamic lands, whatever other Arabic term or terms that may have, at times, been used to refer to it (e.g. *ḥikma*, *kalām*, etc.).

Unsurprisingly, therefore, we find them reflected in even the most sophisticated forms of thought, especially in the thought of those who belong to dominant social groups, which are commonly those most aligned with public orthodoxy, and which happen also to comprise the majority of the intelligentsia.

Indeed, Aristotelian philosophical method has been to start with public dogma, to justify it as much as possible. The difference between this and propaganda is that public orthodoxy is only a starting point, rather than a fixed view subject to no future revision. In all likelihood, public orthodoxy will, at some point, break down under close philosophical examination. But in light of the fact that philosophy intentionally (or perhaps inevitably) begins with *endoxa*, we should be little surprised that its conclusions end up resembling them.

The sincerity of a philosophical investigation and the distinction between it and a propaganda machine can easily be discerned by producing counter examples, examples where philosophical conclusions contradict public orthodoxy. Philosophy in the Islamic world cannot be considered as merely highbrow, religious propaganda. True, the conclusions were often—to some degree—in harmony with prevailing public beliefs, but there were many instances in which harmony gave way to strident discord. Several examples can be supplied from the foregoing pages.

Consider those solutions to the Liar paradox that located the fallacy in mere self-reference, like the solution of Ṭūsī. Philosophers who wished to undermine such solutions challenged them by raising a theological counter example. The Qur’ān is wont to refer to itself. Among the many verses interpreted as self-referential, the first to appear is “That is the book in which there is no doubt, guidance for the God-fearing” (2:2), where “the book” is taken to mean the Qur’ān. Needless to say, since this very verse is part of “the book,” it refers to itself. Defenders of self-reference would then ask Ṭūsī and those who followed in his footsteps, “Are these verses of the Qur’ān neither true nor false, since they are also self-referential, like Liar sentences?” The force of the challenge lies, of course, in the fact that it is well-established religious dogma that *al-qur’ān kulluhu ḥaqq* (“The Qur’ān, all of it, is true”). How then could one suggest that some of its verses are neither true nor false?²⁹⁶

Despite the theological strongarm tactics, adherents of Ṭūsī’s solution maintained their position with equanimity. While I have seen no defender of Ṭūsī’s solution confront such verses of the Qur’ān head on, they do directly dismiss a similar Islamic challenge, the sentence “Every praise is God’s” (conspicuously similar to what is arguably the first verse of the Qur’ān, “Praise is God’s, the Lord of the worlds”). This sentence itself can be interpreted as a praise of God, and as

²⁹⁶ “Know that it follows from Muḥaqqiq Dawānī’s view [which is, in this text, really a version of Ṭūsī’s] that sentences like ‘This sentence is true,’ where ‘this’ refers to that very sentence, are not statements. Such a consequence is false, and therefore the assumption from which it follows must too be false. [‘This sentence is true,’ would not be a statement] since it is self-referential, just as ‘This sentence is false’ would not be, according to Dawānī. The consequence of this view is false since so many similar [self-referential] sentences appear in the Qur’ān, ‘which falsehood approaches neither head-on nor from behind’ (41:42), such as, ‘Indeed, it is a mighty book, which falsehood approaches neither head-on nor from behind—a revelation from the Wise, the Praiseworthy’ (41:41-42); ‘That is the book in which there is no doubt,’ (2:2) when ‘that’ is taken as referring either to the Qur’ān or to the containing chapter; or ‘In truth we sent it down, and in truth it descended’ (17:105); or ‘Indeed, it is the certain truth,’ (69:51), or any other of the innumerable [self-referential] verses. All are ‘true’ and have ‘no doubt’ about them, despite the fact that they refer to themselves as ‘true’ and having ‘no doubt’ about themselves.” Khalīl ar-Raḥmān Muṣṭafābādī, *Taḥqīq Jawāb Al-Ishkāl al-Musammā Bi-Jidhr al-Aṣamm*, ed. Karīm Mukhbish (al-Maṭba‘ al-ḥanafī, 1268s), 18.2-13.

such, they held it to be self-referential. To this challenge, they made no concession but simply reiterated that such sentences are, by virtue of their being self-referential, not statements and, therefore, neither true nor false.²⁹⁷

In another context, regarding the possibility that souls exist pre-eternally before being embodied, Ṭūsī not only applies this principle of ignoring dogmatically motivated arguments but states it explicitly:

I have seen no evidence for [the pre-eternal existence of souls]. There may be some passages in the books of the prophets, may God bless them, in line with these views, but it is possible that they could be interpreted away (*yaḥtamil al-ta'wīl*). In any case, the investigation is not scriptural (*sam'ī*) such that prophetic texts ought to be consulted.²⁹⁸

In this regard, Ṭūsī is, as he frequently turns out to be, a follower of Avicenna, who writes,

It is clear from all this, then, that religious doctrines have come down to address the masses with what they can understand, simplifying what they can't using comparisons and analogies. Were it any other way, religious doctrine would be of no use at all. Hence, how could the *prima facie* sense of religious doctrine act as proof in [philosophical] matters?²⁹⁹

The totality of all relations paradox furnishes us with another counter-example to the thesis that philosophy in the Islamic world was merely highbrow religious propaganda or in any way deeply religious. Three solutions to the paradox involved the claim that there simply could be no such totality as the totality of all relations. An objection levelled against some of these three solutions—which nonetheless equally applies to all of them—was that God's knowledge is just such a totality, and surely no one would claim it doesn't exist. Some philosophers made attempts to resolve the objection, but with little success.³⁰⁰ In any case, none (to my knowledge) abandoned their solution in the face of the theological objection.

Another counter-example pertaining to God's knowledge is found in the literature on the paradox of the absolute unknown. Some philosophers claimed that there could be no absolute unknowns, since, if they existed, they would be known insofar as they were *known* to exist. This claim was rejected by several others, however, who instead maintained that it was perfectly possible for something to exist and yet be entirely unknown to exist and indeed unknown in all other ways. But if something is, in the widest sense, absolutely unknown, it must be unknown to God as well. While no author ever answers such a theological objection, it is more significant to note that no author even raises it. The logic of the discussion could very well have inspired the objection in the mind of one of these philosophers, yet it was not discussed: no philosopher who insisted that absolute unknowns could exist felt compelled to defend himself against a potential threat of divine ignorance.

²⁹⁷ For an example of this in Mullā Mubīn, see Rezakhany, "Jalāl ad-Dīn ad-Dawānī's Solution," 210n81.

²⁹⁸ Muḥammad b. Ishāq Ṣadr al-Dīn al-Qūnawī and Naṣīr al-Dīn Muḥammad b. Muḥammad Ṭūsī, *al-Murāsālāt bayna Ṣadr al-Dīn al-Qūnawī wa-Naṣīr al-Dīn al-Ṭūsī*, trans. Gudrun Schubert, Bibliotheca Islamica 43 (Beirut: Franz Steiner Verlag, 1995), 119.10-120.1.

²⁹⁹ Avicenna, *al-Aḥwāṣīya fī amr al-ma'ād*, ed. Ḥasan 'Āṣī (Beirut: al-Mu'assasa al-jāmi'iyya li-l-dirasa wa-l-nashr wa-l-tawzī', 1984), 103.1-4.

³⁰⁰ Shams al-Dīn Ibn Mubārakshāh et al., *Hāshiyat Sharḥ Hikmat al-'ayn*, vol. 1 (Kazan: al-Maṭba'a al-Mīriyya, 1903), 292.15-ultimate.

In sum, *falsafa* and Islam, I am convinced, are not related in quite the way some might think. Doubtless, they *are* related but only as closely as ‘Western values’ and ‘contemporary Western philosophy’ are (or ‘secularism,’ ‘democracy,’ ‘science,’ and ‘contemporary Western philosophy’ are). There is no problem with calling *falsafa* “Islamic philosophy,” so long as it is “Islamic” only in the sense that, e.g., Western philosophy is “Western,” which is to say that it is “Islamic” in the sense of emerging from an Islamic culture (or some similar sense), rather than in the sense of being committed to Islamic theological dogma despite any evidence to the contrary.³⁰¹ There are many other arguments in favor of the view that Islam and “Islamic philosophy” are not so closely related as today is sometimes thought. Most of these, however, lie outside the scope of this study, and we will therefore have to suffice ourselves with the one here sketched.

Our study of paradoxes sheds light on the philosophical method commonly pursued in the post-classical Islamic world. While any characteristics of that method described here would have to be tentative, given the limited scope of the data, I am nonetheless confident that they will be shown to be highly generalizable for the philosophical enterprise as a whole, when one day the requisite evidence has accumulated.

Some of the chief characteristics of the philosophical method pursued by the authors here surveyed—which I strongly suspect would generalize to the philosophical enterprise in the Islamic world more broadly—are the following:

1. **Problem-based approach (aporetic):** Their approach to philosophy was problem based in at least two senses. In the first, philosophy itself was, to an extent, a conglomeration of problems. At the highest level, there was the problem of understanding reality, whether external or mental. Theories were then proposed to solve this problem. These theories, in turn, would generate their own problems. The paradoxes of this study are problems of just this sort. Such problems again demanded solutions, which were then met with objections. These objections represent the second sense in which the philosophical enterprise was problem-based or aporetic. The initial drive of philosophical inquiry was problem solving just as the most immediate and low-level, day-to-day operations of philosophy were problem solving: posing objections and resolving them.
2. **Cumulative:** The philosophical tradition of the Islamic world had a very good memory (relative to at least some other traditions). Participants in the tradition were, to a great extent, aware of what had been written before. Since the appearance of originality was not, it seems to me, highly valued, they did not attempt to conceal the fact that much of their work was reproduced from the work of prior scholars. That is not to say that originality was not at all pursued. It was merely implicitly acknowledged—and sensibly so—that originality was a good that could normally be acquired only in small quantities. In most cases, philosophical writing was reproduced (not necessarily verbatim) from prior work. The cumulative nature of the enterprise made progress both obvious and easy to gauge.

³⁰¹ For other discussions of what to call *falsafa*, see Peter Adamson and Richard C. Taylor, eds., *The Cambridge Companion to Arabic Philosophy* (Cambridge: Cambridge University Press, 2005), 3–4; Khaled El-Rouayheb and Sabine Schmidtke, eds., *The Oxford Handbook of Islamic Philosophy* (New York, NY: Oxford University Press, 2017), 5.

3. **Scientific:** Relatedly, the relatively dry and organized format of philosophical works made progress clearly recognizable and highly measurable. Philosophy in general, it seems to me, ranges on a spectrum between “scientific” and “literary.” At the utmost reaches of the “scientific” end of the spectrum, a philosophy is (a) dryly and exactly phrased as well as meticulously organized, (b) focused on technical topics inaccessible to those without specialized training, and (c) justified with great rigor. At the polar opposite, on the furthest end of the “literary” side, are authors like Voltaire or Ibn Ṭufayl. Their conclusions are stated more than proven, and their style is highly engaging and accessible, if at times lacking precision. Fewest and perhaps greatest are those philosophers who have almost perfectly straddled the spectrum’s midpoint. They beguile their readers with eloquence and wit as they discuss those philosophical problems closest to the hearts of all men, whether specialists or laymen, without sacrificing the rigor of their arguments or the technicality of their topics. A good example is Schopenhauer. Philosophers between the midpoint and the scientific end are Aristotle, Locke, and Kant, as well as almost all (to my knowledge) post-classical philosophers of the Islamic world. Those between the midpoint and the furthest reaches of the “literary” side are Plato, Abū Bakr al-Rāzī, Abū Ḥayyān al-Tawḥīdī, Descartes, and Berkeley.

For the most part, philosophy in the Islamic world clearly belongs at the extreme of the “scientific” end of the spectrum. It is written (a) in very dry terms and in a highly organized fashion. The tradition is notoriously focused on precise phrasing such that commentators would derive much philosophical significance from a sentence based on the exact wording used, and it was restricted to specific subjects which were organized in a rather set format, both in terms of the order of the topics as well as in terms of the way a given topic was explored. Rare indeed are the times that a philosophical point is illustrated with metaphor or that a literary flourish embellishes a discussion.³⁰² The tradition (b) dwells heavily on questions that cannot even be posed without extensive education, as opposed to questions that are—to some extent—posed by those who may even be illiterate. It would (c) pride itself on adducing proof for every theorem. Indeed, even theorems whose proofs should have been (cynically speaking) laxer, on account of their religious importance—such as the proof that there could only be one necessary existent—were subject to stringent demands for evidence and difficult objections (like *shubḥat al-tawḥīd*).³⁰³

4. **Encapsulated:** Related to the tradition’s heavily scientific bent, rarely was one topic explicitly connected to another. Rather, when a topic was treated, it was more or less treated on its own terms, with little integration with another.

These characteristics are significant in the main because they are insignificant. I am hard pressed to believe that the difference between the Islamic tradition and any other genuinely philosophical tradition is significantly greater than the difference between any other tradition and another. Indeed, one simple historical fact should lead us to hypothesize that it is highly similar to most other traditions of Western philosophy: it too is descended from the Greek tradition. Furthermore, it can hardly be doubted that a (1) problem-based approach is a necessary feature of

³⁰² Of course, it is no contradiction to simultaneously acknowledge that many literary techniques were used in certain parts of philosophical works that had little bearing on the philosophy itself.

³⁰³ R. Pourjavady and S. Schmidtke, *A Jewish Philosopher of Baghdad: ‘Izz al-Dawla Ibn Kammūna (d. 683/1284)* (Brill, 2006), 37–50.

philosophy—or any other sophisticated, theoretical subject. The philosophical tradition of the Islamic world differs perhaps most profoundly with other well-known traditions in that it seldom pretended to originality by hiding its sources in changed formatting or style. Otherwise, most traditions and individual philosophers within traditions vary according to how scientific or encapsulated their approach is. It happened that the post-classical tradition very much tended towards being scientific and encapsulated, not radically dissimilar from the tradition of twentieth century analytic philosophy, for example.

Even though it continues to gain traction and has nearly assumed the status of orthodoxy among specialists, the thesis that the philosophical enterprise of the Islamic world did not fall into dilapidation after the 13th century is, in a way, rather radical. It is a rejection of past scholarly authority and, if taken seriously, has far-reaching epistemic implications. Until relatively recently, most orientalists were agreed in their judgement that nothing of great intellectual worth was produced by the post-classical Islamic world.³⁰⁴

By overturning this verdict, scholars open a door to skepticism about (1) received authority in general and about (2) the possibility of knowing who the great minds of the past were. How is it that so many respected and learned orientalists made such a monstrous blunder of judgement? For decades, not one of these esteemed scholars recognized what is now deemed a grave error. Surely the peer-review process ought to have prevented such a mistake? How do orientalists' inheritors—the Islamicists and Arabists of today—know they are not making blunders similar in kind and degree to those of their intellectual forebears?

Just as (1) is a point of concern for any discipline that is even marginally historically aware of itself, (2) is a crippling concern for any intellectual who wants to benefit from the insights of the past. There are oceans of manuscripts in libraries all over the world. How do we know which ones to read, to edit, to publish? If we spent lifetimes and lifetimes, we could not evaluate all of them. To a certain extent, to overturn the judgement of the past about who the worthiest thinkers of their times were is an act of great hubris and folly: they read all of the works of their own generation and the former generation, as did the generation before theirs. Is our own generation so learned that we dare throw out the accumulated conclusions of so many past generations of the intelligentsia? How could we, a single generation, be more informed than so many other generations?

To determine the case definitively, a comprehensive historiographical study would be needed in order to identify the source of error in the research of so many great orientalists. Why could they not see, as so many do today, that the Islamic world indeed did continue to produce intellectually stimulating research? Why did so many learned professors just a single generation ago insist on precisely the opposite conclusion?

Although it would require another dissertation to answer these questions in full, this dissertation has already made a strong case that the Islamic world *did* continue to produce

³⁰⁴ El-Rouayheb, *The Development of Arabic Logic (1200-1800)*, 17. See also Khaled El-Rouayheb, *Relational Syllogisms and the History of Arabic Logic, 900-1900* (Leiden: Brill, 2010), 2–3. Sonja Brentjes, “The Prison of Categories—‘Decline’ and Its Company,” in *Islamic Philosophy, Science, Culture, and Religion: Studies in Honor of Dimitri Gutas*, ed. Felicitas Opwis, David Reisman, and Dimitri Gutas, Islamic Philosophy, Theology and Science, Volume 83 (Leiden: Brill, 2012).

intellectually stimulating research. The foregoing chapters have carefully documented the contributions of the Islamic world to the study of paradoxes of self-reference. They have severely weakened at least two aspects of the hypothesis that the later Islamic world produced little of intellectual merit. The first is that what may have appeared to be intellectual production was actually thinly disguised religion, and the second is that what genuine intellectual activity existed was meagre and merely a stagnant repetition of previous work.

Paradoxes of self-reference are in almost no way linked to religion in general or Islam in particular. In fact, as I explained above, the few times explicitly Islamic issues are raised in the literature, philosophers never make concessions to dogmatically motivated objections. Rack my brains as I might, I cannot see how the research I have here documented is in any non-trivial way “religious.” That is to say, the thesis that the intellectual output of the later Islamic world was exclusively religious is demonstrably false. Further studies would be needed to disprove more qualified versions of the same thesis. But the fact that such non-religious research on paradoxes demonstrably continued well into the 19th century should arouse our expectation that it continued in other disciplines too.

In light of the foregoing chapters, it is difficult to maintain the view that intellectual activity in the later Islamic world was meagre. The topic here studied, paradoxes of self-reference, is one of the most technical and sophisticated in philosophy. All three paradoxes bear on fundamental philosophical assumptions and have wide-ranging theoretical consequences. It takes a highly developed and refined philosophical enterprise to recognize such problems and to devote such vast intellectual resources to their solution. The interest in these problems throughout the eastern Islamic world was immense.

In addition to the great interest the intelligentsia of the Islamic world exhibited in these learned matters, they also continued to pursue original research on them. Due to the scientific quality of the philosophical enterprise in the Islamic world, philosophical progress is easy to measure. For each paradox, several original solutions were proposed over the course of six or seven centuries, and usually as many objections to those solutions were levelled. To suggest the research was stagnant is not only infeasible due to the large number of original solutions, but also because of the cacophony of diverging opinion and the continuous barrage of objections from all sides: no solution for any of the three paradoxes here studied received anything even approaching unanimous acceptance. Without a widely accepted view on a question, maintaining that the research was stagnant becomes difficult.

In addition to taking us through the annals of intellectual history and exercising our minds with some of the subtlest, most complex, and longest-standing problems of philosophy, this study has imparted more general lessons. The argument for calling *falsafa* “Islamic philosophy” has sustained a blow. The study of paradoxes of self-reference in the Islamic world would suggest that there was little substantively “Islamic” about the philosophical enterprise there pursued. Rather, *falsafa* is no more “Islamic” (in the sense of defending Islam without concern for evidence) than contemporary analytic or continental philosophy is “Western,” “secular,” “democratic,” or “scientific” (in the sense of defending what such labels may stand for no matter the contrary evidence).

Beyond showing that *falsafa* is unlikely to be in any non-trivial sense profoundly “Islamic,” the study also further characterized the philosophical enterprise of the Islamic world. We found that it is problem-based, cumulative, encapsulated, and “scientific” (as opposed to “literary”). While traditions of philosophy vary according to almost all of these criteria, it is unclear to what extent *falsafa* is particularly unique among them or radically different from the rest. I am inclined to favor the hypothesis that it is not much different from any other philosophical tradition than one is from another.

Finally, we addressed the shift in opinion of the Islamicists of the last century on the question of post-classical intellectual decline. A study of paradoxes in the Islamic world shows that the hypothesis that the entire intellectual edifice came crumbling down after the 12th, 13th, 14th, or 15th century is untenable. At worst, the research here presented forces the decline hypothesis to be significantly qualified, and at best, it indicates that it is likely to be completely false.

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