Title

Role of ion diamagnetic effects in the generation of large scale flows in toroidal ion temperature gradient mode turbulence

Journal

PHYSICS OF PLASMAS, 7(10)

ISSN

1070-664X

Authors

Smolyakov, AI
Diamond, PH
Medvedev, MV

Publication Date

2000-10-01

DOI

10.1063/1.1289514

Peer reviewed

Role of ion diamagnetic effects in the generation of large scale flows in toroidal ion temperature gradient mode turbulence

A. I. Smolyakov[a)]
Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, Saskatchewan S7N5E2 Canada

P. H. Diamond and M. V. Medvedev[b)]
Department of Physics, University of California at San Diego, La Jolla, California 92093

(Received 28 February 2000; accepted 26 June 2000)

The secondary instability of large scale flows, such as zonal flows and streamers, in toroidal ion temperature gradient turbulence is investigated. It is shown that diamagnetic effects (finite pressure fluctuations), such as those for toroidal ion temperature gradient modes, significantly increase the total Reynolds force. In general, pressure fluctuations have a finite phase shift relative to the electrostatic potential. It is shown that both parts (in-phase and out-of-phase) contribute to the diamagnetic Reynolds stress tensor. Also, the out-of-phase component of the pressure fluctuations is responsible for anomalous energy flux. It is shown here, that a specific contribution of the out-of-phase component to the Reynolds stress tensor allows us to decouple zonal flow dynamics from slow evolution of the ion pressure profile. General equations for the zonal flow and streamer instabilities in toroidal ion temperature gradient turbulence are derived and the growth rates are determined. © *2000 American Institute of Physics*. [S1070-664X(00)02210-2]

I. INTRODUCTION

The transfer of wave energy towards the long wavelength region and the formation of large scale structures (zonal flows and convective cells) is a result of the well-known inverse cascade in two-dimensional and quasi two-dimensional fluids.[1,2] Such large scale structures are frequently observed in the turbulent motions of plasmas and geostrophic fluids[3,4] (see also references in Ref. 4). The strongly sheared flow associated with such localized structures leads to the turbulence suppression and enhancement of confinement in a tokamak that has been extensively studied, both theoretically and experimentally, in recent years.[5–14] It appears that zonal flows[15,16] are an element of drift wave dynamics that is crucial for the reduction and regulation of anomalous transport in a tokamak. [Zonal flows are defined here as poloidal and toroidally symmetric ($q_z = q_\theta = 0$) perturbations with a finite radial scale q_r^{-1} larger than the scale of the underlying small scale turbulence, $q_r \ll k_r$, $\mathbf{q}$ is the wave vector for large scale motions, $\mathbf{k}$ is the wave vector of small scale turbulence, and r, θ, and z are axis of a straight cylindrical tokamak.] The general theory of zonal flows and the self-regulation of the drift-wave turbulence in a tokamak has been presented in Ref. 17 (see also related works in Refs. 18–28). In the present paper, we extend the theory of zonal flows with special emphasis on toroidal ion temperature gradient (TITG) driven turbulence. We analyze the development of zonal flows and streamers (defined here as large scale structures with $q_r \ll q_\theta$) in TITG turbulence by using the basic fluid equations and seek to clarify the specific role of pressure fluctuations and finite Larmor radius (FLR) effects.

The growth of zonal flow can be attributed to the effect of Reynolds forces generated by small scale fluctuations.[29] For drift wave turbulence the Reynolds force is determined by the nonlinear part of the ion polarization (inertial) drift. For plasmas with finite pressure fluctuations, such as toroidal ion temperature gradient driven turbulence, the contribution of pressure fluctuations to the ion polarization drift is significant and must be taken into account. We show that finite pressure fluctuations significantly increase the total Reynolds force. In general, pressure fluctuations have a finite phase shift with respect to the electrostatic potential. We show here that both parts (in-phase and out-of-phase) contribute to the diamagnetic Reynolds stress tensor. The in-phase part provides a dominant contribution. The out-of-phase component of the pressure fluctuations is smaller, assuming the scale separation between the small scale fluctuations and large scale flows. The contribution of the out-of-phase component, however, is important to decouple zonal flow evolution from the slow evolution of ion pressure profile. It turns out that contribution of the out-of-phase component in the Reynolds stress tensor is similar to the contribution to the anomalous energy flux which cancel the slow pressure evolution from the vorticity equation.

In its simplest form, the generation of poloidal plasma flow by turbulence is conveniently described by the energy conservation relation (or the Poynting theorem) that can be used to relate the turbulence-induced Reynolds stress to the radial components of the wave energy density.[29] This formulation assumes that the underlying the drift-wave turbulence is driven by an instability that can be described by the dissipative (imaginary) part of the dielectric dispersion function.

[a)]Electronic mail: smolyakov@sask.usask.ca
[b)]Present address: Harvard–Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138.

The fluid toroidal ion temperature gradient driven instability belongs to the class of reactive instabilities excited in systems with a real dispersion function (another example of such a system is the beam instability in a cold plasma). For such a system, an alternative description in terms of the generalized wave action integral[30] is more suitable. We show that the wave-action conservation for TITG fluctuations can be written in terms of ''normal mode variables'' for the unstable modes and identify the relevant variables.

The rest of this paper is organized as follows. In Sec. II we discuss the basic properties of the TITG driven instability. In Sec. III the diamagnetic Reynolds stress is derived and coupling to the energy transport is discussed. In Sec. IV we derive equations describing the instability of large scale flows (zonal flows and streamers). We conclude in Sec. V with a discussion of the results.

II. TOROIDAL ION TEMPERATURE GRADIENT DRIVEN MODES

The basic dynamics of the toroidal ion temperature gradient driven mode is described by following fluid equations:

$$\frac{\partial}{\partial t}\frac{n_i}{n_0}-V_{*i}\frac{\partial}{\partial y}\frac{e\phi}{T_i}+\mathbf{V}_E\cdot\nabla\frac{n_i}{n_0}+\mathbf{V}_{Di}\cdot\nabla\left(\frac{e\phi}{T_i}+\frac{p_i}{p_0}+\frac{T_i}{T_0}\right)-\rho_i^2\nabla_\perp\cdot\frac{d_0}{dt}\nabla_\perp\left(\frac{e\phi}{T_i}+\frac{p_i}{p_0}\right)=0, \tag{1}$$

$$\frac{3}{2}\left(\frac{\partial}{\partial t}\frac{p_i}{p_0}-V_{*p}\frac{\partial}{\partial y}\frac{e\phi}{T_i}+\mathbf{V}_E\cdot\nabla\frac{p_i}{p_0}\right)+\frac{5}{2}\mathbf{V}_{Di}\cdot\nabla\left(\frac{e\phi}{T_i}+\frac{p_i}{p_0}+\frac{T_i}{T_0}\right)+\frac{5}{2}\rho_i^2\nabla_\perp\cdot\frac{d_0}{dt}\nabla_\perp\left(\frac{e\phi}{T_i}+\frac{p_i}{p_0}+\frac{T_i}{T_0}\right)=0. \tag{2}$$

Within the fluid model, the FLR effects are determined by the inertial and gyroviscous plasma flows[31] including the inertial corrections to the ion heat flux.[32,33] Equations (1) and (2) provide the ion density and pressure response which is valid for arbitrary values of the $k^2\rho^2$ parameter. One can shows that expressions for the ion density and pressure found from (1) and (2) correspond to the Pade approximations to the exact kinetic expressions.[32,33] In the paper we will use Eqs. (1) and (2) in the limit of small $k^2\rho^2<1$. Using a Boltzmann distribution for the electron density and linearizing (1) and (2) we obtain

$$\left(\omega_{*i}-\omega_D+\frac{1}{\tau}\omega+k^2\rho^2[\omega-\omega_{*p}]\right)\tilde{\phi}-\omega_D\tilde{p}=0, \tag{3}$$

$$\left(\omega_{*p}-\frac{5}{3}\omega_D\left(1-\frac{1}{\tau}\right)+\frac{5}{3}k^2\rho^2[\omega-\omega_{*i}(1+2\eta_i)]\right)\tilde{\phi}+\left(\omega-\frac{10}{3}\omega_D\right)\tilde{p}=0. \tag{4}$$

Equations (3) and (4) may be also recovered from the gyrofluid equations derived in Ref. 34.

Here, electrostatic potential and plasma pressure are normalized as follows $e\tilde{\phi}/T_i\to\tilde{\phi}$, $\tilde{p}_i/p_0\to\tilde{p}_i$, and $\omega_D=\mathbf{k}\cdot\mathbf{V}_D$, $\mathbf{V}_D=2cT_i\mathbf{b}\times\nabla\ln B/eB_0$, $\omega_*=k_\theta cT_i/eB_0L_n$, $\omega_{*p}=\omega_*(1+\eta_i)$, $\eta_i=\partial\ln T_i/\partial\ln n$. Neglecting FLR effects in Eqs. (3) and (4), one obtains the dispersion equation describing the toroidal ion temperature gradient instability[35]

$$\omega_k=\frac{5}{3}\omega_D-\frac{\tau}{2}(\omega_{*i}-\omega_D)\pm(\omega_D\omega_{*i}\tau)^{1/2}(\eta_{cr}-\eta_i)^{1/2}, \tag{5}$$

where

$$\eta_{\mathrm{cr}}=\frac{2}{3}-\frac{\tau}{2}+\frac{1}{4}\tau\left(\frac{\omega_{*i}}{\omega_D}+\frac{\omega_D}{\omega_{*i}}\right)+\frac{10}{9\tau}\frac{\omega_D}{\omega_{*i}}. \tag{6}$$

The FLR effects play an important role in the dynamics of the mean flow. As was shown in Ref. 17, in the linear regime the instability of the mean flow sets in when the velocity of the mean flow modulations is close to the group velocity of the small scale wave packet. The group velocity, which is to a significant degree determined by the FLR effects, then becomes an important characteristic affecting conditions for the mean flow instability. The FLR effects are also important for basic dynamics of small scale instability, especially close to the marginal stability. Solving for ω one finds for the real part of ω_k (compare it with the kinetic calculations in Refs. 36 and 37)

$$\omega_k=\frac{\omega_{*i}-\omega_D-10\tau^{-1}\omega_D/3}{2(\tau^{-1}+k^2\rho^2)}-\frac{k^2\rho^2}{2}\frac{\omega_{*p}}{(\tau^{-1}+k^2\rho^2)}. \tag{7}$$

For the long wavelength turbulence $k_\perp^2\rho^2\ll 1$, the approximate expressions for the components of the group velocity are

$$V_{gx}=-k_xk_y\rho^2\tau\Big(V_{*i}(1+\tau+\eta_i)-\tau V_D-\tfrac{10}{3}V_D\Big), \tag{8}$$

$$V_{gy}=k_y\frac{\tau}{2}\left(V_{*i}-V_D-\frac{10}{3\tau}V_D\right). \tag{9}$$

III. TWO-SCALE SEPARATION AND DIAMAGNETIC REYNOLDS STRESS

To describe the dynamics of a large-scale plasma flow that varies on a longer time scale compared to the small-scale fluctuations, we employ a multiple scale expansion thus assuming that there is a sufficient spectral gap separating large-scale and small-scale motions. The electrostatic potential is represented as a sum of fluctuating and mean quantities

$$\phi(\mathbf{X},\mathbf{x},T,t)=\bar{\phi}(\mathbf{X},T)+\tilde{\phi}(\mathbf{X},x,T,t), \tag{10}$$

where $\bar{\phi}(\mathbf{X},T)$ is the mean flow potential.

In general, a similar representation can be done for plasma pressure; we assume, however, that the evolution of the mean plasma pressure is slower than the evolution of the mean plasma flow, so in this work the mean plasma pressure is kept constant.

The generation of the mean plasma flow is conveniently described by the vorticity equation,[29] which simply follows from plasma quasineutrality condition. Averaging (1) over the magnetic surface and over fast small scales and using the quasineutrality equation, we obtain for the evolution of the mean flow

$$\frac{\partial}{\partial t}(\nabla_\perp^2 \bar{\phi}+\nabla_\perp^2 \bar{p})=-\frac{c}{B_0}\overline{R^\phi}-\frac{c}{B_0}\overline{R^p}, \tag{11}$$

$$\overline{R^\phi}=\overline{\mathbf{b}\cdot\nabla\tilde{\phi}\times\nabla\nabla_\perp^2\tilde{\phi}}, \tag{12}$$

$$\overline{R^p}=\overline{\nabla_\perp\cdot[(\mathbf{b}\cdot\nabla\tilde{\phi}\times\nabla)\nabla_\perp\tilde{p}]}. \tag{13}$$

Here $\bar{\phi}$ and $\bar{p}$ are the averaged, and $\tilde{\phi}$ and $\tilde{p}$ are fluctuating parts of the plasma potential and pressure, respectively. The first term on the right-hand side of (11) is the standard Reynolds force and the second one is the diamagnetic Reynolds force due to fluctuating ion pressure.

The Reynolds forces (12) and (13) can be written

$$R^\phi=(\partial_y^2-\partial_x^2)(\partial_x\phi\partial_y\phi)+\partial_x\partial_y((\partial_x\phi)^2-(\partial_y\phi)^2), \tag{14}$$

$$R^p=\partial_y^2(\partial_y p\partial_x\phi)-\partial_x^2(\partial_x p\partial_y\phi)+\partial_x\partial_y(\partial_x p\partial_x\phi-\partial_y p\partial_y\phi). \tag{15}$$

After averaging over the fast scale, we obtain in the leading order the Reynolds stress in the form

$$\overline{R^\phi}=(\nabla_y^2-\nabla_x^2)(\overline{\partial_x\phi\partial_y\phi})+\nabla_x\nabla_y(\overline{(\partial_x\phi)^2}-\overline{(\partial_y\phi)^2}), \tag{16}$$

$$\overline{R^p}=\nabla_y^2(\overline{\partial_y p\partial_x\phi})-\nabla_x^2(\overline{\partial_x p\partial_y\phi}) +\nabla_x\nabla_y(\overline{\partial_x p\partial_x\phi}-\overline{\partial_y p\partial_y\phi}), \tag{17}$$

where we use ∇ for the derivatives over the slow (large scale) and ∂ for the derivatives over the fast (small scale) variables.

As follows from (11), the evolution of the shear flow is coupled to the slow evolution of the pressure profile via the second term on the left-hand side of Eq. (11). It turns out, however, that $\partial\nabla_\perp^2\bar{p}/\partial t$ term is cancelled out by additional contribution to the diamagnetic Reynolds stress tensor (15).

We illustrate this for zonal flows when $\nabla_x \gg \nabla_y$; the situation for large scale streamers for which $\nabla_x \ll \nabla_y$ is similar.

Expanding (17) to the next order we have

$$R^p=-\nabla_x^2(\overline{\partial_x p\partial_y\phi})-\nabla_x^2(\overline{\nabla_x p\partial_y\phi}). \tag{18}$$

The first term in this expression describes a contribution from the pressure fluctuations which are in-phase with fluctuations of the electrostatic potential. The second term is due to the out-of-phase part. The out-of-phase part of pressure fluctuations also contributes to the energy evolution equation, as shown here:

$$\frac{\partial}{\partial t}\bar{p}+\nabla_x(\overline{\tilde{V}_{Ex}\tilde{p}})=\frac{\partial}{\partial t}\bar{p}+(\overline{\tilde{V}_{Ex}\nabla_x\tilde{p}})=\frac{\partial}{\partial t}\bar{p}-\frac{c}{B_0}\overline{\partial_y\tilde{\phi}\nabla_x\tilde{p}}=0. \tag{19}$$

Comparing the last terms in (18) and (19), we find that the out-of-phase contribution cancels the slow pressure evolution term on the left-hand side of Eq. (11). Thus, the only remaining contribution of pressures fluctuations to the Reynolds stress tensor is due to the in-phase component. Such cancellation will not be complete for transient processes where the energy source on the right-hand side of (19) is finite.

Calculation of the mean quantities $\overline{\partial_i\phi\partial_j\phi}$ and $\overline{\partial_i p\partial_j\phi}$ is most conveniently done by employing the notion of the generalized adiabatic invariant N_k.[24–26,30,38]

The wave kinetic equation for the quanta density of the generalized wave action N_k allows us to determine the modulations of N_k due the mean flow–small scale fluctuations interaction. In TITG turbulence we deal with two-field ($\tilde{\phi}_k$ and $\tilde{p}_k$) perturbations and we have to determine a useful combination of $\tilde{\phi}_k$ and $\tilde{p}_k$ to form N_k. Such combination is provided by ''normal variables.'' An appropriate adiabatic invariant for the toroidal ion temperature gradient turbulence then can be obtained in terms of such variables. Thus, instead of the potential $\tilde{\phi}_k$ and pressure $\tilde{p}_k$, we introduce the normal variables $\psi_k=\tilde{\phi}_k+\alpha_k\tilde{p}_k$ as follows. First, we write our basic continuity and energy equations by making the Fourier transformation in space only,

$$\frac{\partial}{\partial t}\tilde{\phi}_k-i\tau(\omega_{*i}-\omega_D)\tilde{\phi}_k+i\omega_D\tau\tilde{p}_k=0, \tag{20}$$

$$\frac{\partial}{\partial t}\tilde{p}_k+i\frac{10}{3}\omega_D\tilde{p}_i-i\left(\omega_{*i}(1+\eta_i)-\frac{5}{3}\omega_D+\frac{5}{3\tau}\omega_D\right)\tilde{\phi}=0. \tag{21}$$

Multiplying the second equation by α_k and adding it to (21) we obtain

$$\frac{\partial}{\partial t}(\tilde{\phi}_k+\alpha_k\tilde{p}_k) +i\omega_k\tilde{\phi}_k-\frac{\omega_D\tau+10\alpha_k\omega_D/3}{\tau(\omega_{*i}-\omega_D)+\alpha_k[\omega_{*p}-5\omega_D/3+5\omega_D/(3\tau)]} \times\tilde{p}_k=0. \tag{22}$$

Here ω_k is the normal mode frequency

$$\omega_k=-\left(\tau(\omega_{*i}-\omega_D)+\alpha_k\left[\omega_{*p}-\frac{5}{3}\omega_D+\frac{5}{3\tau}\omega_D\right]\right). \tag{23}$$

The coupling coefficient α_k is found from Eq. (22),

$$\alpha_k=-\frac{\omega_D\tau+10\alpha_k\omega_D/3}{\tau(\omega_{*i}-\omega_D)+\alpha_k[\omega_{*p}-5\omega_D/3+5\omega_D/(3\tau)]}, \tag{24}$$

which gives

$$\alpha_k[\omega_{*p}-5\omega_D/3+5\omega_D/(3\tau)] =-\frac{5}{3}\omega_D-\frac{\tau}{2}(\omega_{*i}-\omega_D)\pm(\omega_D\omega_{*i}\tau)^{1/2}(\eta_{cr}-\eta_i)^{1/2}. \tag{25}$$

Using this equation and the linear dispersion relation we obtain for ψ_k,

$$\psi_k=\tilde{\phi}_k-\frac{-5\omega_D/3-\tau(\omega_{*i}-\omega_D)/2+i\gamma_k}{(\omega_{*p}-5\omega_D/3+5\omega_D/(3\tau))}\tilde{p}_k. \tag{26}$$

From Eq. (4) we find the following linear relation between the plasma pressure and the potential,

$$\tilde{p}_k=\frac{\omega_{*p}-5\omega_D/3+5\omega_D/(3\tau)}{-\omega+10\omega_D/3}\tilde{\phi}_k. \tag{27}$$

Finally from (26) and (27) we obtain

$$\psi_k=\tilde{\phi}_k+\frac{-\triangle_k-i\gamma_k}{\triangle_k-i\gamma_k}\tilde{\phi}_k=\frac{-2i\gamma_k}{\triangle_k-i\gamma_k}\tilde{\phi}_k, \tag{28}$$

where

$$\triangle_k=-\triangle_{-k}=\frac{5}{3}\omega_D+\frac{\tau}{2}(\omega_{*i}-\omega_D), \tag{29}$$

and $\gamma_k=\gamma_{-k}$ is the growth rate of the TITG instability defined in Eq. (5). Note the normality property of the ψ_k variables,

$$N_k\equiv|\psi_k|^2=\psi_k\psi_{-k}=\frac{4\gamma_k^2}{\gamma_k^2+\triangle_k^2}|\tilde{\phi}_k|^2, \tag{30}$$

that does not hold for an arbitrary combination of $\tilde{\phi}_k+\alpha_k\tilde{p}_k$ due to a complex phase shift between $\tilde{\phi}_k$ and $\tilde{p}_k$ for the unstable modes. The N_k variable can be used now to describe the interaction with the large scale flow. The equation for N_k is obtained below.

In the main order, the interaction of the small scale fluctuations with the mean flow is due to the advection terms

$$\frac{\partial}{\partial t}\tilde{\phi}-(V_{*i}-V_D)\frac{\partial}{\partial y}\tilde{\phi}+V_D\tau\frac{\partial}{\partial y}\tilde{p}=-\overline{\mathbf{V}}_E\cdot\nabla\tilde{\phi}, \tag{31}$$

$$\frac{\partial}{\partial t}\tilde{p}+\frac{10}{3}V_D\frac{\partial}{\partial y}\tilde{p}_i-\left(V_{*p}-\frac{5}{3}V_D+\frac{5}{3\tau}V_D\right)\frac{\partial}{\partial y}\tilde{\phi}$$
$$=-\overline{\mathbf{V}}_E\cdot\nabla\tilde{p}_i. \tag{32}$$

In the Fourier space we have

$$\frac{\partial}{\partial t}\phi_k^>-i\tau(\omega_{*i}-\omega_D)\phi_k^>+i\omega_D\tau p_k^>$$
$$=-\int d^2pL_{p,k-p}\phi_p^<\phi_{k-p}^>, \tag{33}$$

$$\frac{\partial}{\partial t}p_k^>+i\frac{10}{3}\omega_Dp_k^>-i\left(\omega_{*p}-\frac{5}{3}\omega_D+\frac{5}{3\tau}\omega_D\right)\phi_k^>$$
$$=-\int d^2pL_{p,k-p}\phi_p^<p_{k-p}^>, \tag{34}$$

where we split the fields $(\phi,p)=X$ into the large-scale $X_k^<$ and small-scale $X_k^>$ components; $X_k^<=0$ outside a shell $|\mathbf{k}|<\varepsilon\ll 1$, $X_k^>=0$ for $|\mathbf{k}|<\varepsilon$.

Multiplying (34) by α_k and adding it to (33) we have

$$\frac{\partial}{\partial t}\psi_k+i\omega_k\psi_k=-\int d^2pL_{p,k-p}\phi_p^<(\phi_{k-p}^>+\alpha_kp_{k-p}^>). \tag{35}$$

By expanding in small $\mathbf{p}$, the left-handside can be transformed to obtain

$$\frac{\partial}{\partial t}\psi_k+i\omega_k\psi_k=-\int d^2pL_{p,k-p}\phi_p^<$$
$$\times\left(\psi_{k-p}^>+p_{k-p}^>\frac{\partial\alpha_{k-p}}{\partial(\mathbf{k-p})}\cdot\mathbf{p}\right). \tag{36}$$

We note that in general the convection of the turbulence by the mean flow destroys the normal variables (30). The last term on the right-hand side (RHS) of the equation is responsible for mixing of normal variables structure due to the advection of the plasma pressure by the mean flow. For the simplest TITG model in neglect of the FLR effects, the function α_k is a piecewise-constant function with a discontinuity at $k=0$. Since, due to the scale separation assumption, the function $p_{k-p}^>$ is zero in the shell around $k-p=0$, and the contribution of the discontinuity to the second term in (36) can be neglected. Thus, in this case (neglecting the FLR terms) the canonical variables are preserved in the presence of the mean flow.

In this work we neglect the FLR correction in the advective term so that the coupling matrix is

$$L_{k_1,k_2}=-\frac{c}{B_0}\mathbf{b}\cdot\mathbf{k}_1\times\mathbf{k}_2. \tag{37}$$

Then from (36) and following the procedure of Ref. 30 one obtains the equation describing the evolution of the adiabatic invariant in the TITG turbulence,

$$\frac{\partial}{\partial t}N_k(\mathbf{x},t)+\frac{\partial}{\partial\mathbf{k}}(\omega_k+\mathbf{k}\cdot\mathbf{V}_0)\cdot\frac{\partial N_k}{\partial\mathbf{x}}-\frac{\partial}{\partial\mathbf{x}}(\mathbf{k}\cdot\mathbf{V}_0)\cdot\frac{\partial}{\partial\mathbf{k}}N_k$$
$$=2\gamma_kN_k-\widehat{St}(N_k,N_k), \tag{38}$$

where $N_k=|\psi_k|^2$. Equation (38) generalizes the wave kinetic equation and Poynting theorem[29] for the case of the unstable toroidal ion temperature gradient driven mode in the presence of the mean plasma flow. The last term in Eq. (38) represents the ''collision'' operator describing nonlinear self-interaction of small scales, leading to the nonlinear frequency shift $\triangle\omega_k$, $\widehat{St}(N_k,N_k)=\triangle\omega_kN_k$, so that $\omega_k\to\omega_k+\triangle\omega_k$. The exact term of this operator is not important for us (it is given in Refs. 18 and 27), so the role of this nonlinear interaction is to balance the linear growth rate on the right-hand side, so that the stationary spectrum can be established in the equilibrium, $2\gamma_kN_k^0-\widehat{St}(N_k^0,N_k^0)=0$. In our theory below we consider small deviations of the spectrum function $\tilde{N}_k$ from the equilibrium N_k^0, $N_k=N_k^0+\tilde{N}_k$.

IV. ZONAL FLOW AND STREAMER INSTABILITIES

Equation (11) for the evolution of the mean flow can be written in a general two-dimensional form

$$\frac{\partial}{\partial t}\nabla^2\bar{\phi}=\frac{c}{B_0}(1+\delta)\nabla_i\nabla_j\varepsilon_{jk}\overline{\partial_i\tilde{\phi}\partial_k\tilde{\phi}}, \tag{39}$$

where $\varepsilon_{12}=-\varepsilon_{21}$, $\varepsilon_{22}=\varepsilon_{11}=0$, $i=(1,2)=(r,\theta)$, and δ is the parameter that describes the diamagnetic enhancement of the Reynolds force due to temperature fluctuation. In the lowest order in the $k^2\rho^2$ parameter, δ is independent of the wave vector and can be written from Eq. (27) as

$$\delta=\frac{2}{\tau}\,\frac{\omega_{*i}(1+\eta_i)-5\omega_D(1-\tau^{-1})/3}{\omega_{*i}+\omega_D(10\tau^{-1}/3-1)}. \tag{40}$$

The correlator $\overline{\partial_i\tilde{\phi}\partial_k\tilde{\phi}}$ is written using the normal variables

$$\overline{\partial_i\tilde{\phi}\partial_k\tilde{\phi}}=\int d^2k k_i k_k|\tilde{\phi}_k|^2=\int d^2k k_i k_k\frac{\gamma_k^2+\triangle_k^2}{4\gamma_k^2}N_k. \tag{41}$$

Closure for the modulations of $\tilde{N}_k$ in terms of the mean flow potential $\bar{\phi}$ is provided by Eq. (38).

The instability of the zonal flow is related to the in-phase part of $\tilde{N}_k$ which is a perturbation of the spectral density function N_k. It is calculated from (38) as

$$\tilde{N}_k=R(\Omega-\mathbf{q}\cdot\mathbf{V}_g)\frac{\partial}{\partial\mathbf{x}}(\mathbf{k}\cdot\mathbf{V}_0)\cdot\frac{\partial}{\partial\mathbf{k}}N_k^0, \tag{42}$$

where $R(\Omega-\mathbf{q}\cdot\mathbf{V}_g)$ is the resonance broadened action response function,[17]

$$R(\Omega-\mathbf{q}\cdot\mathbf{V}_g)=\frac{\triangle\omega_k}{\triangle\omega_k^2+(\Omega-\mathbf{q}\cdot\mathbf{V}_g)^2}. \tag{43}$$

Here, $\triangle\omega_k$ is the total decorrelation frequency which may also include the linear growth rate and a nonlinear frequency shift. For $\triangle\omega_k\rightarrow 0$, the resonance broadening function reduces to a delta-function, while for very broad-band turbulence $R(\Omega-\mathbf{q}\cdot\mathbf{V}_g)\rightarrow 1/\triangle\omega_k$.

Note that small scale fluctuations may also provide a nonlinear frequency shift to the linear frequency shift $\omega_k\rightarrow\omega_k+\alpha N_k$.[39,40] These would modify the resonance response function in Eq. (42),

$$R(\Omega-\mathbf{q}\cdot\mathbf{V}_g)=\frac{1}{\triangle\omega_k+i\left(\Omega-\mathbf{q}\cdot\mathbf{V}_g-\alpha\mathbf{q}\cdot\dfrac{\partial N_k^0}{\partial\mathbf{k}}\right)}, \tag{44}$$

where we retain the notation $\triangle\omega_k$ for the nonlinear decorrelation rate that is imaginary, in contrast to the real frequency shift αN_k. In the regime of broad-band turbulence, only the nonlinear decorrelation rate is important, leading to the expression $R\rightarrow 1/\triangle\omega_k$. For a narrow spectrum, the frequency shift gives a nonlinear correction to the group velocity. This correction would lead to higher order terms (quadratic in N_k^0) in the expression for the growth rate of the zonal flow instability. We neglect such effects in our theory, retaining only the leading order terms that are linear in N_k^0. The higher order nonlinearity generated by the nonlinear frequency shift may lead to the wave trapping and formation of strongly nonlinear soliton structures.[28,40]

Substituting (42) into (39) one finds the growth rate $\gamma=q_r^2(1+\delta)D_{rr}$ (Ref. 17) for the zonal flow instability, where

$$D_{rr}=-\left(\frac{c}{B_0}\right)^2\int R(\Omega-\mathbf{q}\cdot\mathbf{V}_g)k_\theta^2 k_r\frac{\partial}{\partial k_r}I_k^0 d^2k. \tag{45}$$

Note that the condition

$$k_r\frac{\partial}{\partial k_r}I_k^0<0,$$

which is typically satisfied for the drift-wave turbulence, is required for the instability. The equilibrium spectral density I_k^0 here is defined as $I_k^0=|\phi_k|^2$.

In the opposite limit, $q_\theta\gg q_r\rightarrow 0$, Eq. (39) describes a generation of streamers with the increment $\gamma=q_r^2(1+\delta)D_{\theta\theta}$,

$$D_{\theta\theta}=-\left(\frac{c}{B_0}\right)^2\int R(\Omega-\mathbf{q}\cdot\mathbf{V}_g)k_r^2 k_\theta\frac{\partial}{\partial k_\theta}I_k^0 d^2k. \tag{46}$$

V. SUMMARY AND DISCUSSION

We have investigated the structure of the diamagnetic Reynolds stress tensor in a plasma with finite pressure fluctuations. We have shown that both, in-phase and out-of-phase, components of pressure fluctuations (with respect to the potential fluctuations) are important in calculations of the diamagnetic stress tensor. When the energy source can be neglected, the evolution of the shear flow decouples from the slow evolution of the pressure profile. Such decoupling occurs due to the effects of the out-of-phase part of pressure fluctuations, which give a similar contribution to the energy flux. In general case, the energy source term will be present in the equation for the shear flow dynamics (11). It is generally smaller than other terms as the ratio $\nabla_x/\partial_x<1$, which is the scale separation parameter. The scale separation is not always well pronounced in a number of cases.[6] Sudden changes in the energy source profile may lead to disbalance between this part of the Reynolds stress [second term in (18)] and slow evolution of the pressure [second term on the left-hand side of (11)], thus affecting the large scale flow dynamics.

As follows from (42), the instability of the mean flow occurs due to its interaction with the wave packet. This interaction is strongest for $\Omega\simeq\mathbf{q}\cdot(\partial\omega_k/\partial\mathbf{k})$. For quasi-steady-state evolution of the mean flow $\Omega\ll\mathbf{q}\cdot(\partial\omega_k/\partial\mathbf{k})$, the resonant condition approximately reduces to

$$\mathbf{q}\cdot\mathbf{V}_g\simeq 0, \tag{47}$$

so that the wave vector of the most unstable large scale perturbation is perpendicular to the group velocity of the small scale fluctuations. Note that in this case, a finite nonlinear broadening is important to sustain the instability as follows from the form of the resonance function (43).

For the long wavelength turbulence $k_\perp^2\rho^2\ll 1$, from (8) and (9) one can obtain the following relation between the components of the group velocity:

$$\left|\frac{V_{gx}}{V_{gy}}\right|\simeq k_x k_y\rho\ll 1. \tag{48}$$

This condition and (47) generally mean that the most unstable components of the large scale flow have $q_r\gg q_\theta$. Thus, the long wavelength turbulence predominantly leads to generation of the zonal flows. On the contrary, the generation of streamers, $q_r\ll q_\theta$, occurs for the opposite case, namely,

$V_{gx} \geqslant V_{gy}$, that generally requires shift of the maximum of the energy distribution toward shorter wavelengths, $k_\perp^2 \rho^2 \geqslant 1$.

The dynamics of zonal flows in the toroidal ion temperature gradient driven turbulence has been investigated here. It was shown that the diamagnetic contribution significantly enhances the net Reynolds force. This enhancement is described by Eqs. (39) and (40). As noted above, the generation of streamers is favored for the short wavelength turbulence, while the long wavelength turbulence is more likely to produce zonal flows.

ACKNOWLEDGMENTS

This research was supported by Natural Sciences and Engineering Research Council of Canada and U.S. Department of Energy Grant No. FG03-88ER53275. Useful discussions with M. N. Rosenbluth, F. L. Hinton, Z. Lin, and T. S. Hahm are acknowledged with gratitude.

[1] R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Sov. J. Plasma Phys. **4**, 306 (1978).
[2] A. Hasegawa and M. Wakatani, Phys. Rev. Lett. **59**, 1581 (1987).
[3] T. Soomere, Phys. Rev. Lett. **75**, 2440 (1995).
[4] F. H. Busse, Chaos **4**, 123 (1994).
[5] H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids B **2**, 1 (1990).
[6] T. S. Hahm, M. A. Beer, Z. Lin *et al.*, Phys. Plasmas **6**, 922 (1999).
[7] R. E. Waltz, R. L. Dewar, and X. Garbet, Phys. Plasmas **5**, 1784 (1998).
[8] T. S. Hahm and K. H. Burrell, Phys. Plasmas **2**, 1648 (1995).
[9] X. Garbet, Plasma Phys. Controlled Fusion **39**, B91 (1997).
[10] T. S. Hahm, K. H. Burrell, C. M. Greenfield *et al.*, Plasma Phys. Controlled Fusion **40**, 657 (1998); **38**, 1427 (1996).
[11] R. D. Sydora, V. K. Decyk, and J. M. Dawson, Plasma Phys. Controlled Fusion **38**, A281 (1996).
[12] R. Waltz, G. Kerbel, and J. Milovich, Phys. Plasmas **1**, 2229 (1994).
[13] G. Hammet, M. Beer, W. Dorland, S. C. Cowley, and S. A. Smith, Plasma Phys. Controlled Fusion **35**, 973 (1993).
[14] A. M. Dimits, J. A. Byers, T. J. Williams *et al.*, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1994), Vol. III, p. 457.
[15] M. N. Rosenbluth and F. L. Hinton, Phys. Rev. Lett. **80**, 724 (1998).
[16] Z. Lin, T. S. Hahm, W. W. Lee, W. M. Tang, and R. B. White, Science **281**, 1835 (1998).
[17] P. H. Diamond, M. N. Rosenbluth, F. L. Hinton, M. Malkov, J. Fleischer, and A. Smolyakov, in *Plasma Physics and Controlled Nuclear Fusion Research, 18th IAEA Fusion Energy Conference*, Yokohama, Japan, 1998 (International Atomic Energy Agency, Vienna, 1998), p. IAEA-CN-69/TH3/1.
[18] A. Muhm, A. M. Pukhov, K. H. Spatschek, and V. Tsytovich, Phys. Fluids B **4**, 336 (1992).
[19] A. V. Tur, A. V. Chechkin, and V. V. Yanovsky, Phys. Fluids B **4**, 3513 (1992).
[20] A. V. Chechkin, M. I. Kopp, V. V. Yanovsky, and A. V. Tur, JETP **86**, 357 (1998).
[21] E. A. Fedutenko, Phys. Rev. E **53**, 493 (1996).
[22] E. A. Fedutenko, Phys. Lett. A **200**, 134 (1995).
[23] A. V. Gruzinov, P. H. Diamond, and V. B. Lebedev, Phys. Plasmas **1**, 3148 (1994).
[24] V. B. Lebedev, P. H. Diamond, V. D. Shapiro, and G. I. Soloviev, Phys. Plasmas **2**, 4420 (1995).
[25] A. I. Dyachenko, S. V. Nazarenko, and V. E. Zakharov, Phys. Lett. A **165**, 330 (1995).
[26] B. Dubrulle and S. V. Nazarenko, Physica D **110**, 123 (1997).
[27] A. M. Balk, S. V. Nazarenko, and V. E. Zakharov, Sov. Phys. JETP **71**, 249 (1990).
[28] A. I. Smolyakov, P. H. Diamond, and M. Malkov, Phys. Rev. Lett. **84**, 491 (2000).
[29] P. H. Diamond and Y.-B. Kim, Phys. Fluids B **3**, 1626 (1991).
[30] A. I. Smolyakov and P. H. Diamond, Phys. Plasmas **6**, 4410 (1999).
[31] A. I. Smolyakov, Can. J. Phys. **76**, 321 (1998).
[32] B. N. Kuvshinov and A. B. Mikhailovskii, Plasma Phys. Rep. **22**, 529 (1996).
[33] I. O. Pogutse, A. I. Smolyakov, and A. Hirose, J. Plasma Phys. **60**, 133 (1998).
[34] A. Brizard, Phys. Fluids B **4**, 1213 (1992).
[35] A. Jarmen, P. Andersson, and J. Weiland, Nucl. Fusion **27**, 941 (1987).
[36] N. Mattor, Phys. Fluids B **3**, 1913 (1991).
[37] H. Biglari, P. Diamond, and M. N. Rosenbluth, Phys. Fluids B **1**, 109 (1989).
[38] N. Mattor and P. H. Diamond, Phys. Plasmas **1**, 4002 (1994).
[39] A. Hasegawa and T. Sato, *Space Plasma Physics* (Springer-Verlag, Berlin, 1989), p. 104.
[40] A. Hasegawa, Phys. Fluids **20**, 2155 (1977).