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ENERGY SPECTRA FROM “BEAM–TARGET” NUCLEAR REACTIONS IN MAGNETIC FUSION DEVICES

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Analytical expressions for the energy distribution of fusion products created in “beam–target” fusion reactions in a strong magnetic field are derived.

In controlled fusion experiments, the largest fusion reaction rates [1] and the most easily measured fusion spectra are achieved in plasmas with non-thermal populations. Measurements of these spectra can provide information about the “tail” (non-thermal part) of the ion distribution function that may be difficult to obtain using other diagnostic techniques. For example, recent measurements of the energy distribution of 15 MeV protons produced in the $d(^3\text{He}, p)\alpha$ fusion reaction during fast wave (ICRF) heating in the PLT tokamak provided the first experimental evidence that the ^3He ions accelerated by the waves are anisotropic in velocity space [2]. Expressions relating the temperature of thermal reactants to the width of the fusion spectrum are well known [3,4], but only one analytical treatment relating the distribution of non-thermal reactants to the energy distribution of fusion reaction products has been published [5] and this work contains errors [6]. Other workers have studied the fusion spectra produced by non-thermal plasmas numerically [7] but this approach, though accurate, is tedious and provides little physical insight into the factors affecting the fusion product energy distribution. This paper extends previous analytical work [5] by considering explicitly the effect of a strong magnetic field on the fusion spectra produced in “beam–target” reactions and by carrying the calculation to higher order in the ratio of beam energy to fusion energy. Under some conditions, the derived expressions permit a measured spectrum to be related directly to the beam-ion distribution that produced it.

For a reaction $2(1, 3)4$ where particles 1, 2, 3, 4 are the projectile, target, and products, respectively, the energy of particle 3 using non-relativistic kinematics is [3]

$$E_3 = \frac{m_4}{m_3 + m_4} (Q + K) + V \cos \theta \sqrt{\frac{2m_3 m_4}{m_3 + m_4} (Q + K) + \frac{1}{2} m_3 V^2}, \quad (1)$$

where Q is the fusion energy, $K = \frac{1}{2} m_1 m_2 v^2 / (m_1 + m_2)$ is the relative kinetic energy, $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ is the relative velocity, $\mathbf{V} = (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2) / (m_1 + m_2)$ is the center-of-mass (cm) velocity, and θ is the angle between $\mathbf{V}$ and the cm velocity of particle 3. Our objective is to deduce the energy distribution of product 3, $F(E_3)$, given the reactant distributions $f_1(\mathbf{v}_1)$ and $f_2(\mathbf{v}_2)$ and the reaction probability $\sigma v(v, \theta)$. We assume that the fusion cross section is isotropic in angle, i.e., $\sigma = \sigma(v)$, which is valid for the $d(t, n)\alpha$ and $d(^3\text{He}, p)\alpha$ fusion reactions below the resonance energy [8,9]. In laboratory plasmas, the fusion energy Q is several orders of magnitude larger than the reactant kinetic or cm energy so we order the equation in half-integer powers of the small parameter $\epsilon = m_3 V^2 / Q$ and neglect terms that enter in order $\epsilon^{3/2}$. For most cases of interest $\sigma(v)$ is a rapidly increasing function of the relative velocity so that the weighting factor $\sigma v f_1(\mathbf{v}_1) f_2(\mathbf{v}_2)$ is largest when v is maximized. For ion distributions f_1 and f_2 with comparable velocities, such as in a thermonuclear plasma, v is largest when the reactants collide head on. This implies that most reactions occur with the cm velocity $V \approx 0$, which from eq. (1) implies that the resulting fusion-product

energy distribution is peaked about $E_3 = m_4(Q + K)/(m_3 + m_4)$ [3]. Here, however, we consider the special case of “beam–target” reactions for which $v_1 \gg v_2$, which implies $\sigma(v) \approx \sigma(v_1)$ independent of v_2 . Under these conditions, it is only the distribution function of the beam and the cross section that determine the fusion-product energy distribution. The general expression for the fusion-product energy distribution function $F(E_3)$ becomes

$$\int_{E_1}^{E_u} F(E_3) dE_3 \propto \int \sigma(v_1) v_1 f_1(v_1) H(v_1) dv_1, \quad (2)$$

where

$$H(v_1) = \begin{cases} 1, & \text{if } E_1 < E_3(v_1) < E_u; \\ 0, & \text{otherwise.} \end{cases}$$

Due to a particle's fast gyromotion in a magnetically confined plasma, the particle distribution function is independent of direction in the plane perpendicular to the magnetic field, $f(v_\perp, v_\parallel) = f(v_\perp, v_\parallel)$. We denote the angle made by the magnetic field vector and the beam particle velocity by χ_1 and the angle between the field and the velocity vector of the fusion product by χ_3 . The cm angle θ can be reexpressed in terms of the laboratory angle ϕ between the projectile velocity v_1 and the product velocity v_3 according to

$$\cos \theta = \cos \phi \sqrt{1 - (k_0 \sin \phi)^2} - k_0 \sin^2 \phi, \quad (3)$$

where

$$k_0 \equiv V \sqrt{\frac{m_3(m_3 + m_4)}{2m_4(Q + K)}}.$$

The next step is to carry out the angular integration in eq. (2). Define a reduced fusion-product energy distribution function $\hat{F}(E)$. For the special case of a monoenergetic beam, $f_1(v_1) = \delta(v - v_b)g(v_1/v_1)$. Since $\hat{F}$ is a monotonic function of the lab frame angle ϕ ,

$$\int_0^E \hat{F}(E') dE' = \int_\phi^\pi g(\phi') d\phi'. \quad (4)$$

Differentiating eq. (4) with respect to E yields

$$\begin{aligned} \hat{F}(E) &= -(\mathrm{d}E/\mathrm{d}\phi)^{-1} g(\phi) \\ &= \left[V \sqrt{\frac{2m_3m_4(Q + K)}{m_3 + m_4}} \left(\frac{1 + k_0^2 \cos 2\phi}{\sqrt{1 - k_0^2 \sin^2 \phi}} + 2k_0 \cos \phi \right) \sin \phi \right]^{-1} g(\phi) \\ &= \left\{ V \sqrt{\frac{2m_3m_4Q}{m_3 + m_4}} [1 + 2k_0 \cos \phi + O(\epsilon)] \sin \phi \right\}^{-1} g(\phi), \end{aligned} \quad (5)$$

where we have used eqs. (1) and (3) in evaluating $\mathrm{d}E/\mathrm{d}\phi$. For an isotropic monoenergetic beam $g(\phi) = \sin \phi$ and

$$\int_{E_1}^{E_u} \hat{F}(E) dE = \ln \left(\frac{1 + 2k_0 \frac{(E_u - E_0^i)}{\Delta E}}{1 + 2k_0 \frac{(E_l - E_0^i)}{\Delta E}} \right) / \ln \left(\frac{1 + 2k_0}{1 - 2k_0} \right), \quad (6)$$

for

$$|E - E_0^i| < \Delta E,$$

where

$$E_0^i = \frac{m_4}{m_3 + m_4} (Q + K) + \frac{1}{2} m_3 V^2$$

and

$$\Delta E = V \sqrt{\frac{2m_3 m_4}{m_3 + m_4} (Q + K)}.$$

The analysis for an anisotropic beam is similar. We consider a monoenergetic beam that is unidirectional in pitch angle χ but isotropic in gyroangle ϕ , $g(\phi)d\phi = \delta(\chi - \chi_1)d\chi d\phi$. Reexpressing $\cos \theta$ (eq. (3)) in terms of χ_1 , χ_3 , and ϕ and proceeding as in the isotropic case yields the fusion-product energy distribution produced by an anisotropic monoenergetic beam,

$$\int_{E_i}^{E_u} \hat{F}(E) dE = \frac{1}{\pi} \left[\sin^{-1} \left(\frac{E - E_0^a}{k_1 \Delta E} \right) + k_2 \sqrt{1 - \left(\frac{E - E_0^a}{k_1 \Delta E} \right)^2} \right]_{E=E_i}^{E_u}, \quad (7)$$

for

$$|E - E_0^a| < k_1 \Delta E,$$

where

$$E_0^a = E_0^i + \Delta E [\cos \chi_1 \cos \chi_3 - k_0 (\cos^2 \chi_1 \sin^2 \chi_3 + \sin^2 \chi_1 \cos^2 \chi_3)]$$

$$k_1 = \sin \chi_1 \sin \chi_3 (1 + 2k_0 \cos \chi_1 \cos \chi_3),$$

and

$$k_2 = k_0 \sin \chi_1 \sin \chi_3 / (1 + 2k_0 \cos \chi_1 \cos \chi_3).$$

The accuracy of eq. (7) was tested by integrating it numerically over $\sin \chi_1 d\chi_1$ to recover the isotropic distribution eq. (6). The final result for the fusion-product energy distribution function is found by taking the weighted average of reduced distribution functions

$$F(E_3) = \int dE_1 \sigma(E_1) E_1 f_1(E_1) \hat{F}(E_3, E_1). \quad (8)$$

The reduced distribution functions eqs. (7) and (6) are plotted in figs. 1a and 1b. The physical origin of the twin lobes (fig. 1a) for a perpendicular beam distribution viewed in the perpendicular plane is that the probability of a reaction is constant over a gyro period but more of the orbit forms an angle θ near $|\cos \theta| = 1$ than near $\sin \theta = 0$ (fig. 1c). Similar results for the fusion spectra from a monoenergetic beam were obtained by Lehner (figs. 2 and 4 of ref. [5]).

If the beam distribution function $f_1(E_1)$ decreases rapidly with energy [e.g., if $f_1(E_1)$ is a Boltzmann distribution], then the weighting factor $\sigma(E_1) E_1 f_1(E_1)$ in eq. (8) peaks strongly for some energy $E_1 = E_{\text{peak}}$ and the fusion-product energy distribution function is approximately

$$F(E_3) \simeq \hat{F}(E_3, E_{\text{peak}}). \quad (9)$$

As predicted by eq. (9), spectra from realistic beam distributions look like smoothed monoenergetic spectra (fig. 2).

The expressions, eqs. (6)–(9), have found application in analysis of 15 MeV proton spectra measured during fast wave (ICRF) heating of ^3He ions in the PLT tokamak [2]. In those experiments, spectra similar to the one plotted in fig. 1a were observed with a detector oriented perpendicular to the field ($\chi_3 = \pi/2$), which was taken as evidence that $\geq 90\%$ of the ^3He ions accelerated to ~ 200 keV by the ICRF was

Fig. 1. (a) Spectrum of protons emitted perpendicular to the magnetic field ($\chi_3 = \pi/2$) by $d(^3\text{He}, p)\alpha$ fusion reactions between an anisotropic ($\chi_1 = \pi/2$) monoenergetic (200 keV) ^3He beam and a cold deuterium plasma. The asymmetry between the higher energy and lower energy peaks originates in the transformation from the cm frame to the laboratory frame. (b) Spectrum of protons emitted perpendicular to the magnetic field by $d(^3\text{He}, p)\alpha$ fusion reactions between an isotropic monoenergetic (200 keV) ^3He beam and a cold deuterium plasma. (c) Physical explanation for the twin-lobed structure of a proton spectrum produced by an anisotropic perpendicular beam. Although the probability that a reaction occurs is constant during a beam ion's circular gyromotion, $v_1 \cdot v_3$ does not vary uniformly during a gyro period, causing more ions to be emitted with a maximal Doppler shift than without any shift.

Fig. 2. Spectrum of protons emitted at an angle $\chi_3 = \pi/3$ with respect to the magnetic field in $d(^3\text{He}, p)\alpha$ fusion reactions between anisotropic ^3He ions and a cold deuterium plasma for various angles χ_1 of the ^3He ions with respect to the field (eq. (7)). A distribution in energy of $f_{\text{He}}(E_{\text{He}}) \propto \exp(-E_{\text{He}}/50 \text{ keV})$ is assumed for the ^3He ions and an analytical fit [10] is employed for the cross section σ in eq. (8).

anisotropic in velocity space. Although the data from this experiment provided unambiguous evidence of beam anisotropy, the information was insufficient to establish the direction of ^3He anisotropy. In fig. 2, possible spectra from a similar experiment with a detector oriented at $\chi_3 = \pi/3$ are plotted. The great variation in the relative amplitude of the two peaks as a function of the angle of ^3He velocity χ_1 (fig. 2) indicates that, with properly oriented detectors, spectral measurements can diagnose both the direction of fast beam ions and their degree of anisotropy.

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References

- [1] J.D. Strachan et al., in: Nuclear Data for Science and Technology ed., K.H. Bockhoff (ECSC Brussels, 1983) p. 313.
- [2] W.W. Heidbrink, Nucl. Fusion 24 (1984) 636.
- [3] H. Brysk, Plasma Phys. 15 (1973) 611, and references therein.
- [4] G. Lehner and F. Pohl, Inst. Plasmaphysik Rep. IPP I/60 (1967).
- [5] G. Lehner, Z. Physik. 232 (1970) 174.
- [6] In sects. IIId and IIIf of ref. [5], the expressions for the width of the fusion spectra are incorrect. Rather than

$$(\Delta v)_{\text{total}} = 2 \frac{m_A u_{0A} + m_B u_{0B}}{m_A + m_B},$$
 eq. (67) should read

$$(\Delta v)_{\text{total}}^2 = 2 \frac{m_A u_{0A} + m_B u_{0B}}{m_A + m_B} v_0,$$
 and similarly for the other expressions in sects. IIId and IIIf.
- [7] H.H. Towner and D.L. Jassby, Trans. Am. Nucl. Soc. 22 (1975) 74.
- [8] J.P. Conner, T.W. Bonner, and J.R. Smith, Phys. Rev. 88 (1952) 468.
- [9] T.W. Bonner, J.P. Conner and A.B. Lillie, Phys. Rev. 88 (1952) 473.
- [10] G.H. Miley, H. Towner and N. Ivich, University of Illinois Rept. COO-2218-17 (1974).