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Essays on Electricity Markets in Presence of Strategic Prosumers

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ESSAYS ON ELECTRICITY MARKETS IN PRESENCE OF STRATEGIC PROSUMERS

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by

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Abstract

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by

Sepehr Ramyar

Prosumers, with the ability to act both as a supplier and a consumer in a power market, have received considerable attention recently. Having distributed energy resources, their capability to operate in an isolated mode, separated from the main grid, has also been promoted as a vital option to enhance the power system's resilience. One emerging concern is the prosumer's ability to manipulate the power market as a buyer or as a seller. This thesis vets the outcomes of a power market in presence of strategic prosumers and formulates electricity markets in different frameworks depending on the prosumer's strategy to obtain the equilibria representing the market outcome.

This thesis posits a situation in which a strategic prosumer owns a renewable unit with variant output and a dispatchable backup unit, and participates in a power market following a price-taker, quantity-based, or Stackelberg strategy. The prosumer is assumed to maximize its benefit by deciding amount of power to buy from or sell into the main grid, amount of renewable power to forego consumption, and amount of power to produce from backup unit. The interaction of prosumers and the main grid is modeled through shifting the residual supply curve in the wholesale market, thereby avoiding the possible numerical issues when aggregating their demand horizontally with consumers. The model is applied to a case study of the IEEE 24-bus RTS as an illustrative example.

Prosumers in the market are also subject to uncertain renewable output and, as they participate in the day-ahead market, decide how much energy to consume, sell, or purchase, considering the potential imbalance due to uncertain output. The question of prosumer's risk preferences, taking into account the uncertainty in its renewable output, is investigated and the dynamics between prosumer's risk decisions and its implications for the prosumer's profit maximization are examined in a chance-constrained framework.

This thesis illustrates power market outcomes in presence of strategic prosumers following price-taker, quantity-based, and Stackelberg strategies exercised by the prosumer and shows existence of market-clearing equilibria in the case of perfect and imperfect competition and demonstrate that the prosumer's position in the market does not depend on its strategy along with the fact that the prosumer is able to attain higher pay-off in the price-taker case compared to the quantity-based strategy. This thesis also shows the impact of prosumer's Stackelberg strategy on rent distribution and price formation in the marketplace in a chance-constrained framework to account for uncertainty of its renewable output. Finally, the thesis investigates prosumer risk attitudes and quantifies the impact of renewable output uncertainty and imbalance prices on prosumer's risk preferences.

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Chapter 1

Introduction

1.1 Introduction and Background

Energy markets across the world are at an inflection point. The need for sustainability and mitigating climate change impacts have led to fundamental changes in the design and operation of power markets. The traditional demand-side paradigm in electricity markets has been challenged by a growing body of customers with renewable and distributed energy resources (DER) such as solar panels, storage, and electric vehicles.

As the demand-side becomes more flexible and engaged in the power market, new entities and forms of participation in the energy market emerge. Particularly, we see the advent of *prosumers*, i.e., market participants who are capable of concurrent generation and consumption of energy as opposed to the conventional consumers or suppliers that have traditionally engaged the market on either the demand or supply side. The impact of prosumers on power markets are amplified by aggregators that integrate demand response (DR) and DERs over wide geographic and temporal spectra and offer bundled energy products to the market which has recently been facilitated by FERC Order 2222, encouraging further adoption of DER technologies. With an increasing share of consumers becoming prosumers, it is crucial to investigate the implications for the design and operation of power markets.

The heterogeneity in placement and diffusion of renewables and DERs enables economic opportunities for prosumers far beyond those available to conventional consumers and, as a result, significantly changes the dynamics in electricity markets, offering incentives to the new entities in the marketplace. Therefore, as prosumers are introduced into power systems, market outcomes would likely be different from those of a power market with conventional consumers, especially if prosumers' behavior start to deviate from price-taker assumption. At the same time, given the novelty of prosumers in the marketplace and a potential absence of a mature regulatory framework, these new entities might find themselves subject to less oversight. This calls for an analysis of whether, and how, prosumers could impact power market outcomes.

Moreover, prosumers face inherent uncertainty of their generation capacity from renewables, which highlights the importance of risk attitudes in the decision making of prosumers. As prosumers establish a more prominent position in power systems, their behavior and the way they manage risks will have important ramifications on the quantities traded in the electricity market and the overall market outcomes. The degree of prosumer risk aversion also highlights how it reacts to different market conditions, capturing part of its decision making process.

This thesis attempts to answer the questions on how introduction of prosumers into power markets could impact market outcomes: What are some strategies the prosumer can potentially exercise? and what would market outcomes look like following each strategy? How do market outcomes change in each case depending on the level of renewable capacity available to the prosumer? How does the inherent uncertainty in prosumer's generation resources affect its decisions and tolerance to risk, and by extension, outcomes in the marketplace?

1.2 Outline of Dissertation and Contributions

This thesis is constituted of three papers that address different dimensions concerning modeling a power market in presence of strategic prosumers. While there is much to be studied to cover all questions on the future of power markets with substantial levels of renewable and distributed energy resources, this thesis covers a subset of problems related to the impact of introducing prosumers into power markets using an equilibrium approach.

Chapter 2[1] sets the stage by introducing the mathematical components and the analytical frameworks, which will be used throughout the thesis to model and represent the power market. The prosumers are assumed to be a price-taker or exercise quantity-based strategies. This is achieved by formulating the problem using a complementarity

[1]The material in this chapter have been published at IEEE Transactions on Power Systems. Link: https://ieeexplore.ieee.org/abstract/document/8815858

framework that allows prices to be determined endogenously. In the perfect competition case, the prosumer, along with other market participants, takes the power prices as given and decides on the level of its output or consumption. In the Cournot case, the prosumer possesses information about conventional consumers' demand function and incorporates that extra information into its problem. This chapter illustrates the existence of market equilibrium and shows how the prosumer's position in the market does not depend on its strategy.

Chapter 3[2] formulates the problem based on a different informational structure. Specifically, the prosumer is able to internalize the best response of every other market participant into its own profit maximization problem. This problem is formalized based on a Stackelberg game where the prosumer is the leader, and other market participants are considered as followers. In order words, as the leader, the prosumer is able to internalize the best response of the followers, including consumers, generators, and the grid owner. The uncertainty associated with the renewable output is explicitly modeled by formulating the problem as a Distributionally Robust Chance-Constrained (DRCC) Mathematical Program with Equilibrium Constraints (MPEC). In this chapter, we find that prosumer's strategy depends on the magnitude of renewable generation uncertainty and the degree of risk aversion, which jointly affect the "perceived" quantity of available renewables. Similar to the risk-neutral cases, the risk-averse Stackelberg case always yields a higher payoff for the prosumer compared to the Cournot and perfect competition

[2]The material in this chapter is from a paper currently under review at Springer Energy Systems journal. An earlier version of this chapter was a best paper finalist and published in Hawaii International Conference on System Sciences 2020 (HICSS 53): `https://scholarspace.manoa.hawaii.edu/handle/10125/64122`

cases.

Following the framework established in Chapter 3, Chapter 4 proposes an alternative modeling framework in which prosumers endogenously determine their risk attitudes through optimization in a setup with a day-ahead market and real-time imbalance settlement4[3]. We show that the effect of uncertainty can be managed in a power market when prosumers are allowed to internalize the risk by deciding optimal risk attitudes in order to maximize their profits. We also quantify the forgone profit when prosumers' risk preference deviates from the optimum. The results highlight the fact that endogenizing risk preferences can be a useful tool to manage risks in the power market.

[3]The material in this chapter has been organized in a paper currently under review at IEEE Transactions on Engineering Management

Chapter 2

A Power Market Model in Presence of Strategic Prosumers

2.1 Background and Literature Review

Electric power markets are undergoing rapid and fundamental transformations. The urge for an increase in renewable capacity and generation, in part owing to the efforts of mitigating climate change and pursuing sustainability, has led to significant changes in the design and operation of modern power grids. With the availability of smart meters together with advances in IT, a growing body of customers with renewable power generation capabilities combined with emerging distributed technologies, such as electric vehicles and storage, have altered the conventional demand-side paradigm in electricity markets.

This major shift in power markets towards a more engaged and flexible demand-side involvement has direct impacts on the behavior and participation of various agents in the market. Specifically, we see the advent of *prosumers*, i.e., agents who are capable of concurrent generation and consumption of power as opposed to the conventional consumers or suppliers who only participate in one side of the market. Given an increasing proportion of customers in the power market transforming into this emerging entity, with the duality of consumption and generation, it is expected to have significant implications on the design and operation of the future competitive power market[1][1].

The interactions between prosumers and the wholesale power market are facilitated by aggregators who collect and integrate demand response (DR) and distributed energy resources (DER) at the distribution level and offer the *aggregated* energy bundle as a product to the wholesale market. Aggregators install and operate renewable facilities, such as solar panels, energy storage, electric vehicle charging, and smart energy management systems and are responsible for operation of generating fleet over a wide and diverse set of households and geographical areas constituting a substantial distributed generation and energy management capability[2, 3]. This provides an economic leverage for prosumers participating in the wholesale power markets far beyond ordinary customers as they are capable of integrating considerable resources over space and time, and at the same time, also fundamentally changes the business models of the electricity markets.

[1] For example, recent focus of the power engineering community has been on developing a platform that allows a distribution system operator (DSO) to coordinate and to align with prosumers and an independent system operator (ISO) at the transmission level to facilitate energy transactions. In particular, the final ruling of the FERC Order 745 stipulates that demand response resources participating in an organized wholesale energy market must be compensated for the service they provide to the energy market at the market price for energy, namely the locational marginal price (LMP). Moreover, issues related to the DER aggregation reforms have been discussed by the FERC.

However, the introduction of prosumers into wholesale power markets poses several challenges in terms of operation and planning of a power system, in part driven by economic incentives offered to new participants in the power market. The operation and planning of a power system with increased participation of prosumers would shift the market's focus towards a more distribution level paradigm. This would consequently affect decisions on transmission grid expansion and investments on distribution and power generation facilities[4]. Furthermore, the economic incentives for market participants and the resulting market outcomes might change considerably as more participants are capable of concurrent generation and consumption. In other words, the market outcomes in presence of a prosumer would likely be different from those of a market with conventional consumers, especially when prosumers are allowed to deviate from price-taking assumptions. As these entities are relatively new to the market, they might be subject to relatively less oversight, partly as the result of underdeveloped regulatory framework to address their behavior.

Given the recent paradigm shift in the power markets towards an architecture with an increased presence of prosumers, an interesting question is how this emerging entity, i.e., prosumers with the ability of acting as both a producer and a consumer, might interact with the wholesale market and affect market outcomes as well as other entities in the market.

The impact of strategic prosumers on electricity markets has received some attention in the power systems literature. For example, [5] examines how a demand aggregator, operating a conventional generator and a green energy management system, affects the wholesale market by considering the aggregator exercise a quantity-based or Cournot strategy. This paper, however, does not account for the capability of concurrent generation and consumption by the prosumers, thereby underestimating the ability of the prosumers affecting the market.

In a more recent paper, [6] considers a problem with a different information structure by postulating a load aggregator as a leader while other entities, i.e., producers, consumers and the grid operator, are followers in a Stackelberg setting. The load aggregator operates renewables, a wind source for example, and contemplates to "spillover" or "curtail" its wind power to reduce energy offering into the wholesale market in order to push up the wholesale power prices. Similar to [5], buyer's market power is not considered in the analysis. As prosumers are expected to play a crucial role in the future power market, especially with their continuous growth in the market, models that explicitly formulate prosumer's behavior and endogenize power price formation will prove to be an important tool to assess its impact on market outcomes.

Other papers have also contributed to modeling prosumers' behavior. Authors in [7] implement a two-stage stochastic programming approach to optimize a prosumer's bidding (first stage) and scheduling decisions (second stage) with the objective of minimizing the prosumer's expected cost. However, the power prices are assumed to be exogenous, and the paper fails to reflect the interplay between prosumer's decisions and price formation at the wholesale market.

Reference [8] investigates demand response participation in the wholesale power market in which a DR aggregator offers contracts to customers based on physical constraints and capabilities, such as storage, on-site generation, load shifting, and load shedding and maximizes its expected payoff. While power price paths are simulated based on time series and artificial neural network techniques, it is subject to the same limitation as [7]. Another study examines optimal contract design between a retailer and an end-user when facing uncertain power prices [9]. A power retailer here, to some extent, is similar to a prosumer as it is capable of both purchasing and selling electricity. The authors in [9], however, treat the wholesale prices as exogenous (similar to [7, 8]) and take the contract price as the decision variable.

Therefore, a common thread of the existing literature is to treat the wholesale power prices as given, and focus their attention on finding optimal contracts with customers or dispatch schedules while maximizing expected payoff. In other words, the dynamics of the interplay between the prosumers' strategic actions on the wholesale power prices is commonly not considered. Prosumers' strategic actions will play an important role in the future because the number of prosumers is expected to grow significantly. This is in part facilitated by the emerging decentralized and layered marketplace, such as DSOs to govern and facilitate energy transactions, together with prominent incentive-compatible business models to minimize transaction cost and maximize business opportunity [6, 10].

This chapter extends the existing work by Hobbs [11] with an explicit formulation of the prosumer's problem in a bottom-up complementarity framework, which allows interactions of the prosumers with other entities in the market, e.g., conventional generators, consumers, and the grid operator, to be investigated. The prosumers can be either a seller or a buyer, acting strategically or competitively, as oppose to merely sellers as in [5, 7]. Power prices and transmission charges, and decisions of all the entities in the market are endogenously determined, rather than exogenously given as in [7, 9].[2] (Therefore, our model considers only the high-voltage transmission network, abstracting from representing low-voltage distribution network.) In particular, our formulation does not *ex ante* fixate the prosumer's role, either as a producer or a consumer, in the market, but, instead, allows solutions of the model to decide which one of the two roles the prosumer should assume when maximizing its benefit. That is, whether the prosumer sells power into or buys power from the wholesale market in equilibrium is not known before solving the model. Moreover, our analysis, which explicitly decouples the prosumer's marginal benefit and the bulk energy consumers' willingness-to-pay without *a priori* fixation of prosumer's role, a producer or a consumer, also advances bottom-up modeling of prosumers' behavior. In fact, how to treat prosumers' demand in the model when their role in the equilibrium in the bulk energy market is unknown a priori is actually not trivial. Finally, rigorous proof of the existence of equilibria are provided along with discussions on uniqueness of the solutions to enhance our understanding the properties of the models. Thus, this chapter advances current knowledge of studying prosumers' behavior by allowing an endogenous treatment of power price formation process and simultaneously modeling the prosumers as both a buyer and a

[2]Bottom-up complementarity models formulated based on game-theoretical framework and built upon individual entities' optimization problems have emerged as a popular tool to assess the impact of newly enacted regulations, proposed market designs, emerging technologies, and other considerations in the energy sector. The strengths of this model lies in its ability to incorporate heterogeneity in generating technologies, physical systems, e.g., transmission, various institutions and emerging entities in analyses. Examples include [12, 13].

seller.

This chapter is organized as follows. Section 2.2 gives a detailed formulation of optimization problems faced by each entity in the market, their first-order conditions as well as market clearing conditions that define equilibrium. The models developed in Section 2.2 is then applied to a case study of IEEE 24-bus Test System in Section 2.3. We report main results, provide proofs of solution properties, and generalize our findings in two propositions that emerge from our analyses. Additional numerical simulations are conducted to illustrate our findings. Concluding remarks are given in Section 2.4. We document our proofs of the three propositions in the Appendix A.

2.2 Analytical Model

Our work is based on work by Hobbs [11] and extends his work by introducing prosumers in the model. We use capital letters to indicate parameters and sets. Lowercase letters refer to variables and indices. Dual variables are designated with Greek lower-case letters. In the following presentation, "$x \perp y$" implies $x^T y = 0$.

2.2.1 Individual Optimization Problems

This section proceeds as follows. First, we introduce the optimization problem faced by each entity in the market, including prosumers, producers, the grid operator, and an arbitrageur. Second, we derive the Karush-Kuhn-Tucker (KKT) conditions associated with each variable in the optimization problem. Third, the collection of KKT conditions together with market clearing conditions will characterize a market equilibrium problem in form of a linear complementarity problem, which can then be solved using complementarity solvers, e.g., PATH [14].

2.2.1.1 Consumers

Consumers are assumed to be price-taking agents, and their willingness-to-pay for power is represented by the inverse function in the complementarity form in (2.1):

$$0 \leq d_i \perp p_i - \left(P_i^0 - (P_i^0/Q_i^0)d_i\right) \geq 0, \qquad \forall i = 1, \ldots, I \tag{2.1}$$

where P_i^0 and Q_i^0 represent the vertical and horizontal intercepts of the inverse demand function, respectively, at demand node i. The vertical intercept, also referred to as choke price, indicates that consumption drops to zero when price exceeds P_i^0. The function is positive but decreasing in d_i, the demand quantity for consumer at node i. When there is no regular consumer in node i, we then model that location with a sufficiently small P_i^0 so that the quantity demanded, d_i, is equal to zero in equilibrium.

2.2.1.2 Prosumers

The prosumer at node i possesses renewable energy generation capacity with a negligible short-run marginal cost.[3] We assume that prosumers only engage in power

[3]Individual behind-the-meter prosumers, e.g., owner of roof-top solar, might have limited access to the wholesale or bulk market and be subject to a tariff that does not reflect value of their surplus energy. We assume that our prosumer problem is the result of the aggregation of a large number of end-prosumers, thereby allowing them interact with the bulk market directly. Thus, in a way, we model end-prosumers and the aggregator as a joint entity. One can think about that end-prosumers, who are subject to uncertain level of renewable output, enter bilateral agreement, a contract, with an aggregator while allowing the aggregator to operate their aggregated dispatchable capacity. In this case, the premium associated with the bilateral contracts will be an internal wealth transfer between end-prosumers and the aggregator.

sales or purchases at their local node. [4] This assumption is also consistent with the layered grid structure envisioned in [15]. Thus, the wheeling cost will cancel out in this. That is, the prosumer gets paid by ω_i when moving power to the hub and pays ω_i when selling from the hub to node i. The output from renewable is denoted by K_i, which is uncertain because it is limited by available natural resources, e.g., solar and wind. Meanwhile, it also owns a dispatchable or backup resource with a capacity of G_i in order to hedge against uncertain output K_i.

For our purposes, the prosumer's benefit function of consuming electricity around level K_i is given by $B_i(l_i)$, where l_i corresponds to the quantity consumed by prosumer when renewable output equals K_i (Fig.2.1).[5] It represents a local benefit function centered around consumption level at K_i. As a prosumer engages in the market, directly through bilateral trading with firms, there is limited opportunity for the market to solicit prosumers' preferences through market settlements, i.e., a preference revelation process. The benefit function $B_i(\cdot)$ is assumed to be increasing and strictly concave. The monotonicity of $B_i(\cdot)$ indicates that the prosumer's objective function is increasing in the level of consumption.

We posit that a prosumer maximizes its profit by deciding a) the amount of power to buy from ($z_{fi} < 0$) or sell to ($z_{fi} > 0$) firm f in node i through bilat-

[4] Allowing prosumers to sell surplus power from its local node i to other locations is expected to produce the same market outcomes. This is because the price difference between two nodes, e.g., i and j, is equal to the transmission cost of moving power from i to j. Thus, while selling power to node j might earn extra revenue (i.e., $p_j - p_i$), it will be exactly offset by the transmission cost; see, for example, [11] for the equivalence between Poolco and bilateral markets.

[5] B_i is entirely separate and different from $p_i(d_i)$, which represents willingness-to-pay or the marginal benefit of consumers in the wholesale market. The interaction of $B_i(l_i)$ with the main grid is through shifting the wholesale's supply to left (right) when purchasing from (selling to) the wholesale market.

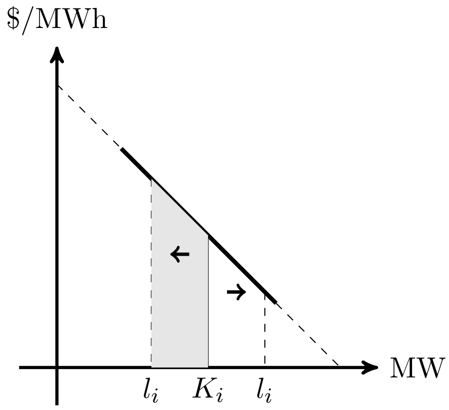

Figure 2.1: An illustration of prosumer's marginal benefit function

eral contracts[6], b) amount of forgone consumption, $K_i - l_i$, and c) amount of power to be generated from the backup dispatchable technology, g_i. The prosumer faces a price-responsive demand characterized by its marginal benefit function. Its maximal consumption is capped by the horizontal intercept of its marginal benefit function.

The optimization problem faced by the prosumer at node i is displayed as follows. As mentioned earlier, the Greek variables within the parenthesis to the right of an equation render the corresponding dual variable.

[6]Because the equivalence between a power market based on pool-type transactions and on bi-lateral contracts have been alluded to in [11], we believe that our assumption herein is reasonable and can be seen as an extension.

$$\underset{z_{fi}, l_i \geq 0, g_i \geq 0}{\text{maximize}} \quad p_i \left(\sum_f z_{fi} \right) - \int_{l_i}^{K_i} B_i'(x)dx - C_i^g(g_i) \tag{2.2a}$$

subject to

$$\sum_f (z_{fi}) + l_i - K_i - g_i = 0 \qquad (\delta_i), \tag{2.2b}$$

$$g_i \leq G_i. \qquad (\kappa_i) \tag{2.2c}$$

The three terms in the objective function of (2.2), in order, correspond to revenue (+) or cost (-) from transactions with the wholesale market, foregone benefit (if $K_i > l_i$) or incremental benefit (if $l_i > K_i$) of consuming power, and generation costs incurred from backup generation, respectively. Two constraints are associated with the prosumers' problem. (2.2b) states that the sum of renewable output K_i and self generation g_i net of sales to the wholesale market or $\sum_f z_{fi}$ equals the quantity consumed l_i. (2.2c) limits the prosumer's conventional backup generator output, g_i, by its capacity G_i. Note that the transactions of prosumer with the wholesale market does not involve the wheeling charge w_i since it only sells or buys from the node where it produces its power. That is, the prosumer gets paid by w_i when moving power to the hub and pay w_i when selling from the hub to the node i. This way of modeling prosumers is consistent with the layer structure of future power market discussed in paper [15].

When a prosumer is modeled in our analysis as a price-taker, it takes the price p_i as given and decides on (z_{fi}, l_i, and g_i) accordingly. However, when a prosumer in our model is designated as a strategic entity, it realizes that by "contracting" some of its procurement of power, it could lower the power price, thereby exercising buyer's market power. On the contrary, it is also aware that if it reduces power sales slightly, it might be able to push up power prices, thereby exercising seller's market power. This highlights the capability of the model to capture the duality of a prosumer in a unified framework. As we demonstrate later, which of the two strategies would be implemented depends on the prosumer's net position in the energy market, which is affected by the zero marginal cost renewable output K_i. While a prosumer only participates in the wholesale market indirectly through bilateral contracts rather than, say directly submitting bids into the market, one can assume that it acquires "strategic" knowledge through its repeated observations of power price clearance processes of the wholesale market.

Knowing that a prosumer can manipulate the power market through changes in procurement or purchase quantities, we then re-write (2.2) as

$$\underset{z_{fi}, l_i \geq 0, g_i \geq 0}{\text{maximize}} \quad p_i(z_{1i}, z_{2i}, \cdots, z_{Fi}) \left(\sum_f z_{fi} \right) - \int_{l_i}^{K_i} B_i'(x)dx - C_i^g(g_i) \qquad \text{(2.2a*)}$$

by representing p_i as a function of z_{fi}. One way of representing prosumer's ability to manipulate the wholesale power market in the model is by treating its belief as a parameter based on conjecture variation approach. One benefit of using this approach is that the parameter can be altered in order to explore the impact of a prosumer's belief of its "manipulating" strength on market outcomes. However, this approach is mainly useful in a situation when the demand function of underlying commodity is unobservable. An example of this is modeling market power of tradable pollution permit market where the demand for tradable permits is actually implied from output decisions of generators in the power market [13]. Because 1) our interests lie in understanding market outcomes when a prosumer behaves compatibly with a producer, and 2) classic results indicating that quantity pre-commitment and Bertrand competition yield Cournot outcomes [16], we believe a Cournot or quantity-based formulation is more apt for our analysis.

Therefore, the first-order conditions associated with prosumers then can then be displayed as follows.

$$\text{For } z_{fi} : p_i - \delta_i = 0, \forall f, i \tag{2.3a}$$

$$\text{For } z_{fi} : p_i - (P_i^0/Q_i^0) \sum_f z_{fi} - \delta_i = 0, \forall f, i \tag{2.3a*}$$

$$0 \le l_i \perp A_i^0 - B_i^0 l_i - \delta_i \le 0, \forall i \tag{2.3b}$$

$$0 \le g_i \perp -C_i^{g\prime} - \kappa_i + \delta_i \le 0, \forall i \tag{2.3c}$$

$$\text{For } \delta_i : l_i - K_i - g_i + \sum_f z_{fi} = 0, \forall i \tag{2.3d}$$

$$0 \le \kappa_i \perp g_i - G_i \le 0, \forall i \tag{2.3e}$$

Here, $A_i^0 - B_i^0 l_i$ is a linear representation of the prosumer's marginal benefit of consumption that is positive and decreasing in l_i which reflects the monotonicity and concavity of the prosumer's benefit function, $B_i(\cdot)$, respectively. As a reminder, the perpendicular sign ($\perp$), for example in (2.3a), means that the inner product of z_{fi} and $p_i - \delta + \mu_i$ is zero. In other words, either z_{fi} is equal to zero or $p_i - \delta + \mu_i$, but not both. A total of five conditions are associated with the prosumers' problem with four primal variables and three dual variables $(z_{fi}, b_{fi}, l_i, g_i, \mu_i, \delta_i, \kappa_i)$.

In cases when the prosumer exercises market power, the first-order conditions of variables z_{fi} are given in (2.3a*). Comparing (2.3a) to (2.3a*), the difference is $(P_i^0/Q_i^0)\sum_f z_{fi}$. This term acts similarly to those under the standard Cournot formulation, which makes the market equilibrium to be different from the outcomes of a perfect competition characterization of the market. The sign defined by $\sum_f z_{fi}$ means the prosumers is either exercising seller's or buyer's market power. Note that when prosumers are price-takers, their consumption l_i is related to the its marginal benefit $A_i^0 - B_i^0 l_i$. When $l_i > 0$, (2.3b) indicates that $A_i^0 - B_i^0 l_i = \delta_i$, which is equal to p_i from (2.3a). That is, l_i is implicitly capped by $A_i^0 - B_i^0 l_i = p_i$ or the marginal benefit equals the equilibrium price.

2.2.1.3 Producers

As alluded to earlier, we assume suppliers or firms are price-takers in the wholesale power market as they are constantly subject to rigorous regulatory oversight.[7] We assume that firm f maximizes its profit by deciding the output x_{fih} and sales s_{fi}.

[7]For example, the PJM market is reported to be competitive, i.e., prices set by marginal offering units close to their marginal costs [17]. Likewise, the day-ahead market in California is generally competitive [18]. However, regulator and market monitor are always concerned about the exercise of market power in local load pocket due to congestion is always a concern, see [17] and [18].

A supplier f's problem is given as follows:

$$\underset{s_{fi} \geq 0, x_{fih} \geq 0}{\text{maximize}} \quad \sum_i (p_i - w_i)(s_{fi} - z_{fi}) - \sum_{fih} (C_{fih}(x_{fih}) - w_i x_{fih}) \tag{2.4a}$$

subject to

$$x_{fih} \leq X_{fih}, \forall i, h \in H_{fi} \qquad (\rho_{fih}), \tag{2.4b}$$

$$\sum_i (s_{fi} - z_{fi}) - \sum_{i, h \in H_{fi}} x_{fih} = 0 \qquad (\theta_f) \tag{2.4c}$$

The first term in the objective function (2.4) is the revenue received from power sales $s_{fi} - z_{fi}$ while paying for the wheeling charge w_i. The second term gives generation cost, minus transmission charge $-w_i$, effectively representing a payment received by the generator from the grid operator for its service of providing counterflow to de-congest the line from i to hub. The cost function C_{fih} is convex and marginally increasing as in the literature [19]. Here, $C(\cdot)$ is the quadratic cost function for generation and w_i is the wheeling cost charged by the grid owner to move power from hub to node i. Solving the optimization problem, the generator decides on its output level.

Turning to the constraints, (2.4b) limits the output x_{fih} to be less than its capacity X_{fih}. (2.4c) assures that total power sales equal its supply while accounting for its bilateral transactions with the prosumers. More specifically, when z_{fi} is negative, (2.4c) suggests that additional x_{fih} needs to be produced by the generator to satisfy demand other than s_{fi}. This effectively reduces the amount of power available to the power pool, thereby, expectedly, driving up the wholesale prices. Similarly, when z_{fi} is positive, output from firm f is reduced as a portion of the wholesale demand is met by the prosumers, hence prices are expected to be lower in this case. This formulation of conventional generators allows the model to decouple the bulk energy demand, defining p_i in (2.1), from the prosumers' marginal benefit function B_i' in (2.2a).

The KKT conditions of the producer f in the wholesale market are summarized as follows:

$$0 \le s_{fi} \perp p_i - w_i - \theta_f \le 0, \forall i \tag{2.5a}$$

$$0 \le x_{fih} \perp -C'(x_{fih}) + w_i - \rho_{fih} + \theta_f \le 0, \forall i, h \in H_{fi} \tag{2.5b}$$

$$\text{For } \theta_f : \sum_i (s_{fi} - z_{fi}) - \sum_{i,h \in H_{fi}} x_{fih} = 0, \ \forall f \tag{2.5c}$$

$$0 \le \rho_{fih} \perp x_{fih} - X_{fih} \le 0, \forall i, h \in H_{fi} \tag{2.5d}$$

2.2.1.4 Grid Operator

The grid owner operates the power network and decides on the allocation of transmission resources while charging producers w_i to move power from hub to node i. The optimization problem faced by the grid operator is given in (2.6).

$$\underset{y_i}{\text{maximize}} \quad \sum_i w_i y_i \tag{2.6a}$$

subject to

$$-T_k \le \sum_i PTDF_{ki} y_i \le T_k \qquad (\lambda_k). \tag{2.6b}$$

The grid operator is a price-taker with respect to w_i and aims to maximize its revenue by deciding y_i given the power flow in each line k is within its thermal limit T_k. Similar to [19], power flows in the network are governed by the power distribution transfer factor (PTDF) based on linearized Directed-Current principle[20]. In this context, the grid operator maximizes the value obtained from the sales of nodal transmission rights based on the topology of the network [21]. The grid operator represents the behavior of the transmission operator or *line owner* that seeks to maximize the value of its network given the set of prices w_i [22]. The grid operator's KKT conditions are then given as follows:

$$w_i - \sum_k PTDF_{ki}(\lambda_k^+ - \lambda_k^-) = 0 \qquad \forall i \tag{2.7a}$$

$$0 \leq \lambda_k^+ \perp \sum_i PTDF_{ki} y_i - T_k \leq 0 \qquad \forall k \tag{2.7b}$$

$$0 \leq \lambda_k^- \perp -\sum_i PTDF_{ki} y_i - T_k \leq 0 \qquad \forall k \tag{2.7c}$$

2.2.1.5 Arbitrager

We include an arbitrager in our model, as it has been shown that solutions from models with an arbitrager are equivalent to that of a POOL-type power market when the market is imperfectly competitive [11]. Moreover, [23] proved that when considering an arbitrager, the cost of moving power from node i to j will equal the price difference between nodes or $p_j - p_i$. The implicit assumption here is that the arbitrager has full knowledge of power prices at each node. An arbitrager moves power from a bus where the market price is lower to one with a higher price. The arbitrager's optimization problem is as follows:

$$\underset{a_i}{\text{maximize}} \quad \sum_i (p_i - w_i) a_i \tag{2.8a}$$

subject to

$$\sum_i a_i = 0. \qquad (p^{hub}) \tag{2.8b}$$

One constraint, (2.8b), is associated with this problem, guaranteeing total sales would equal total purchases, with its dual variable denoting the market price at hub or p^{hub}. The arbitrager's KKT conditions are given in (2.9):

$$p_i - w_i - p^{Hub} = 0 \qquad \forall i \tag{2.9a}$$

$$\sum_i a_i = 0. \tag{2.9b}$$

2.2.1.6 Market Clearing Conditions

While each market participant's optimization problem represents its behavior in the wholesale market, the market clearing conditions tie them all together and ensure the balance between demand and supply. This is shown in (2.10).

$$\sum_f s_{fi} + a_i - \sum_{f,h \in H_{fi}} x_{fih} - \sum_f z_{fi} = y_i, (\omega_i), \forall i \tag{2.10}$$

Note that the first two terms together, $\sum_f s_{fi} + a_i$, equals the demand at node i: d_i, as in (2.1) to determine the whole price at node i. The collection of all KKT conditions for each market participant (2.3)–(2.9) in addition to the market clearing condition (2.10) forms the set of equalities and inequalities, termed as a mixed complementarity problem (MiCP), which cjaracterizes a market equilibrium [24, 25]. The MiCP is formulated in AMPL and solved using the PATH solver [14].

In the following we state the existence of a market equilibrium. The proof is provided in Appendix A.1.

Proposition 1. *(Existence) Assume that the prosumer's marginal benefit function* $B_i'(\cdot) : \Re^1 \to \Re^1$ *is continuous and monotonically decreasing. Further assume that the prosumer's generation cost function* $C_i^g(\cdot)$ *and the supplier's cost function* $C_{fih}(\cdot)$ *are continuously differentiable, for all* $i = 1, \ldots, I$*,* $f = 1, \ldots, F$*, and* $h \in H_{fi}$*. Then a market equilibrium exists, which is defined as the collection of primal variables* $(z, l, g, s, x, y, a, w, p)$ *and the dual variables* $(\delta, \kappa, \theta, \rho, \lambda, p^{Hub})$ *that simultaneously satisfy the optimality conditions* (2.3)*,* (2.5)*,* (2.7)*,* (2.9)*, together with* (2.10).

2.3 Numerical Case Studies

2.3.1 Data, Assumptions and Scenarios

To analyze the power market outcomes in presence of strategic prosumers, the IEEE Reliability Test System (RTS 24-Bus) [26] is used. The topology of the system consists of 24 buses, 38 transmission lines, and 17 constant-power loads with a total of 2,850 MW. We aggregate 32 generators into 13 generators by combining those with the same marginal cost and located at the same node. Six generation units, however, are excluded from the dataset since they are hydro power units operating at their maximum output of 50 MW [27]. Because the wholesale market is assumed to be perfectly competitive, we assume that all the generators are owned by a single firm. In order to be able to analyze the impact of transmission congestion, the capacity of line 7 in the test case is reduced to 150 MW. The marginal cost of generation is represented by a quadratic function parameterized by intercept C_0 and slope C_1.

Furthermore, a prosumer is assumed to be located at node 1 with the same preferences for power consumption as consumers located in that node. That is, both prosumer and consumers in node 1 are assumed to have the same demand function. The prosumer owns a renewable generating unit that produces a varying amount of power (contingent on available natural resources) and a dispatchable unit as a backup option.

The RTS 24-Bus case is first formulated as a least-cost minimization problem and solved with fixed nodal load in order to get dual variables associated with load constraints. The dual variables together with an assumed price elasticity of -0.2 is then used to calculate P_i^0 and Q_i^0. The magnitude of price elasticity of demand is comparable with what has been reported in [28].

We examine a total of six scenarios, varied by the level of renewable output owned by the prosumers as well as the strategic assumption of the prosumers. More specifically, renewable output K_1 is assumed to have three levels: 25, 50, and 120 MW. The prosumer is designated as either a price-taker or as a strategic entity while other entities in the market are assumed to be price-takers.[8] Additional sensitivity analyses

[8] These three levels of renewable outputs are carefully selected so that the prosumers will be in a long as well as short position in the resulting equilibria. Moreover, K_i is capped above in order to prevent the prosumer's marginal benefit from becoming negative. One explicable justification of this assumption is that a prosumer, with the goal of energy self-reliance, is less likely to install excessive

are conducted in order to numerically illustrate two propositions.

2.3.2 Main Results

Tables 2.1 and 2.2 report the main results of our analysis, involving perfect competition and strategic prosumers, respectively. We organize each of the two tables into three parts, corresponding to outcomes associated with the prosumers, wholesale power market, and economic rent distribution. Each table also contains three columns (a)–(c), from left to right, respectively, for cases with 25, 50 and 120 MW of renewable output. To facilitate our expositions, we define a prosumer's net position as follows. A *short* position if the prosumer engages in the market to purchase power, i.e., $\sum_f z_{fi} < 0$. A *long* position is when the prosumer engages in the market to sell power, i.e., $\sum_f z_{fi} > 0$. With this definition in mind, we report prosumer's net sale (+) or purchase (–) to/from the power pool, consumed energy or load, the self generation from backup generation, and its surplus. We also report a number of variables associated with the power market, including total generation, total demand, power price in node 1, and sale-weighted power price, which is computed as $\sum_i p_i d_i / \sum_i d_i$. Note that the difference between total power generation and total power demand is equal to the power purchase by the prosumer. The last section summarizes the economic rent distribution in the power pool, including that of consumers, producers and grid operator.

A number of observations emerge from these two tables. When the renewable

renewable capacity with an effective output much greater than its expected demand. If possessing a considerable amount of renewables, the prosumer, mostly likely, will always be in a long position and act as a producer. That case would be less interesting.

Table 2.1: Results under Perfect Competition Cases

Variables \Scenarios	(a)	(b)	(c)
Renewable output [MW]	25	50	120
Prosumer's sale(+)/purchase(-) [MWh]	-66.27	-44.11	19.96
Prosumer's load [MWh]	102.02	103.09	105.32
Prosumer's generation [MWh]	10.75	8.98	5.25
Marginal cost of backup [$/MWh]	45.75	43.98	40.28
Prosumer's surplus [$K]	9.89	11.05	14.05
Total power demand [MWh]	2,847.32	2,851.35	2,858.81
Total power production [MWh]	2,913.59	2,895.45	2,838.85
Power price in node 1 [$/MWh]	45.75	43.98	40.28
Sale-weighted power price [$/MWh]	35.52	35.41	35.17
Producers' surplus [$K]	39.23	41.12	45.72
Consumers' surplus [$K]	255.74	256.26	257.27
Grid operator's revenue [$K]	10.18	8.50	5.08
Social Surplus [$K]	305.16	305.87	308.07

Table 2.2: Results under Strategic Prosumer Cases

Variables \Scenarios	(a)	(b)	(c)
Renewable output [MWh]	25	50	120
Prosumer's sale(+)/purchase(-) [MWh]	-19.65	-12.97	7.52
Prosumer's load [MWh]	84.49	91.38	112.48
Prosumer's generation [MWh]	39.84	28.41	0.00
Marginal cost of backup [$/MWh]	74.84	63.41	35.00
Prosumer surplus [$K]	9.23	10.78	13.99
Total power demand [MWh]	2,855.25	2,855.85	2,857.69
Total power production [MWh]	2,874.91	2,868.82	2,850.17
Power price in node 1 [$/MWh]	42.21	41.88	40.88
Sale-weighted power price [$/MWh]	42.21	35.26	35.21
Producers' surplus [$K]	43.08	43.52	44.89
Consumers' surplus [$K]	256.78	256.87	257.11
Grid operator's revenue [$K]	6.82	6.52	5.62
Social Surplus [$K]	306.68	306.91	307.62

output is equal to 25 and 50 MW, either as a price-taker or a strategic entity, the prosumer's load is met by self generation plus purchase from the power pool. Thus, in both cases, the prosumer is in a short position. Epitomized by column (a) in Table 2.1, the prosumer's load, 102.02MW, is met by 25MW from renewables, a power purchase of 66.27 MW from the power pool, and self generation of 10.75 MW from the backup unit. Intuitively, other than renewables, there are two competing power sources available to the prosumer, one is by self generation, and the other is by power purchases from the pool. These two options are perfect substitutes for each other, and the profit-maximization principle requires the prosumer to use the option, among the two, that has a lower cost or utilize them both insofar that the marginal cost of two options become equal when market is perfectly competitive. Indeed, Table 2.1 indicates that the prosumer decides to produce the backup option to the level such that its marginal cost is equal to the pool power price in all scenarios.

However, it is not the case when the prosumer is designated as a strategic agent. In particular, while the prosumer remains in a short position with renewable output equal to 25 MW and 50 MW in columns (a) and (b) in Table 2.2, its power consumption of 84.49 MW in (a) is supplied by 25 MW from renewables, 19.65 MW from power purchase, and 39.84 MW from self generation, respectively. The marginal cost of the backup generation, in this scenario, is actually significantly higher than the power price in node 1 by a margin of \$32.63/MWh (=74.84 − 42.21) or 77%. It is this self "over generation" that allows the prosumer to lower its power procurement, suppressing power demand in the power pool, which leads to a lower power price in node 1 compared to its correspondent in Table 2.1, i.e., 42.21 v.s. \$47.74/MWh. A similar tactic is applied by the prosumer in column (b) of Table 2.2 to reduce the power price from \$43.98/MWh in Table 2.1 to \$41.88/MWh.

Furthermore, the power price in node 1 as well as the sales-weighted average power price of the market decline in accordance with the increases in renewable output. Because renewable energy represents a zero marginal cost resource, its abundance suppresses power prices in the market. For instance, the power price in node 1 (the sale-weighted average power price) in Table 2.1 decreases from \$45.75/MWh (\$35.52/MWh) in (a) to \$40.28/MWh (\$35.17/MWh); a similar trend can also be observed in Table 2.2. Given the current parameter setting, the prosumer's foregone benefit of not consuming renewable energy available to it is equal to zero as the prosumer's load in equilibrium is greater than renewable output (see Fig.2.1).[9] The prosumer, as expected, benefits from zero marginal cost renewables as its surplus increases are commensurate with incremental output from renewables.

With the prosumers participating in the market, the total generation and total consumption (excluding the prosumers) in the power market are not equivalent, depending on the prosumer's net position in the market. When the prosumer is in a short position, e.g., scenarios 25 MW and 50 MW in columns (a) and (b), the total generation from producers is greater than the total consumption by consumers, with excessive generation purchased by the prosumer. For instance, as indicated in scenario (a)

[9] Of course, had the valuation of the power by the consumers in the power pool been significantly higher than that of the prosumers, it is possible to observe the prosumer forego some consumption in order to profit from the power pool.

in Table 2.1, a total of 66.27 MW ($= 2,913.59 - 2,847.32$) is procured by the prosumers.

Now we turn to the economic rent analysis. Interestingly, the prosumer is worse off in Table 2.2 compared to Table 2.1. This is in part because producers and consumers in the power pool are designated as price takers. Generally, there are two counteracting forces that jointly determine the market equilibrium. When the prosumer exercises buyer's market power to lower the cost of its procurement, with an attempt to lower power prices, consumers will increase their quantity demanded when seeing lower prices, earning additional economic rent, thereby working against the prosumer. Had the consumers been with a fixed demand or less price responsive, the prosumer will more likely succeed in the attempt to manipulate the power prices in its favor. On the contrary, when the prosumer is in a long position, its effort to exercise seller's market power in order to push up the power prices is also likely to be thwarted by the increases in power sales from the price-taking producers. Producers remain benefiting from the prosumer's strategy when the prosumer is in a long position as they both would prefer higher power prices. For instance, when the prosumer is in a relatively "longer" position, the producers' ability to negate the impacts from the prosumer would be more than offset by the ability of the prosumer to exercise buyer's market power to lower the power prices, leading to a lower surplus \$44.89K in Table 2.2 compared to \$45.72K in Table 2.1. Moving from columns (a) to (b), when the prosumer is in a relatively "weaker" short position, the producers in the wholesale market would then benefit from the prosumers' strategy, leading to a higher surplus in Table 2.2 than that of Table 2.1. Overall, considering columns from (a) to (c) with increasing more renewables, elevation in the surplus by the consumers and producers leads to increases in the social surplus.

2.3.3 Sensitivity Analyses

This section summarizes the main results of the sensitivity analyses on the simulation framework. The focus has been placed on learning the underlying strategies used by the prosumer and the consequential impacts on the equilibra that characterize market outcomes. This is done by uniformly altering the renewable output K_i from 25 to approximately 120 MW to generalize our findings.[10] The sensitivity analyses in this section serves two purposes. One is to explore the impact of the prosumers on the market when they own either *relatively* small or large size of renewable asset. The second one is to understand the impact of renewable stochasticity on the market outcomes. In particular, we assume that there is an expectation on the renewable output by the prosumers. Thus, a high value of K_i corresponds to the situation where output from the renewables is greater than the expectation while small value of K_i is for the situation that output from the renewables is less than the expectation. We summarize the findings formally in two propositions while illustrating the results numerically in Figs. 2.2–2.3. The proofs of these two propositions can be found in the appendices.

Proposition 2. *If a prosumer is in a short (long) position as a price-taker, it will also be in a short (long) position as a strategic entity, and vice versa.*

As alluded to earlier, the prosumers' net position cannot be determined *a*

[10]We limit our attention to those cases with a positive marginal benefit possessed by the prosumers. A larger K_i beyond 120 MW or so will lead the prosumers' marginal benefit to be negative. We therefore rule those cases out.

priori and is the consequence of market dynamics and interactions. This proposition suggests that the strategy executed by the prosumers would only impact the quantity of their decision variables $\sum_f z_{fi}$, the MW of power to buy from or sell to the main grid, but not its net position, i.e., buy (-) or purchase (+). Fig.2.2 illustrates Proposition 2 numerically by plotting the prosumers' net position against the renewable output K_i, where solid and dashed lines represent the scenarios with the prosumers as a price-taker and a strategic entity, respectively. We also plot a horizontal line linking K_i to the case when the prosumers behave in an isolated or island mode without any interaction with the power pool or $\sum_f z_{fi} = 0$. Proposition 2 states that regardless of the choice of strategy, if a prosumer is in a short position as a price-taker, it will also be in a short position as a strategic entity. That is, for a given K_i, both lines will stay at the same side (below or above) separated by the horizontal line.

Proposition 3. *A prosumer is better off by participating in the market as a price-taker rather than a strategic entity exercising Cournot strategy.*

Fig. 2.3 illustrates Proposition 3 numerically by graphing the prosumers' surplus against renewable output K_i under both price taking and strategic scenarios and reminiscent of the observation that when the prosumer is in a short position, contracting the power procurement in order to lower the power price is not an economically viable strategy when the other participants in the market act as price takers. The initial gap of the prosumer's surplus between the two scenarios begins with around \$5K when $K_i = 25$MW. The gap then shrinks with an increase of renewables output K_i, and

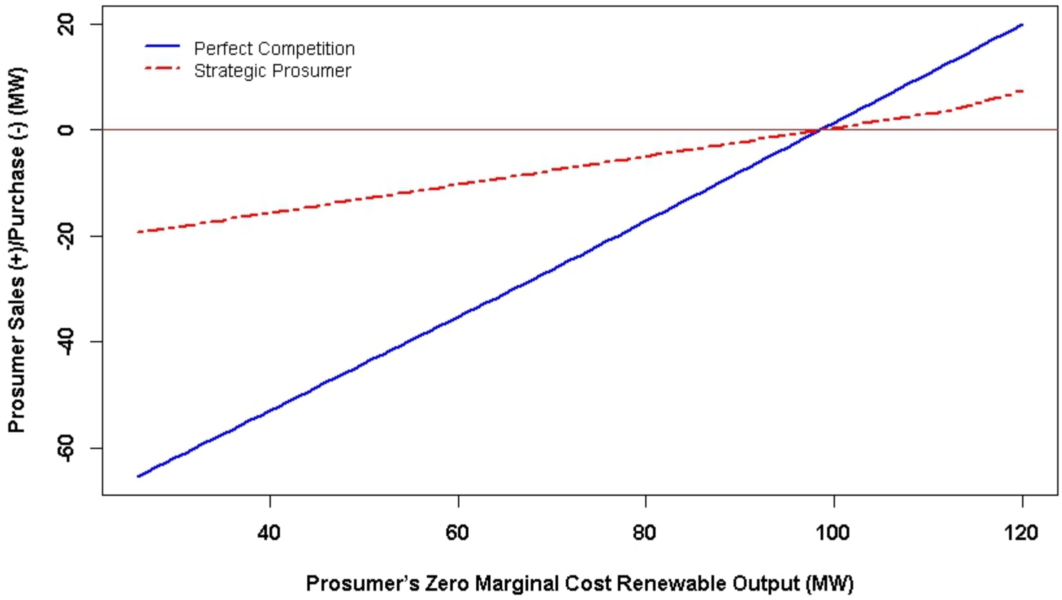

Figure 2.2: Plot of prosumer's net position against renewables output K_i under the price-taker and strategic entity scenarios

eventually asymptotically goes to zero when $K_i = 85$ MW. Fig. 2.3 also shows that the line indicating a prosumers is a price-taker either overlaps with or lies above that of a strategic entity which is a visual illustration of Proposition 3.

2.4 Conclusions

Prosumers' ability to act as a producer and a consumer, a duality that is not commonly seen if not unprecedented in the sector, also creates new opportunity or challenge to the energy sector. This chapter extends the existing work by explicitly formulating the optimization problem faced by a prosumer in a complementarity problem.

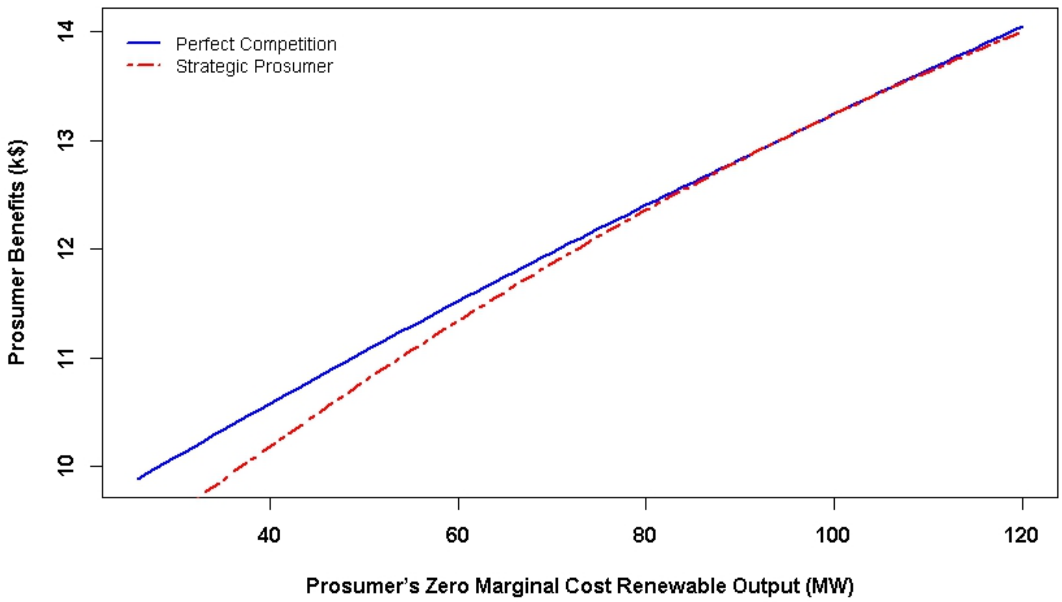

Figure 2.3: Impact of prosumer strategy on prosumer surplus under different levels of renewable outputs

We conclude that exercising market power will not alter a prosumer's net position in equilibrium. That is, if the prosumer is in a short position as a price-taker, i.e., buy power from the main grid, it will also be in a short position had it been a strategic entity. The chapter also discovers that while the prosumer is capable of manipulating the power prices by either exercising buyer's (seller's) power to lower (increase) power prices, it actually is better off if acting as a price-taker while other entities in the market behave competitively. Finally, our analysis concludes that when the renewable output is low (so that the prosumer needs to purchase power from the main grid in equilibrium), consumers could benefit from lower power prices at the expense of producers as the prosumer decreases its power procurement from the power market in order to lower the power prices. On the other hand, as renewable output increases, thus the prosumer becomes a net seller to the main grid; its economic incentive is then more aligned with other conventional suppliers.

Our analysis is subject to a number of limitations. First, we did not consider the possibility that the prosumer operates an energy storage system. In reality, many prosumers might own and operate energy storage equipment, e.g., electrical vehicles, in order to take advantage of lower power prices during off-peak periods. Accounting for this will call for a multiple-period model with a consideration of cross-elasticity of energy demand among time periods in order to examine the effect of power price in one time period on the demand other time periods. Second, we maintain the assumption that market participants, other than the prosumer, are price takers. While the model in Section 2.2 is readily modified to simulate strategic behavior of conventional producers, allowing other producers behave strategically might complicate the analysis so that we might find it difficult to isolate the impact induced by the prosumers. Third, while we simulate different levels of renewable outputs, our analysis is essentially deterministic. Moving to a stochastic modeling framework, for example, by using scenario paths of renewable outputs and correlated demand, will, undoubtedly, be more realistically to represent the reality faced by the power market. Fourth, an aggregator with adequate trading experience might be able to engage in spatial arbitraging to explore price difference in different location. We leave the aforementioned considerations to our future work.

Chapter 3

Risk-Averse and Strategic Prosumers: A Distributionally Robust Chance-Constrained MPEC Approach

3.1 Introduction

The power sector is undergoing rapid transformations in terms of available technologies and architecture. Driven by a need for decarbonization, sustainability, and resilience, we have witnessed a remarkable move towards advancing and deploying distributed renewable resources as well as harnessing price-responsiveness of energy demand. These include both demand response, storage and other flexible resources, which altogether form the broader concept of distributed energy resources (DERs). This paradigm shift towards a more engaged demand-side challenges the conventional, top-down power grid architecture based on supply-side and calls for a new market design for the power sector(e.g., FERC Order 745 and 2222). In particular, as new agents, such as "prosumers" with the ability of concurrent generation and consumption, are introduced to the power sector, their presence is expected to alter economic incentives, which might create opportunities for manipulations, thereby undermining efficiency of the power market.

While behind-the-meter households or end-prosumers may have limited access to the main grid, the ruling under the FERC 2222 allows the integration of multiple DERs owned by different entities with different sizes and diverse technologies to participate in the regionally organized wholesale energy, capacity, and ancillary services markets alongside traditional resources [29]. We refer to those entities with direct access to the main grid as "prosumers" in order to distinguish them from "end-prosumers." This would also affect decisions on transmission grid expansion and investments on distribution and power generation facilities [30].

The transactions between prosumers and the wholesale power market are empowered by the presence of aggregators at the distribution level. Examples include community choice aggregators, which are popular in California and other states. Aggregators could be responsible for operation of generation assets, e.g., solar panels, storage, and electric vehicle charging, over wide geographical areas and diverse types of households that constitute a substantial distributed generation and energy management capability [31, 3]. This provides an economic leverage for prosumers participating in the wholesale power markets far beyond ordinary customers as they are capable of manipulating considerable resources over time and space, which at the same time, also fundamentally changes the business models within the electricity market [32]. To align incentives with the desired outcomes of the power market, it is imperative to understand how this new entity, the prosumer, might impact market outcomes given current market conditions.

A recent thread of literature has also focused on the role of aggregators as middle-men in the power sector that operate DERs on behalf of owners and their interactions with the wholesale power market [31, 3]. The heterogeneity in terms of geographical placement and type of resources owned by the prosumers grants them a competitive advantage, as the information is likely to be private, only known by prosumers. This also calls for a careful examination of DERs market power potential in the market [33], and other studies have shown that even low levels of wind penetration could enable strategic manipulation, leading to efficiency loss [34, 35].[1]

Regression-based analysis is a common approach used by researchers when empirically examining the extent of *ex post* market power. However, it is less useful for vetting the potential of market power when existing data are not yet available. On the other hand, game-theoretical models based on bottom-up formulations have extensively been used to evaluate *ex ante* electricity market outcomes, see, for example [36], [11]. The strength of these models is that they allow representing the interactions among different market participants, especially new ones, while considering market rules and other institutional settings. A number of studies have applied this approach to

[1] In fact, a recent report concludes that, by 2050, prosumers can produce twice as much power as nuclear production now in Europe, thereby rendering them a dominant role in the energy landscape (`https://www.greenbird.com/news/utility-death-spiral`)

address outcomes of a power market considering information asymmetry related to aggregators. For instance, [37] examines the impact of a DR aggregator operating a green energy management system in the wholesale market by implementing a quantity-based (Cournot) strategy. The paper, however, i) does not reflect the dual nature of prosumers (i.e., concurrent generation and consumption), and ii) is limited to Cournot strategy by the aggregator, which is just one of the several strategies at the aggregator's disposal. The Stackelberg game has long been used to model sequential-move games or leader-follower situations [38, 39, 40, 41, 42, 43, 44, 45]. In a more recent work, a game in the electricity market is modeled in a Stackelberg setting where the aggregator is the leader, and the grid operator along with other producers are the followers [6]. The prosumer in these studies is basically modeled as a supplier and is unable to reflect the buyer's power that a prosumer can demonstrate. Therefore, while leader-follower models, such as [6], can provide useful illustrations of how prosumers could exercise market power, formulations that explicitly capture the dual nature of prosumers and endogenize power price formation are more desirable.

This chapter builds on existing work to examine prosumers' market power potential [45, 46, 47]. Particularly, our contribution is to extend the models in [46] and [47] by developing a distributionally robust chance-constrained mathematical program with equilibrium constraints (MPEC) for a leader-follower or Stackelberg setting. The various conclusions under the assumption of risk-neutrality in [47] cannot be directly applicable to or generalized to the risk-averse situation when the impact of the uncertainty of renewables and risk attitude is considered. A distributionally robust chance constraint approach was applied to the optimal power flow (OPF) model, where the chance constrained OPF limits the probability of violating transmission constraints [48]. However, to our best knowledge, our work is the first attempt to develop a distributionally robust chance-constrained MPEC for a leader-follower setting, focusing on a risk-averse leader-prosumer in the power sector. The non-convexity of the MPEC resulting from the bilinear terms in the upper-level problem's objective function and from the optimality conditions of the lower-level problem is overcome using Wolfe duality and disjunctive constraints (see appendices). The problem is then recast as a mixed integer quadratic program (MIQP). When designating prosumer as a leader in a leader-follower framework, the prosumer possesses an advantage of information asymmetry that allows it to internalize the reactions of other market participants into its own profit maximization problem. Similar to [46, 47], the model does not fixate the role of the prosumer as a producer (long position) or as a consumer (short position), but allows the solutions to decide what its role should be in order to maximize its profit.

The rest of this chapter is organized as follows. In Section 3.2, the leader-follower formulation of the prosumer's problem is introduced. A case study based on the IEEE 24-bus system is implemented in Section 3.3. The outcomes of analyses based on altering the amount of renewable output are presented in Section 3.4. We conclude the chapter in Section 3.5.

3.2 Model Setup

We introduce in this section the prosumer's problem in the upper-level and the lower-level optimization problems faced by the grid operator. In what follows, we first introduce the notations that are used throughout this chapter and then explain how each agent participates in the market. We denote I as the set of nodes and L as the set of transmission lines consisting of elements in ordered pairs of distinct nodes. F is the set of generation firms, while H is the set of generation units, and $H_{fi} \subset H$ is the set of generation units owned by firm f at node i. Greek letters render the corresponding dual variable.

3.2.1 Prosumer's Problem

We assume that the prosumer is constituted by bundling of a large number of individual DERs, thereby allowing them to interact with the bulk market directly, consistent with the stipulation by the recent FERC Order 2222 [29]. Thus, in a way, we model end-prosumers and the aggregator as a joint entity.[2]

The renewable generation output at node i is denoted by a random variable $\tilde{K}_i$, which is uncertain because it is dependent on available natural resources, e.g., solar and wind. [3] We assume that the distribution $\mathbb{P}_i$ of $\tilde{K}_i$ belongs to a set $\mathcal{P}_i$ of distributions,

[2]For instance, prosumers are allowed to participate in a day-ahead market but are subject to a fixed retail or contracted rate in real time, similar to the situation faced by several EU countries, e.g., Italy, Netherlands, and Belgium [49].

[3]Two concerns here, namely, privacy and truth-telling, are worth more discussion. We believe that neither should be a significant concern in the current context. Considering that end-prosumers enter a contract with the aggregator, e.g., OhmConnect in [50], the contract will then specify the types of necessary private information from end-prosumers and how the information will be handled. Moreover, given that the aggregator is tasked to maximize the joint profit of participants, non-truth-telling by

where the first and second moments are known, i.e., $\mathbb{E}(\tilde{K}_i) = K_i$ and $\mathbb{V}(\tilde{K}_i) = \sigma_i^2$, respectively, but without exact knowledge of the probability distributions. Meanwhile, the prosumer owns a dispatchable or backup resource, e.g., on-site diesel generator or energy storage, that supplies power g_i with an increasing and strictly convex cost function $C_i^g(g_i)$ and with capacity of G_i in order to hedge against uncertain output $\tilde{K}_i$. Specifically, we assume a quadratic cost function $C_i^g(g_i) = D_i^{g0} g_i + \frac{C_i^{g0}}{2} g_i^2$. Yet, its supply would not be able to fully back up the intermittent renewable output. Here, g_i can be associated with an energy storage, and its marginal cost can be interpreted as the costs of withdrawing energy from the grid, accounting for battery discharge depreciation.

For the purposes of this study, the prosumer's benefit function of consuming electricity at node i is given by $B_i^l(l_i)$, where l_i corresponds to the self-consumption at each node. The benefit function $B_i^l(l_i)$ is assumed to be increasing and strictly concave. The monotonicity of $B_i^l(l_i)$ indicates that the prosumer's objective function is increasing in the level of consumption. Specifically, we assume a quadratic benefit function $B_i^l(l_i) = A_i^{l0} l_i - \frac{A_i^{l0}}{2B_i^{l0}} l_i^2$. We posit that the prosumer maximizes its profit by deciding i) the amount of power to buy from ($z_i < 0$) or sell to ($z_i > 0$) in node i at price p_i. ii) the amount of its own power consumption l_i, and iii) the amount of power to be generated from the backup dispatchable technology or g_i. We also assume that the prosumer is only allowed to sell/buy power locally, i.e., at each node, which is consistent with the future grid's layered structure [32].[4]

end-prosumers would undermine the aggregator's ability to maximize the joint profit on their behalf.

[4] Had the prosumers been modeled to allow to sell surplus power from its local node i to other locations, it is expected to produce the same market outcomes [46].

We then formulate a distributionally robust chance-constrained problem of the prosumer/aggregator facing an uncertain renewable output $\tilde{K}_i$ as follows:

$$\underset{z_i, l_i \geq 0, g_i \geq 0}{\text{maximize}} \quad \sum_i \Big(p_i z_i + B_i^l(l_i) - C_i^g(g_i)\Big) + \sum_i \mathbb{E}\Big[P_i^c\Big(\tilde{K}_i - z_i - l_i + g_i\Big)\Big] \quad (3.1a)$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i\Big(z_i + l_i - \tilde{K}_i - g_i \leq 0\Big) \geq 1 - R_i \qquad (\delta_i), \quad (3.1b)$$

$$g_i \leq G_i. \qquad (\kappa_i), \quad (3.1c)$$

$$l_i, g_i \geq 0. \quad (3.1d)$$

The three terms in the first line of the objective function (3.1a) correspond to revenue (+) or cost (−) from transactions in the day-ahead wholesale market, benefit of consuming power, and generation costs incurred from backup resource, respectively [5]. The second line gives the expected cost/revenue in real time, where P_i^c is the fixed retail rate or the contracted price between the aggregator and the utility. Three constraints are associated with the prosumers' problem. Constraint (3.1b) is the distributionally robust chance constraint of the prosumer. It states that the sum of renewable output $\tilde{K}_i$ and self generation g_i net of transactions with the wholesale day-ahead market, i.e., z_i, has to be equal or greater than the self-consumption l_i with probability of at least $1 - R_i$ for any distribution in $\mathcal{P}_i$. δ_i, the dual variable of the distributionally robust chance constraint of the prosumer and represents the marginal impact of risk

[5]The interaction of the prosumer with the bulk day-ahead energy market is modeled through shifting of supply curves and sales decisions of conventional producers. An alternative way of modeling this situation is to horizontally aggregate consumers' and prosumers' demand curves. However, this aggregation might lead to kinked demand curves, which poses numerical difficulties, see [51] for example.

tolerance level on the expected benefit of the prosumers in (3.1a). Constraint (3.1c) limits the output g_i by its capacity G_i. κ_i in (3.1c) is the dual variable of prosumer's dispatchable generation limit, which renders the scarce rent of the on-site generation capacity. Constraint (3.1d) states the non-negativity of the variables for prosumer's consumption and backup generation.

3.2.2 Lower-Level Problem

We next introduce the lower-level problem at which the grid operator takes supply bids from suppliers and demand bids from consumers/load serving entities and maximizes the bulk market's social surplus subjected to prosumer's decision z_i. Let x_{fih}, d_i, and y_i denote the power output produced by generation unit h at node i owned by firm f, the quantity demanded by consumers at node i, and the power injection/withdrawal at node i, respectively. We assume a) an increasing and strictly concave benefit function $B_i(d_i) = P_i^0 d_i - \frac{P_i^0}{2Q_i^0} d_i^2$ for consumers, which is separated from that of the prosumer, $B_i^l(l_i)$ in (2.2a), and b) an increasing and strictly convex cost function $C_{fih}(x_{fih}) = D_{fih}^0 x_{fih} + \frac{C_{fih}^0}{2} x_{fih}^2$ for generation.

$$\underset{x_{fih}, d_i, y_i}{\text{maximize}} \quad \sum_i B_i(d_i) - \sum_{f,i,h \in H_{fi}} C_{fih}(x_{fih}) \tag{3.2a}$$

subject to

$$x_{fih} \le X_{fih} \quad (\beta_{fih}), \forall f, i, h \in H_{fi}, \tag{3.2b}$$

$$\sum_i PTDF_{ki} y_i \le T_k \quad (\lambda_k^+), \forall k, \tag{3.2c}$$

$$-\sum_i PTDF_{ki} y_i \leq T_k \quad (\lambda_k^-), \forall k \quad , \tag{3.2d}$$

$$d_i - \sum_{f,h \in H_{fi}} x_{fih} - z_i = y_i \quad (\eta_i), \forall i, \tag{3.2e}$$

$$\sum_i y_i = 0 \quad (\theta) \quad , \tag{3.2f}$$

$$x_{fih} \geq 0 \quad (\varepsilon_{fih}), \forall f, i, h \in H_{fi} \quad , \tag{3.2g}$$

$$d_i \geq 0 \quad (\xi_i), \forall i \tag{3.2h}$$

The lower-level problem is the social surplus maximization problem faced by the grid operator and is formulated in (3.2). The lossless linearized DC flow is applied to modeling power flow in the transmission network using the power transfer distribution factor ($PTDF_{ki}$). Constraints (3.2b)–(3.2d) limit generation capacity (X_{fih}), and transmission capacity (T_k). Constraint (3.2e) is the nodal balance with prosumer's transaction (z_i) The inclusion of this constraint shifts the demand of conventional consumers in the wholesale market operated by the ISO, given the decision z_i of the leader-prosumer in the upper level. Specifically, it follows from (3.2e) that the inverse demand (or marginal benefit) function of conventional consumers can be represented as $B_i'(d_i) = B_i'(\sum_{f,h} x_{fih} + z_i + y_i)$ in equilibrium. When the prosumer purchases $z_i (< 0)$ from node i, the effective "wholesale" demand increases or shifts to the right by the absolute value of z_i, reflecting the demand from both the conventional consumers and the prosumer. Similarly, if the prosumer sells $z_i (> 0)$ to node i instead of purchase, then the wholesale demand decreases or shifts to the left by z_i through (3.2e). Note also that the benefit function of the prosumer, $B_i^l(l_i)$, does not appear in the objective function of the ISO in (3.2), but this does not affect the outcome in the lower-level problem because the leader-prosumer's decision l_i and resulting $B_i^l(l_i)$ in the upper level are taken as given (and hence exogenous) by the grid operator in the lower level. The balance between supply and demand is implied in (3.2f). Notice that the social surplus maximization problem does not include sales of each generation firm as a decision variable but rather decides on their output (x_{fih}) and the sales/purchases by the prosumer in node i (z_i). However, once x_{fih} and z_i are decided by the solution of the problem, the sales balance for the generation firm holds automatically and would be consistent with (3.2e) and (3.2f). Constraints (3.2g)–(3.2h) state the non-negativity of generation and consumption, respectively. Given that the lower level is a concave programming problem, the solutions can be represented by its optimality conditions as follows:

$$-C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f, i, h \in H_{fi}, \tag{3.3a}$$

$$B'_i(d_i) - \eta_i + \xi_i = 0 \quad \forall i, \tag{3.3b}$$

$$-\sum_k (\lambda_k^+ - \lambda_k^-) PTDF_{ki} + \eta_i - \theta = 0 \quad \forall i, \tag{3.3c}$$

$$0 \leq \beta_{fih} \perp x_{fih} - X_{fih} \leq 0 \quad \forall f, i, h \in H_{fi}, \tag{3.3d}$$

$$0 \leq \lambda_k^+ \perp \sum_i PTDF_{ki} y_i - T_k \leq 0 \quad \forall k, \tag{3.3e}$$

$$0 \leq \lambda_k^- \perp -\sum_i PTDF_{ki} y_i - T_k \leq 0 \quad \forall k, \tag{3.3f}$$

$$d_i - \sum_{f,h \in H_{fi}} x_{fih} - z_i - y_i = 0 \quad \forall i, \tag{3.3g}$$

$$\sum_i y_i = 0, \tag{3.3h}$$

$$0 \leq \varepsilon_{fih} \perp x_{fih} \geq 0 \quad \forall f, i, h \in H_{fi}, \tag{3.3i}$$

$$0 \leq \xi_i \perp d_i \geq 0 \quad \forall i \tag{3.3j}$$

3.2.3 Distributionally Robust Chance-Constrained MPEC Formulation

This section introduces the leader-follower formulation of the Stackelberg game in the context of prosumers in an electricity market. Here, the upper-level problem is the prosumer's benefit maximization, and the lower-level problem is a collection of complementarity or optimality conditions in the market derived from the grid operator's social surplus maximization problem. The resulting problem of the prosumer is cast as a distributionally robust chance-constrained mathematical program with equilibrium constraints (or DRCC-MPEC) in (3.4):

$$\underset{\Phi \cup \Omega \cup \Lambda}{\text{maximize}} \quad \sum_i \Big(\eta_i z_i + B_i^l(l_i) - C_i^g(g_i)\Big) + \sum_i \mathbb{E}\Big[P_i^c\Big(\tilde{K}_i - z_i - l_i + g_i\Big)\Big] \tag{3.4a}$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i\Big(z_i + l_i - \tilde{K}_i - g_i \leq 0\Big) \geq 1 - R_i \qquad (\delta_i), \tag{3.4b}$$

$$g_i \leq G_i. \qquad (\kappa_i), \tag{3.4c}$$

$$l_i, g_i \geq 0., \tag{3.4d}$$

$$(3.3a) - (3.3j)$$

where $\Phi = \{z_i, l_i, g_i\}$, $\Omega = \{x_{fih}, d_i, y_i\}$, and $\Lambda = \{\beta_{fih}, \lambda_k^+, \lambda_k^-, \eta_i, \theta, \varepsilon_{fih}, \xi_i\}$. The first line of the objective function is the net benefit of the prosumer with equilibrium power prices (η_i) derived from the dual variable associated with the nodal balance constraint (3.2e) in the lower-level problem. Constraints (3.4b)–(3.4d) indicate the operational constraints of the prosumer. Constraints (3.3a) – (3.3j), which are inherited from the optimality conditions of the lower-level problem, form the complementarity problem characterizing the equilibrium of the market.

The problem specified in (3.4) is difficult to solve due to at least three reasons. First, (3.4b) considers probability constraints over all possible distributions with given moments. Second, (3.4) is neither linear nor concave because of the bilinear term $\sum_{fi} \eta_i z_i$. Third, the feasible region is not convex because of the complementarity conditions. We address the first issue of uncertainty by replacing (3.4b) with a robust counterpart of the distributionally robust chance constraint indicated in (3.5) as in [52].

$$\sigma_i\sqrt{\frac{1-R_i}{R_i}} + z_i + l_i - g_i - K_i \leq 0 \quad \forall i \tag{3.5}$$

Moreover, we overcome the second issue by using Wolfe's strong duality to concavify the bilinear term in the objective function (see Appendices). To do this, we can substitute the bilinear term in the objective function using the equality (3.6):

$$\sum_i \eta_i z_i = \sum_i B_i'(d_i)d_i - \sum_k (\lambda_k^+ + \lambda_k^-)T_k - \sum_{f,i,h\in H_{fi}} \big(C_{fih}'(x_{fih})x_{fih} + \beta_{fih}X_{fih}\big) \tag{3.6}$$

We finally linearize the complementarity conditions by disjunctive constraints [53]. Consequently, the DRCC-MPEC (3.4) is re-cast as a mixed integer quadratic program (MIQP).

3.2.4 Perfectly Competitive and Cournot Models

In contrast to the Stackelberg leader-follower formulation in Section 3.2.3, perfect or imperfect (Cournot) competition discussed in chapter 2 entail a different information structure. Particularly, it involves solving simultaneously the prosumer's problem in (3.1a)–(3.1d) and the lower-level problem in (3.2). As discussed in Section 3.2.3, we derive a robust counterpart of the distributionally robust chance constraint for the prosumer's problem. Then, the overall problem can be solved by the collection of first-order conditions of the prosumer's and the lower level problem. This forms a problem known as a mixed linear complementarity problem (MLCP). In the Cournot case, the prosumer is fully aware of wholesale's demand function and able to withhold output to drive up power prices. Similarly, the problem can be formulated as an MLCP. Perfect competition and Cournot formulations for the prosumer along with their theoretical properties and existence of solutions are discussed in [46].

3.3 Numerical Analysis

3.3.1 Data, Assumptions, and Scenarios

The model is applied to the IEEE Reliability Test System (RTS 24-Bus) [54]. The topology of the system consists of 24 buses, 38 transmission lines, and 17 constant-power loads with a total of 2,850 MW. We aggregate 32 generators into 13 generators by combining those with the same marginal cost and located at the same node. Six generation units, however, are excluded from the dataset since they are hydropower units, which operate at their maximum output of 50 MW [27]. In order to analyze the impact of transmission congestion, the capacity of line 7, between nodes 3 and 24, in the test case is reduced to 150 MW. The marginal cost of generation is represented by a quadratic function parameterized by D^0_{fih} and C^0_{fih} as the coefficient for the linear and quadratic terms, respectively. Furthermore, the prosumer, or the leader, located at node 1 is assumed to have the same demand function as consumers in that node. The prosumer owns a renewable generation unit that produces varying amounts of power (contingent on available natural resources) and a dispatchable unit as a backup option. The RTS 24-Bus case is first formulated as a least-cost minimization problem and solved with fixed nodal load in order to compute dual variables associated with load constraints to parameterize the demand. The dual variables together with an assumed price elasticity of -0.2 are then used to calculate the demand parameters, P_i^0 and Q_i^0. The magnitude of price elasticity of demand is comparable with the literature [28].

We examine three scenarios in detail, varied by the levels of renewable output from the units owned by the prosumer with $R = 0.9$. Mean renewable output K_1 is assumed to have three levels, 25, 50, and 120 MW, with their uncertainties characterized by their standard deviation σ_1, which is 20% of the mean (K_1). These levels are chosen carefully to show results in both short and long positions. Finally, we also simulate cases with mean renewable output changing from 25 MW to 120 MW in order to understand its overall impact on economic rent among entities.

3.3.2 Results

Table 3.1 summarizes market outcomes when the prosumer is formulated as a Stackelberg leader for three scenarios, 25, 50, and 120 MW of mean renewable generation. We also report outcomes from perfect competition and Cournot in Tables 3.2 and 3.3, respectively. As indicated in the first row of Table 3.1, the prosumer changes from purchase (-) to sale (+) with increased levels of renewable output. For cases of 25 and 50 MW (columns a–b), the prosumer buys 60.83 MWh and 42.26 MWh, respectively, acting as a consumer or in a *short* position. As expected, the quantity of the purchases decreases as the prosumer's renewable output grows. In column (c), where mean renewable output is equal to 120 MW, the prosumer lies in a *long* position in equilibrium and sells (positive quantity) 11.69 MWh to the power market. The prosumer's purchase quantities in columns (a) and (b) are in between those of perfect and Cournot competition indicated in Tables 3.2 and 3.3. Akin to the short position case, prosumer's sales in the Stackelberg equilibrium is less than the 12.54 MWh in Table 3.2 and larger than 3.61 MWh in Table 3.3.

Table 3.1: Results under Stackelberg Leader Prosumer Cases

Variables \Scenarios	(a)	(b)	(c)
Mean renewable output [MW]	25	50	120
Prosumer's sale(+)/purchase(-) [MWh]	-60.83	-42.26	11.69
Prosumer's load [MWh]	99.35	101.14	105.42
Prosumer's generation [MWh]	15.19	12.21	5.11
Marginal cost of backup [$/MWh]	50.17	47.21	40.11
Prosumer surplus [$K]	9.77	10.91	13.73
Total power demand [MWh]	2,848.31	2,851.68	2,858.07
Total power production [MWh]	2,909.13	2,893.94	2,846.38
Power price in node 1 [$/MWh]	45.32	43.83	40.68
Sales-weighted power price [$/MWh]	35.49	35.39	35.19
Grid operator's revenue [$K]	9.77	8.35	5.44
Producer surplus [$K]	41.86	42.77	44.76
Consumer surplus [$K]	255.86	256.30	257.17
Wholesale social surplus [$K]	307.50	307.42	307.37

Prosumer's quantity demanded, or load, is indicated in the second row of Table 3.1. It increases as the prosumer has more renewable generation resources available and is equal to 99.35, 101.14, and 105.42 MWh for columns (a)–(c), respectively. Having more zero marginal cost renewables by the prosumer (moving from column (a) to (c)) implicitly shifts the market supply curve to the right, leading to an increase in electricity consumption. The prosumer's quantity demanded is also in between those of perfect and Cournot competition illustrated in Tables 3.2 and 3.3: less than 101.95 MWh in column

Table 3.2: Results under Perfect Competition Cases

Variables \Scenarios	(a)	(b)	(c)
Mean renewable output [MW]	25	50	120
Prosumer's sale(+)/purchase(-) [MWh]	-67.74	-47.06	12.54
Prosumer's load [MWh]	101.95	102.94	105.10
Prosumer's generation [MWh]	10.87	9.22	5.64
Marginal cost of backup [$/MWh]	45.87	44.22	40.64
Prosumer surplus [$K]	9.75	10.89	13.73
Total power demand [MWh]	2,847.05	2,850.81	2,858.15
Total power production [MWh]	2,914.79	2,897.87	2,845.61
Power price in node 1 [$/MWh]	45.87	44.22	40.64
Sales-weighted power price [$/MWh]	35.52	35.42	35.19
Grid operator's revenue [$K]	10.30	8.72	5.40
Producer surplus [$K]	39.11	40.86	45.23
Consumer surplus [$K]	255.71	256.18	257.18
Wholesale social surplus [$K]	305.12	305.76	308.81

(a) of Table 3.2 for perfect competition yet higher than 84.03 MWh for the Cournot case in the same column in Table 3.3. The same observation emerges in column (b). In column (c) where the prosumer's mean renewable output is 120 MW, the prosumer's demand in the Stackelberg equilibrium remains in between those of perfect and Cournot competition. However, in this case, the prosumer's quantity demanded is bounded above by the Cournot case (108.46 MWh) and bounded below by perfect competition (105.10 MWh) and asymptotically approaches that of the perfect competition. We address its implication when discussing profit earned by the prosumers. With more renewable available, the prosumer decreases generation from the dispatchable unit as it requires less generation from the backup unit in light of higher levels of renewable capacity. This effectively reduces the marginal cost of the backup unit as more renewable resources become available as shown in Table 3.1 where the marginal cost of the backup unit decreases from 50.17 $/MWh in column (a) to 40.11 $/MWh in column (c).

Turning to prosumer's profit, the prosumer benefits from having more renewable generation output. This is because having more zero marginal cost resources, the prosumer is able to rely less on backup unit (lower operating cost), sell more to (buying less from) the market, and obtain a higher payoff. As seen in Table 3.1 (also in Figure 3.2), the prosumer surplus follows an increasing trend of 9.77, 10.91, and 13.73 ($K) for 25, 50, and 120 MW of mean renewable output, respectively. The prosumer's benefit, unlike aforementioned market outcomes that lay in between price-taker and Cournot strategies, is always the highest in Stackelberg equilibrium. For instance, as noted in Tables 3.2 and 3.3, the prosumer's profit in column (a) is 9.75 and 9.12 $K for the prefect and Cournot competition cases, respectively, which are lower than 9.77 $K for the Stackelberg in Table 3.1. The difference, however, narrows as the prosumer owns more renewable generation capacity, suggesting that a) the prosumer's market power is diminished with increasing amount of mean renewable output K_i, and b) it is easier for the prosumers to exercise buyer's market power (i.e., short position) than seller's market power (i.e., long position) as the latter is likely to be offset by other conventional producers.

The power price in node 1, where the prosumer resides, is directly affected by the available renewables: it drops with an increasing amount of renewables. For example, when the mean renewable output is 25 MW, power price at node 1 is $45.32/MWh, which is reduced to $43.83/MWh and further to $40.68/MWh if the mean renewable generation is 50 and 120 MW, respectively. The same impact over the entire grid can also be observed as the sales-weighted power price is reduced as the prosumer possesses

Table 3.3: Results under Cournot Prosumer Cases

Variables \Scenarios	(a)	(b)	(c)
Mean renewable output [MW]	25	50	120
Prosumer's sale(+)/purchase(-) [MWh]	-20.10	-13.86	3.61
Prosumer's load [MWh]	84.03	90.46	108.46
Prosumer's generation [MWh]	40.59	29.93	0.07
Marginal cost of backup [$/MWh]	75.59	64.93	35.07
Prosumer surplus [$K]	9.12	10.58	13.71
Total power demand [MWh]	2,855.21	2,855.77	2,857.34
Total power production [MWh]	2,875.31	2,869.63	2,853.73
Power price in node 1 [$/MWh]	42.23	41.92	41.07
Sales-weighted power price [$/MWh]	35.28	35.27	35.22
Grid operator's revenue [$K]	6.84	6.56	5.79
Producer surplus [$K]	43.04	43.46	44.63
Consumer surplus [$K]	256.87	256.86	257.07
Wholesale social surplus [$K]	306.75	306.88	307.49

more renewable generation output. As indicated in Table 3.1, the sales-weighted power price is reduced from $35.49/MWh in column (a) to $35.39/MWh and $35.19/MWh in columns (b) and (c), respectively. The power price in node 1 as well as the sales-weighted power price in the Stackelberg equilibrium is sandwiched between those of perfect and Cournot competition. However, whether it is sandwiched from above or blow by the perfect competition and Cournot cases depends on the prosumer's net position in the equilibrium.

As the sales-weighted power price decreases with higher levels of renewable output owned by the prosumer, the total demand in the market increases when more zero-marginal-cost resources become available. As shown in Table 3.1, the total power demanded follows an increasing trend of 2,848.31 to 2,851.68 and 2,858.07 MWh from columns (a) to (c) as the prosumer's renewable output increases. Similarly, with more renewables, the prosumer engages in less purchases from the market (if in the short position) or sells more to the market (if in the long position), thereby mitigating the need for generation from conventional producers, hence reducing the total power generation from conventional units. This is illustrated in Table 3.1, where total power production from the wholesale market is decreased from 2,909.31 MWh in the case of 25 MW renewable to 2,893.94 MWh and 2,846.38 MWh for 50 and 120 MW, respectively. Since less generation from conventional units means that the grid operator would need to move less power around in the network, the grid operator's revenue decreases with higher levels of renewable capacity for the prosumer. Table 3.1 illustrates the fact that the grid operator's revenue ($K) is reduced from 9.77 when renewable output is equal to 25 MW to 8.35 and 5.44 when mean renewable output is 50 and 120 MW, respectively.

For producers in the wholesale market, although the supply curve's shift to the right (due to increased zero-marginal-cost renewable) effectively lowers the equilibrium prices ("Sales-weighted prices" rows), lower producer revenues induced by lower power prices are more than made up for by lower transmission costs paid to the grid operator, leading to an increase in profits. For instance, the producer's surplus in Table 3.1 increases from 41.86 K$ in the 25MW case to 42.77 K$ and 44.76 K$ for 50MW and 120 MW cases, respectively. A comparison of Tables 3.1–3.3 suggests that the producer surplus under the Stackelberg case lies between the other two cases. In particular, it is bounded above by the Cournot (perfect competition) case when the prosumer is in short (long) position.

The sales-weighted power prices are lowest (highest) in Cournot case under the short (long) position. For instance, the sales-weighted power price under the 25MW scenario is \$35.28/MWh, \$35.52/MWh, and \$35.49/MWh for Cournot, perfect competition, and Stackelberg case, respectively. This is mainly because when the prosumer buys less from the main grid under the Cournot case, it effectively shifts the wholesale supply to the right, thereby lowering the power prices. A reversal of order among three cases is observed in column (c) in Tables 3.1–3.3. With increased consumption (demand) and lower prices, the consumers are poised to gain from increased levels of prosumer's renewable output. Indeed, this is what emerges in Table 3.1 where the consumer surplus (in K\$) increases monotonically from 255.86 to 256.30 and 257.17 in columns (a), (b), and (c). Comparing Tables 3.1–3.3 can also be justified by the sales-weighted power prices.

The difference of the wholesale social surplus between the perfect competition case and the other two cases can be explained by the amount of energy that is available to the wholesale market. More specifically, the more energy that is available to the grid, the higher the wholesale social surplus. For instance, the prosumer purchases more from the grid in columns (a) and (b) in Table 3.2 under the perfect competition case, i.e., 67.74 MWh and 47.06 MWh, respectively, than the Stackelberg case of 60.83 MWh and 42.26 MWh in Table 3.1, which leads to lower wholesale social surplus. On the other hand, the prosumer sells more as indicated in (c) in Table 3.2 than that in Table 3.1, thereby leading to higher social surplus under the perfect competition case.

Nevertheless, the comparison between the Stackelberg case in Table 3.1 and the Cournot case in Table 3.3 deserves more attention. As alluded to earlier, a leader can fully and correctly anticipate other market participants' response its actions, i.e., producers, consumers, and the grid operator that would be the followers in this setting. The prosumer is in a more advantageous position (possessing more valuable information) in the Stackelberg setting than in the Cournot case. When the prosumer is in a short position with power purchases from the grid in columns (a) and (b), conventional consumers are worse off by 1.01 \$k (= 256.87 − 255.86) and 0.56 \$k (=256.86 − 256.30), respectively, competing with the prosumer.

To understand the impacts on producers, we then calculate the "output-weighted power price," since the producer surplus is also tied to the power sales to the prosumer.[6]. They are equal to \$32.37/MWh (\$32.95/MWh), \$32.63/MWh (\$33.01/MWh), and \$33.26/MWh (\$33.18/MWh) for the Stackelberg (Cournot) case for 25 MW, 50 MW, and 120 MW of mean renewable generation, respectively. The lower output-weighted power price under the Stackelberg case leads to lower producer surplus in Table 3.1 compared to that in Table 3.3. However, the decline in the producer and consumer surplus is more than compensated by the increase in the gird operator's revenue (because of more power purchases by the prosumer), which results in higher wholesale social surplus in Table 3.1. Finally, when the prosumer is in a long position, selling power into the grid as a producer, the other price-taking producers directly benefit from the leader's action, increasing their profit from \$44.63k in Table 3.3 to \$44.76k in Table 3.1. As the prosumer decides to sell more by a margin of 8.08 MWh (= 11.69 − 3.61) in Table 3.1, the consumer surplus increases marginally by 0.1 \$k (= 257.17 − 257.07).

[6]The output-weighted power price is defined by $\frac{\sum_{f,i,h \in H_{fi}} p_i x_{fih}}{\sum_{f,i,h \in H_{fi}} x_{fih}}$, which is similar to but different from the sales-weighted power price

However, the increase in the producer and consumer surplus is then more than offset by the decline in the grid operator's revenue, leading to a lower wholesale social surplus.

3.4 Comparative Analyses

This section further analyzes the impact of prosumer's presence in power markets. First, we investigate the effects of prosumer's mean renewable output (K_i) on its net position in equilibrium. Next, we examine how the uncertainty of renewable output (σ_i) and the degree of prosumer's risk aversion (R_i) affect the market outcomes.

We compare the outcomes of the Stackelberg case to perfect competition and Cournot cases by varying the levels of mean renewable output K_1 in node 1. Fig. 3.1 plots the prosumer's sale (+) or purchase (-) in perfect competition, Cournot, and Stackelberg cases against the zero-marginal cost renewable output in x-axis from 25 to 120 MW. The horizontal dotted line crossing zero on the y-axis indicates the *island mode* where the prosumer is isolated from the grid. Fig. 3.1 indicates that, in the short position when the prosumer purchases from the grid, for any level of renewable output, the quantity purchased under the Stackelberg case is sandwiched between PC and Cournot. The same phenomenon is observed for the long position when the prosumer sells power to the grid. In other words, the prosumer formulated as a Stackelberg leader reduces purchases (sales) in the short (long) position compared to the case of perfect competition but increase purchases (sales) in the short (long) position compared to the Cournot case. The results are broadly consistent with economic theory that Cournot

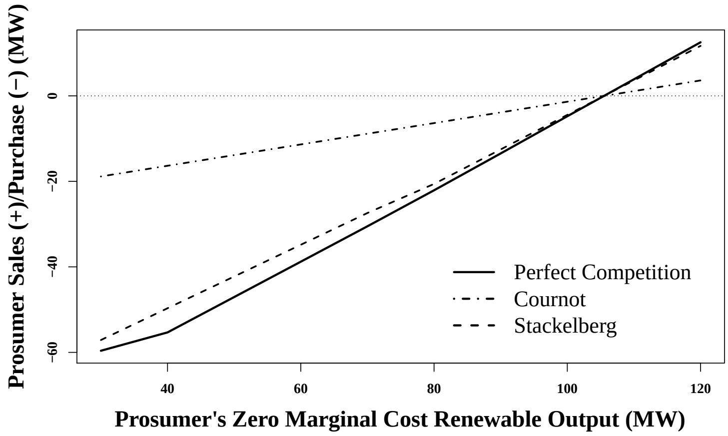

Figure 3.1: Prosumer's sales (+) or purchase (-) under various levels of mean renewable output

case represents the most aggressive case as producers reduce their sales in order to push up prices [55].

The prosumer's profit is also plotted in Fig. 3.2 against different levels of renewable output. Although the lines are not discernible between perfect competition and Stackelberg cases, in fact, for any level of renewable capacity, the prosumer profits in the Stackelberg equilibrium is higher than that of perfect competition and Cournot as in Tables in Section 3.3.2. This observation is corroborated with economic theory that in a Stackelberg case where the leader enjoys more information, the prosumer, as the leader, should perform better compared to the other two cases.

We next turn our attention to the uncertainty of renewable output and the degree of prosumer's risk aversion. Constraint (3.5), which associates prosumer's mean renewable output with the risk parameters, can be rewritten as follows:

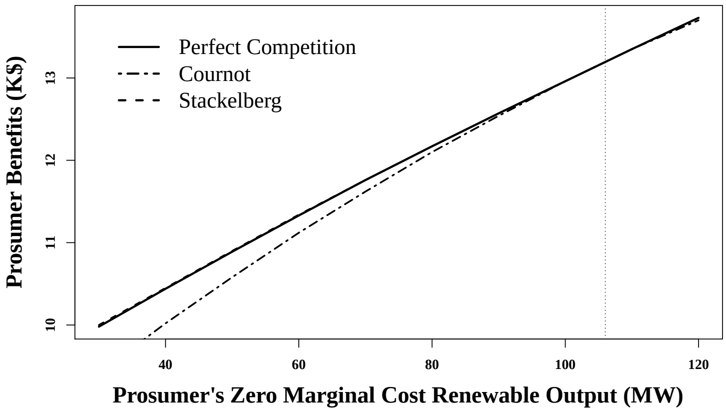

Figure 3.2: Prosumer's profit under various levels of mean renewable output

$$z_i + l_i - g_i \leq K_i^{perceived} \quad \forall i \tag{3.7}$$

where $K_i^{perceived} = K_i - \sigma_i \sqrt{\frac{1-R_i}{R_i}}$ is dubbed "perceived" renewable output for given R_i and σ_i. Since $\sigma_i \sqrt{\frac{1-R_i}{R_i}}$ is non-negative, $K_i^{perceived} \leq K_i$ holds true. Here, $K_i^{perceived}$ may be interpreted as a "certainty equivalent" of uncertain renewable output, jointly affected by R_i and σ_i. In particular, the more risk averse the prosumer (i.e., smaller R_i) and/or the greater the uncertainty of renewable output (i.e., larger σ_i), the lower the perceived renewable output (i.e., $K_i^{perceived}$), thereby inducing the prosumer to act more conservatively.

Tables 3.4 and 3.5 summarize the market outcomes when changing the prosumer's risk aversion ($R_1 = 0.1, 0.5, 0.99$) and the uncertainty parameters ($\sigma_1 = 1, 5, 25$), respectively. We maintain $\sigma_1 = 10$ MW and $R_1 = 0.9$ at their baseline in Tables 3.4

and 3.5, respectively, while $K_1 = 50$ MW.

Table 3.4: Results under Stackelberg Leader Prosumer with Different Risk Aversion Parameters ($\sigma_1 = 10$ MW, $K_1 = 50$ MW)

Variables \Scenarios	(a)	(b)	(c)
Risk Aversion Parameter (R_1)	0.10	0.50	0.99
Perceived output ($K_1^{perceived}$) [MW]	20	40	48.99
Prosumer's sale(+)/purchase(-) [MWh]	-63.48	-47.56	-40.41
Prosumer's load [MWh]	99.09	100.63	101.32
Prosumer's generation [MWh]	15.61	13.06	11.91
Marginal cost of backup [$/MWh]	50.61	48.06	46.92
Prosumer surplus [$K]	9.60	10.59	11.02
Total power demand [MWh]	2,847.83	2,850.72	2,852.02
Total power production [MWh]	2,911.31	2,898.28	2,892.42
Power price in node 1 [$/MWh]	45.53	44.25	43.68
Sales-weighted power price [$/MWh]	35.50	35.42	35.38
Grid operator's revenue [$K]	9.97	8.76	8.21
Producer surplus [$K]	41.73	42.51	42.85
Consumer surplus [$K]	255.80	256.17	256.34
Wholesale social surplus [$K]	307.51	307.44	307.41

Columns (a), (b), and (c) in Table 3.4 correspond to the market outcomes of R_1 equal to 0.10, 0.50, and 0.99, respectively. The perceived output $K_1^{perceived}$ as reported in Table 3.4 increases from 20 MW in column (a) to 48.99 MW in column (c). With increases of R_1 from 0.10 to 0.50, and to 0.99, the prosumer becomes less risk averse or more risk seeking, thereby purchasing less energy from the wholesale market, from 63.48 MWh to 47.56 MWh, and to 40.41 MWh as shown in columns (a)–(c). The fact that less energy is purchased by the prosumer in column (c) implies that more energy is available in the wholesale market, leading to lower sales-weigheted power prices (i.e., $35.38/MWh in column (c) vs. $35.50/MWh in column (a)) and higher consumer surplus ($256.34K in (c) vs. $255.80K in (a)).

The final set of results concerning the impact of uncertainty in renewable output (σ_1) is reported in Table 3.5. Columns (a) through (c) display the outcomes when

Table 3.5: Results under Stackelberg Leader Prosumer with Different Uncertainty of Renewable Output ($R_1 = 0.9$, $K_1 = 50$ MW)

Variables \Scenarios	(a)	(b)	(c)
Uncertainty of renewables (σ_1) [MW]	1	5	25
Perceived output ($K_1^{perceived}$) [MW]	49.67	48.33	41.67
Prosumer's sale(+)/purchase(-) [MWh]	-39.87	-40.93	-46.24
Prosumer's load [MWh]	101.37	101.27	101.76
Prosumer's generation [MWh]	11.83	12.00	12.85
Marginal cost of backup [$/MWh]	46.83	47.00	47.85
Prosumer surplus [$K]	11.05	10.98	10.67
Total power demand [MWh]	2,852.12	2,851.92	2,850.96
Total power production [MWh]	2,891.99	2,892.86	2,897.20
Power price in node 1 [$/MWh]	43.64	43.73	44.15
Sales-weighted power price [$/MWh]	35.38	35.39	35.42
Grid operator's revenue [$K]	8.17	8.25	8.66
Producer surplus [$K]	42.88	42.83	42.57
Consumer surplus [$K]	256.36	256.33	256.20
Wholesale social surplus [$K]	307.41	307.42	307.44

increasing uncertainty σ_1 from 1 MW to 25 MW. With increases in σ_1, the perceived output $K_1^{perceived}$ decreases from 49.67 MW in column (a) to 41.67 MW in column (c). A smaller $K_1^{perceived}$ forces the prosumer to purchase more energy from the wholesale market in column (c), and hence leaves less energy available to the consumers in the grid. This, in turn, leads to higher sales-weighted prices from \$35.38/MWh in column (a) to \$35.42/MWh in column (c), even only marginally. As a result, consumers become worse off, subject to a decline of \$0.16K (=\$256.36K-\$256.20K) in consumer surplus. [7]

[7] The impacts of R_1 and σ_1 when the prosumer is in a long position, selling energy to the grid, can be analyzed in a similar way. In particular, a larger $K_1^{perceived}$ as in Table 3.4 encourages the prosumer to sell more energy, which is expected to lower sales-weighted power prices and makes consumers better off. On the other hand, a smaller $K_1^{perceived}$ as in Table 3.5 induces the prosumer to sell less energy, resulting in lower consumer surplus with higher sales-weighted power prices.

3.5 Conclusion

This chapter extends the existing work [46, 47] and focuses on examining the role of risk-averse prosumers in the electricity market by formulating a risk-averse prosumer as a leader using distributionally robust chance-constrained MPEC approach and compare the results to those where the prosumer is designated as a price-taker and a Cournot entity in the market. The results indicate that market outcomes are affected by the risk-averse prosumer's strategy and the amount of its possessed renewables. Possessing relatively low (high) renewables, the prosumer behaves as consumers (producers) and purchases from (sells to) the main grid. The outcomes of the Stackelberg competition lie in between those of Cournot (least competitive) and perfect competition (most competitive) scenarios. Under the relatively high amount of renewables, the outcomes asymptotically approach that of the perfect competition case. However, although the impact of the strategic prosumer on each of the market participants in the Stackelberg case is in between the perfect and Cournot competition cases, its impact on the wholesale market's social surplus (excluding prosumer's surplus) is ambiguous. More specifically, in the short position, the prosumer in the Stackelberg equilibrium increases the social surplus, higher than perfect competition and Cournot. On the other hand, in the long position, the prosumer's strategy leads to a decline in social surplus, lowest compared to perfect competition and Cournot cases. Our contribution therefore lies on developing a distributionally robust chance-constrained MPEC approach to model risk-averse prosumers when facing uncertain renewables. Finally, the model is ready to include other energy products in the electric power sector, where the prosumers can offer their services. For instance, a spinning reserve market or a market for fast-ramping products can be formulated in a similar way to [56]. Our analysis also highlights the importance of understanding the role of prosumers, acting as a producer or a consumer in equilibrium, when evaluating its interaction with and its impacts on the wholesale market.

Chapter 4

Prosumers, Endogenous Risk Aversion, and Optimization

4.1 Introduction

Electricity markets are evolving rapidly and fundamentally, in response to the growing need for renewable capacity and generation, mainly reinforced by the efforts to mitigate climate change and pursue sustainability. This has resulted in significant changes in the design and operation of modern power grids. As more smart meters and digital grid technologies become available, we witness more facilitation, and hence the willingness toward renewable power generation, such as a solar photovoltaic (PV) system, among traditional consumers. This trend, in conjunction with a variety of distributed energy resources (DER), such as electric vehicles (EV) and storage, has challenged the traditional supply-centric paradigm and illustrated a new reality focused on the demand side in electricity markets.

As the demand side of energy markets becomes increasingly engaged, the behavior and strategies of the once-idle demand-side participants begin to affect the functioning of electricity markets more significantly. Particularly, we observe the emergence of *prosumers*, that is, agents who are capable of concurrent generation and consumption of power, as opposed to conventional consumers or suppliers that would traditionally participate in one side of the market only. As more conventional consumers become prosumers, the collective effect on the design and operation of electricity markets becomes paramount [1]. Prosumer engagements in electricity markets are amplified by aggregators that integrate demand response (DR) and DER. offering bundled energy products to the market [31] [3]. This trend has recently been accelerated by FERC Order 2222, which paves the way for the increased adoption of DER technologies [57].

Prosumers with non-dispatchable renewable power sources face inherent uncertainty of natural resources, such as solar and wind. Hence, individuals' attitudes toward risk play an important role in the decision making of prosumers, similar to investors in financial markets. Within the research communities of economics, finance, and engineering, the assumption of constant and exogenous risk preference has been standard practice [58], [59]. However, in recent years, such a standard assumption has been considered unrealistic because it occasionally fails to describe historical data, particularly in financial markets.[1] For instance, [61] argues that a model with constant risk aversion

[1]Related to this point, Alan Greenspan suggested in his speech at the Federal Reserve Bank of Kansas City that an increase in the market value of asset claims partly depends on changing investors' attitudes toward risk [60].

cannot replicate the key statistical properties observed in real financial markets. This study demonstrates that a time-varying risk-aversion parameter that responds to unexpected excess returns can replicate historical data. In contrast, [62] developed a general equilibrium model in which heterogeneous preferences are endogenously determined in markets. They empirically show that the risk aversion parameters vary across households under different market conditions by comparing landed and landless households in Bangladesh.

Given the increasing influence of prosumers with uncertain renewable power sources, their behavioral and risk assessments have become a key focus, with possible ramifications for the outcomes of current electricity markets [47, 46]. Prosumer attitudes toward risk have a direct bearing on the quantities they supply and demand, and hence, overall market outcomes, given the rapid penetration of renewable energy sources. The degree of prosumer risk aversion is expected to respond and adapt to the surrounding economic environment and market conditions as in the case of investors in the financial field [61]. Therefore, a modeling framework that captures the decision making of prosumers in the context of endogenous risk attitudes under uncertainty is of particular importance.

Previous studies have addressed some dimensions of risk elements in electricity markets involving prosumers. In [63], the authors investigated the risks involved in community energy markets, with a particular focus on peer-to-peer and energy-sharing mechanisms. The authors developed a conditional value-at-risk (CVaR) model for household and community PV systems and formulated the problem in terms of a stochastic game. Similarly, the authors in [64] formulated a cooperative game theory framework for energy hubs and community energy systems using CVaR with a profit-sharing scheme. In [65], the authors proposed a decision method for energy bids in the day-ahead energy market, and evaluated the risks related to the different decisions of prosumers in the microgrid.

However, existing studies on the stochastic approaches for prosumers are scant and mostly assume constant and exogenous risk preference. These studies fall short of 1) explicitly capturing the formation of the risk tolerance of prosumers and 2) modeling how this factor affects their decisions and, consequently, market outcomes. In contrast, we propose an alternative modeling framework in which prosumers endogenously determine their risk attitudes through optimization. Specifically, in our model, the degree of the risk aversion of prosumers is determined by their surplus maximization problem, which is formulated as a distributionally robust chance-constrained problem with uncertain renewable outputs. To the best of our knowledge, this is the first study to derive a prosumer risk-aversion parameter (as a decision variable) by optimization, thereby advancing the modeling approach in addition to its management implications. Such a framework has been scarce, even in other fields, such as finance, with the exception of [66], who models an investor that endogenously chooses his/her risk preference by maximizing the probability of achieving wealth that grows faster than the target growth rate. In contrast to the simple model of investor probability maximization in [66], we investigate a distributionally robust chance-constrained framework, which is a more recent strand of research on uncertainty that focuses on the endogenous decision making of prosumers.

Specifically, we model prosumers who own renewable generation systems and make decisions in a day-ahead wholesale power market, anticipating the effect of their decisions on the other participants in the market. This situation is expressed as a Stackelberg leader-follower game, which results in the generation of a mathematical program with equilibrium constraints (MPEC). The problem is formulated in a distributionally robust chance-constrained framework to account for the uncertainty of the renewable generation of prosumers. Within this framework, prosumers maximize their surplus by adjusting their risk attitudes. Our solution approach applies the Wolfe duality to the lower-level problem to concavify the bilinear term in the objective function of the upper-level problem. Finally, we demonstrate how optimally adjusted risk aversion by prosumers affects outcomes in the wholesale power market using the IEEE Reliability Test System (RTS 24-Bus) [54].

This chapter proceeds as follows. In Section 4.2, we formulate a distributionally robust chance-constrained MPEC, in which leader prosumers determine their risk preferences. Section 4.3 further presents a case study based on an IEEE 24-bus system. Section 4.4 concludes the study.

4.2 Model Setup

We consider a Stackelberg framework in which the prosumer is the leader and the other market participants are followers. This Stackelberg game is formulated as a mathematical program with equilibrium constraints (MPEC) formally defined as in (4.1):

$$\text{minimize} \quad f(x, y) \tag{4.1a}$$

subject to

$$(x, y) \in Z, \tag{4.1b}$$

$$y \in \mathcal{S}(x) \tag{4.1c}$$

In this setup, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $f : \mathbb{R}^{n+m} \to \mathbb{R}$, $F : \mathbb{R}^{n+m} \to \mathbb{R}^m$, and $Z \subseteq \mathbb{R}^{n+m}$. We also define a set-valued mapping $C : \mathbb{R}^n \to \mathbb{R}^m$, where $C(x)$ is a closed convex subset of $\mathbb{R}^m$ for each $x \in \mathbb{R}^n$, and let X be the projection of Z onto $\mathbb{R}^n$. $\mathcal{S}(x)$ is the solution to the variational inequality defined as $(v - y)^\top F(x, y) \geq 0, \quad \forall v \in C(x)$ [67].

An MPEC can be regarded as an optimization problem faced by a leader (upper-level problem), whose actions affect the equilibrium of a market (lower-level problem), which consequently affects the leader's objective. The next Section discusses this issue in detail.

4.2.1 Bi-Level Problem of Power Market with Prosumers

In this section, we describe the prosumer's problem in the upper- and lower-level problems faced by the grid operator. Let I denote the set of nodes (or locations) and let L be the set of transmission lines comprising elements in ordered pairs of distinct nodes. Here, F is the set of conventional generation firms, H is the set of generation units, and $H_{fi} \subset H$ denotes the set of generation units owned by firm f at node i. Finally, we note that the Greek variables within parentheses to the right of the equation render the corresponding dual variable.

4.2.1.1 Upper-Level Problem

In this study, the prosumer makes a decision in a day-ahead wholesale power market, anticipating its effect on other participants. Therefore, the prosumer can be modeled as the leader in a Stackelberg game.[2] The prosumer at node i is assumed to possess non-dispatchable renewable capacity with a negligible short-run marginal cost. The output from renewable sources is denoted by a random variable $\tilde{K}_i$, which is uncertain because it is dependent on available natural resources such as solar and wind. We assume that the distribution $\mathbb{P}_i$ of $\tilde{K}_i$ belongs to a set $\mathcal{P}_i$ of distributions, where the first and second moments are known, that is, $\mathbb{E}(\tilde{K}_i) = K_i$ and $\mathbb{V}(\tilde{K}_i) = \sigma_i^2$, respectively, but without exact knowledge of the probability distributions. In contrast, the prosumer also owns a dispatchable or backup resource, for example, an on-site diesel generator, that supplies power g_i with an increasing and strictly convex cost function $C_i^g(g_i)$ and capacity of G_i to hedge against uncertain output $\tilde{K}_i$. Specifically, we assume a quadratic cost function, $C_i^g(g_i) = D_i^{g0} g_i + \frac{C_i^{g0}}{2} g_i^2$. However, its supply would not be able to fully compensate for the intermittent renewable output.

Regarding the demand side, the prosumer's benefit function of consuming elec-

[2]The individual "behind-the-meter" prosumers, such as the owner of a rooftop solar panel, might have limited access to the wholesale or bulk market and may be subject to fixed tariffs when selling their surplus power back to the grid. We assume that the prosumer (or aggregator) that we present here is a result of the aggregation of a large number of prosumers, thereby allowing them to interact with the bulk market directly.

tricity at node i is given by $B_i^l(l_i)$, where l_i corresponds to the self-consumption at each node. Benefit function $B_i^l(l_i)$ is assumed to be increasing and strictly concave. Specifically, we assume a quadratic benefit function, $B_i^l(l_i) = A_i^{l0} l_i - \frac{A_i^{l0}}{2B_i^{l0}} l_i^2$, for a relevant range of consumption. We assume that the prosumer is only allowed to sell or buy power locally, that is, at each node, which is consistent with the layered structure of a future grid [32].

We posit that the prosumer maximizes its surplus by determining four types of variables: i) its risk attitude/preference or tolerance $r_i \in [0, 1]$, in which a smaller r_i indicates that the prosumer is more risk averse; ii) the amount of traded power z_i, buying from ($z_i < 0$) or selling to ($z_i > 0$) in node i at price p_i, iii) the amount of its own power consumption l_i and iv) the amount of power to be generated g_i from the backup dispatchable technology. We further formulate a distributionally robust chance-constrained problem for the prosumer facing an uncertain renewable output $\tilde{K}_i$ as follows:

$$\underset{r_i, z_i, l_i, g_i}{\text{maximize}} \quad \sum_i \Big(p_i z_i + B_i^l(l_i) - C_i^g(g_i)\Big) + \sum_i \mathbb{E}\Big[P_i^c\Big(\tilde{K}_i - z_i - l_i + g_i\Big)\Big] \tag{4.2a}$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i\Big(z_i + l_i - \tilde{K}_i - g_i\Big) \geq 1 - r_i \qquad (\delta_i), \tag{4.2b}$$

$$g_i \leq G_i. \qquad (\kappa_i), \tag{4.2c}$$

$$r_i \leq 1 \qquad (\nu_i), \tag{4.2d}$$

$$r_i, l_i, g_i \geq 0. \tag{4.2e}$$

The three terms in the first line of the objective function (2.2a) correspond to revenue (+) or cost (−) from transactions in the day-ahead wholesale market, the benefit of consuming power, and the generation costs incurred from backup resources, respectively.[3] The second line provides the expected cost/revenue in real time, where P_i^c is a fixed or contracted retail rate between the prosumer and utility for node i. Here, we envision a situation in which the prosumer is allowed to participate in a day-ahead market but is subject to a fixed retail rate in real time, similar to the situation faced by several EU countries such as Italy, the Netherlands, and Belgium [49]. P_i^c can also be regarded as a real-time imbalance price to finally settle a shortage or excess of energy for the prosumer.

Constraint (2.2b) is the distributionally robust chance constraint of the prosumer. It states that the sum of the renewable output $\tilde{K}_i$ and self-generation g_i, net of

[3]The interaction of the prosumer with the bulk day-ahead energy market is modeled by shifting the supply curves and sales decisions of conventional producers. An alternative way of modeling this situation is to horizontally aggregate the demand curves of consumers and prosumers. However, this aggregation might result in kinked demand curves, which pose numerical difficulties [51].

transactions with the wholesale day-ahead market, that is, z_i, is equal to or greater than the self-consumption l_i with probability $1 - r_i$ or greater for any distributions in $\mathcal{P}_i$. Because the renewable output $\tilde{K}_i$ is a random variable, its realization in real time inherently varies, depending on the weather condition. The prosumer who is conscious of risk would be concerned about bad scenarios with the realization of very low renewable output because it will incur costs to settle the imbalance of power in real time. This is analogous to investors who are cautious about market volatility and are concerned about possible losses owing to the realization of low returns on their financial assets. We posit that the prosumer decides their risk attitude by adjusting r_i in constraint (4.2b), where a smaller r_i means more risk averse. Constraints (4.2c)–(4.2e) specify the ranges of the four decision variables, that is, r_i, z_i, l_i, and g_i.

4.2.1.2 Lower-Level Problem

We further introduce the lower-level problem in which the grid operator takes bids from suppliers and consumers/load serving entities and maximizes the social surplus of the wholesale market, subjected to prosumer's decision z_i. Let x_{fih}, d_i, and y_i denote the power output produced by generation unit h at node i owned by firm f, the quantity demanded by consumers at node i, and the power injection/withdrawal at node i, respectively. We assume an increasing and strictly concave benefit function $B_i(d_i) = P_i^0 d_i - \frac{P_i^0}{2Q_i^0} d_i^2$ for consumers, and an increasing and strictly convex cost function $C_{fih}(x_{fih}) = D_{fih}^0 x_{fih} + \frac{C_{fih}^0}{2} x_{fih}^2$ for generation. The lower-level problem is represented as the maximization of the social surplus (i.e., benefit minus cost) faced by the grid operator:

$$\underset{x_{fih},\, d_i,\, y_i}{\text{maximize}} \quad \sum_i B_i(d_i) - \sum_{f,i,h\in H_{fi}} C_{fih}(x_{fih}) \tag{4.3a}$$

subject to

$$x_{fih} \leq X_{fih} \quad (\beta_{fih}), \forall f, i, h \in H_{fi}\,, \tag{4.3b}$$

$$\sum_i PTDF_{ki} y_i \leq T_k \quad (\lambda_k^+), \forall k, \tag{4.3c}$$

$$-\sum_i PTDF_{ki} y_i \leq T_k \quad (\lambda_k^-), \forall k \quad , \tag{4.3d}$$

$$d_i - \sum_{f,h\in H_{fi}} x_{fih} - z_i = y_i \quad (\eta_i), \forall i, \tag{4.3e}$$

$$\sum_i y_i = 0 \quad (\theta) \qquad\qquad , \tag{4.3f}$$

$$x_{fih} \geq 0 \quad (\varepsilon_{fih}), \forall f, i, h \in H_{fi} \qquad , \tag{4.3g}$$

$$d_i \geq 0 \quad (\xi_i), \forall i \tag{4.3h}$$

Constraints (4.3b)–(4.3d) limit the variables according to the generation capacity (X_{fih}) and the transmission capacity (T_k), along with the power transfer distribution factor ($PTDF_{ki}$). Constraint (3.2e) represents the nodal balance with the prosumer's transaction (z_i) embedded. The balance between supply and demand is implied by (4.3f). Constraints (4.3g)–(4.3h) represent the non-negativity of generation and consumption, respectively. Given that the lower level is a concave programming problem, the solutions can be represented by its optimality conditions, which form a mixed complementarity problem characterizing the equilibrium of the market:

$$-C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f, i, h \in H_{fi}, \tag{4.4a}$$

$$B'_i(d_i) - \eta_i + \xi_i = 0 \quad \forall i, \tag{4.4b}$$

$$-\sum_k (\lambda_k^+ - \lambda_k^-) PTDF_{ki} + \eta_i - \theta = 0 \quad \forall i, \tag{4.4c}$$

$$0 \leq \beta_{fih} \perp x_{fih} - X_{fih} \leq 0 \quad \forall f, i, h \in H_{fi}, \tag{4.4d}$$

$$0 \leq \lambda_k^+ \perp \sum_i PTDF_{ki} y_i - T_k \leq 0 \quad \forall k, \tag{4.4e}$$

$$0 \leq \lambda_k^- \perp -\sum_i PTDF_{ki} y_i - T_k \leq 0 \quad \forall k, \tag{4.4f}$$

$$d_i - \sum_{f, h \in H_{fi}} x_{fih} - z_i - y_i = 0 \quad \forall i, \tag{4.4g}$$

$$\sum_i y_i = 0, \tag{4.4h}$$

$$0 \leq \varepsilon_{fih} \perp x_{fih} \geq 0 \quad \forall f, i, h \in H_{fi}, \tag{4.4i}$$

$$0 \leq \xi_i \perp d_i \geq 0 \quad \forall i \tag{4.4j}$$

4.2.2 Distributionally Robust Chance-Constrained MPEC

Based on the upper- and lower-level problems in Section 4.2.1, the prosumer's problem is now cast as a distributionally robust chance-constrained MPEC as follows:

$$\underset{\Phi \cup \Omega \cup \Lambda}{\text{maximize}} \quad \sum_i \Big(\eta_i z_i + B_i^l(l_i) - C_i^g(g_i)\Big) + \sum_i \mathbb{E}\Big[P_i^c\Big(\tilde{K}_i - z_i - l_i + g_i\Big)\Big] \tag{4.5a}$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i\Big(z_i + l_i - \tilde{K}_i - g_i\Big) \geq 1 - r_i \qquad (\delta_i), \tag{4.5b}$$

$$g_i \leq G_i. \qquad (\kappa_i), \tag{4.5c}$$

$(4.2c) - (4.2e)$,

$(4.4a) - (4.4j)$

where $\Phi = \{r_i, z_i, l_i, g_i\}$, $\Omega = \{x_{fih}, d_i, y_i\}$, and $\Lambda = \{\beta_{fih}, \lambda_k^+, \lambda_k^-, \eta_i, \theta, \varepsilon_{fih}, \xi_i\}$. In the first line of the objective function, the net benefit of the prosumer is expressed using the equilibrium power price η_i (instead of p_i), which is the dual variable associated with the nodal balance constraint (4.3e) in the lower-level problem.

Problem (4.5) is difficult to solve due to the non-concave objective function with the bilinear term resulting from the product of $\sum_i \eta_i z_i$, a product of the primal variable in the upper-level problem and the dual variable in the lower-level problem. We propose a method that applies the Wolfe duality to the lower-level problem. Using the strong duality and constraints in the Wolfe dual formulation of the lower-level problem, we show that the bilinear term in the objective function of the upper-level problem can be concavified as follows (see Appendix A):

$$\sum_i \eta_i z_i = \sum_i B_i'(d_i)d_i - \sum_k (\lambda_k^+ + \lambda_k^-)T_k \tag{4.6}$$
$$- \sum_{f,i,h\in H_{fi}} \big(C'_{fih}(x_{fih})x_{fih} + \beta_{fih}X_{fih}\big)$$

Furthermore, since (4.5b) considers probability constraints over all possible distributions with given moments, we simplify the formulation, as in [52], by replacing (4.5b) with (4.7):

$$z_i + l_i - g_i - K_i + \sigma_i\sqrt{\frac{1-r_i}{r_i}} \leq 0 \quad \forall i \tag{4.7}$$

$K_i - \sigma_i\sqrt{\frac{1-r_i}{r_i}}$ in (4.7) can be interpreted as a "risk-derated" renewable output perceived by the prosumer, which is assumed to be non-negative:

$$K_i - \sigma_i\sqrt{\frac{1-r_i}{r_i}} \geq 0 \quad \forall i \tag{4.8}$$

In our context, $t_i = \sigma_i\sqrt{\frac{1-r_i}{r_i}}$ represents a "risk-averse reservation," also referred to as a "safety parameter" in the chance-constrained optimization literature. This term can be regarded as the buffer amount of energy perceived by the prosumer in order to hedge against a situation in which the realized value of the renewable output in real time is lower than expected. A risk-averse prosumer with small r_i attempts to maintain sufficient reservation output t_i when making decisions in the day-ahead market, whereas a risk-neutral prosumer with $r_i = 1$ perceives the expected output K_i without any reservations, i.e., $t_i = 0$.

Problem (4.5), with the transformation of (4.6) and (4.7), in conjunction with (4.8) can be then solved using off-the-shelf nonlinear programming solvers, such as

Knitro [68].[4]

4.3 Case Study

4.3.1 Data and Assumptions

The model was applied to an IEEE reliability test system (RTS 24-Bus) [54]. The topology of the system comprises 24 buses, 38 transmission lines, and 17 constant-power loads with a total of 2,850 MW. We aggregate the 32 generators into 13 generators by integrating those with the same marginal cost and located at the same node. However, six generation units were excluded from the dataset because they are hydropower units, which operate at a maximum output of 50 MW [27]. To analyze the effect of transmission congestion, the capacity of line 7 between nodes 3 and 24 in the test case was reduced to 150 MW. The generation cost is represented by a quadratic function parameterized by D^0_{fih} and C^0_{fih} as coefficients for the linear and quadratic terms, respectively. Furthermore, the prosumer, or the leader, located at node 1 is assumed to have the same demand function as the consumers in that node. The prosumer owns a renewable generation unit that produces varying amounts of power (contingent on available natural resources), and a dispatchable unit as a backup option. The RTS 24-Bus case was first formulated as a least-cost minimization problem and solved with a fixed nodal load to compute dual variables associated with load constraints. Further, the dual variables, in conjunction with an assumed price elasticity of -0.2 are used to calculate

[4]The complementarity conditions represented in the constraints of problem (4.5) can also be linearized using disjunctive constraints [53], resulting in a mixed integer quadratic program (MIQP).

the demand parameters, P_i^0 and Q_i^0. The magnitude of the price elasticity of demand is comparable to that in the literature [28]. Hereafter, we omit index i for the variables and parameters associated with the prosumer, focusing on node 1. Several cases are considered by varying the imbalance price P^c (\$20/MWh and \$60/MWh) and expected renewable output $K = \mathbb{E}(\tilde{K})$ (25 MWh and 110 MWh) with uncertainty characterized by the standard deviation σ (20% and 80% of the expected renewable output K). For example, the uncertainty corresponding to σ =20% of K =110 MWh is equal to 22 MWh ($= 0.2 \times 110$). The results are then presented in the next section.

4.3.2 Main Results

The main results are summarized in Tables 4.1–4.4. Each table with the same layout contains the results pertaining to the prosumer, wholesale, and economic rent distributions. We further focus our discussion mainly on the behavior of a prosumer. Note that for prosumer surplus, we include the expected surplus from the imbalance settlement, which is equal to $P^c t$, but exclude it when calculating the "day-ahead" total social surplus as the real-time settlement does not occur in the day-ahead market and it is just an expected transfer between two parties (i.e., cancelled out). In the following tables, the expected revenue (or cost) of imbalance, which is equal to $P^c t$ is excluded from the total "day-ahead" social surplus, while included when calculating prosumer surplus.

Table 4.1 reports the case where P^c =\$20/MWh and σ account for 20% (of K). Facing a relatively low price of P^c =\$20/MWh in real time, the prosumer finds it economically undesirable to maintain a "risk-averse reservation" in the day-ahead market. This is because the realization of a disappointing scenario with very low renewable output would not result in significant monetary stress for the prosumer to be exposed to real-time settlement for energy imbalance. Therefore, as demonstrated in Table 4.1, the prosumer prefers less risk-averse attitude, even behaving risk neutrally by choosing $r^* = 1$. The prosumer is in short (long) position when expected renewable output K is 25 MWh (110 MWh), buying 59.5 MWh from (selling 9.96 MWh into) the grid. With a more expected renewable output of $K = 110$ MWh, the prosumer consumes more while selling excess energy into the day-ahead market, which results in a larger amount of prosumer surplus. Overall, the system benefits from a higher expected renewable output, thereby resulting in a greater total day-ahead social surplus.

Table 4.1: Results: the case where $P^c = \$20$/MWh and $\sigma = 20\%$ (of K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(–) [MWh]	-59.50	9.96
Prosumer's load [MWh]	99.48	105.32
Prosumer's backup generation [MWh]	14.97	5.28
Power price at node 1 [$/MWh]	45.21	40.77
Prosumer's risk-averse reservation [MWh]	0	0
Prosumer's optimal risk (r^*)	1	1
Expected revenue(+)/cost(–) of imbalance [$K]	0	0
Prosumer surplus [$K]	9.85	13.65
Conventional consumption [MWh]	2,848.55	2,857.91
Conventional generation [MWh]	2,908.05	2,847.95
Grid operator's revenue [$K]	9.67	5.52
Producer surplus [$K]	41.92	44.71
Consumer surplus [$K]	255.89	257.14
Wholesale social surplus [$K]	307.49	307.36
Total day-ahead social surplus [$K]	317.35	321.02

Table 4.2 shows the outcomes when $P^c = \$60$/WMh and σ is 20% (of K). A relatively high price of $P^c = \$60$/WMh, which is associated with a risk of significant

expenses for the energy imbalance settlement, induces the prosumer to behave conservatively with r^* close to zero by holding a considerable amount of a "risk-averse reservation" to hedge against the worse case of very low renewable output in real time. The prosumer's reservation becomes as high as the expected renewable output, thereby resulting in a risk-derated output of 0. Consequently, the prosumer increases energy purchases from the grid in the day-ahead market even when 110 MWh of renewable output is expected. This is in contrast to Table 4.1, in which the prosumer behaves risk neutrally, selling 9.96 MWh to the grid under K =110 MWh. The backup generation of the prosumer also increases compared with that in Table 4.1. When the expected renewable output increases, the risk-averse prosumer adjusts their reservation by the same amount, thereby resulting in the same rent distribution in the day-ahead market for K =25 MWh and 110 MWh in Table 4.2.

In Tables 4.3–4.4, σ increases from 20% to 80% (of K), while maintaining the same setup for P^c, as shown in Tables 4.1–4.2. The observations in Tables 4.1–4.2 also emerge in Tables 4.3–4.4. This implies that the degree of uncertainty would not affect the market outcomes when the prosumer can endogenously determine or "internalize" their risk attitude to achieve their maximized surplus. The outcomes depend mainly on P^c rather than on σ. Consequently, the outcomes in Tables 4.3 (4.4) are equivalent to those in Tables 4.1 (4.2).

In Fig. 4.1, we further elaborate on the effect of imbalance price P^c on the optimal risk attitude r^* under $\sigma = 20\%$ (of K). Fig. 4.1 demonstrates that the prices for energy imbalance settlement play key roles for the decision of the prosumer's optimal

Table 4.2: Results: the case where P^c =\$60/MWh and σ =20% (of K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(-) [MWh]	-79.39	-79.39
Prosumer's load [MWh]	97.56	97.56
Prosumer's backup generation [MWh]	18.16	18.16
Power price at node 1 [\$/MWh]	46.80	46.80
Prosumer's risk-averse reservation [MWh]	25	110
Prosumer's optimal risk (r^*)	0.038	0.038
Expected revenue(+)/cost(−) of imbalance [\$K]	1.5	6.6
Prosumer surplus [\$K]	10.06	15.16
Conventional consumption [MWh]	2,844.93	2,844.93
Conventional generation [MWh]	2,924.33	2,924.33
Grid operator's revenue [\$K]	11.19	11.19
Producer surplus [\$K]	40.96	40.96
Consumer surplus [\$K]	255.45	255.45
Wholesale social surplus [\$K]	307.60	307.60
Total day-ahead social surplus [\$K]	316.17	316.17

Table 4.3: Results: the case where P^c =\$20/MWh and σ =80% (of K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(-) [MWh]	-59.50	9.96
Prosumer's load [MWh]	99.48	105.32
Prosumer's backup generation [MWh]	14.97	5.28
Power price at node 1 [\$/MWh]	45.21	40.77
Prosumer's risk-averse reservation [MWh]	0	0
Prosumer's optimal risk (r^*)	1	1
Expected revenue(+)/cost(−) of imbalance [\$K]	0	0
Prosumer surplus [\$K]	9.85	13.65
Conventional consumption [MWh]	2,848.55	2,857.91
Conventional generation [MWh]	2,908.05	2,847.95
Grid operator's revenue [\$K]	9.67	5.52
Producer surplus [\$K]	41.92	44.71
Consumer surplus [\$K]	255.89	257.14
Wholesale social surplus [\$K]	307.49	307.36
Total day-ahead social surplus [\$K]	317.35	321.02

risk attitude, indicating that a relatively low imbalance price induces a less risk-averse behavior (i.e., greater r^*), and vice versa. Particularly, for the range of P^c between \$40/MWh and \$60/MWh, the prosumer with K =25 MWh finds it optimal to choose a less risk-averse attitude than that under K =110 MWh. This suggests that when

Table 4.4: Results: the case where P^c =\$60/MWh and σ =80% (of K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(-) [MWh]	-79.39	-79.39
Prosumer's load [MWh]	97.56	97.56
Prosumer's backup generation [MWh]	18.16	18.16
Power price at node 1 [\$/MWh]	46.80	46.80
Prosumer's risk-averse reservation [MWh]	25	110
Prosumer's optimal risk (r^*)	0.390	0.390
Expected revenue(+)/cost(–) of imbalance [\$K]	1.5	6.6
Prosumer surplus [\$K]	10.06	15.16
Conventional consumption [MWh]	2,844.93	2,844.93
Conventional generation [MWh]	2,924.33	2,924.33
Grid operator's revenue [\$K]	11.19	11.19
Producer surplus [\$K]	40.96	40.96
Consumer surplus [\$K]	255.45	255.45
Wholesale social surplus [\$K]	307.60	307.60
Total day-ahead social surplus [\$K]	316.17	316.17

facing moderate P^c and a smaller magnitude of uncertainty (0.2×25 MWh), prosumers behave less risk averse. Outside this range of P^c, r^* is equivalent between K =25 and 110 MWh.

4.3.3 Relative risk profit loss

This section presents the results concerning the effect of a prosumer's decision abougt the risk attitude on its profit. We define the "relative profit loss" in (4.9) to quantify the extent of a prosumer's forgone profit when choosing a suboptimal risk attitude r in comparison with the maximum profit under optimal r^* as follows:

$$\textit{relative profit loss} = \frac{\pi(r^*) - \pi(r)}{\pi(r^*)} \times 100\% \tag{4.9}$$

where $\pi(r^*)$ denotes a prosumer's profit under the optimal risk attitude r^* and $\pi(r)$ is its profit for an arbitrary level of r.

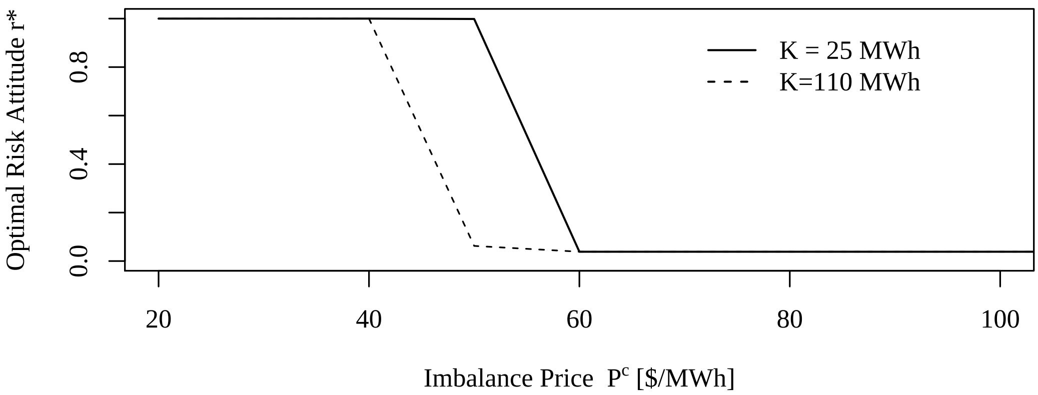

Figure 4.1: Plot of optimal risk attitude r^* against imbalance price P^c under $\sigma = 20\%$ (of K)

Fig. 4.2 plots the relative profit loss defined in (4.9) in y-axis against different values of risk tolerance (r) in x-axis for P^c equal to \$20/MWh in (a) and \$60/MWh in (b), respectively. All the scenarios shown in Fig. 4.2 assume that the degree of uncertainty σ is equal to 20% of K. The values in parentheses correspond to the optimal r^* in each case.

Several observations have emerged. As demonstrated in Tables 4.1–4.2 and Fig. 4.1, a relatively low imbalance price of P^c =\$20/MWh makes the prosumer behave risk neutrally by choosing $r^* = 1$, while a relatively high price of P^c =\$60/MWh induces a risk-averse attitude with $r^* = 0.038$ close to 0. Relative profit loss depends on the deviation of r from the optimum r^*. In Fig. 4.2(a), given $r^* = 1$ under P^c =\$20/MWh, the relative profit losses display decreasing curves in r. By contrast, the losses shown in Fig. 4.2(b) increase in r given that r^* is close to 0 under P^c =\$60/MWh. Additionally,

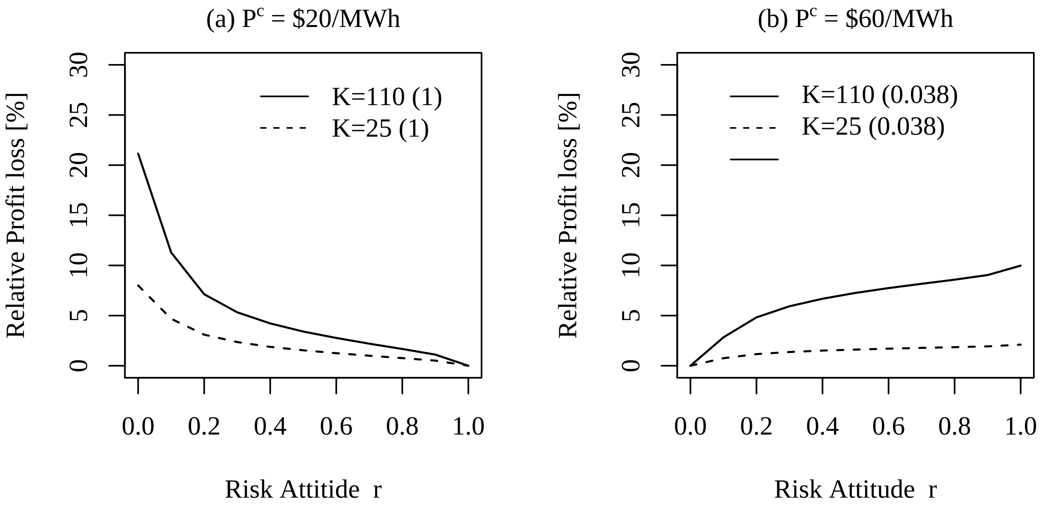

Figure 4.2: Plot of relative profit loss against prosumer's risk attitude r under $\sigma = 20\%$ (of K)

the relative profit loss is larger for a greater expected renewable output of K =110 MWh compared with the case of K =25 MWh in both figures.

We further examine how σ, or the degree of renewable output uncertainty, affects the relative profit loss. Fig. 4.3 plots the relative profit loss against risk attitude r under various levels of σ with four combinations of K and P^c.[5] Figs. 4.3(a)–(b) show the cases of P^c =\$20/MWh, in which the optimal risk attitude of the prosumer is $r^* = 1$, that is, to behave risk neutrally. For a given suboptimal r, when $r^* = 1$ is optimal, the prosumer worsens with a larger value of relative profit loss as the degree of uncertainty increases. In contrast, Figs. 4.3(c)–(d) illustrate the cases of P^c =\$60/MWh, in which the optimum for the prosumer is to choose the risk-averse attitude of $r^* < 1$. Contrary to the case of P^c =\$20/MWh, for a given suboptimal r, the risk-averse prosumer is

[5]Note that condition (4.8) is violated below a threshold value of r under each scenario. We visualize only some relevant ranges in the figures.

generally better off with a lower value of relative profit loss as the degree of uncertainty increases. Therefore, Figs. 4.3(a)–(b) imply that conservative risk attitude when P^c is lower is more costly in the face of a situation with greater uncertainty (larger σ and t), while the prosumer should have behaved risk neutrally ($t = 0$) at the optimum. In contrast, as shown in Figs. 4.3(c)–(d), failure in adequately adjusting risk attitude when P^c is greater is more costly in the face of a smaller degree of uncertainty (smaller σ and t), resulting in an insufficient risk-averse reservation. Finally, the overall relative profit loss is greater under K =110 MWh than under K =25 MWh, intuitively indicating that the cost of getting wrong is greater with a larger K.

4.4 Conclusions

Entities like prosumers in the electric power sector increasingly confront various risks in the real world, including those induced by climate change, as exemplified by the Texas market in February 2021 [69]. Empirical evidence suggests that decision makers can modify their risk preferences in different situations, changing environments, and volatile market conditions [70]. In the current context, a prosumer participating in the day-ahead wholesale power market can maximize its profit by optimally adjusting the risk preference by further considering uncertain renewable resources. This study proposes a distributionally robust chance-constrained MPEC approach to examine the prosumer's endogenous decision concerning their risk attitude. We overcome the non-concavity of the prosumer's objective function by applying Wolfe duality to the lower-

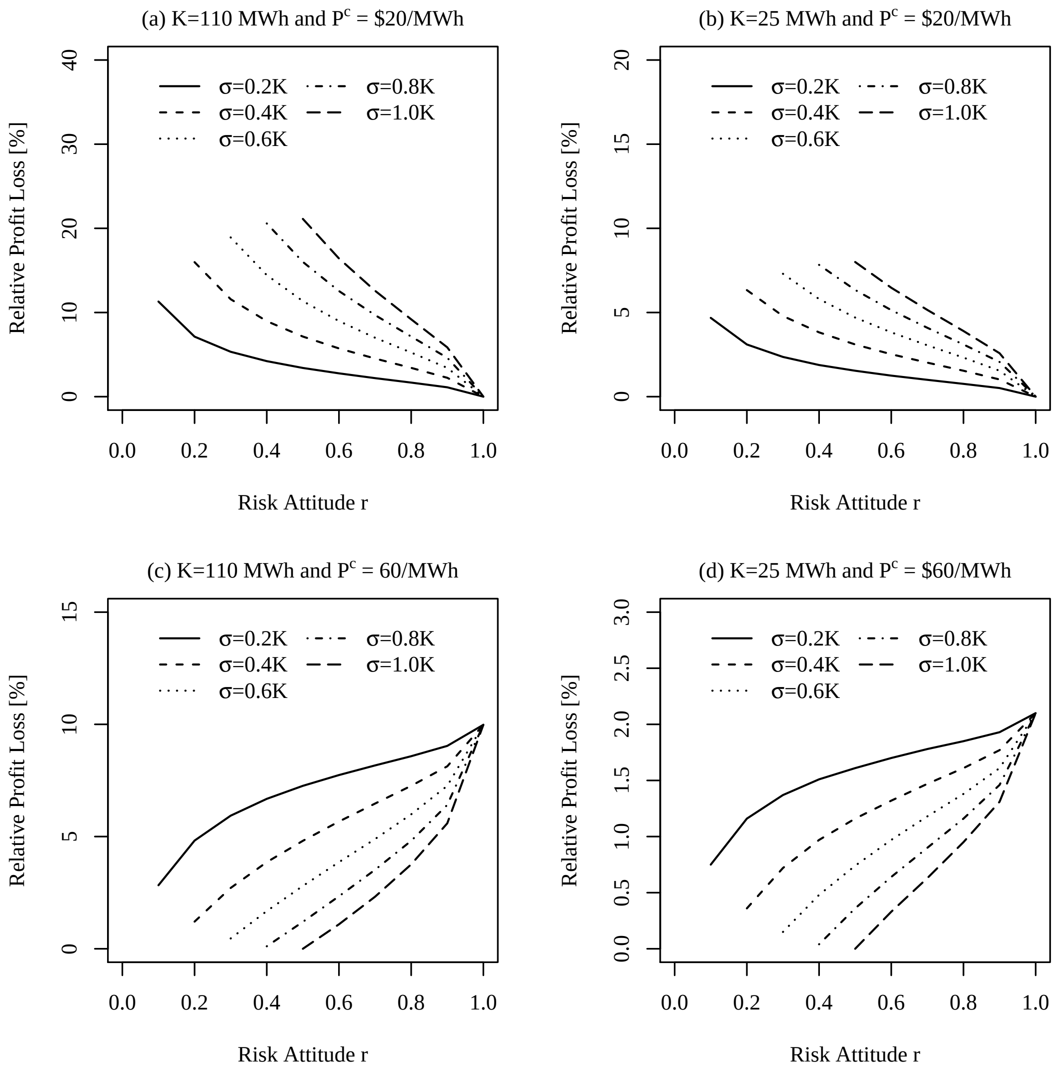

Figure 4.3: Profit loss ratios for various levels of σ

level problem of the grid operator.

The model is applied to a case study based on the IEEE RTS 24-bus system. Our analysis indicates that when allowing a prosumer to endogenously decide their risk preference in the face of uncertain renewable output with energy imbalance settlement, the prosumer can effectively hedge market risk by behaving more conservatively upon perceiving a “risk-derated” output. We demonstrate that the degree of uncertainty plays an insignificant role in determining market outcomes if a prosumer can internalize their risk attitude optimally based on the “risk-averse reservation.” This insight suggests that endogenizing risk preference can be a useful tool for managing risk in an engineering market system.

Therefore, our contribution lies in developing a distributionally robust chance-constrained MPEC framework to model the endogenous decision making of risk preferences under uncertainty. Our analysis also highlights the importance of understanding the prosumer’s risk aversion when evaluating its interaction with, and its effects on a power market built upon an engineering system.

Chapter 5

Conclusions and Future Work

5.1 Summary

This thesis examines the question of modeling an electricity market in presence of strategic prosumers using and equilibrium approach to evaluate market outcomes and investigate the dynamics involved.

Chapter 2 focuses on studying market outcomes when the prosumer is modeled as a price-taker or a strategic entity exercising a quantity-based strategy in the market formulated as mixed-complementarity problems. The model is applied to a IEEE 24-bus test case. This chapter demonstrates the existence of market clearing equilibria and that the position of the prosumer in the market, i.e. whether buying from or selling to the market, does not depend on its strategy. Furthermore, surprisingly, the prosumer is better off acting as a price-taker compared to the quantity-based strategy.

Chapter 3, examines a situation in which the prosumer acts a Stackelberg leader in the market, fully aware of how other entities will react to its decision while also considering uncertainty associated with renewable output. The problem is formulated as a Distributionally Robust Chance-Constrained (DRCC) Mathematical Program with Equilibrium Constraints (MPEC) where the first-order conditions of other market participants/followers, representing their best response, are embedded in the in the leader's problem. The analysis concludes that prosumer's decisions depend on the magnitude of renewable generation uncertainty and its degree of risk aversion. Furthermore, in the short position, in need of purchasing power from the market to meet its demand, the prosumer in the Stackelberg equilibrium increases the social surplus, higher than perfect and imperfect competition. This is because the strategic prosumer is able to lower the average market prices leading to higher consumption and social surplus. On the other hand, in the long position, the prosumer's strategy leads to a decline in social surplus, lowest compared to perfect competition and Cournot cases, due to higher average market prices. These results are new to the literature, thereby contributing to knowledge concerning electricity market in presence of strategic prosumers.

Chapter 4 develops an alternative modeling framework in which prosumers endogenously determine their risk attitude through optimization in the power sector with a day-ahead market and real-time imbalance settlement. The decisions made by prosumers and other participants in the wholesale power market are expressed as a Stackelberg leader-follower game, resulting in a mathematical program with equilibrium constraints. The problem is formulated in a distributionally robust chance-constrained framework to account for renewable generation uncertainty. The chapter concludes that the effect of uncertainty can be managed in the market when prosumers are allowed to internalize risk by determining optimal risk attitudes to maximize their profits, thereby providing insights in terms of managing risks in the energy sector.

5.2 Future Work

Moving forward, addressing a number of research questions can help extend the present work into a more comprehensive toolkit for analyzing power markets with high levels of renewable and distributed energy resources:

First, this thesis mostly focuses on a *static* analysis of a power market. In other words, making the assumption that all actions occur at the same time when characterizing market outcomes. While this approach is useful for describing interactions among entities in the marketplace and, naturally, it can be extended to accommodate multiple time periods, as a next step, it would be relevant to seek answers for the question of how strategies employed by the market participants and the resulting market outcomes would evolve over time. This question is motivated by the fact that market participants observe each other's behavior in the marketplace and learn and adapt to changing market environments. Capturing longer-term evolution of market outcomes as a result of introducing prosumers in the power market will require adopting new tools and modeling frameworks. Data-driven approaches, such as reinforcement learning, can be helpful in capturing how agents in the market learn and respond to the environment.

Another potential extension of the model beyond the static case is to incorporate more temporal elements to account for storage capabilities. This is motivated by the increase of storage availability at both retail and utility scale. Including time intervals may reduce interpretability and increase the complexity of the solution yet make the model more suitable for answering other emerging questions in the power sector, e.g., prosumer's operational decisions.

Finally, this thesis assumes the prosumer to be the only entity in a position to behave strategically in the marketplace. In reality, however, other market participants may may have an incentive not to be price-takers and choose to manipulate the market. Designating multiple market participants as strategic entities, or even considering coalitions among them, could be considered by explicitly modeling theses strategic relationships in the marketplace or implicitly inferred from data-driven simulations of the market. In either case, the choice of which entities to designate as strategic will, to a large extent, depend on the specifics of a market design and characteristics of those market players (such as share of demand or supply, informational advantages, etc.). The modeling approach established in this thesis provides a framework that allows those considerations to be incorporated in future research.

Bibliography

[1] Yael Parag and Benjamin K. Sovacool. Electricity market design for the prosumer era. *Nature Energy*, 1(4):16032, April 2016.

[2] Anthony Papavasiliou and Shmuel S Oren. Large-scale integration of deferrable demand and renewable energy sources. *IEEE Transactions on Power Systems*, 29(1):489–499, 2014.

[3] Lazaros Gkatzikis, Iordanis Koutsopoulos, and Theodoros Salonidis. The role of aggregators in smart grid demand response markets. *IEEE Journal on Selected Areas in Communications*, 31(7):1247–1257, 2013.

[4] Ruoyang Li, Qiuwei Wu, and Shmuel S Oren. Distribution locational marginal pricing for optimal electric vehicle charging management. *IEEE Transactions on Power Systems*, 29(1):203–211, 2014.

[5] C. Chen, S. Kishore, Z. Wang, M. Alizadeh, and A. Scaglione. How will demand response aggregators affect electricity markets? A Cournot game analysis. In *2012 5th International Symposium on Communications, Control and Signal Processing*, pages 1–6, May 2012.

[6] N. Azizan Ruhi, K. Dvijotham, N. Chen, and A. Wierman. Opportunities for Price Manipulation by Aggregators in Electricity Markets. *IEEE Transactions on Smart Grid*, pages 1–1, 2017.

[7] Stig Ødegaard Ottesen, Asgeir Tomasgard, and Stein-Erik Fleten. Prosumer bidding and scheduling in electricity markets. *Energy*, 94:828–843, January 2016.

[8] M. Parvania, M. Fotuhi-Firuzabad, and M. Shahidehpour. Optimal Demand Response Aggregation in Wholesale Electricity Markets. *IEEE Transactions on Smart Grid*, 4(4):1957–1965, December 2013.

[9] S. A. Gabriel, A. J. Conejo, M. A. Plazas, and S. Balakrishnan. Optimal price and quantity determination for retail electric power contracts. *IEEE Transactions on Power Systems*, 21(1):180–187, February 2006.

[10] J. E. Contreras-Ocãna, Miguel A. Ortega-Vazquez, and Baosen Zhang. Participation of an Energy Storage Aggregator in Electricity Markets. *IEEE Transactions on Smart Grid*, Accetped, 2019.

[11] B. E. Hobbs. Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets. *IEEE Transactions on Power Systems*, 16(2):194–202, May 2001.

[12] Udi Helman and Benjamin F Hobbs. Large-scale market power modeling: Analysis of the us eastern interconnection and regulatory applications. *IEEE Transactions on Power Systems*, 25(3):1434–1448, 2010.

[13] Yihsu Chen and B. F. Hobbs. An oligopolistic power market model with tradable NOx permits. *IEEE Transactions on Power Systems*, 20(1):119–129, February 2005.

[14] Steven P Dirkse and Michael C Ferris. The PATH solver: a nommonotone stabilization scheme for mixed complementarity problems. *Optimization Methods and Software*, 5(2):123–156, 1995.

[15] Lorenzo Kristov. The Bottom-Up (R)Evaolution of the Electric Power System: The Pathway to the Integrated-Decenralized System. *IEEE Power & Energy Manazine*, March/April:42–49, 2019.

[16] D.M. Kreps and J.A. Scheinkman. Quantity precommitment and bertrand competition yield cournot outcomes. *Bell Journal of Economics*, 14(2):326–337, 1983.

[17] PJM Interconnection. PJM state of the market –2017, January 2019.

[18] California ISO. Annual report on market issues and performance, January 2019.

[19] Benjamin F. Hobbs and Jong-Shi Pang. Spatial oligopolistic equilibria with arbitrage, shared resources, and price function conjectures. *Mathematical Programming*, 101(1):57–94, August 2004.

[20] Fred C. Schweppe, Michael C. Caramani, Richard D. Tabors, and Roger E. Bohn. *Spot Pricing of Electricity.* Kluwer Academic Publishers, 1998.

[21] Olivier Daxhelet and Yves Smeers. Variational Inequality Models of Restructured

Electricity Systems. In *Complementarity: Applications, Algorithms and Extensions*, Applied Optimization, pages 85–120. Springer, Boston, MA, 2001.

[22] Shmuel S. Oren. Authority and Responsibility of the ISO: Objectives, Options and Tradeoffs. In *Designing Competitive Electricity Markets*, International Series in Operations Research & Management Science, pages 79–95. Springer, Boston, MA, 1998.

[23] Carolyn Metzler, Benjamin F. Hobbs, and Jong-Shi Pang. Nash-Cournot Equilibria in Power Markets on a Linearized DC Network with Arbitrage: Formulations and Properties. *Networks and Spatial Economics*, 3(2):123–150, June 2003.

[24] Stephen Boyd and Lieven Vandenberghe. *Convex optimization*. Cambridge university press, 2004.

[25] Steven A Gabriel, Antonio J Conejo, J David Fuller, Benjamin F Hobbs, and Carlos Ruiz. Equilibria and complementarity problems. In *Complementarity Modeling in Energy Markets*, pages 127–179. Springer, 2013.

[26] A.W. Scheider Jr. The IEEE Reliability Test System-1996. *IEEE Transaction on Power System*, 1999.

[27] Jing Wang, N Encinas Redondo, and Francisco D Galiana. Demand-side reserve offers in joint energy/reserve electricity markets. *IEEE Transactions on Power Systems*, 18(4):1300–1306, 2003.

[28] Inês M Lima Azevedo, M Granger Morgan, and Lester Lave. Residential and

regional electricity consumption in the US and EU: How much will higher prices reduce co_2 emissions? *The Electricity Journal*, 24(1):21–29, 2011.

[29] Federal Energy Regulatory Commission. *FERC Opens Wholesale Markets to Distributed Resources: Landmark Action Breaks Down Barriers to Emerging Technologies, Boosts Competition*, 2020.

[30] Ruoyang Li, Qiuwei Wu, and Shmuel S Oren. Distribution locational marginal pricing for optimal electric vehicle charging management. *IEEE Transactions on Power Systems*, 29(1):203–211, 2013.

[31] Anthony Papavasiliou and Shmuel S. Oren. Large-scale integration of deferrable demand and renewable energy sources. *IEEE Transactions on Power Systems*, 29(1):489–499, 2013.

[32] Lorenzo Kristov. The bottom-up (r) evolution of the electric power system: The pathway to the integrated-decentralized system. *IEEE Power and Energy Magazine*, 17(2):42–49, 2019.

[33] Subhonmesh Bose, Chenye Wu, Yunjian Xu, Adam Wierman, and Hamed Mohsenian-Rad. A unifying market power measure for deregulated transmission-constrained electricity markets. *IEEE Transactions on Power Systems*, 30(5):2338–2348, 2014.

[34] Khaled Alshehri, Mariola Ndrio, and Subhonmesh Bose Tamer Basar. The impact

of aggregating distributed energy resources on electricity market efficiency. In *Annual Conference on Information Sciences and Systems (CISS)*, 2019.

[35] Yang Yu, Baosen Zhang, and Ram Rajagopal. Do wind power producers have market power and exercise it? In *2014 IEEE PES General Meeting*, pages 1–5. IEEE, 2014.

[36] C. J. Day, B. F. Hobbs, and Jong-Shi Pang. Oligopolistic competition in power networks: A conjectured supply function approach. *IEEE Transactions on Power Systems*, 17(3):597–607, August 2002.

[37] Chen Chen, Shalinee Kishore, Zhifang Wang, Mahnoosh Alizadeh, and Anna Scaglione. How will demand response aggregators affect electricity markets?—a cournot game analysis. In *2012 5th International Symposium on Communications, Control and Signal Processing*, pages 1–6. IEEE, 2012.

[38] Yihsu Chen, Benjamin F Hobbs, Sven Leyffer, and Todd S Munson. Leader-follower equilibria for electric power and NOx allowances markets. *Computational Management Science*, 3(4):307–330, 2006.

[39] Robert Entriken and Steve Wan. Agent-based simulation of an automatic mitigation procedure. In *Proceedings of the 38th Annual Hawaii International Conference on System Sciences*, pages 92c–92c. IEEE, 2005.

[40] R Entriken and S Wan. Pushing capacity payments forward: agent-based simu-

lation of an available capacity market. Technical report, EPRI Technical Report (Available at www. epriweb. com/public . . . , 2003.

[41] Benjamin F Hobbs, Carolyn B Metzler, and J-S Pang. Strategic gaming analysis for electric power systems: An MPEC approach. *IEEE Transactions on Power Systems*, 15(2):638–645, 2000.

[42] Xinmin Hu, Daniel Ralph, Eric K Ralph, Peter Bardsley, Michael C Ferris, et al. Electricity generation with looped transmission networks: Bidding to an ISO. *Research Paper*, 16, 2004.

[43] Guido Pepermans. Ramsey pricing in a congested network with market power in generation: A numerical illustration for Belgium. *KU Leuven CES; Leuven Working Papers*, pages 1–38, 2004.

[44] Steven A Gabriel and Florian U Leuthold. Solving discretely-constrained MPEC problems with applications in electric power markets. *Energy Economics*, 32(1):3–14, 2010.

[45] Sauleh Siddiqui and Steven A Gabriel. An sos1-based approach for solving mpecs with a natural gas market application. *Networks and Spatial Economics*, 13(2):205–227, 2013.

[46] Sepehr Ramyar, Andrew Liu, and Yihsu Chen. A power market model in presence of strategic prosumers. *IEEE Transactions on Power Systems*, 35(2):898–908, 2020.

[47] S. Ramyar and Y. Chen. Leader-follower equilibria for power markets in presence of prosumers. In *HICSS*, 2020.

[48] M. Lubin, Y. Dvorkin, and S. Backhaus. A robust approach to chance constrained optimal power flow with renewable generation. *IEEE Transactions on Power Systems*, 31(5):3840–3849, 2016.

[49] Campos Inês, Pontes Luz Guilherme, Marín-González Esther, Gährs Swantje, Hall Stephen, and Holstenkamp Lars. Regulatory challenges and opportunities for collective renewable energy prosumers in the eu. *Energy Policy*, 138:111212, 2020.

[50] PV Magazine. World's largest residential virtual power plant coming from Alphabet-backed SIP and OhmConnect, 2020.

[51] Benjamin F Hobbs and Jong-Shi Pang. Nash-Cournot equilibria in electric power markets with piecewise linear demand functions and joint constraints. *Operations Research*, 55(1):113–127, 2007.

[52] Giuseppe Carlo Calafiore and Laurent El Ghaoui. On distributionally robust chance-constrained linear programs. *Journal of Optimization Theory and Applications*, 130(1):1–22, 2006.

[53] José Fortuny-Amat and Bruce McCarl. A representation and economic interpretation of a two-level programming problem. *Journal of the operational Research Society*, 32(9):783–792, 1981.

[54] AW Schneider. The IEEE reliability test system-1996: A report prepared by the reliability test system task force of the application of probability methods subcommittee. discussion. *IEEE Transactions on Power Systems*, 14(3):1010–1020, 1999.

[55] Robert S Gibbons. *Game Theory for Applied Economists.* Princeton University Press, 1992.

[56] Yihsu Chen and B. F. Hobbs. An oligopolistic power market model with tradable NO_x permits. *IEEE Transactions on Power Systems*, 20(1):119–129, 2005.

[57] Echo D Cartwright. Ferc order 2222 gives boost to ders, 2020.

[58] Markus K. Brunnermeier and Stefan Nagel. Do wealth fluctuations generate time-varying risk aversion? micro-evidence on individuals. *American Economic Review*, 98(3):713–736, 2008.

[59] Pierre-André Chiappori and Monica Paiella. Relative risk aversion is constant: Evidence from panel data. *Journal of the European Economic Association*, 9(6):1021–1052, 2011.

[60] Alan Greenspan. Reflections on central banking. Symposium by Federal Reserve Bank of Kansas City, Jackson Hole, http://www.federalreserve.gov/boarddocs/speeches/ 2005/20050826/default.htm, 2005.

[61] Michele Berardi. Endogenous time-varying risk aversion and asset returns. *Journal of Evolutionary Economics*, 26(3):581–601, 2016.

[62] Ignacio Palacios-Huerta and Tano J. Santos. A theory of markets, institutions, and endogenous preferences. *Journal of Public Economics*, 88:601–627, 2004.

[63] Shichang Cui, Yan-Wu Wang, Chaojie Li, and Jiang-Wen Xiao. Prosumer community: A risk aversion energy sharing model. *IEEE Transactions on Sustainable Energy*, 11(2):828–838, 2019.

[64] Li Ma, Nian Liu, Jianhua Zhang, and Lingfeng Wang. Real-time rolling horizon energy management for the energy-hub-coordinated prosumer community from a cooperative perspective. *IEEE Transactions on Power Systems*, 34(2):1227–1242, 2018.

[65] Gabriella Ferruzzi, Guido Cervone, Luca Delle Monache, Giorgio Graditi, and Francesca Jacobone. Optimal bidding in a day-ahead energy market for micro grid under uncertainty in renewable energy production. *Energy*, 106:194–202, 2016.

[66] Michael Stutzer. Portfolio choice with endogenous utility: A large deviations approach. *Journal of Econometrics*, 116:365–386, 2003.

[67] Zhi-Quan Luo, Jong-Shi Pang, and Daniel Ralph. *Mathematical programs with equilibrium constraints*. Cambridge University Press, 1996.

[68] Richard H. Byrd, Jorge Nocedal, and Richard A. Waltz. Knitro: An integrated

package for nonlinear optimization. In Di Pillo G. and Roma M., editors, *Large-scale Nonlinear Optimization*. Springer, Boston, MA, 2006.

[69] Texas Tribune. Texas power outages: Nearly half the state experiencing water disruptions as power grid operator says it's making progress.

[70] Daniel Kahneman and Amos Tversky. Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2):263–291, 1979.

[71] V. Virasjoki, A.S. Siddiqui, B Zakeri, and A. Salo. Market power with combined heat and power production in the nordic energy system. *IEEE Transactions on Power Systems*, 33(5):5263–5275, 2018.

[72] University of California Energy Institute. Stabilizing Utilities in an Unstable Climate, January 2019.

[73] Yael Parag and Benjamin K. Sovacool. Electricity market design for the prosumer era. *Nature Energy*, April(1), 2016.

[74] Daniel Kahneman, Jack L. Knetsch, and Richard H. Thaler. Anomalies: The endowment effect, loss aversion, and status quo bias. *Journal of Economic Perspectives*, 5(1):193–206, 1991.

[75] Makoto Tanaka. Transmission-constrained oligopoly in the Japanese electricity market. *Energy Economics*, 31(5):690–701, 2009.

[76] James Bushnell. A mixed complementarity model of hydrothermal electricity competition in the western united states. *Operations research*, 51(1):80–93, 2003.

[77] Ingeborg Graabak, Qiuwei Wu, Leif Warland, and Zhaoxi Liu. Optimal planning of the nordic transmission system with 100% electric vehicle penetration of passenger cars by 2050. *Energy*, 107:648–660, 2016.

[78] Anthony Papavasiliou, Haitham Hindi, and Daniel Greene. Market-based control mechanisms for electric power demand response. In *Decision and Control (CDC), 2010 49th IEEE Conference on*, pages 1891–1898. IEEE, 2010.

[79] Farrokh Rahimi and Ali Ipakchi. Demand response as a market resource under the smart grid paradigm. *IEEE Transactions on Smart Grid*, 1(1):82–88, 2010.

[80] Zofia Lukszo and Esther H Park Lee. Demand side and dispatchable power plants with electric mobility. In *Smart Grids from a Global Perspective*, pages 163–177. Springer, 2016.

[81] Martin J Osborne et al. *An introduction to game theory*, volume 3. Oxford university press New York, 2004.

[82] Steven A Gabriel, Antonio J Conejo, J David Fuller, Benjamin F Hobbs, and Carlos Ruiz. Optimality and complementarity. In *Complementarity Modeling in Energy Markets*, pages 31–69. Springer, 2013.

[83] Q Binh Dam, AP Sakis Meliopoulos, Gerald Thomas Heydt, and Anjan Bose. A breaker-oriented, three-phase ieee 24-substation test system. *IEEE Transactions on Power Systems*, 25(1):59–67, 2010.

[84] Nima Amjady, Jamshid Aghaei, and Heidar Ali Shayanfar. Stochastic multiobjective market clearing of joint energy and reserves auctions ensuring power system security. *IEEE Transactions on Power Systems*, 24(4):1841–1854, 2009.

[85] Robert Fourer, David M Gay, and Brian Kernighan. *Ampl*, volume 117. Boyd & Fraser Danvers, MA, 1993.

[86] Andreu Mas-Colell, Michael Dennis Whinston, Jerry R Green, et al. *Microeconomic theory*, volume 1. Oxford university press New York, 1995.

[87] S. Grijalva and M. U. Tariq. Prosumer-based smart grid architecture enables a flat, sustainable electricity industry. In *ISGT 2011*, pages 1–6, January 2011.

[88] Yael Parag and Benjamin K. Sovacool. Electricity market design for the prosumer era. *Nature Energy*, 1(4):16032, April 2016.

[89] H. Vogt, H. Weiss, P. Spiess, and A. P. Karduck. Market-based prosumer participation in the smart grid. In *4th IEEE International Conference on Digital Ecosystems and Technologies*, pages 592–597, April 2010.

[90] Peter Cramton. Electricity market design. *Oxford Review of Economic Policy*, 33(4):589–612, November 2017.

[91] An oligopolistic power market model with tradable NO/sub x/ permits - IEEE Journals & Magazine. https://ieeexplore.ieee.org/abstract/document/1388501/.

[92] Paul Joskow and Jean Tirole. Reliability and competitive electricity markets. *The Rand Journal of Economics*, 38(1):60–84, 2007.

[93] Ali Pinar, Juan Meza, Vaibhav Donde, and Bernard Lesieutre. Optimization strategies for the vulnerability analysis of the electric power grid. *SIAM Journal on Optimization*, 20(4):1786–1810, 2010.

[94] Andrew L Liu and Benjamin F Hobbs. Tacit collusion games in pool-based electricity markets under transmission constraints. *Mathematical Programming*, 140(2):351–379, 2013.

[95] Yihsu Chen, Andrew L Liu, and Benjamin F Hobbs. Economic and emissions implications of load-based, source-based, and first-seller emissions trading programs under california ab32. *Operations Research*, 59(3):696–712, 2011.

[96] Carolyn Metzler, Benjamin F Hobbs, and Jong-Shi Pang. Nash-cournot equilibria in power markets on a linearized dc network with arbitrage: Formulations and properties. *Networks and Spatial Economics*, 3(2):123–150, 2003.

[97] Jong-Shi Pang and Gesualdo Scutari. Nonconvex games with side constraints. *SIAM Journal on Optimization*, 21(4):1491–1522, 2011.

[98] Jacques Gauvin. A necessary and sufficient regularity condition to have bounded multipliers in nonconvex programming. *Mathematical Programming*, 12(1):136–138, 1977.

[99] R. Fletcher. *Practical Methods of Optimization.* Wiley, Chichester, 1987.

[100] David G Luenberger et al. Investment science. *OUP Catalogue*, 1997.

Appendix A

Proof of Propositions in Chapter 2

A.1 Proof of Proposition 1

We prove existence by showing that all the primal variables without explicit bounds, s, z, y, w and a, are all implicitly bounded. With the continuity assumptions of all the objective functions, we know that all the optimization problems involved, (2.2), (2.4), (2.6), and (2.8), have an optimal solution by the well-known Weierstrass' extreme value theorem, so long as the problems are feasible. Then since the constraints in all the optimization problems are linear, the linear constraint qualification holds, which guarantees the existence of dual variables satisfying the complementarity conditions in (2.3), (2.5), (2.7), and (2.9).

Note that feasibility here is not an issue since by letting $l_i = K_i$, $i = 1, \ldots, I$ and all other variables (g, z, s, x, y and a) to be 0, all the optimization problems' constraints, together with the flow balance constraint (2.10), can be satisfied. Hence, the set of joint feasibility of the optimization problems (2.2), (2.4), (2.6), (2.8), and (2.10) is not empty.

Next, we start with the arbitrager's problem (2.8) first. It has been shown in [96] (Equation (6) in [96]) that the two sets of equations in (2.9), together with the expression of the market price p_i as in (2.1), can uniquely determine the value for p^h and a_i, $i = 1, \ldots, I$, with a given set of values of w_i and s_{fi}, $i = 1, \ldots, I$, $f = 1, \ldots, F$. For simplicity, we denote a_i as $a_i(w, s)$, indicating that a_i is a linear function of w and s.

We next consider the grid operator's problem (2.6). It has been shown in [93] that the Mangasarian-Fromovitz Constraint Qualification (MFCQ) holds for the grid operator's problem at any feasible point. It is well-known that MFCQ is equivalent to the set of multipliers to be bounded [98]. Hence, $\lambda_k^{+,-}$ are bounded for $k = 1, \ldots, R$ in (2.7), which implies that w_i is bounded for $i = 1, \ldots, I$.

Now consider the inverse demand function at $i = 1, \ldots, I$:

$$p_i = P_i^0 - \frac{P_i^0}{Q_i^0} d_i = P_i^0 - \frac{P_i^0}{Q_i^0} \left(\sum_f s_{fi} + a_i(w, s) \right),$$

which determines the wholesale electricity price at each node i. In an equilibrium, we must have $0 \leq p_i \leq P_i^0$. If $p_i > P_i^0$ for some i, then the demand $d_i < 0$, which is impossible; on the other hand, if $p_i < 0$, then suppliers can simply choose to produce nothing (i.e., $x = s = 0$) to avoid a net loss in profit. Hence, we can only consider the set of s and w within which that yield a p_i within $[0, P_i^0]$ for each i. Based on the expression of $a_i(w, s)$ with respect to s (as in (6) in [96]), p_i can be re-written as a linear function of s and w, with coefficient of s_fi all non-positive. Then the boundness of w_i

and p_i, together with $s_{fi} \geq 0$, implies the boundedness of $s_f i$, for each f and i. Hence, the supplier's problem (2.4) assumes a finite optimal solution.

For the prosumer's problem (2.2), even though the z variables are not explicitly bounded (and hence the l variables are not bounded above), the level set of the problem must be bounded. This is so since (2.2) is a maximization problem; if $\sum_f z_{fi}$ goes to $-\infty$, the corresponding l_i will become $+\infty$, which will make the first term $p_i(\sum_f z_{fi})$ and the second term $-\int_{l_i}^{K_i} B_i'(x)dx$ in the objective function become negative infinity (since $B_i'(x)$ is a decreasing function by the assumption). This cannot happen to a feasible maximization problem. Hence (2.2) also assumes a finite optimal solution.

The finite optimal solutions of all optimization problems with linear constraints, together with the fact of a non-empty joint feasible set, yields the existence of a market equilibrium.

A.2 Proof of Proposition 2

For easy exposition, we assume there are consumers, a producer and a prosumer in the market. We then drop the subscripts accordingly. Recall the prosumer's optimization problem in (2.2) and definition of its z ($z > 0$: long position and $z < 0$: short position), assuming that (2.2b) is binding, we can rewrite (2.2a) as follows:

$$\underset{z}{\text{maximize}} \quad pz + B(K - z) \tag{A.1a}$$

We further assume that the prosumer only owns zero marginal cost renewables, i.e., $G = 0$ so that constraint (2.2c) is omitted. We will then show how the result can be extended to the case a prosumer with dispatchable generation.

Similarly, the producer's optimization problem can be simplified and rewritten as:

$$\underset{s}{\text{maximize}} \quad ps - C(s) \tag{A.2a}$$

Here, s represents the producer's sales. For simplicity, we assume that $C_0 = 0$ so that cost takes a form of $C(s) = \frac{1}{2}cs^2$. Furthermore, given (2.1) and $G = 0$, consumers quantity demanded equals $d = s + z$.

<u>Price-Taker Prosumer</u> Taking the first order condition of (A.1) and (A.2), together with $d = s + z$, we have three conditions and three unknown (z, s, d):

$$p = A^0 - B^0(K - z) \tag{A.3a}$$

$$p = cs \tag{A.3b}$$

$$p = P^0 - (P^0/Q^0)d \tag{A.3c}$$

Solving the system of equations in (A.3) for z, we have:

$$z = \frac{P^0 c - (\frac{P^0}{Q^0} + c)(A^0 - B^0 K)}{B^0(\frac{P^0}{Q^0} + c) + \frac{P^0}{Q^0}c} \tag{A.4}$$

<u>Strategic Prosumer</u> Accounting for the prosumers' market power, we re-write (A.1) as follows:

$$\underset{z}{\text{maximize}} \quad [P^0 - (\frac{P^0}{Q^0})(s + z)]z + B(K - z) \tag{A.5a}$$

The first order conditions for the three agents under market power would then be:

$$P^0 - \frac{P^0}{Q^0}(s+z) - \frac{P^0}{Q^0}t = A^0 - B^0(K-z) \tag{A.6a}$$

$$(A.3b) \quad \& \quad (A.3c)$$

Solving the system of equations again for z, we have:

$$z = \frac{P^0c - (\frac{P^0}{Q^0}+c)(A^0 - B^0K)}{B^0(\frac{P^0}{Q^0}+c) + \frac{P^0}{Q^0}(\frac{P^0}{Q^0}+2c)} \tag{A.7}$$

The denominator of (A.4) and (A.7) are positive given that all the parameters are positive. Thus, given that the numerators in (A.4) and (A.7) are equivalent, this suggests that sign of z, the prosumer's net sale, under both cases will be the same. Namely, If a prosumer is in a short (long) position as a price-taker, he/she will also be in a short (long) as a strategic entity, and vice versa.

When the prosumer also owns a dispatchable with a marginal cost of c', z under perfect competition and market power cases can be expressed as (A.8) and (A.9), respectively.

$$z = \frac{P^0(c + \frac{cB^0}{c'}) - (\frac{P^0}{Q^0}+c)(A^0 - B^0K)}{B^0(\frac{P^0}{Q^0}\frac{c}{c'} + \frac{P^0}{Q^0}+c) + \frac{P^0}{Q^0}c} \tag{A.8}$$

$$z = \frac{P^0(c + \frac{cB^0}{c'}) - (\frac{P^0}{Q^0}+c)(A^0 - B^0K)}{B^0(\frac{P^0}{Q^0}+c) + \frac{P^0}{Q^0}(\frac{P^0}{Q^0}+2c) + \frac{\frac{P^0}{Q^0}B^0c}{c'}} \tag{A.9}$$

Again, we have the two numerators to be identical and the denominators are both positive. Hence the sign of z would not depend on the choice of prosumer's strategy.

A.3 Proof of Proposition 3

By the same token as the proof of Proposition 2, we denote the power price and the prosumer's net position z for the perfect competition and market power cases by subscripts c and m, respectively.

Price-Taker Prosumer the prosumer surplus defined in (A.1a) is equal to $p_c z_c + B^0(K - z_c)$. By substituting p_c and z_c from the solution to (A.3), we get $p_c z_c + B^0(K - z_c) = \frac{B^0}{2} z_c^2$.

Strategic Prosumer Similarly, substituting p_m and z_m from the solution to (A.6), we have $p_m z_m + B^0(K - z_m) = \frac{B^0}{2} z_m^2$.

Finally, substitute (A.4) and (A.7), respectively, for z_c and z_m, it implies that $\frac{B^0}{2} z_m^2 < \frac{B^0}{2} z_c^2$. Now, since t_c is given by (A.4) and t_m by (A.7), it is evident that:

$$\frac{\beta}{2} t_m^2 < \frac{\beta}{2} t_c^2,$$

Accordingly, we conclude that $p_c z_c + B^0(K - z_c) > p_m z_m + B^0(K - z_m)$. That is, a prosumer is better off with a higher profit when behaving as a price taker regardless of its position in the market.

A.4 Model Equivalence

We consider a situation where prosumer in node i enters a bilateral contract with the aggregator in i to purchase firm energy (l_i^c) at a contract price (p_i^c). By doing so, the prosumer relinquishes its control over dispatch unit, g_i, to the aggregator. The optimization problem faced by the prosumer i is as follows:

$$\underset{l_i \geq 0, l_i^p \geq 0}{\text{maximize}} \quad -p_i^c l_i^p - \int_{l_i}^{K_i} B_i'(x)dx \tag{A.10a}$$

subject to

$$l_i - l_i^p \leq 0 \qquad (\epsilon_i). \tag{A.10b}$$

The first-order conditions for l_i and l_i^p are, respectively, displayed as follows:

$$0 \leq l_i \perp A_i^0 - B_i^0 l_i - \epsilon_i \leq 0 \tag{A.11a}$$

$$0 \leq l_i^p \perp -p_i^c + \epsilon_i \leq 0 \tag{A.11b}$$

The aggregator i decides 1) amount of energy l_i^a to contract with prosumers, 2) amount of energy to purchase from ($z_{fi} < 0$) or sell to ($z_{fi} > 0$) the wholesale market, and 3) amount of g_i to generate while subject to sales and output constraints.

$$\underset{z_{fi}, l_i^a \geq 0, g_i \geq 0}{\text{maximize}} \quad p_i^c l_i^a + p_i \sum_f z_{fi} - \sum_i C_i^g(g_i) \tag{A.12a}$$

subject to

$$\sum_f z_{fi} + l_i^a - K_i - g_i = 0 \qquad (\delta_i), \tag{A.12b}$$

$$g_i \leq G_i \qquad (\kappa_i) \tag{A.12c}$$

The first-order conditions of the aggregator i's problem for variables z_{fi}, l_i^a, g_i, and κ_i

are summarized as follows.

$$p_i - \delta_i = 0 \tag{A.13a}$$

$$0 \le l_i^a \perp p_i^c - \delta_i \le 0 \tag{A.13b}$$

$$0 \le g_i \perp -C_i^{g\prime} + \delta_i - \kappa_i \le 0 \tag{A.13c}$$

$$0 \le \kappa_i \perp g_i - G_i \le 0 \tag{A.13d}$$

At equilibrium, $l_i^a = l_i^p$ with p_i^c defining the contract premium:

$$l_i^a = l_i^c, \qquad (p_i^c) \tag{A.14}$$

Assuming that $l_i^a = l_i^p > 0$, we have i) $p_i^c - \delta_i = 0$ from (A.13b), which is equivalent to (2.3a) and ii) $p_i^c = \epsilon_i$ from (A.11b). Thus, we can conclude that $\epsilon_i = \delta_i$. (A.11a) can then be written as $0 \le l_i \perp A_i^0 - B_i^0 l_i - \delta_i \le 0$, which is equivalent to (2.3b). Moreover, (A.13a)=(2.3a), (A.13c)=(2.3c), and (A.13d)=(2.3e). This establishes the equivalence of the model in Section (2.2.1.2) and the model here.

Appendix A

Derivation of Problem Reformulations in Chapter 3

A.1 Wolfe Duality of Lower-Level Problem

Since the lower-level problem is a concave program, we can obtain the optimal solution by solving its dual problem. Particularly, for a general concave program $\max_x\{f(x) : g(x) \leq 0\}$, the corresponding Wolfe dual is $\min_{x,\lambda}\{\mathcal{L}(x,\lambda) : \nabla_x\mathcal{L}(x,\lambda) = 0, \lambda \geq 0\}$, where $\nabla_x\mathcal{L}(x,\lambda)$ are the gradients of the Lagrangian $\mathcal{L}(x,\lambda) = f(x) - \lambda^\top g(x)$. For a concave (or convex) programming problem, strong duality holds between the primal and Wolfe dual problems. Consequently, the Wolfe dual of the social welfare maximization problem in the lower level is:

$$\min_{\Omega \cup \Lambda} \quad \sum_i B_i(d_i) - \sum_{f,i,h \in H_{fi}} C_{fih}(x_{fih}) \tag{A-1a}$$

$$- \sum_{f,i,h \in H_{fi}} \beta_{fih}(x_{fih} - X_{fih}) - \sum_k \lambda_k^+ (\sum_i PTDF_{ki} - T_k)$$

$$- \sum_k \lambda_k^- (\sum_i -PTDF_{ki} y_i - T_k) - \sum_i \eta_i d_i + \sum_i \eta_i z_i$$

$$+ \sum_{f,i,h \in H_{fi}} \eta_i x_{fih} + \sum_i \eta_i y_i - \theta \sum_i y_i + \sum_{f,i,h \in H_{fi}} \varepsilon_{fih} x_{fih}$$

$$+ \sum_i \xi_i d_i$$

subject to

$$- C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f, i, h \in H_{fi} \tag{A-1b}$$

$$B'_i(d_i) - \eta_i + \xi_i = 0 \quad \forall i \tag{A-1c}$$

$$- \sum_k (\lambda_k^+ - \lambda_k^-) PTDF_{ki} + \eta_i - \theta = 0 \quad \forall i \tag{A-1d}$$

$$\beta_{fih}, \varepsilon_{fih} \geq 0 \quad \forall f, i, h \in H_{fi} \tag{A-1e}$$

$$\lambda_k^+, \lambda_k^- \geq 0 \qquad \forall k \tag{A-1f}$$

$$\xi_i \geq 0 \qquad \forall i \tag{A-1g}$$

Note that non-negativity constraints are imposed on $\{\beta_{fih}, \lambda_k^+, \lambda_k^-, \varepsilon_{fih}, \xi_i\}$, which are associated with the inequality constraints in the primal problem. Otherwise, variables are unrestricted.

Given the concavity of the social welfare maximization problem and that strong duality holds, the original objective functions and (A-1a) have the same value. Thus:

$$
\begin{aligned}
&-\sum_{f,i,h\in H_{fi}}\beta_{fih}(x_{fig}-X_{fih})+\sum_{f,i,h\in H_{fi}}\varepsilon_{fih}x_{fih} \\
&-\sum_k\lambda_k^+(\sum_i PTDF_{ki}y_i-T_k)+\sum_i\xi_i d_i \\
&-\sum_k\lambda_k^-(\sum_i -PTDF_{ki}y_i-T_k)-\theta\sum_i y_i \\
&-\sum_i\eta_i d_i+\sum_{f,i,h\in H_{fi}}\eta_i x_{fih}+\sum_i\eta_i z_i+\sum_i\eta_i y_i=0
\end{aligned}
\tag{A-2}
$$

Using constraints (A-1b)-(A-1d), we further simplify (A-2). In particular, from (A-1b) we have:

$$
\begin{aligned}
&\big(-C'_{fih}(x_{fih})-\beta_{fih}+\eta_i+\varepsilon_{fih}\big)x_{fih}=0 \\
&\sum_{f,i,h\in H_{fi}}(-\beta_{fih}+\eta_i+\varepsilon_{fih})x_{fih}=\sum_{f,i,h\in H_{fi}}C'_{fih}(x_{fih})x_{fih}
\end{aligned}
\tag{A-3}
$$

From (A-1c), we derive:

$$
\begin{aligned}
&\big(B'_i(d_i)-\eta_i+\xi_i\big)d_i=0 \\
&\sum_i\big(-\eta_i+\xi_i\big)d_i=-\sum_i B'_i(d_i)d_i
\end{aligned}
\tag{A-4}
$$

Fom (A-1d), we obtain:

$$
\begin{aligned}
&\Big(-\sum_k(\lambda_k^+-\lambda_k^-)PTDF_k+\eta_i-\theta\Big)y_i=0 \\
&\sum_i\big(\eta_i-\theta\big)y_i=\sum_{k,i}(\lambda_k^+-\lambda_k^-)PTDF_k y_i
\end{aligned}
\tag{A-5}
$$

Substituting (A-3), (A-4), and (A-5) into (A-2), we can rewrite the bilinear term $\sum_i \eta_i z_i$ as follows:

$$\sum_i \eta_i z_i = \sum_i B_i'(d_i)d_i - \sum_k (\lambda_k^+ + \lambda_k^-)T_k \tag{A-6}$$
$$- \sum_{f,i,h \in H_{fi}} \big(C_{fih}'(x_{fih})x_{fih} + \beta_{fih}X_{fih}\big)$$

A.2 Concavification of Objective Function in MPEC

We can substitute (A-6) with the bilinear term $\sum_i \eta_i z_i$ in the original objective function of the MPEC. Along with other constraints, the objective function of MPEC, or the Stackelberg leader formulation for the prosumer, is then rewritten as follows:

$$\max_{\Phi \cup \Omega \cup \Lambda} \sum_i B_i'(d_i)d_i - \sum_k (\lambda_k^+ + \lambda_k^-)T_k \tag{B-1}$$
$$- \sum_{f,i,h \in H_{fi}} \big(C_{fih}'(x_{fih})x_{fih} + \beta_{fih}X_{fih}\big)$$
$$+ \sum_i \big(B_i^l(l_i) - C_i^g(g_i)\big) + \sum_i P_i^c\Big(K_i - z_i - l_i + g_i\Big)$$

A.3 MIQP Reformulation

In MPEC formulation (B-1), we overcome the bilinear terms in the objective function using Wolfe's duality. As a final step, we further remove non-convex terms caused by the complementarity conditions from the lower-level problem by utilizing disjunctive constraints. With binary variables $\{\bar{r}_{fih}, r_k^+, r_k^-, r_{fih}, \hat{r}_i\}$ and positive big constants $\{M_1, M_2, M_3, M_4, M_5\}$, we reformulate the MPEC into an MIQP as follows:

$$\max_{\Phi\cup\Omega\cup\Lambda\cup\Psi} \sum_i B_i'(d_i)d_i - \sum_k(\lambda_k^+ + \lambda_k^-)T_k \tag{C-1a}$$

$$- \sum_{f,i,h\in H_{fi}} \big(C'_{fih}(x_{fih})x_{fih} + \beta_{fih}X_{fih}\big)$$

$$+ \sum_i \big(B_i^l(l_i) - C_i^g(g_i)\big) + \sum_i P_i^c\Big(K_i - z_i - l_i + g_i\Big)$$

subject to

$$\sigma_i\sqrt{\frac{1-R_i}{R_i}} + z_i + l_i - g_i - K_i \leq 0 \qquad \forall i \tag{C-1b}$$

$$g_i \leq G_i \qquad \forall i \tag{C-1c}$$

$$l_i, g_i \geq 0 \qquad \forall i \tag{C-1d}$$

$$-C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f,i,h\in H_{fi} \tag{C-1e}$$

$$B_i'(d_i) - \eta_i + \xi_i = 0 \qquad \forall i \tag{C-1f}$$

$$-\sum_k(\lambda_k^+ - \lambda_k^-)PTDF_{ki} + \eta_i - \theta = 0 \qquad \forall i \tag{C-1g}$$

$$0 \leq -(x_{fih} - X_{fih}) \leq M_1\bar{r}_{fih} \qquad \forall f,i,h\in H_{fi} \tag{C-1h}$$

$$0 \leq \beta_{fih} \leq M_1(1-\bar{r}_{fih}) \qquad \forall f,i,h\in H_{fi} \tag{C-1i}$$

$$d_i - \sum_{f,h\in H_{fi}} x_{fih} - z_i - y_i = 0 \qquad \forall i \tag{C-1j}$$

$$0 \leq -\Big(\sum_i PTDF_{ki}y_i - T_k\Big) \leq M_2 r_k^+ \qquad \forall k \tag{C-1k}$$

$$0 \leq \lambda_k^+ \leq M_2(1-r_k^+) \qquad \forall k \tag{C-1l}$$

$$0 \leq -\Big(-\sum_i PTDF_{ki}y_i - T_k\Big) \leq M_3 r_k^- \qquad \forall k \tag{C-1m}$$

$$0 \leq \lambda_k^- \leq M_3(1-r_k^-) \qquad \forall k \tag{C-1n}$$

$$\sum_i y_i = 0 \tag{C-1o}$$

$$0 \leq x_{fih} \leq M_4 r_{fih} \qquad \forall f,i,h\in H_{fi} \tag{C-1p}$$

$$0 \leq \varepsilon_{fih} \leq M_4(1-r_{fih}) \qquad \forall f,i,h\in H_{fi} \tag{C-1q}$$

$$0 \leq d_i \leq M_5\hat{r}_i \qquad \forall i \tag{C-1r}$$

$$0 \leq \xi_i \leq M_5(1-\hat{r}_i) \qquad \forall i \tag{C-1s}$$

This is now the mixed-integer quadratic program equivalent of the original optimization problem faced by the leader (prosumer).