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Abstract

Game Theory Applications in Socially Responsible Operations and Operations-Marketing Interface

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This dissertation includes three chapters: (1) Farmers' Information Management in Developing Countries - A Highly Asymmetric Information Structure. (2) Information Provision Policies for Improving Farmer Welfare in Developing Countries: Heterogeneous Farmers and Market Selection. (3) Role of Exchangeable Tickets in the Optimal Menu Design for Airline Tickets.

The first chapter studies farmers' information management and utilization problems in developing countries. In these countries, governments, non-governmental organizations, and social entrepreneurs are disseminating agriculture information to farmers to improve their welfare. However, instead of having direct access to the information, farmers usually acquire information from local social networks, and, thus, they may have very different information channels. In this paper, we establish a general framework that accommodates highly asymmetric information structures to capture the fact that information is transmitted indirectly through the social network. In our model, a bipartite graph describes which subset of signals is accessible to a farmer. We characterize a unique Bayesian Nash equilibrium and express farmers' strategies and expected profits in closed forms. We discuss properties of this equilibrium and show that asymmetric information structures can lead to various novel results. We also conduct comprehensive studies on the equilibrium in the "weak signal limit", where signals are subject to substantial noise. We examine the government's optimal information allocation in this limit when its goal is to maximize (1) farmers' total profits or (2) the social welfare.

In the second chapter, we examine the impact of information provision policies on farmer welfare in developing countries where farmers lack relevant and timely information for making informed decisions regarding which crop to grow and which market to sell in. In addition to heterogeneous farmers, we consider the case when farmers are price takers and yet the price of each crop (or the price in each market) is a linearly decreasing function of the total sales quantity. When market information is offered free-of-charge, we show that: (a)

providing information is always beneficial to farmers at the *individual level*; and (b) providing information to all farmers may not be welfare maximizing at the *aggregate level*. To maximize farmer welfare, it is optimal to provide information to a targeted group of farmers who are located far away from either market. However, to overcome perceived unfairness among farmers, we show that the government should provide information to all farmers at a nominal fee so that the farmers will adopt the intended optimal provision policy willingly. We extend our analysis to examine different issues including: precision of market information, and information dissemination via a for-profit company.

The third chapter examines the optimal menu design problem with three types of tickets: refundable, nonrefundable, and exchangeable tickets. We identify the role of exchangeable tickets in trapping consumers and show that for the seller, it is inherently more profitable than the other two types of tickets. On the other hand, increasing the flexibility of exchangeable tickets may dampen the seller's profitability. The analysis also reveals that cancellation fees are used as instruments to adjust the differences between consumers' willingness to pay, and when they are adopted, the seller has less incentive to sell nonrefundable tickets. Our results also explain why menu offerings, while prevalent in the airline industry, are so scant in commodity goods markets.

1 Introduction

This dissertation includes three chapters of game theory applications in socially responsible operations and operations-marketing interface: (1) Farmers' Information Management in Developing Countries - A Highly Asymmetric Information Structure. (2) Information Provision Policies for Improving Farmer Welfare in Developing Countries: Heterogeneous Farmers and Market Selection. (3) Role of Exchangeable Tickets in the Optimal Menu Design for Airline Tickets.[1]

The first chapter studies farmers' information management and utilization problems in developing countries. Imagine the following situation: "a poor farmer in rural India is listening to his neighbor who tells him that the paddy demand will be high next year, and it will be better for them to start producing more. However, he recalls that he received a text message yesterday from a non-governmental organization, which suggested him to reduce the production quantity of paddy to avoid the possible loss due to the delay of monsoon. He ponders what he should do..."

This imaginary situation in fact happens frequently in developing countries. In these countries, micro-entrepreneurs (e.g., farmers, fishermen, and itinerant workers) usually suffer from lack of information. For example, in rural India, farmers frequently miss the opportunities to sell their products at a higher price and only earn a small portion of the value of their products (Sodhi and Tang (2013)). To help these micro-entrepreneurs (farmers hereafter), governments, several non-governmental organizations (NGOs), and even some for-profit enterprises are disseminating relevant information through various information and communication technologies (ICT). For example, the Indian government implements agricultural extension programs to disseminate information about technologies for crop production and general market trends. Reuters Market Light, which is a for-profit company, sells customized agricultural information (including crop advisory, weather forecasts, local market price information, etc.) to farmers through mobile phones[2]. See also Nokia Life Tools, Mali Shambani, and Iffco Kisan Sanchar.

Although information from various sources is provided in order to help farmers make better decisions, not all farmers have direct access to the information. As the story we mentioned above, in many situations, information is spread through the social network. According to Ellen McCullough[3], "most people tend to think that technology information flows to farmers through a direct pipeline from scientists, but that isn't true[4]." In McCullough and Matson (2011), the authors examine the evolution of knowledge systems (networks) of the

[1]This dissertation contains works co-authored with professors Ying-Ju Chen and Christopher S. Tang.

[2]Information from http://www.reutersmarketlight.com

[3]A former research fellow at Stanford's Program on Food Security and the Environment, now at the Bill and Melinda Gates Foundation.

[4]Information from http://news.stanford.edu/news/2011/june/understanding-farmer-networks-060211.html

Yaqui Valley, Mexico, and trace information flows within this system. See also Conley and Udry (2010) and Bandiera and Rasul (2006) about how farmers learn from their information neighbors. Grameen Foundation also utilizes the local network to disseminate information to farmers. This NGO constructs a database of agricultural information. It identifies, recruits, and trains rural community members as Community Knowledge Workers (CKWs), who can query the database using their smartphone with the custom built application. A farmer can access the content of this database either by sending SMS-based queries directly from her phone or by visiting the CKW near to her[5].

When information is transmitted indirectly through a social network, which signals are available to a farmer depends on her position in the network. In this situation, farmers are likely to have highly asymmetric information channels. For example, farmers can have very different amounts of information (different numbers of signals). Also, a farmer might have information overlapping with some farmers on a signal, and information overlapping with another set of farmers on another signal. With this kind of asymmetric information structures resulted from the social network, some research questions naturally arise. How should farmers respond to different signals? What are the characteristics of the equilibrium? How does the equilibrium change with the information structure and the related parameters? Does the asymmetry of information structures lead to new phenomena? Finally, if the government has some new information or has budgets to improve signals, what should it do to improve farmers' total profits, and what should it do to maximize the social welfare?

To address the above questions, we construct a general environment with complex information structures. In our model, farmers can raise different types of crops at the same time, and they need to make production decisions on all of these crops. The market situation of each type of crops is uncertain and it depends on some fundamental factors. The information we mentioned above is signals about each fundamental factor. Finally, a bipartite graph describes which subset of signals is accessible to a farmer. As an illustrative example, farmers need to decide the amounts of paddy and corn they want to produce. The markets of paddy and corn both depend on factors such as weather, global economic condition, the trend of consumers' preference, etc. For each factor, there are some signals that farmers can use to infer its realization. e.g., for the trend of consumers' preference, there are surveys made by different institutions. Each farmer makes production decisions based on signals she receives. Although signals are disseminated by different means in different networks, we avoid the complex information transmission process and focus on the resulting asymmetric information structures.

We identify the unique Bayesian Nash equilibrium, and express farmers' strategies and expected profits in close forms. These closed-form characterizations allow us to examine several properties of the equilibrium:

1. Compared to receiving no signals, observing signals can create a nonnegative extra expected profit to a farmer. Hence, a farmer's expected profit when she cannot observe

[5]Information from http://www.ckw.applab.org/section/index

any signal is a lower bound of her expected profit.

2. Different markets and different fundamentals can be treated separately; and, thus, we only need to consider situations with one market and one fundamental. This natural separation allows us to explicitly examine the farmers' optimal production strategies in response to information provision and the corresponding profit/welfare implications.

3. When some signals are observed by and only by the same set of farmers, they can be combined as a whole signal with a higher precision. Thus, allocating the government's budgets to improve signals is somehow equivalent to releasing new signals.

4. A limit on the extent to which farmers can utilize the signals is provided. This limit implies that farmers' responses can mitigate the fluctuation of the market demand. However, this mitigation is not as strong as that when all farmers know exactly the realized value of the fundamental.

5. We apply our results to a special and realistic case wherein farmers can be separated into two groups such that there is no information overlapping between these two groups[6]. In this situation, a farmer (in group 1) can decide her strategy in a simplified way as follows. She firstly ignores farmers in group 2 to decide a strategy, and then modifies this strategy according to a scaling factor. This scaling factor depends on O_2, a number which serves as the sufficient statistic for farmers in group 1 to make decisions in response to group 2. This property has two implications: (1) If this farmer can guess O_2 by her past experience, she need not worry about the detailed information structure in group 2. This result tells us that when making decisions, a farmer need not take the information structure for farmers in the other part of the world into account. (2) It suggests a simple rule of thumb for farmers to *update* their production strategies when the information structure in other parts of the world changes.

We then use two simple examples to illustrate that an asymmetric information structure can lead to various novel results:

- A farmer may react *adversely* to signals. i.e., she may produce more when observing a pessimistic signal, and reduce the production quantity when the signal suggests a high demand. This happens when the incremental information from this signal regarding the market demand is small, but it provides important information about other farmers' beliefs, and, thus, the market supply. In this situation, an optimistic (pessimistic) realization of this signal is not necessarily representing a good (bad) market demand, but it indicates a high (low) aggregate production quantity. Therefore, reacting adversely to this signal is a natural choice for this farmer.

[6]For example, farmers in different provinces or different countries may have very different sources of information.

- A farmer may benefit from the improvement of signals she cannot observe. This result sounds counterintuitive because usually a farmer suffers when her competitors get more information and have higher competitive advantages against her (see Vives (1984)). However, we find that when a signal is improved, farmers who can observe this signal will put more attention to it, and less attention to their other signals. In this situation, other farmers might be able to utilize those signals better, and this is a potential opportunity for them to earn more profit.

- Farmers may want to share information with other farmers[7]. This observation is different from the literature (please see Gal-Or (1985), in which there is no information sharing in equilibrium, and see also Vives (1984) and Li (1985)). We find that the asymmetric information structure might make a farmer willing to share her signal with another farmer. The intuition is that this farmer might be able to utilize this signal better if the farmer whom she shares the signal with should respond to it adversely.

- While conventional wisdom suggests that information is beneficial, we find that farmers may suffer from receiving previously inaccessible signals. A farmer might suffer when she receives a previously unobservable signal and should react adversely to it. In this situation, she hopes that she never observes this signal: once she observes it (and other farmers know this), other farmers will change their strategies accordingly, and she is forced to react adversely to it.

We also consider the weak signal limit, in which the variances of signals' noises are high compared to the variance of the fundamentals. In this limit, we can have a closer examination on the equilibrium, and subsequently derive valuable insights from these characterizations. One of our findings is that in this weak signal limit, a farmer may benefit from the improvement of a signal she cannot observe, and we provide a necessary condition for this to happen. We also examine the government's optimal information or budgets allocation for the purpose of improving (i) farmers' total profits or (ii) the social welfare (which includes consumers' surplus) in this limit. We consider the situation in which the government has some new signals to release or has some budgets to improve signals. We find that to improve farmers' total profits, the government should allocate all of its resources to (and only to) the farmer who receives most signals on which she has "moderate" competitions with other farmers. We establish an index to determine which farmer should get the resources. On the other hand, the government should release all of its resources to all farmers if its goal is to maximize the social welfare.

In the second chapter, we examine the impact of information provision policies on farmer welfare in developing countries. Poor farmers in developing countries lack relevant and timely information for deciding which crop (paddy or sorghum) to grow during the planting season

[7]To be more specific, "before" the signals realize, a farmer might be able to increase her expected profit by deciding to share some of her signals with another farmer. Note that she should decide whether to share information with others before the signals realize. This is the assumption used in most papers in the literature.

or which market to sell in during the harvest season. To alleviate poverty, governments in countries such as India and Kenya are offering crop advisory and market price information via radio/television programs, online portals, and hotline services. The reader is referred to Chen and Tang (2013) for details of different services provided by the governments, non-governmental organizations (NGOs), and for-profit companies such as Nokia Life Tools and Reuters Market Light (Tang and Sheth (2013)).

To improve farmers' total profit, there is a belief that governments/NGOs should disseminate market information as widely (and precisely) as possible. Chen and Tang (2013) investigate a situation in which *homogeneous* farmers engage in a *Cournot competition* in a *single market* for a *single crop*. They claim that, when farmers have no private signals, providing public signal to all farmers is always beneficial.[8] This claim provides an analytical justification for various government initiatives that call for wide dissemination of market information. However, will this claim continue to ring true when the following assumptions adopted by Chen and Tang (2013) do not hold?

First, Chen and Tang (2013) assume that each farmer is a *price setter*, which is reasonable for large farmers (or farmer cooperatives) whose large quantity production can influence the market price. However, this assumption is less appropriate for smallholder farmers that are commonly observed in India and other developing countries. This observation has motivated us to develop a model in which each smallholder farmer is a *price taker* in the sense that he has no influence on the market price on the individual level. (However, on the aggregate level, the market price drops as the total production quantity increases.)

Second, it is commonly assumed in the existing literature that all agents (farmers) are *homogeneous* so that all farmers have the same payoff for producing the same crop and selling in the same market (see, e.g., Morris and Shin (2002), Cornand and Heinemann (2008), Angeletos and Pavan (2007)), and Chen and Tang (2013)). In our model, we shall consider the case when each farmer can select one of the two crops to grow (or one of the two markets to sell in). Also, these farmers are *heterogeneous* in terms of the inherent preference for growing a certain crop or for selling in a certain market. For example, farmers who have easy (difficult) access to water may prefer growing paddy (sorghum), and farmers would prefer to sell in a nearby market due to transportation issues.

As an initial attempt, we develop a stylized model that incorporates heterogeneous farmers who need to select one of the two markets to sell in (or one of the two crops to grow). We adopt the Hotelling model in which there is a continuous type of infinitesimal farmers located uniformly along a line over $[-0.5, 0.5]$ (c.f., Lilien et al. (1992)).[9] We consider

[8]However, when farmers have private signals, Morris and Shin (2002), Cornand and Heinemann (2008), and Chen and Tang (2013) find that providing public signal to all farmers can be harmful in terms of farmers' total profit. To mitigate this harmful effect, various researchers suggest that the government should: (1) reduce the precision of the public signal, or (2) reduce the number of farmers receiving the public signal.

[9]In a later subsection, we shall extend our analysis to the case when farmer's location is not uniform but a symmetric distribution at the origin 0 and show that our main results continue to

two markets (or two crops) located at both ends of the line that we label as the right market (crop) r and the left market (crop) l. (While our model can be applied to market (or crop) selection, we shall use the term market selection throughout this chapter to ease our exposition.)

We use infinitesimal farmers to capture the fact that each smallholder farmer with 1 unit of production capacity has no impact on the market price; i.e., each farmer is a *price taker*. However, the total quantity to be sold in each market q_i has a direct impact on the price in each market $i = r, l$. We shall assume that the market price p_i is a linearly decreasing function of the total quantity q_i to be sold in the market i so that $p_i = a_i - bq_i$. The price uncertainty is captured by the random intercept a_i, which is a standard modeling setup as considered in Gal-Or (1985) and Li (1985).

Associated with each farmer located at $\theta \in [-0.5, 0.5]$, we normalize the unit production cost to 0. However, there is an imputed cost for each farmer located at θ to sell in each market i. In the context of which market to sell in, this imputed cost can be interpreted as his transportation cost that is measured by the "distance" between his farm location and the market location.[10] Given the market price and the imputed transportation cost, each farmer needs to select the market to sell in.

In this chapter, we consider the case when the government has imperfect signal x_i about a_i, the intercept of the market price function for each market $i = l, r$. To improve farmers' expected total profit, the government needs to decide on the information provision policy (R, ρ), where $R \subset [-0.5, 0.5]$ is the *range* of farmers who receive market signals; and $\rho \leq 1$ is the *percentage* of farmers in R who receive market signals.

For any given information provision policy $\delta = (R, \rho)$, ρ percentage of farmers located within the range R will receive the market signals (x_l, x_r) about the market prices of market l and r, respectively; and all other farmers receive no market signals. By examining the profit function of each farmer, we determine the market selection rule that each farmer will follow in equilibrium. By examining the ex-ante expected total profit of all farmers in equilibrium, we determine the optimal information provision policy $\delta^* = (R^*, \rho^*)$. Our analysis enables us to establish the following results:

1. For any information provision policy $\delta = (R, \rho)$, there exists a unique threshold $\tau^{(\delta)}$ so that a farmer who receives market signals will sell in the left market l if he is located at $\theta < \tau^{(\delta)}$, and sell in the right market r; otherwise. Also, farmers who receive no signals will follow the threshold market selection rule that has the *origin 0* as the threshold.

2. Market signals can improve farmer welfare.

3. To maximize farmers' total profit, providing market signals to all farmers may not be

hold.

[10]In the context of which crop to grow, the notion of the "distance" can be interpreted as the farmer-specific capability (e.g., knowledge, soil condition, etc.) for growing each crop.

optimal; i.e., the optimal information provision policy may call for limited dissemination.

4. To ensure fairness among farmers, the government should offer information to all farmers at a nominal fee so that farmers will adopt the intended optimal provision policy willingly.

The first result is intuitive because farmers would prefer to sell in a nearby market unless the market signals indicate the higher selling price in the farther market would outweigh the additional transportation cost. The second result is also intuitive because market signals can enable farmers to make better decisions. Regarding the third result, there are two forces that drive our third result to be different from the results that support distributing information to all farmers (Chen et al. (2013) and Morris and Shin (2002)). The first force is caused by the fact that we consider two potential markets for heterogeneous farmers to sell in. In this case, providing information to more farmers may not improve farmers' total profit. To elaborate, consider the case when farmers receive no information. In this case, each farmer will sell in the nearby market and the sales quantity in each market is identical (due to symmetry). Now, suppose the government provides information to all farmers. Then each farmer will selfishly choose the market to sell in to increase his own earning without considering the impact on the total profit of all other farmers. Due to the fact that the more promising market can attract more farmers, each farmer who chooses this market creates a negative externality on the total profit of all other farmers. Consequently, we show that providing information to all farmers may not maximize farmers' total profit. This arises because in most situations, it leads to too many farmers who switch from the nearby market to the more promising but farther market.

The second force is caused by the fact that farmers are *price takers* and they have no control of the market price as individuals. In this situation, when a farmer selfishly selects the market that would yield a higher profit for himself, he creates *serious negative externality* that affects other farmers' profit.[11] Therefore, providing information to all farmers may not be optimal.

As it turns out, the third result can be interpreted in the context of Braess's paradox – a well known result in traffic equilibrium. Specifically, Braess states that, when the drivers choose their route selfishly, the overall system performance can deteriorate by adding one extra road to the network.[12] If we interpret providing information as adding more roads, farmer's market selection as driver's route selection, and farmers' profit (resulting from

[11] On the contrary, in the models considered by Chen et al. (2013) and Morris and Shin (2002), each farmer is a *price setter* so that his decision can generate negative externality for other farmers and himself. Hence, each farmer becomes less selfish.

[12] The authors are grateful to Professors Philip Kaminsky and Max Shen of UC Berkeley for suggesting this connection. Steinberg and Zangwill (1983) present necessary and sufficient conditions for *Braess' Paradox* to occur in a general transportation network. The reader is referred to Steinberg and Zangwill (1983) and the reference therein for details.

farmers' market selection) as travel time (resulting from drivers' route selection), then the third result resembles the Braess's paradox even though our model deals with uncertain market condition and noisy market signals.

The fourth result examines a way to operationalize the optimal provision policy. Specifically, instead of limiting farmer's information access, the government can make information accessible to all farmers at a nominal fee. By selecting the fee carefully, it will entice farmers to adopt the intended optimal provision policy willingly without the fear of treating any farmer unfairly.

We extend our analysis to the case when information is distributed through a for-profit company. Because the objective of a for-profit company is to maximize its profit instead of the farmer welfare, we show that the company will charge a higher price than that of the nominal fee charged by the government. Consequently, fewer farmers would find it beneficial to purchase the information from the company. We discuss what the government can do to achieve the intended optimal provision policy. We also extend our base model to examine different issues including: precision of market information, correlated markets, and the more general distribution of farmers.

The third chapter examines the optimal menu design for airline tickets with three types of tickets: refundable, nonrefundable, and exchangeable tickets. When consumers must book the tickets to secure their seats, they usually have no idea whether they should change their schedule or not in the future. A natural solution for the seller to fight against this valuation uncertainty is to offer flexible tickets. Airline companies have long recognized such benefits and in practice offer tickets with different flexibilities. For example, when a consumer wants to purchase a ticket from United Airlines, she can choose to purchase the ticket with Lowest Available Fare, Flexible Fare, or Unrestricted Fare. Tickets with different fares lead to the same service (flying the consumer to the destination), but provide different flexibilities. For instance, a consumer who purchased a ticket with Flexible Fare or Unrestricted Fare can cancel the ticket and get the money back if she needs to change her plan. However, she cannot get the refund if she purchased the ticket with Lowest Available Fare. Also, a consumer might need to pay the cancellation/change fee when she wants to change a ticket purchased with Flexible Fare, but this cancellation/change fee can be waived if the ticket is purchased with Unrestricted Fare.[13] Similar practices can be found in American Airlines, Delta Air Lines, US Airways, Southwest Airlines, etc.

While menus are offered pervasively in the airline industry, the nature of the menu design remains unclear. For example, we note that the menu offerings *differ substantially* across companies. US Airways provides a menu with only two fares. Southwest Airlines provides tickets with three fares, which are all changeable without change/cancellation fees. All AirTran Airways fares are not refundable and a $75 fee applies to any change. The change/cancellation fees also vary across companies. Between the two extreme cases above (all with (without) change/cancellation fees in AirTran Airways (Southwest Airlines)), Amer-

[13]Source: www.united.com, retracted June 2013.

ican Airlines charges \$200 for domestic itinerary changes if the ticket is purchased at Lowest Fare - Choice, and there is no change fee on all other fares. Moreover, even within the same airline company, sometimes the menus and change/cancellation fees are different for different air routes.

The above discrepancy naturally gives rise to some research questions. What is the optimal menu design when the seller can provide tickets with different refund policies? What is the characteristic of different refund policies in this profit maximization process? How should the seller use change/cancellation fees to maximize his profit? Finally, although this kind of menu designs is widely adopted in the airline ticket market, why is it so scant, if such ever exists, in the commodity goods markets? In this chapter, we establish a stylized model to shed some light on the above questions. We consider the situation in which a monopolistic seller intends to sell his tickets to heterogeneous consumers. Although there are many aspects of flexibility, in this work we focus on the two most salient features: (1) whether it is refundable, and (2) if it is not refundable, whether it can be exchanged for a new ticket. These two aspects lead to three different types of tickets. If a refundable ticket is cancelled, the consumer can get the refunded money. On the other hand, a nonrefundable ticket cannot be cancelled; thus, a consumer gets nothing if she purchased a nonrefundable ticket and decides to change her plan. Finally, if an exchangeable ticket is cancelled, the consumer gets some *credits* which can be used to purchase another ticket in the future.[14]

In the basic model, we abstract away change/cancellation fees to focus on the intrinsic values of these refund policies. We find that the profitability[15] of exchangeable tickets is increasing in the ticket price, but profitabilities of refundable tickets and nonrefundable tickets do not depend on the prices. Also, we identify a pecking order in terms of profitability: exchangeable tickets are always intrinsically more profitable than refundable tickets, which are intrinsically more profitable than nonrefundable tickets. As a result, in the optimal strategy, the seller always sells exchangeable tickets to some consumers. This is in line with our observation that exchangeable tickets are used more frequently by sellers than the other two types of tickets in the real world. For instance, United Airlines, Southwest Airlines, American Airlines, AirTran Airways, and Spirit Airlines all provide exchangeable tickets. However, all these companies do not sell nonrefundable tickets to consumers directly. Also, AirTran Airways and Spirit Airlines do not offer refundable tickets.[16] We provide concrete operating regimes in which the seller should offer only one type of tickets or multiple types for consumers to self-select. From the revenue maximization perspective, if the seller provides exchangeable tickets to multiple segments of consumers, it is not necessarily beneficial to increase the flexibility of exchangeable tickets. This might be one of the reasons why sellers sometimes set a finite time horizon beyond which the refunded credits will expire.

[14]The usage of these credits is typically subject to some restrictions set by the seller.

[15]When we use the terms "profitable" and "profitability", it is from the seller's perspective.

[16]Information retracted from companies' websites in June 2013: United Airlines, Southwest Airlines, American Airlines, AirTran Airways, and Spirit Airlines. Additional information was collected via first-hand conversations with the customer services departments of United Airlines, American Airlines, AirTran Airways, and Spirit Airlines.

We then extend our analysis to accommodate cancellation fees.[17] We find that the pecking order identified above is robust against the inclusion of cancellation fees. However, charging a positive cancellation fee will indirectly hurt the profitability of exchangeable tickets, but it has no influence on the profitability of refundable tickets. We note that cancellation fees are used as instruments to adjust the differences between consumers' willingness to pay. A smaller difference between consumers' willingness to pay is desirable since this might lead to less information rent the seller should give to consumers. As a result, the seller should charge a cancellation fee on refundable tickets whenever he believes that this can reduce the difference between consumers' willingness to pay for refundable tickets. On the other hand, charging a positive cancellation fee upon canceling an exchangeable ticket will hurt its profitability, and, hence, the seller should not charge this fee unless the benefit outweighs this side effect. In particular, when the seller sells exchangeable tickets to only one segment, he should not charge a positive cancellation fee on exchangeable tickets. We also prove that when charging cancellation fees is allowed, the seller has a lower incentive to sell nonrefundable tickets. This result coincides with our observation that in the real world, major airline companies rarely sell nonrefundable tickets to consumers directly; instead, they allow consumers to cancel the tickets for monetary refund or credits, and in some cases charge cancellation fees.

Our framework also provides a possible answer to the substantial difference on refund policies in the airline ticket and commodity goods markets. We argue that in commodity goods markets, the seller will not consider refundable goods and exchangeable goods at the same time. That is, in some situations, the seller only considers refundable goods and nonrefundable goods. In other situations, the seller only considers exchangeable goods and nonrefundable goods. We show that offering a menu may be profitable only when consumers with a lower valuation upon using the product have a higher probability to be satisfied by the product. This feature has been a fixture in the airline industry, but it is not widely observed in markets of other commodity products. We provide detailed explanations of this discrepancy in Subsection 4.5, and articulate why it drives the substantially different strategies in these markets.

[17]In practice, many companies (e.g., AirTran Airways, Southwest Airlines, US Airways, and Spirit Airlines) choose to charge the same fee upon canceling and upon changing the ticket. Hence, in this work, we only consider the *cancellation fees* consumers should pay if they want to change or cancel their refundable or exchangeable tickets.

2 Farmers' Information Management in Developing Countries - A Highly Asymmetric Information Structure

In developing countries, governments, non-governmental organizations, and even some social entrepreneurs are disseminating agriculture information to farmers to improve their welfare. However, instead of having direct access to the information, farmers usually acquire information from local social networks, and, thus, they may have very different information channels. We establish a general framework that accommodates highly asymmetric information structures to study farmers' information management and utilization problems. In our model, a bipartite graph describes which subset of signals is accessible to a farmer.

We characterize a unique Bayesian Nash equilibrium and express farmers' strategies and expected profits in closed forms. We discuss properties of this equilibrium and show that asymmetric information structures can lead to various novel results. For example, a farmer may produce more (less) when observing a pessimistic (optimistic) signal, may benefit from the improvement of a signal she cannot observe, may want to share her signal with others, and may become worse off when another farmer releases a signal to her. We conduct comprehensive studies on the equilibrium in the "weak signal limit", where signals are subject to substantial noise. We examine the government's optimal information allocation in this limit when its goal is to maximize farmers' total profits or the social welfare. To improve farmers' total profits, the government should provide all its information to (and only to) one farmer. We establish an index to determine which farmer should get the information. In contrast, to maximize the social welfare, the government should provide all its information to all farmers.

The remainder of this chapter is organized as follows. Subsection 2.1 reviews the related literature, and Subsection 2.2 lays out the model settings. Subsection 2.3 presents the equilibrium and some of its properties. We then use two illustrative examples in Subsection 2.4 to show that asymmetric information structures can lead to various novel results. In Subsection 2.5, we study the weak signal limit. We only present the main results, and all the proofs are relegated to the appendix.

2.1 Literature Review

This chapter is related to *socially responsible operations*. This research stream studies how a social enterprise makes the poor as producers (micro-entrepreneurs) and helps them by enabling financial, information, demand, or supply flows for them (see Sodhi and Tang (2013) and Sodhi and Tang (2011)). Although there are some empirical and experimental papers concerning the influence of providing information to farmers in developing countries (see, e.g., Mittal et al. (2010), Fafchamps and Minten (2012), and Parker et al. (2012)), only a few theoretical papers are in this area. Chen et al. (2013) investigate the influence of the

ITC e-Choupal network on farmers production and selling strategy. An et al. (2015) examine the benefit for a farmer to join formal or informal cooperatives. In Chen et al. (2015), the authors examine farmers' incentive to share information in a voice-based forum, Avaaj Otalo. In this chapter, we study the influence of the information flow on farmers' strategies and expected profits, and the government's optimal information allocation. Compared with Chen and Tang (2013), we adopt an asymmetric information structure to address the fact that usually information is transmitted through the social network. This flexible framework also allows us to examine farmers' incentive to share information with others, and how the central planner can selectively allocate the available information to achieve his objective.

In this chapter, we examine how farmers interpret and respond to various signals. In this light, this chapter is related to the literature about information management. Our model is a generalized version of the second stage of the model in Gal-Or (1985). In that paper, the author examines firms' incentive to share information, and shows that there is no information sharing in the unique Nash equilibrium. In Gal-Or (1986) and Vives (1984), the authors adopt different model settings and find that whether sharing information is optimal or not depends on the nature of competition (Cournot or Bertrand) and the source of uncertainty (common or private uncertainty). Please see Raith (1996) for a comprehensive survey of this research stream. In this chapter, we show that asymmetric information structures can also make farmers willing to share information. In a two-tier supply chain setting (one upstream firm and many downstream firms), Li (2002) studies the downstream firms' incentive to share information vertically. The author finds two effects of information sharing, and identifies conditions under which information may be traded. See also Ha et al. (2011) and Li and Zhang (2008).

As for the interaction between public and private information, Morris and Shin (2002) study the impact of public information in a setting where agents' actions are strategic complementary. They show that when private information exists, increasing public disclosure may hurt the social welfare. Although reducing the precision of the public information might be able to prevent the adverse effect of providing public information on social welfare, Cornand and Heinemann (2008) show that another possible instrument is to restrict the number of receivers. That is, releasing the information to some but not all agents. In a general framework with externalities, strategic complementarity or substitutability, Angeletos and Pavan (2007) study how agents use the information (public and private) in equilibrium under different circumstances, and examine the influence of information on social welfare. Colombo et al. (2012) study the social value of public information in a similar framework as that in Angeletos and Pavan (2007), but they endogenize the acquisition of private information (where each agent can choose the precision of his/her private information). They also examine the relation between the efficiencies in the acquisition and in the use of information. In this chapter, we find that in the weak signal limit, if the government's goal is to maximize farmers' total profit, it wants to provide as precise as possible signals. However, it will release these signals to one farmer only. Note that our work is fundamentally different from all aforementioned papers because we adopt a highly asymmetric information structure.

2.2 Model Setting

We consider farmers' competitive production decisions in an information complex environment, in which a bipartite graph describes which subset of signals is accessible to a farmer. In this chapter, to illustrate our results more clearly, all matrices and vectors are written in bold. For any matrix $\boldsymbol{A}$, we use $(\boldsymbol{A})_{ij}$ to represent the ij-th element in $\boldsymbol{A}$. For any row or column vector $\boldsymbol{v}$, we use $(\boldsymbol{v})_i$ to represent the i-th element in $\boldsymbol{v}$. Also, we use $\boldsymbol{A}^T$ to denote the transpose matrix of $\boldsymbol{A}$. We will use Figure 1 as an illustrative example to help us explain the model settings more clearly.

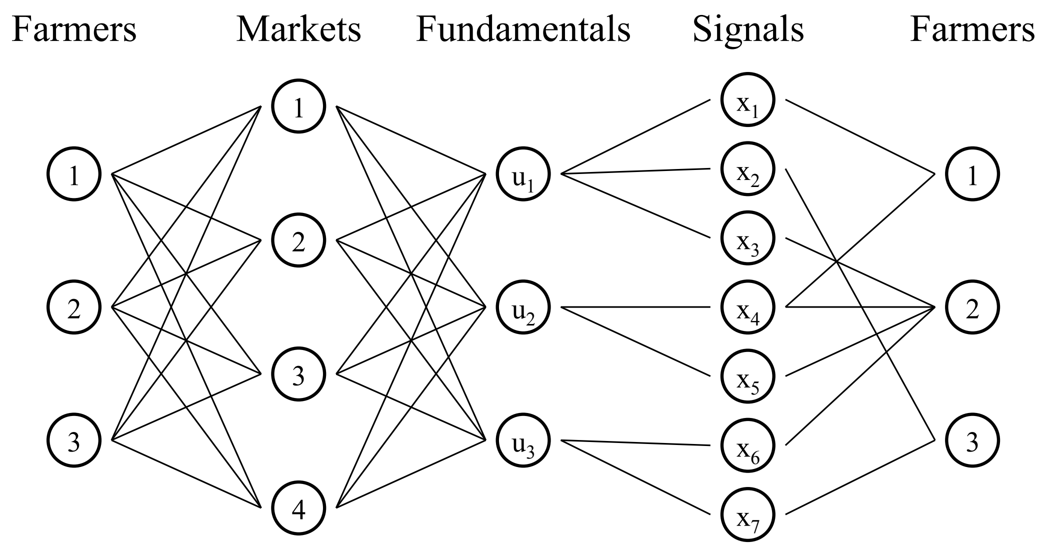


Figure 1: Illustrative example for the model setting.

Farmers and markets. In our model, n_c farmers play Cournot competition in n_m different products, and each product is sold in one market. For example, in Figure 1, 3 farmers play Cournot competitions in 4 markets. Let p_i be the price of product i. The inverse demand is expressed as $p_i = a_i - b_i Q_i$, where Q_i is the aggregate quantity of product i and a_i is an uncertain demand intercept. Let $\boldsymbol{P}$ be the price vector such that $(\boldsymbol{P})_i = p_i$, let $\boldsymbol{a}$ be the demand intercept vector such that $(\boldsymbol{a})_i = a_i$, let $\boldsymbol{Q}$ be the aggregate production quantity vector such that $(\boldsymbol{Q})_i = Q_i$, and let $\boldsymbol{b}$ be an n_m-by-n_m diagonal matrix where $(\boldsymbol{b})_{ii} = b_i$. Then, we can express the prices of these products in a compact form: $\boldsymbol{P} = \boldsymbol{a} - \boldsymbol{b}\boldsymbol{Q}$.

Fundamentals. The demand intercept in each market is uncertain and is decided by n_f fundamentals. For example, the first fundamental might be the global economic condition, and the second fundamental might be the trend of consumers' preference. In Figure 1, the 4 markets are influenced by 3 fundamentals. Let u_i be the value of fundamental i, and let $\boldsymbol{u}$ be the fundamental vector such that $(\boldsymbol{u})_i = u_i$. We assume markets are influenced by fundamentals in a linear way and describe the influence by: $\boldsymbol{a} = \boldsymbol{a_0} + \boldsymbol{\psi}\boldsymbol{u}$, where $\boldsymbol{a_0}$ is a constant vector, and $\boldsymbol{\psi}$ is a constant n_m-by-n_f matrix. Here, we assume that these fundamentals are independent of each other. Otherwise, we can express them in terms of another set of more fundamental factors which are independent from each other. We assume that these fundamentals are normally distributed: $u_i \sim N(0, \sigma_{fi}^2)$ $\forall i$. Each fundamental has a mean zero because any nonzero mean value can be absorbed into $\boldsymbol{a_0}$. We assume that $\boldsymbol{a_0}$ is large compared to the variations of fundamentals so that the prices are almost always positive. This assumption is widely used in the literature. See, e.g., Gal-Or (1985) and Vives (1984).

Signals. For each fundamental i, there are n_{si} signals. For instance, if a fundamental is the trend of consumers' preference, signals about it might be surveys made by different institutions. Let $X_1 = \{x_1, x_2, ..., x_{n_{s1}}\}$ be the set of signals of fundamental 1, $X_2 = \{x_{n_{s1}+1}, x_{n_{s1}+2}, ..., x_{n_{s1}+n_{s2}}\}$ be the set of signals of fundamental 2, and so on. For example, in Figure 1, signals 1, 2, and 3 are about fundamental 1, and, thus, $X_1 = \{x_1, x_2, x_3\}$. Let $n_s = \sum_{i=1}^{n_f} n_{si}$ be the total number of signals. Also, let $\boldsymbol{x}$ be the signal vector such that $(\boldsymbol{x})_i = x_i$. For each fundamental k, we assume that $x_i = u_k + \epsilon_i$ $\forall x_i \in X_k$, where $\epsilon_i \sim N(0, \sigma_{si}^2)$ is the noise of this signal and is independent of all other random variables. For notational convenience, we use $\beta_i \equiv 1/\sigma_{si}^2$ to denote the precision of signal i, and use $\alpha_j \equiv 1/\sigma_{fj}^2$ to denote the "intrinsic certainty" of fundamental j. Furthermore, we define an n_s-by-n_s diagonal matrix $\boldsymbol{\beta}$ such that $(\boldsymbol{\beta})_{ii} = \beta_i$, and an n_f-by-n_f diagonal matrix $\boldsymbol{\alpha}$ such that $(\boldsymbol{\alpha})_{ii} = \alpha_i$. We also define an n_s-by-n_f matrix $\boldsymbol{T}$ such that for all i and j, $(\boldsymbol{T})_{ij} = 1$ if $x_i \in X_j$ and $(\boldsymbol{T})_{ij} = 0$ if $x_i \notin X_j$.

Information channels. Each farmer receives some signals. We use I_j to denote the set of signals observed by farmer j. For example, in Figure 1, farmer 1 can observe signals 1 and 4. Therefore, $I_1 = \{x_1, x_4\}$. We adopt a general information channel structure[18]. Hence, it is possible that a signal is observed by only one farmer (a private signal), by some but not all farmers (a partial-public signal), or by all farmers (a public signal). We assume that the information structure is common knowledge. That is, a farmer knows that which signal(s) is observed by which farmer(s), even though she cannot know the realized values of the signals she cannot observe.[19] For each farmer j, we define an n_s-by-n_s diagonal matrix $\boldsymbol{D_j}$ such that $(\boldsymbol{D_j})_{ii} = 1$ if $x_i \in I_j$ and $(\boldsymbol{D_j})_{ii} = 0$ otherwise, and we define an information amount vector $\boldsymbol{m_j}$ such that $(\boldsymbol{m_j})_k = \sum_{i \in \{N: x_i \in I_j \cap X_k\}} \beta_i$ is the sum of precisions of farmer j's signals about fundamental k. We define an n_s-by-n_s diagonal matrix $\boldsymbol{L_j}$ in the following way: For all i, if $x_i \in I_j$, $(\boldsymbol{L_j})_{ii} = \frac{\beta_i}{\alpha_k + (\boldsymbol{m_j})_k}$, where k is the index of the fundamental signal i is related to ($x_i \in X_k$). If $x_i \notin I_j$, $(\boldsymbol{L_j})_{ii} = 0$. We also define $\boldsymbol{D} \equiv \sum_{j=1}^{n_c} \boldsymbol{D_j}$, and define $d_i \equiv (\boldsymbol{D})_{ii}$, which is the number of farmers who receive signal i. Finally, we define $\boldsymbol{I}$ as an n_s-by-n_s identity matrix.

Production. After observing the signals she receives, each farmer j decides her production quantity vector $\boldsymbol{q_j}$, where $(\boldsymbol{q_j})_i$ is the amount of product i she produces. Therefore,

[18]The only constraint is that there is at least one signal which is observed by at least one farmer. We use this constraint to rule out the trivial situation in which all signals are unobservable to all farmers. Note that in our setting fundamentals remain uncertain conditional on all relevant signals.

[19]This common knowledge assumption serves as the basis of the subsequent game-theoretic analysis. It implies that all players in our setup have the correct understanding of the game.

we have $Q_i = \sum_{j=1}^{n_c} (\boldsymbol{q_j})_i$ and $\boldsymbol{Q} = \sum_{j=1}^{n_c} \boldsymbol{q_j}$. Without loss of generality, we normalize farmers' cost of producing each product to zero. For each farmer, observing signals affects her decisions in two ways. First, she can form better forecasts for the market demand intercepts. Second, if a signal is received by her and other farmers, she can "infer" their beliefs through these signals. Other farmers' beliefs are important since the market prices depend not only on the market demand intercepts but also on the aggregate production quantities. We have the following Lemma 1 about a farmer's expectation of fundamentals and signals:

Lemma 1. *For farmer j, $E(\boldsymbol{u}|I_j) = \boldsymbol{T}^T \boldsymbol{L_j} \boldsymbol{D_j} \boldsymbol{x}$, and $Var(u_k|I_j) = \frac{1}{\alpha_k + (\boldsymbol{m_j})_k}$. Also, for all k and all $x_i \in X_k$, $E(x_i|I_j) = E(u_k|I_j)$ if $x_i \notin I_j$, and $E(x_i|I_j) = x_i$ if $x_i \in I_j$.*

Since farmer j's expected value of any fundamental k is linear in the signals with a coefficient $(\boldsymbol{L_j})_{ii}$ for x_i if $x_i \in X_k$ and 0 for x_i if $x_i \notin X_k$, we can interpret $(\boldsymbol{L_j})_{ii}$ as the weight farmer j puts on signal i.

Notation	Meaning
n_c, n_m, and n_f	Number of farmers, markets (products), and fundamentals.
$\boldsymbol{P}$, $\boldsymbol{a}$, and $\boldsymbol{Q}$	The price, demand intercept, and aggregate production quantity vectors.
$\boldsymbol{b}$	An n_m-by-n_m diagonal matrix where $(\boldsymbol{b})_{ii} = b_i$. ($\boldsymbol{P} = \boldsymbol{a} - \boldsymbol{bQ}$.)
$\boldsymbol{q_j}$	Farmer j's production quantity vector.
$\boldsymbol{u}$ and $\boldsymbol{\psi}$	$\boldsymbol{u}$ is the fundamental vector and $\boldsymbol{\psi}$ is an n_m-by-n_f matrix describing the influence of fundamentals on the demand intercepts. ($\boldsymbol{a} = \boldsymbol{a_0} + \boldsymbol{\psi u}$.)
$\boldsymbol{x}$	The signal vector where $(\boldsymbol{x})_i = x_i$.
X_k and n_{sk}	The set and the number of signals about fundamental k.
α_k and β_i	The intrinsic certainty of fundamental k and the precision of signal i.
$\boldsymbol{\alpha}$	An n_f-by-n_f diagonal matrix where $(\boldsymbol{\alpha})_{ii} = \alpha_i$.
$\boldsymbol{\beta}$	An n_s-by-n_s diagonal matrix where $(\boldsymbol{\beta})_{ii} = \beta_i$.
$\boldsymbol{T}$	An n_s-by-n_f matrix such that $\forall i\ j$, $(\boldsymbol{T})_{ij} = 1$ if $x_i \in X_j$ and $(\boldsymbol{T})_{ij} = 0$ if $x_i \notin X_j$.
I_j	The set of signals farmer j can observe.
$\boldsymbol{m_j}$	The information amount vector for farmer j where $(\boldsymbol{m_j})_k = \sum_{i \in \{N : x_i \in I_j \cap X_k\}} \beta_i$, the sum of precisions of signals about fundamental k she can observe.
$\boldsymbol{D_j}$	An n_s-by-n_s diagonal matrix where $(\boldsymbol{D_j})_{ii} = 1$ if $x_i \in I_j$ and $(\boldsymbol{D_j})_{ii} = 0$ otherwise.
$\boldsymbol{L_j}$	An n_s-by-n_s diagonal matrix such that if $x_i \in I_j$, $(\boldsymbol{L_j})_{ii} = \frac{\beta_i}{\alpha_k + (\boldsymbol{m_j})_k}$, where k is the index of the fundamental signal i is related to ($x_i \in X_k$). If $x_i \notin I_j$, $(\boldsymbol{L_j})_{ii} = 0$.
$\boldsymbol{D}$ and d_i	$\boldsymbol{D} = \sum_{j=1}^{n_c} \boldsymbol{D_j}$, and $d_i = (\boldsymbol{D})_{ii}$, which is the number of farmers who can observe signal i.
$\boldsymbol{I}$	An n_s-by-n_s identity matrix.

Table 1: Summary of notation.

Timing. The sequence of events is as follows. (1) Nature chooses the values of fundamentals and the noise of each signal. (2) Each farmer receives the signals she can observe. (3) Each farmer chooses her production quantities simultaneously. (4) Markets are cleared at prices $\boldsymbol{P} = \boldsymbol{a} - \boldsymbol{bQ}$. The model setting is common knowledge for all farmers, although they cannot observe the realized values of fundamentals and the signals they do not receive.

Because farmers can only get incomplete information, the appropriate solution concept is Bayesian Nash equilibrium (Fudenberg and Tirole (1991)). We will identify and analyze the unique Bayesian Nash equilibrium in the next subsection.

2.3 Equilibrium Analysis

In this subsection, we start with identifying the equilibrium and examining farmers' strategies and expected profits in the equilibrium in subsection 2.3.1. Then, we describe some properties of the equilibrium in subsection 2.3.2.

2.3.1 The Equilibrium, Farmers' Strategies, and Farmers' Expected Profits

We first characterize the equilibrium in the following proposition.

Proposition 1. *There is a unique Bayesian Nash equilibrium. In this equilibrium, each farmer j responds to signals in a linear way and chooses $\boldsymbol{q_j} = \boldsymbol{C_j} + \boldsymbol{B_j D_j x}$, where $\boldsymbol{C_j} = \frac{1}{n_c+1}\boldsymbol{b}^{-1}\boldsymbol{a_0}$, and*

$$\begin{aligned}\boldsymbol{B_j D_j} = \;& \boldsymbol{b}^{-1}\boldsymbol{\psi T}^T \boldsymbol{L_j D_j} \\ & -(\boldsymbol{b}^{-1}\boldsymbol{\psi T}^T \textstyle\sum_i \boldsymbol{L_i D_i})\Big[\boldsymbol{I} + \sum_k \Big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_k})\boldsymbol{TT}^T\boldsymbol{L_k}\Big)\boldsymbol{D_k}\Big]^{-1} \\ & \times\Big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{TT}^T\boldsymbol{L_j}\Big)\boldsymbol{D_j}.\end{aligned} \tag{1}$$

In this unique Bayesian Nash equilibrium, each farmer j has an expected profit of

$$\frac{1}{(n_c+1)^2}\boldsymbol{a_0}^T\boldsymbol{b}^{-1}\boldsymbol{a_0} + Tr\Big[\boldsymbol{b B_j D_j}(\boldsymbol{\beta}^{-1} + \boldsymbol{T\alpha}^{-1}\boldsymbol{T}^T)\boldsymbol{D_j}^T\boldsymbol{B_j}^T\Big]. \tag{2}$$

According to this proposition, farmers always produce a base amount of each product, and make adjustments according to the signals in a linear way. With respect to each signal i and product m, farmer j has a response coefficient $(\boldsymbol{B_j D_j})_{mi}$. As a consistency check, we note that if $x_i \notin I_j$, $(\boldsymbol{B_j D_j})_{mi} = 0 \;\; \forall m$. i.e., farmer j cannot utilize signal i if she does not observe it. Also, we can separate each farmer's expected profit into two parts, the base profit and the fluctuating profit, by its dependence on parameters related to the uncertainty. The base profit is the expected profit a farmer can earn if she does not receive any signal, and the fluctuating profit is the extra expected profit resulted from observing signals. We have the following corollary.

Corollary 1. *A farmer's fluctuating profit is always nonnegative. Therefore, compared to receiving no signals, observing signals never makes a farmer worse off. Also, although a farmer's expected profit depends on the information structure, the base profit, which is independent of the information structure, defines the lower bound of a farmer's expected profit.*

The intuition is as follows: when a farmer *deviates* and chooses not to respond to any signal, she can get an expected profit which is the same as that when she cannot observe any signal. This results from the fact that the distributions of all fundamentals and noises are symmetric about zero. As a result, the base profit is the lower bound of her expected profit. As for the value of the fluctuating profit, we have the following Corollary 2.

Corollary 2. *Given fixed $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, a farmer gets a higher expected profit from a specific market if in this market, in equilibrium (1) she reacts more aggressively to a specific signal. That is, a higher $|(\boldsymbol{B_j D_j})_{mi}|$ for signal i. Or (2) the sum of her response coefficients to the signals about a specific fundamental is more aggressive. That is, a higher $|\sum_{i \in \{N: x_i \in X_k\}} (\boldsymbol{B_j D_j})_{mi}|$ for fundamental k.*

This is an intuitive result: If a farmer is more confident about a signal or a fundamental, she will react more aggressively to it and enjoy a higher expected profit from utilizing it. Note that here we use the term "aggressive" instead of the term "positive" because it is possible that farmers respond to signals adversely. We will illustrate this point later.

2.3.2 Equilibrium Properties

After expressing farmers' strategies and expected profits in equilibrium in close forms, now we can examine some properties of this equilibrium.

Proposition 2. *Different markets and different fundamentals can be treated separately. That is,*

(1) When deciding the response coefficient to a signal $x_i \in X_k$ in a market m, a farmer can decide it as if there is only this fundamental k and this market m.

(2) Each farmer's expected profit can be decomposed into base profits in each of the markets and fluctuating profits pertaining to each of the market-fundamental pairs, which can be calculated as if there is only the corresponding fundamental and market.

Due to Proposition 2, when we study the properties of the equilibrium, we can treat each fundamental and each market *separately*. Therefore, from now on, we concentrate on the situation with only one fundamental and one market. Note that now a_0, b, ψ, and α are all scalars. Also, $\boldsymbol{T}$ becomes an n_s-by-1 array, and $\boldsymbol{B_j D_j}$ becomes a 1-by-n_s array for all j.

Lemma 2. *Signals can be combined and treated as a whole signal if they are received by and only by the same set of farmers.*

This is a very intuitive result: if some signals are observed by and only by the same group of farmers, a farmer's final production quantity should be the same no matter she treats these signals as separate signals or as a whole signal.

We have the next lemma about the upper bound and lower bound of the sum of response coefficients of all farmers. These provide a sensible limit on the extent to which farmers can utilize the signals.

Lemma 3. $0 < b\psi^{-1} \sum_j \boldsymbol{B_j D_j T} < \frac{n_c}{n_c+1}$.

Given a realized value, u, of the fundamental, the price $P = \frac{1}{n_c+1}a_0 + \psi u - b\sum_j \boldsymbol{B_j D_j T} u - b\sum_j \boldsymbol{B_j D_j \epsilon}$ is normally distributed with a mean $\frac{1}{n_c+1}a_0 + \psi u(1 - b\psi^{-1}\sum_j \boldsymbol{B_j D_j T})$. Hence, Lemma 3 has the following implication.

Corollary 3. *Farmers' responses can mitigate the fluctuation of the market demand intercept. However, this mitigation is not as strong as that when all farmers know exactly the realized value of the fundamental.*

This corollary says that when the realized demand intercept is high (low), the expected aggregate production quantity becomes higher (lower). Therefore, farmers' responses can mitigate the fluctuation of the market demand intercept. However, because of the uncertainty from noises of signals, farmers do not respond to the signals as confidently as when all farmers know exactly the realized value of the fundamental.

In a social network problem, when the network is unconnected, usually we can find that agents in different components can be decoupled. In this work, the corresponding situation is that farmers can be separated into different groups and no signal is observed by farmers from different groups. i.e., each signal belongs to a group and is transmitted exclusively within the network of this group. We have the following proposition about how to decouple farmers in different groups.

Proposition 3. *If farmers can be separated into two disjoint groups, groups* 1 *and* 2*, such that no signal is received by farmers from different groups, then a farmer (say farmer j in group* 1*) has response coefficients:* $\boldsymbol{B_j D_j} = \frac{1-O_2}{1-O_1O_2}\boldsymbol{B'_j D'_j}$*, where for each k,* $\boldsymbol{B'_k D'_k}$ *are farmer k's response coefficients if farmers in the other group do not exist, and* $O_i = b\psi^{-1}\sum_{k\in(group\ i)} \boldsymbol{B'_k D'_k T}$*.*

In reality, it is hard to believe that when making production decisions, farmers take into account the complex information structures of farmers in different provinces or different countries. Proposition 3 implies that what a farmer in group 1 does is to ignore farmers in group 2 first and find her optimal strategy. Then, she calculates O_1, which is proportional to the sum of response coefficients of all farmers in group 1 to all signals as if group 2 does not exist. She should also guess the value of O_2, the counterpart of O_1 in group 2. Then she just multiplies her response coefficients by $\frac{1-O_2}{1-O_1O_2}$. Because O_1 and O_2 are both between zero and one, she suppresses her response to the signals. It is possible that she can guess O_2 from the past experience. In this situation, when deciding the strategy, a farmer need not worry about the information structure of farmers in other parts of the world. Another implication of Proposition 3 is that when the information structure for farmers in the other group changes (e.g. a new signal is released to them), a farmer only needs to update the corresponding O_i to get her new optimal strategy.

2.4 Illustrative Example

In this subsection, we use simple examples to illustrate that highly asymmetric information channels can lead to various novel results.

Example 1. Suppose there are three farmers engaged in Cournot competition in one market which is influenced by one fundamental. There are three signals about this fundamental. Farmer 1 can observe signals 1 and 2. However, farmer 2 can only observe signal 2 and farmer 3 can only observe signal 3. Using our notation, $I_1 = \{x_1, x_2\}$, $I_2 = \{x_2\}$, and $I_3 = \{x_3\}$.

With the formulas in Proposition 1, we can calculate farmers' response coefficients and expected profits directly. We have the following observations.

Observation 1. *Farmers may react adversely to the signals. That is, a farmer may produce more when a signal suggests a low market demand intercept while produce less when the signal is optimistic.*

At first glance, this observation sounds counterintuitive because usually farmers produce more when the signal implies a high demand, and reduce the production quantity when facing a pessimistic signal. However, a signal not only provides farmers the information about the market demand, but also the information about other farmers' beliefs, and, hence, their production quantities. In this example, if signal 1 is much more precise than signal 2, signal 2 cannot tell farmer 1 much about the fundamental and she puts very low weight on it. On the other hand, farmer 2 only receives signal 2, so she puts higher weight on it. When a specific signal exhibits such weight difference, the second effect might dominate the first one. That is, signal 2 cannot tell farmer 1 much about the market demand, but it indicates farmer 1 what farmer 2 believes. By reacting adversely to signal 2, farmer 1 can avoid the harsh competition with farmer 2 when this signal suggests a high demand. She can also grasp more profit in the empty market when the signal suggests a low demand and farmer 2 reduces her production quantity accordingly.

In reality, sometimes we can observe the phenomena that many farmers receive the same signal suggesting a high demand next year, and increase their production quantities naively. However, the market ends up with a very high aggregate quantity and the price drops significantly. In this situation, sophisticated farmers might want to react adversely to the signal in order to avoid this herding effect.

One might think that a farmer suffers when another farmer gets more information and has more competitive advantage against her (see Vives (1984)). However, the following observation says that this does not always hold.

Observation 2. *A farmer may benefit from the improvement of a signal she cannot observe.*

In this example, we find that farmer 2 might benefit when signal 1 becomes more precise. The rough intuition is that when signal 1 becomes more precise, farmer 1 will pay more attention to it and less attention to signal 2. In this situation, farmer 2 can utilize signal 2 better and enjoy more profit. We relegate the detail discussion to Proposition 6.

Example 2. In this example, all settings are the same as Example 1 except that farmer 3 can also observe signal 2. One can think of this as farmer 1 or 2 decides to share this signal with farmer 3. (Note that they should decide whether to share it with farmer 3 before the signals realize.) Then we have the following observations.

Observation 3. *A farmer may benefit from sharing some of her signals with another farmer.*

Different from the literature (please see Gal-Or (1985), in which there is no information sharing in equilibrium, and see also Vives (1984) and Li (1985)), we find that the highly asymmetric information channels may make farmers willing to share their signals with others. In this example, if signal 3 is much more precise than the other two signals, farmer 3 might need to react adversely to signal 2. In this situation, farmer 1 and farmer 2 might be able to utilize signal 2 better and get a higher profit compared to Example 1.

This incentive to share information potentially creates issues when one intends to evaluate the benefit of information provision via experiments. It is well-known that randomized controlled trials are effective in evaluating social programs. However, if farmers would strategically share information with others, the separation between treatment and controlled groups is no longer clean. This spillover therefore may contaminate the experiment design.

Observation 4. *A farmer may be worse off when receiving a previously unobservable signal.*

This observation differs from the conventional wisdom that information is beneficial. In the situation described in Observation 3, farmer 3 might be worse off when she is forced to react adversely to signal 2. In this situation, she hopes that she never observes this signal: once she observes it (and other farmers know this), they will modify their strategies accordingly, and her optimal strategy becomes reacting adversely to it.

2.5 Weak Signal Expansion

Although we have farmers' strategies and expected profits in close forms, these formulas are too complicated for us to get more insightful results. The main difficulty comes from the inverse matrix in Equation (1) that describes farmers' strategies. To get more insight into the equilibrium, in this subsection we focus on the weak signal limit. In this regime, the precisions of the signals are low compared to the intrinsic certainty of the fundamental ($\beta_i \ll \alpha \ \forall i$), and this allows us to expand the inverse matrix. The benefit of this weak signal expansion is two-fold. First, in this regime, we can express farmers' strategies and expected profits explicitly. This helps us examine the properties we are interested in directly, and get useful insights. Second, we can build upon the explicit characterizations in this regime to examine other higher-level decisions, in particular, how the government should allocate its information.

We present the expansion of farmers' strategies in Subsection 2.5.1, and discuss more equilibrium properties in Subsection 2.5.2. Finally, the government's optimal information or budgets allocation is examined in Subsection 2.5.3.

2.5.1 Expansion of Farmers' Strategies

We expand the inverse matrix in Equation (1) in the following Lemma 4.

Lemma 4. *In this weak signal limit, we can expand the inverse matrix in Equation (1) into the following form:*

$$\begin{aligned}
&\left[\boldsymbol{I} + \textstyle\sum_{j=1}^{n_c} \left(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\right)\boldsymbol{D_j}\right]^{-1} \\
&= (\boldsymbol{I}+\boldsymbol{D})^{-1}\Big\{\boldsymbol{I} \quad - \textstyle\sum_{j=1}^{n_c}(\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\boldsymbol{D_j}(\boldsymbol{I}+\boldsymbol{D})^{-1} \\
&\qquad\qquad\qquad + \Big[\textstyle\sum_{j=1}^{n_c}(\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\boldsymbol{D_j}(\boldsymbol{I}+\boldsymbol{D})^{-1}\Big]^2 \\
&\qquad\qquad\qquad - \ldots\Big\}
\end{aligned} \tag{3}$$

The difficult part in the inverse matrix comes from the fact that in making decisions, famers should guess unobservable signals, guess other farmers' guess of unobservable signals, guess other farmers' guess of other farmers' guess of unobservable signals, and so on. In this weak signal limit, the higher-order terms are less important and, thus, the expansion is valid[20].

Beyond the above mathematical explanations, we now offer an economic justification for this expansion. When farmers are not sophisticated enough, it is possible that they only go through a few steps of thinking (see Camerer et al. (2004)). In this situation, we only need to count the first few orders. We expand farmers' strategies in terms of $\{\boldsymbol{L_j}\}$'s in the following Proposition 4.

Proposition 4. *The first-order term in* $\boldsymbol{B_j}\boldsymbol{D_j}$ *is*

$$b^{-1}\psi\boldsymbol{T}^T\Big\{\boldsymbol{L_j}\boldsymbol{D_j} - \sum_{i=1}^{n_c}\boldsymbol{L_i}\boldsymbol{D_i}(\boldsymbol{I}+\boldsymbol{D})^{-1}\boldsymbol{D_j}\Big\}. \tag{4}$$

And the second-order term in $\boldsymbol{B_j}\boldsymbol{D_j}$ *is*

$$-b^{-1}\psi\boldsymbol{T}^T\sum_{i=1}^{n_c}\boldsymbol{L_i}\boldsymbol{D_i}(\boldsymbol{I}+\boldsymbol{D})^{-1}\Big\{(\boldsymbol{I}-\boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\boldsymbol{D_j} - \sum_{k=1}^{n_c}(\boldsymbol{I}-\boldsymbol{D_k})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_k}\boldsymbol{D_k}(\boldsymbol{I}+\boldsymbol{D})^{-1}\boldsymbol{D_j}\Big\}. \tag{5}$$

[20]Please see A.1.10 for more discussions.

The first-order term in $\boldsymbol{B_j D_j}$ includes two forces. First, the farmer j wants to make her production decision according to the signals. i.e., when a signal suggests a high (low) demand, she wants to produce more (less). However, this response is mitigated because she knows that some farmers also receive the same signal and the market fluctuation will be alleviated by their actions.

The second-order term in $\boldsymbol{B_j D_j}$ also includes two parts. The first part is farmer j's response to the expected first-order responses other farmers make to the signals farmer j cannot observe. This farmer j's response is again mitigated because other farmers who receive the same signals will go through similar adjustments.

2.5.2 More Properties of The Equilibrium

After expanding farmers' strategies, now we can have a closer examination of the equilibrium. We find the following properties of the equilibrium in the weak signal limit.

Proposition 5. *Farmers always react positively to the observable signals in the weak signal limit.*

In the weak signal limit, the difference between the amount of information each farmer gathered is small, compared to the intrinsic certainty of the fundamental. In this situation, farmers' weights on a specific signal are close, and, hence, they only need to "adjust" their responses, but need not react in the opposite direction. Also, the strength of a farmer's response coefficient to a specific signal is increasing in the signal precision, decreasing in the intrinsic certainty of the fundamental, and decreasing in the number of farmers who can also observe this signal.

Proposition 6. *A farmer j may benefit from the improvement of a signal i she cannot observe. However, in the weak signal limit, there is a necessary condition: $\forall k$ s.t. $x_i \in I_k,\ I_j \subset I_k$.*

If farmer j cannot observe the improved signal i, this improvement has two effects on her expected profit. First, farmers who receive signal i will reduce the weight they put on other signals, and farmer j can utilize these signals better if she can observe these signals. The second effect results from the fact that the aggregate response coefficient becomes higher now. In this situation, farmer j should withhold her response coefficients. These two effects are of the second order, but the second effect weakly dominates the first one. They have the same magnitude only when the first effect is maximized. That is, each farmer who can observe signal i should be able to observe all the signals farmer j observes. In this situation, these two effects cancel out each other, and we should focus on the third-order terms.

Proposition 7. *In the weak signal limit, farmers always benefit from the improvement of signals they can observe. Also, farmers' total profits always increase when signals are improved.*

If a signal i is improved, farmers who can observe this signal get higher expected profits because they can have better utilizations of this signal. This effect is of the first order. Although the improvement of signal i also has influence on how they respond to other signals, this is only the second-order effect and is negligible. Also, its influence on other farmers is at most of the second order. Therefore, farmers' total profits are always increasing in the precisions of signals in the weak signal limit.

Proposition 8. *In the weak signal limit, farmers never want to share their signals with other farmers, and farmers always benefit from observing more signals.*

In the weak signal limit, no farmer needs to react adversely to any signal. Therefore, sharing a signal with other farmers only results in more competitors who can exploit the signal, and, thus, no farmer wants to share information with others. On the other hand, when observing a previously unobservable signal, a farmer always utilizes it to earn a higher profit.

2.5.3 Government's Decision

In this subsection, we examine the government's optimal information provision (or budgets allocation to improve signals) when its goal is to improve farmers' total profits or to improve the social welfare. First of all, we argue that allocating the government's budgets to improve signals is somehow equivalent to releasing new signals. Because of Lemma 2, we have the following corollary.

Corollary 4. *Improving the precision of a signal i by δ has the same outcome as releasing a new signal with precision δ to and only to the farmers who can observe signal i.*

Because of this equivalence, we assume that the cost of increasing one unit of precision is the same across different signals and does not depend on the current precision. The following proposition says that the government should not let multiple farmers share the same information if its goal is to improve farmers' total profits.

Proposition 9. *In the weak signal limit, when a new signal is released (to at least one farmer), farmers' total profits become higher. However, this increment is decreasing in the number of farmers who receive this new signal.*

This proposition leads to the following implication.

Corollary 5. *In the weak signal limit, when the government wants to improve farmers' total profits:*

1. If the government has multiple new signals, it should release all of them. However, each signal should be released to only one farmer.

2. If the government has budgets to improve signals, it should spend all budgets to improve private signals (or the signals observed by the least number (but non-zero) of farmers).

From Proposition 9, we know that to improve farmers' total profits, releasing a new signal to one farmer is better than releasing it to multiple farmers and better than abandoning it. Therefore, the government should release all of its signals, and each signal should be observed by only one farmer. On the other hand, due to Corollary 4 and Proposition 9, the government should spend all of its budgets to improve signals which are observed by the least number (but non-zero) of farmers.

The following lemma is about the marginal benefit to farmers' expected total profits ($TP \equiv \sum_l$(farmer l's expected profit)) of improving a signal i which is observed by farmer j only (or releasing a new weak signal to farmer j only).

Lemma 5. *In the weak signal limit, for any signal i which is observed by farmer j only, we have (up to the second order):*

$$\frac{\partial TP}{\partial \beta_i} = b^{-1}\psi^2 \Big\{ \frac{1}{4\alpha^2} - \frac{1}{2\alpha^3} \boldsymbol{T}^T \boldsymbol{\beta} (\boldsymbol{I} + \boldsymbol{D})^{-2} \big(3\boldsymbol{D} + \boldsymbol{D}^2 + \boldsymbol{D_j} - \boldsymbol{D}\boldsymbol{D_j} \big) \boldsymbol{T} \Big\}. \tag{6}$$

With Lemma 5, we establish an index

$$f_j \equiv \sum_{i \in \{N : x_i \in I_j\}} \beta_i \frac{d_i - 1}{(1 + d_i)^2} \tag{7}$$

that determines which farmer(s) should get the new signals or which private signal(s) should be improved.

Proposition 10. *In the weak signal limit, to improve farmers' total profits, if the government has some new signals, it should release all of them to the farmer j with the highest f_j. If the government has some budgets to improve the signals and there are more than one private signals, it should improve the private signal of the farmer j with the highest f_j.*

This index appears to be novel to the literature. It is the weighted sum of precisions of signals observable to a farmer, in which the precision of each observable signal i is weighted by $\frac{d_i - 1}{(1+d_i)^2}$. This factor is non-monotone (unimodal) and reaches its peak at $d_i = 3$. Hence, this proposition claims that the new signals should be given to the farmer who receives most signals on which she has "moderate" competitions with other farmers. The intuition is as follows. When a farmer j receives a new signal, she will put less weight on her other signals (say k), and farmers who can observe signal k may benefit. The aggregate benefit is increasing in the precision of signal k. However, its dependence on the number of farmers who can observe signal k is non-monotone (proportional to $\frac{d_k - 1}{(1+d_k)^2}$). Note that $\{f_j\}$'s do not depend on private signals. Therefore, all new signals should be allocated to the same farmer, and all budgets should be used to improve the same private signal.

The following proposition is about the government's policy when its goal is to improve the social welfare (which includes consumers' surplus).

Proposition 11. *In the weak signal limit, to improve the social welfare, the government should allocate all of its signals (budgets) to all farmers (to improve the signal observed by most farmers).*

When the government's goal is to improve the social welfare, it wants to provide all of its information to all farmers. Note that this is the opposite of the policy it uses to maximize farmers' total profit. Thus, one should be cautious when devising the information provision policy under different objectives.

3 Information Provision Policies for Improving Farmer Welfare in Developing Countries: Heterogeneous Farmers and Market Selection

In this chapter, we examine the impact of information provision policies on farmer welfare in developing countries where farmers lack relevant and timely information for making informed decisions regarding which crop to grow and which market to sell in. In addition to heterogeneous farmers, we consider the case when farmers are price takers and yet the price of each crop (or the price in each market) is a linearly decreasing function of the total sales quantity. When market information is offered free-of-charge, we show that: (a) providing information is always beneficial to farmers at the *individual level*; and (b) providing information to all farmers may not be welfare maximizing at the *aggregate level*. To maximize farmer welfare, it is optimal to provide information to a targeted group of farmers who are located far away from either market. However, to overcome perceived unfairness among farmers, we show that the government should provide information to all farmers at a nominal fee so that the farmers will adopt the intended optimal provision policy willingly. We extend our analysis to examine different issues including: precision of market information, and information dissemination via a for-profit company.

The remainder of this chapter is organized as follows. Subsection 3.1 reviews the related literature, and Subsection 3.2 presents the model settings. In Subsection 3.3, we analyze the farmer's market selection rule in equilibrium and the farmer welfare. Subsection 3.4 examines different information provision policies and identifies the optimal provision policy. Extensions, including a for-profit company's strategy, precision reduction as an instrument, and the robustness check, are discussed in Subsection 3.5. We only present the main results, and all proofs are provided in the appendix.

3.1 Literature Review

This chapter is related to an emerging stream of research in socially responsible operations. Sodhi and Tang (2013) propose that a sustainable way to alleviate poverty is to engage the poor as producers or distributors by enabling them with financial, information, and material flows along the supply chain. Recent research articles that examine the implications of disseminating agricultural information via mobile phones or Internet include the following. First, Fafchamps and Minten (2012) and Mittal et al. (2010) find mixed results regarding the benefits of disseminating information through mobile phones. By examining the impact of disseminating market price information via mobile phones in India, Parker et al. (2012) provide empirical evidence about the reduction of geographic price dispersion of crops in rural communities. Chen et al. (2013) investigate the implication of ITC e-Choupals, an Internet platform that provides market price and crop advisory information.

In economics literature, Morris and Shin (2002) study the impact of public information when agents want to align themselves with the underlying fundamentals but they also want to coordinate with others due to the strategic complementarity in their actions. The authors find that if agents have no private information, public information can increase the welfare. However, in the presence of private signals with substantial precisions, public information might be harmful. In this situation, reducing the precision of the public signal can improve the welfare. Instead of reducing signal precision, Cornand and Heinemann (2008) suggest that reducing the number of agents receiving signals might be a better choice. In a "farmer" context (Cournot competition in which farmers' actions are strategic substitutes), Chen and Tang (2013) and Zhou et al. (2013) also reach similar results.

Our work complements the existing literature in the following manner. All analytical models in the literature rely on three key assumptions: (a) farmers are price setters in the sense that they engage in Cournot competition so that their production quantity can influence the price; (b) farmers are homogeneous in terms of their transaction costs; and (c) all farmers produce the same crop (or sell in the same market). We relax all three assumptions in this work and find that, even without private information, providing information to all farmers may not be optimal (in terms of farmers' total profit) and it may be optimal for the government to provide limited access to farmers. However, this information allocation might create fairness concerns, and we show that the government can ensure fairness by providing information access to all farmers at a nominal fee so that farmers will adopt the intended optimal provision policy willingly.

3.2 Model Description

In the base model, we focus on the case when the government has to determine an effective information provision policy that is intended to maximize farmers' total profit. (In a later subsection, we extend our analysis to the case when a for-profit company needs to decide on the information provision policy and the service subscription fee that maximize the firm's profit.) Our model consists of the following elements:

1. **Heterogeneous Farmers.** We adopt the Hotelling model by assuming that there is a continuous type of infinitesimal farmers distributing uniformly over a line $[-0.5, 0.5]$.[21] Each smallholder farmer can produce up to 1 unit. Also, each farmer is a price taker: his production quantity is too small to influence the market price. However, the total quantity produced by the farmers is large enough to affect the market price.

2. **Two Markets.** There are two markets located at the opposite ends of the line: the "left" market l is located at -0.5 and the "right" market r is located at 0.5.

3. **Uncertain Market Price.** For each market i, $i = l, r$, the unit market price $p_i = a_i - bq_i$, where q_i is the total quantity to be sold by the farmers in market i and b is price elasticity. Also, a_i is the intercept (or market size) so that $a_i = A + u_i$ $(i \in \{l, r\})$, where A is the mean value of the intercept and $u_i \sim N(0, \sigma^2)$ represents the uncertainty of the intercept in market i.[22] We define $\alpha \equiv 1/\sigma^2$ to denote the "intrinsic certainty" of the intercepts.

4. **Farmer's Profit Function without Market Information.** For each farmer located at $\theta \in [-0.5, 0.5]$, we normalize his unit production cost to 0. However, there is a *transportation cost* for this farmer to sell in market i that depends on the "distance" between his location θ and the market i that he sells in. By accounting for the uncertain market price in market i, the profit for a farmer who is located at $\theta \in [-0.5, 0.5]$ and sells one unit in market i is given by $\hat{\pi}_0(\theta, i)$, where

$$\begin{aligned} \hat{\pi}_0(\theta, l) &= a_l - bq_l - t(0.5 + \theta), \text{ and} \\ \hat{\pi}_0(\theta, r) &= a_r - bq_r - t(0.5 - \theta). \end{aligned} \quad (8)$$

5. **Market Signals.** For each market $i = l, r$, the government has a noisy signal x_i about the uncertainty of the market intercept, u_i, where $x_i = u_i + \epsilon_i$. We assume that $\epsilon_i \sim N(0, s^2)$ and ϵ_l and ϵ_r are independent. By using $\beta \equiv 1/s^2$ to denote the precision of market signals, it is easy to check that, for $i = l, r$,

$$\begin{aligned} E(u_i|(x_l, x_r)) &= \frac{\beta}{\alpha + \beta} \cdot x_i, \text{ and} \\ Var(u_i|(x_l, x_r)) &= \frac{1}{\alpha + \beta}. \end{aligned} \quad (9)$$

   By noting that $\frac{1}{\alpha+\beta} < \frac{1}{\alpha} = \sigma^2$, we can conclude that market signals enable farmers to obtain more accurate forecast about the intercept.

[21] We shall extend our analysis to the case when the distribution is not uniform but a general symmetric distribution.

[22] Following a standard assumption used in the literature, we assume that A is sufficiently large so that the p_i is almost always positive (Gal-Or (1985) and Vives (1984)). Also, we assume that u_l and u_r are independent. In Subsection 3.5.3, we show our results continue to hold when u_l and u_r are correlated.

6. **Government Information Provision Policy.** With the possession of market signals (x_l, x_r), we shall focus on a class of provision policy δ that can be specified by two decisions $K \in [0, 0.5]$, and $\rho \in [0, 1]$ so that ρ percent of farmers located within $[-K, K]$ will receive the market signals.[23]

The sequence of events goes as follows. The government first sets provision policy $\delta = (K, \rho)$. The market signals (x_l, x_r) are realized, and the government disseminates them according to policy δ. For each farmer located at θ, he will use the market signals (x_l, x_r) he receives (if any) to select the market to sell in. Once the total quantity to be sold in each market (q_i) is determined, the market price $p_i = A + (u_i|(x_l, x_r)) - bq_i$ is realized in market i for $i = l, r$, and the farmers' total profit is also realized. For ease of reference, we provide a summary of our notation in Table 2.

[23]We did consider a more general class of provision policy δ that can be specified by two decisions $R \subset [-0.5, 0.5]$ and $\rho \leq 1$ so that ρ percentage of farmers located within R will receive the market signals. For tractability, we shall focus on the case when R is symmetric about the origin 0. When R consists of a finite number of closed intervals, we show that each provision policy (R, ρ) is dominated by a corresponding provision policy under which R is a continuous interval so that $R = [-K, K]$, where $K \in [0, 0.5]$. We omit the details here, but the reader is referred to the online supporting material for details. Therefore, it suffices for us to focus on a class of provision policy $\delta = (K, \rho)$, where $K \in [0, 0.5]$, and $\rho \in [0, 1]$ throughout this chapter.

Notation	Meaning
p_i, a_i, and q_i	The price, intercept, and aggregate quantity in market i so that $p_i = a_i - bq_i$, where $i \in \{l, r\}$.
A	The expected value of the intercept in each market.
u_i	The uncertainty of the intercept in market i. ($a_i = A + u_i$.)
α	The intrinsic certainty of the intercept in each market.
x_i	The signal about u_i.
β	The precision of each signal.
K	The range $[-K, K]$ in which the government provides signals.
ρ	The percentage of farmers located within $[-K, K]$ who receive the market signals.
$\tau^{(\delta)}$	The market selection threshold associated with policy δ for farmers with signals.
$\pi^{(\delta)}(\theta; x_l, x_r)$ (or $\pi_0^{(\delta)}(\theta; x_l, x_r)$)	The *ex-post* expected profit of each farmer at θ with (or without) signals conditional on (x_l, x_r) under policy δ.
$w^{(\delta)}(x_l, x_r)$	The *ex-post* expected total profit of *all farmers* conditional on (x_l, x_r) under policy δ.
$\Pi^{(\delta)}(\theta)/\Pi_0^{(\delta)}(\theta)$	The *ex-ante* expected profit of each farmer at θ with/without signals under policy δ.
$W^{(\delta)}$	The *ex-ante* expected total profit of *all farmers* under policy δ.

Table 2: Summary of notation.

To examine the value of information and the value of centralized control, we shall examine three benchmark provision policies:

1. **Centralized Control Policy (C):** To maximize farmers' total profit, the government uses market signals to assign the market for each farmer to sell in. (Besides policy (C), all other provision policies are implemented under a *decentralized* system.)

2. **Full Information with Decentralized Control Policy (F1):** $\delta^{(F1)} = (K = 0.5, \rho = 1)$. All farmers receive signals, and each farmer selects the market that maximizes his profit.

3. **No Information Policy (F0):** $\delta^{(F0)} = (K = 0.5, \rho = 0)$. Farmers receive no signals.

The above three benchmark provision policies enable us to examine the value of information and the value of centralized control by comparing the ex-ante expected farmers' total profit. We also consider two additional provision policies:

1. **Full Range with Partial Intensity Policy (F$\boldsymbol{\rho}$):** $\delta^{(F\rho)} = (K = 0.5, \rho \in [0,1])$. Each farmer receives signals with probability ρ, where ρ is selected by the government.

2. **General Policy (K$\boldsymbol{\rho}$):** $\delta^{(K\rho)} = (K \in [0, 0.5], \rho \in [0,1])$. The government selects both K and ρ so that farmers located over the region $[-K, K]$ will receive signals with probability ρ, and farmers located over $[-0.5, -K) \cup (K, 0.5]$ will receive no signals. In this case, the optimal provision policy $\delta^* \equiv (K^*, \rho^*) = \arg\max\{W^{(\delta)} : \delta = (K, \rho),\ K \in [0, 0.5],\ \rho \in [0,1]\}$.

3.3 Analysis: Farmer's Market Selection and Farmer's Profit

We use backward induction to analyze a Stackelberg game in which the government acts as the leader and the farmers act as followers. Specifically, we first examine the market selection rule that each farmer will adopt in equilibrium. Anticipating the market selection rule, the government determines the optimal provision policy $\delta^* = (K^*, \rho^*)$ that maximizes farmers' total profit.

3.3.1 Farmer's Threshold Market Selection Rule under Provision Policy $\boldsymbol{\delta}$

Upon disseminating market signals (x_l, x_r) according to policy $\delta = (K, \rho)$, we now examine each farmer's market selection rule. By considering the fact that farmers are price takers and risk-neutral, it is immediately clear that each farmer will produce and sell one unit in exactly one market. Therefore, for any provision policy $\delta = (K, \rho)$, each farmer located at θ will select the market to sell in that yields the higher expected profit. The comparison of the expected profits between two markets yields the following *threshold market selection rule*:

Lemma 6. *For any provision policy $\delta = (K, \rho)$, each farmer who is located at θ will adopt the following threshold market selection rule in equilibrium:*

1. *If the farmer receives signals (x_l, x_r), then he will sell in the left market l if $\theta < \tau^{(\delta)}(x_l, x_r)$ and sell in the right market r if $\theta \geq \tau^{(\delta)}(x_l, x_r)$, where:*

$$\tau^{(\delta)}(x_l, x_r) = \max\{-K, \min\{K, \frac{1}{2(\rho b + t)} \cdot \frac{\beta}{\alpha + \beta} \cdot (x_l - x_r)\}\}. \tag{10}$$

2. *If the farmer receives no signals, then he will sell in the left market l if $\theta < 0$. Otherwise, he will sell in the right market r.*

Lemma 6 is based on the following intuition. Consider a farmer who is located at $\theta \in [0, K]$. First, if he receives no signals, then both markets have the same expected selling price. To reduce transportation cost, he would prefer to sell his crop in the nearby market r. This explains the second statement. Next, suppose this farmer receives signals that has $x_l > x_r$ (i.e., the expected selling price in market l is higher than that of market r). Also, suppose x_l is sufficiently larger than x_r so that the threshold $\tau^{(\delta)}(x_l, x_r) \in (\theta, K]$. Then the first statement states that it is more profitable for this farmer to "switch" from selling in market r to market l. This is because the higher selling price to be obtained in market l would outweigh the extra transportation cost to be incurred from switching.

Therefore, relative to the No Information Policy (F0) that has $\tau^{(F0)} = 0$,[24] the number of switchers (i.e., farmers who receive signals and then switch from selling in the nearby market to the farther away market) under policy $\delta = (K, \rho)$ is equal to $\rho \cdot |\tau^{(\delta)}(x_l, x_r) - \tau^{(F0)}| = \rho \cdot |\tau^{(\delta)}(x_l, x_r)|$. As we shall see, the number of switchers associated with a provision policy will play an important role in explaining some of the results later.

3.3.2 Farmer's Profit Function under Provision Policy δ

For any provision policy δ, we now determine the expected profit of each farmer. In preparation, let us examine the sales quantity to be sold in each market q_i when all farmers follow the threshold rules as stated in Lemma 6 under any policy δ. To do so, let us consider Figure 2 along with the threshold selection rules as stated in Lemma 6. (For ease of exposition, let us consider the case when market signals satisfy $x_l > x_r$ so that $\tau^{(\delta)} \in (0, K]$.) In this case, we can describe each farmer's market selection in equilibrium as follows: (1) a farmer who locates over the region $[-0.5, 0)$ will sell in market l regardless of whether he receives signals or not; (2) a farmer who locates over the region $[\tau^{(\delta)}, 0.5]$ will sell in market r regardless of whether he receives signals or not; and (3) a farmer who locates over the region $[0, \tau^{(\delta)})$ will sell in market l if he receives signals (with probability ρ), and will sell in market r if he receives no signals (with probability $(1 - \rho)$).

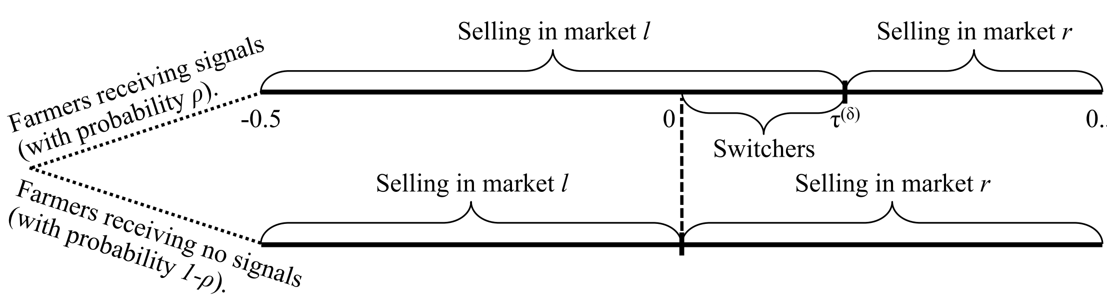


Figure 2: Farmers' market selection rules.

By using the fact that the farmers are located uniformly over $[-0.5, 0.5]$, it is easy to

[24]For ease of notation, we use $\tau^{(F0)} \equiv 0$ to denote the threshold adopted by all farmers under the No Information Policy.

check from Figure 2 that the sales quantities $q_l^{(\delta)} = 0.5 + \rho\tau^{(\delta)}$ and $q_r^{(\delta)} = 0.5 - \rho\tau^{(\delta)}$. Hence, we can apply (8), $q_l^{(\delta)}$ and $q_r^{(\delta)}$ to determine each farmer's (ex-post) expected profit as follows. First, for a farmer located at θ who receives no signals, his (ex-post) expected profit can be expressed as:[25]

$$\pi_0^{(\delta)}(\theta; x_l, x_r) = \begin{cases} E(\hat{\pi}_0(\theta, l)|(x_l, x_r)) = A + E(u_l|(x_l, x_r)) - b(0.5 + \rho\tau^{(\delta)}) - t(0.5 + \theta), \text{ if } \theta < 0, \\ E(\hat{\pi}_0(\theta, r)|(x_l, x_r)) = A + E(u_r|(x_l, x_r)) - b(0.5 - \rho\tau^{(\delta)}) - t(0.5 - \theta), \text{ if } \theta \geq 0. \end{cases} \tag{11}$$

Also, we can apply (10) and (11) to determine this farmer's ex-ante expected profit $\Pi_0^{(\delta)}(\theta) \equiv E_{(x_l,x_r)}(\pi_0^{(\delta)}(\theta; x_l, x_r))$.

Second, for a farmer located at θ who receives signals (x_l, x_r) under policy δ, we can apply (8), $q_l^{(\delta)}$ and $q_r^{(\delta)}$ along with the threshold rule to determine his ex-post expected profit as:

$$\pi^{(\delta)}(\theta; x_l, x_r) = \begin{cases} E(\hat{\pi}_0(\theta, l)|(x_l, x_r)) = A + E(u_l|(x_l, x_r)) - b(0.5 + \rho\tau^{(\delta)}) - t(0.5 + \theta), \text{ if } \theta < \tau^{(\delta)}, \\ E(\hat{\pi}_0(\theta, r)|(x_l, x_r)) = A + E(u_r|(x_l, x_r)) - b(0.5 - \rho\tau^{(\delta)}) - t(0.5 - \theta), \text{ if } \theta \geq \tau^{(\delta)}. \end{cases} \tag{12}$$

Also, we can use (10) and (12) to determine this farmer's ex-ante expected profit $\Pi^{(\delta)}(\theta) \equiv E_{(x_l,x_r)}(\pi^{(\delta)}(\theta; x_l, x_r))$. The following lemma examines the properties of a farmer's ex-ante expected profit under any provision policy δ.

Lemma 7. *For any policy $\delta = (K, \rho)$, the ex-ante expected profit associated with a farmer who is located at θ has the following properties:*

1. *If he receives no signals, then his ex-ante expected profit $\Pi_0^{(\delta)}(\theta) = A - 0.5b - 0.5t + t|\theta|$, where $\Pi_0^{(\delta)}(\theta)$ is increasing in $|\theta|$.*

2. *If he receives signals, then his ex-ante expected profit*

$$\Pi^{(\delta)}(\theta) = \Pi_0^{(\delta)}(\theta) + \sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}\frac{t}{\rho b+t}exp\Big[-\frac{\alpha(\rho b+t)^2(\alpha+\beta)}{\beta}\theta^2\Big] - 2t|\theta|\Phi\Big[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}|\theta|\Big] + \sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}\frac{\rho b}{\rho b+t}exp\Big[-\frac{\alpha(\rho b+t)^2(\alpha+\beta)}{\beta}K^2\Big] - 2\rho bK\Phi\Big[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}K\Big]. \tag{13}$$

where $\Phi(x) \equiv \int_{-\infty}^{x}\sqrt{\frac{\alpha\beta}{4\pi(\alpha+\beta)}}exp\Big[-\frac{\alpha\beta}{4(\alpha+\beta)}y^2\Big]\mathrm{d}y$. Note that $\Pi^{(\delta)}(\theta)$ is increasing in $|\theta|$.

[25]For a farmer who receives no signals, his market selection is based on the comparison between $E(\hat{\pi}_0(\theta, l))$ and $E(\hat{\pi}_0(\theta, r))$, which does not depend on (x_l, x_r). However, after he selects the market to sell in, his ex-post expected profit depends on the market selection of other farmers, which depends on (x_l, x_r) via the threshold $\tau^{(\delta)}(x_l, x_r)$.

3. $\Pi^{(\delta)}(\theta) > \Pi_0^{(\delta)}(\theta)$.

4. $\Pi^{(\delta)}(\theta) - \Pi_0^{(\delta)}(\theta)$ *is decreasing in* $|\theta|$.

The results stated in Lemma 7 is depicted in Figure 3. Specifically, when a farmer is located close to the origin so that $|\theta|$ is small (i.e., he is located far away from either markets), he incurs a higher transportation cost regardless of the market he sells in. As such, his expected profit is lower than other farmers who are located farther away from the origin so that $|\theta|$ is large (i.e., who are located near one of the markets). This explains why the ex-ante expected profit is increasing in $|\theta|$.

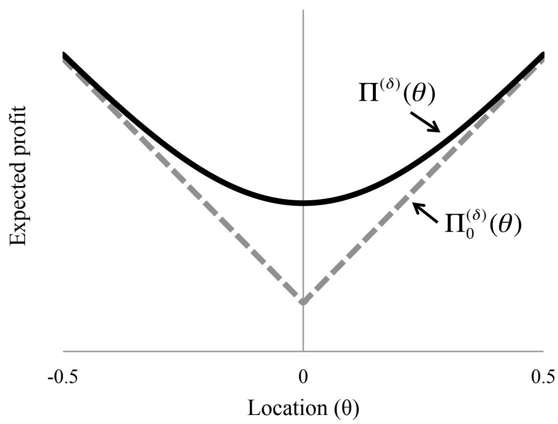


Figure 3: Farmer's ex-ante expected profit.

Next, by noting that a farmer can use the market signals to make better informed market selection decision, statement 3 of Lemma 7 reveals that each farmer can earn a higher ex-ante expected profit when he receives signals. Finally, because of the inherent transportation cost, farmers who are located near a particular market would sell in the nearby market unless the signals suggest the market price in the farther away market is much higher. This observation explains statement 4 of Lemma 7: market signals are less beneficial to those farmers who are located near a particular market (i.e., when $|\theta|$ is large).

3.3.3 Farmers' Expected Total Profit under Provision Policy δ

By using the farmer's ex-post expected profit function given in (11) and (12) along with the threshold $\tau^{(\delta)} \in [-K, K]$ associated with policy $\delta = (K, \rho)$ and Figure 2, we can determine the (ex-post) expected total profit of all farmers, $w^{(\delta)}(x_l, x_r)$, where:

$$w^{(\delta)}(x_l, x_r) = \int_{-0.5}^{0.5} \pi_0^{(\delta)}(\theta; x_l, x_r)\mathrm{d}\theta + \rho \int_{-K}^{K} (\pi^{(\delta)}(\theta; x_l, x_r) - \pi_0^{(\delta)}(\theta; x_l, x_r))\mathrm{d}\theta. \quad (14)$$

From this, we can determine the (ex-ante) expected total profit of all farmers, $W^{(\delta)}$, where:

$$W^{(\delta)} = E_{(x_l,x_r)}\,[w^{(\delta)}(x_l,x_r)]. \tag{15}$$

3.4 Analysis: Comparisons of Provision Policies

By examining the (ex-post) and (ex-ante) expected total profit functions given in (14) and (15), we now examine the implications of those three benchmark provision policies (F0), (F1), and (C), as well as the other two policies (Fρ) and (Kρ).

3.4.1 Benchmark Provision Policies

To begin, recall from Lemma 6 that all farmers who receive no signals will select the market to sell in according to the threshold associated with the No Information Policy $\tau^{(F0)} = 0$. We now determine the thresholds associated with the other two benchmark policies (C) and (F1). Under policy (F1), $\delta = (K = 0.5, \rho = 1)$. By substituting $K = 0.5$ and $\rho = 1$ into (10), we get:

$$\tau^{(F1)}(x_l,x_r) = \max\{-0.5, \min\{0.5, \frac{1}{2(b+t)} \cdot \frac{\beta}{\alpha+\beta} \cdot (x_l - x_r)\}\}. \tag{16}$$

Next, under policy (C), the government can first use the realized market signals (x_l, x_r), (11), (12), and (14) to determine the (ex-post) expected total profit $w^{(C)}(\tau; x_l, x_r)$ associated with any threshold τ by setting $K = 0.5$ and $\rho = 1$.[26] Then the government determines the threshold $\tau^{(C)}$ that maximizes $w^{(C)}(\tau; x_l, x_r)$ (i.e., $\tau^{(C)}(x_l, x_r) = \arg\max\{w^{(C)}(\tau; x_l, x_r) : \tau \in [-0.5, 0.5]\}$), where

$$\tau^{(C)}(x_l,x_r) = \max\{-0.5, \min\{0.5, \frac{1}{2(2b+t)} \cdot \frac{\beta}{\alpha+\beta} \cdot (x_l - x_r)\}\}. \tag{17}$$

Finally, under the centralized control policy (C), the government asks each farmer to follow the threshold rule according to $\tau^{(C)}(x_l, x_r)$.

By using the fact that $\tau^{(F0)} = 0$ along with (16) and (17), we establish the following lemma:

Lemma 8. *When $x_l > x_r$, $0 = \tau^{(F0)} < \tau^{(C)} \leq \tau^{(F1)}$. When $x_l < x_r$, $0 = \tau^{(F0)} > \tau^{(C)} \geq \tau^{(F1)}$.*

While the Lemma 8 is established algebraically, it can be interpreted by using the aforementioned notion of "switchers" as depicted in Figure 2. Specifically, Lemma 8 reveals

[26] The government should choose a threshold structure. Otherwise, we can always find a pair of farmers who sell in different markets, such that if they exchange the markets they sell in, famers' total profit becomes higher.

that the number of switchers under policy (F1) is higher than that under policy (C) with centralized control. To explain this result, suppose that $x_l > x_r$. Under policy (F1), each farmer uses information to select the market selfishly without any concern of the total profit. Therefore, starting from the origin 0, each farmer with $\theta > 0$ will "switch" from the nearby market r to market l that is farther away until $\theta = \tau^{(F1)}$. By doing so, each switcher will earn a higher profit, but the farmers' total profit may decrease because of the negative externality he exerts on others (due to extra selling quantity in the market that is farther away). By noting the fact that this negative externality is managed carefully under policy (C) when the market selection is centrally controlled by the government, the number of switchers associated with policy (C) will be lower than that of policy (F1). This explains why $0 = \tau^{(F0)} < \tau^{(C)} \leq \tau^{(F1)}$ when $x_l > x_r$.

By using the thresholds associated with all three benchmark policies as stated above, we can use (14) to compare the ex-post expected total profits in the following proposition.[27]

Proposition 12. *For any realized signals (x_l, x_r), we can compare the ex-post expected total profits associated with policies (C), (F1), and (F0) to examine the following issues:*

1. *Value of Centralized Control:* $w^{(C)} - w^{(F1)} = (2b + t) \cdot (\tau^{(F1)} - \tau^{(C)})^2 \geq 0.$
2. *Value of Information:* $w^{(F1)} - w^{(F0)} = t \cdot (\tau^{(F1)} - \tau^{(F0)})^2 = t(\tau^{(F1)})^2 \geq 0.$

The first statement of Proposition 12 indicates that, when all farmers receive signals under policies (F1) and (C), the government can improve farmers' "ex-post" expected total profit by controlling the market selection of each farmer centrally. Also, this improvement is based on the square of the distance between two thresholds $\tau^{(F1)}$ and $\tau^{(C)}$, which is equivalent to the square of the difference in the number of switchers between policies $(F1)$ and (C). The second statement reveals that, relative to the case of no information, providing signals to all farmers can improve farmers' "ex-post" expected total profit and this improvement is based on the square of the distance between two thresholds $\tau^{(F1)}$ and $\tau^{(F0)} = 0$, which equals to the square of the number of switchers under policy $(F1)$.

Because Proposition 12 holds for any realized signals, we can use the "sample path analysis" to argue that controlling farmer's market selection centrally and providing information to farmers will improve farmers' "ex-ante" expected total profit. Also, by noting that $\tau^{(F1)}$ and $\tau^{(C)}$ given in (16) and (17) depend on (x_l, x_r), we can use the results as stated in Proposition 12 to compare the ex-ante expected total profits associated with policies (C), (F1) and (F0) by computing: $W^{(C)} - W^{(F1)} = E_{(x_l,x_r)}[w^{(C)} - w^{(F1)}]$ and $W^{(F1)} - W^{(F0)} = E_{(x_l,x_r)}[w^{(F1)} - w^{(F0)}]$.

Proposition 13. *By comparing the farmers' (ex-ante) expected total profits under three benchmark policies, we can examine the following issues:*

[27] For ease of exposition, we shall examine the case when b, t, or α is high enough or β is low enough so that $\tau^{(C)}$ and $\tau^{(F1)}$ lie within $(-0.5, 0.5)$ almost surely. However, when the thresholds are truncated at ± 0.5, the expressions are slightly different but the qualitative characteristics remain the same.

1. *Value of Centralized Control:* $W^{(C)} - W^{(F1)} = \frac{b^2}{2(b+t)^2(2b+t)} \frac{\beta}{\alpha(\alpha+\beta)} > 0$. *The value of centralized control is decreasing in* α *and* t*, and increasing in* β.

2. *Value of Information:* $W^{(F1)} - W^{(F0)} = \frac{t}{2(b+t)^2} \frac{\beta}{\alpha(\alpha+\beta)} > 0$. *The value of information is decreasing in* α *and* b*, and increasing in* β.

Proposition 13 has the following implications. Information provision and centralized control both become more valuable when market conditions are more uncertain (i.e., when α is decreasing), and when signals are more precise (i.e., when β is increasing). Also, recall from Proposition 12 that the value of centralized control is increasing in $(\tau^{(F1)} - \tau^{(C)})^2$. When the transportation cost t becomes higher, farmers are more concerned about transportation cost than the market price when deciding which market to sell in. As such, $|\tau^{(F1)} - \tau^{(C)}|$ becomes smaller. This explains why the value of centralized control is decreasing in t. Finally, when price elasticity b is large, the total quantity to be sold in each market outweighs the impact of market signals on the market price. As such, information has less value when b is large.

In summary, we find that controlling farmer's market selection centrally can improve farmers' (ex-ante) expected total profit. More importantly, providing market information will improve farmers' (ex-ante) expected total profit. This result motivates us to examine whether it is optimal to provide information to all farmers. In other words, would the Full Information with Decentralized Control Policy (F1) be the optimal provision policy? We examine this question next.

3.4.2 Partial Intensity Policy (F$\boldsymbol{\rho}$)

To examine whether the government should provide information to all farmers, we now examine policy (Fρ) that generalizes policy (F1). Under policy (Fρ), $K = 0.5$ and ρ is a decision variable, and policy (Fρ) becomes policy (F1) when $\rho = 1$. Apply (10) along with the fact that $K = 0.5$, the threshold corresponding to policy (Fρ) is given as:

$$\tau^{(F\rho)}(x_l, x_r) \quad = \quad \max\{-0.5, \min\{0.5, \frac{1}{2(\rho b + t)} \cdot \frac{\beta}{\alpha+\beta} \cdot (x_l - x_r)\}\}. \tag{18}$$

Compare the threshold $\tau^{(F\rho)}(x_l, x_r)$ under policy (Fρ) along with the threshold $\tau^{(F1)}(x_l, x_r)$ given in (16), it is easy to check that $\tau^{(F\rho)}(x_l, x_r) \geq \tau^{(F1)}(x_l, x_r)$ when $x_l > x_r$. However, by noting that ρ percentage of farmers receive signals who will select the market according to threshold $\tau^{(F\rho)}(x_l, x_r)$ and $(1-\rho)$ percentage of farmers receive no signals who will select the market according to threshold $\tau^{(F0)} = 0$, the number of switchers under policy (Fρ) is equal to $\rho \cdot |\tau^{(F\rho)}(x_l, x_r)| \leq |\tau^{(F1)}(x_l, x_r)|$ for any realized (x_l, x_r). It follows from Proposition 12 and Lemma 8 that farmers' ex-post expected total profit under policy (F1) is lower than that of under policy (C) because there are too many switchers under policy (F1). Therefore, it is possible for the government to improve farmers' total profit by selecting $\rho < 1$ so that the number of switchers under policy (Fρ) is smaller than that of under policy (F1). We first substitute threshold $\tau^{(F\rho)}$ into (15) to determine $W^{(F\rho)}$; i.e., the ex-ante expected total profit under policy (Fρ). Then we determine the optimal $\rho^* = \arg\max\{W^{(F\rho)} : \rho \in [0,1]\}$ and establish the following results:

Proposition 14. *Under the provision policy (Fρ), it is possible to have $\rho^* < 1$. For instance, if b, t, or α is large enough or β is small enough so that $\tau^{(F\rho)}$ lies within $(-0.5, 0.5)$ almost surely and if $t < b$, then $\rho^* = \frac{t}{b} < 1$.*

Proposition 14 reveals that, to maximize farmers' ex-ante expected total profit, it may not be optimal to provide information to all farmers. This result is different from the results obtained by Chen et al. (2013) and Morris and Shin (2002) that support distributing information to all farmers. There are two key factors that drive this key result. The first factor is caused by the fact that we consider two potential markets for heterogeneous farmers to sell in. In this case, providing information to more farmers may not improve farmers' total profit. To elaborate, consider the case when farmers receive no information. In this case, each farmer will sell in the nearby market and the sales quantity in each market is identical (due to symmetry). Now, suppose the government provides information to all farmers. Then each farmer will selfishly choose the market to increase his own profit. Consequently, as stated in the intuition of Lemma 8, it is possible that farmers' total profit is reduced by the fact that too many farmers selfishly switch to sell in the farther away market without considering the impact on the profit of other farmers.

The second factor is caused by the fact that farmers are price takers and they have no control of the market price as individuals. In this situation, when a farmer selfishly selects the market that would yield a higher profit for himself, he creates serious negative externality that affects other farmers' profit[28]. Due to the negative externality, it may not be optimal to provide information to all farmers. Even though our model deals with uncertain market condition and noisy market signals in a different context, our result resembles a well known result in traffic equilibrium that is known as the Braess's paradox. Specifically, Braess states that, when the drivers choose their route selfishly, the overall system performance can deteriorate by adding one extra road to the network.

In summary, Proposition 14 reveals that, when information is disseminated to the entire range $[-0.5, 0.5]$, it may not be optimal to set the intensity level $\rho = 1$ so that all farmers receive signals. This result makes us wonder if the government should disseminate information to the entire range. We examine this issue next.

3.4.3 General Policy (Kρ)

We now examine the general policy (Kρ) under which the government selects the range $[-K, K]$ and the intensity level ρ. To determine the optimal general policy, the government

[28]On the contrary, in the models considered by Chen et al. (2013) and Morris and Shin (2002), farmers are price setters who can control the market price by selecting their production quantity. Consequently, the negative externality imposed by each farmer is reduced because each farmer's decision will have direct impact on his own profit.

solves the following problem: $\max_{K\in[0,0.5],\rho\in[0,1]} W^{(K\rho)}$, where $W^{(K\rho)}$ can be determined by using (10) and (15). The following proposition summarizes our findings:

Proposition 15. *Under the general provision policy ($K\rho$), the optimal value K^* and ρ^* are given as follows:*

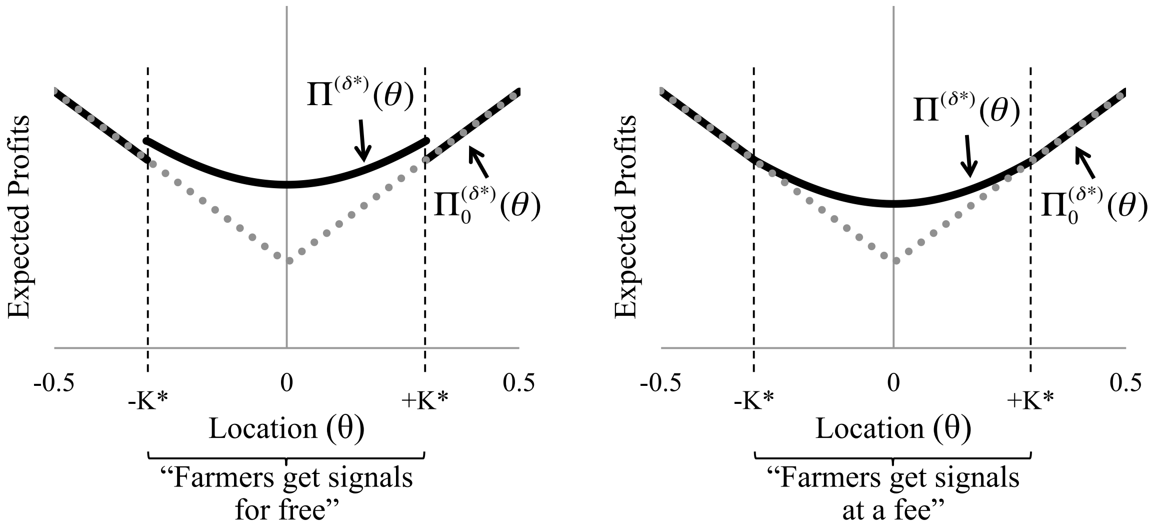
1. *The optimal intensity level $\rho^* = 1$.*
2. *The optimal range is specified by $[-K^*, K^*]$, where $K^* = \min\{\bar{K}, 0.5\}$ and $\bar{K}$ maximizes $W^{(K\rho)}$ for the case when $\rho = 1$. Also, K^* is increasing in β, and decreasing in α and b.*

Akin to Proposition 14, Proposition 15 suggests that it might not be optimal to provide information to all farmers. However, Proposition 15 states that it is more beneficial (in terms of improving farmers' ex-ante expected total profit) by limiting information access over a targeted range $[-K^*, K^*]$ at a full intensity level $\rho^* = 1$. To explain this result, observe from Lemma 7 that farmers located near the origin would benefit more from receiving signals than those framers who are located far from the origin. This observation implies that, instead of disseminating information to farmers over the entire range $[-0.5, 0.5]$ at a lower intensity level $\rho < 1$ under policy (Fρ), it is actually better to limit information access to a target range $[-K^*, K^*]$ with full intensity $\rho^* = 1$.

3.4.4 Perceived Unfairness and Nominal Fees

While the optimal general policy (K^*, ρ^*) enables the government to maximize farmers' ex-ante expected total profit, some farmers may object to this policy because of the perceived unfairness. To elaborate, suppose the government offers information according to the optimal policy $\delta^* = (K^*, \rho^*)$ as given in Proposition 15. First, for any farmer who lies outside the range $[-K^*, K^*]$, he will receive no signals and will earn an ex-ante expected profit $\Pi_0^{(\delta^*)}(\theta)$. Second, for any farmer who lies within the range $[-K^*, K^*]$, he will receive signals and will earn an ex-ante expected profit $\Pi^{(\delta^*)}(\theta)$. Therefore, following from Lemma 7 and Figure 3, we can trace the ex-ante expected profit of each farmer under the optimal general policy (K^*, ρ^*) as depicted in Figure 4(a).

Because information provision is truncated at $-K^*$ and K^*, the profit function is discontinuous. Also, farmers who located just outside the range $[-K^*, K^*]$ would feel being treated unfairly by the government due to "income reversal": they now earn less than their neighbors who are located just inside the range. To mitigate this perceived unfairness, we show that the government can offer signal access to all farmers at a nominal fee $p(K^*)$ so that the farmers will adopt the intended optimal provision policy willingly, and the resulting farmer's profit function is a continuous function without income reversal. The way to implement this fee-based service is as follows. First, the government announces that each farmer can gain access to market information at a nominal fee $p(K^*)$. Second, recall from



(a) Free Service. (b) Fee-based Service.

Figure 4: Solid line: farmer's ex-ante expected profit when signals are provided (a) free of charge; or (b) at a nominal fee. Dashed line: farmer's ex-ante expected profit when there is no information provision.

Lemma 7 that information is more valuable to farmers who are located near the origin and less valuable to farmers who are located far away from the origin. Therefore, the government can set $p(K^*)$ so that: (1) farmers who are located within the range $(-K^*, K^*)$ will purchase the signals; (2) farmers who are located outside the range $[-K^*, K^*]$ will not purchase the signals; and (3) farmers who are located at $-K^*$ and K^* are indifferent. The last observation reveals that we can use $\Pi_0^{(\delta^*)}(\theta)$ and $\Pi^{(\delta^*)}(\theta)$ given in Lemma 7 to determine $p(K^*)$, where:

$$\Pi_0^{(\delta^*)}(K^*) = \Pi^{(\delta^*)}(K^*) - p(K^*).$$

By charging a nominal fee $p(K^*)$, the government can implement the optimal policy ($K^*, \rho^* = 1$) while the resulting farmer's profit function is a continuous function without income reversal (Figure 4(b)). Also, the fee collected by the government can be used as farm subsidies for all farmers or as a way to recoup some of the investment cost associated with information acquisition and information dissemination. More importantly, relative to the case without information, each farmer who used to earn a lower profit due to his disadvantaged location can now earn a higher profit. Therefore, implementing the optimal policy with a nominal fee enables the government to reduce income inequality without incurring perceived unfairness (in terms of restricted information access and income reversal).

3.5 Extensions

In this subsection, we first extend our analysis to examine the case when information is disseminated by a for-profit company (instead of the government). Then we extend our base model to examine the following issues: (1) the impact of signal precision β; (2) the implication of correlated markets; and (3) the implication of the uniform distribution of farmers.

3.5.1 Information dissemination through a for-profit company

We now consider the case when information is disseminated by a for-profit company. This company can determine the advertising intensity ρ_c to the full range $[-0.5, 0.5]$[29], and it can also determine the service fee p_c. Each farmer who receives the advertisement can decide whether to purchase the signals. By using the same argument as presented in the last subsection, we can show that the company's strategy is equivalent to determining its provision policy $\delta = (K_c, \rho_c)$ and setting a fee $p_c(K_c, \rho_c)$ so that all farmers within the range $(-K_c, K_c)$ who receive the advertisement will purchase the signals; and no farmers outside the range $[-K_c, K_c]$ will purchase the signals (regardless of whether they receive the advertisement or not). Also, upon receiving the advertisement, farmers who are located at $-K_c$ and K_c are indifferent between purchasing the signals or not.

By noting that $2\rho_c K_c$ farmers will purchase the signals, the company can determine the optimal policy that maximizes its own profit by solving the following problem:

$$\max_{(K_c, \rho_c)} \{p_c(K_c, \rho_c) \cdot 2\rho_c K_c\},$$

subject to $\Pi_0^{(\delta)}(K_c) = \Pi^{(\delta)}(K_c) - p_c(K_c, \rho_c)$. The constraint ensures that farmers who are located at $-K_c$ and K_c are indifferent between purchasing the signals or not. By solving the company's problem, we obtain the following results:

Proposition 16. *When information is disseminated through a for-profit company, the company's optimal provision policy $\delta^* = (K_c^*, \rho_c^*)$ can be described as follows:*

1. *The optimal intensity level $\rho_c^* = 1$.*

2. *The optimal range is specified by $[-K_c^*, K_c^*]$, where $K_c^* = \min\{\bar{K}_c, 0.5\}$ and $\bar{K}_c$ is the solution to the company's problem.*

3. *K_c^* is increasing in β, and decreasing in α, b, and t.*

4. *$K_c^* \leq K^*$.*

[29] It is easy to show that the company's strategy does not change even if it can also decide the range $R_c \subseteq [-0.5, 0.5]$ to advertise in.

Proposition 16 resembles the results as stated in Proposition 15. However, it reveals one important fact. If the government leaves the company to its own devices, the company will disseminate information to a small range of farmers (because $K_c^* \leq K^*$). Consequently, the situation deviates from the social optimum. (That is, farmers' expected total earning from selling their crop is not maximized. Note that this is the total surplus shared by farmers and the company because farmers use part of it to purchase the signals.) To fix this problem, the government should provide incentive (e.g., tax credits) to induce the company to set $p_c^* = p(K^*)$ (or equivalently, $K_c^* = K^*$) so that farmers' expected total earning from selling their crop is maximized. We shall defer the analysis of different incentive schemes as future research.

3.5.2 Implication of signal precision

When farmers have private signals, Chen and Tang (2013) show that disseminating public signals to farmers can be hurtful. To reduce this harmful effect of public signals, Chen and Tang (2013) argued that one should consider: (a) reducing the number of farmers who receive public signals; or (b) reducing the precision of the public signals. While we do not consider private signals in our model, Proposition 15 reveals that the government can maximize farmers' ex-ante expected total profit by reducing information access to farmers within $[-K^*, K^*]$. Therefore, we wonder if the government should also reduce the precision of the public signals. The following proposition clarifies this issue: the government should always provide the most precise signals as possible.

Proposition 17. *Under any provision policy $\delta = (K, \rho)$, farmers' ex-ante expected total profit $W^{(\delta)}$ is increasing in the precision level of the signals β.*

3.5.3 Correlated Markets

In our model, we assume that the market price $p_l = A + u_l - bq_l$ and $p_r = A + u_r - bq_r$ are independent. We now relax this assumption by considering the case when (u_l, u_r) are bi-variate normal with the same mean 0, the same variance σ^2, and a correlation coefficient g.

To analyze this extension, we can use exactly the same approach as before to obtain the same results by simply replacing $\alpha \equiv 1/\sigma^2$ with $\alpha' \equiv \frac{\alpha}{1-g}$. By noting that α' is increasing in the correlation coefficient g, we can apply Proposition 15 to show that K^* is decreasing in g. Therefore, the optimal range for disseminating information $[-K^*, K^*]$ decreases as the market intercepts become more positively correlated (i.e., as g increases).

To explain this result, let us use the fact that each farmer's market selection hinges on the market price difference that is measured in terms of $(u_l - u_r)$, where $(u_l - u_r)$ has a mean 0 and a variance $2\frac{1-g}{\alpha}$. Therefore, when the markets are more positively correlated (i.e., as g increases), the variance of $(u_l - u_r)$ becomes smaller. As such, the numbers of switchers under both policies (C) and (F1) decrease. Therefore, to discourage unproductive switching, it is optimal for the government to reduce the optimal range for disseminating information $[-K^*, K^*]$ when market intercepts become more positively correlated.

3.5.4 A more general distribution of farmers

In our model, we adopt the Hotelling model by assuming that all farmers are uniformly distributed over $[-0.5, 0.5]$. We now relax this assumption by considering a more general distribution of farmers. Specifically, we assume that the density of farmers at $\theta \in [-0.5, 0.5]$ is given as $h(\theta)$, where $0 < h(\theta) < \infty$ and $h(\theta)$ is symmetric about the origin 0 so that $h(\theta) = h(-\theta)$.

Proposition 18. *When farmers are distributed according to a symmetric density function $h(\theta)$ over $[-0.5, 0.5]$, we obtain the following results:*

1. *The (ex-post) expected total profits and thresholds associated with policies (C), (F1), and (F0) satisfy:*
   (a) *Value of Centralized Control: $w^{(C)} - w^{(F1)} \geq 0$.*
   (b) *When $x_l > x_r$, $0 = \tau^{(F0)} < \tau^{(C)} \leq \tau^{(F1)}$. When $x_l < x_r$, $0 = \tau^{(F0)} > \tau^{(C)} \geq \tau^{(F1)}$.*
2. *The (ex-ante) expected total profits associated with policies (C), (F1), and (F0) satisfy:*
   (a) *Value of Centralized Control: $W^{(C)} - W^{(F1)} > 0$.*
   (b) *Value of Information: $W^{(F1)} - W^{(F0)} > 0$.*
3. *Under the provision policy ($F\rho$), it is possible to have $\rho^* < 1$.*

Proposition 18 reveals that, even when the farmers are generally distributed over $[-0.5, 0.5]$, the qualitative results as stated in Lemma 8, statement 1 in Proposition 12, and Propositions 13 and 14 continue to hold. Therefore, even when farmers are not uniformly distributed over $[-0.5, 0.5]$, farmers' expected total profit is still hurt by the fact that there are too many switchers under policy (F1), and providing information to all farmers may not be optimal.

4 Role of Exchangeable Tickets in the Optimal Menu Design for Airline Tickets

While menus are offered pervasively in the airline industry, the nature of the menu design remains unclear. For example, the menu offerings differ substantially across companies and across different air routes within the same company. To have a better understanding in the menu offerings, we build a stylized model to examine a monopolistic seller's optimal menu design with three types of tickets: refundable, nonrefundable, and exchangeable tickets. The role of exchangeable tickets is of particular interest as it is less explored in the academic literature, but is prevalent in practice. We show that for the seller, the exchangeable ticket is inherently more profitable than the other two types of tickets; thus, it is an indispensable element of the optimal menu. On the other hand, increasing the flexibility of exchangeable tickets may dampen the seller's profitability. The analysis also reveals that cancellation fees are used as instruments to adjust the differences between consumers' willingness to pay, and when they are adopted, the seller has less incentive to sell nonrefundable tickets. We provide a possible explanation for why menu offerings, while prevalent in the airline industry, are so scant in commodity goods markets.

The remainder of this chapter is organized as follows. Subsection 4.1 is the literature review. Subsection 4.2 lays out the model settings, and Subsection 4.3 presents the analysis, results, and discussions of the basic model. We study the extensions in Subsection 4.4 and Subsection 4.5. All the proofs are relegated to the appendix.

4.1 Literature Review

Valuation uncertainty is a critical concern for operations and marketing researchers and practitioners. Therefore, various instruments have been proposed to mitigate its impact (see, e.g., Swinney (2011), Crocker and Letizia (2013), and Cui et al. (2014b)). One pervasively adopted instrument is allowing product returns. In Davis et al. (1995), the authors develop a stylized model to determine conditions under which money-back guarantees work best in terms of enhancing profits and social welfare. Hess et al. (1996) argue that some consumers purchase the products with no intention of keeping them, and the seller can control this kind of behavior in a profitable way by charging a nonrefundable restocking fee. Chu et al. (1998) compare three refund policies: no questions asked, no refunds, and verifiable problems only. They find that no questions asked is most efficient in handling the consumer opportunism. Hsiao and Chen (2014) also compare two widely used refund policies, money-back guarantee and hassle-free policies. The optimal return policy designs are studied under various circumstances (see, e.g., Shulman et al. (2011), Guo (2009), Hsiao and Chen (2012), Shulman et al. (2009), and Shulman et al. (2010)).

While these papers focus on a single return policy, we examine the optimal "menu" design of return policies. In this light, our work is also closely related to the literature on "principal-agent" problems. In that literature, as consumers (agents) from different segments are ex-ante indistinguishable to the seller (principal), a menu should be offered to the consumers to induce them to disclose their types. This principal-agent framework has been applied in various areas, including supply chain contracting (Li and Scheller-Wolf (2011) and Ha (2001)), price discrimination (Moorthy (1984) and Yayla-Küllü et al. (2011)), procurement and production planning (Yang et al. (2009)), and dynamic contracting (Kakade et al. (2013) and Zhang and Zenios (2008)).

In this work, tickets are essentially the same, and the menu consists of different flexibilities instead of different qualities. Therefore, this work is related to Cui et al. (2014a), in which a regular product with conditional upgrade is provided as an extra choice. This work is not the first one combining the menu design and return policies. In Courty and Li (2000), the authors study the optimal menu of refund contracts. In their model, the seller provides contracts with different prices and refund amounts for consumers to choose from. Also, in Heiman et al. (2002), the authors model the money back guarantees (MBGs) as options and study the optimal menu design. In their model, the seller can choose to sell all the products without MBGs, to sell all the products with MBGs, or to sell the products and MBGs separately. However, all the aforementioned papers do not examine the "exchangeable" return policy we consider in this chapter. To the best of our knowledge, this work serves as the first-cut analytical study of the "exchangeable" return policy.

4.2 Basic Model

We consider a stylized model in which a monopolistic seller intends to sell his tickets to heterogeneous consumers. We adopt the following model setting.

Types of tickets. At the time of ticket reservation, potential consumers are unable to ex ante know whether they should change their schedule or not in the future. Given the presence of consumers' valuation uncertainty, a natural solution for the seller is to offer flexible tickets. As mentioned in the introduction, we consider three types of tickets. If a consumer has purchased a *refundable* ticket but eventually needs to change her plan, she can cancel this ticket and get the refund.[30] If a consumer has purchased a *nonrefundable* ticket but eventually needs to change her plan, this ticket becomes useless for her. She cannot get anything from this ticket.[31]

The third kind of tickets is the *exchangeable* ticket. If a consumer has purchased an exchangeable ticket but eventually needs to change her plan, she can cancel this ticket and get some *credits*, which can be used to purchase another ticket.[32] The usage of these credits

[30]This type of tickets corresponds to the *Unrestricted Fare* and the *Flexible Fare* in United Airlines, the *Business Select Fare* and the *Anytime Fare* in Southwest Airlines, and the *Flexible Fare* in US Airways. Note that United Airlines (and likewise for Southwest Airlines) sets some minor differences between its tickets to induce finer price discriminations. These differences are less relevant to our research questions. Thus, we omit them in our model.

[31]More precisely, this type of tickets is non-refundable and non-exchangeable. This corresponds to Delta Air Lines' bulk fare or tour fare tickets issued through certain third-party websites and travel agents, and the *Name Your Own Price* reservations on Priceline.com.

[32]This type of tickets is non-refundable but exchangeable. In the industry, this type of tickets is sometimes called the "non-refundable tickets" because the firms want to emphasize the non-refundable feature. However, in this work, we use the term "exchangeable ticket" because we want to emphasize that this ticket is non-refundable but exchangeable. This type of tickets is corresponding to the *Lowest Available Fare* in United Airlines, the *Wanna Get Away Fare* in Southwest Airlines, and the *Non-refundable Fare* in US Airways.

is typically restricted, in particular within a time window. For example, the airline company may specify a period "T = six months" such that the consumer can use these credits within six months. If it turns out that the consumer wants another ticket, the credits can be used as real money. This model setting captures the idea that for consumers, refundable tickets are more flexible than exchangeable tickets.

In this chapter, we assume that if a consumer wants to change her refundable (exchangeable) ticket, she will cancel it and then use the refunded money (credits) to purchase the new ticket. By this assumption, we can combine two situations: consumers want to make some minor changes to the original ticket, and consumers want to cancel the original ticket and reserve a new independent one (now or in the future). This assumption is without loss of generality when the fee of canceling the ticket is not higher than the fee of changing it. In practice, many companies (e.g., AirTran Airways, Southwest Airlines, US Airways, and Spirit Airlines) choose to charge the same fee upon canceling and upon changing the ticket. Because of this assumption, we only consider the *cancellation fees* the consumers should pay if they want to change or cancel their refundable or exchangeable tickets.

In the basic model, the seller is required to provide full refund in cash (or in credits) to the consumers if a refundable (or exchangeable) ticket is cancelled. In other words, the seller cannot charge any cancellation fee from the consumers who bought a refundable or exchangeable ticket and want to cancel it. We will relax this constraint in Subsection 4.4.

Consumers. Each consumer intends to purchase (at most) one ticket. Our model setting features three dimensions of heterogeneity, which will be introduced later. To provide the simplest possible ground for these heterogeneities, we assume that there are two consumer segments, high and low, and within each segment consumers are homogeneous. This two-segment setup facilitates the desired tractability and is commonly adopted in the vast literature (e.g. Davis et al. (1995), Gerstner and Holthausen (1986), Raju et al. (1990), Hsiao and Chen (2012), and Varian (1980)). The proportions of the high and low segments are respectively q_h and q_l, where $q_l + q_h = 1$ and $0 < q_h < 1$. A low-segment consumer obtains valuation V_l upon using the ticket, whereas a high-segment consumer obtains valuation V_h ($0 < V_l < V_h$).

When purchasing the ticket, a low-segment (high-segment) consumer has a pre-specified schedule in mind, but only with a probability α_l (α_h) she can follow her original plan in the future. If a consumer has purchased the ticket but eventually changes her plan, the ticket becomes useless and yields a null payoff to her. This uncertainty about potential schedule changes can be resolved only after the consumer has decided whether to purchase the ticket. In reality, consumers must book the tickets in advance in order to secure their seats, but there might be some last-minute change of plans before they take off.

If a consumer needs to change her plan, with some probability she will need to purchase another ticket within T. In this situation, she can use the refunded credits (if any). This demand for a new ticket might rise before canceling the ticket. (For example, the consumer still needs to take the trip, but she wants to change the boarding time.) It is also possible that the demand appears in the future. (For example, after the original trip is cancelled, the consumer finds that she needs to take another independent trip.) We set the aggregate probability of using the refunded credits (if any) to be β_i for a segment-i consumer, given that her original schedule is changed[33].

While in practice ticket selling is a truly dynamic continuous-time process, in this work we use a simplified two-period setting as follows. In the first period, consumers decide whether to purchase the tickets without knowing their true valuations (whether they should change their schedule or not). In the second period, after seeing the valuation realization, each consumer may cancel the ticket (if permitted by the seller). We abstract away consumers' hassle cost of canceling refundable or exchangeable tickets. This is because in the real world, usually consumers can cancel a refundable or exchangeable ticket through the Internet or by simply making a phone call.

Salvage value and capacity constraint on the tickets. If a ticket is cancelled in period 2, the seller can get a salvage value, s_2, by selling it to another consumer. This may be facilitated by last-minute deals for some bargain hunters, or through some third-party opaque selling channels. On the other hand, if a ticket is not sold at the beginning (period 1), the seller can get a salvage value, s_1, by selling it through other channels. One can think of this as allocating a block of unsold seats to the travel agents or some long-term collaborating companies. Alternatively, it is also possible that these seats can be used to supply the airline alliances (such as Star Alliance, SkyTeam, and Oneworld) in exchange of future favors (see, e.g., Hu et al. (2013)).

In the real world, the seller usually faces capacity constraints on airline tickets. In this work, we assume that tickets are enough to supply the consumers in question. In essence, they are the first choices for the seller before seeking other alternative channels or buyers. That is, they are the premium buyers that the seller should focus on. We further assume that $\alpha_h V_h > s_1$ and $\alpha_l V_l > s_1$. These assumptions rule out the trivial cases wherein the seller has the incentive not to sell the nonrefundable tickets to one or both segments. Also, these assumptions make sure that selling tickets can always increase the social welfare. It is worth mentioning that some unsold/cancelled tickets are in fact worthless if the capacity constraint is not binding when they are sold through other channels. However, in the selling stage (the first period), what we know is only the probability that the capacity constraint will eventually be binding. Hence, s_1 and s_2 are in fact the *expected* salvage values of unsold/cancelled tickets[34], and the capacity constraint is encoded in them.

[33] In fact, it is also possible that the consumers, who purchased another types of tickets and changed their plans, want to purchase another tickets from this seller. However, as long as these probabilities are less than $\{\beta_i\}'s$, this effect can be included in the extra cost of exchangeable tickets (we will introduce this later), and our results hold qualitatively. Note that this is a reasonable assumption because consumers have higher incentive to make further purchase if they have credits which will expire after certain time point.

[34] Note that s_1 and s_2 might depend on the number of unsold/cancelled tickets, but, even so, our results still hold qualitatively.

In reality some hassle costs may arise during the refund or exchange process. Nonetheless, this is typically on the airlines' side and can be easily incorporated in s_2. We assume that this cost is small so that $s_2 > 0$. We also assume that $s_1 > s_2$. Otherwise, the seller can keep the unsold tickets to period 2 and then sell those as the cancelled tickets. By doing so, he can generate a value higher than s_2 because in this case he need not absorb the hassle costs of the refund or exchange processes. Finally, without loss of generality, the costs of the tickets are normalized to zero.

Extra cost of exchanging tickets. If a consumer cancels an exchangeable ticket, and eventually wants to use the refunded credits to purchase another ticket, the seller needs to pay an extra cost c. The parameter includes the processing cost and the opportunity cost of the new ticket, which might otherwise be sold to another consumer. Here we assume that c is lower than the price of exchangeable tickets. This is because usually consumers are required to pay the price difference when the new ticket is more expensive than the cancelled one.[35]

Seller's policies. The seller is risk neutral and therefore aims at maximizing his expected payoff. He can decide in which form (or forms) he wants to sell the tickets. For instance, he can choose to sell two types of tickets, refundable and exchangeable tickets, at prices P_r and P_e, respectively, if doing so is optimal for him.

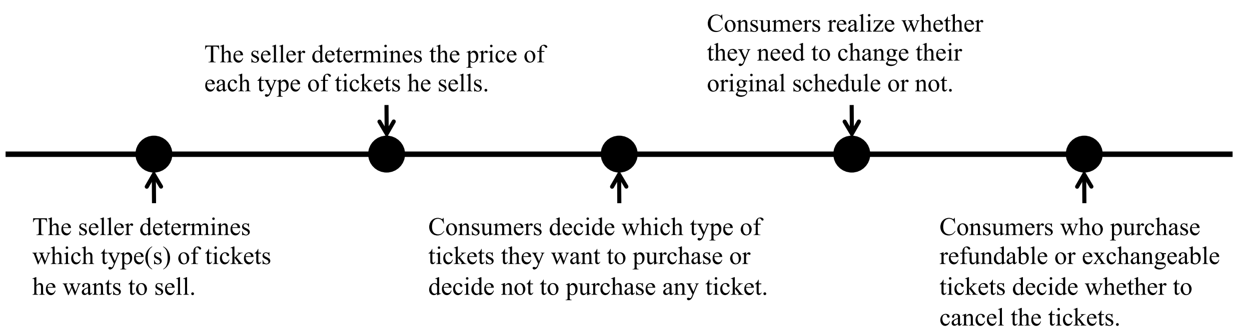


Figure 5: Sequence of events.

Timing. The sequence of events is as follows. 1) The seller determines which type(s) of tickets he wants to sell. 2) The seller determines the price of each type of tickets he sells. 3)

[35]Here, we assume that c is independent of the amount of credits (K) to illustrate our results more clearly. This assumption is reasonable in situations in which c only contains the processing cost, which does not depend on K. However, in some special situations, we should also count the opportunity cost. (e.g., when the capacity constraint of the new ticket is binding or when the consumer will purchase the new ticket even if she does not have these credits.) When c is a function of K, our results still hold (qualitatively) under the following assumptions: (1) $c(K) < K$ and (2) $c'(K) < 1$ (see A.3.9 for further discussions). Note that (1) holds because credits are less valuable than real money. Also, (2) holds because: (a) in any situation, it is unreasonable to have a cost increasing faster than K, and (b) in some situations, the opportunity cost of the new ticket is zero, so there is only the processing cost, which is independent of K.

Consumers decide whether to purchase the ticket. If multiple types of tickets are available, consumers should also decide which kind of tickets they want to buy. 4) Consumers realize whether they need to change their original schedule or not. 5) Consumers who purchase refundable or exchangeable tickets decide whether to cancel the tickets. We exclude the information asymmetry issue and risk aversion to rule out the two well-documented effects, signaling and insurance, in the economic and marketing literature.

Notation	Meaning
V_i	The segment-i consumers' utility upon using the ticket, where $i \in \{l, h\}$.
α_i	Probability of following the original plan for the segment-i consumers, where $i \in \{l, h\}$.
β_i	Probability of using the refunded credits (if any) for the segment-i consumers, given that they should change their original plan, where $i \in \{l, h\}$.
q_i	Proportion of the segment-i consumers, where $i \in \{l, h\}$.
s_j	Salvage value (for the seller) of tickets unsold/cancelled in period j, where $j \in \{1, 2\}$.
c	The extra cost to the seller when a consumer who cancelled an exchangeable ticket wants to purchase another ticket with the refunded credits.
P_k	The price of the type-k ticket, where $k \in \{r(\textit{refundable}), e(\textit{exchangeable}), n(\textit{nonrefundable})\}$.

Table 3: Summary of notation.

4.3 Analysis

In this subsection, we will examine the seller's optimal strategy and its intuition. We start with some preliminary analysis in Subsection 4.3.1. These preparations are helpful not only in finding the seller's optimal strategy but also in having better understanding of the characteristics of different types of tickets.

4.3.1 Preparation

By backward induction, before examining the seller's optimal strategy, we need to decide consumers' strategies first. We start with consumers' optimal reaction when they know whether they need to change their schedule or not, given that they have purchased a certain type of tickets. We can then calculate consumers' willingness to pay (WTP hereafter) for each type of tickets and their optimal choices when multiple types of tickets are available.

Nonrefundable ticket. If a segment-i consumer purchased a nonrefundable ticket and then wants to change her plan, she cannot get anything back. Therefore, her expected utility from purchasing this ticket is $\alpha_i V_i - P_n$. Her WTP for a nonrefundable ticket is $\alpha_i V_i$.

Refundable ticket. When a refundable ticket is cancelled, the consumer can get a refund P_r, which is the ticket price since the seller provides full refund. It can be shown that a segment-i consumer will not purchase the refundable ticket if $P_r > V_i$. On the other hand, if $P_r \leq V_i$, a segment-i consumer who purchases a refundable ticket will cancel it if and only if she needs to change her plan after the uncertainty is resolved. Hence, when $P_r \leq V_i$, a segment-i consumer's expected utility from buying the ticket is $\alpha_i V_i + (1 - \alpha_i) P_r - P_r$, and her WTP for a refundable ticket is V_i.

Exchangeable ticket. If an exchangeable ticket is cancelled, the consumer can get P_e credits, which can be used as real money if she wants to purchase another ticket. It can be shown that the segment-i consumers will not purchase the exchangeable tickets when $P_e > V_i$. On the other hand, when $P_e \leq V_i$, a segment-i consumer who purchases an exchangeable ticket will cancel it if and only if she needs to change her plan after the uncertainty is resolved. Hence, when $P_e \leq V_i$, a segment-i consumer's expected utility from buying the ticket is $\alpha_i V_i + (1 - \alpha_i)\beta_i P_e - P_e$. Her WTP for an exchangeable ticket is $\frac{\alpha_i V_i}{1-\beta_i+\alpha_i\beta_i}$. Note that $\beta_i P_e$ is her expected utility from the refunded credits. (With a probability β_i, she will use these credits to substitute for the same amount of money.) Similarly, if this consumer cancels a refundable ticket, she gets an expected utility P_r from the refunded "money", P_r. By considering consumers' expected utilities from refunded credits and money, different types of tickets become comparable. Also, this is the reason why consumers' expected utilities from buying refundable tickets do not depend on $\{\beta_i\}'s$.

From the above discussions, for consumers a refundable ticket is more flexible than an exchangeable ticket, which is more flexible than a nonrefundable ticket. We say that a ticket is *acceptable* for a consumer if its price is below or equal to her WTP. Apparently, if none of the tickets is acceptable, she chooses not to purchase any ticket. Amongst the acceptable tickets, a consumer shall choose the most favorable one (i.e., the one that yields the highest expected payoff). This then leads to various consumers' incentive compatible constraints, which are listed in the appendix.

Seller's profit. Given the consumers' strategies, now we can calculate the seller's profit of selling a segment-i consumer a specific type of tickets at a certain price, and the total surplus generated by this ticket.

Ticket Type	Seller's profit of selling a segment-i consumer this ticket at P_k ($k \in \{r, e, n\}$).	The total surplus (shared by the seller and consumers) generated by this ticket.
Nonrefundable	P_n	$\alpha_i V_i$
Refundable	$\alpha_i P_r + (1-\alpha_i)s_2$	$\alpha_i V_i + (1-\alpha_i)s_2$
Exchangeable	$P_e + (1-\alpha_i)(s_2 - \beta_i c)$	$\alpha_i V_i + (1-\alpha_i)s_2 + (1-\alpha_i)\beta_i(P_e - c)$

Table 4: Seller's profit of selling a segment-i consumer a specific type of tickets at a certain price, and the total surplus generated by this ticket.

From the above results, we have the following Lemma 9.

Lemma 9. *The profitability of exchangeable tickets is increasing in the ticket price, but the profitabilities of refundable tickets and nonrefundable tickets do not depend on the prices. Also, for the seller, exchangeable tickets are always intrinsically more profitable than refundable tickets, which are intrinsically more profitable than nonrefundable tickets.*

The intuition is as follows. Given the same amount of surplus the seller leaves to consumers, the seller earns more profits if the tickets can create more benefit shared by these two parties.[36] The surplus created by a nonrefundable ticket comes entirely from its realized value for consumers. On the other hand, the surplus created by a refundable ticket contains two parts - its realized value for consumers and its salvage value for the seller when it is cancelled. Recall that it is cancelled if and only if the consumer who purchased it needs to change her plan and has a null utility upon keeping it. This salvage value for the seller is positive; thus, refundable tickets are intrinsically more profitable than nonrefundable tickets. Note that these surpluses do not depend on ticket prices.

Finally, compared to refundable tickets, there is one more source from which exchangeable tickets can generate surplus (shared by the seller and the consumers): exchangeable tickets help the seller lock in consumers' money. When a consumer cancelled an exchangeable ticket and wants to purchase another ticket with the refunded credits, she needs to purchase it from this particular seller. In contrast, if a consumer cancelled a refundable ticket and wants another ticket, she can purchase it from other sellers[37]. This effect is increasing in the ticket price, and it is always positive. As a result, exchangeable tickets are always intrinsically more profitable than refundable tickets.

[36] Recall that $c < P_e$ and $s_2 > 0$.

[37] Note that although it is possible that a consumer, who purchased a refundable or nonrefundable ticket and changed her plan, chooses to purchase the new ticket from this seller, we account for this possibility in c, and our results are robust. The underlying reason is that consumers have higher incentive to make further purchase if they have credits which will expire after certain time point.

4.3.2 Results and Discussions

With these preparations, now we can characterize the optimal strategy for the seller. The seller can decide the type(s) of tickets he sells, and also the corresponding price(s). From now on, we use "the T_hT_l strategy" to refer to the optimal one among the seller's strategies which induce the segment-i consumers to purchase the T_i type tickets, where $i \in \{l, h\}$ and $T_l, T_h \in \{R(\textit{Refundable}), E(\textit{Exchangeable}), N(\textit{Nonrefundable}), O(\textit{no tickets})\}$. For example, the EO strategy is the optimal one among the strategies which make the high-segment consumers purchase the exchangeable tickets and the low-segment consumers not purchase any tickets.

We now investigate the structural properties of the optimal strategy for the seller.

Lemma 10. *The seller always sells exchangeable tickets to at least one segment.*

This lemma can be generalized to the following form: When there are multiple segments of consumers and multiple types of tickets, the seller always sells the type of tickets which can create the most surplus to at least one segment. This result is a direct implication of Lemma 9 and it coincides with our observation that the exchangeable tickets are used more frequently by the sellers than the other two types of tickets in the real world. For example, United Airlines, Southwest Airlines, American Airlines, AirTran Airways, and Spirit Airlines all provide exchangeable tickets, but some of them do not provide the other two types of tickets.

Because exchangeable tickets are the most profitable type of tickets, the seller's optimal strategy largely hinges on which segment has a higher WTP for exchangeable tickets. As a result, we split all possible situations into two scenarios as in the following definition.

Definition 1. *All the possible situations are categorized into the following two scenarios:*

Scenario H: The high-segment consumers have a higher WTP for exchangeable tickets than the low-segment consumers. (That is, $\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} > \frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l}$*.)*

Scenario L: The high-segment consumers do not have a higher WTP for exchangeable tickets than the low-segment consumers. (That is, $\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} \leq \frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l}$*.)*

Note that a segment-i consumer's WTP for exchangeable tickets is increasing in β_i, the probability that the refunded credits will be used. Hence, we can find that a situation with a higher β_h (or a lower β_l) are more likely to be in Scenario H. The next proposition provides a full characterization of the seller's possible optimal strategies.

Proposition 19. *In Scenario H, the seller should choose his optimal strategy from the following four strategies, depending on which one is feasible and gives him the highest profit.*

The Strategy	Feasibility Condition	Profit
EE	always feasible	$q_h[\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_h)(s_2 - \beta_h c)]$ $+q_l[\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_l)(s_2 - \beta_l c)]$
EO	always feasible	$q_h[\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)]$ $+q_l s_1$
ER	$\frac{1-\beta_l+\alpha_l\beta_l}{\alpha_l} > \frac{1-\beta_h+\alpha_h\beta_h}{\alpha_h}$	$q_h[\frac{\alpha_h V_l}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)]$ $+q_l[\alpha_l V_l + (1-\alpha_l)s_2]$
EN	$1-\beta_l+\alpha_l\beta_l > 1-\beta_h+\alpha_h\beta_h$	$q_h[\frac{min[\alpha_h V_h, \alpha_l V_l]}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)]$ $+q_l[\alpha_l V_l]$

Table 5: The candidates for the optimal strategy in Scenario H.

On the other hand, in Scenario L, the seller should choose his optimal strategy from the following two strategies:

The Strategy	Feasibility Condition	Profit
EE	always feasible	$q_h[\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)]$ $+q_l[\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_l)(s_2 - \beta_l c)]$
RE	always feasible	$q_h[\alpha_h V_h + (1-\alpha_h)s_2]$ $+q_l[\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_l)(s_2 - \beta_l c)]$

Table 6: The candidates for the optimal strategy in Scenario L.

The profit from each segment is decided by two components - the total surplus generated by the tickets, and the surplus the seller leaves to the segment. When both segments of consumers have the same WTPs for exchangeable tickets, the seller can earn the highest possible profit by the EE strategy. This is because the total surplus is maximized (by selling both segments exchangeable tickets at the highest possible price) and no information rent should be given to the consumers (since their WTPs are the same). On the other hand, when consumers from different segments do not have the same WTPs for exchangeable tickets, we have the following observation.

Corollary 6. *The seller always sells exchangeable tickets to the segment of consumers with a higher WTP for exchangeable tickets. As for the other segment of consumers, the seller can choose:*

1. A pooling strategy (the EE strategy): The seller sells exchangeable tickets to both segments.

2. A segmentation strategy (the ER, EN, and EO strategies in Scenario H, or the RE strategy in Scenario L): The seller sells another type of tickets to or just ignores the segment with a lower WTP for exchangeable tickets.

When facing heterogeneous consumers, it is commonly recommended that the seller shall use a menu of tickets (contracts) to induce them to self-select. Thus, segmentation strategies are rather expected. However, Corollary 6 suggests the possibility of *abandoning* the menu: sometimes it can be optimal for the seller to offer exchangeable tickets to both segments. This pooling strategy arises because exchangeable tickets are intrinsically more profitable than the other two types of tickets. To illustrate the intuition of this corollary, we use Scenario H as an example.

If the seller chooses the pooling strategy, he sells exchangeable tickets to both segments. In this EE strategy, he gets the highest possible profit from the low segment but loses some potential profits from the high segment. If he wants more profits from the high segment, he needs to decouple these two segments by choosing a segmentation strategy.

If the low segment is very unimportant, a natural choice for the seller is to ignore this segment. By doing so, he can get the highest possible profit from the high segment although he will lose the market of the low segment. If the low segment is of moderate importance so he does not want to give up this market completely, he can choose to sell another type of tickets to this segment. Here, we use the ER strategy as an example, and the situation for the EN strategy is similar. If this ER strategy is feasible, comparing to the EE strategy, now the seller can get more profit from the high segment by charging a higher price for the exchangeable tickets. However, he loses some profits from the low segment because refundable tickets create less total surplus than exchangeable tickets. For the seller, whether to choose a segmentation strategy is ultimately determined by the battle between the gain from the high segment and the loss from the low segment.

When the low-segment consumers have a higher WTP for exchangeable tickets, the seller's pooling and segmentation strategies are analogues of those in the previous situation except for one major difference - when he wants to choose a segmentation strategy, only the RE strategy is considered. In this strategy, tickets are sold at the targeted consumers' WTPs. Because consumers from each segment have a higher WTP for the tickets designed for them than consumers from the other segment, all IC constraints are satisfied automatically and this strategy is always feasible. In this strategy, the seller grasps all the surplus from the high segment and gets the highest possible profit from the low segment. Because of this result and the fact that selling refundable tickets is intrinsically more profitable than selling nonrefundable tickets or selling no tickets, the NE and the OE strategies are always strictly dominated by the RE strategy and are thereby never considered.

The next corollary says that blindly making exchangeable tickets as flexible as possible might be suboptimal.

Corollary 7. *An increase in the flexibility of exchangeable tickets may hurt the seller's profit.*

As an application of Corollary 7, increasing T, the period within which the refunded credits are valid, can increase both $\{\beta_i\}$'s. Although it can increase the value of exchangeable tickets, Corollary 7 explains why we can still observe finite-length T's in the real world.

Intuitively, an increase in $\{\beta_i\}$'s can increase the total surplus, and, hence, the seller's profit. However, when the seller uses the pooling (EE) strategy, this naive conjecture might become invalid. To illustrate this, we use Scenario H as an example. In this situation, the price of exchangeable tickets is decided by the low-segment consumers' WTP for exchangeable tickets only. Therefore, an increase in β_l helps the seller rise P_e. Although it also increases the cost of serving the low segment,[38] the revenue increment from the low segment itself is enough to cover this extra cost. Hence, this has a positive effect on the seller's profit. On the other hand, an increase in β_h cannot help the seller increase the price but only results in a higher cost of serving the high segment.

In reality, usually the seller cannot change β_h and β_l directly and separately, and what he can do is to change these two parameters together by some instruments. For instance, increasing T will increase both β_h and β_l, and in our example, this is not profitable when the loss from the higher β_h dominates the gain from the higher β_l.[39]

4.4 Cancellation Fee

In addition to the huge variety of tickets, another frequently observed instrument used in the airline ticket market is the cancellation fee. In this subsection, we relax the constraint that the seller cannot charge the cancellation fee when a consumer wants to cancel a refundable or exchangeable ticket. The sequence of events is the same, except that in stage 2 the seller also determines the cancellation fee f_r (f_e) if he chooses to sell refundable (exchangeable) tickets.

In this extension, our goal is to find out what is the optimal strategy for the seller when charging cancellation fees is allowed. We shall focus on the case in which $P_r \geq f_r$ and $P_e \geq f_e$. Otherwise, there is no incentive for consumers to cancel the tickets, and these tickets degenerate to the nonrefundable ones. On the other hand, we assume that $f_r \geq 0$ and $f_e \geq 0$ since in practice, it is unreasonable that the seller overly compensates the consumers for changing their plans; this also eliminates the possibility of consumer arbitrage (i.e., purchasing the ticket simply for the refund).

When a refundable ticket is cancelled, the seller provides only partial refund, $P_r - f_r$, to the consumers. On the other hand, when an exchangeable ticket is cancelled, we assume that consumers need to pay the cancellation fee only when they want to use the credits.[40] That is, consumers need not pay the fee if they do not use the credits. In compliance with

[38] Recall that β_i is the probability that a segment-i consumer will ask for another ticket from the seller given that she had cancelled her initial exchangeable ticket.

[39] Please see A.3.9 for another force which can enhance Corollary 7.

[40] For instance, United Airlines and US Airways charge the cancellation fee only when consumers want to use the refunded credits. Moreover, they require that the credits cannot be used to pay this fee. This fee is sometimes called the "rebooking fee" or the "reissue fee" because it should be paid when consumers want to use the refunded credits to purchase a new ticket.

the practice, a consumer should not have any further loss if she cancelled an exchangeable ticket and eventually does not use the refunded credits.[41]

After incorporating the cancellation fees, there are some minor modifications to consumers' expected utilities, consumers' WTPs, and the seller's profits. These are presented in the appendix, and here we only summarize the main results. The following lemma articulates the influences of cancellation fees on the profitabilities of refundable and exchangeable tickets.

Lemma 11. *Charging a cancellation fee, f_r, does not hurt the profitability of refundable tickets, but charging a cancellation fee, f_e, reduces the profitability of exchangeable tickets. However, even with a positive f_e, exchangeable tickets are still more profitable than refundable tickets.*

The intuition is as follows. For refundable tickets, the cancellation fee is a net transfer between the seller and consumers, which leaves the total surplus unaffected.[42] On the other hand, for exchangeable tickets, although charging a cancellation fee again has no explicit influence on the total surplus, it leads to a price reduction. (Given a fixed amount of surplus the seller leaves to the consumers, the higher f_e is, the lower P_e should be.) In this situation, the amount of money trapped by the seller is reduced, which in turn lowers the total surplus. Finally, exchangeable tickets can still help the seller trap consumers' money, even though this effect is mitigated by f_e. Hence, exchangeable tickets are still always more profitable than refundable tickets.

The following Lemma 12 and Lemma 13 discuss the designs of f_e and f_r, respectively.

Lemma 12. *When charging a cancellation fee, f_e, upon canceling an exchangeable ticket is allowed:*

1. *If at optimality the seller sells exchangeable tickets to only one segment, then $f_e = 0$.*
2. *The seller might be able to improve the EE strategy by charging a positive f_e.*

The first part of this lemma is valid even when there are more than two segments. The intuition is as follows. In any specific strategy, if the seller sells exchangeable tickets to only one segment with $f_e > 0$, he can strictly improve his profit by the following method. He reduces f_e and increases P_e so that the segment of consumers who initially purchase exchangeable tickets can get the same amount of surplus as before. On the other hand, he does not change the prices of all other tickets. In this process, this segment of consumers will continue purchasing exchangeable tickets and the seller earns more profit from this segment since the total surplus generated by exchangeable tickets is increasing in P_e. He can continue

[41]Note that this cancellation fee is different from the extra money a consumer needs to pay when the new ticket is more expensive than the original one.

[42]Even with a positive cancellation fee, the total surplus created by a refundable or exchangeable ticket when it is sold to a segment-i consumer is the same as that in Table 4.

increasing his profit by reducing f_e until $f_e = 0$ or another segment of consumers switches to exchangeable tickets. If another segment of consumers switches to exchangeable tickets, this switch is beneficial to the seller since exchangeable tickets are more profitable than other types of tickets. As a result, selling exchangeable tickets to only one segment with $f_e > 0$ is never optimal.

On the other hand, when multiple segments of consumers purchase exchangeable tickets, the seller might want to charge a positive f_e. For instance, in our model, the seller might be able to improve the EE strategy by charging a positive f_e. This arises because charging the cancellation fee, f_e, might help the seller reduce the difference between consumers' WTPs for exchangeable tickets. With a smaller difference between consumers' WTPs, the seller can give less information rent to the segment of consumers who are willing to pay more. However, charging a positive f_e also indirectly results in a less total surplus created by exchangeable tickets.

When the first effect exists (that is, a positive f_e can indeed reduce the difference between consumers' WTPs) and dominates the second effect, the seller wants to increase f_e to make consumers' WTPs as close as possible, subject to the constraints that f_e cannot exceed $\alpha_h V_h$ and $\alpha_l V_l$ (see A.3.4). On the other hand, when the second effect is dominant, the seller should choose $f_e = 0$.

Lemma 13. *In Scenario H, the seller might be able to improve the profitability and feasibility of the ER strategy by charging a positive f_r.*

On the other hand, in Scenario L, charging a positive f_r has no influence on the profitability and feasibility of the RE strategy as long as the high-segment consumers still have a higher WTP for refundable tickets than the low-segment consumers.

Charging a cancellation fee, f_r, might help the seller reduce the difference between consumers' WTPs for refundable tickets. In Scenario H, this means a lower information rent the seller should give to the high segment when using the ER strategy. Hence, the seller will want to use f_r to make consumers' WTPs for refundable tickets as close as possible, subject to the constraint that f_r cannot exceed $\alpha_l V_l$ (see A.3.4). Note that if increasing f_r enlarges the difference between consumers' WTPs for refundable tickets, then the seller chooses $f_r = 0$. On the other hand, in Scenario L, this benefit vanishes because no information rent is given to consumers in the RE strategy.

Lemma 12 and Lemma 13 identify the salient role of cancellation fees: they are used as instruments to adjust the differences between consumers' WTPs. A smaller difference between consumers' WTPs is desirable since this might lead to less information rent the seller should give to consumers.

Proposition 20. *When charging cancellation fees is allowed, the seller's optimal strategy is:*

In Scenario H, the seller should choose his optimal strategy from the EE, ER, and EO strategies, depending on which one is feasible and gives him the highest profit.

On the other hand, in Scenario L, the seller should choose his optimal strategy from the EE and RE strategies, depending on which one gives him the highest profit.

The details of these strategies (optimal prices and cancellation fees, feasibility conditions, and profits) are listed in the appendix. After incorporating the cancellation fees, f_r and f_e, the profitability rankings of these three types of tickets are still the same as before. As a result, the set of candidates for the optimal strategy is similar to that in the basic model. The seller again needs to choose a pooling strategy or a segmentation strategy.

When he chooses the pooling strategy, he might be able to use f_e to improve the EE strategy. On the other hand, when he chooses a segmentation strategy, he sets $f_e = 0$ according to Lemma 12. He sells exchangeable tickets to the segment of consumers with a higher WTP for exchangeable tickets, and uses another type of tickets to distract or just ignores the other segment. This part is similar to that in Proposition 19 except for two differences occurring in Scenario H. First, the seller might be able to use f_r to improve the ER strategy (Lemma 13). Second, the seller will not consider using nonrefundable tickets to distract the low segment. That is, the EN strategy is never optimal. We have the following corollary about this observation.

Corollary 8. *When charging cancellation fees, f_r and f_e, is allowed, the seller has a lower incentive to sell nonrefundable tickets.*

This corollary coincides with our observation that in the real world, major airline companies rarely sell nonrefundable tickets to consumers directly. They usually sell refundable or exchangeable tickets with or without cancellation fees to consumers. The intuition of this corollary is that after using cancellation fees as instruments, refundable tickets and exchangeable tickets can be strictly better substitutions for nonrefundable tickets. For example, when f_r is very close to P_r, consumers will have similar expected utilities from purchasing refundable tickets or nonrefundable tickets, although they will cancel the refundable tickets when they need to change their plans. In this situation, refundable tickets can do strictly better than nonrefundable tickets, and, hence, the seller has less incentive to sell nonrefundable tickets.

Note that this result becomes invalid when the salvage value of the cancelled tickets for the seller is negative ($s_2 < 0$). This situation happens when the cost of the canceling and refunding processes is too high comparing to the expected profit of selling the cancelled tickets again. This might be the reason why we can still observe nonrefundable tickets sold by third-party agents, like priceline.com and cheapoair.com, in the real world.

4.5 Commodity Good

While offering a menu to induce consumers' self-selection is a central premise in the academic literature, a natural question is why this kind of menu design is not widely observed in commodity goods markets despite its ubiquity in the airline ticket market. In commodity goods markets, products are usually sold at one price with one return policy. In this subsection, we apply our framework to commodity goods markets. We hope this can provide some rationale behind the industry practice.

We use the same model settings as before and now the seller can sell his products to consumers in three forms: refundable goods, exchangeable goods, and nonrefundable goods. We assume that the seller should give consumers a replacement if the product cannot function well. (This is required by law in most places (such as the United States).) As a result, the only reason for consumers to return the product is that they do not like the product after purchasing it.

First of all, the following observations seem sensible to us: One major difference between the airline ticket market and commodity goods markets is the uncertainty about future consumptions. In commodity goods markets, consumers have a clear picture about their future purchasing behavior. That is, while making the purchasing decision, they are pretty sure whether they will purchase other items in the future in case of returning the product. Therefore, we will have $\beta_i = 0$ or $\beta_i = 1$. Here, we assume that $\beta_h = \beta_l$, because usually the value of β_i depends mainly on characteristics of the *seller* instead of consumers. For example, if the seller is a large grocery store, then consumers, independent of their types, are certain that they will purchase other items from this seller in the future ($\beta_h = \beta_l = 1$). On the other hand, if the seller is a souvenir store in a remote region, the value of the refunded credits might be even lower than the hassle cost of returning the product, especially when the consumer will not visit the place again in the near future. In this case, $\beta_h = \beta_l = 0$.

When $\beta_h = \beta_l = 0$, exchangeable goods are equivalent to nonrefundable goods for both the consumers and the seller. Hence, the effective situation is that the seller only considers refundable goods and nonrefundable goods. On the other hand, when $\beta_h = \beta_l = 1$, for consumers, exchangeable goods are equivalent to refundable goods. Therefore, the seller only chooses the more profitable type of goods. However, profitabilities of exchangeable and refundable goods are very close. This is because when $\beta_i = 1$, if the segment-i consumers return the products and then purchase other items in the future with the refunded credits, they just use the credits to substitute for real money they should pay. (Note that they will make these future purchases no matter whether they have these credits or not.) In other words, exchangeable goods are not as effective in trapping consumers' money as in the airline ticket market. Hence, which type of goods is more profitable depends on other costs which are not modeled in our model. (e.g., the difference between the cost of refunding real cash and the cost of refunding credits, or the influence of different return policies on consumer relationship management.) Based on the above discussions, we highlight the result in Observation 5.

Observation 5. *The seller does not consider selling exchangeable goods and selling refundable goods at the same time.*[43]

[43] That is, in some situations, the seller only considers refundable goods and nonrefundable goods. In other situations, the seller only considers exchangeable goods and nonrefundable goods.

Here, we assume that the seller only considers refundable goods and nonrefundable goods since refundable goods are more generally observed in most commodity goods markets than exchangeable goods. We have the following proposition about a necessary condition for a menu to be optimal. Note that the following analysis is also applicable to markets of the other case.

Proposition 21. *When the seller can choose to sell his products in the form of refundable goods or nonrefundable goods, a necessary condition for the optimal strategy to be a menu is* $\alpha_h < \alpha_l$.[44]

Although $\alpha_h < \alpha_l$ is very common in the airline industry, it is not widely observed in markets of other commodity products. In the commodity goods markets, consumers with higher valuations upon using the product usually have an equal or better understanding about the product than consumers with lower valuations. Hence, comparing to consumers with lower valuations, consumers with higher valuations usually have at least the same probability to be satisfied by the product. As a result, providing a menu is not profitable for the seller in this kind of markets.

5 Conclusion

In the first chapter, we build a general framework which can accommodate highly asymmetric information structures, multiple markets, and multiple fundamentals. We identify the unique Bayesian Nash equilibrium, and express farmers' strategies and expected profits in close forms. Some properties of this equilibrium have been examined. For example, compared to receiving no signals, observing signals creates nonnegative extra expected profit to a farmer; and, hence, a farmer's expected profit when she cannot observe any signal defines a lower bound of her expected profit. Our another finding is that different markets and different fundamentals can be treated separately. This result reduces the difficulty of analyzing the equilibrium because we can focus on models with only one market and one fundamental. We also find that if some signals are released to and only to the same set of farmers, they can be combined into a signal with better precision. This intuitive finding helps us establish the equivalence between information provision and signals improvement when discussing the government's information or budgets allocation.

We provide a limit on the extent to which farmers can utilize the signals, and it implies that farmers' responses can mitigate the fluctuation of the market demand. However, this mitigation is weaker than that when all farmers know exactly the realized value of the fundamental. We then show that if farmers can be separated into two disjoint groups such that no signal is released to farmers from different groups, then farmers from one group need not consider the complex information channels in the other group. They just ignore

[44] To be more specific, when $\alpha_h \geq \alpha_l$, the seller chooses his optimal strategy from the *RR* or the *RO* strategies.

the other group and get their response coefficients, and then multiply these coefficients by a factor. This factor is decided by two numbers, and each of them is proportional to the sum of all response coefficients in one group. The number of the group a farmer belongs to can be calculated by the farmer, and the number of the other group might be guessed according to the farmer's past experience. In this situation, farmers need not worry about the information structure for farmers in other parts of the world.

To illustrate that highly asymmetric information channels can lead to various novel results, we provide two simple examples. In these examples, we observe that farmers may respond to signals adversely, farmers may benefit from the improvement of signals they cannot observe, farmers may be willing to share their signals with others, and farmers may suffer from being shared signals with. To get more insight, we turn our attention to the weak signal limit. In this limit, we can expand farmers' strategies and have a closer examination on the equilibrium. We also study the government's optimal allocation of its information or budgets when its goal is to improve farmers' total profits or the social welfare.

In this chapter, we only study the Cournot competition (quantity competition) with common uncertainties (demand uncertainties). It is very possible that highly asymmetric information structures will lead to more interesting results in the Bertran competition (price competition) or when the uncertainties come from each farmer (private uncertainties/cost uncertainties). In the literature, it has been shown that the equilibrium properties highly depends on the nature of competition and the source of uncertainties. Hence, the influence of asymmetric information structures on the equilibrium might be altered tremendously under these different settings. Another possible extension is to endogenize the information acquisition. We can assume that farmers are in a social network with a general structure, and let farmers decide whether they want to transmit their information to others through this network. This setting may be able to capture the reality better. Also, it can help us get better understanding in the social network.

In the second chapter, we present a model to analyze the government's information provision policy for the case when there are heterogeneous farmers who need to select one of the two markets to sell in (or select one of the two crops to grow). When farmers are price takers, our analysis indicates information is always beneficial to individual farmers. Specifically, farmers who are located far away from both markets will benefit the most from information access.

To maximize farmers' expected total profit, it may not be optimal for the government to provide information to all farmers. Instead, it is optimal for the government to provide information to all farmers who are located in a targeted range. To implement such an optimal provision policy in a fair manner, we show that the government should impose a nominal fee for signal access. We consider other extensions including correlated markets and a more general distribution of farmers as robustness checks, and show that most results continue to hold.

We extend our analysis to the case when information is disseminated by a for-profit company and show that the company would reduce the information access to a smaller range of farmers so as to maximize its revenue. Therefore, to entice the company to provide information access to the optimal range of farmers that maximizes farmers' expected total earning from selling their crop, the government should develop the right incentive to the company. The development of an incentive scheme would be an interesting topic for future research. Also, another future research topic would be the issue of private signals. In the presence of private signals, the simple threshold market selection rule is no longer an equilibrium strategy. Without a simple threshold market selection rule, it would be technically challenging to determine the optimal information provision policy.

In the third chapter, we establish a stylized model to examine a monopolistic seller's optimal menu design when three types of tickets (refundable, nonrefundable, and exchangeable tickets) are available. We find that the profitability of exchangeable tickets is increasing in the ticket price, but the profitabilities of refundable and nonrefundable tickets are independent of the prices. Also, we identify a pecking order in terms of profitability: exchangeable tickets are always more profitable than refundable tickets, which are more profitable than nonrefundable tickets. Therefore, the seller always sells exchangeable tickets to some consumers in the optimal menu design. Moreover, we note that if the seller sells exchangeable tickets to multiple segments of consumers, it is not necessarily beneficial for him to increase the flexibility of exchangeable tickets.

We then incorporate cancellation fees into our model. We find that although charging a positive cancellation fee has no influence on the profitability of refundable tickets, it will indirectly hurt the profitability of exchangeable tickets. However, the pecking order identified above remains the same. Our another finding is that when charging cancellation fees is allowed, the seller has less incentive to sell nonrefundable tickets.

Finally, we map our model to commodity goods markets and show that offering a menu may be optimal only when consumers with a lower valuation upon using the product are more likely to be satisfied by it. Though this feature has been a fixture in the airline industry, it is not a common observation in the markets of other commodity products. As a result, our framework provides a possible reason why menus of refunds/returns are rarely adopted in commodity goods markets.

We make some simplifications in our model in order to get a sharper insight and illustrate our results more clearly. Removing these simplifications can lead to some interesting future extensions. First, in practice, ticket selling is a truly dynamic continuous-time process, and in such a context, exchangeable tickets might result in more interesting benefits and drawbacks to the seller. The interplays between different types of tickets and the seller's overbooking strategy might also be interesting. Second, while our analysis documents the first-order effect of price discrimination using different tickets, the seller can make finer price discriminations based on different scenarios. For instance, he can charge a higher fee upon canceling a ticket than changing a ticket. It is also possible for him to require advance purchase upon certain types of tickets. Finally, in this chapter, we assume that consumers' utilities of using the credits are the same as using real money, and treat the seller's cost of serving consumers who want to use the credits as given. Endogenizing these future transactions with credits will give us better understanding in this "exchangeable" refund policy, but it requires a more refined dynamic setting for credit redemption.

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A Appendix

A.1 Proofs of Chapter 1: Farmers' Information Management in Developing Countries - A Highly Asymmetric Information Structure

A.1.1 Proof of Lemma 1

Given the realized values of signals in I_j, the probability density function of u_k is

$$\begin{aligned} f(u_k = y|I_j) = &\ A_0 exp\Big[-\tfrac{1}{2}\alpha_k y^2\Big] \times \textstyle\prod_{i\in\{N:x_i\in I_j\cap X_k\}} exp\Big[-\tfrac{1}{2}\beta_i(x_i-y)^2\Big] \\ = &\ A_1 exp\Big[-\tfrac{1}{2}\big(\alpha_k + (\boldsymbol{m_j})_k\big)\big(y - \textstyle\sum_{i\in\{N:x_i\in I_j\cap X_k\}} \frac{\beta_i x_i}{\alpha_k+(\boldsymbol{m_j})_k}\big)^2\Big] \\ = &\ A_1 exp\Big[-\tfrac{1}{2}\big(\alpha_k + (\boldsymbol{m_j})_k\big)\big(y - (\boldsymbol{T}^T\boldsymbol{L_j}\boldsymbol{D_j}\boldsymbol{x})_k\big)^2\Big], \end{aligned} \tag{19}$$

where A_0 and A_1 are the normalizing constants which do not depend on y. From the above equation and the fact that $E(\epsilon_i) = 0 \ \ \forall i$, we can get Lemma 1.

A.1.2 Proof of Proposition 1

Each farmer j's goal is to maximize her expected profit:

$$\max_{\boldsymbol{q_j}} \Big[E\big((\boldsymbol{a}-\boldsymbol{b}\sum_i \boldsymbol{q_i})^T\boldsymbol{q_j}|I_j\big)\Big] = \max_{\boldsymbol{q_j}} \Big[\big(\boldsymbol{a_0}^T + E(\boldsymbol{u}|I_j)^T\boldsymbol{\psi}^T\big)\boldsymbol{q_j} - \sum_{i\neq j} E(\boldsymbol{q_i}|I_j)^T\boldsymbol{b}^T\boldsymbol{q_j} - \boldsymbol{q_j}^T\boldsymbol{b}^T\boldsymbol{q_j}\big)\Big].$$

Because $-\boldsymbol{b}$ is a negative definite matrix, the second-order sufficient condition holds. The first-order condition yields

$$\boldsymbol{q_j} = \boldsymbol{b}^{-1}\boldsymbol{a_0} + \boldsymbol{b}^{-1}\boldsymbol{\psi}E(\boldsymbol{u}|I_j) - \sum_{i\neq j} E(\boldsymbol{q_i}|I_j) - \boldsymbol{q_j}. \tag{20}$$

First, we assume that farmers take linear strategies in which $\boldsymbol{q_i} = \boldsymbol{C_i} + \boldsymbol{B_i}\boldsymbol{D_i}\boldsymbol{x} \ \ \forall i$. By this assumption and Lemma 1, we have:

$$\begin{aligned}
&\boldsymbol{C_j} + \boldsymbol{B_j D_j x} = \boldsymbol{b}^{-1}\boldsymbol{a_0} + \boldsymbol{b}^{-1}\boldsymbol{\psi T}^T \boldsymbol{L_j D_j x} - \sum_i \Big[\boldsymbol{C_i} + \boldsymbol{B_i D_i}\big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{TT}^T\boldsymbol{L_j}\big)\boldsymbol{D_j x}\Big] \\
&\Rightarrow \boldsymbol{C_j} = \frac{1}{n_c+1}\boldsymbol{b}^{-1}\boldsymbol{a_0} \\
&\Rightarrow \boldsymbol{B_j D_j} = \boldsymbol{b}^{-1}\boldsymbol{\psi T}^T\boldsymbol{L_j D_j} - \sum_i \boldsymbol{B_i D_i}\big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{TT}^T\boldsymbol{L_j}\big)\boldsymbol{D_j} \\
&\Rightarrow \sum_j \boldsymbol{B_j D_j} = \boldsymbol{b}^{-1}\boldsymbol{\psi T}^T \sum_j \boldsymbol{L_j D_j} - \sum_i \boldsymbol{B_i D_i}\sum_j \big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{TT}^T\boldsymbol{L_j}\big)\boldsymbol{D_j} \\
&\Rightarrow \sum_l \boldsymbol{B_l D_l} = \boldsymbol{b}^{-1}\boldsymbol{\psi T}^T \sum_i \boldsymbol{L_i D_i}\Big\{I + \sum_k \big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_k})\boldsymbol{TT}^T\boldsymbol{L_k}\big)\boldsymbol{D_k}\Big\}^{-1}
\end{aligned} \tag{21}$$

From the above equations, we can get $\boldsymbol{C_j}$ and $\boldsymbol{B_j D_j}$. Then we can check that this is indeed a Bayesian Nash equilibrium.

Next, we want to prove that this is a unique Bayesian Nash equilibrium. If there is an equilibrium in which each farmer j follows a strategy $\boldsymbol{q_j}(I_j)$, these $\{\boldsymbol{q_j}(I_j)\}$'s must satisfy Equation (20). Define $\boldsymbol{g_j}(I_j) = \boldsymbol{q_j}(I_j) - \boldsymbol{C_j} - \boldsymbol{B_j D_j x}$. Note that $\boldsymbol{B_j D_j x}$ only depends on signals in I_j. From Equations (20) and (21), we have

$$\begin{aligned}
\boldsymbol{g_j}(I_j) =& \boldsymbol{q_j}(I_j) - \boldsymbol{C_j} - \boldsymbol{B_j D_j x} \\
=& \boldsymbol{b}^{-1}\boldsymbol{a_0} + \boldsymbol{b}^{-1}\boldsymbol{\psi}E(\boldsymbol{u}|I_j) - \sum_{i \neq j} E(\boldsymbol{q_i}(I_i)|I_j) - \boldsymbol{q_j}(I_j) \\
&- \frac{1}{n_c+1}\boldsymbol{b}^{-1}\boldsymbol{a_0} - \boldsymbol{b}^{-1}\boldsymbol{\psi T}^T\boldsymbol{L_j D_j x} + \sum_i \boldsymbol{B_i D_i}\big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{TT}^T\boldsymbol{L_j}\big)\boldsymbol{D_j x} \\
=& -\sum_i \Big[E(\boldsymbol{q_i}(I_i)|I_j) - \boldsymbol{C_i} - \boldsymbol{B_i D_i}\big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{TT}^T\boldsymbol{L_j}\big)\boldsymbol{D_j x}\Big] \\
=& -\sum_i E\Big(\boldsymbol{q_i}(I_i) - \boldsymbol{C_i} - \boldsymbol{B_i D_i x}\big|I_j\Big) = -\sum_i E\Big(\boldsymbol{g_i}(I_i)\big|I_j\Big).
\end{aligned} \tag{22}$$

Following similar arguments as Lemma 2 and Proposition 1 in Li (1985), for each market l, $(\boldsymbol{g_j}(I_j))_l = 0$ almost surely for all j.

Given the realized values of signals in I_j, farmer j's expected profit is $E(\boldsymbol{P}^T\boldsymbol{q_j}|I_j) = \big(\boldsymbol{a_0} + \boldsymbol{\psi}E(\boldsymbol{u}|I_j) - \boldsymbol{b}\sum_{i \neq j} E(\boldsymbol{q_i}|I_j) - \boldsymbol{b q_j}\big)^T \boldsymbol{q_j} = \boldsymbol{q_j}^T\boldsymbol{b q_j}$, where $\boldsymbol{q_j}$ is her optimal production quantity we get above. Therefore, her expected profit is

$$\begin{aligned}
E\big(E(\boldsymbol{P}^T\boldsymbol{q_j}|I_j)\big) &= E\big((\boldsymbol{x}^T\boldsymbol{D_j}^T\boldsymbol{B_j}^T + \boldsymbol{C_j}^T)\boldsymbol{b}(\boldsymbol{C_j} + \boldsymbol{B_j D_j x})\big) \\
&= \frac{1}{(n_c+1)^2}\boldsymbol{a_0}^T\boldsymbol{b}^{-1}\boldsymbol{a_0} + Tr(\boldsymbol{D_j}^T\boldsymbol{B_j}^T\boldsymbol{b B_j D_j \beta}^{-1}) \\
&\quad + Tr(\boldsymbol{T}^T\boldsymbol{D_j}^T\boldsymbol{B_j}^T\boldsymbol{b B_j D_j T \alpha}^{-1}) \\
&= \frac{1}{(n_c+1)^2}\boldsymbol{a_0}^T\boldsymbol{b}^{-1}\boldsymbol{a_0} + Tr\big(\boldsymbol{b B_j D_j}(\boldsymbol{\beta}^{-1} + \boldsymbol{T\alpha}^{-1}\boldsymbol{T}^T)\boldsymbol{D_j}^T\boldsymbol{B_j}^T\big).
\end{aligned} \tag{23}$$

A.1.3 Proof of Proposition 2

From the formula of farmers' strategies, we can find that farmers' response coefficients in a market do not depend on other markets. (Note that the lth row in $\boldsymbol{B_j D_j}$ do not depend on the mth rows in $\boldsymbol{\psi}$ and $\boldsymbol{b}$ for all l and all $m \neq l$.)

Also, $\boldsymbol{L_j D_j}$, $\sum_i \boldsymbol{L_i D_i}$, $\left[\boldsymbol{I} + \sum_k \left(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_k})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_k}\right)\boldsymbol{D_k}\right]^{-1}$, and $\left(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\right)\boldsymbol{D_j}$ are all block-diagonal matrices. For each k, the kkth blocks in these matrices are of size $n_{sk} \times n_{sk}$ and depend on fundamental k only. i.e., they do not depend on parameters of other fundamentals. This property is also inherited by $\boldsymbol{L_j D_j} - \sum_i \boldsymbol{L_i D_i}\left[\boldsymbol{I} + \sum_k \left(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_k})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_k}\right)\boldsymbol{D_k}\right]^{-1}\left(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\right)\boldsymbol{D_j}$. Hence, for all k, the response coefficients to signals about fundamental k depend on this fundamental k only and are independent of other fundamentals.

From the above result, farmers' expected profits can be decomposed into base profits pertaining to each of the markets and fluctuating profits pertaining to each of the market-fundamental pairs.

A.1.4 Proof of Lemma 2

Without loss of generality, suppose signals 1 and 2 are observed by and only by the same set of farmers (G_1), and let G_2 be the set of farmers who cannot observe these signals. (If there are n ($n > 2$) signals which are observed by and only by the same set of farmers, we can combine them in $n-1$ steps, and in each step we combine 2 signals.)

Define v_i as the unit vector such that $(v_i)_i = 1$ and $(v_i)_j = 0 \quad \forall j \neq i$. Multiplying both sides of the fourth line in Equation (21) by v_1 and by v_2, we can find that $\sum_j (\boldsymbol{B_j D_j})_1/\beta_1 = \sum_j (\boldsymbol{B_j D_j})_2/\beta_2$. Combining this result with the third line in Equation (21), we have $(\boldsymbol{B_j D_j})_1/\beta_1 = (\boldsymbol{B_j D_j})_2/\beta_2 \quad \forall j \in G_1$. Let $(\boldsymbol{B_j D_j})_1 = \beta_1 f_j$ and $(\boldsymbol{B_j D_j})_2 = \beta_2 f_j \quad \forall j \in G_1$.

We can combine signals 1 and 2 into a whole signal by defining $x_1' \equiv \frac{\beta_1 x_1 + \beta_2 x_2}{\beta_1 + \beta_2} = u + \frac{\beta_1 \epsilon_1 + \beta_2 \epsilon_2}{\beta_1 + \beta_2} = u + \epsilon_1'$. Since $\epsilon_1' \sim N(0, 1/(\beta_1 + \beta_2))$, this new signal is of the same form as other signals and its precision is $\beta_1 + \beta_2$. Note that all the information about the fundamental in signals 1 and 2 is encoded in this new signal. This is because the distribution of the fundamental conditional on signals 1 and 2 is the same as that conditional on the new signal.

Suppose farmers in G_1 receive signal x_1' instead of signals 1 and 2, by using the third line in Equation (21), it can be shown that farmers' strategies are as follows: Farmers in G_2 have the same response coefficients to observable signals as before. For each farmer j in G_1, her response coefficient to x_1' is the sum of her response coefficients to signals 1 and 2

$(\beta_1 f_j + \beta_2 f_j)$, and her response coefficients to other signals are the same as before.

From our results in the last three paragraphs, given any realized signals, farmers' production quantities and expected profits are the same no matter farmers in G_1 respond to signals 1 and 2 or respond to the processed signal x_1'. One thing worth mentioning is that combining these results with the formula in Proposition 1, farmers' expected profits remain the same if signals 1 and 2 are replaced by a more precise signal with precision $\beta_1 + \beta_2$.

A.1.5 Proof of Lemma 3

Without losing generality, let $\psi > 0$. (Note that $b\psi^{-1}\sum_j \boldsymbol{B_j D_j T}$ does not depend on the sign of ψ.) Suppose that $b\psi^{-1}\sum_j \boldsymbol{B_j D_j T} \geq \frac{n_c}{n_c+1}$. Define a set $K \equiv \{k \in N | (\sum_j \boldsymbol{B_j D_j})_k > 0\}$ and a set $L \equiv \{l \in N | \exists k \in K \ s.t. \ (\boldsymbol{B_l D_l})_k > 0\}$. We have $K \neq \emptyset$ and $L \neq \emptyset$.

It can be shown that $\forall l \in L$, $\boldsymbol{B_l D_l T} < \frac{1}{n_c+1} b^{-1}\psi$. Otherwise, $\exists l \in L$ and $k \in K \ s.t. \ \boldsymbol{B_l D_l T} \geq \frac{1}{n_c+1} b^{-1}\psi \wedge (\boldsymbol{B_l D_l})_k > 0$. If the realized signals take the following form $(\boldsymbol{x})_k = 1$ and $(\boldsymbol{x})_m = \frac{\beta_k}{\alpha+\beta_k} \ \forall m \neq k$, we can find that

$$\begin{aligned}
&b^{-1}\psi \boldsymbol{T}^T \boldsymbol{L_l D_l x} - \sum_i \boldsymbol{B_i D_i}\big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_l})\boldsymbol{T}\boldsymbol{T}^T \boldsymbol{L_l}\big)\boldsymbol{D_l x} \\
&= b^{-1}\psi \frac{\beta_k}{\alpha + \beta_k} - \sum_i \boldsymbol{B_i D_i T}\frac{\beta_k}{\alpha+\beta_k} - (\sum_i \boldsymbol{B_i D_i})_k (1 - \frac{\beta_k}{\alpha+\beta_k}) \\
&< \frac{1}{n_c+1} b^{-1}\psi \frac{\beta_k}{\alpha+\beta_k} < \boldsymbol{B_l D_l T}\frac{\beta_k}{\alpha+\beta_k} + (\boldsymbol{B_l D_l})_k (1 - \frac{\beta_k}{\alpha+\beta_k}) = \boldsymbol{B_l D_l x}.
\end{aligned} \tag{24}$$

The third line in Equation (21) is violated. This is a contradiction.

Next, we want to show that $\exists l \in L$ and $q \notin K \ s.t. \ (\boldsymbol{B_l D_l})_q < 0$. This follows from the fact that $\sum_{k\in K}\sum_{j\in L}(\boldsymbol{B_j D_j})_k \geq \sum_{k\in K}\sum_j (\boldsymbol{B_j D_j})_k \geq \sum_j \boldsymbol{B_j D_j T} \geq \frac{n_c}{n_c+1} b^{-1}\psi > \sum_{j\in L} \boldsymbol{B_j D_j T}$.

For this l, $\exists k \in K \ s.t. \ (\boldsymbol{B_l D_l})_k > 0$. If the realized signals take the following form $(\boldsymbol{x})_k = \beta_q$, $(\boldsymbol{x})_q = -\beta_k$, and $(\boldsymbol{x})_m = 0 \ \forall m \in N \ s.t. \ m \neq k \wedge m \neq q$, we can find that

$$\begin{aligned}
&b^{-1}\psi \boldsymbol{T}^T \boldsymbol{L_l D_l x} - \sum_i \boldsymbol{B_i D_i}\big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_l})\boldsymbol{T}\boldsymbol{T}^T \boldsymbol{L_l}\big)\boldsymbol{D_l x} \\
&= -(\sum_i \boldsymbol{B_i D_i})_k \beta_q - (\sum_i \boldsymbol{B_i D_i})_q (-\beta_k) \\
&< 0 < (\boldsymbol{B_l D_l})_k \beta_q + (\boldsymbol{B_l D_l})_q (-\beta_k) = \boldsymbol{B_l D_l x}.
\end{aligned} \tag{25}$$

The third line in Equation (21) is violated. This is a contradiction. Therefore, we get $b\psi^{-1}\sum_j \boldsymbol{B_j D_j T} < \frac{n_c}{n_c+1}$.

According to similar arguments, we can prove that $0 < b\psi^{-1}\sum_j \boldsymbol{B_j D_j T}$.

A.1.6 Proof of Proposition 3

We can directly check that the third line in Equation (21) is satisfied by the strategy constructed in Proposition 3. Hence, this is indeed the strategy in the unique Bayesian Nash equilibrium.

A.1.7 Instance of Observation 1

In Example 1, let $b = \psi = 1$, $\alpha = 0.3$, $\beta_1 = 1$, and $\beta_2 = \beta_3 = 0.1$. We can get $\boldsymbol{B_1 D_1} = (0.330448, -0.00550747, 0)$.

A.1.8 Instance of Observation 2

In Example 1, let $b = \psi = 1$, $\alpha = 0.3$, $\beta_1 = 1$, and $\beta_2 = \beta_3 = 0.1$. Farmer 2's expected profit is $0.0792683 + 0.0625a_0^2$. If $\beta_1 = 3$, farmer 2's expected profit becomes $0.0806057 + 0.0625a_0^2$.

A.1.9 Instance of Observations 3 and 4

Let $b = \psi = 1$, $\alpha = 0.3$, $\beta_1 = \beta_2 = 0.1$, and $\beta_3 = 1$. In Example 1, farmer 2's expected profit is $0.0426193 + 0.0625a_0^2$ and farmer 3's expected profit is $0.448056 + 0.0625a_0^2$. On the other hand, in Example 2, farmer 2's expected profit is $0.0520833 + 0.0625a_0^2$ and farmer 3's expected profit is $0.440972 + 0.0625a_0^2$.

A.1.10 Proof of Lemma 4

$$\begin{aligned}
&\left[\boldsymbol{I} + \sum_{j=1}^{n_c}\Big(\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\Big)\boldsymbol{D_j}\right]^{-1} \\
=&\left[(\boldsymbol{I} + \boldsymbol{D}) + \sum_{j=1}^{n_c}(\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\boldsymbol{D_j}\right]^{-1} \\
=&(\boldsymbol{I} + \boldsymbol{D})^{-1}\left[\boldsymbol{I} + \sum_{j=1}^{n_c}(\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\boldsymbol{D_j}(\boldsymbol{I} + \boldsymbol{D})^{-1}\right]^{-1}
\end{aligned}$$

Let $\boldsymbol{A} = \sum_{j=1}^{n_c}(\boldsymbol{I} - \boldsymbol{D_j})\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{L_j}\boldsymbol{D_j}(\boldsymbol{I} + \boldsymbol{D})^{-1}$, this lemma just describes the expansion of $(\boldsymbol{I} + \boldsymbol{A})^{-1}$. To make this expansion valid, we should have $\lim_{n\to+\infty}\boldsymbol{A}^n = 0$. Let $C(\boldsymbol{G}) = \max_m \sum_l (\boldsymbol{G})_{lm}$ be the largest column sum in an arbitrary matrix $\boldsymbol{G}$. We want to show that if all elements in $\boldsymbol{A}$ is nonnegative (which is satisfied already) and $C(\boldsymbol{A}) < 1$, then $\lim_{n\to+\infty}\boldsymbol{A}^n = 0$.

Let $C(\boldsymbol{A}) = M < 1$, then for any m, $\sum_l(\boldsymbol{A}^2)_{lm} = \sum_l\sum_i(\boldsymbol{A})_{li}(\boldsymbol{A})_{im} = \sum_i\sum_l(\boldsymbol{A})_{li}(\boldsymbol{A})_{im}$ $\leq \sum_i M(\boldsymbol{A})_{im} \leq M^2$. Hence, $C(\boldsymbol{A}^2) \leq M^2$. Also, for any m, $\sum_l(\boldsymbol{A}^3)_{lm} = \sum_l\sum_i(\boldsymbol{A}^2)_{li}(\boldsymbol{A})_{im}$

$= \sum_i \sum_l (\boldsymbol{A}^2)_{li} (\boldsymbol{A})_{im} \leq \sum_i M^2 (\boldsymbol{A})_{im} \leq M^3$, and, thus, $C(\boldsymbol{A}^3) \leq M^3$. By continuing this process, we can show that $C(\boldsymbol{A}^n) \leq M^n \ \ \forall n$. Combining this with the fact that all elements in $\boldsymbol{A}$ is nonnegative, we show that $\lim_{n \to +\infty} \boldsymbol{A}^n = 0$. In conclusion, to make the expansion valid, a sufficient condition is that all column sums in $\sum_{j=1}^{n_c} (\boldsymbol{I} - \boldsymbol{D_j}) \boldsymbol{T} \boldsymbol{T}^T \boldsymbol{L_j} \boldsymbol{D_j} (\boldsymbol{I} + \boldsymbol{D})^{-1}$ are less than one. In this weak signal limit, we have $\beta_i \ll \alpha \ \ \forall i$, and, thus, this sufficient condition is satisfied. The intuition of this sufficient condition is as follows: for each farmer j and any $x_i \in I_j$, $(\boldsymbol{L_j})_{ii}$ is the weight she puts on signal i when conjecturing the value of the fundamental, and, hence, the values of signals she cannot observe. Therefore, the column sum of the i-th column in $(\boldsymbol{I} - \boldsymbol{D_j}) \boldsymbol{T} \boldsymbol{T}^T \boldsymbol{L_j} \boldsymbol{D_j}$ is the aggregate weight farmer j puts on signal i when guessing the unobservable signals. This sufficient condition ensures that for each signal i, the average aggregate weight farmers, who can observe this signal, put on it when guessing unobservable signals is not too high (lower than $\frac{1+d_i}{d_i}$).

A.1.11 Proof of Proposition 4

This follows directly from Proposition 1 and Lemma 4.

A.1.12 Proof of Proposition 5

We can further expand farmers' strategies in terms of $\alpha^{-1}\boldsymbol{\beta}$. The first-order term in $\boldsymbol{B_j} \boldsymbol{D_j}$ is

$$b^{-1} \psi \boldsymbol{T}^T \alpha^{-1} \boldsymbol{\beta} (\boldsymbol{I} + \boldsymbol{D})^{-1} \boldsymbol{D_j}. \tag{26}$$

And the second-order term in $\boldsymbol{B_j} \boldsymbol{D_j}$ is

$$\begin{aligned} &- b^{-1} \psi \alpha^{-2} \boldsymbol{T}^T \boldsymbol{\beta} (\boldsymbol{I} + \boldsymbol{D})^{-1} \boldsymbol{D} \boldsymbol{T} \boldsymbol{T}^T \boldsymbol{\beta} (\boldsymbol{I} + \boldsymbol{D})^{-1} \boldsymbol{D_j} \\ &- b^{-1} \psi \alpha^{-2} \boldsymbol{T}^T \boldsymbol{\beta} (\boldsymbol{I} + \boldsymbol{D})^{-1} \boldsymbol{D_j} \boldsymbol{T} \boldsymbol{T}^T \boldsymbol{\beta} \boldsymbol{D_j} \\ &+ b^{-1} \psi \alpha^{-2} \sum_i \boldsymbol{T}^T \boldsymbol{\beta} (\boldsymbol{I} + \boldsymbol{D})^{-1} \boldsymbol{D_i} \boldsymbol{T} \boldsymbol{T}^T \boldsymbol{\beta} \boldsymbol{D_i} (\boldsymbol{I} + \boldsymbol{D})^{-1} \boldsymbol{D_j}. \end{aligned} \tag{27}$$

We can find that the first-order terms in farmers response coefficients to observable signals are nonzero and in the same sign with ψ.

A.1.13 Proof of Propositions 6, 7, 8, 9, 10, and Lemma 5

With the aid of Equations (2), (26), and (27), these results can be obtained after some straightforward but tedious calculations.

A.1.14 Proof of Proposition 11

We can express the expected social welfare as

$$\begin{aligned}
E\big(\frac{(2a-bQ)Q}{2}\big) &= E(a_0Q) + E(u\psi Q) - \frac{1}{2}E(QbQ) \\
&= \frac{n_c^2+2n_c}{2(n_c+1)^2}a_0b^{-1}a_0 + Tr(\psi\sum_j \boldsymbol{B_j D_j T}\alpha^{-1}) \\
&\quad - \frac{1}{2}Tr\big(b(\sum_j \boldsymbol{B_j D_j})(\boldsymbol{T}\alpha^{-1}\boldsymbol{T}^T + \boldsymbol{\beta}^{-1})(\sum_j \boldsymbol{B_j D_j})^T\big).
\end{aligned} \tag{28}$$

From this equation and Equations (26) and (27), we can prove this proposition.

A.2 Proofs of Chapter 2: Information Provision Policies for Improving Farmer Welfare in Developing Countries: Heterogeneous Farmers and Market Selection

A.2.1 Proof of Lemma 6

First, we prove that farmers who receive signals follow a threshold market selection rule. If not, there exist realized signals (x_l, x_r), and a pair of farmers who receive signals, farmers 1 and 2, at θ_1 and θ_2 ($\theta_2 > \theta_1$), such that farmer 1 chooses market r and farmer 2 chooses market l. Because farmer 1 chooses market r, we have $E(p_r|x_l,x_r) - t(0.5-\theta_1) \geq E(p_l|x_l,x_r) - t(0.5+\theta_1)$. As a result, $E(p_r|x_l,x_r) - t(0.5-\theta_2) > E(p_l|x_l,x_r) - t(0.5+\theta_2)$ and farmer 2 should also choose market r. This is a contradiction.

According to a similar argument, farmers without signals also follow a threshold market selection rule. Because we focus on the equilibrium which is symmetric about the origin 0, for farmers without signals, the expected numbers of the farmers who receive signals in the two markets are the same. As a result, the origin 0 is chosen as the threshold. On the other hand, farmers with signals have the correct belief about the actions of farmers without signals.

Given the realized signals (x_l, x_r), if the threshold $\tau^{(\delta)}$ lies within $(-K, K)$, a farmer at $\tau^{(\delta)}$ who receives signals should be indifferent about the market to sell in:

$$\begin{aligned}
& A + \tfrac{\beta x_l}{\alpha+\beta} - b(\rho(\tfrac{1}{2}+\tau^{(\delta)}) + (1-\rho)\tfrac{1}{2}) - t(\tfrac{1}{2}+\tau^{(\delta)}) \\
= \ & A + \tfrac{\beta x_r}{\alpha+\beta} - b(\rho(\tfrac{1}{2}-\tau^{(\delta)}) + (1-\rho)\tfrac{1}{2}) - t(\tfrac{1}{2}-\tau^{(\delta)}).
\end{aligned} \tag{29}$$

Therefore, we have $\tau^{(\delta)} = \frac{1}{2(\rho b+t)} \cdot \frac{\beta}{\alpha+\beta} \cdot (x_l - x_r)$. Note that this $\tau^{(\delta)}$ is truncated at K and $-K$. In these situations, the signals suggest that one market is much better than the other one and all farmers who receive signals select the better market.

A.2.2 Proof of Lemma 7

The first statement is from (1) $\Pi_0^{(\delta)}(\theta) = E_{(x_l,x_r)}(\pi_0^{(\delta)}(\theta; x_l, x_r))$; (2) $E_{(x_l,x_r)}(E(u_i|(x_l, x_r))) = E_{(x_l,x_r)}(\frac{\beta x_i}{\alpha+\beta}) = 0$ for $i = l, r$; and (3) $E_{(x_l,x_r)}(\tau^{(\delta)}) = 0$.

The third statement is obtained by comparing (11) and (12). Without loss of generality, for a farmer at $\theta \geq 0$, we can observe that $\pi_0^{(\delta)}(\theta; x_l, x_r) = \pi^{(\delta)}(\theta; x_l, x_r)$ when $\theta \geq \tau^{(\delta)}$. When $\theta < \tau^{(\delta)}$ (this situation happens with a positive probability), it is easy to check that $\pi_0^{(\delta)}(\theta; x_l, x_r) < \pi^{(\delta)}(\theta; x_l, x_r)$.

As for the second statement, from the above argument, for a farmer at $\theta \geq 0$, we have:

$$\begin{aligned}
& \Pi^{(\delta)}(\theta) - \Pi_0^{(\delta)}(\theta) = E_{(x_l,x_r)}(\pi^{(\delta)}(\theta; x_l, x_r) - \pi_0^{(\delta)}(\theta; x_l, x_r)) \\
= & \int_{-\infty}^{\infty} \int_{x_r+2(\rho b+t)\frac{\alpha+\beta}{\beta}\theta}^{x_r+2(\rho b+t)\frac{\alpha+\beta}{\beta}K} \left[\frac{\beta}{\alpha+\beta}(x_l - x_r) - \frac{\rho b}{\rho b+t}\frac{\beta}{\alpha+\beta}(x_l - x_r) - 2t\theta\right] f_X(x_l) f_X(x_r) \mathrm{d}x_l \mathrm{d}x_r \\
+ & \int_{-\infty}^{\infty} \int_{x_r+2(\rho b+t)\frac{\alpha+\beta}{\beta}K}^{\infty} \left[\frac{\beta}{\alpha+\beta}(x_l - x_r) - 2\rho bK - 2t\theta\right] f_X(x_l) f_X(x_r) \mathrm{d}x_l \mathrm{d}x_r \\
= & \int_{-\infty}^{\infty} \int_{2(\rho b+t)\frac{\alpha+\beta}{\beta}\theta}^{2(\rho b+t)\frac{\alpha+\beta}{\beta}K} \left[\frac{t}{\rho b+t}\frac{\beta}{\alpha+\beta}y - 2t\theta\right] f_Y(y) f_W(w) \mathrm{d}y \mathrm{d}w \\
+ & \int_{-\infty}^{\infty} \int_{2(\rho b+t)\frac{\alpha+\beta}{\beta}K}^{\infty} \left[\frac{\beta}{\alpha+\beta}y - 2\rho bK - 2t\theta\right] f_Y(y) f_W(w) \mathrm{d}y \mathrm{d}w \\
= & \sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}\frac{t}{\rho b+t} exp\left[-\frac{\alpha(\rho b+t)^2(\alpha+\beta)}{\beta}\theta^2\right] - 2t\theta\Phi\left[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}\theta\right] \\
+ & \sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}\frac{\rho b}{\rho b+t} exp\left[-\frac{\alpha(\rho b+t)^2(\alpha+\beta)}{\beta}K^2\right] - 2\rho bK\Phi\left[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}K\right],
\end{aligned} \tag{30}$$

where $y \equiv x_l - x_r$ and $w \equiv x_l + x_r$. Also, $f_X(x_l)$, $f_X(x_r)$, $f_Y(y)$, and $f_W(w)$ are the probability density functions of x_l, x_r, y, and w.

From a direct integration, we can get

$$\begin{aligned}
W^{(\delta)} = & \frac{\rho t}{(\rho b+t)^2}\frac{\beta}{\alpha(\alpha+\beta)}\left[\frac{1}{2} - \Phi\left[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}K\right]\right] - 2\rho(2\rho b + t)K^2\Phi\left[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}K\right] \\
+ & \frac{\rho(2\rho b+t)}{(\rho b+t)}\sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}K exp\left[-\frac{\alpha(\rho b+t)^2(\alpha+\beta)}{\beta}K^2\right] + A - \frac{1}{4}t - \frac{1}{2}b.
\end{aligned} \tag{31}$$

Finally, without loss of generality, when $\theta > 0$, $\frac{\mathrm{d}\Pi^{(\delta)}(\theta)}{\mathrm{d}\theta} = -2t\Phi\left[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}\theta\right] + t > 0$ and $\frac{\mathrm{d}\Pi_0^{(\delta)}(\theta)}{\mathrm{d}\theta} = t$. The remaining parts of this lemma are proved.

A.2.3 Proof of Lemma 8

This lemma follows from (16) and (17).

A.2.4 Proof of Proposition 12

This proposition follows from (11), (12), (14), (16), (17), and some algebra.

A.2.5 Proof of Proposition 13

This proposition follows from Proposition 12 and the fact that $E_{(x_l,x_r)}((x_l - x_r)^2) = \frac{2(\alpha+\beta)}{\alpha\beta}$.

A.2.6 Proof of Proposition 14

We provide an example to support the first part: Suppose $\alpha = \beta = 1$, $b = 1$, and $t = 0.5$, we have $W^{(F1)} - W^{(F0)} = 0.072555 < 0.077870 = W^{(F\rho)}(\rho = 0.7) - W^{(F0)}$. The second part follows from (33) with $K = 0.5$. Note that in the approximation in this proposition, the K' in (33) goes to infinity.

A.2.7 Proof of Proposition 15

We have:

$$\begin{aligned}
\frac{\partial W^{(K\rho)}(K,\rho)}{\partial K} &= 2\rho\sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}exp\Big[-\frac{\alpha(\rho b+t)^2(\alpha+\beta)}{\beta}K^2\Big] - 4\rho(2\rho b+t)K\Phi\Big[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}K\Big] \\
&= 2\rho\sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}exp\Big[-\frac{1}{2}K'^2\Big] - \frac{4\rho(2\rho b+t)}{\rho b+t}\sqrt{\frac{\beta}{2\alpha(\alpha+\beta)}}K'\Psi\Big[-K'\Big] \\
&= 2\rho\sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}exp\Big[-\frac{1}{2}K'^2\Big]\left\{1 - \frac{(2\rho b+t)}{\rho b+t}\sqrt{2\pi}\frac{K'\Psi\Big[-K'\Big]}{exp\Big[-\frac{1}{2}K'^2\Big]}\right\}
\end{aligned} \tag{32}$$

and

$$\begin{aligned}
\frac{\partial W^{(K\rho)}(K,\rho)}{\partial \rho} &= \frac{t(-\rho b+t)}{(\rho b+t)^3}\frac{\beta}{\alpha(\alpha+\beta)}\left[\frac{1}{2} - \Phi\Big[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}K\Big]\right] - (8\rho b+2t)K^2\Phi\Big[-\frac{2(\rho b+t)(\alpha+\beta)}{\beta}K\Big] \\
&\quad + \frac{2\rho^2b^2+5\rho bt+t^2}{(\rho b+t)^2}\sqrt{\frac{\beta}{\pi\alpha(\alpha+\beta)}}Kexp\Big[-\frac{\alpha(\rho b+t)^2(\alpha+\beta)}{\beta}K^2\Big] \\
&= \frac{t(-\rho b+t)}{(\rho b+t)^3}\frac{\beta}{\alpha(\alpha+\beta)}\left[\frac{1}{2} - \Psi\Big[-K'\Big]\right] - \frac{8\rho b+2t}{(\rho b+t)^2}\frac{\beta}{2\alpha(\alpha+\beta)}K'^2\Psi\Big[-K'\Big] \\
&\quad + \frac{2\rho^2b^2+5\rho bt+t^2}{(\rho b+t)^3}\sqrt{\frac{1}{2\pi}}\frac{\beta}{\alpha(\alpha+\beta)}K'exp\Big[-\frac{1}{2}K'^2\Big],
\end{aligned} \tag{33}$$

where $K' = \sqrt{\frac{2\alpha(\alpha+\beta)}{\beta}}(\rho b+t)K$ and $\Psi(x) \equiv \int_{-\infty}^{x}\sqrt{\frac{1}{2\pi}}exp\Big[-\frac{1}{2}y^2\Big]\mathrm{d}y$. Let (K^*, ρ^*) be the optimal solution, and let K'^* be the corresponding K'. We have $\frac{\partial W^{(K\rho)}}{\partial K}(K^*, \rho^*) \geq 0$, where $\frac{\partial W^{(K\rho)}}{\partial K}(K^*, \rho^*) = 0$ if $K^* < 0.5$. Note that $K = 0$ is never optimal. Since $\frac{\partial W^{(K\rho)}}{\partial K}(K^*, \rho^*) \geq 0$, we have:

$$\begin{aligned} & \frac{\partial W^{(K\rho)}}{\partial \rho}(K^*, \rho^*) \\ \geq \; & \frac{t}{(\rho^* b+t)^3} \frac{\beta}{\alpha(\alpha+\beta)} \Bigg\{ (-\rho^* b + t) \Bigg[\frac{1}{2} - \Psi\Big[- K'^* \Big] \Bigg] + \frac{\rho^* b(3\rho^* b+t)}{(2\rho^* b+t)} \sqrt{\frac{1}{2\pi}} K'^* exp\Big[-\frac{1}{2} K'^{*2} \Big] \Bigg\} \end{aligned} \tag{34}$$

When $\rho^* b \leq t$, we have $\frac{\partial W^{(K\rho)}}{\partial \rho}(K^*, \rho^*) > 0$. On the other hand, when $\rho^* b > t$, we have:

$$\begin{aligned} & \frac{\partial W^{(K\rho)}}{\partial \rho}(K^*, \rho^*) \\ \geq \; & \frac{t}{(\rho^* b+t)^3} \frac{\beta}{\alpha(\alpha+\beta)} (\rho^* b - t) \Bigg\{ - \Bigg[\frac{1}{2} - \Psi\Big[- K'^* \Big] \Bigg] + \frac{\rho^* b(3\rho^* b+t)}{(2\rho^* b+t)(\rho^* b - t)} \sqrt{\frac{1}{2\pi}} K'^* exp\Big[-\frac{1}{2} K'^{*2} \Big] \Bigg\} \\ \geq \; & \frac{t}{(\rho^* b+t)^3} \frac{\beta}{\alpha(\alpha+\beta)} (\rho^* b - t) \Bigg\{ - \Bigg[\frac{1}{2} - \Psi\Big[- K'^* \Big] \Bigg] + \frac{3}{2} \sqrt{\frac{1}{2\pi}} K'^* exp\Big[-\frac{1}{2} K'^{*2} \Big] \Bigg\}. \end{aligned} \tag{35}$$

When $\rho^* b > t$, $1 - \frac{3}{2}\sqrt{2\pi}\frac{K'^*\Psi\left[-K'^*\right]}{exp\left[-\frac{1}{2}K'^{*2}\right]} > 1 - \frac{(2\rho^* b+t)}{\rho^* b+t}\sqrt{2\pi}\frac{K'^*\Psi\left[-K'^*\right]}{exp\left[-\frac{1}{2}K'^{*2}\right]} \geq 0$. It can be shown numerically that $K'^* < 1.0361$. ($\frac{K'\Psi\left[-K'\right]}{exp\left[-\frac{1}{2}K'^2\right]}$ is increasing in K'.) It can be shown numerically that $\frac{\partial W^{(K\rho)}}{\partial \rho}(K^*, \rho^*) > 0$ within this range. Since $\frac{\partial W^{(K\rho)}}{\partial \rho}(K^*, \rho^*)$ is always greater than zero, we have $\rho^* = 1$.

Due to the fact that $\frac{K'\Psi\left[-K'\right]}{exp\left[-\frac{1}{2}K'^2\right]}$ is increasing in K', there exists a $\bar{K} > 0$ such that $K^* = \min[\bar{K}, 0.5]$. To be more specific, $W^{(K\rho)}(K, \rho = 1)$ is increasing in K when $K < \bar{K}$ and decreasing in K when $K > \bar{K}$.

As for the last part, let $\bar{K}'$ be the K' corresponding to $\bar{K}$. $\bar{K}'$ is independent of α and β, and is decreasing in b. Also, given a fixed $\bar{K}'$, $\bar{K}$ is increasing in β, and is decreasing in α and b.

A.2.8 Proof of Proposition 16

We define $CP(K_c, \rho_c)$ as the company's profit, and $W^{(FPC)}(K_c, \rho_c)$ as farmers' ex-ante expected total profit associated with the company's policy $\delta = (K_c, \rho_c)$. We have:

$$\begin{aligned}
CP(K_c,\rho_c) =& \; 2K_c\rho_c[\Pi^{(\delta)}(K_c) - \Pi_0^{(\delta)}(K_c)] \\
=& \; 2K_c\rho_c\Bigg\{\sqrt{\tfrac{\beta}{\pi\alpha(\alpha+\beta)}}exp\Big[-\tfrac{\alpha(\rho_c b+t)^2(\alpha+\beta)}{\beta}K_c^2\Big] \\
& -2(\rho_c b+t)K_c\Phi\Big[-\tfrac{2(\rho_c b+t)(\alpha+\beta)}{\beta}K_c\Big]\Bigg\}, \\
W^{(FPC)}(K_c,\rho_c) =& \; W^{(K\rho)}(K_c,\rho_c) - CP(K_c,\rho_c), \\
\tfrac{\partial CP(K_c,\rho_c)}{\partial K_c} =& \; 2\rho_c\sqrt{\tfrac{\beta}{\pi\alpha(\alpha+\beta)}}exp\Big[-\tfrac{1}{2}K_c'^2\Big]\Bigg\{1-2\sqrt{2\pi}\tfrac{K_c'\Psi\big[-K_c'\big]}{exp\big[-\frac{1}{2}K_c'^2\big]}\Bigg\}, \\
\tfrac{\partial CP(K_c,\rho_c)}{\partial \rho_c} =& \; 2K_c\sqrt{\tfrac{\beta}{\pi\alpha(\alpha+\beta)}}exp\Big[-\tfrac{1}{2}K_c'^2\Big]\Bigg\{1-\tfrac{2\rho_c b+t}{\rho_c b+t}\sqrt{2\pi}\tfrac{K_c'\Psi\big[-K_c'\big]}{exp\big[-\frac{1}{2}K_c'^2\big]}\Bigg\},
\end{aligned} \tag{36}$$

where $K_c' = \sqrt{\frac{2\alpha(\alpha+\beta)}{\beta}}(\rho_c b+t)K_c$.

Because $\frac{\partial CP}{\partial K_c}(K_c^*, \rho_c^*) \geq 0$, $\frac{\partial CP}{\partial \rho_c}(K_c^*, \rho_c^*)$ is greater than zero. As a result, we have $\rho_c^* = 1$. The second statement of this proposition follows from the fact that $\frac{K_c'\Psi\big[-K_c'\big]}{exp\big[-\frac{1}{2}K_c'^2\big]}$ is increasing in K_c', and the third statement results from a similar argument as that in the proof of Proposition 15. The fourth statement comes from the fact that $\bar{K}_c < \bar{K}$, which is obtained by comparing (32) and the third equation in (36).

A.2.9 Proof of Proposition 17

A direct calculation of $\frac{\partial \Pi^{(\delta)}}{\partial \beta}$ shows that $\Pi^{(\delta)}(\theta)$ is always increasing in β for any θ and any $\delta = (K, \rho)$.

A.2.10 Proof of Proposition 18

Statement 1(a) follows from the fact that under policy (C), the government is maximizing farmers' ex-post expected total profit according to the signals. As for statement 1(b), note that even when farmer are distributed according to a symmetric distribution, they still follow threshold market selection rules under any provision policy. Without loss of generality, we assume that $x_l > x_r$. In this situation, it is easy to see that $\tau^{(C)} > 0$. Also, for the farmer at $\tau^{(C)}$, switching to market l results in a positive individual profit increment to him, and creates a negative externality on other farmers' total profit. On the other hand, under policy

(F1), suppose $\tau^{(F1)} < \tau^{(C)}$. The farmer at $\tau^{(C)}$ should switch to market l because (1) there are fewer farmers in market l than that under policy (C) and (2) he does not care about other farmers' profit under policy (F1). This is a contradiction. Therefore, we have $\tau^{(F1)} \geq \tau^{(C)}$.

Statement 2(a) is obtained because (1) $w^{(C)} - w^{(F1)} \geq 0 \; \forall (x_l, x_r)$ and (2) $\exists (x_l, x_r)$ such that $w^{(C)} - w^{(F1)} > 0$. Statement 2(b) follows from Statements (1) and (3) in Lemma 7, which continue to hold under a general symmetric distribution of farmers. (Statement (1) in Lemma 7 holds because for farmers without signals, the expected number of farmers in each market is still 0.5. Statement (3) in Lemma 7 is due to the following two observations. First, given any (x_l, x_r), a farmer who observes signals can always deviate and choose to behave as a farmer without signals (and, hence, $\pi^{(\delta)}(\theta; x_l, x_r) \geq \pi_0^{(\delta)}(\theta; x_l, x_r) \; \forall (x_l, x_r)$). Second, there exist situations in which switching can generate more profit (i.e., $\exists (x_l, x_r)$ $s.t.$ $\pi^{(\delta)}(\theta; x_l, x_r) > \pi_0^{(\delta)}(\theta; x_l, x_r)$).

Statement (3) is supported by the example we provide in the proof of Proposition 14. Note that in that example, we have $\rho^* < 1$ as long as $h(\theta)$ does not deviate from the uniform distribution too much.

A.3 Proofs of Chapter 3: Role of Exchangeable Tickets in the Optimal Menu Design for Airline Tickets

A.3.1 Proof of Subsection 4.3.1

Refundable ticket. With ticket refund, an additional stage of decision making arises. After the valuation uncertainty is resolved, the consumers who bought a refundable ticket will cancel it if and only if the realized value is lower than the refund amount, P_r. Given this post-purchase strategy, if $P_r > V_i$, a segment-i consumer who bought the ticket will cancel it no matter whether she should change her schedule or not. However, if this is the case, the consumer will get nothing and she should not purchase it at all.

Exchangeable ticket. After the uncertainty is resolved, the consumers who bought an exchangeable ticket will cancel it if and only if the realized value is lower than the expected value of the refunded P_e credits. If $P_e > V_i$, a segment-i consumer always gets a nonpositive (or even negative when the possibility of using the credits is not one) utility from purchasing this ticket, no matter which post-purchase action she takes. Hence, the segment-i consumers will not purchase the ticket when $P_e > V_i$. When $P_e \leq V_i$, a segment-i consumer who bought the ticket will not cancel it when she need not change her plan. However, if she needs to change her plan, she will cancel it. (Even if she does not have an immediate demand for another ticket, the possibility for her to need another ticket in the future T period is still positive and, hence, the credits are valuable for her.)

Incentive compatible constraints. When refundable, exchangeable, and nonrefundable tickets are acceptable to a segment-i consumer at prices P_r, P_e, and P_n, respectively, she will prefer refundable tickets to exchangeable tickets if and only if $(1-\beta_i+\alpha_i\beta_i)P_e \geq \alpha_i P_r$; she will prefer exchangeable tickets to nonrefundable tickets if and only if $P_n \geq (1-\beta_i+\alpha_i\beta_i)P_e$; she will prefer refundable tickets to nonrefundable tickets if and only if $P_n \geq \alpha_i P_r$.

A.3.2 Proof of Lemma 10

For any strategy in which the seller does not sell exchangeable tickets (or the most profitable tickets) to any segment, the seller can strictly increase his profit in the following way: Starting from the original strategy, he fixes the prices of all types of tickets which have been purchased by consumers. He starts to provide exchangeable tickets (or the most profitable tickets) at an extremely high price, and then gradually reduces this price until at least one segment switches to these tickets. By doing so, he can create a new feasible strategy in which consumers will get the same amount of surplus as before (or a surplus higher than before by an infinitesimal amount) and he can earn a strictly higher profit because exchangeable tickets (or the most profitable tickets) can create more surplus.

A.3.3 Proof of Proposition 19

According to Lemma 10, the seller must sell exchangeable tickets to at least one segment in the optimal strategy. As a result, we only need to consider the EE, ER, EN, EO, RE, NE, and OE strategies. First, in Scenario H, we will show that only the EE, ER, EN, and EO strategies are the possible candidates for the optimal strategy.

The EE strategy. This strategy is always feasible. The seller charges $P_e = \frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l}$, which is the low-segment consumers' WTP for exchangeable tickets, and his profit is $q_h[\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_h)(s_2-\beta_h c)] + q_l[\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_l)(s_2-\beta_l c)]$.

The ER strategy. This strategy is feasible if and only if $\frac{1-\beta_l+\alpha_l\beta_l}{\alpha_l} > \frac{1-\beta_h+\alpha_h\beta_h}{\alpha_h}$. Under this condition, the low segment is easier to be attracted by refundable tickets than the high segment, given any specific price of exchangeable tickets. In this strategy, the seller charges $P_r = V_l$ and leaves a null surplus to the low-segment consumers. He also charges $P_e = \frac{\alpha_h V_l}{1-\beta_h+\alpha_h\beta_h}$ and gives the high-segment consumers a positive surplus, which is barely enough to keep them from switching to refundable tickets. On the other hand, P_e is higher than the low-segment consumers' WTP for exchangeable tickets, and, hence, they will not switch to exchangeable tickets. The seller's profit is $q_h[\frac{\alpha_h V_l}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2-\beta_h c)] + q_l[\alpha_l V_l + (1-\alpha_l)s_2]$.

The EN strategy. This strategy is feasible if and only if $1-\beta_l+\alpha_l\beta_l > 1-\beta_h+\alpha_h\beta_h$. Under this condition, the low segment is easier to be attracted by nonrefundable tickets than the high segment, given any specific price of exchangeable tickets. In this strategy, the seller charges $P_n = \alpha_l V_l$ and leaves a null surplus to the low-segment consumers. He also charges $P_e = \frac{min[\alpha_h V_h, \alpha_l V_l]}{1-\beta_h+\alpha_h\beta_h}$ and gives each high-segment consumer a surplus $max[\alpha_h V_h - \alpha_l V_l, 0]$, which is barely enough to keep them from switching to nonrefundable tickets. On the other hand, P_e is higher than the low-segment consumers' WTP for exchangeable tickets, and, thus, they will not switch to exchangeable tickets. The seller's profit is $q_h[\frac{min[\alpha_h V_h, \alpha_l V_l]}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)] + q_l[\alpha_l V_l]$.

The EO **strategy.** This strategy is always feasible. In this strategy, the seller simply ignores the low segment and charges $P_e = \frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h}$, which is the high-segment consumers' WTP for exchangeable tickets. His profit is $q_h[\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)] + q_l s_1$.

The RE **strategy.** This strategy, if feasible, is never optimal. Since the low segment has a lower WTP for exchangeable tickets, the seller should leave the high-segment consumers a certain amount of surplus to keep them from switching to exchangeable tickets. The seller can be strictly better off by simply removing the option of refundable tickets and let all consumers purchase exchangeable tickets. By doing so, consumers will not get a higher surplus, and the seller can get a strictly higher profit since exchangeable tickets can create more surplus.

The NE **strategy.** According to a similar argument as that for the RE strategy above, this strategy, if feasible, is always strictly dominated by the EE strategy.

The OE **strategy.** This strategy is never feasible since the high segment has a higher WTP for exchangeable tickets than the low segment.

Second, in Scenario L, we will show that only the EE and RE strategies are the possible candidates for the optimal strategy.

The EE **strategy.** This strategy is always feasible. The seller charges $P_e = \frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h}$, which is the high-segment consumers' WTP for exchangeable tickets. The profit is $q_h[\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)] + q_l[\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_l)(s_2 - \beta_l c)]$.

The RE **strategy.** This strategy is feasible if and only if $\frac{1-\beta_h+\alpha_h\beta_h}{\alpha_h} > \frac{1-\beta_l+\alpha_l\beta_l}{\alpha_l}$. However, this condition is implied by the fact that $\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} \leq \frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l}$ and $V_h > V_l$. As a result, this strategy is always feasible. In this strategy, the seller charges $P_r = V_h$ and $P_e = \frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l}$ and earn a profit of $q_h[\alpha_h V_h + (1-\alpha_h)s_2] + q_l[\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_l)(s_2 - \beta_l c)]$. One thing worth mentioning is that the seller does not leave any surplus to the consumers in this strategy.

Other strategies. In the RE strategy, the seller sells refundable tickets to the high-segment consumers and grasps all the surplus from them. He also gets the highest possible profit from the low segment. As a result, except the RE strategy, only strategies in which the seller sells exchangeable tickets to the high segment have the possibility to weakly dominate the RE strategy. Therefore, except the RE strategy, we only need to consider the EE, ER, EN, and EO strategies. The ER strategy is never feasible since $\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} \leq \frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l}$ and $V_h > V_l$ imply $\frac{1-\beta_h+\alpha_h\beta_h}{\alpha_h} > \frac{1-\beta_l+\alpha_l\beta_l}{\alpha_l}$. According to a similar argument as that for the RE strategy in Scenario H, the EN strategy, if feasible, is always strictly dominated by the EE strategy. Finally, the EO strategy is never feasible.

A.3.4 Preparation for Cancellation Fee

Refundable tickets. When $P_r > V_i$, a segment-i consumer cannot get any benefit (and has a net loss if $f_r > 0$) by purchasing the refundable ticket. Hence, in this situation, she will not purchase the ticket. On the other hand, when $P_r \leq V_i$, a segment-i consumer who purchases a refundable ticket will cancel the ticket if and only if she needs to change her plan after the uncertainty is resolved. In this situation, a segment-i consumer's expected utility from buying the ticket is $\alpha_i V_i + (1-\alpha_i)(P_r - f_r) - P_r$. Given a cancellation fee f_r, her WTP for a refundable ticket is $V_i - \frac{1-\alpha_i}{\alpha_i} f_r$. Note that because of the constraint $P_r \geq f_r$, if the seller wants to sell refundable tickets to the segment-i consumers, he should have $f_r \leq \alpha_i V_i$.

Exchangeable tickets. According to a similar argument, a segment-i consumer will not purchase the exchangeable ticket if $P_e > V_i$. On the other hand, when $P_e \leq V_i$, a segment-i consumer who purchases an exchangeable ticket will cancel the ticket if and only if she needs to change her plan after the uncertainty is resolved. In this situation, a segment-i consumer's expected utility from buying the ticket is $\alpha_i V_i + (1-\alpha_i)\beta_i(P_e - f_e) - P_e$. Given a cancellation fee f_e, her WTP for an exchangeable ticket is $\frac{\alpha_i V_i - (1-\alpha_i)\beta_i f_e}{1-\beta_i+\alpha_i\beta_i}$. Note that because of the constraint $P_e \geq f_e$, if the seller wants to sell exchangeable tickets to the segment-i consumers, he should have $f_e \leq \alpha_i V_i$.

Given a cancellation fee f_r, by selling refundable tickets at the price P_r to the segment-i consumers, the seller can earn a profit $q_i(\alpha_i P_r + (1-\alpha_i)(s_2 + f_r))$. On the other hand, given a cancellation fee f_e, by selling exchangeable tickets at the price P_e to the segment-i consumers, the seller can earn a profit $q_i[P_e + (1-\alpha_i)(s_2 - \beta_i(c - f_e))]$.

A.3.5 Proof of the second part of Lemma 12

First, we focus on Scenario H. Given a f_e, if the high-segment consumers still have a higher WTP for exchangeable tickets, the seller's profit when using the EE strategy is $q_h[\frac{\alpha_l V_l - (1-\alpha_l)\beta_l f_e}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_h)(s_2 - \beta_h(c - f_e))] + q_l[\frac{\alpha_l V_l - (1-\alpha_l)\beta_l f_e}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_l)(s_2 - \beta_l(c - f_e))]$. This profit is strictly increasing in f_e if and only if $q_h[(1-\alpha_h)\beta_h - \frac{(1-\alpha_l)\beta_l}{1-\beta_l+\alpha_l\beta_l}] + q_l[(1-\alpha_l)\beta_l - \frac{(1-\alpha_l)\beta_l}{1-\beta_l+\alpha_l\beta_l}] > 0$.

Starting from $f_e = 0$, if increasing f_e is profitable, the seller will want to increase f_e up to the point at which these two segments have the same WTPs for exchangeable tickets. At this point, the seller can grasp all the surplus from consumers, and, hence, any further increase in f_e only results in a loss of profit. However, there are two constraints on f_e, $f_e \leq \alpha_h V_h$ and $f_e \leq \alpha_l V_l$. Therefore, the seller should increase f_e until the two segments have the same WTPs or f_e reaches any one of its upper bounds. When $\alpha_h V_h < \alpha_l V_l$, the point at which the two segments have the same WTPs will come before $\alpha_h V_h$. As a result, the seller will increase f_e up to $min[\frac{\alpha_h V_h(1-\beta_l+\alpha_l\beta_l) - \alpha_l V_l(1-\beta_h+\alpha_h\beta_h)}{(1-\alpha_h)\beta_h - (1-\alpha_l)\beta_l}, \alpha_l V_l]$. On the other hand, if increasing f_e is not profitable, the seller should choose $f_e = 0$.

Next, in Scenario L, according to a similar argument as above, the seller should choose

$f_e = min[\frac{\alpha_l V_l(1-\beta_h+\alpha_h\beta_h)-\alpha_h V_h(1-\beta_l+\alpha_l\beta_l)}{(1-\alpha_l)\beta_l-(1-\alpha_h)\beta_h}, \alpha_h V_h]$ if $q_h[(1-\alpha_h)\beta_h - \frac{(1-\alpha_h)\beta_h}{1-\beta_h+\alpha_h\beta_h}] + q_l[(1-\alpha_l)\beta_l - \frac{(1-\alpha_h)\beta_h}{1-\beta_h+\alpha_h\beta_h}] > 0$, and $f_e = 0$ otherwise.

A.3.6 Proof of Lemma 13

First, we focus on Scenario H. When the seller wants to use the ER strategy, according to Lemma 12, he chooses $f_e = 0$. Given a specific P_e, the ER strategy is feasible if and only if $\frac{1-\beta_l+\alpha_l\beta_l}{\alpha_l}P_e - \frac{1-\alpha_l}{\alpha_l}f_r > \frac{1-\beta_h+\alpha_h\beta_h}{\alpha_h}P_e - \frac{1-\alpha_h}{\alpha_h}f_r$. This feasibility condition not only depends on the parameters but also depends on the price of exchangeable tickets. However, the seller has no incentive to charge a P_e lower than the low-segment consumers' WTP for exchangeable tickets to pursue the feasibility of the ER strategy, (since this strategy will be strictly dominated by the EE strategy). As a result, we change the feasibility condition to $V_l - \frac{1-\alpha_l}{\alpha_l}f_r > \frac{1-\beta_h+\alpha_h\beta_h}{\alpha_h}\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} - \frac{1-\alpha_h}{\alpha_h}f_r$. That is, when P_e equals the low-segment consumers' WTP for exchangeable tickets, the low segment is easier to be attracted by refundable tickets. When $\alpha_h < \alpha_l$, this feasibility is increasing in f_r.

When using the ER strategy, if the high-segment consumers have a higher WTP for refundable tickets than the low-segment consumers, the information rent given to the high-segment consumers is increasing in the difference between the two segments' WTPs for refundable tickets. Hence, the seller always wants to use f_r to reduce this difference. Increasing f_r can indeed reduce this difference if and only if $\alpha_h < \alpha_l$. Hence, if $\alpha_h < \alpha_l$, the seller wants to increase f_r up to the point at which these two segments have the same WTPs for refundable tickets. However, f_r cannot exceed $\alpha_l V_l$. Therefore, the seller should increase f_r up to $min[\frac{\alpha_h\alpha_l(V_h-V_l)}{\alpha_l-\alpha_h}, \alpha_l V_l]$. This optimal f_r can also help the seller maximize the feasibility of the ER strategy. Note that this optimal f_r will not exceed $\alpha_h V_h$ since if $\alpha_h V_h < \alpha_l V_l$, we will have $\frac{\alpha_h\alpha_l(V_h-V_l)}{\alpha_l-\alpha_h} < \alpha_h V_h$. On the other hand, if $\alpha_h \geq \alpha_l$, increasing f_r can neither improve the feasibility of the ER strategy nor reduce the difference between consumers' WTPs. Hence, in this situation, the seller should charge $f_r = 0$.

Second, in Scenario L, when using the RE strategy, again the seller should choose $f_e = 0$. However, he can charge a positive f_r. As long as the high-segment consumers still have a higher WTP for refundable tickets than the low-segment consumers, the RE strategy is feasible, and the seller can grasp all the surplus from the high segment and get the highest possible profit from the low segment in this strategy. Since a positive f_r does not hurt the refundable tickets' profitability, the seller can get the same amount of profit as the case in which $f_r = 0$. Hence, charging a positive f_r has no influence on the profitability and feasibility of the RE strategy in this range.

A.3.7 Proof of Proposition 20

According to Lemma 10 and Lemma 11, again, we only need to consider the EE, ER, EN, EO, RE, NE, and OE strategies. First, in Scenario H, we will show that only the EE, ER, and EO strategies are the possible candidates for the optimal strategy.

The EE strategy. This strategy is always feasible. According to the proof of the second part of Lemma 12, the seller should charge $P_e = \frac{\alpha_l V_l - (1-\alpha_l)\beta_l f_e}{1-\beta_l+\alpha_l\beta_l}$, and his profit is

$$q_h[\frac{\alpha_l V_l - (1-\alpha_l)\beta_l f_e}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_h)(s_2 - \beta_h(c-f_e))] + q_l[\frac{\alpha_l V_l - (1-\alpha_l)\beta_l f_e}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_l)(s_2 - \beta_l(c-f_e))],$$

where $f_e = min[\frac{\alpha_h V_h(1-\beta_l+\alpha_l\beta_l) - \alpha_l V_l(1-\beta_h+\alpha_h\beta_h)}{(1-\alpha_h)\beta_h-(1-\alpha_l)\beta_l}, \alpha_l V_l]$ if $q_h[(1-\alpha_h)\beta_h - \frac{(1-\alpha_l)\beta_l}{1-\beta_l+\alpha_l\beta_l}] + q_l[(1-\alpha_l)\beta_l - \frac{(1-\alpha_l)\beta_l}{1-\beta_l+\alpha_l\beta_l}] > 0$, and $f_e = 0$ otherwise.

The ER strategy. According to Lemma 12, the seller should choose $f_e = 0$. Then according to the proof of Lemma 13, he will choose $f_r = min[\frac{\alpha_h\alpha_l(V_h-V_l)}{\alpha_l-\alpha_h}, \alpha_l V_l]$ if $\alpha_h < \alpha_l$, and $f_r = 0$ otherwise. This strategy is feasible if and only if $V_l - \frac{1-\alpha_l}{\alpha_l} f_r > \frac{1-\beta_h+\alpha_h\beta_h}{\alpha_h}\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} - \frac{1-\alpha_h}{\alpha_h} f_r$. If this strategy is feasible, the seller should charge $P_r = V_l - \frac{1-\alpha_l}{\alpha_l} f_r$ and $P_e = \frac{\alpha_h V_l + [\frac{\alpha_l-\alpha_h}{\alpha_l}] f_r}{1-\beta_h+\alpha_h\beta_h}$, and his profit is $q_h[\frac{\alpha_h V_l + [\frac{\alpha_l-\alpha_h}{\alpha_l}] f_r}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)] + q_l(\alpha_l V_l + (1-\alpha_l)s_2)$.

The EN strategy. According to Lemma 12, the seller should choose $f_e = 0$ and, hence, this strategy is the same as that in the basic model. This strategy is feasible if and only if $1 - \beta_l + \alpha_l\beta_l > 1 - \beta_h + \alpha_h\beta_h$. His profit is $q_h[\frac{min[\alpha_h V_h, \alpha_l V_l]}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)] + q_l[\alpha_l V_l]$. However, this strategy, if feasible, is always strictly dominated by the ER strategy.

The EO strategy. According to Lemma 12, the seller should choose $f_e = 0$ and, hence, this strategy is the same as that in the basic model. This strategy is always feasible with a profit $q_h[\frac{\alpha_h V_h}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h c)] + q_l s_1$.

The RE, NE, and OE strategies. According to Lemma 12, the seller should choose $f_e = 0$, and the high-segment consumers have a higher WTP for exchangeable tickets. Due to a similar argument as that in the proof of Proposition 19, these strategies are never optimal.

In Scenario L, only the EE and RE strategies are possible candidates for the optimal strategy.

The EE strategy. This strategy is always feasible. According to the proof of the second part of Lemma 12, the seller should charge $P_e = \frac{\alpha_h V_h - (1-\alpha_h)\beta_h f_e}{1-\beta_h+\alpha_h\beta_h}$ and his profit is

$$q_h[\frac{\alpha_h V_h - (1-\alpha_h)\beta_h f_e}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_h)(s_2 - \beta_h(c-f_e))] + q_l[\frac{\alpha_h V_h - (1-\alpha_h)\beta_h f_e}{1-\beta_h+\alpha_h\beta_h} + (1-\alpha_l)(s_2 - \beta_l(c-f_e))],$$

where $f_e = min[\frac{\alpha_l V_l(1-\beta_h+\alpha_h\beta_h) - \alpha_h V_h(1-\beta_l+\alpha_l\beta_l)}{(1-\alpha_l)\beta_l-(1-\alpha_h)\beta_h}, \alpha_h V_h]$ if $q_h[(1-\alpha_h)\beta_h - \frac{(1-\alpha_h)\beta_h}{1-\beta_h+\alpha_h\beta_h}] + q_l[(1-\alpha_l)\beta_l - \frac{(1-\alpha_h)\beta_h}{1-\beta_h+\alpha_h\beta_h}] > 0$, and $f_e = 0$ otherwise.

The RE strategy. According to Lemma 12 and Lemma 13, the seller should choose $f_e = 0$ and the situation is the same as that in the basic model as long as the high-segment consumers still have a higher WTP for refundable tickets than the low-segment consumers. In this regime, this strategy is always feasible, and its profit is $q_h[\alpha_h V_h + (1-\alpha_h)s_2] + q_l[\frac{\alpha_l V_l}{1-\beta_l+\alpha_l\beta_l} + (1-\alpha_l)(s_2 - \beta_l c)]$.

The other strategies. According to a similar argument as that in the proof of Proposition 19, except the RE strategy, we only need to consider the EE, ER, EN, and EO strategies. Again, we can find that the ER, EN, and EO strategies are always strictly dominated or infeasible.

A.3.8 Proof of Proposition 21

According to an argument similar to that in Lemma 10, the seller only considers the RR, RN, RO, NR, and OR strategies here. (Note that refundable goods are the most profitable goods now.)

Now, we want to show that when $\alpha_h \geq \alpha_l$, providing a menu (the RN or the NR strategy) is never optimal. First, in this situation, the RN strategy is never feasible. Second, when $\alpha_h \geq \alpha_l$, the high-segment consumers always have a higher WTP for refundable goods than the low-segment consumers even with a positive f_r. Hence, according to a similar argument as that for the RE strategy in Scenario H in Proposition 19, the NR strategy, if it is feasible, is always strictly dominated by the RR strategy. Note that the OR strategy is also never feasible. Therefore, when $\alpha_h \geq \alpha_l$, the seller chooses the optimal strategy from the RR or the RO strategies.

A.3.9 Extra cost of exchanging tickets as a function of the amount of credits

The surplus created by an exchangeable ticket sold to a segment-i consumer is $\alpha_i V_i + (1 - \alpha_i)s_2 + (1-\alpha_i)\beta_i(P_e - c(P_e))$. Because $P_e - c(P_e)$ is still increasing in P_e and always positive, Lemma 9 and Lemma 10 do not change. The proof of Proposition 19 highly depends on the above two lemmas, so it also remains the same except that c in the profits should be changed to $c(P_e)$. Note that Corollary 6 and Corollary 7, which are from Proposition 19, also remain the same as before.

Moreover, we want to note that increasing the flexibility of exchangeable tickets increases the opportunity cost part in $c(P_e)$. This is because in this situation a consumer has a higher probability to be able to use the refunded credits to substitute the real money she would pay if she does not have these credits. Corollary 7 is enhanced by this force when we stop treating c as a constant.

Lemma 11, Lemma 12, Lemma 13, Proposition 20, and Corollary 8 also remain the same, except that when deciding the optimal f_e in the EE strategy, we should also take into account that c implicitly depends on f_e. Finally, Proposition 21 has nothing to do with c, so it does not change.