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UPPER-HYBRID SOLITONS

Allan N. Kaufman and Lennart Stenflo

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UPPER-HYBRID SOLITONS

Allan N. Kaufman

Physics Department and Lawrence Berkeley Laboratory
University of California, Berkeley, CA 94720, U. S. A.

and

Lennart Stenflo

Department of Plasma Physics
Umeå University
90187 Umeå, Sweden

August 19, 1974

ABSTRACT

A nonlinear upper hybrid wave with negative dispersion propagates across a magnetic field as a soliton, when its pulse speed exceeds the magnetosonic speed.

Because of the current interest in envelope solitons,[1] which are stable modulation pulses of a high frequency field, we here inquire into the effect of an ambient magnetic field on the characteristics of soliton propagation.

In this study, we limit our considerations to propagation across a uniform magnetic field. The high-frequency carrier wave is taken to be an upper-hybrid longitudinal wave, which has negative dispersion $D \equiv \partial^2\omega/\partial k^2$ (in contrast to a Langmuir wave) at $k = 0$, if the plasma frequency ω_e is less than the electron gyrofrequency Ω_e . The modulation envelope exerts a ponderomotive force on the plasma; if this force has low frequency and wave number (relative to the ion gyrofrequency and inverse gyroradius), it drives a magnetosonic wave, with relative density and magnetic field perturbations equal. Whereas the Langmuir soliton is known to exist only for subsonic pulse speed,[2] we shall see that the upper-hybrid soliton, because of its negative dispersion, can exist only for super-(magneto)-sonic speeds.

With the ambient field $\underset{\sim}{B}_0$ in the z-direction, and propagation in the x-direction only, we express the modulated upper-hybrid field (in the x-direction) as

$$E(x, t) \exp(-i\omega_0 t) + \text{c.c.},$$

with $\omega_0^2 = \omega_{e0}^2 + \Omega_{e0}^2$; i.e., ω_0 is determined by the unperturbed density and magnetic field. The modulation produces a low-frequency ponderomotive potential energy[3]

$$\psi(x, t) = \frac{e^2 |E|^2 (x, t)}{m(\omega_0^2 - \Omega_e^2)} ,$$

or to lowest order in $|E|^2$,

$$\psi(x, t) = \frac{|E|^2 (x, t)}{4\pi\ n_0} , \tag{1}$$

in terms of the unperturbed density n_0 . The force density $- n_0 \nabla\psi = - \nabla|E|^2/4\pi$ then acts on the plasma; in the MHD description, the plasma acceleration is

$$n\, m_i \dot{\underset{\sim}{u}} = c^{-1} \underset{\sim}{j} \times \underset{\sim}{B} - \nabla |E|^2/4\pi , \tag{2}$$

if we ignore particle pressure in the interests of simplicity. The plasma flow induces magnetic field perturbation:

$$\partial\underset{\sim}{B}/\partial t = \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B}) , \tag{3}$$

tying field to matter (n/B = const.). Linearizing (2) and (3) and eliminating $\underset{\sim}{u}$, we obtain for spatial variations perpendicular to $\underset{\sim}{B}_0$:

$$(\nabla^2 - c_A^{-2}\partial^2/\partial t^2)(\delta B/B_0) = -\nabla^2|E|^2/B_0^2 , \tag{4}$$

where c_A is the Alfvén[4] speed: $c_A^2 \equiv B_0^2/4\pi\, n_0\, m_i$. To obtain the evolution equation for the modulation $E(x, t)$, we consider the upper-hybrid linear dispersion relation for $k\rho_e \ll 1$ ($\rho_e \equiv$ electron thermal gyroradius):

$$\omega = \omega_{UH} + \frac{1}{2} D k^2 , \tag{5}$$

where $D \equiv \partial^2\omega/\partial k^2$ is the dispersion, and is negative when $\omega_e < \Omega_e$. For $\omega_e \ll \Omega_e$, $D = -\omega_e^2\rho_e^2/\Omega_e$. We interpret the upper-hybrid frequency in the eikonal sense:

$$\omega_{UH}^2 = \omega_e^2(x, t) + \Omega_e^2(x, t);$$

for linear perturbations,

$$\begin{aligned}\omega_0\, \delta\omega_{UH} &= \omega_{e0}\, \delta\omega_e + \Omega_{e0}\, \delta\Omega_e \\ &= \frac{1}{2}\omega_{e0}^2(\delta n/n_0) + \Omega_{e0}^2(\delta B/B_0) \\ &= (\frac{1}{2}\omega_{e0}^2 + \Omega_{e0}^2)(\delta B/B_0) .\end{aligned} \tag{6}$$

Combining (5) and (6), we see that the modulation frequency is

$$\delta\omega = \frac{1}{2} Dk^2 + \mu\, \omega_0(\delta B/B_0) , \tag{7}$$

where $\mu \equiv (\frac{1}{2}\omega_{e0}^2 + \Omega_{e0}^2)/(\omega_{e0}^2 + \Omega_{e0}^2) \approx 1$ for $\omega_e \ll \Omega_e$. Interpreting (7) as an operator on $E(x, t)$, we obtain

$$i\,\partial E/\partial t \;=\; -\tfrac{1}{2} D\;\partial^2 E/\partial x^2 \;+\; \mu\,\omega_0(\delta B/B_0)E\;. \qquad (8)$$

For a pulse moving (without change of shape) at the group velocity $V = Dk$, i.e., $|E|(x, t)$ a function of $x - Vt$, the driven solution of (4) is

$$\delta B/B_0 \;=\; (|E|^2/B_0{}^2)(M^2 - 1)^{-1}\;, \qquad (9)$$

where $M \equiv V/c_A$ is the Mach number of the pulse. Substitution of (9) into (8) yields

$$i\,\partial E/\partial t \;=\; -\tfrac{1}{2} D\;\partial^2 E/\partial x^2 \;+\; \nu|E|^2 E\;, \qquad (10)$$

the standard form of the nonlinear Schrödinger equation, with

$$\nu \;\equiv\; \mu\,\omega_0\,B_0{}^{-2}(M^2 - 1)^{-1}\;. \qquad (11)$$

The soliton solution of (10) is well-known:[1]

$$E(x, t) \;=\; A\,\mathrm{sech}\,\kappa(x - Vt)\,\exp(-i\,\Omega\,t + ikx)\;, \qquad (12)$$

with

$$V \;=\; Dk\;,$$
$$\Omega \;=\; \tfrac{1}{2} D(k^2 - \kappa^2)\;,$$
$$A \;=\; \kappa(-\,D/\nu)^{1/2}\;. \qquad (13)$$

Condition (13) shows that D and ν must have opposite signs; since D is negative here, ν must be positive. Hence, from (11), solitons exist only at supersonic speeds $(M > 1)$.

We note that the two conditions $V > c_A$ and $k\rho_e < 1$ impose lower and upper bounds on k. Consistency then demands

$$4\pi n_0 T_e/B_0^{\ 2} \gg (m_e/m_i)(\Omega_e/\omega_e)^4.$$

Also, the condition (for MHD) that $\kappa V \ll \Omega_i$ shows that $\kappa \ll k$; i.e., the pulse is many wave-lengths wide.

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FOOTNOTES AND REFERENCES

1. For a survey, see A. Scott, F. Chu, and D. McLaughlin, Proc. IEEE 61, 1443 (1973).
2. V. Karpman, Plasma Physics 13, 477 (1971).
3. For a review, see H. Motz and C. Watson, Adv. in Electronics and Electron Phys. 23, 153 (1967).
4. The Alfvén speed must be replaced by the magnetosonic speed, if the particle pressure is included.

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LAWRENCE BERKELEY LABORATORY
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