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Integration of Fault Detection and Identification into a Fault Tolerant Automated Highway System: Final Report

**Robert H Chen, Hok K. Ng, Jason L. Speyer,
D. Lewis Mingori**

University of California Los Angeles

**California PATH Research Report
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Integration of Fault Detection and Identification into a Fault Tolerant Automated Highway System

Agreement No. 65A0013, M.O.U. 315

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Summary

This report is continuation of the work of (Douglas *et al.*, 1996) and (Douglas *et al.*, 1997) which concerns vehicle fault detection and identification. A vehicle health monitoring approach based on analytical redundancy is described. Fault detection filters and parity equations use the control commands and sensor measurements to generate the residuals which have a unique static pattern in response to each fault. This allows the faults not only to be detected, but also identified. Sensor noise, process disturbances, system parameter variations, unmodeled dynamics and nonlinearities can distort these static patterns. A Shirayev sequential probability ratio test that has been extended to multiple hypotheses examines the fault detection filter and parity equation residuals and generates the probability of the presence of a fault. A point design of fault detection filters and parity equations is developed for the longitudinal dynamics of the PATH Buick LeSabre. Fault detection filters are evaluated using empirical data obtained from U.C. Berkeley. The preliminary evaluation is promising in that the fault detection filters can detect and identify actuator and sensor faults as expected even under various disturbances and uncertainties.

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Chapter 1

Introduction

A proposed transportation system with vehicles traveling at high speed, in close formation and under automatic control demands a high degree of system reliability. This requires a health monitoring system capable of detecting a fault as it occurs, identifying the faulty component and determining a course of action that restores safe operation of the system. In this report, a vehicle health monitoring approach based on analytical redundancy is described. A system view of vehicle health monitoring and management is summarized by Figure 1.1. Vehicle dynamics are driven by throttle, brake and steering commands, and various unmeasured exogenous influences such as road noise and actuator faults. Sensors measure a possible nonlinear function of the dynamic states and are corrupted by noise, biases and faults of their own. The vehicle health monitoring system uses the control commands and sensor measurements to generate the conditional probability of each fault hypothesis. The fault hypothesis probabilities are generated in two stages. In the first stage, a residual generator formed as a combination of fault detection filters and parity equations generates the residuals which have a unique static pattern with a given fault or no-fault condition. In the second stage, a residual processor interrogates the residuals by matching the residuals to one of several known patterns. The pattern matching is done with a probabilistically based algorithm so the residual processor generates the fault hypothesis probabilities rather than a simple binary announcement. A simple threshold mapping could be added very easily to produce a binary announcement if that were needed. The fault hypothesis probabilities are passed to a vehicle health management system developed by U.C. Berkeley. The vehicle health management system determines the impact of the possible fault on safe vehicle operation and adjusts control laws if necessary to accommodate a degraded operating condition.

In Chapter 2, the nonlinear vehicle simulation of the PATH Buick LeSabre is briefly discussed.

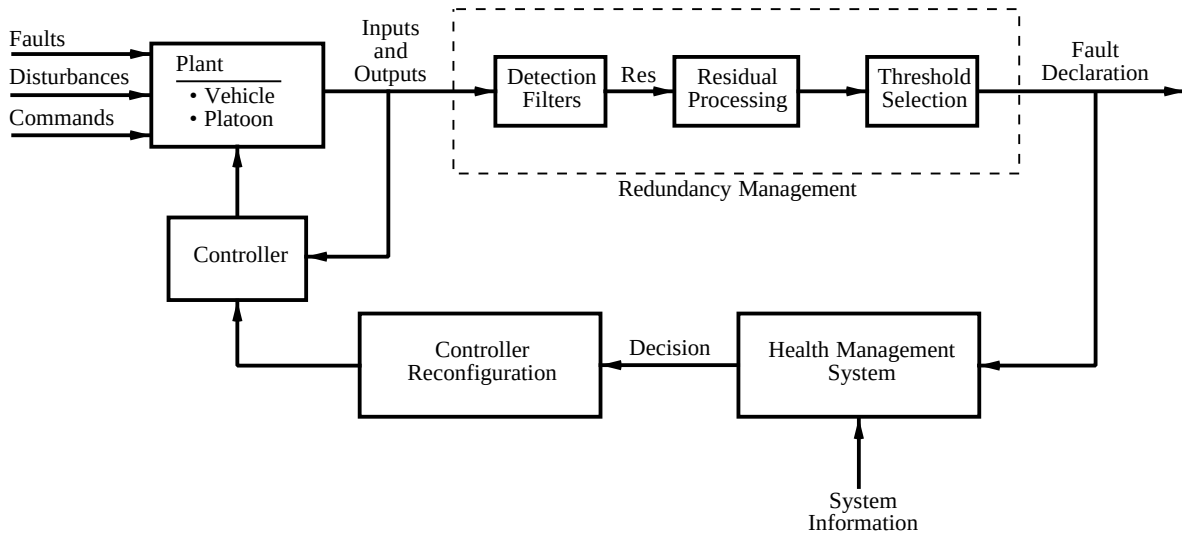


Figure 1.1: A system view of vehicle health monitoring and management

The PATH Buick LeSabre has two control inputs (throttle and brake) and seven measurements (manifold pressure, manifold temperature, engine speed, longitudinal velocity, longitudinal acceleration, throttle and brake) for the longitudinal dynamics of the vehicle. Since the fault detection filter is model-based, linear vehicle models are derived for the purpose of fault detection filter design.

In Chapter 3, the background of the fault detection filter is briefly discussed. The idea of the fault detection filter is to combine control commands and sensor measurements with known system dynamics to obtain an analytical redundancy. The fault detection filter is designed to have an invariant subspace structure that forces the residual to take on a prescribed and fixed direction in response to a fault. References (Massoumnia, 1986; White and Speyer, 1987; Douglas and Speyer, 1996, 1999) describe the fault detection filter in detail and some of our early results in defining fault detection filter design algorithms.

In Chapter 4, fault detection filters are developed for the longitudinal dynamics of the PATH Buick LeSabre to detect and identify the throttle actuator, brake actuator, manifold pressure sensor, manifold temperature sensor, engine speed sensor, longitudinal velocity sensor (i.e., front and rear wheel speed sensors) and longitudinal accelerometer faults. Parity equations are developed to detect the throttle actuator, brake actuator, throttle sensor, brake sensor, manifold air mass sensor and manifold pressure sensor faults. By combining the residuals generated by the fault

detection filters and parity equations, the residuals have a unique static pattern in response to each fault. Therefore, a fault in any actuators or sensors on the PATH Buick LeSabre that control or measure the longitudinal dynamics of the vehicle can be detected and identified.

In Chapter 5, fault detection filters and parity equations are evaluated using empirical data obtained from U.C. Berkeley. The preliminary evaluation is promising in that the fault detection filters can detect and identify actuator and sensor faults as expected even under various disturbances and uncertainties including sensor noise, road noise, system parameter variations, unmodeled dynamics and nonlinearities.

In Chapter 6, a multiple model Shiriyayev sequential probability ratio test (SPRT) (Malladi and Speyer, 1999) is used as a residual processing scheme. Tests using a high-fidelity, nonlinear vehicle simulation show that modeled sensor and actuator faults are detected and identified quickly in the presence of simulated road noise, vehicle nonlinear dynamics and off design point vehicle operation. Preliminary tests indicated a transient false alarm in the manifold temperature sensor after a small ramp fault was added to the manifold air mass sensor. The transient false alarm lasted only for a few seconds after which the correct fault was identified. Further work showed the cause of the false alarm to be a correlation in the noise statistics of the filter residual. The Shiriyayev SPRT residual processing algorithm assumes that the residual noise is white. Recent work has focused on whitening the filter residuals before processing by the Shiriyayev SPRT. Current nonlinear simulation studies show this approach to be successful: no false alarms are raised and all faults are detected and isolated quickly.

In Chapter 7, a new fault detection filter design algorithm (Chen and Speyer, 2000) is presented. The design algorithm is based on an optimization problem where the transmission from the target fault, the fault to be detected, is maximized and the transmission from the nuisance faults, the faults to be blocked, is minimized. Furthermore, the transmission from the sensor noise, process noise and plant uncertainties is minimized. Therefore, the geometric structure of the fault detection filter is approximated in the presence of these disturbances to any degree determined by the designer by using the weightings of the transmissions. Furthermore, this approach allows extension to linear time-varying systems.

Chapter 2

Vehicle Model and Nonlinear Simulation

The fault detection and identification approach considered here is model based. The high-fidelity six degree of freedom nonlinear model derived in (Douglas *et al.*, 1996, 1997) wherein the vehicle is modeled as a rigid body which can perform translational and rotational motion in three dimensions is modified to simulate a Buick LeSabre by using the parameters and engine data from U.C. Berkeley. This model also allows for arbitrary variations in road profile and road noise. The linear model used for fault detection and identification is obtained from the nonlinear model. An object-oriented vehicle simulation is implemented in C++ and is currently hosted on an Apple Macintosh PowerPC 8100 computer.

The nonlinear vehicle model in (Douglas *et al.*, 1996, 1997) can be expressed in the form

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= Cx + D\dot{x}\end{aligned}$$

where x is the state vector, u is the input vector and y is the measurement vector. In this model, some of the relationships between variables such as engine model are described by look-up tables. Because of this feature, a numerical procedure must be used to obtain a linearized model. Linear models for the longitudinal vehicle dynamics are derived numerically from the nonlinear vehicle simulation using a central differences method. The models are described in Sections 2.1 and 2.2. The derivation method is described in (Douglas *et al.*, 1996, 1997).

2.1 Linear Model

The linearized longitudinal dynamics of the vehicle are derived numerically from a high-fidelity nonlinear simulation using a central differences method. The nonlinear model and the central differences method are described in detail in (Douglas *et al.*, 1996, 1997). The linearization is done at a single nominal operating point of 27 meters per second (equal to 60.75 miles per hour) with the car traveling straight ahead. Since the car is not in a turn, the linear longitudinal dynamics decouple completely from the linear lateral dynamics. The longitudinal model has thirteen states and three inputs. Two of the inputs, throttle and brake actuator commands are regarded as controls. The third input is the manifold temperature and is regarded as a known, that is a measured, exogenous input.

States:	m_a : Manifold air mass.
	w_e : Engine speed.
	v_x : Longitudinal velocity.
	z : Vertical position.
	v_z : Vertical velocity.
	θ : Pitch angle.
	q : Pitch rate.
	$\bar{w}_f$: Sume of the front wheel speeds.
	$\bar{w}_r$: Sume of the rear wheel speeds.
	$\bar{F}_f$: Sume of the front suspension forces.
	$\bar{F}_r$: Sume of the rear suspension forces.
	α : Throttle angle.
	T_b : Brake torque.
Control inputs:	u_α : Commanded throttle angle.
	u_{T_b} : Commanded brake torque.
Exogenous input:	w_{T_m} : Manifold temperature.

2.2 Linear Model Reduction

Before using the linear model for filter design, a model order reduction is done to simplify the model without significant loss of accuracy. The model order reduction is done by dynamic truncation with a steady-state correction, a process described in more detail in (Douglas *et al.*, 1996, 1997).

The thirteenth-order longitudinal model has eigenvalues: -238.83 , -142.86 , -125.66 , -68.67 , -54.90 , -47.66 , -14.03 , $-4.53 \pm 12.59i$, $-2.14 \pm 7.53i$, -1.25 and -0.0326 . Observe that six of these eigenvalues are significantly faster than the rest. By inspection of the eigenvectors, it is determined that the fast eigenvalues are associated with the states $\bar{\omega}_f$, $\bar{\omega}_r$, $\bar{F}_f$, $\bar{F}_r$, ω_e and α . First, the derivatives of the fast states $\bar{\omega}_f$, $\bar{\omega}_r$, $\bar{F}_f$, $\bar{F}_r$, ω_e and α are set to zero. Then, the linear dynamic equations are solved for the fast states in terms of the remaining states: m_a , v_x , z , v_z , θ , q and T_b . The result is substituted into the state equations of the remaining states. However, when this procedure is followed, the frequency responses of the reduced and full-order models do not match very well. Therefore, the same procedure is repeated with only five states $\bar{\omega}_f$, $\bar{\omega}_r$, $\bar{F}_f$, $\bar{F}_r$ and α to be dropped. The eigenvalues of the eighth-order reduced-order longitudinal model are -49.43 , -14.02 , $-5.24 \pm 11.51i$, $-2.53 \pm 7.11i$, -1.25 and -0.0348 which are close to the eigenvalues of the full-order longitudinal model. Also the frequency responses of the reduced and full-order models match closely.

2.3 Vehicle Measurements

There are seven measurements for the PATH Buick LeSabre.

y_{pm} : Manifold pressure sensor.

y_{ω_e} : Engine speed sensor.

y_{T_m} : Manifold temperature sensor.

y_{v_x} : Longitudinal velocity sensor.

y_{a_x} : Longitudinal accelerometer.

y_{α} : Throttle sensor.

y_{T_b} : Brake sensor.

Note that the longitudinal velocity measurement is obtained from the front and rear wheel speed sensors. Since the empirical data obtained from U.C. Berkeley has only the longitudinal velocity measurement, but not the front and rear wheel speed measurements, the front and rear wheel speed sensors are conveniently denoted as the longitudinal velocity sensor here. Also note that the throttle and brake sensors y_α and y_{T_b} measure control inputs rather than states and the manifold temperature sensor y_{T_m} measures an exogenous input. Therefore, there are only four sensors that provide measurements linearly related to the vehicle longitudinal states: y_{pm} , y_{ω_e} , y_{v_x} and y_{a_x} .

Chapter 3

Design Algorithms for Fault Detection Filter

Analytic redundancy is an approach to health monitoring that compares dissimilar instruments using a detailed system model. The approach is to find dynamic or algebraic relationships between sensors and actuators. That is, information provided by a monitored sensor is, in some form, also provided by other sensors or, through the dynamics, by actuator commands. A popular approach to analytical redundancy is the detection filter which was first introduced by (Beard, 1971) and refined by (Jones, 1973). It is also known as Beard-Jones detection filter. A geometric interpretation and a spectral analysis of the detection filter are given in (Massoumnia, 1986) and (White and Speyer, 1987), respectively. Design algorithms have been developed (Douglas and Speyer, 1996, 1999; Chen and Speyer, 2002) which improve the detection filter robustness. The idea of a detection filter is to place the reachable subspace of each fault into invariant subspaces which do not overlap each other. Then, when a nonzero residual is detected, a fault can be announced and identified by projecting the residual onto each of the invariant subspaces. In this way, multiple faults can be monitored in one filter.

A linear time-invariant system model with q faults used for fault detection filter design has the form

$$\dot{x} = Ax + Bu + \sum_{i=1}^q F_i m_i \quad (3.1a)$$

$$y = Cx + Du \quad (3.1b)$$

where the input u and the output y both are known. The fault signatures F_i are known, fixed and model the directional characteristics of the faults. The fault modes m_i model the unknown time-

varying amplitude of the faults and do not have to take scalar values. Before the fault detection filter design can begin, a system model with faults has to be found with the form (3.1).

In Section 3.1, the sensor and actuator fault models are given. In Section 3.2, Beard-Jones detection filter is briefly discussed.

3.1 Modeling of Sensor and Actuator Faults

Seven sensors and two actuators as described in Chapter 2 are to be monitored. The sensors are the manifold pressure sensor y_{pm} , engine speed sensor y_{ω_e} , manifold temperature sensor y_{T_m} , longitudinal velocity sensor y_{v_x} , longitudinal accelerometer y_{a_x} , throttle sensor y_{α} and brake sensor y_{T_b} . The two actuators are the throttle u_{α} and brake u_{T_b} . Two of the sensors y_{α} and y_{T_b} , are monitored with algebraically redundant information while five sensors and two actuators are included in the fault detection filter design.

3.1.1 Sensor Fault Models

Two classes of sensor fault are considered. One measures a linear combination of states. For the longitudinal vehicle dynamics these include y_{pm} , y_{ω_e} , y_{v_x} and y_{a_x} . Another class of sensor fault is one that measures exogenous inputs. The manifold temperature sensor is the only sensor in this class.

The fault of a sensor which measures system states can be modeled as an additive term in the measurement equation

$$y = Cx + E_i\mu_i \quad (3.2)$$

where E_i is a column vector of zeros except for a one in the i^{th} position and where μ_i is an arbitrary time-varying scalar. Since, for fault detection filter design, faults are expressed as additive terms to the system dynamics, a way must be found to convert the E_i sensor fault form of (3.2) to an equivalent F_i form as in (3.1). Let f_i satisfy

$$Cf_i = E_i$$

and define a new state

$$\bar{x} = x + f_i\mu_i$$

Then,

$$y = C\bar{x}$$

and the dynamic equation of $\bar{x}$ is

$$\dot{\bar{x}} = A\bar{x} + Bu + F_i m_i$$

where $F_i = [f_i \quad -Af_i]$ and $m_i = [\dot{\mu}_i \quad \mu_i]^T$. An interpretation of the effect of a sensor fault is f_i is the sensor fault rate $\dot{\mu}_i$ direction and $-Af_i$ is the sensor fault magnitude μ_i direction. This interpretation suggests a possible simplification when information about the spectral content of the sensor fault is available. If it is known that a sensor fault has persistent and significant high frequency components, such as in the case of a noisy sensor, the fault direction could be approximated by the f_i direction alone. Or, if it is known that a sensor fault has only low frequency components, such as in the case of a bias, the fault direction could be approximated by the $-Af_i$ direction alone. For example, if a sensor were to develop a bias, a transient would be likely to appear in both fault directions but, in steady-state, only the residual associated with the faulty sensor should be nonzero.

3.1.2 Actuator Fault Models

A linear model partitioned to isolate first-order actuator dynamics can be expressed as

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & -\omega \end{bmatrix} \begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \end{bmatrix} u + B_\omega \omega_e$$

where x_a is a vector of actuator states and ω_e is an exogenous input. Typically, exogenous inputs are dynamic disturbances such as road noise and wind gusts and are not known or measured. However, the manifold temperature is modeled as a dynamic input and is measured. A fault in this sensor is modeled as a direction given by the associated column of the B_ω matrix.

A fault in a control input is also modeled as an additive term in the system dynamics. In the case of a fault appearing at the input of an actuator, that is the actuator command, the fault has the same direction as the associated column of the $[0, \omega]^T$ matrix. A fault appearing at the output of an actuator, the actuator position, has the same direction as the associated column of the $[B^T, 0]^T$ matrix. In the vehicle model, the throttle actuator dynamics are relatively fast and, in an approximation made here, are removed from the system model. Thus, the throttle control

input is applied directly to the system through a column of the B matrix. However, the brake actuator dynamics are kept because its dynamics are slow. It can be shown that without sensors which can measure actuator states, faults which occur at the input and output of the actuator can not be isolated. Furthermore, both directions are needed to model the actuator fault.

3.2 Beard-Jones Detection Filter

In this section, Beard-Jones detection filter is briefly discussed from the geometric point of view (Massoumnia, 1986; Douglas, 1993). Following the development in Section 3.1, the actuator and sensor faults can be modeled as additive terms in the state equation. Therefore, a linear time-invariant system with q faults can be modeled as

$$\dot{x} = Ax + Bu + \sum_{i=1}^q F_i m_i \quad (3.3a)$$

$$y = Cx \quad (3.3b)$$

Assume F_i are monic so that $m_i \neq 0$ imply $F_i m_i \neq 0$.

The detection filter is a linear observer in the form of

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \quad (3.4)$$

and the residual is

$$r = y - C\hat{x}$$

By using (3.3) and (3.4), the dynamic equation of the error $e = x - \hat{x}$ is

$$\dot{e} = (A - LC)e + \sum_{i=1}^q F_i m_i$$

and the residual can be written as

$$r = Ce$$

The detection filter gain L is chosen such that $A - LC$ is stable and there exists an invariant subspace $\mathcal{T}_i$ for each fault F_i . $\mathcal{T}_i$ is called the minimal (C, A) -unobservability subspace or the detection space of F_i . Assume (C, A) is observable and the invariant zeros of (C, A, F_i) have the same geometric and algebraic multiplicities. $\mathcal{T}_i$ can be found by

$$\mathcal{T}_i = \mathcal{W}_i \oplus \mathcal{V}_i \quad (3.5)$$

where $\mathcal{W}_i$ is the minimal (C, A) -invariant subspace of F_i given by the recursive algorithm

$$\mathcal{W}_i^0 = 0 \quad (3.6a)$$

$$\mathcal{W}_i^{k+1} = \text{Im} F_i \oplus A(\mathcal{W}_i^k \cap \text{Ker } C) \quad (3.6b)$$

and $\mathcal{V}_i$ is spanned by the invariant zero directions of (C, A, F_i) . When $\dim F_i = 1$, the recursive algorithm (3.6) implies

$$\mathcal{W}_i = \text{Im} \begin{bmatrix} F_i & AF_i & \cdots & A^{k_i} F_i \end{bmatrix}$$

where k_i is the smallest non-negative integer such that $CA^{k_i} F_i \neq 0$.

It is assumed that $CT_1 \cdots CT_q$ are independent, that is,

$$CT_i \cap \sum_{j \neq i} CT_j = 0$$

If they are not independent, the faults can only be detected, but not identified. This condition is called output separability. If the faults are not output separable, then usually, the designer needs to discard some faults from the design set. It is also assumed that $(C, A, [F_1 \cdots F_q])$ does not have more invariant zeros than $(C, A, F_1) \cdots (C, A, F_q)$. If it does, the extra invariant zeros will become part of the eigenvalues of $A - LC$. This condition is called mutual detectability. For more details, please refer to (Massoumnia, 1986; Douglas, 1993). For the design algorithms to form the detection filter gain L , please refer to (White and Speyer, 1987; Douglas and Speyer, 1996, 1999; Chen and Speyer, 2002).

When there is no fault, the residual generated by the detection filter is zero after the transient response due to the initial condition error because $A - LC$ is stable. When the fault m_i occurs, the residual becomes nonzero, but only in the direction of CT_i because

$$\begin{aligned} \text{Im} F_i &\subseteq \mathcal{T}_i \\ (A - LC)\mathcal{T}_i &\subseteq \mathcal{T}_i \end{aligned}$$

Hence, the fault can be identified by projecting the residual onto each CT_i by using a projector $\hat{H}_i$ that annihilates $[CT_1 \cdots CT_{i-1} \quad CT_{i+1} \cdots CT_q] \triangleq C\hat{\mathcal{T}}_i$.

$$\hat{H}_i : \mathcal{Y} \rightarrow \mathcal{Y}, \quad \text{Ker } \hat{H}_i = C\hat{\mathcal{T}}_i, \quad \hat{H}_i = I - C\hat{\mathcal{T}}_i[(C\hat{\mathcal{T}}_i)^T C\hat{\mathcal{T}}_i]^{-1}(C\hat{\mathcal{T}}_i)^T$$

where $\mathcal{Y}$ is the output space. The projected residual $\hat{H}_i r$ is nonzero only when the fault m_i is nonzero and is zero even when other faults $m_{j \neq i}$ are nonzero. Therefore, by monitoring $\hat{H}_1 r \cdots \hat{H}_q r$, every fault can be detected and identified.

Chapter 4

Fault Detection Filter Design

Several design considerations arise that are specific to the longitudinal vehicle dynamics fault monitoring problem and are discussed in Section 4.1. In Section 4.2, the structure of the fault monitoring system is discussed. In Section 4.3, the algebraic parity equation is introduced to provide a second component to the fault monitoring system.

4.1 Design Consideration

Several design considerations arise that are specific to the longitudinal vehicle dynamics health monitoring problem. One problem is a conditioning problem that arises from the model order reduction. Another concerns the output separability of the modeled faults. A third problem concerns a reasonable expectation that a fault detection filter should produce a nonzero fault residual for as long as a modeled fault is present.

Ill-conditioned fault direction For all actuator and sensor faults considered here, the detection spaces are given by the fault directions themselves because $CF_i \neq 0$ for every fault. However, in the case of the brake actuator, $CF_{u_{T_b}} \neq 0$ only holds for the reduced, eighth-order model. For the full-order model, $CF_{u_{T_b}} = 0$ so $F_{u_{T_b}}$ should be considered as a very weakly observable direction. Therefore, for fault detection filter design, the brake actuator detection space is taken to be the second-order space given by $\text{Im} [F_{u_{T_b}} AF_{u_{T_b}}]$.

Output separability The output separability design requirement states that the residuals produced by design faults be pairwise linearly independent. Faults that are not output separable generate co-linear residuals and cannot be isolated. For all actuator and sensor faults considered

here, output separability of two faults F_i and F_j is determined by

$$CF_i \cap CF_j = 0 \quad (4.1)$$

Performing the check (4.1) reveals that two pairs of faults are not output separable. The throttle actuator fault F_{u_α} and manifold pressure sensor fault $[f_{y_{pm}} - Af_{y_{pm}}]$ are not output separable because $CF_{u_\alpha} = Cf_{y_{pm}}$. The manifold temperature sensor fault $F_{y_{T_m}}$ and manifold pressure sensor fault are not output separable because $CF_{y_{T_m}} = -CAf_{y_{pm}}$.

First, consider the throttle actuator and manifold pressure sensor faults where $CF_{u_\alpha} = Cf_{y_{pm}}$ indicates that they cannot be isolated. As explained in Section 3.1.1, the direction of the pressure sensor fault magnitude is $-Af_{y_{pm}}$ while the direction of the fault rate is $f_{y_{pm}}$. The throttle actuator and pressure sensor faults become output separable if only the sensor fault magnitude direction is used. This design decision could allow a noisy but zero mean sensor fault to remain undetected through the direction $f_{y_{pm}}$. Also, since the throttle fault detection space is spanned by $F_{u_\alpha} = f_{y_{pm}}$, a pressure sensor fault rate will stimulate the throttle fault residual. However, a throttle actuator fault could never stimulate the pressure sensor fault residual. In summary, as long as the pressure sensor fault spectral components are low frequency, the throttle actuator and manifold pressure sensor faults should be detectable and isolatable.

Next, consider the manifold temperature and pressure sensor faults where $CF_{y_{T_m}} = -CAf_{y_{pm}}$ indicates that they cannot be isolated. Since $-Af_{y_{pm}}$ represents the fault magnitude direction, this direction can not be dropped from the detection space. One remedy is to design a second fault detection filter that does not take the manifold pressure as a measurement. Such a filter will be unaffected by pressure sensor faults but will respond to manifold temperature sensor faults. A problem with this fix is that the throttle actuator and temperature sensor faults are not output separable without a pressure sensor measurement.

Responses of the two filter designs are summarized in Figure 4.1. Each row represents a bias (hard) fault in either the throttle actuator, the pressure sensor or the temperature sensor. The columns are the residual responses to the given fault conditions. The first column is the response of the throttle actuator fault residual of the first filter. The second column is the response of the pressure sensor and temperature sensor fault residuals also from the first filter. The third column is the response of the throttle actuator and temperature sensor fault residuals of the second filter. Therefore, (1,1)-block is the throttle actuator fault residual response of the first detection filter

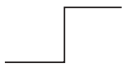
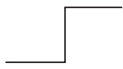
Residual Fault	Detection Filter #1		Detection Filter #2
	Throttle Actuator	Manifold Pressure and Temperature	
Throttle Actuator			
Manifold Pressure			
Manifold Temperature			

Figure 4.1: Fault Signatures

when throttle actuator fault occurs. The second detection filter has more residuals for other faults which are not shown here.

Figure 4.1 shows that neither filter alone can detect and isolate the three faults: the throttle actuator, the pressure sensor and the temperature sensor. Taken together, the two filters produce a pattern unique to each fault so that the faults may be isolated. However, the picture is not yet complete. A problem with the second fault detection filter is described in the next section.

Zero steady-state fault residual It is a reasonable expectation that a fault detection filter should produce a nonzero fault residual for as long as a modeled fault is present. It can be shown (Chen and Speyer, 1999) that a necessary and sufficient condition for a residual to hold a non-zero steady-state value in response to a bias fault is (C, A, F_i) does not have any invariant zeros at the origin. Since $(C, A, F_{y_{T_m}})$ has an invariant zero at origin, the second fault detection filter will not see the temperature sensor faults in the steady state. When a temperature bias fault occurs, the residual responds with only a transient. Figure 4.1 is corrected in Figure 4.2 to illustrate the transitory response. Once again, the fault patterns for the three faults are not unique, at least not in steady-state.

Since a second fault detection filter no longer fixes the output separability problem, another fix is needed. If a manifold air mass sensor is installed on the car, an algebraic relation between the manifold pressure and manifold air mass is useful

$$\text{manifold pressure} - 19.3272 * \text{manifold air mass} = 0 \quad (4.2)$$

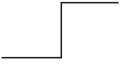
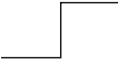
Residual Fault	Detection Filter #1		Detection Filter #2
	Throttle Actuator	Manifold Pressure and Temperature	
Throttle Actuator			
Manifold Pressure			
Manifold Temperature			

Figure 4.2: Fault Signatures

This convenient relation arises from the perfect gas law. The magic number 19.3272 includes the gas constant, a nominal temperature and the manifold volume. Equation (4.2) is a parity equation that is satisfied when the manifold pressure and manifold air mass sensors are working and is not satisfied when either sensor has failed. The parity equation by itself cannot isolate a fault.

By combining the parity equation (4.2) with the first fault detection filter of the last section, a residual pattern unique to each fault is formed and the faults may be isolated. The faults are the throttle actuator, the pressure sensor, the temperature sensor and the air mass sensor. The residual patterns are summarized in Figure 4.3. Each row represents a bias (hard) fault in either the throttle actuator, the pressure sensor, the temperature sensor or the air mass sensor. The columns are the residual responses to the given fault conditions. The first column is the response of the throttle actuator fault residual of the first filter. The second column is the response of the pressure sensor and temperature sensor fault residuals also from the first filter. The third column is the response of the parity equation for the manifold pressure and air mass sensors. The manifold air mass sensor can be replaced by an extra manifold pressure or temperature sensor and a similar parity equation can be formed. The parity equation is discussed further in Section 4.3.

4.2 Fault Detection Filter Configuration

The ability to distinguish one fault from another requires for an observable system that the detection spaces be independent. Therefore, the number of faults that can be detected and identified by a fault detection filter is limited by the size of the state space and the sizes of the detection

Residual Fault	Detection Filter		Parity Equation
	Throttle Actuator	Manifold Pressure And Temperature	
Throttle Actuator			
Manifold Pressure			
Manifold Temperature			
Manifold Air Mass			

Figure 4.3: Fault Signatures

spaces associated with each of the faults. If the problem considered has more faults than can be accommodated by one fault detection filter, then a bank of filters is constructed. The health monitoring system described in this section for a vehicle going straight, considers six system faults: four sensor faults and two actuator faults. Since the reduced-order longitudinal model has eight states and four measurements, clearly more than one fault detection filter is needed. The dimension of the throttle actuator, the manifold pressure sensor and the manifold temperature sensor detection spaces is one. The dimension of the brake actuator and the rest of the sensor faults is two. Therefore, for this problem at least three filters are needed.

One consideration in grouping the faults among the fault detection filters is to group faults which are robust to system nonlinearities. Note that an actuator fault changes the vehicle operating point possibly introducing nonlinear effects into all measurements. The nonlinear effect is small if the residual response is small compared to that for some nominal fault. Also, sensor faults that are open-loop are easily isolated since they do not stimulate any dynamics. One approach to fault grouping is to place actuator and sensor faults into different fault detection filters.

Usually an attempt is made to group as many faults as possible in each filter. When full-order filters are used, this approach minimizes the number of filters needed. When reduced-order filters are used, this approach minimizes the order of each complementary space and, therefore, the order of each reduced-order filter. Note that each fault included in a fault detection filter design imposes more constraints on the filter eigenvectors. Sometimes, the objective of obtaining well-conditioned

filter eigenvectors imposes a tradeoff between robustness and the reduced-order filter size.

Another consideration in grouping the faults is to group faults which are mutually detectable or non-mutually detectable but with the extra invariant zeros on the left-half real axis. In the case of non-mutually detectable faults, the invariant zeros arising from the fault combinations will be the eigenvalues of the filters, that is, some poles of the filters cannot be assigned (Massoumnia, 1986). Therefore, if the extra invariant zeros are in the left-half plane and reasonably slow, they can be part of the eigenvalues of the detection filter. Otherwise, a game-theoretic fault detection filter (Chung and Speyer, 1998) or a generalized least-squares fault detection filter (Chen and Speyer, 2000) will be used. The advantages of these filters are that they are more robust and they do not have the constraint of non-mutually detectable. However, each filter can detect only one fault and therefore more filters are needed.

With all the above considerations in mind, now how many fault detection filters are needed and which faults should go together are decided. There are nineteen possible combinations of these six faults. Four of them are not output separable and eight of them are non-mutually detectable with positive invariant zeros. Therefore, only seven combinations can be used for the Beard-Jones detection filter. However, two of them are not well-conditioned and this left with $[F_{u_\alpha} \ F_{y_{v_x}}]$, $[F_{u_\alpha} \ F_{y_{a_x}}]$, $[F_{u_\alpha} \ F_{y_{pm}}]$, $[F_{y_{v_x}} \ F_{y_{pm}}]$ and $[F_{y_{a_x}} \ F_{y_{pm}}]$. Note that $F_{y_{\omega_e}}$ is not in any of these combinations and therefore generalized least-squares fault detection filters are used. More details of this filter are given in Chapter 7. The detection filter design have two choices, $[F_{u_\alpha} \ F_{y_{a_x}}]$, $[F_{y_{v_x}} \ F_{y_{pm}}]$ and $[F_{u_{T_b}} \ F_{y_{\omega_e}}]$ or $[F_{u_\alpha} \ F_{y_{v_x}}]$, $[F_{y_{a_x}} \ F_{y_{pm}}]$ and $[F_{u_{T_b}} \ F_{y_{\omega_e}}]$. Both cases have been designed and tested with empirical data from U.C. Berkeley. The first combination yields better results. Note that the $[F_{y_{v_x}} \ F_{y_{pm}}]$ filter is also sensitive to faults in the manifold temperature sensor y_{T_m} since manifold temperature and manifold pressure sensor faults are not output separable. The three fault detection filters are given in Table 4.1.

4.3 Algebraic Redundancy

Algebraic parity equations provide a second component to the fault detection and identification system. The following algebraically redundant pairs are available: the throttle sensor y_α and throttle actuator, the brake sensor y_{T_b} and brake actuator u_{T_b} and the manifold pressure sensor y_{pm} and manifold air mass sensor y_{m_a} . Three parity equations are defined:

Fault detection filter 1:	Throttle actuator Longitudinal accelerometer
Fault detection filter 2:	Longitudinal velocity sensor Manifold pressure sensor Manifold temperature sensor
Fault detection filter 3:	Brake actuator Engine speed sensor
Parity equation 1:	Throttle actuator Throttle sensor
Parity equation 2:	Brake actuator Brake sensor
Parity equation 3:	Manifold pressure sensor Manifold air mass sensor

Table 4.1: Faults organized into analytically redundant groups

1. $0 = \text{Throttle sensor } y_\alpha - \text{Throttle actuator } u_\alpha$
2. $0 = \text{Brake sensor } y_{T_b} - \text{Brake actuator } u_{T_b}$
3. $0 = \text{Manifold pressure sensor } y_{pm} - \text{Manifold air mass sensor } y_{m_a} * 19.3272$

None of the parity equations can by itself identify a fault. But by combining the parity equations with the first fault group detection filter of Section 4.2, a unique residual pattern is produced allowing each fault to be isolated. The patterns are summarized in Figure 4.4.

Each row of Figure 4.4 represents a bias (hard) fault in either the throttle actuator, the throttle sensor, the brake actuator, the brake sensor, the manifold pressure sensor, the manifold temperature sensor or the manifold air mass sensor. The columns are the residual responses to the given fault conditions. The first column is the response of the throttle actuator fault residual of the first filter. The second column is the response of the brake actuator fault residual of the third filter. The third column is the response of the pressure sensor and temperature sensor residuals of the second filter. The fourth, fifth and sixth columns are responses of the first, second and third parity equations.

Combining the fault detection filters of Section 4.2 and parity equations in Section 4.3, a set of six analytic redundancy relationships either dynamic or algebraic are presented.

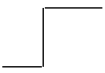
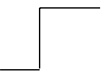
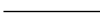
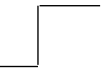
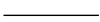
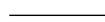
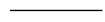
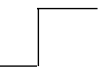
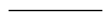
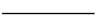
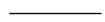
Residual Fault	Throttle Actuator Residual	Brake Actuator Residual	Pressure and Temperature Residual	Parity Equation #1	Parity Equation #2	Parity Equation #3
Throttle Actuator						
Throttle Sensor						
Brake Actuator						
Brake Sensor						
Manifold Pressure						
Manifold Temperature						
Manifold Air Mass						

Figure 4.4: Fault Signatures

Chapter 5

Fault Detection Filter Evaluation Using Empirical Data

Fault detection filter performance is evaluated using the nonlinear simulation discussed in Section 2 and the empirical vehicle data obtained from U.C. Berkeley. The fault detection filters designed in Chapter 4 are first tested on simulated flat smooth and rough roads at an off-nominal condition, then tested by the empirical data. The off-nominal operating condition is with the vehicle traveling straight ahead at 25 meters per second. Recall that the fault detection filters are designed for the vehicle traveling straight ahead at 27 meters per second. Performance is evaluated on the smooth road with respect to robustness to model nonlinearities and on the rough road with respect to model nonlinearities, road roughness and sensor noise. The roughness used is similar to what would be found on a reasonably well maintained freeway. The sensor noise is simulated as white noise. Their variance are derived from the precision of the instrument if available or $\frac{1}{30}$ of the size of the fault used for test which means the error is between $+\frac{1}{10}$ and $-\frac{1}{10}$ of the size of the fault. Finally, fault detection filter performance is evaluated using the empirical data which has 100 seconds. Figure 5.1 shows part of the empirical data, car speed and throttle command. Although the car speed is almost a constant, the throttle command changed a lot (between 6 and 8.5 degrees) because there are some slopes whose magnitude are unknown. Therefore, the detection filter performance is evaluated with respect to robustness to model nonlinearities, road roughness, sensor noise and slope. Note that the empirical data does not have any actuator nor sensor failure. The fault is added to the data when it is used by the fault detection filter because sensor fault does not affect the car and for the actuator fault, this means the actuator command changes, but the actuator does not respond.

With the filters operating at an off-nominal condition, it is expected that the residuals will be nonzero but small even when no fault is present. The question of what should be considered small is answered by comparing the size of a nonzero residual due to non-linearities and the size of a nonzero residual due to a fault. A residual scaling factor is chosen such that when a fault is introduced into the linearized dynamics the magnitude of the corresponding reduced-order fault detection filter residual is one.

Of course, the size of the residual is proportional to the size of the fault. The size of the fault used for finding the residual scaling factors is determined as follows. For most sensors, the size of the fault is given by the difference in magnitude between the sensor output at the nominal and off-nominal steady state operating conditions. For accelerometers, the output is zero in any steady state condition and another method has to be used. The value $1\frac{\text{m}}{\text{sec}^2} \approx 0.1g$ is chosen as a reasonable value for an accelerometer bias fault. For throttle actuator and sensor faults, the size of the faults is one degree. This increases the speed of the car by about 2 meters per second or 4.5 miles per hour. A brake fault is simulated by applying a brake torque just large enough to slow the vehicle from 27 meters per second to 25 meters per second.

Since the empirical data test is more interesting, only this part of the results are shown here. The performance of the fault detection filter for the first fault group which includes throttle actuator fault and longitudinal accelerometer fault is shown in Figures 5.2 and 5.3. Note that the bias faults occur at ten seconds and the associated residuals jump to around one in a reasonably short time. Figure 5.3 shows eight to sixteen seconds of Figure 5.2. The performance of the fault detection filter for the second fault group which includes longitudinal velocity sensor fault and manifold pressure sensor fault is shown in Figures 5.4 and 5.5. The performance of the fault detection filter for the third fault group which includes brake actuator fault and engine speed fault is shown in Figures 5.6 and 5.7.

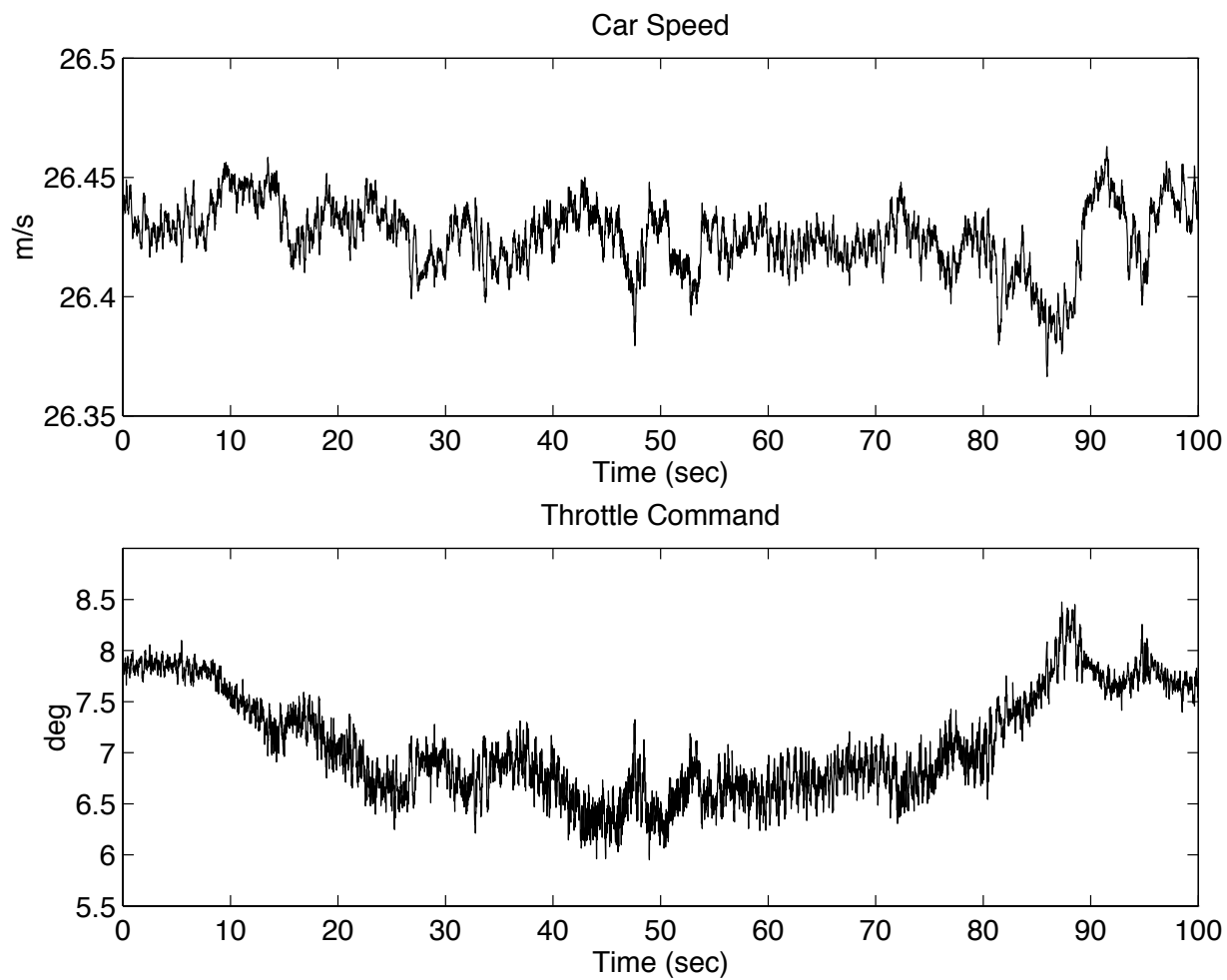


Figure 5.1: Empirical data from U.C. Berkeley

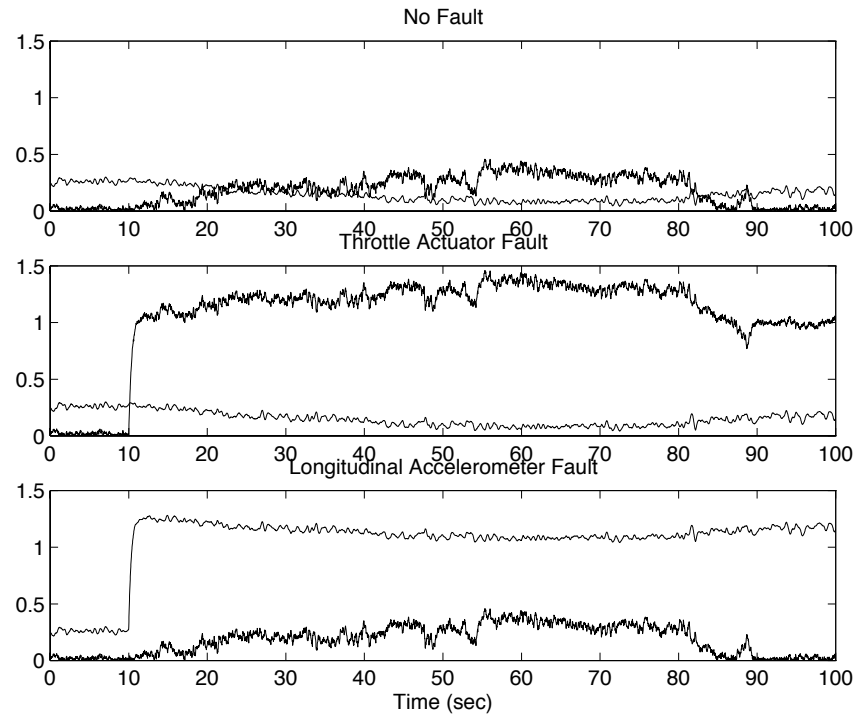


Figure 5.2: Residuals for fault detection filter one

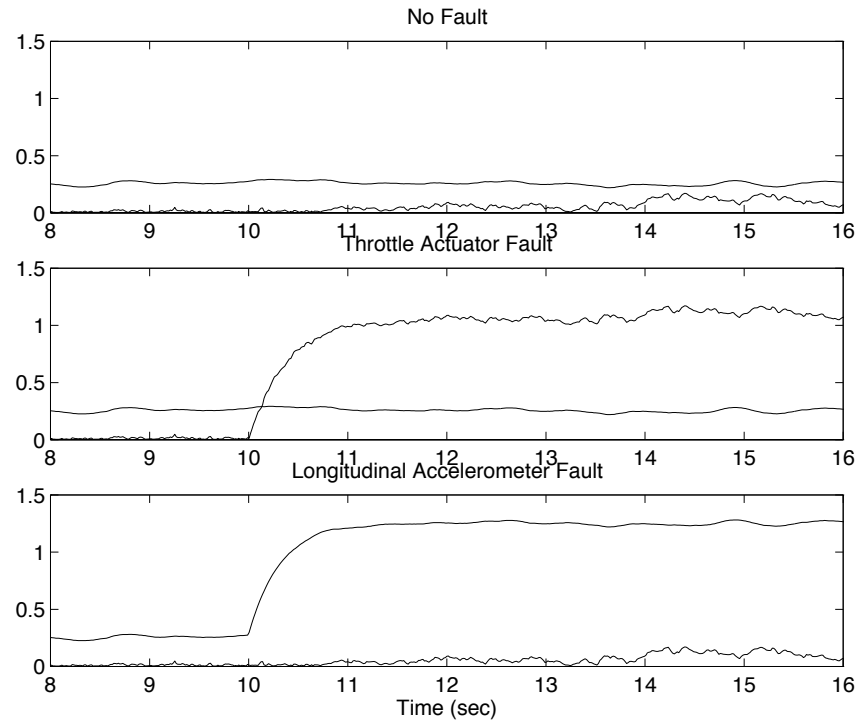


Figure 5.3: Residuals for fault detection filter one

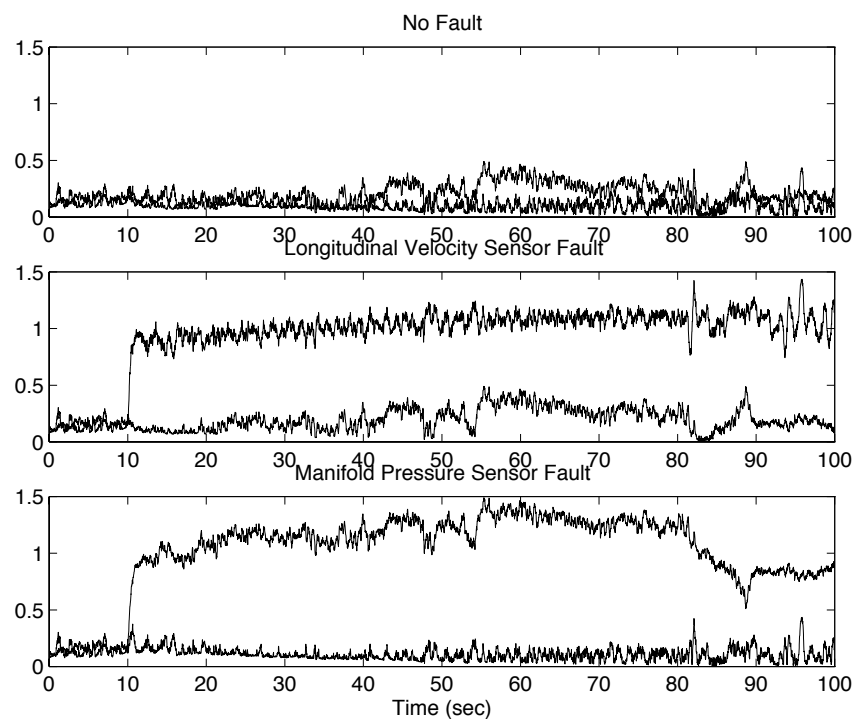


Figure 5.4: Residuals for fault detection filter two

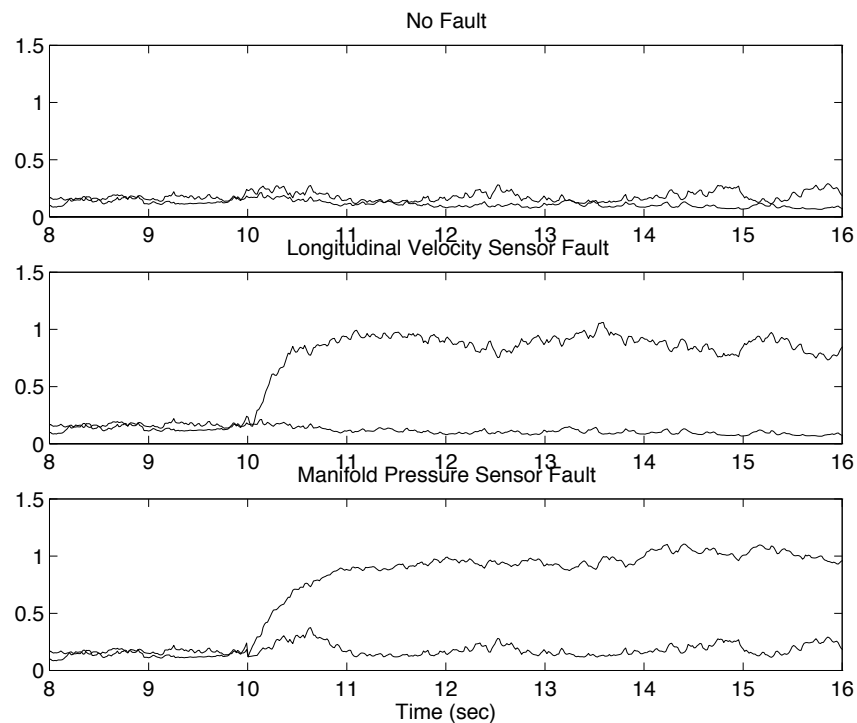


Figure 5.5: Residuals for fault detection filter two

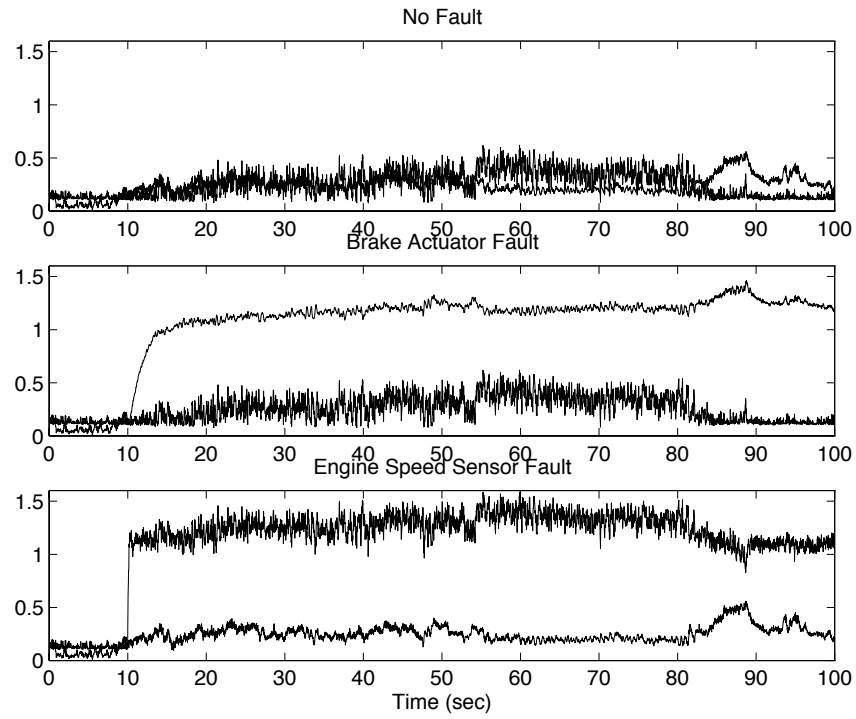


Figure 5.6: Residuals for fault detection filter three

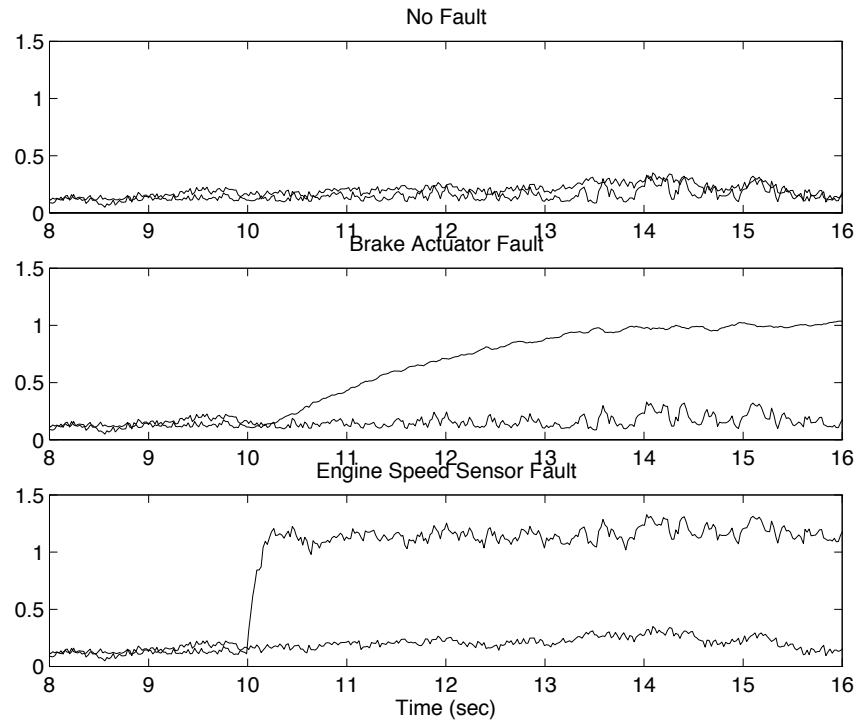


Figure 5.7: Residuals for fault detection filter three

Chapter 6

Residual Processing for Enhanced Fault Detection and Identification

The essential feature of a residual processor is to analyze the residues generated by the fault detection filters and isolate a fault, if it has occurred, with an associated probability. This allows for higher level decision making and the creation of detection thresholds. Nominally, the residual process is zero in the absence of a fault and non-zero otherwise. However, when driven by model uncertainty, nonlinearities, process disturbances and sensor noise, the residual process fails to go to zero even in the absence of faults. This is noted in the simulation studies of the detection filters. Furthermore, the residual process may be nonzero when a fault occurs for which the detection filter is not designed. In this case, the residual directional properties are not defined as the detection filter detects but cannot isolate the fault.

The residual output from all the fault detection filters may be viewed as a pattern which contains information about the presence or absence of a fault. By considering the residual processor design as a static pattern recognition problem, the residual processor could be a Bayesian neural network whose input is the residual process and the outputs, in some sense, approximate the posteriori probabilities of each fault. However, the Bayesian neural network, while easy to implement, is not easily amenable to mathematical analysis. Moreover, due to feedback, the performance deteriorates in the presence of noise at the input.

An approach which appears to perform as well as the Bayesian neural network and is amenable to mathematical analyses is the Shiriyayev sequential probability ratio test (SPRT). A multiple hypothesis Shiriyayev SPRT is derived by adopting a dynamic programming viewpoint and showed that for a certain criterion of optimality, it detects and isolates a fault in minimum time (Malladi

$\mathcal{H}_0$: No fault	$\mathcal{H}_1$: Manifold air mass sensor
$\mathcal{H}_2$: Manifold temperature sensor	$\mathcal{H}_3$: Manifold pressure sensor
$\mathcal{H}_4$: Throttle sensor	$\mathcal{H}_5$: Throttle actuator
$\mathcal{H}_6$: Brake sensor	$\mathcal{H}_7$: Brake actuator
$\mathcal{H}_8$: Engine speed sensor	$\mathcal{H}_9$: Front wheel speed sensor
$\mathcal{H}_{10}$: Rear wheel speed sensor	$\mathcal{H}_{11}$: Longitudinal accelerometer
$\mathcal{H}_{12}$: Vertical accelerometer	

Table 6.1: Fault hypotheses for residual processing

and Speyer, 1999). The fault isolation problem is now solved by assuming that each fault corresponds to a particular hypothesis. The multiple hypothesis Shiriyayev SPRT is described in detail in (Douglas *et al.*, 1996) and (Malladi and Speyer, 1999).

Note that the residual processed here is not generated by the fault detection filters and parity equations in Chapter 4. Instead, the fault detection filters and parity equations in (Douglas *et al.*, 1997) are used because they present a more interesting case. The combined residual processes, $x_k \in \mathbb{R}^{12}$, from the fault detection filters and parity equations are considered to be the measurement sequence for the Shiriyayev SPRT. The measurement sequence is assumed to have a Gaussian distribution and be conditionally independent, that is, once a fault occurs, the measurement process is independent.

There are 13 hypotheses $\{\mathcal{H}_0 \dots \mathcal{H}_{12}\}$, including the no-fault case, listed in Table 6.1. Associated with each hypothesis is a given residual probability density function. The density functions for all hypotheses are constructed as follows. First, since the residuals are assumed for simplicity to have Gaussian distribution (although not required by the theory), the only required statistics are the mean and covariance. Next, a step fault is modeled as a sudden increase in the mean of the residual process. While a ramp fault could be modeled as a gradual increase in the mean, no hypothesis models this type of fault signature directly. Using the nonlinear simulation, apply a design step fault of some particular size to one component at a time and compute the resulting residual mean and covariance matrix.

It is important to state that while the hypothesis statistics are associated with a given design step fault, the statistics remain fixed throughout all residual processor testing. The hypothesis statistics are not recomputed when the size of an applied fault does not match the design fault or when ramp faults are applied.

Suppose the statistics of the residual process $\{x\}$ are exactly modeled by the Shiriyayev SPRT as

$$\text{Under } \mathcal{H}_i : x \sim \mathcal{N}(m_i, \Lambda_i)$$

where m_i and Λ_i are a known mean and covariance. It is not surprising that when a hard fault of the same magnitude as the design step fault is applied in the nonlinear simulation, the residual processor isolates the fault almost immediately. However, real faults have an unknown magnitude and never match the design case so it seems reasonable to evaluate the residual processor by applying ramp faults to the nonlinear simulation.

To illustrate typical results a ramp fault in the manifold air mass sensor is considered. Figure 6.1 shows the residuals given by fault detection filter 1 as a ramp fault is applied to the manifold air mass sensor. The size of the fault is gradually increased from zero at two seconds to the design size of 0.07 kg/s at seven seconds. It is seen that the posteriori probability of a fault in the manifold air mass sensor, hypothesis $\mathcal{H}_1$, becomes one at around five seconds. Figure 6.2 shows the posteriori probability of a fault in the manifold temperature sensor, hypothesis $\mathcal{H}_2$. The probability increases initially, but goes back to zero as the fault size in the manifold air mass sensor increases. This indicates a ramp fault in the air mass sensor, hypothesis ($\mathcal{H}_1$), triggers a false alarm in the temperature sensor, hypothesis ($\mathcal{H}_2$).

A simple lowpass filter does not eliminate this false alarm: one possible reason for this might be the correlation of the residual process. Note that the recursive relation for the Shiriyayev SPRT requires the measurement sequence to be conditionally independent, that is, once a change in hypothesis occurs, the measurement sequence should be independent. Since a Gaussian distribution for the residual process is assumed, it needs to be uncorrelated to satisfy the underlying assumptions of the Shiriyayev SPRT. This calls for an adaptive whitening filter, one which can uncorrelate the residual process in the presence of changing statistics. The AKF developed in (Malladi and Speyer, 1996) is used for this purpose.

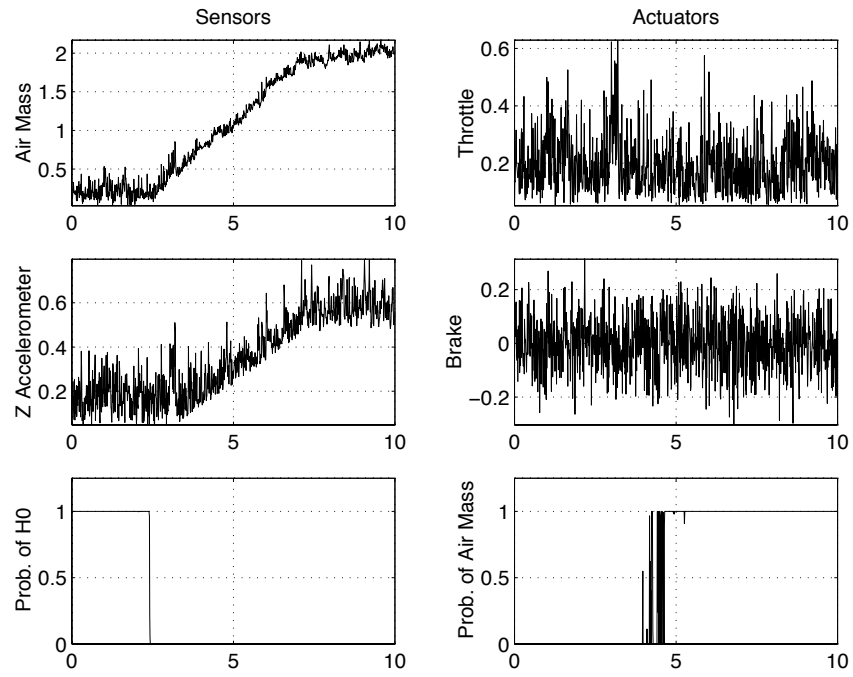


Figure 6.1: Ramp fault in manifold air mass sensor

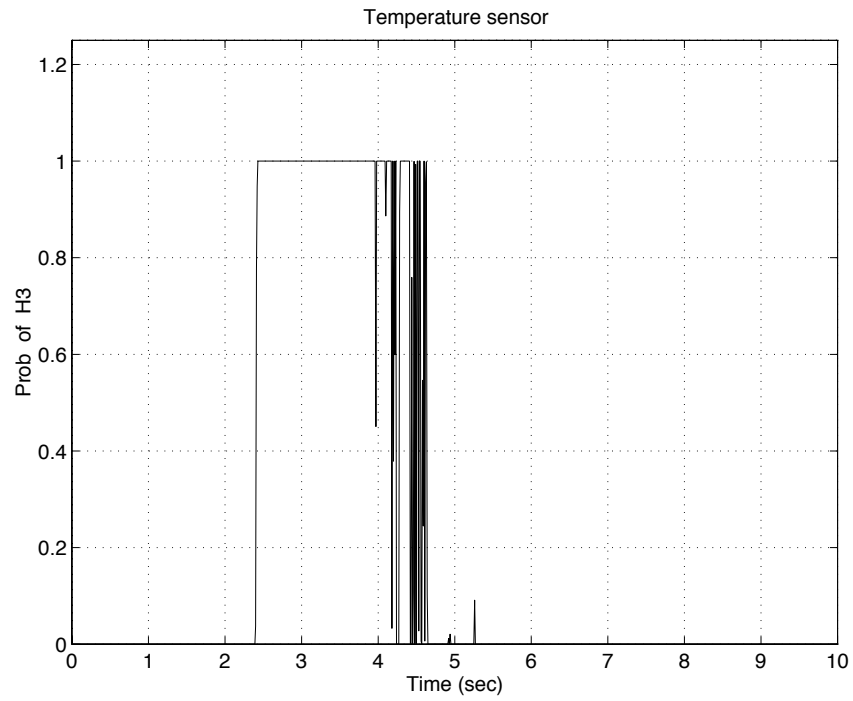


Figure 6.2: False alarm in the manifold temperature sensor as a ramp fault in manifold air mass sensor is applied

The correlated residual is modeled as an Auto-Regressive (AR) process and its coefficients are estimated by the AKF: the innovation process from the filter becomes the new input to the recursive relation of Shiryayev SPRT. Let the residual process from the detection filters be denoted by r_k . Then a state-space equivalent to the AR process can be modeled as:

$$x_{k+1} = A_k x_k + b_k + w_k$$

$$r_k = C_k x_k + d_k + v_k$$

where $C_k = [r_{k-1} | \dots | r_{k-m^{(ar)}}]$ is the measurement matrix $x_k \in \mathbb{R}^{m^{(ar)}}$ are the AR-coefficients, A_k denotes the dynamics of the AR coefficients, and b_k , d_k are appropriate bias vectors. Under each hypothesis $\mathcal{H}_i$,

$$\begin{aligned} A &= I & W_j &= W & d &= d_i \\ C_k &= [r_{k-1} | \dots | r_{k-m^{(ar)}}] & b &= [0 \ 0 \ 0 \ 0 \ 0]^T \end{aligned}$$

Clearly, the matrices A , W , and the order $m^{(ar)}$ of the AR process are design parameters in this model. The innovation process becomes:

$$\nu_k \triangleq r_k - C_k \bar{x}_k$$

Note that the bias d_k has not been included in the innovation process. Therefore, under each $\mathcal{H}_i$

$$\nu_k \approx \mathcal{N}(d_i, C_k M_k C_k^T + V_i)$$

The AKF is tested by inserting the same ramp fault in the air mass sensor as was done earlier. As design parameters, $A = 0.1 \cdot I$ and $W = 0.02 \cdot I$ are chosen. The order of the AR process is varied from 5 to 20. As in Figure 6.1, a 10 second simulation of a ramp fault in the air mass sensor is used where the size of the fault is gradually increased from zero at $t = 2$ sec to the design size of 0.07 kg/s at $t = 7$ sec.

The results of the test are illustrated in Figures 6.3 to 6.5. The false alarm in the temperature sensor is gradually eliminated as the order of the AR process increases. For an AR order of 20, it is almost totally eliminated.

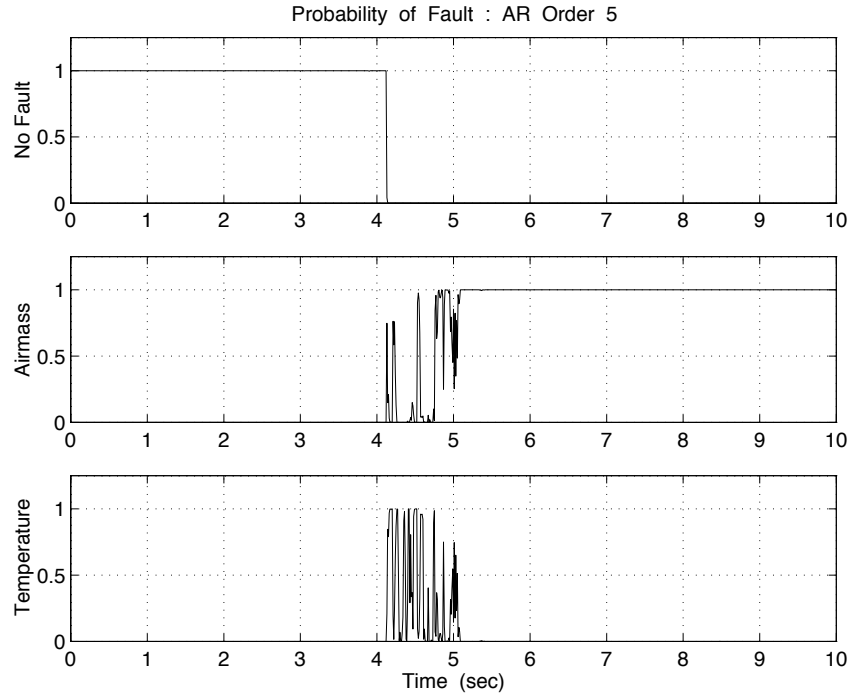


Figure 6.3: False alarm in the manifold temperature sensor: AR order is 5

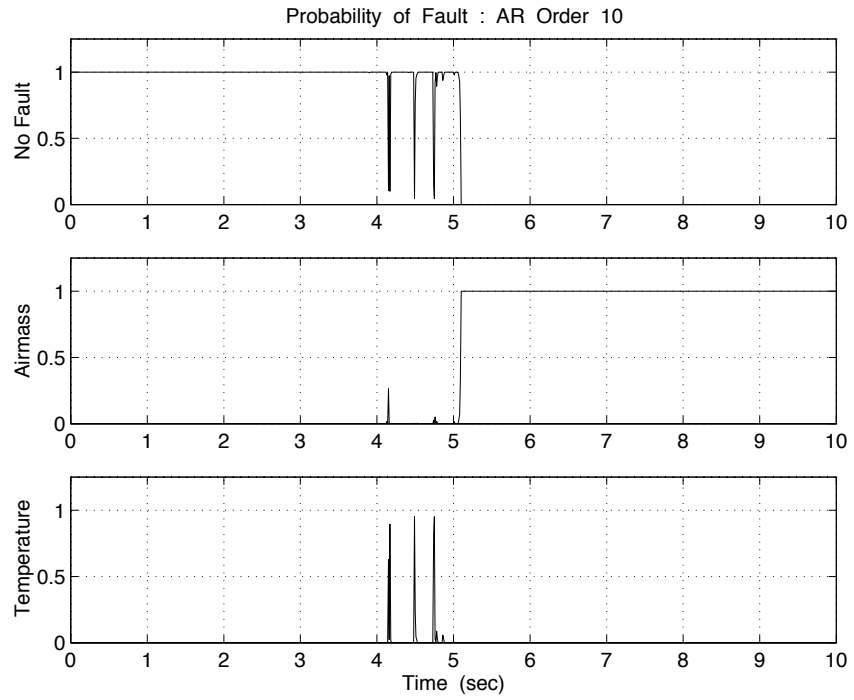


Figure 6.4: False alarm in the manifold temperature sensor: AR order is 10

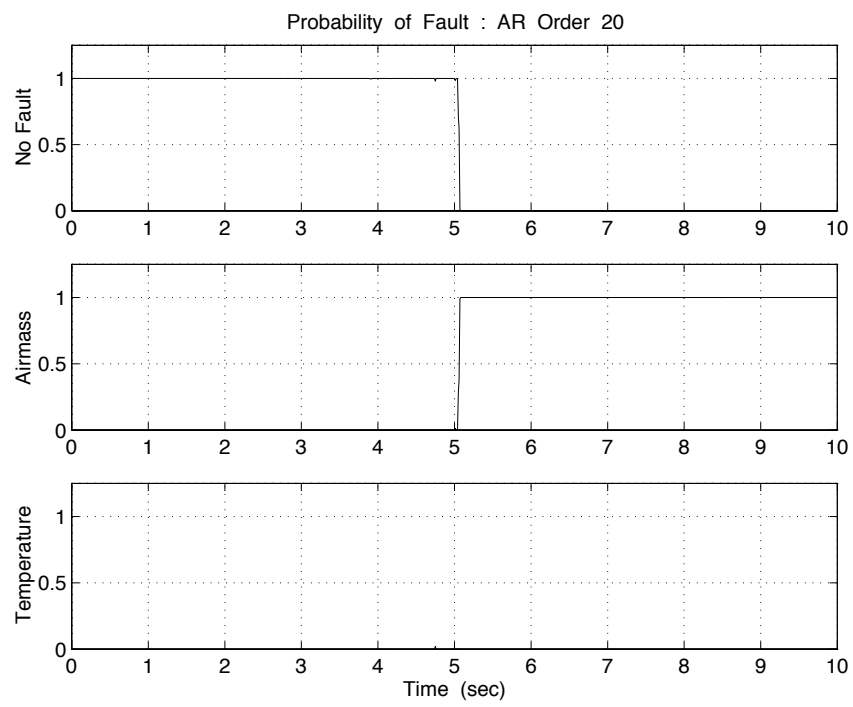


Figure 6.5: False alarm in the manifold temperature sensor: AR order is 20

Chapter 7

A Generalized Least-Squares Fault Detection Filter

In this chapter, a generalized least-squares fault detection filter (Chen and Speyer, 2000), motivated by (Bryson and Ho, 1975; Chung and Speyer, 1998), is presented. A new least-squares problem with an indefinite cost criterion is formulated as a min-max problem by generalizing the least-squares derivation of the Kalman filter (Bryson and Ho, 1975) and allowing the explicit dependence on the target fault which is not presented in (Chung and Speyer, 1998). Since the filter is derived similarly to (Chung and Speyer, 1998), many properties obtained in (Chung and Speyer, 1998) also apply to this filter. However, some new important properties are given. For example, since the target fault direction is now explicitly in the filter gain calculation, a mechanism is provided which enhances the sensitivity of the residual to the target fault. Furthermore, the projector, which annihilates the residual direction associated with the nuisance faults and is assumed in the problem formulation of (Chung and Speyer, 1998), is not required in the derivation of this filter. Finally, the nuisance fault directions are generalized for time-invariant systems so that their associated invariant zero directions are included in the invariant subspace generated by the filter. This prevents the invariant zeros from becoming part of the eigenvalues of the filter. It is also shown that this filter completely blocks the nuisance faults in the limit where the weighting on the nuisance faults is zero. In the limit, the nuisance faults are placed in a minimal (C, A) -unobservability subspace for time-invariant systems and a similar invariant subspace for time-varying systems. A minimal (C, A) -unobservability subspace (Massoumnia, 1986; Massoumnia *et al.*, 1989) implies that there is a projector $\tilde{H}$ induced from the nuisance fault directions such that $(\tilde{H}C, A - LC)$ has an unobservable subspace for some filter gain L . Therefore, the generalized least-squares fault

detection filter becomes equivalent to the unknown input observer in the limit and extends the unknown input observer to the time-varying case. Reduced-order filters are derived in the limit for both time-invariant and time-varying systems. These limiting results are important in ensuring that both fault detection and identification can occur.

The problem is formulated in Section 7.1 and its solution is derived in Section 7.2 (Chung and Speyer, 1998; Bryson and Ho, 1975; Rhee and Speyer, 1991; Banavar and Speyer, 1991). In Section 7.3, some conditions for this problem are derived by using linear matrix inequality (Chung and Speyer, 1998). In Section 7.4, the filter is derived in the limit (Chung and Speyer, 1998; Bell and Jacobson, 1975). In Section 7.5, it is shown that, in the limit, the nuisance faults are placed in an invariant subspace. For time-invariant systems, this subspace is the minimal (C, A) -unobservability subspace. In Section 7.6, reduced-order filters are derived in the limit for both time-invariant and time-varying systems. In Section 7.7, numerical examples are given.

7.1 Problem Formulation

Consider a linear system,

$$\dot{x} = Ax + B_u u + B_w w \quad (7.1a)$$

$$y = Cx + v \quad (7.1b)$$

where u is the control input, y is the measurement, w is the process noise, and v is the sensor noise. All system variables belong to real vector spaces, $x \in \mathcal{X}$, $u \in \mathcal{U}$ and $y \in \mathcal{Y}$. System matrices A , B_u , B_w and C are time-varying and continuously differentiable. Following the development in (Beard, 1971; Massoumnia, 1986; White and Speyer, 1987; Chung and Speyer, 1998), any actuator, sensor and plant fault can be modeled as an additive term in the state equation (7.1a). Therefore, a linear system with q failure modes can be modeled by

$$\dot{x} = Ax + B_u u + B_w w + \sum_{i=1}^q \bar{F}_i \bar{\mu}_i \quad (7.2a)$$

$$y = Cx + v \quad (7.2b)$$

where $\bar{\mu}_i$ belong to real vector spaces, and $\bar{F}_i$ are time-varying and continuously differentiable. The failure modes $\bar{\mu}_i$ are unknown and arbitrary functions of time that are zero when there is no failure. The failure signatures $\bar{F}_i$ are maps that are known. A failure mode $\bar{\mu}_i$ models the time-varying amplitude of a failure while a failure signature $\bar{F}_i$ models the directional characteristics of

a failure. Assume the $\bar{F}_i$ are monic so that $\bar{\mu}_i \neq 0$ imply $\bar{F}_i \bar{\mu}_i \neq 0$. In this paper, the generalized least-squares fault detection filter is designed to detect only one fault and not to be affected by other faults. Therefore, let $\mu_1 = \bar{\mu}_i$ be the target fault and $\mu_2 = [\bar{\mu}_1^T \cdots \bar{\mu}_{i-1}^T \bar{\mu}_{i+1}^T \cdots \bar{\mu}_q^T]^T$ be the nuisance fault (Massoumnia *et al.*, 1989). Then, (7.2) can be rewritten as

$$\dot{x} = Ax + B_u u + B_w w + F_1 \mu_1 + F_2 \mu_2 \quad (7.3a)$$

$$y = Cx + v \quad (7.3b)$$

where $F_1 = \bar{F}_i$ and $F_2 = [\bar{F}_1 \cdots \bar{F}_{i-1} \bar{F}_{i+1} \cdots \bar{F}_q]$.

There are three assumptions about the system (7.3) that are needed in order to have a well-conditioned unknown input observer. Assumption 7.1 is the general requirement to design any linear observer (Kwakernaak and Sivan, 1972). Assumption 7.2 ensures that the target fault can be isolated from the nuisance fault (Massoumnia *et al.*, 1989; Douglas, 1993; Chung and Speyer, 1998). The output separability test is discussed in Remark 5 of Section 7.5. Assumption 7.3 ensures for time-invariant systems, a nonzero residual in steady-state when the target fault occurs (Chen and Speyer, 1999).

Assumption 7.1. For time-varying systems, (C, A) is uniformly observable. For time-invariant systems, (C, A) is detectable.

Assumption 7.2. F_1 and F_2 are output separable.

Assumption 7.3. For time-invariant systems, (C, A, F_1) does not have invariant zero at origin.

The objective of blocking the nuisance fault while detecting the target fault can be achieved by solving the following min-max problem,

$$\min_{\mu_1} \max_{\mu_2} \max_w \max_{x(t_0)} J \quad (7.4)$$

where a generalized least-squares cost criterion is

$$J = \frac{1}{2} \int_{t_0}^t \left(\|\mu_1\|_{Q_1^{-1}}^2 - \|\mu_2\|_{\gamma Q_2^{-1}}^2 - \|w\|_{Q_w^{-1}}^2 - \|y - Cx\|_{V^{-1}}^2 \right) d\tau - \frac{1}{2} \|x(t_0) - \hat{x}_0\|_{\Pi_0}^2 \quad (7.5)$$

subject to (7.3a). Note that, without the minimization with respect to μ_1 , (7.5) reduces to the standard least-squares cost criterion of the Kalman filter (Bryson and Ho, 1975). t is the current

time and y is assumed given. Q_1, Q_2, Q_w, V and Π_0 are positive definite. γ is a non-negative scalar. Note that Q_1, Q_2, Q_w, Π_0 and γ are design parameters to be chosen while V may be physically related to the power spectral density of the sensor noise because of (7.3b) (Bryson and Ho, 1975). The interpretation of the min-max problem is the following. Let μ_1^*, μ_2^*, w^* and $x^*(t_0)$ be the optimal strategies for μ_1, μ_2, w and $x(t_0)$, respectively. Then, $x^*(\tau|Y_t)$, the x associated with μ_1^*, μ_2^*, w^* and $x^*(t_0)$, is the optimal trajectory for x where $\tau \in [t_0, t]$ and given the measurement history $Y_t = \{y(\tau)|t_0 \leq \tau \leq t\}$. Since μ_1 maximizes $y - Cx$ and $\mu_2, w, x(t_0)$ minimizes $y - Cx$, $y - Cx^*$ is made primarily sensitive to μ_1 and minimally sensitive to μ_2, w and $x(t_0)$. However, since x^* is the smoothed estimate of the state, a filtered estimate of the state, called $\hat{x}$, is needed for implementation. From the boundary condition in Section 7.2, at the current time t , $x^*(t|Y_t) = \hat{x}(t)$. Therefore, $y - C\hat{x}$ is primarily sensitive to the target fault and minimally sensitive to the nuisance fault, process noise and initial condition. Note that when Q_1 is larger, $y - C\hat{x}$ is more sensitive to the target fault. When γ is smaller, $y - C\hat{x}$ is less sensitive to the nuisance fault. In (Chung and Speyer, 1998), the cost criterion blocks the nuisance fault, but does not enhance the sensitivity to the target fault. In Section 7.5, it is shown that the filter completely blocks the nuisance fault when γ is zero by placing it into an invariant subspace, called $\text{Ker } S$. Therefore, the residual used for detecting the target fault is

$$r = \hat{H}(y - C\hat{x}) \quad (7.6)$$

where $\hat{x}$, the filtered estimate of the state, is given in Section 7.2 and

$$\hat{H} : \mathcal{Y} \rightarrow \mathcal{Y} \text{ , } \text{Ker } \hat{H} = C \text{Ker } S \text{ , } \hat{H} = I - C \text{Ker } S [(C \text{Ker } S)^T C \text{Ker } S]^{-1} (C \text{Ker } S)^T \quad (7.7)$$

$\text{Ker } S$ is given and discussed in Sections 7.4 and 7.5.

Remark 1. The process noise can be considered as part of the nuisance fault so that it could be completely blocked from the residual. However, the size of the nuisance fault is limited because the target fault and nuisance fault have to be output separable. Therefore, it is not always possible to include every process noise to the nuisance fault. The plant uncertainties can also be considered similarly to the process noise. In (Chung and Speyer, 1998), the process noise and plant uncertainties can only be considered as part of the nuisance fault. $\blacklozenge$

Remark 2. The differential game solved for the game-theoretic fault detection filter (Chung and Speyer, 1998) is

$$\min_{\hat{x}} \max_{\mu_2} \max_y \max_{x(t_0)} J$$

where

$$J = \int_{t_0}^{t_1} \left[\|\hat{H}C(x - \hat{x})\|_Q^2 - \gamma (\|\mu_2\|_{M^{-1}}^2 - \|y - Cx\|_{V^{-1}}^2) \right] dt - \|x(t_0) - \hat{x}_0\|_{\gamma P_0^{-1}}^2$$

subject to

$$\dot{x} = Ax + Bu + F_2\mu_2$$

Note that the target fault is not included in the cost criterion nor the system. Also, the derivation of the filter depends on the projector $\hat{H}$ which is defined apriori. However, there is no need for the generalized least-squares fault detection filter to explicitly introduce a projector in the cost criterion.

◀

7.2 Solution

In this section, the min-max problem given by (7.4) is solved (Chung and Speyer, 1998; Bryson and Ho, 1975; Rhee and Speyer, 1991; Banavar and Speyer, 1991). The variational Hamiltonian of the problem is

$$\begin{aligned} \mathcal{H} = & \frac{1}{2} \left(\|\mu_1\|_{Q_1^{-1}}^2 - \|\mu_2\|_{\gamma Q_2^{-1}}^2 - \|w\|_{Q_w^{-1}}^2 - \|y - Cx\|_{V^{-1}}^2 \right) \\ & + \lambda^T (Ax + B_u u + B_w w + F_1 \mu_1 + F_2 \mu_2) \end{aligned}$$

where $\lambda \in \mathcal{R}^n$ is a continuously differentiable Lagrange multiplier. The first-order necessary conditions (Bryson and Ho, 1975) imply that the optimal strategies for μ_1 , μ_2 , w and the dynamics for λ are

$$\mu_1^* = -Q_1 F_1^T \lambda \tag{7.8a}$$

$$\mu_2^* = \frac{1}{\gamma} Q_2 F_2^T \lambda \tag{7.8b}$$

$$w^* = Q_w B_w^T \lambda \tag{7.8c}$$

$$\dot{\lambda} = -A^T \lambda - C^T V^{-1} (y - Cx) \tag{7.8d}$$

with boundary conditions,

$$\lambda(t_0) = \Pi_0[x^*(t_0) - \hat{x}_0] \quad (7.8e)$$

$$\lambda(t) = 0 \quad (7.8f)$$

By substituting (7.8a), (7.8b) and (7.8c) into (7.3a) and combining with (7.8d), the two-point boundary value problem requires the solution to

$$\begin{bmatrix} \dot{x}^* \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & \frac{1}{\gamma}F_2Q_2F_2^T - F_1Q_1F_1^T + B_wQ_wB_w^T \\ C^TV^{-1}C & -A^T \end{bmatrix} \begin{bmatrix} x^* \\ \lambda \end{bmatrix} + \begin{bmatrix} B_uu \\ -C^TV^{-1}y \end{bmatrix} \quad (7.9)$$

with boundary conditions (7.8e) and (7.8f). Note that x^* is now the state using the optimal strategies (7.8a), (7.8b) and (7.8c). The form of (7.8e) suggests that

$$\lambda = \Pi(x^* - \hat{x}) \quad (7.10)$$

where $\Pi(t_0) = \Pi_0$, $\hat{x}(t_0) = \hat{x}_0$ and $\hat{x}$ is an intermediate state. By differentiating (7.10) and using (7.9),

$$\begin{aligned} 0 &= \left[\dot{\Pi} + \Pi A + A^T \Pi + \Pi \left(\frac{1}{\gamma}F_2Q_2F_2^T - F_1Q_1F_1^T + B_wQ_wB_w^T \right) \Pi - C^TV^{-1}C \right] x^* \\ &\quad - \left[\dot{\Pi} + A^T \Pi + \Pi \left(\frac{1}{\gamma}F_2Q_2F_2^T - F_1Q_1F_1^T + B_wQ_wB_w^T \right) \Pi \right] \hat{x} - \Pi \dot{\hat{x}} + \Pi B_uu + C^TV^{-1}y \end{aligned}$$

By adding and subtracting $\Pi A \hat{x}$ and $C^TV^{-1}C \hat{x}$,

$$\begin{aligned} 0 &= \left[\dot{\Pi} + \Pi A + A^T \Pi + \Pi \left(\frac{1}{\gamma}F_2Q_2F_2^T - F_1Q_1F_1^T + B_wQ_wB_w^T \right) \Pi - C^TV^{-1}C \right] (x^* - \hat{x}) \\ &\quad - \Pi \dot{\hat{x}} + \Pi A \hat{x} + \Pi B_uu + C^TV^{-1}(y - C \hat{x}) \end{aligned}$$

Therefore, (7.10) is a solution to (7.9) if

$$\Pi \dot{\hat{x}} = \Pi A \hat{x} + \Pi B_uu + C^TV^{-1}(y - C \hat{x}) \quad , \quad \hat{x}(t_0) = \hat{x}_0 \quad (7.11)$$

$$-\dot{\Pi} = \Pi A + A^T \Pi + \Pi \left(\frac{1}{\gamma}F_2Q_2F_2^T - F_1Q_1F_1^T + B_wQ_wB_w^T \right) \Pi - C^TV^{-1}C \quad , \quad \Pi(t_0) = \Pi_0 \quad (7.12)$$

By substituting μ_1^* (7.8a), μ_2^* (7.8b), w^* (7.8c) and (7.10) into the cost criterion (7.5),

$$\begin{aligned} J^* &= \frac{1}{2} \int_{t_0}^t \left[- \| x^* - \hat{x} \|_{\Pi \left(\frac{1}{\gamma}F_2Q_2F_2^T - F_1Q_1F_1^T + B_wQ_wB_w^T \right) \Pi}^2 - \| y - Cx^* \|_{V^{-1}}^2 \right] d\tau \\ &\quad - \frac{1}{2} \| x^*(t_0) - \hat{x}_0 \|_{\Pi_0}^2 \end{aligned}$$

By adding the zero term

$$0 = \frac{1}{2} \|x^*(t_0) - \hat{x}_0\|_{\Pi(t_0)}^2 - \frac{1}{2} \|x^*(t) - \hat{x}(t)\|_{\Pi(t)}^2 + \frac{1}{2} \int_{t_0}^t \frac{d}{d\tau} \|x^* - \hat{x}\|_{\Pi}^2 d\tau$$

to J^* ,

$$J^* = \frac{1}{2} \int_{t_0}^t \left[-\|x^* - \hat{x}\|_{\Pi\left(\frac{1}{\gamma}F_2Q_2F_2^T - F_1Q_1F_1^T + B_wQ_wB_w^T\right)\Pi}^2 - \|y - Cx^*\|_{V^{-1}}^2 \right. \\ \left. + (\Pi\dot{x}^* - \Pi\dot{\hat{x}})^T(x^* - \hat{x}) + (x^* - \hat{x})^T\dot{\Pi}(x^* - \hat{x}) + (x^* - \hat{x})^T(\Pi\dot{x}^* - \Pi\dot{\hat{x}}) \right] d\tau$$

Note that $\|x^*(t) - \hat{x}(t)\|_{\Pi(t)}^2 = 0$ because of the boundary condition (7.8f). By substituting $\dot{x}^*$ (7.9), (7.10), (7.11), (7.12) into J^* and expanding $\|y - Cx^*\|_{V^{-1}}^2$ into $\|(y - C\hat{x}) - C(x^* - \hat{x})\|_{V^{-1}}^2$,

$$J^* = -\frac{1}{2} \int_{t_0}^t \|y - C\hat{x}\|_{V^{-1}}^2 d\tau$$

Since $x^* = \hat{x}$ at current time t (7.8f), the generalized least-squares fault detection filter is (7.11). Note that (7.11) is used by the residual (7.6) to detect the target fault.

Remark 3. For a steady-state filter, (7.12) becomes

$$0 = \Pi A + A^T \Pi + \Pi \left(\frac{1}{\gamma} F_2 Q_2 F_2^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T \right) \Pi - C^T V^{-1} C \quad (7.13)$$

The stability of the filter depends on showing that $\hat{x}^T \Pi \hat{x}$ is a Lyapunov function where the coefficient matrix in the estimator equation (7.11) is $A_{cl} \triangleq A - \Pi^{-1} C^T V^{-1} C$. By substituting A_{cl} into (7.13),

$$\Pi A_{cl} + A_{cl}^T \Pi = -\Pi \left(\frac{1}{\gamma} F_2 Q_2 F_2^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T \right) \Pi - C^T V^{-1} C$$

When $Q_1 = 0$, given that $(A, [F_2 \ B_w])$ is uniformly controllable for time-varying systems and stabilizable for time-invariant systems, $\Pi A_{cl} + A_{cl}^T \Pi < 0$ and the filter is exponentially stable for time-varying systems and asymptotically stable for time-invariant systems (Kwakernaak and Sivan, 1972). However, when $Q_1 \neq 0$, $\Pi A_{cl} + A_{cl}^T \Pi$ might become indefinite. This can be interpreted as an attempt to make the residual sensitive to the target fault. If Q_1 is too large, the target fault could destabilize the filter. If $(A, [F_2 \ B_w])$ is not stabilizable, model reduction can be used to remove the uncontrollable and unstable subspace. Note that the game-theoretic fault detection filter (Chung and Speyer, 1998) is always stable because the target fault is not in the problem formulation. $\blacklozenge$

7.3 Conditions for the Nonpositivity of the Cost Criterion

In this section, the cost criterion (7.5) is converted into an equivalent linear matrix inequality from which the sufficient conditions for optimality can be derived (Chung and Speyer, 1998). The linear matrix inequality, associated with the solution optimality, is just the left half of the saddle point inequality,

$$J(\mu_1^*, \mu_2, w, x(t_0)) \leq J(\mu_1^*, \mu_2^*, w^*, x^*(t_0)) = 0 \leq J(\mu_1, \mu_2^*, w^*, x^*(t_0))$$

The asterisk indicates that the optimal strategy is being used for that element.

By using a Lagrange multiplier $(x - \hat{x})^T \Pi$ to adjoin the constraint (7.3a) to the cost criterion (7.5) and substituting μ_1^* (7.8a),

$$J = \frac{1}{2} \int_{t_0}^t \left[-\| \mu_2 \|_{\gamma Q_2^{-1}}^2 - \| w \|_{Q_w^{-1}}^2 - \| y - Cx \|_{V^{-1}}^2 + (x - \hat{x})^T \Pi (Ax + B_u u + B_w w + F_2 \mu_2 - \dot{x}) \right] d\tau - \frac{1}{2} \| x(t_0) - \hat{x}_0 \|_{\Pi_0}^2$$

By adding and subtracting $\frac{1}{2} \int_{t_0}^t (x - \hat{x})^T \Pi A \hat{x} d\tau$ and $\frac{1}{2} \int_{t_0}^t (x - \hat{x})^T \Pi \dot{\hat{x}} d\tau$ to J ,

$$J = \frac{1}{2} \int_{t_0}^t \left[\| x - \hat{x} \|_{\Pi A}^2 - \| \mu_2 \|_{\gamma Q_2^{-1}}^2 - \| w \|_{Q_w^{-1}}^2 - \| y - Cx \|_{V^{-1}}^2 + (x - \hat{x})^T (-\Pi \dot{\hat{x}} + \Pi A \hat{x} + \Pi B_u u + \Pi B_w w + \Pi F_2 \mu_2) - (x - \hat{x})^T \Pi (\dot{x} - \dot{\hat{x}}) \right] d\tau - \frac{1}{2} \| x(t_0) - \hat{x}_0 \|_{\Pi_0}^2$$

By integrating $(x - \hat{x})^T \Pi (\dot{x} - \dot{\hat{x}})$ by parts, substituting (7.3a) and (7.8a), adding and subtracting $\frac{1}{2} \int_{t_0}^t \hat{x}^T A^T \Pi (x - \hat{x}) d\tau$ to J ,

$$J = \frac{1}{2} \int_{t_0}^t \left[\| x - \hat{x} \|_{\Pi + \Pi A + A^T \Pi - \Pi F_1 Q_1 F_1^T \Pi}^2 - \| \mu_2 \|_{\gamma Q_2^{-1}}^2 - \| w \|_{Q_w^{-1}}^2 - \| y - Cx \|_{V^{-1}}^2 + (x - \hat{x})^T + (-\Pi \dot{\hat{x}} + \Pi A \hat{x} + \Pi B_u u + \Pi B_w w + \Pi F_2 \mu_2) + (-\Pi \dot{\hat{x}} + \Pi A \hat{x} + \Pi B_u u + \Pi B_w w + \Pi F_2 \mu_2)^T (x - \hat{x}) \right] d\tau - \frac{1}{2} \| x(t_0) - \hat{x}_0 \|_{\Pi_0 - \Pi(t_0)}^2 - \frac{1}{2} \| x(t) - \hat{x}(t) \|_{\Pi(t)}^2$$

By expanding $\| y - Cx \|_{V^{-1}}^2$ into $\| (y - C\hat{x}) - C(x - \hat{x}) \|_{V^{-1}}^2$ and substituting the generalized least-squares fault detection filter (7.11) into J ,

$$J = \frac{1}{2} \int_{t_0}^t \left\{ \begin{bmatrix} (x - \hat{x})^T & w^T & \mu_2^T \end{bmatrix} W \begin{bmatrix} x - \hat{x} \\ w \\ \mu_2 \end{bmatrix} - \| y - C\hat{x} \|_{V^{-1}}^2 \right\} d\tau - \frac{1}{2} \| x(t_0) - \hat{x}_0 \|_{\Pi_0 - \Pi(t_0)}^2 - \frac{1}{2} \| x(t) - \hat{x}(t) \|_{\Pi(t)}^2$$

where

$$W = \begin{bmatrix} \dot{\Pi} + \Pi A + A^T \Pi - \Pi F_1 Q_1 F_1^T \Pi - C^T V^{-1} C & \Pi B_w & \Pi F_2 \\ B_w^T \Pi & -Q_w^{-1} & 0 \\ F_2^T \Pi & 0 & -\gamma Q_2^{-1} \end{bmatrix}$$

Therefore, the sufficient conditions for $J \leq 0$ are

$$W \leq 0 \tag{7.14}$$

$$\Pi_0 - \Pi(t_0) \geq 0$$

$$\Pi(t) \geq 0$$

In the limit where γ is zero, (7.14) becomes

$$\Pi F_2 = 0 \tag{7.15a}$$

$$\dot{\Pi} + \Pi A + A^T \Pi + \Pi(-F_1 Q_1 F_1^T + B_w Q_w B_w^T) \Pi - C^T V^{-1} C \leq 0 \tag{7.15b}$$

More detail about the limit is discussed in next section.

7.4 Limiting Case

In this section, the min-max problem (7.4) is solved in the limit where γ is zero (Chung and Speyer, 1998; Bell and Jacobson, 1975). It is shown that the solution satisfies the sufficient condition (7.15) derived from the linear matrix inequality. When γ is zero, there is no constraint on μ_2 to minimize $y - Cx$. Therefore, the nuisance fault is completely blocked from the residual which is shown in Sections 7.5 and 7.6.

In the limit, the cost criterion (7.5) becomes

$$J = \frac{1}{2} \int_{t_0}^t \left(\|\mu_1\|_{Q_1^{-1}}^2 - \|w\|_{Q_w^{-1}}^2 - \|y - Cx\|_{V^{-1}}^2 \right) d\tau - \frac{1}{2} \|x(t_0) - \hat{x}_0\|_{\Pi_0}^2 \tag{7.16}$$

This problem is singular with respect to μ_2 . Therefore, the Goh transformation (Bell and Jacobson, 1975) is used to form a nonsingular problem. Let

$$\begin{aligned} \phi_1(\tau) &= \int_{t_0}^{\tau} \mu_2(s) ds \\ \alpha_1 &= x - F_2 \phi_1 \end{aligned} \tag{7.17}$$

By differentiating (7.17) and using (7.3a),

$$\dot{\alpha}_1 = A\alpha_1 + B_u u + B_w w + F_1 \mu_1 + B_1 \phi_1 \tag{7.18}$$

where $B_1 = AF_2 - \dot{F}_2$. By substituting (7.17) into the limiting cost criterion (7.16),

$$J = \frac{1}{2} \int_{t_0}^t \left[\|\mu_1\|_{Q_1^{-1}}^2 - \|w\|_{Q_w^{-1}}^2 - \|\phi_1\|_{F_2^T C^T V^{-1} C F_2}^2 - \|y - C\alpha_1\|_{V^{-1}}^2 + (y - C\alpha_1)^T V^{-1} C F_2 \phi_1 + \phi_1^T F_2^T C^T V^{-1} (y - C\alpha_1) \right] d\tau - \frac{1}{2} \|\alpha_1(t_0^+) + F_2 \phi_1(t_0^+) - \hat{x}_0\|_{\Pi_0}^2 \quad (7.19)$$

Then, the new min-max problem is

$$\min_{\mu_1} \max_{\phi_1} \max_w \max_{\alpha_1(t_0^+)} J \quad (7.20)$$

subject to (7.18).

If $F_2^T C^T V^{-1} C F_2$ fails to be positive definite, (7.20) is still a singular problem with respect to ϕ_1 . Then, the Goh transformation has to be used until the problem becomes nonsingular. If $F_2^T C^T V^{-1} C F_2 = 0$, let

$$\begin{aligned} \phi_2(\tau) &= \int_{t_0}^{\tau} \phi_1(s) ds \\ \alpha_2 &= \alpha_1 - B_1 \phi_2 \end{aligned} \quad (7.21)$$

Then,

$$\dot{\alpha}_2 = A\alpha_2 + B_u u + B_w w + F_1 \mu_1 + B_2 \phi_2 \quad (7.22)$$

where $B_2 = AB_1 - \dot{B}_1$. If $F_2^T C^T V^{-1} C F_2 \geq 0$, the Goh transformation is applied only on the singular part which is discussed in three cases. First, if some column vectors of $C F_2$ is zero, by rearranging the order of the column vectors of F_2 ,

$$F_2^T C^T V^{-1} C F_2 = \begin{bmatrix} \bar{Q} & 0 \\ 0 & 0 \end{bmatrix} \quad (7.23)$$

where $\bar{Q}$ is positive definite. Then, ϕ_1 can be divided into ϕ_{11} and ϕ_{12} such that (7.20) is singular with respect to ϕ_{12} , but not ϕ_{11} . The associated fault directions of ϕ_{11} and ϕ_{12} are denoted by B_{11} and B_{12} , respectively. Then the Goh transformation is applied only on ϕ_{12} ,

$$\begin{aligned} \phi_{22}(\tau) &= \int_{t_0}^{\tau} \phi_{12}(s) ds \\ \alpha_2 &= \alpha_1 - B_{12} \phi_{22} \end{aligned} \quad (7.24)$$

Then,

$$\dot{\alpha}_2 = A\alpha_2 + B_u u + B_w w + F_1 \mu_1 + B_2 \phi_2 \quad (7.25)$$

where $\phi_2 = [\phi_{11}^T \ \phi_{22}^T]^T$ and $B_2 = [B_{11} \ AB_{12} - \dot{B}_{12}]$.

The second case is that $CF_2 \neq 0$, $\text{rank}(CF_2) < \dim(CF_2)$, and $\text{rank } F_2 = \dim F_2$ which imply some column vectors of CF_2 are the linear combinations of others. Then, a new basis will be chosen for F_2 such that some column vectors of the new CF_2 are zero and (7.23) is satisfied. The third case is that $CF_2 \neq 0$ and $\text{rank } F_2 < \dim F_2$ which imply some column vectors of F_2 are the linear combinations of others. Then, a new set of lower-order vectors can be formed for F_2 such that the new $F_2^T C^T V^{-1} CF_2 > 0$. Note that it is possible that these three cases could happen at the same time. Nevertheless, there always exists a transformation for F_2 such that either (7.23) is satisfied or $F_2^T C^T V^{-1} CF_2 > 0$. By substituting (7.21) or (7.24) into (7.19),

$$J = \frac{1}{2} \int_{t_0}^t \left[\mu_1 \|Q_1^{-1}\|^2 - \|w\|_{Q_w^{-1}}^2 - \|\phi_2\|_{B_1^T C^T V^{-1} C B_1}^2 - \|y - C\alpha_2\|_{V^{-1}}^2 + (y - C\alpha_2)^T V^{-1} C B_1 \phi_2 \right. \\ \left. + \phi_2^T B_1^T C^T V^{-1} (y - C\alpha_2) \right] d\tau - \frac{1}{2} \|\alpha_2(t_0^+) + [F_2 \ B_1][\phi_1(t_0^+)^T \ \phi_2(t_0^+)^T]^T - \hat{x}_0\|_{\Pi_0}^2$$

Then, the new min-max problem is

$$\min_{\mu_1} \max_{\phi_2} \max_w \max_{\alpha_2(t_0^+)} J$$

subject to (7.22) or (7.25).

The transformation process stops if the weighting on ϕ_2 , $B_1^T C^T V^{-1} C B_1$, is positive definite. Otherwise, continue the transformation until there exists B_k such that the weighting on ϕ_k , $B_{k-1}^T C^T V^{-1} C B_{k-1}$, is positive definite. Then, in the limit, the min-max problem (7.4) becomes

$$\min_{\mu_1} \max_{\phi_k} \max_w \max_{\alpha_k(t_0^+)} J$$

where

$$J = \frac{1}{2} \int_{t_0}^t \left[\mu_1 \|Q_1^{-1}\|^2 - \|w\|_{Q_w^{-1}}^2 - \|\phi_k\|_{B_{k-1}^T C^T V^{-1} C B_{k-1}}^2 - \|y - C\alpha_k\|_{V^{-1}}^2 \right. \\ \left. + (y - C\alpha_k)^T V^{-1} C B_{k-1} \phi_k + \phi_k^T B_{k-1}^T C^T V^{-1} (y - C\alpha_k) \right] d\tau \\ - \frac{1}{2} \|\alpha_k(t_0^+) + \bar{B}\bar{\phi}(t_0^+) - \hat{x}_0\|_{\Pi_0}^2 \quad (7.26)$$

and $\bar{B} = [F_2 \ B_1 \ B_2 \ \cdots \ B_{k-1}]$, $\bar{\phi} = [\phi_1^T \ \phi_2^T \ \cdots \ \phi_k^T]^T$ subject to

$$\dot{\alpha}_k = A\alpha_k + B_u u + B_w w + F_1 \mu_1 + B_k \phi_k \quad (7.27)$$

The variational Hamiltonian of the problem is

$$\begin{aligned}\mathcal{H} = & \frac{1}{2} \left[\| \mu_1 \|^2_{Q_1^{-1}} - \| w \|^2_{Q_w^{-1}} - \| \phi_k \|^2_{B_{k-1}^T C^T V^{-1} C B_{k-1}} - \| y - C \alpha_k \|^2_{V^{-1}} \right. \\ & + (y - C \alpha_k)^T V^{-1} C B_{k-1} \phi_k + \phi_k^T B_{k-1}^T C^T V^{-1} (y - C \alpha_k) \left. \right] \\ & + \lambda^T (A \alpha_k + B_u u + B_w w + F_1 \mu_1 + B_k \phi_k)\end{aligned}$$

where $\lambda \in \mathcal{R}^n$ is a continuously differentiable Lagrange multiplier. The first-order necessary conditions imply that the optimal strategies for μ_1 , ϕ_k , w and the dynamics for λ are

$$\mu_1^* = -Q_1 F_1^T \lambda \quad (7.28a)$$

$$\phi_k^* = (B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} [B_k^T \lambda + B_{k-1}^T C^T V^{-1} (y - C \alpha_k)] \quad (7.28b)$$

$$w^* = Q_w B_w^T \lambda \quad (7.28c)$$

$$\dot{\lambda} = -A^T \lambda - C^T V^{-1} (y - C \alpha_k) + C^T V^{-1} C B_{k-1} \phi_k^* \quad (7.28d)$$

with boundary conditions,

$$\bar{\phi}^*(t_0^+) = -(\bar{B}^T \Pi_0 \bar{B})^{-1} \bar{B}^T \Pi_0 [\alpha_k^*(t_0^+) - \hat{x}_0] \quad (7.28e)$$

$$\lambda(t_0^+) = \Pi_0 [\alpha_k^*(t_0^+) + \bar{B} \bar{\phi}^*(t_0^+) - \hat{x}_0] \quad (7.28f)$$

$$\lambda(t) = 0 \quad (7.28g)$$

By substituting (7.28e) into (7.28f),

$$\lambda(t_0^+) = [\Pi_0 - \Pi_0 \bar{B} (\bar{B}^T \Pi_0 \bar{B})^{-1} \bar{B}^T \Pi_0] [\alpha_k^*(t_0^+) - \hat{x}_0] \quad (7.28h)$$

By substituting (7.28a), (7.28b), (7.28c) into (7.27) and (7.28b) into (7.28d), a two-point boundary value problem with boundary conditions (7.28g) and (7.28h) results for satisfying

$$\begin{aligned} \begin{bmatrix} \dot{\alpha}_k^* \\ \dot{\lambda} \end{bmatrix} = & \begin{bmatrix} \bar{A} & B_k (B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T \\ C^T \bar{H}^T V^{-1} \bar{H} C & -\bar{A}^T \end{bmatrix} \begin{bmatrix} \alpha_k^* \\ \lambda \end{bmatrix} \\ & + \begin{bmatrix} B_u u + B_k (B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T C^T V^{-1} y \\ -C^T \bar{H}^T V^{-1} \bar{H} y \end{bmatrix} \end{aligned} \quad (7.29)$$

where $\bar{A} = A - B_k (B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T C^T V^{-1} C$ and $\bar{H} = I - C B_{k-1} (B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_{k-1}^T C^T V^{-1}$. Note that α_k^* is now the state using the optimal strategies (7.28a), (7.28b) and (7.28c). The form of (7.28h) suggests that

$$\lambda = S(\alpha_k^* - \hat{x}) \quad (7.30)$$

where

$$S(t_0^+) = \Pi_0 - \Pi_0 \bar{B} (\bar{B}^T \Pi_0 \bar{B})^{-1} \bar{B}^T \Pi_0 \quad (7.31a)$$

$$\hat{x}(t_0^+) = \hat{x}_0 \quad (7.31b)$$

and $\hat{x}$ is an intermediate state. By differentiating (7.30) and using (7.29),

$$\begin{aligned} 0 = & \left\{ \dot{S} + S\bar{A} + \bar{A}^T S + S[B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T] S \right. \\ & \left. - C^T \bar{H}^T V^{-1} \bar{H} C \right\} \alpha_k^* \\ & - \left\{ \dot{S} + \bar{A}^T S + S[B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T] S \right\} \hat{x} \\ & - S\dot{\hat{x}} + S B_u u + [S B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T C^T V^{-1} + C^T \bar{H}^T V^{-1} \bar{H}] y \end{aligned}$$

By adding and subtracting $S\bar{A}\hat{x}$ and $C^T \bar{H}^T V^{-1} \bar{H} C \hat{x}$,

$$\begin{aligned} 0 = & \left\{ \dot{S} + S\bar{A} + \bar{A}^T S + S[B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T] S \right. \\ & \left. - C^T \bar{H}^T V^{-1} \bar{H} C \right\} (\alpha_k^* - \hat{x}) - S\dot{\hat{x}} + S\bar{A}\hat{x} + S B_u u + [S B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T C^T V^{-1} \\ & + C^T \bar{H}^T V^{-1} \bar{H}] (y - C\hat{x}) \end{aligned}$$

Therefore, (7.30) is a solution to (7.29) if

$$S\dot{\hat{x}} = S\bar{A}\hat{x} + S B_u u + [S B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T C^T V^{-1} + C^T \bar{H}^T V^{-1} \bar{H}] (y - C\hat{x}) \quad (7.32)$$

$$\begin{aligned} -\dot{S} = & S[A - B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T C^T V^{-1} C] \\ & + [A - B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T C^T V^{-1} C]^T S \\ & + S[B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T] S - C^T \bar{H}^T V^{-1} \bar{H} C \end{aligned} \quad (7.33)$$

subject to (7.31).

By substituting μ_1^* (7.28a), ϕ_k^* (7.28b), w^* (7.28c), $\bar{\phi}^*(t_0^+)$ (7.28e) and (7.30) into the limiting cost criterion (7.26),

$$\begin{aligned} J^* = & \frac{1}{2} \int_{t_0}^t \left\{ -\| \alpha_k^* - \hat{x} \|_{S[B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T] S}^2 - \| y - C\alpha_k^* \|_{\bar{H}^T V^{-1} \bar{H}}^2 \right\} d\tau \\ & - \frac{1}{2} \| \alpha_k^*(t_0^+) - \hat{x}_0 \|_{\Pi_0 - \Pi_0 \bar{B} (\bar{B}^T \Pi_0 \bar{B})^{-1} \bar{B}^T \Pi_0}^2 \end{aligned}$$

By adding the zero term

$$0 = \frac{1}{2} \| \alpha_k^*(t_0^+) - \hat{x}_0 \|_{S(t_0^+)}^2 - \frac{1}{2} \| \alpha_k^*(t) - \hat{x}(t) \|_{S(t)}^2 + \frac{1}{2} \int_{t_0}^t \frac{d}{d\tau} \| \alpha_k^* - \hat{x} \|_S^2 d\tau$$

to J^* ,

$$J^* = \frac{1}{2} \int_{t_0}^t \left\{ -\| \alpha_k^* - \hat{x} \|^2_{S[B_k(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T] S} - \| y - C \alpha_k^* \|^2_{\bar{H}^T V^{-1} \bar{H}} \right. \\ \left. + (S \dot{\alpha}_k^* - S \dot{\hat{x}})^T (\alpha_k^* - \hat{x}) + (\alpha_k^* - \hat{x})^T \dot{S} (\alpha_k^* - \hat{x}) + (\alpha_k^* - \hat{x})^T (S \dot{\alpha}_k^* - S \dot{\hat{x}}) \right\} d\tau$$

Note that $\| \alpha_k^*(t) - \hat{x}(t) \|_{S(t)}^2 = 0$ because of the boundary condition (7.28g). By substituting $\dot{\alpha}_k^*$ (7.29), (7.30), (7.32), (7.33) into J^* and expanding $\| y - C \alpha_k^* \|^2_{\bar{H}^T V^{-1} \bar{H}}$ into $\| (y - C \hat{x}) - C(\alpha_k^* - \hat{x}) \|^2_{\bar{H}^T V^{-1} \bar{H}}$,

$$J^* = -\frac{1}{2} \int_{t_0}^t \| y - C \hat{x} \|^2_{\bar{H}^T V^{-1} \bar{H}} d\tau$$

Since $\alpha_k^* = \hat{x}$ at current time t (7.28g), the limiting generalized least-squares fault detection filter is (7.32). However, (7.32) can not be used because S has a null space which is shown in Theorem 7.1. Therefore, a reduced-order filter for (7.32) is derived in Section 7.6. Note that the second half of the optimization problem is solved differently as (Chung and Speyer, 1998).

Theorem 7.1 shows that the limiting Riccati matrix S has a null space and satisfies the sufficient condition (7.15) derived from the linear matrix inequality which implies that S is the limit of Π .

Theorem 7.1.

$$S \begin{bmatrix} B_{k-1} & \cdots & B_1 & F_2 \end{bmatrix} = 0 \\ \dot{S} + SA + A^T S + S(-F_1 Q_1 F_1^T + B_w Q_w B_w^T) S - C^T V^{-1} C \leq 0$$

Proof. By multiplying (7.33) by B_{k-1} from the right and subtracting $S \dot{B}_{k-1}$ from both sides,

$$\frac{d}{d\tau}(S B_{k-1}) = \\ -[A^T + S(-F_1 Q_1 F_1^T + B_w Q_w B_w^T) + (S B_k - C^T V^{-1} C B_{k-1})(B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} B_k^T] S B_{k-1}$$

This is a homogeneous differential equation and the boundary condition is zero because $S(t_0^+) \bar{B} = 0$ from (7.31a) and B_{k-1} is contained in $\bar{B}$. Therefore $S B_{k-1} = 0$. Similarly, by multiplying (7.33) by $B_{k-2}, \dots, B_1$ and F_2 ,

$$S \begin{bmatrix} B_{k-2} & \cdots & B_1 & F_2 \end{bmatrix} = 0$$

To prove the second part of this theorem, (7.33) can be rewritten as

$$\dot{S} + SA + A^T S + S(-F_1 Q_1 F_1^T + B_w Q_w B_w^T) S - C^T V^{-1} C \\ = -(B_k^T S - B_{k-1}^T C^T V^{-1} C)^T (B_{k-1}^T C^T V^{-1} C B_{k-1})^{-1} (B_k^T S - B_{k-1}^T C^T V^{-1} C)$$

and it is nonpositive definite. ◀

7.5 Properties of the Null Space of S

In this section, some properties of the null space of S are given. It is shown that the null space of S is equivalent to the minimal (C, A) -unobservability subspace for time-invariant systems and a similar invariant subspace for time-varying systems. Therefore, the limiting generalized least-squares fault detection filter is equivalent to the unknown input observer and extends it to the time-varying case. The minimal (C, A) -unobservability subspace is a subspace which is $(A - LC)$ -invariant and unobservable with respect to $(\tilde{H}C, A - LC)$ for some filter gain L and projector $\tilde{H}$ (Massoumnia *et al.*, 1989). One method for computing the minimal (C, A) -unobservability subspace of F_2 , called $\mathcal{T}_2$ here, is $\mathcal{T}_2 = \mathcal{W}_2 \oplus \mathcal{V}_2$ (Wonham, 1985; Douglas, 1993) where $\mathcal{W}_2 = [F_2 \ B_1 \ \cdots \ B_{k-1}]$ is the minimal (C, A) -invariant subspace of F_2 and $\mathcal{V}_2$ is the subspace spanned by the invariant zero directions of (C, A, F_2) . The associated $\tilde{H}$ is

$$\tilde{H} : \mathcal{Y} \rightarrow \mathcal{Y} \ , \ \text{Ker } \tilde{H} = C\mathcal{T}_2 = CB_{k-1} \ , \ \tilde{H} = I - CB_{k-1}[(CB_{k-1})^T CB_{k-1}]^{-1}(CB_{k-1})^T \quad (7.34)$$

Note that $\text{Ker } \tilde{H} = \text{Ker } \bar{H}$.

Theorem 7.2 shows that the null space of S is a (C, A) -invariant subspace. Theorem 7.3 shows that the null space of S is contained in the unobservable subspace of $(\tilde{H}C, A - LC)$.

Theorem 7.2. $\text{Ker } S$ is a (C, A) -invariant subspace.

Proof. The dynamic equation of the error, $e = x - \hat{x}$, in the absence of the target fault, process noise and sensor noise can be obtained by using (7.3) and (7.32).

$$S\dot{e} = [SA - SB_k(B_{k-1}^T C^T V^{-1} CB_{k-1})^{-1} B_{k-1}^T C^T V^{-1} C - C^T \bar{H}^T V^{-1} \bar{H} C]e$$

because $SF_2 = 0$. By adding $\dot{S}e$ to both sides and using (7.33),

$$\begin{aligned} \frac{d}{d\tau}(Se) = & - \{[A - B_k(B_{k-1}^T C^T V^{-1} CB_{k-1})^{-1} B_{k-1}^T C^T V^{-1} C]^T + \\ & S[B_k(B_{k-1}^T C^T V^{-1} CB_{k-1})^{-1} B_k^T - F_1 Q_1 F_1^T + B_w Q_w B_w^T]\} Se \end{aligned} \quad (7.35)$$

If the error initially lies in $\text{Ker } S$, (7.35) implies that the error will never leave $\text{Ker } S$. Therefore, $\text{Ker } S$ is a (C, A) -invariant subspace. ◐

Theorem 7.3. $\text{Ker } S$ is contained in the unobservable subspace of $(\tilde{H}C, A - LC)$.

Proof. Let $\zeta \in \text{Ker } S$. By multiplying (7.33) by ζ^T from the left and ζ from the right,

$$\frac{d}{d\tau}(\zeta^T S \zeta) = \zeta^T C^T \bar{H}^T V^{-1} \bar{H} C \zeta = 0$$

Then, $\tilde{H}C\zeta = 0$ because $\bar{H}C\zeta = 0$ and $\text{Ker } \tilde{H} = \text{Ker } \bar{H}$. From Theorem 7.2, $\text{Ker } S$ is a (C, A) -invariant subspace. Therefore, $\text{Ker } S$ is contained in the unobservable subspace of $(\tilde{H}C, A - LC)$.

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From Theorem 7.1, $C \text{Ker } S \supseteq CB_{k-1}$. From Theorem 7.3, $C \text{Ker } S \subseteq CB_{k-1}$. Therefore, $C \text{Ker } S = CB_{k-1}$ and $\hat{H}$ (7.7) is equivalent to $\tilde{H}$ (7.34). Note that (7.34) is a better way to form $\hat{H}$ which is used by the residual (7.6) because it does not require the solution to the limiting Riccati equation (7.33).

For time-invariant systems, it is important to discuss the invariant zero directions when designing the fault detection filter. The invariant zeros of (C, A, F_2) will become part of the eigenvalues of the filter if their associated invariant zero directions are not included in the invariant subspace of F_2 (Massoumnia *et al.*, 1989; Douglas, 1993). Therefore, the null space of S needs to include at least the invariant zero directions associated with the invariant zeros on the right-half plane and $j\omega$ -axis. However, the invariant zeros on the left-half plane might become part of the filter eigenvalues since there is no guarantee that their associated invariant zero directions are in the null space of S . It is important that the left-half-plane invariant zeros are not part of the filter eigenvalues because they might be ill-conditioned even though stable. This can be done by modifying the nuisance fault direction to enforce the null space of S to include the invariant zero direction. The invariant zero of (C, A, F_2) is z at which $M(z)$ loses rank where

$$M(z) = \begin{bmatrix} zI - A & F_2 \\ C & 0 \end{bmatrix}$$

An invariant zero direction ν is formed from a partitioning of the null space of $M(z)$ as

$$\begin{bmatrix} zI - A & F_2 \\ C & 0 \end{bmatrix} \begin{bmatrix} \nu \\ \bar{\nu} \end{bmatrix} = 0 \quad (7.36)$$

If one of the column vectors of F_2 , called F_{2i} , has an invariant zero and the invariant zero direction is ν_i , the null space of S includes $\text{span}\{F_{2i} \ A F_{2i} \ \dots \ A^{k_{2i}-1} F_{2i}\}$ from Theorem 7.1 where k_{2i} is the smallest positive integer such that $CA^{k_{2i}-1} F_{2i} \neq 0$. However, ν_i might not be included. By

modifying F_{2i} to ν_i , the null space of S includes $\text{span}\{\nu_i \ A\nu_i \cdots A^{k_{2i}}\nu_i\}$ which is equivalent to $\text{span}\{F_{2i} \ AF_{2i} \cdots A^{k_{2i}-1}F_{2i} \ \nu_i\}$ by (7.36). Therefore, this modification guarantees that the invariant zero direction is in the null space of S . This is demonstrated by the numerical examples in Section 7.7.3. If (C, A, ν_i) has invariant zeros, the same procedure above will be repeated until there is no invariant zero. If the invariant zero is associated with not just one, but several column vectors of F_2 , only one of these vectors will be replaced by the invariant zero direction.

For time-invariant systems, from Theorem 7.1 and modified nuisance fault direction, the null space of S contains the minimal (C, A) -unobservability subspace of F_2 . From Theorem 7.3, the null space of S is contained in the minimal (C, A) -unobservability subspace of F_2 . Therefore, the null space of S is equivalent to the minimal (C, A) -unobservability subspace of F_2 , and the limiting generalized least-squares fault detection filter is equivalent to the unknown input observer. Note that the invariant zero and minimal (C, A) -unobservability subspace are only defined for time-invariant systems. For time-varying systems, Theorems 7.1, 7.2 and 7.3 imply that the null space of S is a similar invariant subspace.

Remark 4. The modification of the nuisance fault direction can apply to the game-theoretic fault detection filter (Chung and Speyer, 1998) so that it could become equivalent to the unknown input observer in the limit. ◀

Remark 5. In order to detect the target fault, F_1 can not intersect the null space of S which is unobservable to the residual. If it does, the target fault will be difficult or impossible to detect even though the filter can still be derived by solving the min-max problem. If F_1 does not intersect the null space of S , F_1 and F_2 are called output separable (Massoumnia *et al.*, 1989), and the output separability test can be stated as

$$CB_{k-1} \cap C\tilde{B}_{\tilde{k}-1} = \emptyset$$

where $\tilde{B}_{\tilde{k}-1}$ is the Goh transformation of F_1 . ◀

7.6 Reduced-Order Filter

In this section, reduced-order filters are derived for the limiting generalized least-squares fault detection filter (7.32) for both time-varying and time-invariant systems. The reduced-order filter

is necessary for implementation because (7.32) can not be used due to the null space of S . It is shown that the reduced-order filter completely blocks the nuisance fault.

Since $S(t)$ is non-negative definite, there exists a state transformation $\Gamma(t)$ such that

$$\Gamma(t)^T S(t) \Gamma(t) = \begin{bmatrix} \bar{S}(t) & 0 \\ 0 & 0 \end{bmatrix} \quad (7.37)$$

where $\bar{S}(t)$ is positive definite. Theorem 7.4 provides a way to form the transformation.

Theorem 7.4. There exists a state transformation $\Gamma(t)$ where

$$\begin{bmatrix} \bar{Z}(t) & \text{Ker } S(t) \end{bmatrix} = \Gamma(t) \begin{bmatrix} Z_1(t) & 0 \\ 0 & Z_2(t) \end{bmatrix} \quad (7.38)$$

$\bar{Z}$ is any $n \times (n - k_2)$ continuously differentiable matrix such that itself and $\text{Ker } S$ span the state space where $n = \dim \mathcal{X}$ and $k_2 = \dim(\text{Ker } S)$. Z_1 and Z_2 are any $(n - k_2) \times (n - k_2)$ and $k_2 \times k_2$ invertible continuously differentiable matrices, respectively. Then, the $\Gamma(t)$ obtained from (7.38) satisfies (7.37).

Proof.

$$\text{Ker } S = \Gamma \begin{bmatrix} 0 \\ Z_2 \end{bmatrix} \Rightarrow S \Gamma \begin{bmatrix} 0 \\ Z_2 \end{bmatrix} = 0 \Rightarrow \Gamma^T S \Gamma \begin{bmatrix} 0 \\ Z_2 \end{bmatrix} = 0$$

Since Z_2 is invertible by definition and $\Gamma^T S \Gamma$ is symmetric, (7.37) is true. ◀

Note that Theorem 7.4 does not define Γ uniquely and Γ can be computed apriori because $\text{Ker } S$ can be obtained apriori.

By applying the transformation to the estimator states,

$$\Gamma^{-1} \hat{x} \triangleq \hat{\eta} = \begin{bmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \end{bmatrix}$$

By multiplying (7.32) by Γ^T from the left, adding $\Gamma^T S \Gamma \dot{\Gamma}^{-1} \hat{x}$ to both sides, and using $\Gamma \Gamma^{-1} = I$,

$$\begin{aligned} \begin{bmatrix} \bar{S} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\hat{\eta}}_1 \\ \dot{\hat{\eta}}_2 \end{bmatrix} &= - \begin{bmatrix} \bar{S} & 0 \\ 0 & 0 \end{bmatrix} \Gamma^{-1} \dot{\Gamma} \begin{bmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \end{bmatrix} + \begin{bmatrix} \bar{S} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \end{bmatrix} + \begin{bmatrix} \bar{S} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} u \\ &+ \left\{ \begin{bmatrix} \bar{S} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \left(\begin{bmatrix} D_1^T & D_2^T \end{bmatrix} \begin{bmatrix} C_1^T \\ C_2^T \end{bmatrix} V^{-1} \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \right)^{-1} \begin{bmatrix} D_1^T & D_2^T \end{bmatrix} \begin{bmatrix} C_1^T \\ C_2^T \end{bmatrix} V^{-1} \right. \\ &\left. + \begin{bmatrix} C_1^T \\ C_2^T \end{bmatrix} \bar{H}^T V^{-1} \bar{H} \right\} \left(y - \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \end{bmatrix} \right) \end{aligned} \quad (7.39)$$

where

$$\begin{aligned}\Gamma^{-1}A\Gamma &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \Gamma^{-1}B_u = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}, \quad C\Gamma = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \\ \Gamma^{-1}B_{k-1} &= \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}, \quad \Gamma^{-1}B_k = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}\end{aligned}$$

Since $SB_{k-1} = 0$ from Theorem 7.1,

$$\Gamma^T S \Gamma \Gamma^{-1} B_{k-1} = \begin{bmatrix} \bar{S} D_1 \\ 0 \end{bmatrix} = 0$$

which implies $D_1 = 0$. Then, (7.39) can be transformed into two equations,

$$\begin{aligned}\bar{S}\dot{\hat{\eta}}_1 &= \bar{S}(A_{11} - \Gamma_{11})\hat{\eta}_1 + \bar{S}(A_{12} - \Gamma_{12})\hat{\eta}_2 + \bar{S}M_1u \\ &+ [\bar{S}G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} + C_1^T \bar{H}^T V^{-1} \bar{H}](y - C_1 \hat{\eta}_1 - C_2 \hat{\eta}_2)\end{aligned}\quad (7.40a)$$

$$0 = C_2^T \bar{H}^T V^{-1} \bar{H}(y - C_1 \hat{\eta}_1 - C_2 \hat{\eta}_2) \quad (7.40b)$$

where

$$\Gamma^{-1}\dot{\Gamma} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix}$$

Note that Γ^{-1} and $\dot{\Gamma}$ can be computed apriori from (7.38). By multiplying (7.33) by Γ^T from the left and Γ from the right, subtracting $\dot{\Gamma}^T S \Gamma$ and $\Gamma S \dot{\Gamma}^T$ to both sides, and using $\Gamma \Gamma^{-1} = I$, the limiting Riccati equation can be transformed into two equations,

$$0 = \bar{S}[A_{12} - \Gamma_{12} - G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} C_2] \quad (7.41)$$

$$\begin{aligned}-\dot{\bar{S}} &= \bar{S}[A_{11} - \Gamma_{11} - G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} C_1] \\ &+ [A_{11} - \Gamma_{11} - G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} C_1]^T \bar{S} \\ &+ \bar{S}[G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} G_1^T - N_1 Q_1 N_1^T + R_1 Q_w R_1^T] \bar{S} - C_1^T \bar{H}^T V^{-1} \bar{H} C_1\end{aligned}\quad (7.42)$$

where

$$\Gamma^{-1}B_w = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}, \quad \Gamma^{-1}F_1 = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

and

$$\Gamma(t_0^+)^T S(t_0^+) \Gamma(t_0^+) = \begin{bmatrix} \bar{S}(t_0^+) & 0 \\ 0 & 0 \end{bmatrix}$$

From (7.40b),

$$\bar{H}C_2 = 0 \quad (7.43)$$

because $y - C_1\hat{\eta}_1 - C_2\hat{\eta}_2$ is arbitrary. By substituting (7.41) and (7.43) into (7.40a), the reduced-order limiting generalized least-squares fault detection filter is

$$\dot{\hat{\eta}}_1 = (A_{11} - \Gamma_{11})\hat{\eta}_1 + M_1u + [G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} + \bar{S}^{-1} C_1^T \bar{H}^T V^{-1} \bar{H}](y - C_1\hat{\eta}_1) \quad (7.44)$$

where $\bar{S}$ is the solution of (7.42). Note that Γ_{11} can be computed apriori. The dimension of the reduced-order filter is $n - k_2$. In the limit, the residual (7.6) becomes

$$r = \hat{H}(y - C_1\hat{\eta}_1) \quad (7.45)$$

because $\hat{H}C_2 = 0$ from (7.43) and $\text{Ker } \hat{H} = \text{Ker } \bar{H}$.

Theorem 7.5 shows that the reduced-order limiting filter (7.44) completely blocks the nuisance fault.

Theorem 7.5. The nuisance fault is completely blocked from the residual (7.45).

Proof. By applying the transformation to the system states,

$$\Gamma^{-1}x \triangleq \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$$

to transform (7.3) into

$$\dot{\eta}_1 = (A_{11} - \Gamma_{11})\eta_1 + (A_{12} - \Gamma_{12})\eta_2 + M_1u + R_1w + N_1\mu_1 \quad (7.46a)$$

$$\dot{\eta}_2 = (A_{21} - \Gamma_{21})\eta_1 + (A_{22} - \Gamma_{22})\eta_2 + M_2u + R_2w + N_2\mu_1 + K_2\mu_2$$

$$y = C_1\eta_1 + C_2\eta_2 + v \quad (7.46b)$$

where

$$\Gamma^{-1}F_2 = \begin{bmatrix} 0 \\ K_2 \end{bmatrix}$$

Then, the residual (7.45) becomes

$$r = \hat{H}(C_1e_1 + v)$$

where $e_1 = \eta_1 - \hat{\eta}_1$. From (7.41),

$$A_{12} - \Gamma_{12} - G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} C_2 = 0 \quad (7.47)$$

because $\bar{S}$ is positive definite. By using (7.44), (7.46) and (7.47),

$$\begin{aligned} \dot{e}_1 = & [A_{11} - \Gamma_{11} - G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} C_1 - \bar{S}^{-1} C_1^T \bar{H}^T V^{-1} \bar{H} C_1] e_1 + R_1 w + N_1 \mu_1 \\ & - [G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} + \bar{S}^{-1} C_1^T \bar{H}^T V^{-1} \bar{H}] v \end{aligned}$$

This shows that the residual is not affected by the nuisance fault. ◀

For time-invariant systems, the reduced-order limiting filter and reduced-order limiting Riccati equation can be derived similarly.

$$\dot{\hat{\eta}}_1 = A_{11} \hat{\eta}_1 + M_1 u + [G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} + \bar{S}^{-1} C_1^T \bar{H}^T V^{-1} \bar{H}] (y - C_1 \hat{\eta}_1) \quad (7.48)$$

$$\begin{aligned} -\dot{\bar{S}} = & \bar{S} [A_{11} - G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} C_1] \\ & + [A_{11} - G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} D_2^T C_2^T V^{-1} C_1]^T \bar{S} \\ & + \bar{S} [G_1(D_2^T C_2^T V^{-1} C_2 D_2)^{-1} G_1^T - N_1 Q_1 N_1^T + R_1 Q_w R_1^T] \bar{S} - C_1^T \bar{H}^T V^{-1} \bar{H} C_1 \end{aligned} \quad (7.49)$$

Note that Γ (7.38) is constant for time-invariant systems because $\text{Ker } S$ is fixed. $\text{Ker } S$ can be obtained from computing the minimal (C, A) -unobservability subspace of F_2 , $\text{Ker } S = \mathcal{T}_2 = \mathcal{W}_2 \oplus \mathcal{V}_2$, instead of solving (7.33).

7.7 Example

In this section, three numerical examples are used to demonstrate the performance of the generalized least-squares fault detection filter. In Section 7.7.1, the filter is applied to a time-invariant system. In Section 7.7.2, the filter is applied to a time-varying system. In Section 7.7.3, the null space of the limiting Riccati matrix S (7.33) is discussed.

7.7.1 Example 1

In this section, two cases for a time-invariant problem are presented. The first one shows that the sensitivity of the filter (7.11) to the nuisance fault decreases when γ is smaller. The second one

shows that the sensitivity of the reduced-order limiting filter (7.48) to the target fault increases when Q_1 is larger. The time-invariant system is from (White and Speyer, 1987).

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 2 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

where F_1 is the target fault direction and F_2 is the nuisance fault direction. There is no process noise.

In the first case, the steady-state solutions to the Riccati equation (7.12) are obtained with weightings chosen as $Q_1 = 1$, $Q_2 = 1$, and $V = I$ when $\gamma = 10^{-4}$ and 10^{-6} , respectively. Figure 7.1 shows the frequency response from both faults to the residual (7.6). The left one is $\gamma = 10^{-4}$, and the right one is $\gamma = 10^{-6}$. The solid lines represent the target fault, and the dashed lines represent the nuisance fault. This example shows that the nuisance fault transmission can be reduced by using a smaller γ while the target fault transmission is not affected.

In the second case, the steady-state solutions to the reduced-order limiting Riccati equation (7.49) are obtained with $V = 10^{-4}I$ when $Q_1 = 0$ and 0.0019, respectively. Figure 7.2 shows the frequency response from the target fault and sensor noise to the residual (7.45). The left one is $Q_1 = 0$, and the right one is $Q_1 = 0.0019$. The solid lines represent the target fault, and the dashed lines represent the sensor noise. This example shows that the sensitivity of the filter to the target fault can be enhanced by using a larger Q_1 . The sensor noise transmission also increases because part of the sensor noise comes through the same direction as the target fault. However, the sensor noise transmission is small compared to the target fault transmission. In either case, the nuisance fault transmission stays zero and is not shown in Figure 7.2. Note that when $Q_1 = 0$, the generalized least-squares fault detection filter is similar to (Chung and Speyer, 1998) which does not enhance the target fault transmission.

7.7.2 Example 2

In this section, the filter (7.11) and the reduced-order limiting filter (7.44) are applied to a time-varying system which is from modifying the time-invariant system in the previous section by adding some time-varying elements to A and F_2 matrices while C and F_1 matrices are the same.

$$A = \begin{bmatrix} -\cos(t) & 3 + 2\sin(t) & 4 \\ 1 & 2 & 3 - 2\cos(t) \\ 5\sin(t) & 2 & 5 + 3\cos(t) \end{bmatrix}, \quad F_2 = \begin{bmatrix} 5 - 2\cos(t) \\ 1 \\ 1 + \sin(t) \end{bmatrix}$$

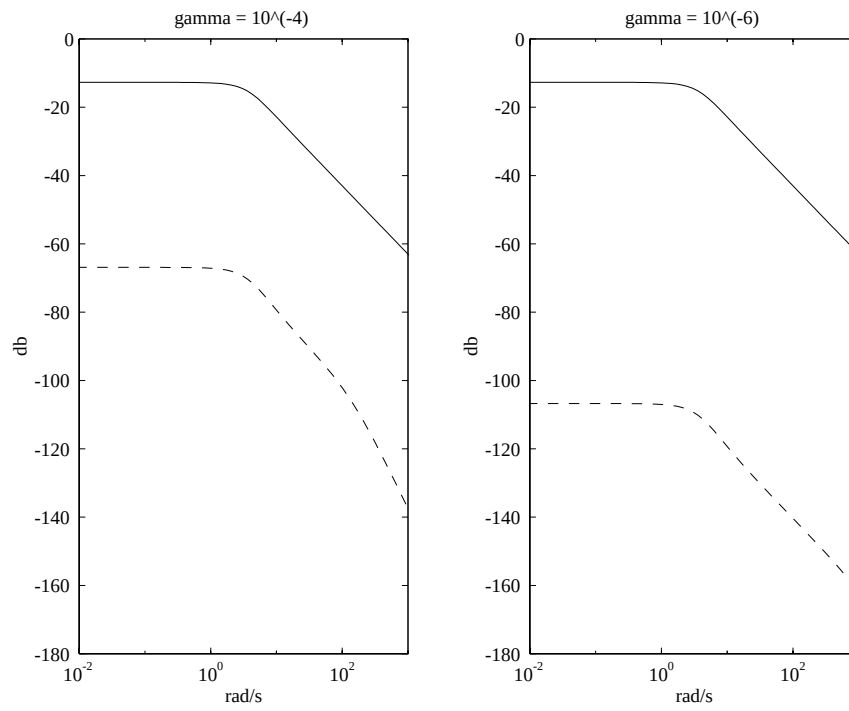


Figure 7.1: Frequency response from both faults to the residual

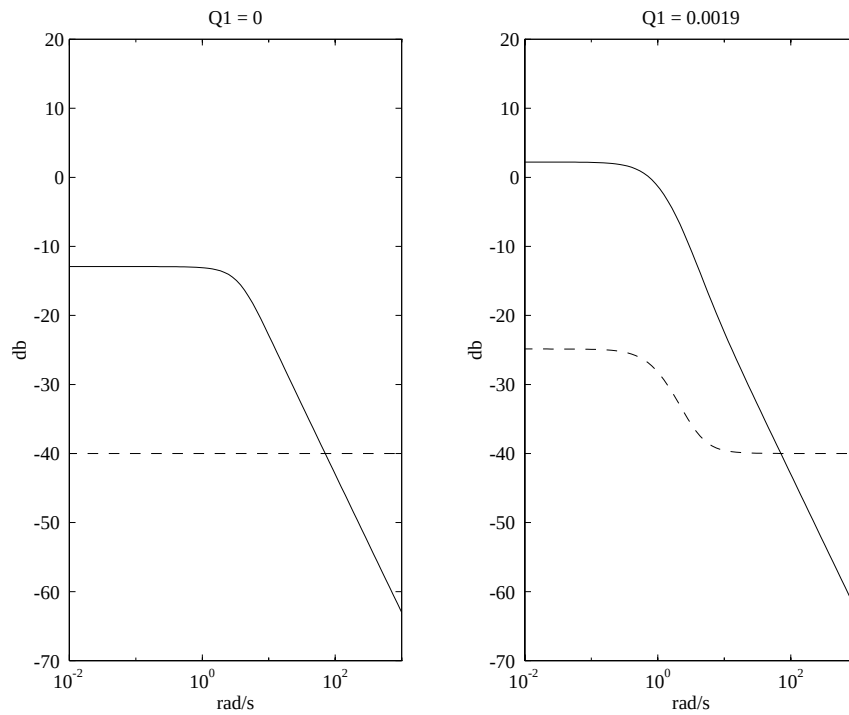


Figure 7.2: Frequency response from target fault and sensor noise to the residual

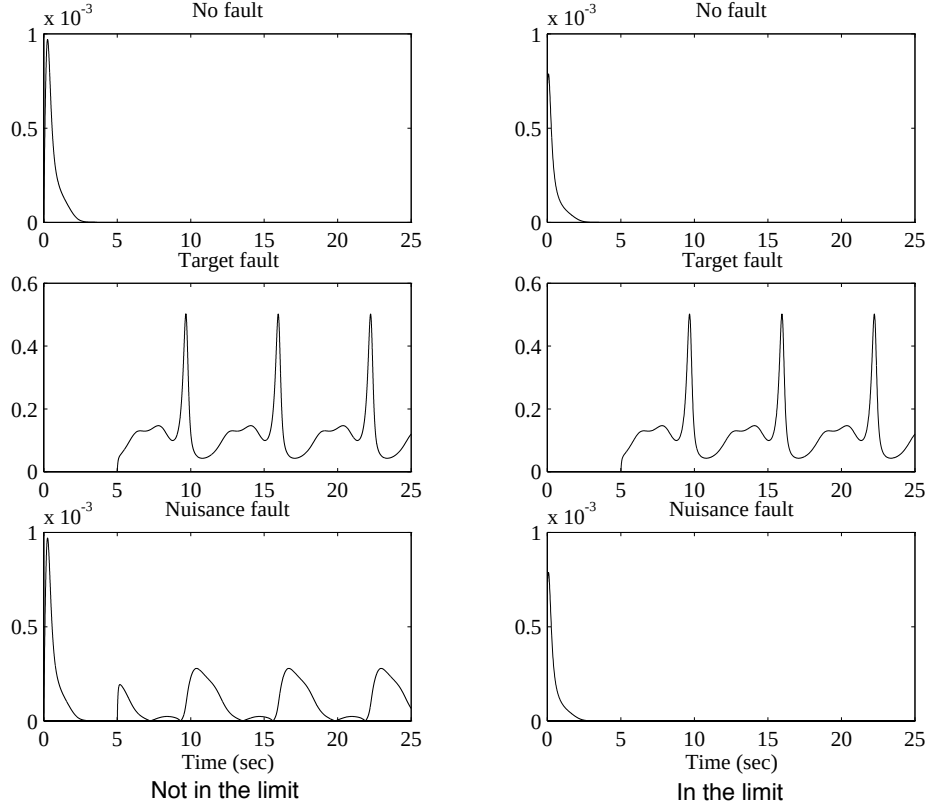


Figure 7.3: Time response of the residual

The Riccati equation (7.12) is solved with $Q_1 = 1$, $Q_2 = 1$, $V = I$ and $\gamma = 10^{-5}$ for $t \in [0, 25]$. The reduced-order limiting Riccati equation (7.42) is solved with the same Q_1 and V . Figure 7.3 shows the time response of the norm of the residuals when there is no fault, a target fault and a nuisance fault, respectively. The faults are unit steps that occur at the fifth second. In each case, there is no sensor noise. The left three figures show the residual (7.6) for the filter (7.11). There is a small nuisance fault transmission because (7.11) is an approximate unknown input observer. The right three figures show the residual (7.45) for the reduced-order limiting filter (7.44). Note that the nuisance fault transmission is zero. There is a transient response until about two seconds due to the initial condition. This example shows that both filters, (7.11) and (7.44), work well for time-varying systems.

7.7.3 Example 3

In this section, three cases are presented to show the properties of the null space of the limiting Riccati matrix S . The first case shows that $\text{Ker } S$ includes the nuisance fault direction and the

invariant zero direction associated with the right-half-plane invariant zero. The second case shows that $\text{Ker } S$ includes only the nuisance fault direction, but not the invariant zero direction associated with the left-half-plane invariant zero. The third case shows that the invariant zero direction associated with the left-half-plane invariant zero is included in $\text{Ker } S$ if the nuisance fault direction is modified. These three cases show that the null space of S is equivalent to the minimal (C, A) -unobservability subspace of F_2 .

In the first case, A and C matrices are the same as the example in Section 7.7.1 and

$$F_1 = \begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

(C, A, F_2) has an invariant zero at 3 and the invariant zero direction is $[1 \ 0 \ 0]^T$. The weightings are chosen as $Q_1 = 1$, $Q_2 = 1$ and $V = I$. The steady-state solutions to the Riccati equation (7.12) when $\gamma = 10^{-6}$ and the limiting Riccati equation (7.33) are

$$\Pi = \begin{bmatrix} 0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0010 & -0.0002 \\ 0.0000 & -0.0002 & 0.0965 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0965 \end{bmatrix}$$

This shows that the nuisance fault direction and the invariant zero direction associated with the right-half-plane invariant zero are in the null space of S .

The second case is from modifying the previous case such that the invariant zero is in the left-half plane instead of the right-half plane. The system matrices are the same except

$$F_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

(C, A, F_2) has an invariant zero at -3 and the invariant zero direction ν is $[1 \ 0 \ 0]^T$. The weightings are the same. The steady-state solutions to the Riccati equations (7.12) and (7.33) are

$$\Pi = \begin{bmatrix} 0.0630 & -0.1885 & 0.0559 \\ -0.1885 & 0.5645 & -0.1674 \\ 0.0559 & -0.1674 & 0.1462 \end{bmatrix}, \quad S = \begin{bmatrix} 0.0626 & -0.1879 & 0.0557 \\ -0.1879 & 0.5637 & -0.1671 \\ 0.0557 & -0.1671 & 0.1460 \end{bmatrix}$$

This shows that the null space of S includes only the nuisance fault direction, but not the invariant zero direction associated with the left-half-plane invariant zero. The filter (7.48) has an eigenvalue at the invariant zero -3.

In the third case, the nuisance fault direction used for the filter design is changed to ν and the weightings are the same. The steady-state solutions to the Riccati equations (7.12) when $\gamma = 10^{-10}$

and (7.33) are

$$\Pi = \begin{bmatrix} 0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0044 & -0.0009 \\ 0.0000 & -0.0009 & 0.0967 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0965 \end{bmatrix}$$

This shows that the null space of S includes both F_2 and ν because $\text{Ker } S = \text{span}\{\nu \ A\nu\} = \text{span}\{F_2 \ \nu\}$. The filter (7.48) does not have any eigenvalue at the invariant zero -3.

Chapter 8

Conclusion

Analytical redundancy is a viable approach to vehicle health monitoring. The fault detection filters developed here as a point design for the longitudinal dynamics of the PATH Buick LeSabre are evaluated using empirical data. The preliminary evaluation is promising in that the fault detection filters can detect and identify actuator and sensor faults as expected even under various disturbances and uncertainties.

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