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Multiple Parton Scattering in Nuclei: Heavy Quark Energy Loss and Modified Fragmentation Functions

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Multiple scattering, induced radiative energy loss and modified fragmentation functions of a heavy quark in nuclear matter are studied within the framework of generalized factorization in perturbative QCD. Modified heavy quark fragmentation functions and energy loss are derived in detail with illustration of the mass dependencies of the Landau-Pomeranchuk-Migdal interference effects and heavy quark energy loss. Due to the quark mass dependence of the gluon formation time, the nuclear size dependencies of nuclear modification of the heavy quark fragmentation function and heavy quark energy loss are found to change from a linear to a quadratic form when the initial energy and momentum scale are increased relative to the quark mass. The radiative energy loss of the heavy quark is also significantly suppressed due to limited cone of gluon radiation imposed by the mass. Medium modification of the heavy quark fragmentation functions is found to be limited to the large z region due to the form of heavy quark fragmentation functions in vacuum.

I. INTRODUCTION

In ultra-relativistic heavy-ion collision, energetic partons from initial hard processes have to propagate through the produced dense medium and therefore suffer multiple scattering and lose a significant amount of energy. Such energy loss or jet quenching has been proposed as a good probe of the hot and dense medium formed in high-energy nuclear collisions [1,2]. It in effect suppresses the final leading hadron distribution from the propagating parton giving rise to modified fragmentation functions and the final hadron spectra [3,4]. Recent theoretical studies [5–9] all show that the effective parton energy loss is proportional to the gluon density of the medium. Therefore measurements of the parton energy loss will enable one to extract the initial gluon density of the produced hot medium. Strong suppression of high transverse momentum hadron spectra is indeed observed by experiments [10–13] at the Relativistic Heavy-Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL). The suppression pattern agrees very well with the jet quenching mechanism [14,15], indicating large parton energy loss in a medium with large initial gluon density. Comparing to jet quenching as measured in deeply inelastic scattering off nuclei, the initial gluon density in central $Au + Au$ collisions at $\sqrt{s} = 200$ GeV is about 30 times higher than that in a cold Au nuclei [16]. Such a high initial gluon density is unprecedented and is a strong indication of the formation of quark gluon plasma.

The extraction of the initial gluon density from jet quenching pattern as measured at RHIC relies on the assumption that it is caused by parton multiple scattering and induced radiation. While such an assumption is based on a solid physical picture and is supported by a multitude of experimental

data [17], it is still important to have additional and independent study of the consequences of parton energy loss. Quenching of heavy quark spectra has been proposed as a special probe because of the unique mass dependence of the energy loss and medium modification of the fragmentation functions [18–21]. In this paper, we will apply the framework of twist expansion that was developed for the study of medium modification of parton fragmentation functions to heavy quarks.

Multiple parton scattering inside nuclei in generally is a higher twist process that involves multiple parton correlations. By dimensional counting, such higher twist processes are power-suppressed in terms of the momentum scale Q^2 involved. Therefore, one can have a systematic expansion of the cross section in $1/Q^2$. Since the probability of multiple scattering increases with the nuclear size, the leading higher twist contribution should be enhanced by $A^{1/3}$. Furthermore, multiple parton correlations involve only the intrinsic properties of the nuclei. They should be independent of the hard processes involved. This twist expansion is referred to as generalized factorization [22]. Such framework has been applied to semi-inclusive DIS on nuclear targets to study nuclear modification of light (massless) quark fragmentation functions and effective parton energy loss [23]. Because of the Landau-Pomeranchuk-Migdal interference [24], the phase space available for the induced gluon radiation is limited that is also proportional to $A^{1/3}$. The final twist-four contribution to the modified fragmentation functions is then proportional to $A^{2/3}/Q^2$ [23]. Such a quadratic dependence on nuclear size is indeed observed in semi-inclusive deep inelastic lepton-nucleus experiments [16,25].

In this paper, we will extend the study of modified fragmentation functions in nuclei [23,26] to heavy quarks when they propagate through nuclear matter. We will derive the modified heavy quark fragmentation functions and the effective energy loss. To demonstrate the effect of quark mass, we will compare the results with the ones for light quarks. One of the most important effect is the reduction of gluon formation time when it is radiated from a slow heavy quark whose virtuality is not much larger than its mass. Such a reduction will effectively eliminate LPM effect and the nuclear size dependence of the modification will become linear in contrast to the case of a light quark. The second mass effect is the significant reduction of induced quark energy loss due to limited gluon radiation angle imposed by the mass. With detailed data analysis of experimental data both in the single electron channel [27] and direct measurement of heavy mesons [28], one could learn more about the parton energy loss mechanism in dense matter.

The results of heavy quark energy loss in the present study were already reported in Ref. [20]. In this work, we will elaborate the detailed derivation and focus on numerical calculations and discussions about modified heavy quark fragmentation functions. The paper is organized as follows. In the next section we will present the theoretical formalism of our calculation including the generalized factorization of twist-4 processes. In Section III we will show in detail the calculation of different contributions to the modified heavy quark fragmentation function and energy loss due to multiple scattering. In Section IV we will numerically evaluate and discuss the modified fragmentation functions of a heavy quark propagating in nuclei. In Section V, discussion and numerical calculation of heavy quark energy loss will be given. We will demonstrate the mass effects by discussing how the dependence on medium size changes from a linear to a quadratic dependence when the energy of the heavy quark and the momentum scale is increased, and the suppression of the energy loss for the heavy quark relative to a light quark due to "dead-cone" effect [18]. Section VI will summarize

our work.

II. GENERALIZED FACTORIZATION

In order to separate the complication of heavy quark production and propagation, we consider a simple process of charm quark production via the charge-current interaction in DIS off a large nucleus. The results can be easily extended to heavy quark propagation in a hot medium. The differential cross section for the semi-inclusive process $\ell(L_1) + A(p) \longrightarrow \nu_\ell(L_2) + H(\ell_H) + X$ can be expressed as

$$E_{L_2} E_{\ell_H} \frac{d\sigma_{\text{DIS}}}{d^3 L_2 d^3 \ell_H} = \frac{G_{\text{F}}^2}{(4\pi)^3 s} L_{\mu\nu}^{cc} E_{\ell_H} \frac{dW^{\mu\nu}}{d^3 \ell_H} \ . \tag{1}$$

Here L_1 and L_2 are the four momenta of the incoming lepton and the outgoing neutrino, ℓ_H the observed heavy meson momentum, $p = [p^+, m_N^2/2p^+, \mathbf{0}_\perp]$ is the momentum per nucleon in the nucleus, m_N is the mass of nucleon and $s = (p + L_1)^2$. G_{F} is the four-fermion coupling constant, and $q = L_2 - L_1 = [-Q^2/2q^-, q^-, \mathbf{0}_\perp]$ the momentum transfer via the exchange of a vector boson $B(q)$. The charge-current leptonic tensor is given by $L_{\mu\nu}^{cc} = 1/2\text{Tr}(\rlap{/}{L}_1 \gamma_\mu(1 - \gamma_5)\rlap{/}{L}_2(1 + \gamma_5)\gamma_\nu)$. We assume $Q^2 \ll M_W^2$. The semi-inclusive hadronic tensor is defined as,

$$E_{\ell_H} \frac{dW_{\mu\nu}}{d^3 \ell_H} = \frac{1}{2} \sum_X \langle A|J_\mu^+|X, H\rangle\langle X, H|J_\nu^{+\dagger}|A\rangle 2\pi\delta^4(q + p - p_X - \ell_H) \tag{2}$$

where $\sum_X$ runs over all possible final states and $J_\mu^+ = \sum_f \bar{\psi}_f \gamma_\mu V \psi_f$ is the hadronic charged current. Here we define $V = (1 - \gamma_5)V_{ij}$ and V_{ij} represents the CKM flavor mixing matrix [29]. We want to clarify that the symbol Q in this paper stands for both the heavy quark flavor and the momentum scale of the exchanged vector boson.

We consider the process of DIS in which $W^\pm$ collides with a light quark q with momentum $k = xp$ producing a heavy quark Q with mass M and momentum ℓ_Q. The heavy quark then fragments into a heavy quark flavored hadron. In this paper, we will take the charm quark as an example and the cases for other heavy quarks will be straightforward. In order to investigate the energy spectrum of charm fragmentation we define the Lorentz-invariants $z = \ell_Q^-/q^-, z_H = \ell_H^-/\ell_c^-$. The leading-twist and lowest order perturbative QCD (pQCD) calculation of heavy quark production gives

$$\frac{dW_{\mu\nu}^{S(0)}}{dz_H} = \sum_q \int dx f_q^A(x) H_{\mu\nu}^{(0)}(x, p, q, M) D_{Q \to H}(z_H) \ , \tag{3}$$

where $f_q^A(x)$ is the quark distribution and $D_{Q \to H}(z_H)$ is the non-perturbative heavy quark fragmentation function in vacuum [30–33]. The hard partonic part is

$$H_{\mu\nu}^{(0)}(k, q, M) = \frac{e_q^2}{2} \text{Tr}(x\rlap{/}{p}\gamma_\mu V(\rlap{/}{q} + x\rlap{/}{p})V^\dagger \gamma_\nu) \frac{2\pi}{2p \cdot q} \delta(x - x_B - x_M) \ , \tag{4}$$

$$x_M = \frac{M^2}{2p^+ q^-} \ , \quad x_B = \frac{Q^2}{2p^+ q^-} \ . \tag{5}$$

In the case when the momentum scale Q is much larger than the heavy quark mass, large logarithms such as $\log(Q^2/M^2)$ arise to all orders of the perturbative expansion, so the fix-order perturbation theory breaks down and a perturbative resummation of large quasi-collinear logs, $\log(Q^2/M^2)$,

should be performed [31,32], which will give the corresponding QCD evolution equations for parton distribution functions and heavy quark fragmentation functions. After considering higher order contributions, the inclusive tensor can be written as [31,34–37]

$$\frac{dW_{\mu\nu}^{S}}{dz_{H}} = \sum_{q} \int dx f_{q}^{A}(x,\mu^{2}) H_{\mu\nu}^{(0)}(x,p,q,M) D_{Q\to H}(z_{H},\mu^{2}),\tag{6}$$

where $D_{Q\to H}(z_{H},\mu^{2})$ satisfies the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [38] QCD evolution equations.

In a nuclear medium, the propagating heavy quark in DIS will experience additional scattering with other partons from the nucleus. The rescattering may induce additional gluon radiation and cause the heavy quark to lose energy. Such induced gluon radiation will effectively give rise to additional terms in the evolution equation leading to the modification of the heavy quark fragmentation functions in a medium. These are higher-twist corrections since they involve higher-twist parton matrix elements and are power-suppressed. We will consider those contributions that involve two-parton correlations from two different nucleons inside the nucleus. They are proportional to the size of the nucleus [39] and thus are enhanced by a nuclear factor $A^{1/3}$ as compared to two-parton correlations in a nucleon. As in previous studies [23], we will neglect those contributions that are not enhanced by the nuclear medium.

FIG. 1. A typical diagram for quark-gluon re-scattering processes with three possible cuts, central(C), left(L) and right(R).

For the production of heavy quarks there are usually two kinds of mechanisms: intrinsic and extrinsic (via gluon fusion) heavy quark production [40]. Since we are only interested in rescattering and induced gluon radiation of the heavy quark after it is produced in DIS we will only consider a simple case of intrinsic heavy quark production via charged-current weak interaction. In this case, after small modification we can still employ the generalized factorization of multiple scattering processes [22]. In this formalism with collinear approximation, the double scattering contribution to radiative correction from processes like the one illustrated in Fig. 1 can be written in the following form,

$$\frac{dW_{\mu\nu}^{D}}{dz_{H}} = \sum_{q} \int_{z_{H}}^{1} \frac{dz}{z} D_{Q\to H}(z_{H}/z) \int \frac{dy^{-}}{2\pi} dy_{1}^{-} dy_{2}^{-} \frac{d^{2}y_{T}}{(2\pi)^{2}} d^{2}k_{T} \overline{H}_{\mu\nu}^{D}(y^{-},y_{1}^{-},y_{2}^{-},k_{T},p,q,M,z)$$

$$\times e^{-i\vec{k}_{T}\cdot\vec{y}_{T}} \frac{1}{2} \langle A|\bar{\psi}_{q}(0)\,\gamma^{+}\,A^{+}(y_{2}^{-},0_{T})\,A^{+}(y_{1}^{-},y_{T})\,\psi_{q}(y^{-})|A\rangle \ . \tag{7}$$

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Here $\overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, M, z)$ is the Fourier transform of the partonic hard part $\widetilde{H}_{\mu\nu}(x, x_1, x_2, k_T, p, q, M, z)$ in momentum space,

$$\overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, M, z) = \int dx \frac{dx_1}{2\pi} \frac{dx_2}{2\pi} e^{ix_1 p^+ y^- + ix_2 p^+ y_1^- + i(x - x_1 - x_2)p^+ y_2^-}$$
$$\times \widetilde{H}^D_{\mu\nu}(x, x_1, x_2, k_T, p, q, M, z), \tag{8}$$

where $k = [xp^+, 0, \vec{0}_\perp]$, $k_1 = [x_1 p^+, 0, \vec{0}_\perp]$, $k_2 = [x_2 p^+, 0, \vec{k}_T]$, $k_3 = [x_3 p^+, 0, \vec{k}_T]$, and k_T is the relative transverse momentum carried by the second parton in the double scattering. We assume that k_T is small and therefore can make collinear expansion of the hard partonic cross section with respect to the transverse momentum of the initial partons. The first term in the collinear expansion gives the eikonal contribution to the leading-twist results, making the matrix element in the single scattering process gauge invariant, while the second (or linear) term vanishes for unpolarized initial and final states after integration over k_T. The leading term in the collinear expansion that contributes to the double scattering process comes from the quadratic term in the collinear expansion,

$$\frac{dW^D_{\mu\nu}}{dz_H} = \sum_q \int_{z_H}^1 \frac{dz}{z} D_{Q \to H}(z_H/z) \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \frac{1}{2} \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma{}^+(y_2^-) F^{+\sigma}(y_1^-) \psi_q(y^-) | A \rangle$$
$$\times \left(-\frac{1}{2} g^{\alpha\beta} \right) \left[\frac{1}{2} \frac{\partial^2}{\partial k_T^\alpha \partial k_T^\beta} \overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, M, z) \right]_{k_T = 0}. \tag{9}$$

There are many diagrams involving double parton scattering. The hard part of the partonic scattering for each diagram, $\widetilde{H}_{\mu\nu}(x, x_1, x_2, k_T, p, q, M, z)$, always contains two δ-functions from the on-shell conditions of the two cut-propagators. These δ-functions, together with the contour integrations which contain different sets of poles in the un-cut propagators, will determine the values of the momentum fractions x, x_1, and x_2 [23]. The phase factors in $\overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, M, z)$ [Eq. (8)] can then be factored out, which will be combined with the partonic fields in Eq. (9) to form twist-four partonic matrix elements or two-parton correlations. The double scattering corrections in Eq. (9) can then be factorized into the product of fragmentation functions, twist-four partonic matrix elements and the partonic hard scattering cross section.

III. DOUBLE SCATTERING AND INDUCED GLUON RADIATION OFF A HEAVY QUARK

According to the generalized factorization theorem in Eq. (9), we should calculate the hard part of parton multiple scattering. We will assume an axial gauge $n \cdot A = 0$ with $n = [1, 0^-, \vec{0}_\perp]$. The hard part of quark-gluon double scattering has a total of 23 cut diagrams as illustrated in Figs. 1-21.

We take the central cut diagram in Fig. 1 as an example to show how to calculate the hard part. With the conventional Feynman rule in the Standard Model one can write down the hard partonic part of the central cut-diagram of Fig. 1,

$$\overline{H}^D_{C\,\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, z) = \int dx \frac{dx_1}{2\pi} \frac{dx_2}{2\pi} e^{ix_1 p^+ y^- + ix_2 p^+ y_1^- + i(x - x_1 - x_2)p^+ y_2^-} \int \frac{d^4\ell}{(2\pi)^4}$$
$$\times \frac{p^{+2}}{2} \mathrm{Tr}\left[x\not{p}\gamma_\mu V n^\sigma n^\rho \widehat{H}_{\sigma\rho} V^\dagger \gamma_\nu \right] 2\pi\delta_+(\ell^2) \delta(1 - z - \frac{\ell^-}{q^-}). \tag{10}$$

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$$\widehat{H}_{\sigma\rho} = \frac{C_F}{2N_c}g^4 \frac{\gamma\cdot(q+k_1)+M}{(q+k_1)^2 - M^2 - i\epsilon}\gamma_\alpha \frac{\gamma\cdot(q+k_1-\ell)+M}{(q+k_1-\ell)^2-M^2-i\epsilon}\gamma_\sigma(\gamma\cdot\ell_Q + M)\gamma_\rho$$
$$\times\, \varepsilon^{\alpha\beta}(\ell)\frac{\gamma\cdot(q+k-\ell)+M}{(q+k-\ell)^2-M^2+i\epsilon}\gamma_\beta\frac{\gamma\cdot(q+k)+M}{(q+k)^2-M^2+i\epsilon}\, 2\pi\delta_+(\ell_Q^2 - M^2)\,, \tag{11}$$

where $k^2 = k_1^2 = 0$, $p^2 = m_N^2$, $\ell_Q^2 = M^2$, m_N and M are the nucleon mass and the produced heavy quark mass, respectively. In addition, $\varepsilon^{\alpha\beta}(\ell)$ is the polarization tensor of a gluon propagator in the axial gauge and ℓ, $\ell_Q = q + k_1 + k_2 - \ell$ are the 4-momenta carried by the gluon and the final heavy quark, respectively. $z = \ell_Q^-/q^-$ is the fraction of longitudinal momentum (the large minus component) carried by the final heavy quark after gluon radiation.

In order to simplify the calculation of the trace part and extract the leading contribution in the limit $\ell_T \to 0$ and $k_T \to 0$, we also apply the collinear approximation to complete the trace of the product of γ-matrices as

$$n^\sigma \widehat{H}_{\sigma\rho}n^\rho \approx \frac{\gamma\cdot\ell_Q + M}{4\ell_Q^-}\mathrm{Tr}\left[\gamma^- n^\sigma \widehat{H}_{\sigma\rho}n^\rho\right]\,. \tag{12}$$

Therefore we have

$$\frac{1}{2}\mathrm{Tr}\left[x\slashed{p}\gamma_\mu V n^\sigma n^\rho \widehat{H}_{\sigma\rho}V^\dagger\gamma_\nu\right]$$
$$\approx \frac{1}{2}\mathrm{Tr}\left[x\slashed{p}\gamma_\mu V(\gamma\cdot\ell_Q + M)V^\dagger\gamma_\nu\right]\frac{1}{4\ell_Q^-}\mathrm{Tr}\left[\gamma^- n^\sigma \widehat{H}_{\sigma\rho}n^\rho\right]\,. \tag{13}$$

After carrying out momentum integration in x, x_1, x_2 and $\ell^\pm$ with the help of contour integration and δ-functions, the partonic hard part can be factorized into the product of the matrix $H_{\mu\nu}^{(0)}(k,q)$ of vector boson and quark scattering, and the quark-gluon rescattering part $\overline{H}^D$,

$$\overline{H}_{\mu\nu}^D(y^-, y_1^-, y_2^-, k_T, p, q, M, z) = \int dx H_{\mu\nu}^{(0)}(k, q, M)\,\overline{H}^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z)\,, \tag{14}$$

where $H_{\mu\nu}^{(0)}(k, q, M)$ is defined in Eq. (4). Contributions from all the diagrams have this factorized from. Therefore, we need only list the rescattering part $\overline{H}^D$ for different diagrams in the following.

We also define the momentum fractions

$$x_L = \frac{\ell_T^2}{2p^+q^- z(1-z)}\,,\quad x_D = \frac{k_T^2 - 2\vec{k}_T\cdot\vec{\ell}_T}{2p^+q^- z}\,, \tag{15}$$

as in the light quark case. Then, for the central-cut diagram in Fig. 1 we have [23],

$$\overline{H}_{1,C}^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z) = \int d\ell_T^2\, \frac{(1+z^2)\ell_T^2 + (1-z)^4 M^2}{(1-z)(\ell_T^2 + (1-z)^2 M^2)^2}$$
$$\times\, \frac{\alpha_s}{2\pi}C_F\frac{2\pi\alpha_s}{N_c}\overline{I}_{1,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z)\,, \tag{16}$$

$$\overline{I}_{1,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + ix_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y^- - y_1^-)$$
$$\times\left[1 - e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-}\right]\left[1 - e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_1^-)}\right]\,. \tag{17}$$

The above contribution resembles the cross section of dipole scattering and contains essentially four terms. The first diagonal term corresponds to the so-called hard-soft process where the gluon

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radiation is induced by the hard scattering between the vector boson B and an initial quark with momentum k. The quark is knocked off-shell by the B boson and becomes on-shell again after radiating a gluon. Afterwards the on-shell quark (or the radiated gluon) will have a secondary scattering with another soft gluon from the nucleus. The second diagonal term is due to the so-called double hard process where the quark is on-shell after the first hard scattering with the vector boson. The gluon radiation is then induced by the scattering of the quark with another gluon that carries finite momentum fraction $x_L+(1-z)x_M/z+x_D$. The two off-diagonal terms are interferences between hard-soft and double hard processes. The cancellation between the two diagonal and off-diagonal terms essentially gives rise to the LPM interference for the induced gluon radiation. From Eq. (A2), we get the formation time for gluon radiation from a heavy quark,

$$\tau_f^Q \equiv \frac{1}{(x_L + (1-z)x_M/z)p^+} \, . \tag{18}$$

In the limit of collinear radiation ($x_L \to 0$) or when the formation time of the gluon radiation, τ_f^Q, is much larger than the nuclear size, the two processes (soft-hard and double hard) have destructive interference, leading to the LPM interference effect [24]. It is interesting to note that the formation time of gluon radiation off a heavy quark τ_f^Q is shorter than that off a light quark [23]

$$\tau_f^q \equiv \frac{1}{x_L p^+} \, . \tag{19}$$

This is simply because the formation time for gluon radiation is only relative to the propagation of the quark. The velocity of low energy heavy quarks is much smaller than that of a light quark. The corresponding gluon formation time is also smaller. One should then expect the LPM interference effect to be significantly reduced for intermediate energy heavy quarks. It can be shown that this phenomenon will also modify the dependence of the heavy quark energy loss on the nuclear size, which will be discussed in detail in Section V.

In addition to the central-cut diagram, we also should take into account the asymmetrical cut-diagrams in Fig. 1, which represent interference between single and triple scattering. We note that the trace part is the same as in the central-cut diagram. The difference is just the phase factor. Thus we have from these cut diagrams,

$$\begin{aligned}
\overline{I}_{1,L}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) &= -e^{i(x+x_L)p^+y^- + ix_D p^+(y_1^- - y_2^-)}\theta(y_1^- - y_2^-)\theta(y^- - y_1^-) \\
&\quad \times (1 - e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)}) \, , \tag{20}
\end{aligned}$$

$$\begin{aligned}
\overline{I}_{1,R}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) &= -e^{i(x+x_L)p^+y^- + ix_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y_2^- - y_1^-) \\
&\quad \times (1 - e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}) \, . \tag{21}
\end{aligned}$$

With the same procedure we can obtain the contributions of all other cut diagrams, which are listed in Appendix A.

IV. MODIFIED HEAVY QUARK FRAGMENTATION FUNCTION

To calculate the leading twist-four contribution to the semi-inclusive cross section according to the generalized factorization formula in Eq. (7), one has to expand the hard partonic cross section

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in k_T. We can rearrange different contributions according to the θ-functions in $\overline{H}^D_{C,R,L}$ (the sum of all the contributions from central-cut, right-cut or left-cut diagrams) and define

$$\overline{H}^D = \int d\ell_T^2 \frac{\alpha_s}{2\pi} e^{i(x+x_L)p^+y^- + ix_D p^+(y_1^- - y_2^-)} \frac{2\pi\alpha_s}{N_c}$$
$$\times \left[H_C^D \theta(-y_2^-)\theta(y^- - y_1^-) - H_R^D \theta(-y_2^-)\theta(y_2^- - y_1^-) - H_L^D \theta(y^- - y_1^-)\theta(y_1^- - y_2^-) \right] . \quad (22)$$

From Eqs. (A1)-(A36), we can check that

$$\overline{H}^D_C(k_T = 0) = \overline{H}^D_R(k_T = 0) = \overline{H}^D_L(k_T = 0) = C_F \frac{(1+z^2)\ell_T^2 + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2]^2} . \quad (23)$$

Since one can reorganize the θ-functions as

$$\int dy_1^- dy_2^- \left[\theta(-y_2^-)\theta(y_2^- - y_1^-) + \theta(y^- - y_1^-)\theta(y_1^- - y_2^-) - \theta(-y_2^-)\theta(y^- - y_1^-) \right]$$
$$= \int_0^{y^-} dy_1^- \int_0^{y_1^-} dy_2^- , \quad (24)$$

one finds that $\overline{H}^D(k_T = 0)$ in Eq. (7) just gives the eikonal contribution to the next-leading-order correction of single scattering which can be gauged away since it does not correspond to any physical double scattering.

The leading contributions to the quark-gluon rescattering result from the quadratic term in the k_T expansion of $\overline{H}^D$,

$$\nabla_{k_T}^2 H^D_{C(L,R)}|_{k_T=0} = 4C_A \frac{1+z^2}{1-z} \frac{\ell_T^4}{[\ell_T^2 + (1-z)^2 M^2]^4} \widetilde{H^D}_{C(L,R)} + \mathcal{O}(x_B/Q^2\ell_T^2) , \quad (25)$$

$$\widetilde{H_C^D} = c_1(z,\ell_T^2,M^2)(1 - e^{-i(x_L+(1-z)x_M/z)p^+y_2^-})(1 - e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)})$$
$$+ c_2(z,\ell_T^2,M^2) \left[e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}(1 - e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)}) \right.$$
$$\left. + e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)}(1 - e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}) \right]$$
$$+ c_3(z,\ell_T^2,M^2) e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)} e^{-i(x_L+(1-z)x_M/z)p^+y_2^-} \quad (26)$$

$$\widetilde{H_C^L} = c_4(z,\ell_T^2,M^2)(e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)} - e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_2^-)})$$
$$+ c_5(z,\ell_T^2,M^2)(1 - e^{-i(x_L+(1-z)x_M/z)p^+(y^- - y_1^-)}) \quad (27)$$

$$\widetilde{H_C^R} = c_4(z,\ell_T^2,M^2)(e^{-i(x_L+(1-z)x_M/z)p^+y_2^-} - e^{-i(x_L+(1-z)x_M/z)p^+y_1^-})$$
$$+ c_5(z,\ell_T^2,M^2)(1 - e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}), \quad (28)$$

where the coefficient $c_i(z,\ell_T^2,M^2), i = 1,2,3,4,5$ are polynomial functions of M^2/ℓ_T^2,

$$c_1(z,\ell_T^2,M^2) = 1 + \frac{(1-z)^2(z^2 - 6z + 1)}{1+z^2} \frac{M^2}{\ell_T^2} + \frac{2z(1-z)^4}{1+z^2} \frac{M^4}{\ell_T^4} , \quad (29)$$

$$c_2(z,\ell_T^2,M^2) = \frac{(1-z)}{2} \left\{ 1 - \left[\frac{(1-z)(2z^3 - 5z + 8z - 1)}{(1+z^2)} + \frac{2C_F}{C_A}(1-z)^3 \right] \frac{M^2}{\ell_T^2} \right.$$
$$\left. - \left[\frac{z(1-z)^4(3z-1)}{(1+z^2)} + \frac{2C_F}{C_A} \frac{(1-z)^7}{(1+z^2)} \right] \frac{M^4}{\ell_T^4} \right\} , \quad (30)$$

$$c_3(z,\ell_T^2,M^2) = \frac{C_F(1-z)^2}{C_A} \left[1 - \frac{8z(1-z)^2}{1+z^2} \frac{M^2}{\ell_T^2} - \frac{(1-z)^4(z^2 - 4z + 1)}{1+z^2} \frac{M^4}{\ell_T^4} \right] , \quad (31)$$

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$$c_4(z, \ell_T^2, M^2) = (1-z)^2 \frac{M^2}{\ell_T^2} \left[1 + \frac{(1-z)^4}{1+z^2} M^2/\ell_T^2 \right] \left[\frac{C_F}{C_A} (1-z)^2 + 2z - 1 \right] , \tag{32}$$

$$c_5(z, \ell_T^2, M^2) = (1-z)^2 \frac{M^2}{\ell_T^2} \left[1 + \frac{(1-z)^4}{1+z^2} \frac{M^2}{\ell_T^2} \right] . \tag{33}$$

In the limit of $M^2 = 0$, one can recover from Eqs. (25)-(28) the results for light quark multiple scattering with complete calculation beyond helicity amplitude approximation [26].

Compared with the results of gluon radiation from the light quark multiple scattering [23,26] which have an overall form $1/\ell_T^4$, the radiative gluon spectrum from a heavy quark in Eq. (25) is suppressed by a factor

$$f_{Q/q} = \left[\frac{\ell_T^2}{\ell_T^2 + (1-z)^2 M^2} \right]^4 = \left[1 + \frac{\theta_0^2}{\theta^2} \right]^{-4} , \tag{34}$$

where $\theta_0 = M/q^-$ and $\theta = \ell_T/l^-$ is the angle of the radiated gluon relative to the heavy quark. One can see that the mass of the heavy quark provides a lower bound for the radiation angle of the collinear gluon which dominates the gluon spectrum in the case of radiation off a light quark. This effectively suppresses the gluon radiation at angle smaller than the ratio of the quark mass M to its energy q^-, and thus reduces radiative energy loss of a heavy quark. Such a suppression of small angle gluons is often referred to as the "dead cone" effect [18]. In the result of our current calculation, additional mass effects on the final gluon spectrum are in the mass dependence of coefficient functions $c_i(z, \ell_T^2, M^2)$, $i = 1, 2, 3, 4, 5$ and most importantly the mass dependence of the gluon formation time [Eq. (18)] which will dictate the LPM interference pattern in induced gluon bremsstrahlung off a heavy quark. These additional mass effects will significantly influence the final radiative gluon spectra and modified the so-called "dead cone" effect. In fact, the coefficient functions $c_i(z, \ell_T^2, M^2)$ contain terms like $\frac{M^4}{\ell_T^4}$. Multiplied with the factor in Eq. (34), they give finite contribution to the gluon spectra at $\ell_T = 0$. This in effect will fill up the small angle cone along the direction of the propagating heavy quark with soft gluons, as also pointed out in Ref. [21]. The net "dead cone" effect still results in the significant reduction of the heavy quark energy loss, as numerical results shown in Fig. 9 and Fig. 10 of Section V below.

Substituting Eqs. (25)-(27) in Eq. (9), we have the leading higher-twist contribution to the semi-inclusive tensor of heavy quark fragmentation in DIS off a nucleus,

$$\frac{W_{\mu\nu}^D}{dz_H} = \sum \int dx H_{\mu\nu}^{(0)} \int_{z_H}^1 \frac{dz}{z} D_{Q \to H}(\frac{z_H}{z}) \frac{C_A \alpha_s}{2\pi} \frac{1+z^2}{1-z} \int \frac{d\ell_T^2}{[\ell_T^2 + (1-z)^2 M^2]^4} \ell_T^4$$

$$\times \frac{2\pi\alpha_s}{N_c} T_{qg}^{A,Q}(x, x_L, M^2) + (g - \text{fragmentation}) + (\text{virtual corrections}) , \tag{35}$$

where

$$T_{qg}^{A,Q}(x, x_L, M^2) \equiv T_{qg}^{A,C}(x, x_L, M^2) + T_{qg}^{A,L}(x, x_L, M^2) + T_{qg}^{A,R}(x, x_L, M^2) , \tag{36}$$

$$T_{qg}^{A,C}(x, x_L, M^2) = \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \widetilde{H_C^D} \frac{1}{2} \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma{}^+(y_2^-) F^{+\sigma}(y_1^-) \psi_q(y^-) | A \rangle$$

$$\times e^{i(x+x_L)p^+ y^-} \theta(-y_2^-) \theta(y^- - y_1^-) , \tag{37}$$

$$T_{qg}^{A,L}(x, x_L, M^2) = \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \widetilde{H_L^D} \frac{1}{2} \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma{}^+(y_2^-) F^{+\sigma}(y_1^-) \psi_q(y^-) | A \rangle$$

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$$\times \, e^{i(x+x_L)p^+y^-}\theta(y^- - y_1^-)\theta(y_1^- - y_2^-) \ , \tag{38}$$

$$T_{qg}^{A,R}(x, x_L, M^2) = \int \frac{dy^-}{2\pi}\, dy_1^-\, dy_2^-\, \widetilde{H_R^D}\, \frac{1}{2}\langle A|\bar{\psi}_q(0)\,\gamma^+\, F_{\sigma}{}^+(y_2^-)\, F^{+\sigma}(y_1^-)\,\psi_q(y^-)|A\rangle$$

$$\times \, e^{i(x+x_L)p^+y^-}\theta(-y_2^-)\theta(y_2^- - y_1^-) \ , \tag{39}$$

are twist-four quark-gluon correction functions inside the nucleus with $\widetilde{H_C^D}$, $\widetilde{H_L^D}$ and $\widetilde{H_R^D}$ given in Eqs. (26)-(28). They are all independent with each other because they involve different θ functions. The twist-four parton matrices $T_{qg}^{A,L}$ and $T_{qg}^{A,R}$ are new, which purely result from the mass effect of the heavy quark and involve left and right cut diagrams. These new parton matrices will vanish when we take $M^2 = 0$ [See the definitions in Eqs. (26-28)]. Furthermore, they are proportional to coefficients $c_4(z, \ell_T^2, M^2)$ and $c_5(z, \ell_T^2, M^2)$ which in turn contain an additional factor of $(1-z)^2$. In the case of soft gluon radiation $z \to 1$, these matrix elements are suppressed compared to the matrix elements involved in the central cut diagrams.

During the collinear expansion, we have kept ℓ_T finite and took the limit $k_T \to 0$. As a consequence, the gluon field in one of the twist-four parton matrix elements in Eqs. (37)-(39) carries zero momentum in the soft-hard process. As argued in Ref. [23], this is due to the omission of higher order terms in the collinear expansion. As a remedy to the problem, a subset of the higher-twist terms in the collinear expansion can be resummed to restore the phase factors such as $\exp(ix_T p^+ y^-)$, where $x_T \equiv \langle k_T^2\rangle/2p^+ q^- z$ is related to the intrinsic transverse momentum of the initial partons. Therefore, soft gluon fields in the parton matrix elements will carry a fractional momentum x_T.

Until now we have only considered quark-gluon double scattering in a nucleus. To make a complete calculation we should also take into account the processes of quark-quark scattering. However, it has been shown [23] that the contributions of quark-quark scattering are suppressed by a factor $1/Q^2$ as compared with quark-gluon double scattering. In the heavy quark case, it further involves intrinsic heavy quark distribution inside the nucleus which should be very small as compared to light quark and gluon distributions. Thus, we can completely neglect the contributions of quark-quark scattering for heavy quark propagation.

The virtual corrections in Eq. (35) can be obtained via unitarity requirement similarly as in Ref. [23]. Including these virtual corrections and the single scattering contribution, we can rewrite the semi-inclusive tensor in terms of a modified fragmentation function $\widetilde{D}_{Q\to H}(z_H, \mu^2)$,

$$\frac{dW_{\mu\nu}}{dz_H} = \sum_q \int dx \widetilde{f}_q^A(x, \mu_I^2) H_{\mu\nu}^{(0)}(x, p, q, M)\widetilde{D}_{Q\to H}(z_H, \mu^2) \tag{40}$$

where $\widetilde{f}_q^A(x, \mu^2)$ is the quark distribution function which in principle should also include the higher-twist contribution [41] of the initial state scattering. The modified effective heavy quark fragmentation function is defined as

$$\widetilde{D}_{Q\to H}(z_H, \mu^2) \equiv D_{Q\to H}(z_H, \mu^2)$$

$$+ \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2 + (1-z)^2 M^2} \frac{\alpha_s}{2\pi} \int_{z_H}^1 \frac{dz}{z} \Delta\gamma_{q\to qg}(z, x, x_L, \ell_T^2, M^2) D_{Q\to H}(z_H/z)$$

$$+ \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2 + z^2 M^2} \frac{\alpha_s}{2\pi} \int_{z_H}^1 \frac{dz}{z} \Delta\gamma_{q\to gq}(z, x, x_L, \ell_T^2, M^2) D_{g\to H}(z_H/z) \, , \tag{41}$$

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where $D_{Q \to H}(z_H, \mu^2)$ and $D_{g \to H}(z_H, \mu^2)$ are the leading-twist fragmentation functions of a heavy quark in vacuum. The modified splitting functions are given as

$$\Delta\gamma_{q \to qg}(z, x, x_L, \ell_T^2, M^2) = \left[\frac{1+z^2}{(1-z)_+} T_{qg}^{A,Q}(x, x_L, M^2) + \delta(1-z)\Delta T_{qg}^{A,Q}(x, \ell_T^2, M^2)\right]$$
$$\times \frac{2\pi C_A \alpha_s \ell_T^4}{[\ell_T^2 + (1-z)^2 M^2]^3 N_c \widetilde{f}_q^A(x, \mu_I^2)}, \tag{42}$$

$$\Delta\gamma_{q \to gq}(z, x, x_L, \ell_T^2, M^2) = \Delta\gamma_{q \to qg}(1-z, x, x_L, \ell_T^2, M^2), \tag{43}$$

$$\Delta T_{qg}^{A,Q}(x, \ell_T^2, M^2) \equiv \int_0^1 dz \frac{1}{1-z} \left[2T_{qg}^{A,Q}(x, x_L, M^2)|_{z=1} - (1+z^2)T_{qg}^{A,Q}(x, x_L, M^2)\right]. \tag{44}$$

The above equations are similar to the case of double scattering of a light quark [23,26] except that the splitting functions for gluon radiation of heavy quark are quite different from the one in the light quark case.

To numerically calculate the modified heavy quark fragmentation function and study the energy loss of a heavy quark we have to estimate the twist-four parton matrices $T_{qg}^{A,Q}(x, x_L, M^2)$, which are in principle not calculable and can only be measured independently in experiments similarly as parton distribution functions. Nevertheless, with some hypotheses they can be factorized. Here we will apply the approximation adopted in Refs. [22,23,39],

$$\int \frac{dy^-}{2\pi} dy_1^- dy_2^- e^{ix_1 p^+ y^- + ix_2 p^+ (y_1^- - y_2^-)} \langle A|\bar{\psi}_q(0)\frac{\gamma^+}{2} F_\sigma{}^+(y_2^-) F^{+\sigma}(y_1^-)\psi_q(y^-)|A\rangle \theta(-y_2^-)\theta(y^- - y_1^-)$$
$$\approx \frac{C}{x_A} f_q^A(x_1) x_2 f_g^N(x_2), \tag{45}$$

$$\int \frac{dy^-}{2\pi} dy_1^- dy_2^- e^{ix_1 p^+ y^- + ix_2 p^+ (y_1^- - y_2^-) \pm i(x_L + (1-z)x_M/z)p^+ y_2^-}$$
$$\times \langle A|\bar{\psi}_q(0)\frac{\gamma^+}{2} F_\sigma{}^+(y_2^-) F^{+\sigma}(y_1^-)\psi_q(y^-)|A\rangle \theta(-y_2^-)\theta(y^- - y_1^-)$$
$$\approx \frac{C}{x_A} f_q^A(x_1) x_2 f_g^N(x_2) e^{-(x_L + (1-z)x_M/z)^2/x_A^2}, \tag{46}$$

where $x_A = 1/m_N R_A$, $f_q^A(x)$ is the quark distribution inside a nucleus, $f_g^N(x)$ is the gluon distribution inside a nucleon and C is assumed to be a constant, reflecting the strength of two-parton correlation inside a nucleus.

In soft radiation approximation, $z \to 1$, the parton matrix elements $T_{qg}^{A,L(R)}(x, x_L, M^2)$ from left and right cut diagrams are suppressed and thus can be neglected in our following numerical calculation. According to Eqs. (45) and (46), we have

$$T_{qg}^{A,C}(x, x_L, M^2) \approx \frac{c_1(z, \ell_T^2, M^2) C}{x_A}(1 - e^{-(x_L + (1-z)x_M/z)^2/x_A^2})\left[f_q^A(x + x_L) x_T f_g^N(x_T)\right.$$
$$+ f_q^A(x - (1-z)x_M/z)(x_L + x_T + (1-z)x_M/z)f_g^N(x_L + x_T + (1-z)x_M/z)\right]$$
$$+ \frac{c_2(z, \ell_T^2, M^2) C}{2x_A}\left\{e^{-(x_L + (1-z)x_M/z)^2/x_A^2}\left[f_q^A(x + x_L) x_T f_g^N(x_T)\right.\right.$$
$$+ f_q^A(x - (1-z)x_M/z)(x_L + x_T + (1-z)x_M/z)f_g^N(x_L + x_T + (1-z)x_M/z)\right]$$
$$- 2f_q^A(x - (1-z)x_M/z)(x_L + x_T + (1-z)x_M/z)f_g^N(x_L + x_T + (1-z)x_M/z)\right\}$$
$$+ \frac{c_3(z, \ell_T^2, M^2) C}{x_A}f_q^A(x - (1-z)x_M/z)$$
$$\times (x_L + x_T + (1-z)x_M/z)f_g^N(x_L + x_T + (1-z)x_M/z). \tag{47}$$

To further simplify the calculation, we assume $x_M(1-z)/z, x_L \ll x_T \ll x$. The modified parton matrix elements can be approximated by

$$
\begin{aligned}
& T_{qg}^{A,C}(x, x_L, M^2) \\
& \approx \frac{\widetilde{C}}{x_A} f_q^A(x) \left[(1 - e^{-(x_L+(1-z)x_M/z)^2/x_A^2}) a_1(z, \ell_T^2, M^2) + a_2(z, \ell_T^2, M^2) \right],
\end{aligned}
\tag{48}
$$

where $\widetilde{C} \equiv 2C x_T f_g^N(x_T)$ is a coefficient which should in principle depends on Q^2 and x_T. Here we will simply take it as a constant. In order to simplify notations we have defined

$$
a_1(z, \ell_T^2, M^2) = c_1(z, \ell_T^2, M^2) - c_2(z, \ell_T^2, M^2) ,
\tag{49}
$$

$$
a_2(z, \ell_T^2, M^2) = c_3(z, \ell_T^2, M^2)/2 .
\tag{50}
$$

Under these approximations, the only parameter in our calculation is $\widetilde{C}$ which is also the only parameter that enters into the modified fragmentation functions for light quarks [23]. In the study of experimental data on modified light quark fragmentation in DIS off nuclear targets, a value of this parameter $\widetilde{C} \simeq 0.0060$ was found [16] to describe the data very well. Such a value is also consistent with that extracted from the study of transverse momentum broadening of Drell-Yan processes in $p + A$ collisions [42]. In our following numerical calculations of nuclear modification of heavy quark fragmentation functions we will take the same value.

According to Eq. (41), both heavy quark and gluon fragmentation functions contribute to the modified heavy quark fragmentation function. In order to include the contribution of gluon fragmentation, we have to consider the fragmentation function for heavy quarks in the next-to-leading order pQCD calculation. Here we follow the ansatz in Ref. [35] and express the overall fragmentation function of a parton i into the hadron H as [34,35]:

$$
D_{i \to H}(z_H, \mu) = \int_{z_H}^{1} \frac{dz}{z} D_i^Q(z, \mu) D_{Q \to H}(z_H/z),
\tag{51}
$$

where $D_i^Q(z, \mu)$ is the perturbative fragmentation function (PFF) for a massless parton to fragment into a massive heavy quark Q within pQCD cascade. The perturbative fragmentation function (PFF) satisfies the normal DGLAP QCD evolution equations [38] and $D_{Q \to H}(z_H/z)$ is a non-perturbative fragmentation function, describing the transition from the heavy quark to the heavy meson, $e.g.$, the non-perturbative charm quark fragmentation function into D meson in Eq. (55).

From the next-to-leading order pQCD calculations [31], we can extract the initial conditions of PFF's for the heavy quark at a scale μ_0 of the order of the heavy quark mass M as

$$
D_Q^Q(z, \mu_0) = \delta(1-z) + \frac{\alpha_s(\mu_0)C_F}{2\pi} \left[\frac{1+z^2}{1-z} \left(\log \frac{\mu_0^2}{M^2} - 2\log(1-z) - 1 \right) \right]_+,
\tag{52}
$$

$$
D_g^Q(z, \mu_0) = \frac{\alpha_s(\mu_0)C_A}{2\pi} [z^2 + (1-z)^2] \log \frac{\mu_0^2}{M^2},
\tag{53}
$$

$$
D_{q,\bar{q},\bar{Q}}^Q(z, \mu_0) = 0 .
\tag{54}
$$

With these initial conditions for PFF's and the DGLAP evolution equations we can obtain the PFF functions evolved up to any scale $\mu > \mu_0$. After convoluting PFF with the non-perturbative fragmentation function $D_{Q \to H}(z_H/z)$ in Eq. (51) we can get the fragmentation function $D_{i \to H}(z_H, \mu)$

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in vacuum, which can be applied to calculate the heavy quark production cross section within QCD factorization formula such as Eq. (6) [35,40].

The non-perturbative fragmentation function for the charm quark to fragment into D meson in vacuum can be parameterized in Peterson type functional form [30] as

$$D_{c \to D}(z) = \frac{N}{z[1 - z^{-1} - \varepsilon_c/(1-z)]}, \tag{55}$$

where N normalizes $D_c(z)$ to $\int dz D_c(z) = 1$. The parameter ε_c is related to the heavy quark mass (M_c for charm quark) by $\varepsilon_c = \Lambda^2/M_c^2$ and Λ stands for a hadronic scale.

The PFF at scale $\mu > \mu_0$ can be given by solving the DGLAP equations, which is complicated in numerical calculations and currently there is no parametrization forms available as for the light quark fragmentation function [36]. To simplify the numerical calculations, we choose $\mu_0 = Q$ as the first step for numerical calculations and then obtain $D_c^D(z, Q)$ and $D_g^D(z, Q)$ according to Eq. (51). This approximation is similar to the Approximation Case B used in Ref. [37]. Shown in Fig. 2 as the solid line is the charm quark fragmentation function into D meson at $Q^2 = 10$ GeV^2 after including higher order pQCD corrections. These vacuum heavy quark fragmentation functions will be used as input in our numerical computation of modified charm quark fragmentation function in Eq. (41). We note that in the limit of vanishing quark masses the massless parton model expression should be recovered in principle.

FIG. 2. Charm quark fragmentation function into D meson in vacuum (solid line) and inside a nucleus (dashed line). For the nuclear modification, $x_B = 0.08$ and $x_A = 0.05$ are used.

Shown in Fig. 2 as the dashed line is the modified fragmentation function of a charm quark into the D meson. The value $x_A = 0.05$ corresponds to a nucleus with a radius $R_A = 4.25$ fm. We have taken the charm quark mass $M = 1.5$ GeV. One can see that the modification due to the double scattering in a nucleus for heavy quarks is quite different from light quarks [16,23]. This is mainly caused by the form of heavy quark fragmentation functions in vacuum which peak at large z. Because of the multiple scattering and induced gluon radiation, the position of the peak of the modified fragmentation function is effectively shifted to a smaller value of z. As a consequence, the heavy quark fragmentation function remains unchanged, or even slightly enhanced for a large range

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of fractional momentum z as shown in Fig. (3) by the ratio of modified fragmentation function to the vacuum fragmentation function. The modification only becomes significant and the fragmentation function is suppressed at large z above the position of the peak. This is in sharp contrast to the case of modified light quark fragmentation functions which are suppressed relative to the vacuum form in a very large range of z. Note that the heavy quark fragmentation function is strongly enhanced at very small z, similarly to the case of light quark fragmentation, due to heavy quark pair production from the radiated gluons induced by multiple scattering inside nuclei. However, this enhancement is limited to much smaller z than for light quarks fragmenting into light hadrons.

FIG. 3. Modification factor for the charm quark fragmentation function in a nucleus.

V. HEAVY QUARK ENERGY LOSS

Another advantage of studying multiple scattering of heavy quark in medium is that one can actually measure the heavy quark energy loss by flavor tagging, since the leading quark will remain the heavy flavor which is very unlikely to be absorbed by the medium. Similarly to the study of light quark energy loss, we define the heavy quark energy loss as the energy fraction carried by the induced gluon,

$$
\begin{aligned}
\langle \Delta z_g^Q \rangle (x_B, \mu^2) &= \int_0^{\mu^2} d\ell_T^2 \int_0^1 dz \frac{\alpha_s}{2\pi} (1-z) \frac{\Delta \gamma_{q \to qg}(z, x_B, x_L, \ell_T^2)}{\ell_T^2 + (1-z)^2 M^2} \\
&= \frac{C_A \alpha_s^2}{N_c} \int_0^{\mu^2} d\ell_T^2 \int_0^1 dz \frac{(1+z^2)\ell_T^4}{[\ell_T^2 + (1-z)^2 M^2]^4} \frac{T_{qg}^{A,C}(x_B, x_L, M^2)}{\widetilde{f}_q^A(x_B, \mu_I^2)} \,.
\end{aligned}
\tag{56}
$$

Substituting the approximate expression for the nuclear twist-four parton matrix in Eq. (48), we obtain,

$$
\begin{aligned}
\langle \Delta z_g^Q \rangle (x_B, \mu^2) &= \frac{\widetilde{C} C_A \alpha_s^2}{N_c \, x_A} \int_0^{Q^2} d\ell_T^2 \int_0^1 dz \frac{(1+z^2)\ell_T^4}{[\ell_T^2 + (1-z)^2 M^2]^4} \\
&\quad \times \left[(1 - e^{-\widetilde{x}_L^2/x_A^2}) a_1(z, \ell_T^2, M^2) + a_2(z, \ell_T^2, M^2) \right] \,.
\end{aligned}
\tag{57}
$$

Here we choose the factorization scale as $\mu^2 = Q^2$ and define $\widetilde{x}_L \equiv x_L + (1-z) x_M / z$. Note that the virtual correction in $\Delta \gamma_{q \to qq}$ does not contribute to the energy loss. Also, $\widetilde{x}_L / x_A = L_A^- / \tau_f$

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with $L_A^- = R_A m_N/p^+$ the nuclear size in the chosen frame. The second term proportional to a_2 corresponds to a finite contribution in the factorization limit. This term will survive in the limit of complete LPM cancellation when double scattering acts like a single scattering for induced gluon radiation. We have neglected such a term in the study of light quark propagation since it is proportional to R_A, as compared to the R_A^2 dependence from the first term due to the LPM effect. In this study we have to keep the second term for heavy quark propagation since the first term with the LPM interference effect will have a similar nuclear dependence when the heavy quark mass reduces the gluon formation time for low energy heavy quarks. However, for energetic heavy quark, the mass can become negligible and one should reach the limit of a light quark energy loss.

To elucidate the two different limits, we examine the phase factor in Eq. (57),

$$\frac{(x_L + (1-z)x_M/z)^2}{x_A^2} = \frac{x_B^2[\ell_T^2 + (1-z)^2 M^2]^2}{x_A^2 z^2 (1-z)^2 Q^4} \sim \frac{x_B^2 M^4}{x_A^2 Q^4}, \tag{58}$$

and define it as $T \equiv x_B^2 M^4/x_A^2 Q^4$, which should control the LPM interference effect, and therefore the behavior of the total heavy quark energy loss. There are two distinct limiting behaviors of the energy loss for different values of of x_B, Q^2 and x_A.

When $T \gg 1$ for $Q^2 \ll M^2$ or $x_B \gg x_A$, we have

$$1 - e^{-\widetilde{x}_L^2/x_A^2} \simeq 1,$$

which means there is no LPM interference [20] and we obtain

$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\widetilde{C}\alpha_s^2}{N_c} \frac{x_B}{x_A Q^2}. \tag{59}$$

Since $x_A = 1/m_N R_A$, the heavy quark energy loss in this case depends linearly on the nuclear size R_A as the Bethe-Heitler form in Abelian gauge interaction. Similar results were also derived in the generalized opacity expansion method [19].

In the opposite limit when $Q^2 \gg M^2$ or $x_B \ll x_A$, the quark mass becomes negligible and T will take a moderate value. The gluon formation time can be much larger than the nuclear size and therefore the LPM interference effect will dominate again. In this case, one can make a variable change $x_L \to \widetilde{x}_L$ with $\widetilde{x}_M = (1-z)x_M/z$ and $\widetilde{x}_\mu = \mu^2/2p^+q^- z(1-z) + \widetilde{x}_M$ in Eq. (57) and obtain

$$\langle \Delta z_g^H \rangle (x_B, \mu^2) = \frac{\widetilde{C} C_A \alpha_s^2 x_B}{N_c Q^2 x_A} \int_0^1 dz \frac{1+z^2}{z(1-z)} \int_{\widetilde{x}_M}^{\widetilde{x}_\mu} d\widetilde{x}_L \frac{(\widetilde{x}_L - \widetilde{x}_M)^2}{\widetilde{x}_L^4}$$
$$\times \left[(1 - e^{-\widetilde{x}_L^2/x_A^2}) a_1(z, \ell_T^2, M^2) + a_2(z, \ell_T^2, M^2) \right]. \tag{60}$$

This form is very similar to the one for the light quark energy loss [26]. Since $a_1(z, \ell_T^2, M^2)$ and $a_2(z, \ell_T^2, M^2)$ are dimensionless coefficients, and T have a moderate value, the exponential factor in the above equation coming from the LPM interference regularizes the integration over $\widetilde{x}_L$ and limits $\widetilde{x}_L < x_A$. We obtain $\int d\widetilde{x}_L/\widetilde{x}_L^2 \sim 1/x_A$. With similar argument in Ref. [23] we can conclude that the heavy quark energy loss is proportional to

$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\widetilde{C}\alpha_s^2}{N_c} \frac{x_B}{x_A^2 Q^2}. \tag{61}$$

Therefore, the heavy quark energy loss has a quadratic dependence on the nuclear size when $Q^2 \gg M^2$ or $x_B \ll x_A$. This is understandable because under such condition, the heavy quark becomes again relativistic and should behavior like a light quark in terms of induced energy loss.

To present our numerical calculations of heavy quark energy loss, we rescale the heavy quark energy loss by $\widetilde{C}(Q^2)C_A\alpha_s^2(Q^2)/N_C$ and define

$$d = \langle \Delta z_g^H \rangle \frac{N_C}{\widetilde{C}(Q^2)C_A\alpha_s^2(Q^2)} \ . \tag{62}$$

The R_A dependence of the rescaled heavy quark energy loss is shown in Figs. 4-8, where the points are numerical results, the dashed lines are linear fit to the numerical results and solid curves are quadratic fit. They clearly show that the heavy (charm) quark energy loss has different nuclear size dependencies as Q^2 and x_B change. In Fig. 4, Fig. 5 and Fig. 6 we fix $Q^2 = 10$ GeV^2 and change the value of x_B to demonstrate the nuclear size R_A dependence of heavy quark energy loss for a charm quark ($M = 1.5$ GeV). One notes that when x_B is very small (large heavy quark energy), the charm quark energy loss depends quadratically on R_A (see Fig. 4). However, as we increase x_B (decrease heavy quark energy) a gradual transition from a quadratic dependence on R_A of the energy loss to a linear dependence (see Fig. 5) takes place. When x_B is very large the charm quark energy loss has an all most linear dependence on nuclear size R_A as Fig. 6 illustrates. Similarly, shown in Figs. 6, 7 and 8, we see that even for large values of x_B (where we fix $x_B = 0.15$) or small heavy quark energy, the same transition from linear nuclear size dependence to quadratic dependence of the heavy quark energy loss takes place as we increase Q^2.

FIG. 4. The R_A dependence of heavy quark energy loss for a charm quark.

FIG. 5. The R_A dependence of heavy quark energy loss for a charm quark.

FIG. 6. The R_A dependence of heavy quark energy loss for a charm quark.

FIG. 7. The R_A dependence of heavy quark energy loss for a charm quark.

FIG. 8. The R_A dependence of heavy quark energy loss for a charm quark.

Another mass effect on the induced gluon radiation is the "dead-cone" phenomenon [18] that suppresses the small angle gluon radiation. Since the size of the dead-cone $\theta_0 = M/q^-$ [Eq. (34)], within which the gluon radiation is suppressed, is inversely proportional to the quark's energy, the reduction of energy loss is stronger for a slower heavy quark. For a heavy quark with either a high energy q^- or virtuality Q^2, its radiative energy loss should approach that of a light quark. Since the dimensionless coefficients $a_1(z, \ell_T^2, M^2)$ and $a_2(z, \ell_T^2, M^2)$ in Eq. (60) also depend on the heavy quark mass, they will have addition mass effects on the heavy quark energy loss.

To illustrate the difference of energy loss between heavy quark and light quark and the quark mass effect we define a ratio R as:

$$R \equiv \frac{\langle \Delta z_g^Q \rangle(x_B, \mu^2)}{\langle \Delta z_g^q \rangle(x_B, \mu^2)} \ , \tag{63}$$

where $\langle \Delta z_g^q \rangle(x_B, \mu^2)$ is the light quark energy loss [26] which can be obtained by setting $M = 0$ in Eq. (57) and Eq. (60),

$$\langle \Delta z_g^q \rangle(x_B, \mu^2) = \frac{\widetilde{C}\alpha_s^2}{N_c x_A} \int_0^{Q^2} \frac{d\ell_T^2}{\ell_T^4} \int_0^1 dz\, (1 + z^2)(1 - e^{-x_L^2/x_A^2}) C_A [1 - \frac{1-z}{2}] . \tag{64}$$

In Fig. 9, we show the change of the ratio R with Q^2 for a charm quark ($M = 1.5$ GeV) propagating in a nucleus with $x_A = 0.04$ and $x_B = 0.1$. Please note that in the nuclear parton matrix elements, the fractional momentum in a nucleon is limited to $x_L < 1$ due to the momentum conservation. Even though the Fermi motion effect in a nucleus can allow $x_L > 1$, the parton distribution in this region is still significant suppressed. Thus it provides a natural cut-off for x_L in the numerical integration over z and ℓ_T in Eqs. (57) and (64).

FIG. 9. The Q^2 dependence of R for a charm quark.

We can observe that when Q^2 is not too large, the heavy quark mass effect significantly suppresses the energy loss caused by induced gluon radiation. When $M^2/Q^2 \to 0$, the effect of quark mass becomes negligible and $R \to 1$. This is consistent with the pQCD factorization theorem that when the momentum transfer is very large one can neglect the effect of quark mass.

FIG. 10. The x_B dependence of R for a charm quark.

Shown in Fig. 10 is the x_B (or heavy quark energy) dependence of the ratio between heavy quark and light quark energy loss for fixed $Q^2 = 10$ GeV^2. It is obvious that suppression of heavy quark energy loss due to the "dead-cone" effect of heavy quark mass is most significant when x_B is large (or quark energy is small). When x_B is very small (quark energy is large), the effect of quark mass is small and the quark energy loss approaches that of a light quark.

VI. SUMMARY

Utilizing the generalized factorization of twist-four processes we have studied the nuclear modification of heavy quark fragmentation functions and the energy loss of a heavy quark propagating

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through dense matter after it is produced via a hard process in DIS in the twist expansion approach. Taking into account of the multiple scattering suffered by the heavy quark we have derived the modified heavy quark fragmentation functions related to twist-four corrections with nuclear enhancement. We find that the formation time for gluons radiated from a heavy quark is smaller relative to that of a light quark, since it is always measured against the propagation time of the quark. With certain kinematics when the quark energy and virtuality is small, the gluon formation time can become much smaller than the nuclear size. In this case, the heavy quark energy loss or the nuclear modification of the heavy quark fragmentation functions have a linear nuclear size dependence. We have shown through both analytic deduction and numerical calculation that a gradual transition from a linear to a quadratic nuclear size dependence takes place when one increases the quark's energy or initial virtuality.

We also compared the energy loss of a heavy quark with that of a light quark and demonstrate that the quark mass effect, including the so-called "dead-cone" phenomenon, will significantly suppress the heavy quark energy loss when the momentum transfer is not too large. This heavy quark mass effect will decrease if the heavy quark energy, or the momentum scale Q^2 is much larger than the quark mass. When $M \to 0$, our calculations recover the results for massless quarks in previous studies.

Similar to the case of light quark propagation [16], the results discussed in this work can be easily extended to a hot and dense medium, which will have practical consequences for heavy quark production and suppression in heavy ion collisions. When the data on direct measurement of D-meson spectra in high-energy $A + A$ collisions become available in the near future, one should be able to use the modified fragmentation function in a parton model to study the modification of the D-meson spectra [43] and probe medium properties similarly as one has done for high p_T light hadrons [15]. The different pattern of energy loss for heavy quarks, such as energy and medium size dependence, will not only confirm the unique feature of non-Abelian energy loss but also give more confidence in using jet tomography to study properties of dense matter in heavy-ion collisions.

APPENDIX A

In this appendix we list the calculation results of double scattering of the heavy quark discussed in Section III. There are total 23 cut diagrams as illustrated in Fig. 11-21.

FIG. 11.

There are three three different cuts(central, left, right) in Fig. 11, and their contributions are

$$\overline{H}_1^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z) = \int d\ell_T^2 \frac{(1+z^2)\ell_T^2 + (1-z)^4 M^2}{(1-z)(\ell_T^2 + (1-z)^2 M^2)^2}$$
$$\times \frac{\alpha_s}{2\pi} C_F \frac{2\pi\alpha_s}{N_c} \overline{I}_{1,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z), \qquad \text{(A1)}$$

$$\overline{I}_{1,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + i x_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y^- - y_1^-)$$
$$\times \left[1 - e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-}\right]\left[1 - e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_1^-)}\right], \qquad \text{(A2)}$$

$$\overline{I}_{1,L}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + i x_D p^+(y_1^- - y_2^-)}\theta(y_1^- - y_2^-)\theta(y^- - y_1^-)$$
$$\times \left(1 - e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_1^-)}\right), \qquad \text{(A3)}$$

$$\overline{I}_{1,R}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + i x_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y_2^- - y_1^-)$$
$$\times \left(1 - e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-}\right). \qquad \text{(A4)}$$

FIG. 12.

In Fig. 12, there are two different cuts for induced gluon radiation, central or left. They give,

$$\overline{H}_2^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z)$$
$$= \int d\ell_T^2 \frac{(1+z^2)\vec{\ell}_T \cdot (\vec{\ell}_T - (1-z)\vec{k}_T) + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2][(\vec{\ell}_T - (1-z)\vec{k}_T)^2 + (1-z)^2 M^2]}$$
$$\times \frac{\alpha_s}{2\pi}(C_F - \frac{C_A}{2})\frac{2\pi\alpha_s}{N_c} \overline{I}_2(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z), \qquad \text{(A5)}$$

$$\overline{I}_{2,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + i x_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y^- - y_1^-)$$

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$$\times \left[e^{-i(x_L+(1-z)x_M/z)p^+(y^--y_1^-)} - e^{-i(x_L+(1-z)x_M/z)p^+(y^--y_1^-+y_2^-)} \right] , \qquad (A6)$$

$$\overline{I}_{2,L}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) = e^{i(x+x_L)p^+y^-+ix_Dp^+(y_1^--y_2^-)}\theta(y^--y_1^-)\theta(y_1^--y_2^-)$$
$$\times \left[e^{-i(x_L+(1-z)x_M/z)p^+(y^--y_2^-)+i(x_D^0-x_D)p^+(y_1^--y_2^-)} - e^{-i(x_L+(1-z)x_M/z)p^+(y^--y_1^-)} \right] , \quad (A7)$$

where $x_D^0 = k_T^2/2p^+q^-$.

FIG. 13.

As for the central cut and right cut of Fig. 13, we obtain

$$\overline{H}_3^D(y^-,y_1^-,y_2^-,k_T,x,p,q,M,z)$$
$$= \int d\ell_T^2 \frac{(1+z^2)\vec{\ell_T}\cdot(\vec{\ell_T}-(1-z)\vec{k_T})+(1-z)^4M^2}{(1-z)[\ell_T^2+(1-z)^2M^2][(\vec{\ell_T}-(1-z)\vec{k_T})^2+(1-z)^2M^2]}$$
$$\times \frac{\alpha_s}{2\pi}(C_F-\frac{C_A}{2})\frac{2\pi\alpha_s}{N_c}\overline{I}_3(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) , \qquad (A8)$$

$$\overline{I}_{3,C}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) = e^{i(x+x_L)p^+y^-+ix_Dp^+(y_1^--y_2^-)}\theta(-y_2^-)\theta(y^--y_1^-)$$
$$\times \left[e^{-i(x_L+(1-z)x_M/z)p^+y_2^-} - e^{-i(x_L+(1-z)x_M/z)p^+(y^--y_1^-+y_2^-)} \right] , \qquad (A9)$$

$$\overline{I}_{3,R}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) = e^{i(x+x_L)p^+y^-+ix_Dp^+(y_1^--y_2^-)}\theta(y^--y_1^-)\theta(y_1^--y_2^-)$$
$$\times \left[e^{-i(x_L+(1-z)x_M/z)p^+y_1^-+i(x_D^0-x_D)p^+(y_1^--y_2^-)} - e^{-i(x_L+(1-z)x_M/z)p^+y_2^-} \right] , \qquad (A10)$$

FIG. 14.

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There is only one cut (left cut) in Fig. 14 with the contribution,

$$\overline{H}_{4,C}^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z) = \int d\ell_T^2 \frac{(1+z^2)\ell_T^2 + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2]^2}$$

$$\times \frac{\alpha_s}{2\pi} C_F \frac{2\pi\alpha_s}{N_c} \overline{I}_4(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z), \quad (A11)$$

$$\overline{I}_{4,L}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + ix_D p^+(y_1^- - y_2^-)}\theta(y^- - y_1^-)\theta(y_1^- - y_2^-)$$

$$\times e^{i(x_D^0 - x_D)p^+(y_1^- - y_2^-)} e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_2^-)}. \quad (A12)$$

FIG. 15.

As for Fig. 15, we get

$$\overline{H}_5^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z) = \int d\ell_T^2 \frac{(1+z^2)(\vec{\ell}_T - (1-z)\vec{k}_T)^2 + (1-z)^4 M^2}{(1-z)[(\vec{\ell}_T - (1-z)\vec{k}_T)^2 + (1-z)^2 M^2]^2}$$

$$\times \frac{\alpha_s}{2\pi} C_F \frac{2\pi\alpha_s}{N_c} \overline{I}_5(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z), \quad (A13)$$

$$\overline{I}_{5,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + ix_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y^- - y_1^-)$$

$$\times e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_1^- + y_2^-)}. \quad (A14)$$

FIG. 16.

The contribution from Fig. 16 is

$$\overline{H}_{6,C}^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z) = \int d\ell_T^2 \frac{(1+z^2)\ell_T^2 + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2]^2}$$

$$\times \frac{\alpha_s}{2\pi} C_F \frac{2\pi\alpha_s}{N_c} \overline{I}_6(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z), \quad (A15)$$

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$$\overline{I}_{6,R}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) = -e^{i(x+x_L)p^+y^-+ix_Dp^+(y_1^--y_2^-)}\theta(-y_2^-)\theta(y_2^--y_1^-)$$
$$\times\, e^{i(x_D^0-x_D)p^+(y_1^--y_2^-)}e^{-i(x_L+(1-z)x_M/z)p^+y_1^-}\,. \qquad (A16)$$

FIG. 17.

As for the processes in Fig. 17 we have three possible cuts with the corresponding contributions,

$$\overline{H}_{7,C}^D(y^-,y_1^-,y_2^-,k_T,x,p,q,M,z) = \int d\ell_T^2 \frac{(1+z^2)(\vec{\ell}_T-\vec{k}_T)^2+(1-z)^4M^2}{(1-z)[(\vec{\ell}_T-\vec{k}_T)^2+(1-z)^2M^2]^2}$$
$$\times\, \frac{\alpha_s}{2\pi}\,C_A\,\frac{2\pi\alpha_s}{N_c}\overline{I}_{7,C}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z)\,, \qquad (A17)$$

$$\overline{I}_{7,C}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) = e^{i(x+x_L)p^+y^-+ix_Dp^+(y_1^--y_2^-)}\theta(-y_2^-)\theta(y^--y_1^-)$$
$$\times\, \left[e^{ix_Dp^+y_2^-/(1-z)}-e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}\right]$$
$$\times\, \left[e^{ix_Dp^+(y^--y_1^-)/(1-z)}-e^{-i(x_L+(1-z)x_M/z)p^+(y^--y_1^-)}\right]\,, \qquad (A18)$$

$$\overline{H}_{7,L(R)}^D(y^-,y_1^-,y_2^-,k_T,x,p,q,M,z) = \int d\ell_T^2 \frac{(1+z^2)\ell_T^2+(1-z)^4M^2}{(1-z)[\ell_T^2+(1-z)^2M^2]^2}$$
$$\times\, \frac{\alpha_s}{2\pi}\,C_A\,\frac{2\pi\alpha_s}{N_c}\overline{I}_{7,L(R)}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z)\,, \qquad (A19)$$

$$\overline{I}_{7,L}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) = -e^{i(x+x_L)p^+y^-+ix_Dp^+(y_1^--y_2^-)}\theta(y^--y_1^-)\theta(y_1^--y_2^-)$$
$$\times\, e^{-i(1-z/(1-z))x_Dp^+(y_1^--y_2^-)}$$
$$\times\, \left[1-e^{-i(x_L+(1-z)x_M/z)p^+(y^--y_1^-)}\right]\,, \qquad (A20)$$

$$\overline{I}_{7,R}(y^-,y_1^-,y_2^-,\ell_T,k_T,x,p,q,M,z) = -e^{i(x+x_L)p^+y^-+ix_Dp^+(y_1^--y_2^-)}\theta(-y_2^-)\theta(y_2^--y_1^-)$$
$$\times\, e^{-i(1-z/(1-z))x_Dp^+(y_1^--y_2^-)}\left[1-e^{-i(x_L+(1-z)x_M/z)p^+y_2^-}\right]\,. \quad (A21)$$

FIG. 18.

There are also three cuts in Fig. 18 and we have

$$
\overline{H}_8^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z) = \int d\ell_T^2 \frac{(1+z^2)\vec{\ell_T} \cdot (\vec{\ell_T} - \vec{k_T}) + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2][(\vec{\ell_T} - \vec{k_T})^2 + (1-z)^2 M^2]}
$$
$$
\times \frac{\alpha_s}{2\pi} \frac{C_A}{2} \frac{2\pi\alpha_s}{N_c} \overline{I}_8(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) , \tag{A22}
$$

$$
\overline{I}_{8,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + ix_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y^- - y_1^-)
$$
$$
\times \left[e^{ix_D p^+ y_2^-/(1-z)} - e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-} \right]
$$
$$
\times \left[1 - e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_1^-)} \right] , \tag{A23}
$$

$$
\overline{I}_{8,L}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + ix_D p^+(y_1^- - y_2^-)}\theta(y^- - y_1^-)\theta(y_1^- - y_2^-)
$$
$$
\times e^{-i(1-z/(1-z))x_D p^+(y_1^- - y_2^-)}
$$
$$
\times \left[e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_1^-)} - e^{ix_D p^+(y^- - y_1^-)/(1-z)} \right] , \tag{A24}
$$

$$
\overline{I}_{8,R}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + ix_D p^+(y_1^- - y_2^-)}\theta(-y_2^-)\theta(y_2^- - y_1^-)
$$
$$
\times \left[e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-} - e^{ix_D p^+ y_2^-/(1-z)} \right] . \tag{A25}
$$

FIG. 19.

The contributions from the three cuts in Fig. 19 are

$$
\overline{H}_9^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z) = \int d\ell_T^2 \frac{(1+z^2)\vec{\ell_T} \cdot (\vec{\ell_T} - \vec{k_T}) + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2][(\vec{\ell_T} - \vec{k_T})^2 + (1-z)^2 M^2]}
$$
$$
\times \frac{\alpha_s}{2\pi} \frac{C_A}{2} \frac{2\pi\alpha_s}{N_c} \overline{I}_9(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) , \tag{A26}
$$

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$$\overline{I}_{9,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + i x_D p^+ (y_1^- - y_2^-)} \theta(-y_2^-)\theta(y^- - y_1^-)$$
$$\times \left[e^{i x_D p^+ (y^- - y_1^-)/(1-z)} - e^{-i(x_L + (1-z)x_M/z)p^+ (y^- - y_1^-)} \right]$$
$$\times \left[1 - e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-} \right], \tag{A27}$$

$$\overline{I}_{9,L}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + i x_D p^+ (y_1^- - y_2^-)} \theta(y^- - y_1^-)\theta(y_1^- - y_2^-)$$
$$\times \left[e^{-i(x_L + (1-z)x_M/z)p^+ (y^- - y_1^-)} - e^{i x_D p^+ (y^- - y_1^-)/(1-z)} \right], \tag{A28}$$

$$\overline{I}_{9,R}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = -e^{i(x+x_L)p^+ y^- + i x_D p^+ (y_1^- - y_2^-)} \theta(-y_2^-)\theta(y_2^- - y_1^-)$$
$$\times e^{-i(1-z/(1-z))x_D p^+ (y_1^- - y_2^-)}$$
$$\times \left[e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-} - e^{i x_D p^+ y_2^-/(1-z)} \right], \tag{A29}$$

FIG. 20.

From Fig. 20 with two possible cuts (central or left), we have

$$\overline{H}_{10,C}^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z)$$
$$= \int d\ell_T^2 \frac{(1+z^2)(\vec{\ell_T} - \vec{k_T}) \cdot (\vec{\ell_T} - (1-z)\vec{k_T}) + (1-z)^4 M^2}{(1-z)[(\vec{\ell_T} - \vec{k_T})^2 + (1-z)^2 M^2][(\vec{\ell_T} - (1-z)\vec{k_T})^2 + (1-z)^2 M^2]}$$
$$\times \frac{\alpha_s}{2\pi} \frac{C_A}{2} \frac{2\pi\alpha_s}{N_c} \overline{I}_{10,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z), \tag{A30}$$

$$\overline{I}_{10,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + i x_D p^+ (y_1^- - y_2^-)} \theta(-y_2^-)\theta(y^- - y_1^-)$$
$$\times e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-} \left[e^{i x_D p^+ (y^- - y_1^-)/(1-z)} - e^{-i(x_L + (1-z)x_M/z)p^+ (y^- - y_1^-)} \right], \tag{A31}$$

$$\overline{H}_{10,R}^D(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z)$$
$$= \int d\ell_T^2 \frac{(1+z^2)\vec{\ell_T} \cdot (\vec{\ell_T} - z\vec{k_T}) + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2][(\vec{\ell_T} - z\vec{k_T})^2 + (1-z)^2 M^2]}$$
$$\times \frac{\alpha_s}{2\pi} \frac{C_A}{2} \frac{2\pi\alpha_s}{N_c} \overline{I}_{10,R}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z), \tag{A32}$$

$$\overline{I}_{10,R}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + i x_D p^+ (y_1^- - y_2^-)} \theta(-y_2^-)\theta(y_2^- - y_1^-)$$
$$\times \left[e^{-i(x_D - x_D^0)p^+ (y_1^- - y_2^-) - i(x_L + (1-z)x_M/z)p^+ y_1^-} \right.$$
$$\left. - e^{-i(1-z/(1-z))x_D p^+ (y_1^- - y_2^-) - i(x_L + (1-z)x_M/z)p^+ y_2^-} \right]. \tag{A33}$$

26

FIG. 21.

Finally, Fig. 21 has both central cut and the left cut with contributions as

$$
\overline{H}^D_{11,C}(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z)
$$

$$
= \int d\ell_T^2 \frac{(1+z^2)(\vec{\ell_T} - \vec{k_T}) \cdot (\vec{\ell_T} - (1-z)\vec{k_T}) + (1-z)^4 M^2}{(1-z)[(\vec{\ell_T} - \vec{k_T})^2 + (1-z)^2 M^2][(\vec{\ell_T} - (1-z)\vec{k_T})^2 + (1-z)^2 M^2]}
$$

$$
\times \frac{\alpha_s}{2\pi} \frac{C_A}{2} \frac{2\pi\alpha_s}{N_c} \overline{I}_{11,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) , \tag{A34}
$$

$$
\overline{I}_{11,C}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + i x_D p^+ (y_1^- - y_2^-)} \theta(-y_2^-)\theta(y^- - y_1^-)
$$

$$
\times e^{-i(x_L + (1-z)x_M/z)p^+(y^- - y_1^-)} \left[e^{i x_D p^+ y_2^- /(1-z)} - e^{-i(x_L + (1-z)x_M/z)p^+ y_2^-} \right] , \tag{A35}
$$

$$
\overline{H}^D_{11,L}(y^-, y_1^-, y_2^-, k_T, x, p, q, M, z)
$$

$$
= \int d\ell_T^2 \frac{(1+z^2)\vec{\ell_T} \cdot (\vec{\ell_T} - z\vec{k_T}) + (1-z)^4 M^2}{(1-z)[\ell_T^2 + (1-z)^2 M^2][(\vec{\ell_T} - z\vec{k_T})^2 + (1-z)^2 M^2]}
$$

$$
\times \frac{\alpha_s}{2\pi} \frac{C_A}{2} \frac{2\pi\alpha_s}{N_c} \overline{I}_{11,L}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) , \tag{A36}
$$

$$
\overline{I}_{11,L}(y^-, y_1^-, y_2^-, \ell_T, k_T, x, p, q, M, z) = e^{i(x+x_L)p^+ y^- + i x_D p^+ (y_1^- - y_2^-)} \theta(y^- - y_1^-)\theta(y_1^- - y_2^-)
$$

$$
\times \left[e^{-i(x_D - x_D^0)p^+(y_1^- - y_2^-) - i(x_L + (1-z)x_M/z)p^+(y^- - y_2^-)} \right.
$$

$$
\left. - e^{-i(1 - z/(1-z))x_D p^+(y_1^- - y_2^-) - i(x_L + (1-z)x_M/z)p^+(y^- - y_1^-)} \right] . \tag{A37}
$$

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