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Threshold Shear Strains for Cyclic Degradation and Cyclic Pore Water Pressure Generation in Two Clays

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Abstract: Cyclic threshold shear strains are fundamental cyclic soil properties that have not been fully investigated. To learn more about the threshold shear strains for cyclic degradation, γ_{td} , and cyclic pore water pressure generation, γ_{tp} , in fully saturated clays, nine multistage cyclic strain-controlled NGI direct simple shear tests are conducted on laboratory-made kaolinite clay (PI = 28) and kaolinite-bentonite clay (PI = 55). Three levels of vertical effective consolidation stress, σ'_{vc} (113 kPa, approximately 216 kPa, and approximately 674 kPa); three OCRs (1, 4 and 7.8); and two cyclic loading frequencies, f (0.01 and 0.1 Hz), were applied. In three tests on the normally consolidated (NC) kaolinite clay, γ_{td} varied between 0.012 and 0.014% and γ_{tp} between 0.014 and 0.034%. In two tests on the overconsolidated (OC) kaolinite clay with OCR = 4, γ_{td} was 0.013% and γ_{tp} 0.016 and 0.017%. In two tests on the NC kaolinite-bentonite clay, γ_{td} was 0.013 and 0.016% and γ_{tp} 0.052 and 0.078%. In the test on OC kaolinite-bentonite clay with OCR = 4, γ_{td} was 0.014% and with OCR = 7.8 it was 0.012%. For the same soil γ_{tp} is typically slightly greater than γ_{td} . Clear trends of γ_{td} and γ_{tp} with σ'_{vc} , OCR, and f could not be identified given the relatively small number of tests. The results indicate that if these trends exist they are small. The comparison of the above results with those from the literature shows that γ_{td} for six different soils ranges between 0.006 and 0.05% and γ_{tp} for eight soils between 0.014 and 0.1%, and that there is a modest trend of γ_{td} and moderate trend of γ_{tp} increasing with PI. DOI: 10.1061/(ASCE)GT.1943-5606.0001461. © 2016 American Society of Civil Engineers.

Introduction

When fully saturated soils are subjected to cyclic loading in undrained conditions involving moderate and large cyclic shear strain amplitudes, γ_c , their structure is permanently altered, their stiffness and strength decrease, and their pore water pressure changes permanently with the number of cycles, N . Such cyclic degradation of stiffness and pore water pressure change are among the most important phenomena in soil dynamics (Kramer 1966; Ishihara 1966). Fully saturated sands are the most susceptible to cyclic degradation and significant pore water pressure buildup. Because of an extreme cyclic pore water pressure buildup and resulting drop of effective stresses, sands can completely lose their stiffness and strength and eventually liquefy. On the other end of the spectrum are fully saturated clays of high plasticity, i.e., clays with high plasticity index, PI. Under significant cyclic loading, normally consolidated (NC) clays with high PI degrade relatively little and cyclic pore water pressure buildup is relatively small. Overconsolidated (OC) clays with high PI degrade cyclically even less, while the cyclic pore water pressure may actually decrease with N instead of increase (Andersen et al. 1980; Dobry and Vucetic 1987; Vucetic 1988). Somewhere in between are NC clays of low plasticity that under continuous cyclic loading with moderate to large γ_c can degrade substantially and simultaneously exhibit relatively high permanent excess prewater pressures (Tan and Vucetic 1989).

As opposed to such a destructive behavior under moderate and large γ_c , when fully saturated soils are subjected to very small amplitudes, γ_c , the structure of the soil practically does not change. Consequently, at very small γ_c there are no noticeable cyclic degradation and permanent cyclic pore water pressure changes. The amplitude γ_c below which there is practically no cyclic degradation and above which a noticeable degradation occurs is known as the threshold shear strain for cyclic degradation, γ_{td} (Tabata and Vucetic 2010; Vucetic 1994b). The amplitude γ_c below which there is practically no permanent cyclic pore water pressure change with N and above which such a change is clearly noticeable is known as the threshold shear strain for cyclic pore water pressure generation, γ_{tp} (Dobry et al. 1982). For the same soil, the magnitudes of γ_{td} and γ_{tp} are usually somewhat different. Amplitude γ_{td} is typically smaller than γ_{tp} (Tabata and Vucetic 2010).

Here the term *pore water pressure generation* means the generation of either positive or negative pore water pressures, or the combination of the two during cyclic loading.

The cyclic threshold shear strain concept was introduced by Dobry et al. (1982) for sands and it is now recognized as an important concept in soil dynamics. The cyclic threshold shear strain divides two fundamentally different domains of cyclic soil behavior and, consequently, the approach to the solution of a soil dynamics problem often depends on whether the magnitudes of γ_c are less than or greater than the cyclic threshold shear strains (e.g., Vucetic 1994a, b; Kramer 1996). Besides γ_{td} and γ_{tp} , which are applicable to fully saturated soils cyclically sheared in undrained conditions, there are two other cyclic threshold shear strain amplitudes, the threshold shear strain for cyclic compression, also referred to as the volumetric cyclic threshold shear strain (Yound 1972; Chu and Vucetic 1992; Hsu and Vucetic 2004), and the threshold shear strain for cyclic stiffening (Kim et al. 1991; Stokoe et al. 1995). Because of their relevance, all four threshold shear strains are discussed to different extents in textbooks on geotechnical earthquake engineering and soil behavior, such as those by Kramer (1996), Ishihara (1996), and Mitchell and Soga (2005).

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For example, Kramer (1996) in the section of his book on the initiation of liquefaction discusses the cyclic strain approach introduced by Dobry et al. (1982) and Dobry and Ladd (1980). He explains that, according to the cyclic strain approach, if $\gamma_c < \gamma_{tp}$ no excess cyclic pore water pressure will be generated in fully saturated sands and, consequently, liquefaction cannot be initiated. The liquefaction hazard evaluation would end at that point. On the other hand, if $\gamma_c > \gamma_{tp}$ liquefaction is possible and the liquefaction resistance of the soil must be evaluated. Similarly, Ishihara (1996) presents the results on cyclic degradation and threshold shear strains of clays from the studies by Ohara and Matsuda (1988), Tan and Vucetic (1989), and Vucetic (1994a, b). Ishihara explains that if $\gamma_c < \gamma_{td}$ the cyclic degradation will not occur, and it therefore does not have to be taken into account in soil dynamics analyses. As opposed to that, if $\gamma_c > \gamma_{td}$ the degradation will occur and it should be considered in the analyses. For example, in Vucetic (1994b) it is explained that the selection of the method for seismic site response analysis depends on the relative magnitude of γ_{td} with respect to the magnitudes of γ_c generated by the earthquake at the site. If $\gamma_c < \gamma_{td}$, the equivalent linear analysis that does not account for cyclic degradation, such as SHAKE (Schnabel et al. 1972), is sufficient. If $\gamma_c > \gamma_{td}$, more complex analyses that include cyclic degradation and γ_{td} as one of the input parameters should be used, such as, for example, D-MOD by Matasovic and Vucetic (1993, 1995).

However, in spite of their relevance, the cyclic threshold shear strains have not been adequately investigated, with the exception of γ_{tp} in fully saturated sands, which was extensively studied in connection with the liquefaction of saturated sandy deposits during earthquakes (e.g., Dobry et al. 1982; Dyvik et al. 1984). The level of γ_{td} in sands and its relation to cyclic pore water pressure have been investigated in a systematic manner just recently (e.g., Mortezaie 2012; Vucetic and Mortezaie 2015), while γ_{tp} and γ_{td} in clays have been evaluated on a relatively small number of soils (Hsu and Vucetic 2006; Tabata and Vucetic 2010). A systematic investigation of γ_{td} and γ_{tp} in OC clays has not been conducted, while a number of factors affecting γ_{td} and γ_{tp} in both NC and OC clays still need to be studied. Among the reasons for such a lack of studies focusing directly on the threshold shear strains are the complexities and high costs associated with high quality small-strain cyclic laboratory testing.

The goal of this paper is to present the results of new study on γ_{td} and γ_{tp} in fully saturated clays (Mortezaie 2012) and compare them to the results of previous studies. Nine cyclic tests, designed specifically for the evaluation of γ_{td} and γ_{tp} , were conducted on two laboratory-made NC and OC clays, a kaolinite clay having $PI = 28$ and a kaolinite-bentonite clay having $PI = 55$. The clays were tested in a Norwegian Geotechnical Institute (NGI) type of direct simple shear (DSS) device in the constant-volume equivalent-undrained mode. The tests were multistaged and cyclic strain-controlled, with constant γ_c in each stage and with γ_c greater in every subsequent stage. In every test the magnitude of γ_c varied from stage to stage between approximately 0.003% and 1.0 or 2.0%. The testing program included three levels of vertical effective consolidation stress, σ'_{vc} ; three different overconsolidation ratios, OCR; and two different frequencies of cyclic straining, f .

Clays Tested, Specimen Preparation and Testing Procedure

The clays were prepared in a consolidation tank from thin slurries made of commercially available clay powders and tap water. The kaolinite clay was made of pure kaolinite powder, while the

kaolinite-bentonite clay was made of a mixture of 85% kaolinite powder and 15% bentonite powder (montmorillonite clay minerals) by weight. Such laboratory-made clays are used because they can be easily reproduced from commercial kaolinite and bentonite powders by other investigators who can then repeat the tests to check their accuracy and/or add new results to the existing data. Furthermore, montmorillonite and kaolinite are the most common clay minerals present in natural soils (Mitchell 1976) and, consequently, the behavior of the laboratory-made kaolinite and kaolinite-bentonite clays is similar to that of many natural clays of similar plasticity.

The slurry of both clays was consolidated in three stages into a clay cake, 60 mm high and 180 mm in diameter. The consolidation was slow. The first two stages lasted 1–2 days for the kaolinite clay and 2–3 days for the kaolinite-bentonite clay, which was long enough for the completion of primary consolidation. The third stage lasted up to a week to allow for a substantial secondary compression. Each clay cake was then extruded from the tank and cut vertically into three segments. From each segment a short cylinder specimen, 66.7 mm in diameter and 18 to 19 mm high, was trimmed in the NGI-DSS trimming apparatus and tested.

The kaolinite clay has liquid limit $LL = 61$, plastic limit $PL = 33$, and plasticity index $PI = LL - PL = 28$, and is classified according to the Unified Soil Classification System (USCS) as the high plasticity silt, MH. Although it is classified as silt, this soil is still called clay because it is composed entirely from clay mineral particles and in the Casagrande's Plasticity Chart it plots just below the A line. The kaolinite-bentonite clay has $LL = 93$, $PL = 38$, and $PI = 55$ and is classified as the high plasticity clay, CH.

The NGI-DSS testing apparatus employed in the present investigation was introduced by Bjerrum and Landva (1966) for monotonic loading testing and it has been substantially modified for the present cyclic testing. To enable the application of precise cyclic horizontal displacements and loads, a computer-controlled, closed-loop, servohydraulic system with a set of sensitive displacement transducers and load cells was integrated into the device. To obtain a precise small-strain cyclic soil behavior, the recorded signals were greatly amplified and the raw test records were analyzed and processed to eliminate false loads and deformations inherent to the NGI-DSS testing. This was done according to the procedure described by Mortezaie and Vucetic (2012).

The configuration of the NGI-DSS device specimen setup employed in the present investigation is shown in Fig. 1. The definitions of γ_c and the cyclic shear stress amplitude, τ_c , are also provided in the figure. A typical NGI-DSS specimen is a short cylinder of soil sitting on the porous stone and covered by another porous stone. The porous stones are firmly fixed in the bottom and top caps. Around the vertical boundary the specimen is enclosed tightly in the wire-reinforced rubber membrane. The purpose of the membrane is to greatly restrict (and if properly selected almost prevent) the radial strains during consolidation and shear. Accordingly, in such a setup, the specimen can be consolidated under the conditions close to the K_0 conditions with no lateral strains and can be subsequently sheared under the conditions with practically no lateral strains. Such conditions exist, for example, in horizontally layered level ground prior to and during the cyclic shearing attributable to the vertically propagating seismic shear waves.

The specimen consolidation in the NGI-DSS device was performed in a typical manner in five or more vertical stress increments and for the OC clays several subsequent decrements. In the case of the kaolinite clay, each increment and decrement lasted a minimum of 2 h, which for the compression steps was enough for the completion of primary consolidation and beginning of secondary

Fig. 1. Configuration of the NGI-DSS device specimen setup at UCLA

compression. To achieve the same results for the specimens of the kaolinite-bentonite clay, the loading and unloading steps lasted at least 8 h. After the last increment on NC soil and last decrement on OC soil, the specimen was left for at least 1 day to consolidate well into the secondary compression stage.

If the wire-reinforced rubber membrane is properly selected (stronger membrane for stiffer soil) and the specimen trimming and consolidation properly conducted, the vertical compression curve of the NGI-DSS test should look very similar to the curve obtained for the same soil in the standard consolidometer with the metal ring confinement. This response is verified in Fig. 2 for two OC kaolinite-bentonite clay specimens. They were loaded first to the vertical stresses of 846 and 885 kPa and then unloaded to σ'_{vc} of 211 and 113 kPa, corresponding to OCRs of 4 and 7.8, respectively.

During the cyclic shearing in the NGI-DSS device in the constant-volume equivalent-undrained mode, the volume of the fully saturated specimen is maintained constant and the pressure of the pore water is always zero. In such a test, the vertical total stress during shearing is therefore equal to the vertical effective stress. Consequently, the change of the total vertical stress that is

necessary to maintain the volume of the specimen constant is practically equal to the excess pore water pressure that would have developed in a true undrained test. The change in the total vertical stress during shearing is thus called the equivalent pore water pressure, or simply the pore water pressure, which is denoted herein by Δu . Because of the negligibly small lateral deformations of the specimen as a result of the wire-reinforced rubber membrane confinement, the specimen volume is typically maintained practically constant by just maintaining the height of the specimen constant, which is a standard procedure at NGI and other laboratories conducting the NGI-DSS tests (Iversen 1977). The validation of the NGI-DSS constant-volume equivalent-undrained testing concept is provided in Dyvik et al. (1987). One can also find this validation in Vucetic and Lacasse (1984).

Typical Results

The results of one out of nine multistage, cyclic strain-controlled tests are presented in Figs. 3 and 4. Fig. 3 shows the variation of the shear strain, γ , shear stress τ normalized with σ'_{vc} , $\tau^* = \tau / \sigma'_{vc}$, and

Fig. 2. Comparison between the compression curves obtained for the kaolinite-bentonite clay specimens consolidated in the wire-reinforced rubber membrane in the NGI-DSS test (KaoBen03 and KaoBen04 tests) and in the steel ring of a standard consolidometer

the normalized equivalent pore water pressure, $\Delta u^* = \Delta u / \sigma'_{vc}$, with time. The test had nine cyclic stages with 10 cycles each. The amplitude γ_c ranged from 0.0031 to 0.7%. The small amplitude $\gamma_c = 0.0031\%$ in the first stage is well below the thresholds γ_{td} and γ_{tp} , while $\gamma_c = 0.7\%$ in the ninth stage is well above them. The resulting stress-strain loops are shown in Fig. 4. From Figs. 3 and 4 it is evident that in the last four stages (stages 6 through 9) the cyclic shear stress amplitude, τ_c , decreased with N while the magnitude of the cyclic pore water pressure, Δu_N , increased. Here Δu_N is the equivalent excess pore water pressure recorded at the end of shear strain cycle N . It is also evident that in the first stages 1 to 3 there were no noticeable changes of τ_c and Δu_N . Consequently, in the stages 6 to 9 the amplitudes γ_c are above γ_{td} and γ_{tp} , while in the stages 1 to 3 they are below, indicating that γ_{td} and γ_{tp} are somewhere between the γ_c 's of stages 3 and 6, i.e., somewhere between 0.01 and 0.081%. The process for estimating the actual values of γ_{td} and γ_{tp} from the above results is described in subsequent sections.

Testing Program

The testing program and final test results are presented in Table 1. Between the nine tests there were essentially three different levels of σ'_{vc} (113 kPa, 210 to 222 kPa, and 668 to 680 kPa); three OCRs

Fig. 3. Variation of strain, stress, and pore water pressure with time in a multistage cyclic strain-controlled test on kaolinite clay

Fig. 4. Stress-strain loops of a multistage cyclic strain-controlled test on kaolinite clay

(1, 4, and 7.8); and two frequencies of cyclic loading, f (0.01 and 0.1 Hz). Previous studies show that both the degradation parameter, t , which measures the rate of cyclic degradation with N [defined below by Eq. (2)], and the cyclic pore water pressure, Δu_N , depend on σ'_{vc} , OCR, and f (e.g., Vucetic and Dobry 1988; Tan and Vucetic 1989; Mortezaie and Vucetic 2013). The above variations of parameters σ'_{vc} , OCR, and f may, therefore, allow a preliminary, general evaluation of their effects on γ_{td} and γ_{tp} .

Frequencies of 0.1 and 0.01 Hz are generally low in comparison to the frequencies of displacements in soil dynamics problems involving ground response to earthquakes and machine foundation vibrations (O'Reilly and Brown 1991), while they compare better to the frequencies of ocean waves during big storms acting on the foundations of offshore structures (Schjetne 1976; Ellers 1982). Such relatively low frequencies are employed to have a better control over the application and recording of very small strains, stresses, and equivalent pore water pressures around the thresholds. Furthermore, as concluded at the end of the paper, the frequency seems to have a negligible effect on γ_{td} and γ_{tp} .

Threshold Shear Strain for Cyclic Degradation, γ_{td}

Characterization of Cyclic Degradation and the Procedure for Evaluating γ_{td} from the Results of a Multistage Cyclic Strain-Controlled Test

The relative decrease of τ_c with N in the last four stages 6 to 9 of the test Kao32 shown in Figs. 3 and 4 can be characterized by the degradation index, $\delta(N)$, and the degradation parameter, t , introduced by Idriss et al. (1978)

$$\delta(N) = \frac{G_{SN}(N)}{G_{S1}} = \frac{\frac{\tau_{cN}(N)}{\gamma_c}}{\frac{\tau_{c1}}{\gamma_c}} = \frac{\tau_{cN}(N)}{\tau_{c1}} \quad (1)$$

$$t = -\frac{\log \delta(N)}{\log N} \quad (2)$$

where τ_{cN} and therefore G_{SN} and δ are functions of N ; γ_c , τ_{c1} , and G_{S1} are constants; stress τ_{cN} = average cyclic shear stress amplitude, τ_c , in cycle N of a given stage; $\tau_{c1} = \tau_c$ in the first cycle of the same stage; modulus G_{SN} = average secant shear modulus G_S in cycle N of a particular cyclic stage; and $G_{S1} = G_S$ in the first cycle of the same stage. Modulus G_{SN} can be determined by measuring the slope of the secant line connecting the tips of the cyclic loop, i.e., by measuring the slope of the cyclic loop. The degradation index, δ , therefore describes the relative reduction of G_S with N in a single stage. Fig. 4 shows clearly how the slopes of the cyclic loops in the last stages of the test are decreasing with N , i.e., how G_{SN} degrades with N . Many published results of cyclic strain-controlled tests on different clays show that if δ is plotted versus N in a log-log format, a more or less straight line is obtained. The slope of this line is the average degradation parameter, t , which describes the rate of cyclic degradation with N as a result of the applied constant γ_c .

The procedure to evaluate the magnitude of γ_{td} within the concept of the degradation index δ and parameter t is provided below for the test Kao32 presented in Figs. 3 and 4. Fig. 5(a) shows the variation of δ with N for all nine cyclic stages of the test. Because the curves for the first three stages with very small γ_c fluctuate around $\delta = 1$ in a rather irregular manner and are thus difficult to differentiate, they are plotted separately in Fig. 5(b) in a larger scale. The fluctuation is a consequence of the limitations and difficulties in measuring and recording precisely very small cyclic stresses as a result of the very small cyclic strains. The degradation

Table 1. Testing Program and Threshold Strain Results

Test name	Soil name	Classification symbol	Plasticity index (PI)	Void ratio (e)	Water content, w (%)	Vertical consolidation stress (σ'_{vc}) (kPa)	Overconsolidation ratio (OCR)	Cyclic frequency, f (Hz)	Threshold shear strain for cyclic degradation, γ_{td} (%)	Threshold shear strain for cyclic pore water pressure generation, γ_{tp} (%)	Ratio between γ_{tp} and γ_{td} (γ_{tp}/γ_{td})
Kao32	Kaolinite	MH	28	1.23	46	218	1	0.1	0.012	0.034	2.8
Kao12				1.14	43	222		0.01	0.012	0.026	2.2
Kao33				1.06	40	680		0.01	0.014	0.014	1.0
Kao34				1.07	40	212	4	0.1	0.013	0.016	1.2
Kao11				1.01	38	210		0.01	0.013	0.017	1.3
KaoBen01	Kaolinite-	CH	55	1.42	54	220	1	0.1	0.016	0.052	3.3
KaoBen02	bentonite			1.09	51	668		0.01	0.013	0.078	6.0
KaoBen03				1.14	43	211	4	0.01	0.014	—	—
KaoBen04				1.17	44	113	7.8	0.01	0.012	—	—

Fig. 5. Cyclic degradation in the multistage cyclic strain-controlled test on kaolinite clay: (a) cyclic degradation in all nine stages; (b) cyclic degradation in the first three stages; (c) cyclic degradation in the last six stages

curves for the first two stages do not exhibit any clear trend with N , while the curve for the third stage with $\gamma_c = 0.01\%$ shows just a slight, barely recognizable trend of δ decreasing with N . Accordingly, it can be concluded that the amplitudes γ_c in the first two stages are definitely smaller than γ_{td} , while γ_c in the third stage is almost equal to γ_{td} . Given such results, it appears that γ_{td} can be best evaluated from the degradation trend in the last six cyclic stages, in particular the stages just above the anticipated γ_{td} . The corresponding six curves from the last six stages are replotted in Fig. 5(c).

The degradation parameter, t , is calculated for all 10 cycles in the last six stages according to Eq. (2), resulting in 10 values of t for each stage. The corresponding data points and the curves of their upper and lower bounds are plotted versus γ_c in Fig. 6. From this plot in linear scales an approximate value of γ_{td} can be obtained by extrapolating the trend to $t = 0$. However, a more precise extrapolation can be performed in a semilogarithmic format in which the horizontal axis is stretched in the domain of small γ_c of interest. This presentation is shown in Fig. 7. If the data bounds in Fig. 7 are extrapolated to $t = 0$, the range of the threshold γ_{td} between 0.009 and 0.013% is obtained. To obtain a more practical, single average value of γ_{td} , the extrapolation presented in Fig. 8 is finally

adopted. In Fig. 8 just four cyclic stages above the anticipated γ_{td} are included and each stage is represented with the single data point corresponding to the average degradation parameter obtained for all 10 cycles. The average parameters, t , actually correspond very closely to the slopes of the lines in Fig. 5(c). The three stages below anticipated γ_{td} are also included in Fig. 8, but simply with $t = 0$ data points because their δ values fluctuate around zero [Fig. 5(b)]. The extrapolation of the curve through such t - γ_c data points to $t = 0$ in Fig. 8 seems to be the most meaningful and consistent way of estimating γ_{td} . For test Kao32 such an estimate yields $\gamma_{td} = 0.012\%$. The same procedure is employed below to estimate γ_{td} from the results of eight other multistage cyclic strain-controlled tests on kaolinite and kaolinite-bentonite clays.

Values of Threshold Shear Strain for Cyclic Degradation

The trend lines of the average degradation parameters, t , with γ_c for four other multistage cyclic tests on kaolinite clay constructed according to the above procedure are presented in Fig. 9. The values of the applied σ'_{vc} , OCR, and f are specified at the top of each plot. The values of γ_{td} range between 0.012 and 0.014%, which is a very

Fig. 6. Variation of the degradation parameter, t , with the cyclic shear strain amplitude, γ_c , in the last six stages of the multistage cyclic test on kaolinite clay

Fig. 7. Variation of the degradation parameter, t , with the cyclic shear strain amplitude, γ_c , in the last six stages of the multistage cyclic test on kaolinite clay presented in the semilog format

small variation indeed. The trends of the average degradation parameters, t , with γ_c for the four multistage cyclic tests on kaolinite-bentonite clay are presented in Fig. 10. The values of γ_{td} range between 0.012 and 0.016%. These values again exhibit a rather small variation.

The shapes and slopes of the t versus γ_c curves in Figs. 9 and 10 depend not only on σ'_{vc} , OCR, and f (Mortezaie and Vucetic 2013), but also significantly on the magnitudes and sequence of γ_c applied in a multistage test. These curves therefore differ from the t versus γ_c curves obtained from a series of separate tests on the same soil conducted at the same σ'_{vc} , OCR, and f , but just different γ_c , such as those presented by Idriss et al. (1978), Vucetic and Dobry (1988), and Tan and Vucetic (1989). Consequently, to draw conclusions about various cyclic degradation trends at $\gamma_c > \gamma_{td}$, the

Fig. 8. Variation of the average degradation parameter, t , with the cyclic shear strain amplitude, γ_c , in the multistage cyclic test on kaolinite clay presented in the semilog format

curves in Figs. 9 and 10 cannot be straightforwardly compared to each other or to the curves in the above cited papers.

Summary on γ_{td} Values and Trends

The threshold shear strain for cyclic degradation of kaolinite (MH; PI = 28) and kaolinite-bentonite (CH; PI = 55) clays varies between 0.012 and 0.016% (Table 1), which is a narrow range. Although the average γ_{td} of kaolinite-bentonite clay of 0.0137% is greater than the average 0.0128% of kaolinite clay, the difference is so small that in the case of these two soils it cannot be claimed that γ_{td} is typically greater if PI is significantly greater. In previous investigations of various cyclic threshold shear strains, a general trend of increasing the threshold shear strain with PI has been obtained (e.g., Vucetic 1994b; Hsu and Vucetic 2004, 2006; Tabata and Vucetic 2010). Furthermore, the very small variation of γ_{td} in all nine tests indicates that if the effects of σ'_{vc} , OCR, and f on γ_{td} exist, they must be small to negligible.

Threshold Shear Strain for Cyclic Pore Water Pressure Generation, γ_{tp}

Method for Evaluating γ_{tp} from the Results of a Multistage Cyclic Strain-Controlled Test

Fig. 3 clearly shows that in the last four stages of test Kao32 the cyclic pore water pressure, Δu_N , increased, while in the first four stages it did not, indicating that γ_{tp} is somewhere between 0.02 and 0.081%. To evaluate γ_{tp} from such a multistage cyclic strain-controlled test more precisely, the approach by Hsu and Vucetic (2002, 2006) is used. The approach treats each cyclic stage, i , as a separate cyclic strain-controlled test. According to this approach, the results of all stages treated in this manner are plotted on a single plot in the format of the normalized cyclic pore water pressure change, Δu_{Ni}^* , with the number of cycles, N , versus the logarithm of γ_c , as shown in Fig. 11, and the obtained trend is then exploited to evaluate γ_{tp} . The approach also takes into account that a permanent pore water pressure change that occurs in each stage

Fig. 9. Variation of the average degradation parameter, t , with the cyclic shear strain amplitude, γ_c , in four multistage cyclic tests on kaolinite clay

with $\gamma_c > \gamma_{tp}$ affects the vertical effective stress at the beginning of the subsequent stage. Accordingly, in the above approach the cyclic pore water pressures, Δu_{Ni} , are plotted in the normalized format with respect to the effective vertical stress at the beginning of each stage, σ'_{vi} , which is obtained by subtracting the residual pore water pressure at the end of the previous stage ($i-1$) from the initial effective vertical consolidation stress of the test, σ'_{vc} , listed in Table 1. The normalized cyclic pore water pressure in stage i is thus calculated as $\Delta u_{Ni}^* = (\Delta u_{Ni} / \sigma'_{vi})$.

Threshold Shear Strain for Cyclic Pore Water Pressure Generation in Kaolinite Clay (MH; PI=28)

The results of three multistage cyclic strain-controlled tests on NC kaolinite clay (Kao32, Kao12, and Kao33) are presented in the format described above in Figs. 11–13. To obtain a more accurate estimation of γ_{tp} , only the cyclic stages just below and above γ_{tp} are considered, which is basically no more than three to four stages on each side of γ_{tp} . Each vertical set of data points on these charts corresponds to a single cyclic strain-controlled stage, i , i.e., to the change of the normalized cyclic pore water pressure with N in a single stage.

It can be seen that there is practically no cyclic pore water pressure change in the first three stages of test Kao32 and the first two

stages of tests Kao12 and Kao33. The cyclic threshold strain γ_{tp} is therefore somewhat greater than γ_c applied in these initial stages. In each test, in the subsequent stage beyond these initial stages, there is a small but noticeable change in the normalized cyclic pore water pressure, Δu_{Ni}^* , indicating that γ_{tp} is smaller than γ_c applied in this subsequent stage. In the following stages with greater γ_c , much greater pressures Δu_{Ni}^* consistently develop. In all three tests the increase of Δu_{Ni}^* in the stages with $\gamma_c > \gamma_{tp}$ is very consistent such that γ_{tp} could be relatively easily estimated as the point where the curves corresponding to different N clearly start to diverge. The estimated γ_{tp} values for these three tests, Kao32, Kao12, and Kao33, are 0.034, 0.026, and 0.014%, respectively.

The equivalent estimations of γ_{tp} from tests Kao34 and Kao11 on the OC kaolinite clay with OCR = 4 are presented in Figs. 14 and 15. The corresponding γ_{tp} values are 0.016 and 0.017% respectively. However, the results show that, unlike in the NC kaolinite clay, the cyclic pore water pressure in the kaolinite clay overconsolidated to OCR = 4 decreases with the number of cycles. This trend was expected because it agrees with past investigations, which show that in the OC clays the cyclic pore water pressure may decrease with N , or decrease and then increase, depending on the PI of the clay, level of OCR, and magnitude of γ_c (e.g., Andersen et al. 1980; Matsui et al. 1980; Vucetic et al. 1985; Dobry and Vucetic 1987; Ohara and Matsuda 1988).

Fig. 10. Variation of the average degradation parameter, t , with the cyclic shear strain amplitude, γ_c , in four multistage cyclic tests on kaolinite-bentonite clay

The investigations cited above actually show that such a pore water pressure decrease and associated effective stress increase occur in spite of the fact that clays are cyclically degrading. The time histories of test Kao11, presented in Fig. 16, illustrate clearly such a relationship between the degradation and pore water pressure development. The microstructural changes responsible for such a behavior that seem contrary to the effective stress principle are discussed in detail by Dobry and Vucetic (1987) and are here just briefly summarized. In OC clays, these changes involve mainly the breakage of clay particle bonds from cyclic shearing, which results in the softening of the soil (cyclic degradation) and, in turn, the tendency toward dilation as a result of the strong interparticle repulsion forces generated during preconsolidation. In undrained, constant volume conditions, this tendency toward dilation causes the development of negative pore water pressures. This means that the breakage of bonds that causes the degradation is also responsible for the negative pore water pressures, i.e., that without the breakage of bonds the negative pore water pressures would not develop.

Furthermore, the past studies and Figs. 14–16 show that the cyclic pore water pressure decrease in OC clays is typically very small. Accurate measurements of such small pressure variations,

especially those just above the threshold, are obviously more difficult than measurements of large positive pore water pressures in the NC clays. Consequently, the evaluation of γ_{tp} in OC clays is more challenging and subjective than in normally consolidated clays.

Threshold Shear Strain for Cyclic Pore Water Pressure Generation in Kaolinite-Bentonite Clay (CH; PI=55)

Estimations of γ_{tp} from the two multistage cyclic strain-controlled tests on the NC kaolinite-bentonite clay KaoBen01 and KaoBen02 are presented in Figs. 17 and 18. The data do not seem as consistent as those of the kaolinite clay above, which makes the estimations more challenging. The estimated values of γ_{tp} are 0.052 and 0.078%, respectively.

The values of γ_{tp} in the tests KaoBen03 and KaoBen04 on the OC clay could not be estimated reliably from the recorded data. The equivalent pore water pressure changes were too small in relation to the precision of the NGI-DSS closed-loop device for the specimen height control. As explained above, in the NGI-DSS test the specimen height control is employed to maintain the volume of the

Fig. 11. Change of the normalized equivalent cyclic pore water pressure, Δu_{Ni}^* , with the cyclic shear strain amplitude, γ_c , in the stages of the multistage cyclic strain-controlled test Kao32 on normally consolidated clay

Fig. 13. Change of the normalized equivalent cyclic pore water pressure, Δu_{Ni}^* , with the cyclic shear strain amplitude, γ_c , in the stages of the multistage cyclic strain-controlled test Kao33 on normally consolidated clay

Fig. 12. Change of the normalized equivalent cyclic pore water pressure, Δu_{Ni}^* , with the cyclic shear strain amplitude, γ_c , in the stages of the multistage cyclic strain-controlled test Kao12 on normally consolidated clay

Fig. 14. Change of the normalized equivalent cyclic pore water pressure, Δu_{Ni}^* , with the cyclic shear strain amplitude, γ_c , in the stages of the multistage cyclic strain-controlled test Kao34 on overconsolidated clay

specimen constant. The only reliable results from tests KaoBen03 and KaoBen04, which are available in (Mortezaie 2012), are that in both tests γ_{tp} is somewhere between 0.01 and 0.1%.

Summary on γ_{tp} Values and Trends

The values of γ_{tp} from seven tests, five on kaolinite clay (MH; PI = 28) and two on kaolinite-bentonite clay (CH; PI = 55), are listed in Table 1. The values of γ_{tp} for the kaolinite clay are between 0.014 and 0.034%, which is a moderate range. No particular trends of γ_{tp} with σ'_{vc} , OCR, and f can be identified from the results on kaolinite clay. The values of γ_{tp} for the kaolinite-bentonite clay are 0.052 and 0.078%, i.e., considerably greater than for the kaolinite clay. Such a difference between the two clays confirms the results

of previous studies, which show that γ_{tp} for clays with higher PI is generally greater (Vucetic 1994b; Hsu and Vucetic 2006).

Difference between the Thresholds for Cyclic Degradation and Cyclic Pore Water Pressure Generation

The ratios between γ_{tp} and γ_{td} listed in Table 1 show that in all but one test γ_{tp} is greater than γ_{td} . In one test γ_{td} and γ_{tp} are estimated to be the same. The average (γ_{tp}/γ_{td}) ratio is 2.5, while the largest ratio of 6 is obtained from test KaoBen02. A similar difference was noticed earlier (Tabata and Vucetic 2010). It is evident that the changes of clay structure responsible for the cyclic degradation

Fig. 15. Change of the normalized equivalent cyclic pore water pressure, Δu_{Ni}^* , with the cyclic shear strain amplitude, γ_c , in the stages of the multistage cyclic strain-controlled test Kao11 on overconsolidated clay

Fig. 17. Change of the normalized equivalent cyclic pore water pressure, Δu_{Ni}^* , with the cyclic shear strain amplitude, γ_c , in the stages of the multistage cyclic strain-controlled test KaoBen01 on normally consolidated clay

Fig. 16. Time histories of test Kao11 on overconsolidated kaolinite clay (PI = 28; OCR = 4) showing how in the stages beyond γ_{tp} the cyclic pore water pressure first decreases and then increases while, at the same time, the cyclic degradation takes place

start occurring at smaller γ_c than the changes responsible for the generation of permanent pore water pressure. Somewhat similar phenomena are observed in the cyclic behavior of dry and fully saturated sands. In the cyclic tests on dry sand, Drnevich and Richart (1970) obtained an increase in G_{SN} at γ_c smaller than γ_c necessary to generate the permanent volume change, i.e., at γ_c smaller than the threshold shear strain for cyclic compression (the volumetric cyclic threshold shear strain). In cyclic tests on fully saturated sands, the changes of modulus G_{SN} also start occurring at γ_c smaller than γ_{tp} (Vucetic and Mortezaie 2015).

Comparison to Previously Published Data

The values of γ_{td} obtained earlier on four clays are listed in Table 2 along with the values obtained in the present study. All of these data are plotted versus PI in Fig. 19. Although the previous results on four soils show a clear trend of γ_{td} increasing with PI, when

Fig. 18. Change of the normalized equivalent cyclic pore water pressure, Δu_{Ni}^* , with the cyclic shear strain amplitude, γ_c , in the stages of the multistage cyclic strain-controlled test KaoBen02 on normally consolidated clay

combined with the present data the trend is not as strong. The data in Fig. 19 indicate that γ_{td} for different clays is approximately between 0.006 and 0.05%. As more data become available, this range may change and a stronger trend between γ_{td} and PI may possibly emerge.

The values of γ_{tp} for six clays tested earlier are listed in Table 3 along with the values obtained in the present study. All data are plotted versus PI in Fig. 20. The chart shows that all data points fall roughly between 0.01 and 0.1% and exhibit a modest trend of γ_{tp} increasing with PI.

Comparison of charts in Figs. 19 and 20 show again that γ_{tp} is generally greater than γ_{td} .

As far as the effects of σ'_{vc} , OCR, and f are concerned, from the limited data obtained in the present study no clear effects on γ_{td} and γ_{tp} can be identified. The authors believe that these effects actually do exist but are small, and that they could be obtained from larger-scale studies with many more tests.

Summary and Conclusions

Under moderate to large cyclic shear strains, fully saturated clayey soils experience cyclic degradation of stiffness and permanent pore

Fig. 19. Relationship between the threshold shear strain for cyclic degradation, γ_{td} , and soil's plasticity index, PI, from the cyclic simple shear test data obtained in this study and the study by Tabata and Vucetic (2004, 2010)

Table 2. Threshold Shear Strains for Cyclic Degradation, γ_{td} , Obtained on Four Fine Grained Soils by Tabata and Vucetic (2004, 2010) Using NGI-Type Dual-Specimen Direct Simple Shear Device (Doroudian and Vucetic 1995) and γ_{td} Data Obtained in This Study

Unified soil classification symbol	Plasticity index (PI)	Effective vertical stress prior to cyclic shearing, σ'_{vc} (kPa)	Overconsolidation	Estimated threshold shear strain for cyclic degradation, γ_{td} (%)	Reference
CL-ML	6	276	Not reported	0.006	Tabata and Vucetic (2004, 2010)
ML	12	280		0.015	
CL	26	37		0.040	
CH	47	274		0.050	
MH	28	218	OCR = 1	0.012	This study
		222		0.012	
		680		0.014	
		212	OCR = 4	0.013	
CH	55	210		0.013	
		220	OCR = 1	0.016	
		668		0.013	
		211	OCR = 4	0.014	
		113	OCR = 7.8	0.012	

Table 3. Threshold Shear Strains for Cyclic Pore Water Pressure Generation, γ_{tp} , Obtained Previously by Different Researchers on Six Fine Grained Soils and γ_{tp} Data Obtained in This Study

Unified soil classification symbol	Plasticity index (PI)	Effective vertical stress σ'_{vc} (kPa)	Overconsolidation	Estimated threshold shear strain for cyclic pore water pressure generation γ_{tp} (%)	Test type	Reference
ML-MH	14.4	446	Believed to be small based on only positive Δu_{Ni}	0.050	NGI-type direct simple shear	Hsu and Vucetic (2002, 2006)
MH	20	108		0.023		
		220		0.025		
CH-CL	30	222		0.020		
		666		0.030		
CH	Approximate 40–50	20–26	Believed to be small based on information provided	0.04–0.07	High amplitude resonant column	Andreasson (1979,1981)
MH	55	200–400	OCR = 1	Approximate 0.10	Triaxial	Matsui et al. (1980)
CH/MH	25	49	OCR = 1	0.080	Kjellman-type (Swedish) direct simple shear	Ohara and Matsuda (1988)
MH	28	218	OCR = 1	0.034	NGI-type direct simple shear	This study
		222		0.026		
		680		0.014		
		212	OCR = 4	0.016		
		210		0.017		
CH	55	220	OCR = 1	0.052		
		668		0.078		

water pressure changes. The cyclic shear strain amplitude, γ_c , below which there is practically no cyclic degradation and above which a noticeable degradation occurs, is known as the threshold shear strain for cyclic degradation, γ_{td} . The amplitude γ_c , below which there is practically no permanent cyclic pore water pressure change and above which there is a noticeable change, is known as the threshold shear strain for cyclic pore water pressure generation, γ_{tp} . Nine multistage cyclic strain-controlled NGI direct simple shear tests were conducted on two laboratory-made, fully saturated, normally consolidated (NC) and overconsolidated (OC) clays, a kaolinite clay having PI = 28 and a kaolinite-bentonite clay having PI = 55. Among the nine tests, there were generally three different levels of the vertical effective consolidation stress, σ'_{vc} (113 kPa, 210 to 222 kPa, and 668 to 680 kPa); three OCRs (1, 4, and 7.8); and two frequencies of cyclic loading, f (0.01 and 0.1 Hz).

For the NC kaolinite clay, γ_{td} ranges in three tests between 0.012 and 0.014% and γ_{tp} between 0.014 and 0.034%. For the OC

kaolinite clay with OCR = 4, γ_{td} in two tests is 0.013% and γ_{tp} 0.016 and 0.017%. For the NC kaolinite-bentonite clay, γ_{td} in two tests is 0.013 and 0.016% and γ_{tp} 0.052 and 0.078%. For the OC kaolinite-bentonite clay with OCR = 4, γ_{td} is 0.014%, and with OCR = 7.8 it is 0.012%.

The comparison between the magnitudes of γ_{td} and γ_{tp} confirms previous observations that for the same soil tested under the same conditions, γ_{tp} is greater than γ_{td} . In the present investigation, with the exception of just one test, the (γ_{tp}/γ_{td}) ratios range between 1.2 and 6.

The trends of γ_{td} and γ_{tp} with σ'_{vc} , OCR, and f could not be identified, because for such analyses a much larger testing program is necessary. However, the present results indicate that these effects, if they exist, are small.

A comparison of the present results with those from earlier investigations shows that γ_{td} for six different soils ranges approximately between 0.006 and 0.05%, while γ_{tp} for eight different soils ranges approximately between 0.01 and 0.1%. The comparison also shows a modest trend of γ_{td} and a moderate trend of γ_{tp} increasing with PI.

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Fig. 20. Relationship between the threshold shear strain for cyclic pore water pressure generation, γ_{tp} , and soil's plasticity index, PI, obtained in this and four previous studies

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