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Grand Unified Theory with a Stable Proton*

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Abstract

We demonstrate that a phenomenologically viable four-dimensional grand unified theory with no proton decay can be constructed. This is done in the framework of the minimal non-supersymmetric SU(5) GUT by introducing new representations and separating the physical quark and lepton fields into different multiplets. In such a theory all beyond Standard Model particles are naturally heavy, but one can tune the parameters of the model such that gauge coupling unification is achieved and some of the new particles are at the TeV scale and accessible at the LHC.

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1 Introduction

The idea of grand unification was proposed shortly after the Standard Model (SM) of elementary particles was completely formulated based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ [2, 3, 4, 5, 6]. Grand unification postulates that the three gauge interactions of the SM – the electromagnetic, weak, and strong forces – are the manifestation of a single force at high energies. The first partially unified theory was the Pati-Salam model built on the gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ [7]. Subsequent proposals of complete grand unification were based on SU(5) [8] and SO(10) [9, 10].

Grand unified theories (GUTs) are the holy grail of particle physics, bringing orderliness to the otherwise unrelated particles and interactions of the SM. For the last 40 years it has been commonly believed that in any realistic four-dimensional (4D) GUT the proton cannot be stable. Increasingly stringent experimental bounds on the proton lifetime [11] severely constrained existing GUTs, often excluding their minimal realization [12, 13]. Thus, many have been led to consider instead theories without a single unifying gauge group, loosing the most appealing property of GUTs – complete unification.

We have shown by an explicit construction that 4D GUTs with a stable proton based on a single gauge group that are phenomenologically viable do in fact exist [1]. A discussion of this is presented below.

2 Minimal SU(5)

Since our model is based on the SU(5) gauge group, we first review briefly the key elements of the minimal SU(5) GUT – its particle content, Lagrangian, symmetry breaking pattern and proton decay channels.

2.1 Fermion sector

There are two fermion irreducible SU(5) representations (irreps) containing all SM matter fields of a given family. In terms of left-handed fields these are the 5^c and 10, where "c" denotes charge conjugation. The decomposition of those SU(5) multiplets into representations of the SM gauge group is (for simplicity, we consider only the first generation):

$$5^c = l \oplus d^c \ , \qquad 10 = e^c \oplus q \oplus u^c \ ,$$

where l and q are the SM left-handed lepton doublet and quark doublet, respectively, while e, d and u are the SM right-handed electron, down quark and up quark. The explicit decomposition including the $\mathrm{SU}(3)_c$ and $\mathrm{SU}(2)_L$ indices is provided in the appendix.

2.2 Higgs sector and symmetry breaking

The two scalar irreps in the minimal SU(5) model are:

$$\begin{aligned} 5_H &= H \oplus (3,1)_{-1/3} \ , \\ 24_H &= (1,1)_0 \oplus (1,3)_0 \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{5/6} \oplus (8,1)_0 \ . \end{aligned}$$

Assuming a $\mathcal{Z}_2$ symmetry of the Lagrangian under $24_H \to -24_H$, the part of the scalar potential involving just the adjoint 24_H takes the form

$$V(24_H) = -\tfrac{1}{2}\mu_{24}^2 \,\mathrm{Tr}\Big(24_H^2\Big) + \tfrac{1}{4}a_1\Big[\mathrm{Tr}\Big(24_H^2\Big)\Big]^2 + \tfrac{1}{4}a_2\,\mathrm{Tr}\Big(24_H^4\Big) \, .$$

The 24_H develops a vacuum expectation value (vev) at the GUT scale,

$$\langle\, 24_H \rangle = \tfrac{1}{\sqrt{30}}\, v_{24}\, \mathrm{diag}\,(2,2,2,-3,-3) \, ,$$

which spontaneously breaks the symmetry $\mathrm{SU}(5) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The fields $(3,2)_{-5/6}$ and $(\bar{3},2)_{5/6}$ are the would-be Goldstone bosons of the broken SU(5). The other fields in the 24_H obtain masses on the order of v_{24} and μ_{24}, thus they are all at the GUT scale.

The SM Higgs doublet in the 5_H develops the standard electroweak vev, which further breaks $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$. For the most general form of the scalar potential $V(5_H, 24_H)$ the doublet and triplet in 5_H generically have masses of the order of the GUT scale, and a tuning of parameters is required for the SM Higgs mass to be down at the electroweak scale. This is known as the doublet-triplet splitting problem.

2.3 Gauge bosons

In a theory based on SU(5) there are 24 gauge bosons, A_μ^a, where $a = 1, ..., 24$. Upon SU(5) breaking, those gauge bosons become the 8 gluons, 4 electroweak gauge bosons and the heavy vector gauge bosons $X_\mu = (3,2)_{-5/6}$ and $\overline{X}_\mu = (\bar{3},2)_{5/6}$ with mass

$$m_X = \sqrt{\tfrac{5}{6}}\, g\, v_{24} \ ,$$

where g is the SU(5) gauge coupling constant.

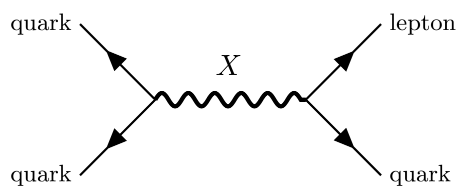

Figure 1: Proton decay mediated by the vector gauge boson $X_\mu = (3,2)_{-5/6}$.

2.4 Quark and lepton masses

The Yukawa sector of the minimal SU(5) is given by

$$\mathcal{L}_Y \;=\; y_5\; 5^c\, 10\; 5^*_H \;+\; y_{10}\, 10\; 10\; 5_H \;\supset\; y_5\, l\, H^* e^c + y_5\, q\, H^* d^c + y_{10}\, q\, H\, u^c$$

and results in the prediction $m_e = m_d$, $m_\mu = m_s$ and $m_\tau = m_b$ at the GUT scale. While the relation $m_\tau = m_b$, after running down to the low scale, is roughly consistent with experimental data, the relations $m_e = m_d$ and $m_\mu = m_s$ are not.

2.5 Proton decay

There are two sources of proton decay in the minimal SU(5) – interactions mediated by the vector gauge bosons X_μ and $\overline{X}_\mu$, and processes involving the color triplet scalar $T = (3,1)_{-1/3}$ from the 5_H.

The vector gauge boson interactions with quarks and leptons arise from the fermion kinetic terms in the Lagrangian,

$$\mathcal{L}_{\rm kin} = i\,{\rm Tr}\Big(\overline{5^c}\, \not{D}\, 5^c\Big) + i\,{\rm Tr}\Big(\overline{10}\, \not{D}\, 10\Big) \;\supset\; g\;\bar{l}\,\not{X}\,d^c + g\;\bar{q}\,\not{X}\,e^c + g\;\overline{u^c}\,\not{X}\,q \;+ {\rm h.c.}.$$

Those terms give rise to dimension-six operators mediating proton decay

$$\mathcal{L}^{(X)}_{\rm dim\,6} = \frac{g^2}{m_X^2}\,(\,\overline{u^c}\gamma_\mu q\,)\Big(\overline{e^c}\gamma^\mu q + \overline{d^c}\gamma^\mu l\Big) \;+\; {\rm h.c.}\;,$$

corresponding to the interaction shown in Fig. 1. The resulting proton decay rate is $\Gamma_p \sim \alpha^2 m_p^5/m_X^4$ and current experimental limits on proton lifetime [11] require

$$m_X \gtrsim 10^{16}\ {\rm GeV}\ .$$

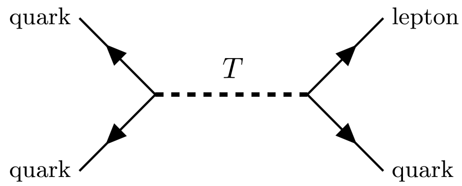

Figure 2: Proton decay mediated by the scalar $T = (3,1)_{-1/3}$.

The color triplet scalar interactions with quarks and leptons are described by the Yukawa terms

$$\mathcal{L}_Y \supset y_5 \; l \, T^* q + y_5 \; d^c \, T^* u^c + y_{10} \; q \, T \, q + y_{10} \, u^c \, T \, e^c \; + \text{ h.c.}$$

and produce the dimension-six operators

$$\mathcal{L}^{(T)}_{\dim 6} = \frac{y_5 \, y_{10}}{m_T^2} \Big[(q \, q) \, (q \, l) + (d^c \, u^c) \, (u^c \, e^c) \Big] \; + \text{ h.c.} \, ,$$

resulting in proton decay shown in Fig. 2. Because of the small Yukawa couplings, consistency with proton lifetime constraints leads to a less stringent bound on m_T than the one on m_X, requiring merely

$$m_T \gtrsim 10^{12} \text{ GeV} \; .$$

We will show now how introducing extra fermion and scalar irreps into the minimal SU(5) GUT can forbid all proton decay channels discussed above, and how to forbid proton decay at any order in perturbation theory.

3 SU(5) without proton decay

We explicitly construct a four-dimensional non-supersymmetric SU(5) GUT in which the proton is stable. The idea is to add new irreps into the minimal SU(5) model and arrange that the physical SM quarks and leptons fall into different multiplets. The new SU(5) irreps introduced are 40-plets and 50-plets, since in their $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ decomposition they contain fields with the quantum numbers of SM quarks, but not the leptons. This allows to rotate the SM quark fields out of the 5 and 10 irreps, such that the leptons still reside in the 5 and 10, but the quarks themselves live entirely in the 40's and 50's. This arrangement prevents the vector gauge bosons X_μ and $\overline{X}_\mu$ as well as the scalar T from connecting quarks to leptons.

3.1 Fermion sector

The new fermion irreps added to the minimal SU(5) model are two vector-like 40-plets and two vector-like 50-plets, so that the complete list of fermion irreps along with their $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ decomposition is [14]:

$$
\begin{aligned}
5^c &= l \oplus D_5^c \ , \\
10 &= e^c \oplus Q_{10} \oplus U_{10}^c \ , \\
40_i &= Q_{40_i} \oplus U_{40_i}^c \oplus (1,2)_{-3/2} \oplus (\bar{3},3)_{-2/3} \oplus (8,1)_1 \oplus (\bar{6},2)_{1/6} \ , \\
\overline{40}_i &= Q_{\overline{40}_i}^c \oplus U_{\overline{40}_i} \oplus (1,2)_{3/2} \oplus (3,3)_{2/3} \oplus (8,1)_{-1} \oplus (6,2)_{-1/6} \ , \\
50_i^c &= D_{50_i}^c \oplus (1,1)_2 \oplus (3,2)_{7/6} \oplus (6,3)_{1/3} \oplus (\bar{6},1)_{-4/3} \oplus (8,2)_{-1/2} \ , \\
\overline{50_i^c} &= D_{\overline{50}_i} \oplus (1,1)_{-2} \oplus (\bar{3},2)_{-7/6} \oplus (\bar{6},3)_{-1/3} \oplus (6,1)_{4/3} \oplus (8,2)_{1/2} \ ,
\end{aligned}
$$

where $i = 1, 2$. Note that D_5^c, Q_{10} and U_{10}^c are not the SM quark fields – they mix with the fields in the same SM representation residing in other SU(5) multiplets, and the SM quarks are their linear combinations. The full decomposition including $\mathrm{SU}(3)_c$ and $\mathrm{SU}(2)_L$ indices is given in the appendix.

3.2 Higgs sector and symmetry breaking

In the scalar sector, instead of the usual 5_H and 24_H, one introduces the irreps 24_H, 45_H and 75_H. Their decomposition into SM multiplets is:

$$
\begin{aligned}
24_H &= (1,1)_0 \oplus (1,3)_0 \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{5/6} \oplus (8,1)_0 \ , \\
45_H &= H \oplus (3,1)_{-1/3} \oplus (3,3)_{-1/3} \oplus (\bar{3},1)_{4/3} \oplus (\bar{3},2)_{-7/6} \oplus (\bar{6},1)_{-1/3} \\
&\quad \oplus (8,2)_{1/2} \ , \\
75_H &= (1,1)_0 \oplus (3,1)_{5/3} \oplus (\bar{3},1)_{-5/3} \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{5/6} \oplus (\bar{6},2)_{-5/6} \\
&\quad \oplus (6,2)_{5/6} \oplus (8,1)_0 \oplus (8,3)_0 \ .
\end{aligned}
$$

The irreps 24_H and 75_H acquire GUT-scale vevs, v_{24} and v_{75}, which break $\mathrm{SU}(5) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and, as explained below, provide GUT-scale masses to all beyond-SM fermions. The SM Higgs in the 45_H develops the standard electroweak vev, breaking the electroweak symmetry down to electromagnetism and resulting in SM quark and lepton masses.

The scalar potential of the theory, under the assumption of invariance under $24_H \to -24_H$ and $75_H \to -75_H$, is given by

$$
\begin{aligned}
\mathcal{L}_H \;=\; & -\tfrac{1}{2}\mu_{24}^2 \mathrm{Tr}(24_H^2) + \tfrac{1}{4}a_1\Big[\mathrm{Tr}(24_H^2)\Big]^2 + \tfrac{1}{4}a_2\mathrm{Tr}(24_H^4) \\
& -\tfrac{1}{2}\mu_{75}^2 \mathrm{Tr}(75_H^2) + \tfrac{1}{4}\sum b_k \mathrm{Tr}(75_H^4)_k + \tfrac{1}{2}\sum g_k \mathrm{Tr}(24_H^2 75_H^2)_k \\
& + M_{45}^2 \mathrm{Tr}\Big(|45_H|^2\Big) + \sum h_k \mathrm{Tr}\Big(24_H^2|45_H|^2\Big)_k + \ldots \,,
\end{aligned}
$$

where $k = 1, 2, 3$ correspond to contractions with the two lowest representations in a given trace combining into a singlet, 2-component tensor and 4-component tensor, respectively. The explicit index contractions are shown in the appendix.

There exists a large region of parameter space for which all components of the 24_H and 75_H have GUT-scale masses, apart from one linear combination of the $(3, 2)_{-5/6}$ fields (from the 24_H and 75_H) and one combination of the $(\bar{3}, 2)_{5/6}$ fields, both remaining massless, since those are the would-be Goldstone bosons of the broken SU(5) [15, 16, 17]. All components of the 45_H are naturally at the GUT scale and a tuning of parameters in the scalar potential is needed to reproduce the SM Higgs mass. This tuning is equivalent to the doublet-triplet splitting problem in the minimal SU(5) and perhaps can be avoided by introducing further SU(5) multiplets [18, 19].

3.3 Fermion mass terms

The Yukawa and pure mass terms in our model are:

$$
\begin{aligned}
\mathcal{L}_Y \;=\; & Y_l\, 5^c 10\, 45_H^* + Y_u^{ij} 40_i\, 40_j\, 45_H + Y_d^{ij} 40_i\, 50_j^c\, 45_H^* + M_{40}^{ij}\, \overline{40}_i\, 40_j \\
& + \lambda_1^{ij} 24_H \overline{40}_i\, 40_j + \lambda_2^{ij}\, \overline{40}_i\, 24_H 40_j + \lambda_3^{i}\, 24_H 10\, \overline{40}_i + \lambda_4^{ij}\, \overline{40}_i\, 75_H 40_j \\
& + \lambda_5^{i}\, 75_H 10\, \overline{40}_i + M_{50}^{ij}\, 50_i^c\, \overline{50_j^c} + \lambda_6^{ij}\, 50_i^c\, 24_H \overline{50_j^c} + \lambda_7^{ij}\, 50_i^c\, 75_H \overline{50_j^c} \\
& + \lambda_8^{i}\, 75_H 5^c\, \overline{50_i^c} + \mathrm{h.c.}\,,
\end{aligned}
$$

where $i, j = 1, 2$ and the coefficients of the only other allowed contractions $10\; 40_i\, 45_H$ are tuned to zero. We will now show that there exists a region of parameter space for which all new fermions have masses at the GUT scale, and at the same time all masses of the SM particles can be recovered.

Focusing on the fields with the quantum numbers of the SM down quark, after SU(5) breaking the relevant mass terms are

$$\mathcal{L}_{\text{mass}} = \begin{pmatrix} D_{\overline{50}_1} & D_{\overline{50}_2} \end{pmatrix} \mathcal{M}_D \begin{pmatrix} D_5^c \\ D_{50_1}^c \\ D_{50_2}^c \end{pmatrix} .$$

Performing a biunitary transformation to the mass eigenstate basis, $\mathcal{M}_D^{\text{diag}} = (R_D)_{2\times 2}\, \mathcal{M}_D\, (L_D)^\dagger_{3\times 3}$, the mass eigenstates are

$$\begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \end{pmatrix} = L_D \begin{pmatrix} D_5^c \\ D_{50_1}^c \\ D_{50_2}^c \end{pmatrix} .$$

In order to rotate the SM down quark out of the 5^c irrep, it is sufficient for the mass eigenstate D_1^c not to contain any admixture of D_5^c. This is accomplished by imposing the condition

$$\det\left(M_{50}^{ij} + \tfrac{1}{3\sqrt{30}} \lambda_6^{ij} v_{24} + \tfrac{1}{3\sqrt{2}} \lambda_7^{ij} v_{75} \right) = 0 \ .$$

This tuning of parameters guarantees that the SM down quark field d^c resides only in the 50_1^c and 50_2^c irreps, i.e.,

$$d^c \equiv D_1^c = L_D^{12} D_{50_1}^c + L_D^{13} D_{50_2}^c \ ,$$

where the coefficients L_D^{12} and L_D^{13} are functions of the Lagrangian parameters. This ensures that d^c does not get its mass from SU(5) breaking. An explicit calculation reveals that for the above choice of parameters all other fields in the 50_1^c and 50_2^c have GUT-scale masses.

The same strategy can be applied to the SM quark doublet and the up quark. The physical q and u^c are rotated out of the 10 irrep and end up as linear combinations of the corresponding fields from the 40_1 and 40_2 irreps. Again, it can be shown that all other fields in the 40_1 and 40_2 develop masses at the GUT scale.

Ultimately, the SM quark and lepton masses originate entirely from electroweak symmetry breaking through the Lagrangian terms

$$\begin{aligned} \mathcal{L}_Y \ &\supset \ Y_l\, 5^c 10\, 45_H^* + Y_u^{ij}\, 40_i\, 40_j\, 45_H + Y_d^{ij}\, 40_i\, 50_j^c\, 45_H^* + \text{h.c.} \\ &\supset \ y_l\, l\, H^* e^c + y_u\, q\, H\, u^c + y_d\, q\, H^* d^c + \text{h.c.} \ . \end{aligned}$$

Contrary to the minimal SU(5) scenario, there is no problematic relation between the electron and down quark masses.

3.4 Proton stability

3.4.1 Tree level

The most dangerous proton decay operators in the standard SU(5) GUT arise from fermion kinetic terms, as discussed earlier, and involve the vector gauge bosons $X_\mu = (3,2)_{-5/6}$ and $\overline{X}_\mu = (\bar{3},2)_{5/6}$. In our model, the corresponding Lagrangian terms are

$$\mathcal{L}_{\text{kin}} = i \sum_R \text{Tr}\left(\overline{R}\, \not{D}\, R\right),$$

with $R = 5^c$, 10, 40_i, $\overline{40}_i$, 50^c_i and $\overline{50^c_i}$. However, since the SM leptons live in the 5 and 10 irreps, whereas the SM quarks live in the 40 and 50 irreps, in our model there are no vertices connecting X_μ or $\overline{X}_\mu$ to a quark and a lepton. This immediately implies that there is no tree-level proton decay through a vector gauge boson exchange.

It is also straightforward to check that our model is free from tree-level proton decay mediated by scalars. For the same reasons as above, the terms

$$\mathcal{L}_Y \supset Y_l\, 5^c 10\, 45^*_H + Y^{ij}_u\, 40_i\, 40_j\, 45_H + Y^{ij}_d\, 40_i\, 50^c_j\, 45^*_H + \text{h.c.}$$

do not result in any vertices connecting the color triplet scalar $T = (3,1)_{-1/3}$ or any other scalar from the 45 irrep to a quark and a lepton. This completes the proof that there is no tree-level proton decay in our model.

3.4.2 Loop level

To investigate proton decay at higher orders in perturbation theory, it is no longer possible to do this on a case by case basis, and a symmetry argument is needed. It turns out that our model does exhibit such a partial discrete symmetry – all Lagrangian terms, apart from $\lambda^i_3 24_H 10\, \overline{40}_i$, $\lambda^i_5 75_H 10\, \overline{40}_i$ and $\lambda^i_8 75_H 5^c\, \overline{50^c_i}$, are invariant upon substituting

$$5^c \to -5^c\,, \qquad 10 \to -10\,.$$

Under this transformation the SM leptons are odd, since they live in the 5 and 10 irreps, whereas the quarks are even, since they reside in other irreps. In proton decay the initial state involves no leptons and no heavy states, so it is even under this transformation, whereas the final state consists of an odd number of leptons and no heavy states, so it is odd. This implies that proton decay is forbidden at any loop order as long as the fields from the 24_H and

75_H are not involved. One cannot set $\lambda_3^i = \lambda_5^i = \lambda_8^i = 0$ to remove the terms not invariant under $5^c \to -5^c,\ 10 \to -10$, since then it would be impossible to rotate the SM quarks out of the 5 and 10 irreps. To forbid the remaining proton decay channels we assume that SU(5) breaking is non-linearly realized [20]. The components of 24_H and 75_H decouple and at the Lagrangian level they are replaced by non-dynamical condensates. The scalar sector of the theory is then described by a nonlinear sigma model [21, 22].

Let us note that an alternative recent proposal [23] uses the same argument to remove the 24_H fields from the spectrum of the minimal SU(5) GUT. That model, however, achieves proton stability by imposing specific gauge conditions that eliminate all beyond-SM fields from the theory, making it indistinguishable from the SM. The only other attempts to construct 4D GUT models based on a single gauge group without proton decay we are aware of [24, 25, 26, 27, 28, 29, 30, 31] are either experimentally excluded by now due to the presence of new light particles with SM charges or suffer from tree-level proton decay mediated by scalars that cannot be removed by invoking non-linear symmetry breaking.

4 Conclusions

We have constructed a four-dimensional grand unified theory based on SU(5) that does not suffer from proton decay at any order in perturbation theory. The idea is to separate the physical quark and lepton fields into different representations of the gauge group. The absence of proton decay at tree level is achieved by adding extra multiplets into the theory and imposing specific relations between the model parameters. Full proton stability requires nonlinear SU(5) breaking.

Another interesting feature of the model is the possibility of having full gauge coupling unification, despite the theory being non-supersymmetric. This can be realized by lowering the masses of some of the scalars in the 45_H to the TeV scale and adding one more scalar representation [32, 33, 34]. This provides the opportunity to test the model at the LHC.

Although our specific construction is based on SU(5), it is meant to serve only as a proof of concept that grand unified theories built on a single gauge group with a stable proton do exist. Perhaps a simpler and more attractive theory of this type can be constructed in the framework of the gauge group SO(10). We hope that our finding will revive the interest in grand unification and open the door to a new branch of model building.

Acknowledgments

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A SU(5) representations

Below we provide the SU(3) × SU(2) × U(1) decomposition of the SU(5) multiplets relevant for our model. The $\alpha, \beta, \gamma, \delta, \sigma$ are SU(3) indices and a, b, c, d are SU(2) indices:

$$5^c_\alpha = (D^c_5)_\alpha \ , \qquad 5^c_a = \epsilon_{ab} l^b \ ,$$

$$10^{\alpha\beta} = \tfrac{1}{\sqrt{2}}\epsilon^{\alpha\beta\gamma}(U^c_{10})_\gamma \ , \qquad 10^{\alpha a} = -\tfrac{1}{\sqrt{2}} Q^{\alpha a}_{10} \ , \qquad 10^{ab} = \tfrac{1}{\sqrt{2}}\epsilon^{ab} e^c \ ,$$

$$24^\alpha_\beta = [(8,1)_0]^\alpha_\beta + \tfrac{2}{\sqrt{30}}\delta^\alpha_\beta (1,1)_0 \ , \qquad 24^\alpha_a = \tfrac{1}{\sqrt{2}}[(3,2)_{-\frac{5}{6}}]^\alpha_a \ ,$$
$$24^a_b = [(1,3)_0]^a_b - \tfrac{3}{\sqrt{30}}\delta^a_b (1,1)_0 \ , \qquad \langle\, 24^A_B \,\rangle = \tfrac{1}{\sqrt{30}} v_{24} \left(2\delta^\alpha_\beta - 3\delta^a_b \right),$$

$$40^{\alpha\beta\gamma}_\delta = \tfrac{1}{3}\epsilon^{\alpha\beta\gamma}(U^c_{40})_\delta \ , \qquad 40^{\alpha\beta\gamma}_a = \tfrac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}[(1,2)_{-\frac{3}{2}}]_a \ ,$$
$$40^{\alpha\beta a}_\gamma = -\tfrac{1}{3}\delta^{[\alpha}_\gamma (Q_{40})^{\beta]a} + \tfrac{1}{\sqrt{6}}\epsilon^{\alpha\beta\delta}[(\bar{6},2)_{\frac{1}{6}}]^a_{\gamma\delta} \ ,$$
$$40^{\alpha\beta a}_b = -\tfrac{1}{6}\epsilon^{\alpha\beta\gamma}\delta^a_b (U^c_{40})_\gamma + \tfrac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}[(\bar{3},3)_{-\frac{2}{3}}]^a_{b\gamma} \ ,$$
$$40^{\alpha ab}_\beta = \tfrac{1}{\sqrt{6}}\epsilon^{ab}[(8,1)_1]^\alpha_\beta \ , \qquad 40^{\alpha ab}_c = \tfrac{1}{3}\epsilon^{ab}(Q_{40})^\alpha_c \ ,$$

$$45^{\alpha\beta}_\gamma = \tfrac{1}{\sqrt{2}}\epsilon^{\alpha\beta\delta}[(\bar{6},1)_{-\frac{1}{3}}]_{\delta\gamma} + \tfrac{1}{\sqrt{2}}\delta^{[\alpha}_\gamma [(3,1)_{-\frac{1}{3}}]^{\beta]} \ ,$$
$$45^{\alpha\beta}_a = \tfrac{1}{\sqrt{2}}\epsilon^{\alpha\beta\gamma}[(\bar{3},2)_{-\frac{7}{6}}]_{\gamma a} \ , \qquad 45^{\alpha a}_\beta = \tfrac{1}{\sqrt{2}}[(8,2)_{\frac{1}{2}}]^{\alpha a}_\beta + \tfrac{1}{2\sqrt{6}}\delta^\alpha_\beta H^a \ ,$$
$$45^{a\alpha}_b = \tfrac{1}{\sqrt{2}}[(3,3)_{-\frac{1}{3}}]^{a\alpha}_b - \tfrac{1}{2\sqrt{2}}\delta^a_b [(3,1)_{-\frac{1}{3}}]^\alpha \ ,$$
$$45^{ab}_\alpha = \tfrac{1}{\sqrt{2}}\epsilon^{ab}[(\bar{3},1)_{\frac{4}{3}}]_\alpha \ , \qquad 45^{ab}_c = -\tfrac{3}{\sqrt{6}}\delta^{[a}_c H^{b]} \ ,$$

$$50^{\alpha\beta\gamma}_{\delta\sigma} = \tfrac{1}{3}\delta^{[\alpha}_{\delta}\delta^{\beta}_{\sigma}D^{\gamma]}_{50}\,,\quad 50^{\alpha\beta a}_{\gamma\delta} = \tfrac{2}{\sqrt{6}}\delta^{[\alpha}_{[\gamma}[(8,2)_{\frac{1}{2}}]^{\beta]a}_{\delta]},\quad 50^{\alpha\beta\gamma}_{ab} = \tfrac{1}{2\sqrt{3}}\epsilon^{\alpha\beta\gamma}\epsilon_{ab}(1,1)_{-2}\,,$$
$$50^{\alpha\beta\gamma}_{\delta a} = \tfrac{1}{2\sqrt{6}}\Big[\delta^{[\gamma}_{\delta}\epsilon^{\alpha\beta]\sigma}[(\bar{3},2)_{-\frac{7}{6}}]_{\sigma a}\,,\quad 50^{\alpha\beta a}_{\gamma b} = \tfrac{1}{2\sqrt{3}}\epsilon^{\alpha\beta\delta}[(\bar{6},3)_{-\frac{1}{3}}]^{a}_{\gamma\delta b} + \tfrac{1}{6}\delta^{a}_{b}\delta^{[\alpha}_{\gamma}D^{\beta]}_{50}\,,$$
$$50^{\alpha\beta a}_{bc} = -\tfrac{1}{\sqrt{6}}\epsilon^{\alpha\beta\sigma}\delta^{a}_{[b}[(\bar{3},2)_{-\frac{7}{6}}]_{c]\sigma}\,,\quad 50^{\alpha ab}_{\beta\gamma} = \tfrac{1}{2\sqrt{3}}\epsilon^{ab}\epsilon_{\beta\gamma\delta}[(6,1)_{\frac{4}{3}}]^{\alpha\delta}\,,$$
$$50^{ab\alpha}_{c\beta} = \tfrac{1}{\sqrt{6}}\delta^{[a}_{c}[(8,2)_{\frac{1}{2}}]^{b]\alpha}_{\beta}\,,\qquad 50^{ab\alpha}_{cd} = \tfrac{1}{3}\delta^{[a}_{c}\delta^{b]}_{d}D^{\alpha}_{50}\,,$$

$$75^{\alpha\beta}_{\gamma\delta} = -\tfrac{1}{\sqrt{3}}\delta^{[\alpha}_{\gamma}[(8,1)_0]^{\beta]}_{\delta} + \tfrac{1}{3\sqrt{2}}\delta^{[\alpha}_{\gamma}\delta^{\beta]}_{\delta}(1,1)_0\,,$$
$$75^{\alpha\beta}_{\gamma a} = \tfrac{1}{2\sqrt{2}}[(\bar{6},2)_{-\frac{5}{6}}]^{\alpha\beta}_{\gamma a} - \tfrac{1}{\sqrt{6}}\delta^{[\alpha}_{\gamma}[(3,2)_{-\frac{5}{6}}]^{\beta]}_{a}\,,\quad 75^{\alpha\beta}_{ab} = \tfrac{1}{2}\epsilon^{\alpha\beta\gamma}\epsilon_{ab}[(\bar{3},1)_{-\frac{5}{3}}]_{\gamma}\,,$$
$$75^{\alpha a}_{\beta b} = \tfrac{1}{4}[(8,3)_0]^{\alpha a}_{\beta b} + \tfrac{1}{2\sqrt{3}}\delta^{a}_{b}[(8,1)_0]^{\alpha}_{\beta} - \tfrac{1}{6\sqrt{2}}\delta^{\alpha}_{\beta}\delta^{a}_{b}(1,1)_0\,,$$
$$75^{a\alpha}_{bc} = \tfrac{2}{\sqrt{6}}\delta^{a}_{[b}[(3,2)_{-\frac{5}{6}}]^{\alpha}_{c]}\,,\qquad 75^{ab}_{cd} = \tfrac{1}{\sqrt{2}}\delta^{[a}_{c}\delta^{b]}_{d}(1,1)_0\,,$$
$$\langle\, 75^{AB}_{CD}\,\rangle = \tfrac{1}{3\sqrt{2}}v_{75}\Big(\delta^{[\alpha}_{\gamma}\delta^{\beta]}_{\delta} + 3\delta^{[a}_{c}\delta^{b]}_{d} - \tfrac{1}{2}\delta^{\alpha}_{\gamma}\delta^{b}_{d} + \tfrac{1}{2}\delta^{\alpha}_{\delta}\delta^{b}_{c} + \tfrac{1}{2}\delta^{\beta}_{\gamma}\delta^{a}_{d} - \tfrac{1}{2}\delta^{\beta}_{\delta}\delta^{a}_{c}\Big)\,.$$

B Scalar potential

Upon writing the indices out explicitly, the scalar potential takes the following form (subscript H was dropped for clarity),

$$\begin{aligned}\mathcal{L}_H \;=\; & -\tfrac{1}{2}\mu^2_{24}24^i_j24^j_i + \tfrac{1}{4}a_1(24^i_j24^j_i)^2 + \tfrac{1}{4}a_2 24^i_j24^j_k24^k_l24^l_i - \tfrac{1}{2}\mu^2_{75}75^{ij}_{kl}75^{kl}_{ij}\\ & + \tfrac{1}{4}b_1(75^{ij}_{kl}75^{kl}_{ij})^2 + \tfrac{1}{4}b_2 75^{ij}_{pq}75^{kl}_{ij}75^{mn}_{kl}75^{pq}_{mn} + \tfrac{1}{4}b_3 75^{ij}_{kl}75^{kl}_{in}75^{mn}_{pq}75^{pq}_{mk}\\ & + \tfrac{1}{4}c_1(24^i_j24^j_i)(75^{kl}_{mn}75^{mn}_{kl}) + \tfrac{1}{4}c_2\,24^i_j24^j_k75^{kl}_{mn}75^{mn}_{li}\\ & + \tfrac{1}{4}c_3\,24^i_j75^{jk}_{il}24^m_n75^{nl}_{mk} + M^2_{45}\,\overline{45}^i_{jk}45^{jk}_i + d_1(24^i_j24^j_i)(\overline{45}^k_{lm}45^{lm}_k)\\ & + d_2\,24^i_j24^j_k\,\overline{45}^k_{lm}45^{lm}_i + d_3\,24^i_j\,\overline{45}^j_{ik}24^l_m45^{mk}_l + \sum e_i\,\mathrm{Tr}(75^2 45^2)_i\,.\end{aligned}$$

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