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# The Armed Peace: A Punctuated Equilibrium Theory of War

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*According to a leading rationalist explanation, war can break out when a large, rapid shift of power causes a credible commitment problem. This mechanism does not specify how inefficient fighting can resolve this cause, so it is an incomplete explanation of war. We present a complete information model of war as a sequence of battles and show that although opportunities for a negotiated settlement arise throughout, the very desirability of peace creates a commitment problem that undermines its likelihood. Because players have incentives to settle as soon as possible, they cannot credibly threaten to fight long enough if an opponent launches a surprise attack. This decreases the expected duration and costs of war and causes mutual deterrence to fail. Fighting's destructiveness improves the credibility of these threats by decreasing the benefits from continuing the war and can eventually lead to peace. In equilibrium players can only terminate war at specific windows of opportunity and fighting results in escalating costs that can leave both players worse off at the time peace is negotiated than a full concession would have before the war began.*

To understand why wars begin, we have to know why they end—the termination of war must involve the resolution of its causes (Blainey 1988, x). In one of the two major rationalist accounts, war occurs when actors have private information about their expected value of war and crisis bargaining fails because they are unable to reveal this information credibly (Fearon 1995; Powell 1999). Fighting then resolves the cause of war by providing additional information that enables actors to coordinate their expectations and negotiate peace (Slantchev 2003). Whereas this constitutes a complete explanation of war in Blainey's sense, there are very good substantive reasons to insist on a theory that does not rely on incomplete information (Powell 2006). The prime candidate is the other major rationalist account: the *commitment problem*. In general, in order to avoid bargaining failure one or both actors must commit to certain courses

of action in the future. The problem arises when these promises or threats are not credible because when the contingency arises the actors have no incentives to abide by them. Such commitments are unbelievable, and as a result cannot achieve peace in the present. Although usually couched in terms of preventive war (a rising challenger cannot promise to deliver future benefits to a declining state in quantity sufficient to offset the latter's gain from waging war while still relatively strong), the mechanism is much more general (Powell 2004).

For instance, imagine a situation in which there are first-strike advantages: one's expected payoff from war is strictly higher when one is the first to attack than when one must absorb an attack before retaliating. As Fearon (1995) shows, if this advantage is sufficiently high, then bargaining will break down because at least one of the actors will not be able to credibly promise not to attack

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when the other is holding back. Powell (2006) demonstrates that in this scenario, actors face the equivalent of a *large, rapid power shift*: one can either strike first or permit the advantage to shift to the opponent by allowing him to achieve first strike. As Powell explains, this makes the mechanism strategically equivalent to the usual preventive war explanation that relies on shifts in power resulting from differential rates of economic or military growth (Organski 1968).

Unfortunately, none of the different specifications of the commitment problem as it currently stands meets Blainey's requirement: the theory does not explain how inefficient fighting can resolve the commitment problem short of the total obliteration of one of the warring states. Because outcomes of such finality are very rare, the theory does not provide a satisfactory answer to this puzzle. But if fighting does not resolve the commitment problem, it is pointless to bear its costs and risks in this context: in the end, the commitment problem will still exist and both states would have suffered tremendously. Moreover, if the opponents can terminate the war without resolving the commitment problem, then what does it mean to say that the war was caused by this problem in the first place?

We have several goals in this article. First, we elucidate Blainey's requirement and explain why it is imperative that our explanations of war are complete and coherent. We then discuss how the commitment problem mechanism fails these requirements and why. This leads us to a model with a stylized representation of war that simultaneously addresses our objections to the traditional approach and allows the commitment problem to arise. Our second goal is to demonstrate using this model that large, rapid shifts of power are not sufficient to cause war. Instead, peace fails when states cannot credibly threaten to impose large enough costs to deter each other from trying to exploit the advantages of surprise attack. The credibility problem arises from the very desirability of peace; states cannot threaten to prolong fighting more than absolutely necessary. However, through its very destructiveness, fighting may enable them to commit to deterrent threats that terminate the war. We show that the size of the power shift is determined by the strategies the actors choose. This implies that an account of war as a commitment problem must explain why actors fail to use strategies that reduce the power shift and avoid fighting. We then use these results to show how rational actors can escalate and incur costs far out of proportion to the gains and how by the time war is settled both actors are likely to be worse off in the resulting settlement than just conceding the prize from the outset without fighting, why wars tend to be settled when actors are nearing exhaustion, and

why peace is only possible at specific junctures during the conflict.

## The Credible Commitment Problem

The *bargaining model of war* posits a fundamental puzzle that any rationalist theory of war must be able to answer. Because fighting destroys resources, the benefits that players can divide after a war are always less than the benefits they could have divided prior to it. This means that it should be possible to locate *ex ante* agreements that would leave both players better off compared to what they can expect to get from fighting. The *inefficiency puzzle* is why actors are unable to reach such an agreement even when their existence is common knowledge (Fearon 1995; Powell 1999).

While most of the recent work has focused on informational asymmetries as a cause of war, there are compelling reasons to consider causal mechanisms that do not depend on incomplete information (Powell 2006). First, uncertainty is ubiquitous and is present in almost all crises, and yet only a few of these escalate into war. Second, the information revelation mechanism cannot deal very well with long wars: to think that it takes many years of near constant interaction for opponents to learn enough about each other is surely stretching the theory. Third, Leventoglu and Tarar (2005) show that private information by itself often merely leads to delay in reaching a negotiated settlement preferable to war, and for bargaining to break down in war requires that players are impatient enough. Fourth, Fey and Ramsay (2007) demonstrate that mutual optimism, which is often at the core of informational explanations such as Blainey's (1988), cannot be the cause of war if both parties must agree to fight for war to occur.

This means that we need an approach that does not rely exclusively on incomplete information. The major explanation of war under complete information that has emerged in the bargaining model of war is the *credible commitment problem* (CCP) caused by large, rapid shifts of power. Powell (2004) derives a condition that ensures that such power shifts will be sufficient to cause war in a wide class of models. Powell (2006) further demonstrates that bargaining indivisibilities, first-strike advantages (Fearon 1995), and bargaining over objects that are sources of military power (Fearon 1996) also turn out to be manifestations of this fundamental mechanism.

To illustrate the CCP, imagine an environment in which an actor,  $S_1$ , expects a significant advantage from initiating war while his opponent,  $S_2$ , is unprepared for fighting. To achieve peace,  $S_2$  must be able to reduce the

temptation to  $S_1$  either by promising future compensation through peaceful transfers or by threatening to wage war that will be sufficiently costly to  $S_1$ . In other words, to avoid war today,  $S_2$  must commit to providing benefits or inflicting costs on  $S_1$  in the future. However, this commitment will not be credible if  $S_2$  has no incentives to follow through on it. If  $S_2$  becomes strong tomorrow, she will not want to provide the promised compensation. If fighting is very costly to  $S_2$  as well, then she will not want to fight with the ferocity necessary to inflict the required costs on  $S_1$ . In either case, the future commitment becomes unbelievable and  $S_1$  cannot be induced to forego war in the present.

For the mechanism to work, the power shift must be sufficiently *large*. That is, the difference in the expected payoffs from war when fighting from a position of strength and fighting from a position of weakness must be so large that it (a) makes war tempting to the strong and (b) makes it impossible for the weak to commit credibly to deliver enough benefits or impose enough costs to offset that difference. The shift must also be sufficiently *rapid*. That is, no negotiation can occur before the advantage shifts from one actor to the other. If actors can adjust piecemeal while the shift is happening, then the declining actor is vulnerable to salami tactics, through which the rising opponent extracts small concessions in a series of steps, avoiding war altogether (Fearon 1996). In the first-strike environment, the power shift is essentially instantaneous: when one decides to forego striking first, one is immediately vulnerable to a first strike by the opponent. This further implies that peace must make both actors simultaneously unwilling to exploit the temporary advantage a first strike would accord them.

Although the CCP mechanism seems to provide an adequate explanation of the outbreak of war, it raises another fundamental puzzle: *how does fighting resolve the commitment problem?* To see why this is important, note that any rationalist explanation of war must meet two minimalist criteria. It must be *complete*, which means it must account for war's outbreak and termination; and it must be *coherent*, which means that its account of termination must explain how fighting has resolved the cause. In other words, a theory of war is complete and coherent if it can identify a cause of war that fighting then resolves. Whereas it can be shown that the asymmetric information explanation is both coherent and complete, the CCP is neither because it is silent on how inefficient fighting can resolve the commitment problem.

One trivial answer is that it does so by eliminating one of the opponents. However, this is empirically untenable: many wars end in negotiated settlements. For example, out of 104 interstate wars between 1816 and 1991, 67 (or 64%) ended short of total military victory

for one side.<sup>1</sup> Most wars end because both sides *agree* to cease hostilities while both could fight on. That is, they strike a bargain where previously they have been unable to do so. Furthermore, if we are to sustain CCP as a *cause* of war, then it must be the case that when players agree to settle, this problem is resolved. If this were not true, then peace would be possible even in the presence of the commitment problem, which would then imply that this problem could not have been the cause of fighting in the first place. Gartzke (1999, 571–72) notes this shortcoming of CCP and even goes on to argue that because it cannot be overcome, any rationalist explanation of war necessarily requires incomplete information. We want to contest this claim by showing at least one way in which war can resolve the commitment problem. We develop a model where the power shift arises from first-strike advantages but since the underlying mechanism is shared by the other ways to generate large, rapid power shifts, the results have broader implications.

We now enumerate several minimal features that the model must have to address the puzzle we have identified. First, we must represent war as a sequence of costly engagements rather than a game-ending costly lottery over exogenous outcomes because we wish to trace the effect of fighting. We must allow for negotiations during fighting, and we must allow for the game to end with a military victory in addition to a peace settlement. Second, we must incorporate large, rapid shifts of power to create an environment where the original CCP can manifest itself. Third, contrary to most existing bargaining models, we should not assume that an agreement automatically ends the game with the division of benefits. Rather, we must allow for the possibility that actors can renege on the agreement and attempt to use the newly acquired resources to extract further concessions. In other words, peace, if it happens, must be endogenous to the model. Finally, ideally we would not want our results to be dependent on a particular choice of a bargaining protocol, and the model must have complete information.

## The Model

Two players, each initially endowed with some capital  $K_i > 0$ , dispute a prize worth  $v \geq 0$ .<sup>2</sup> Each period  $t$  ( $t = 1, 2, \dots$ ) consists of two rounds: bargaining and fighting. Let  $S(t)$  denote the entire surplus at the

<sup>1</sup>Our calculations use Slantchev's (2004) data set, which separates war outcomes into those where one side is unable to continue the military contest and those where both sides agree to a settlement while still physically able to continue fighting (Kecskemeti 1958).

<sup>2</sup>We refer to player 1 (or a generic player  $i$ ) as "he," and player 2 as "she."

beginning of period  $t$ . If players reach an agreement  $(x_1, x_2)$  such that  $x_i \in [0, S(t)]$  and  $x_1 + x_2 = S(t)$  in the bargaining round, they implement it and resources become immediately available for fighting. If they fail to reach an agreement, they keep whatever resources they had at the beginning of the period. After the bargaining round, players simultaneously choose whether to fight. If both choose not to fight, the game ends and the negotiated distribution of resources remains. If at least one player chooses to fight the other, a battle occurs and each loses  $c_i \geq 1$  units of his resources, with  $C = c_1 + c_2$  being the total cost. The game transits to the next period with probability  $(1 - p)$ , or one of the players collapses with probability  $p < 1$ . How this probability is distributed between the two players depends on their actions during the period: if they both choose to fight, then either player 1 collapses with probability  $p_2$  or player 2 collapses with probability  $p_1 = p - p_2$ ; if  $i$  chooses to fight but  $j$  chooses not to, then  $j$  collapses with probability  $p$ , and  $i$  collapses with probability 0 (that is, if there is going to be a battle, it pays to participate in it). Since the game can reach period  $t$  only after  $t - 1$  fights, the total surplus at the beginning of period  $t$  is given by  $S(t) = K_1 + K_2 + v - (t - 1)C$  for every history of the game that leads to  $t$ . How long players can survive fighting depends on their resource endowment. Because capital stocks are finite, the game ends in a finite number of periods.

*Payoffs.* A player who collapses fighting in period  $t$  derives utility 0, and the surviving player gets  $S(t) - C$ ; that is, the victor absorbs the loser's remaining resources. If player  $i$ 's capital stock falls below  $c_i$  in  $t$ , then  $i$  collapses automatically without an additional battle with a payoff of 0, and  $j$ 's payoff is  $S(t)$ . If both players run out of resources at the same time, they collapse simultaneously and split the total surplus equally, each obtaining  $S(t)/2$ . If both players choose not to fight at time  $t$ , the game ends in peace and each keeps his share as agreed upon. Since there is no confrontation in this case, there is no capital loss.

*Strategies.* A history of the game is the list of acceptance/rejection decisions and the decisions to fight. Formally, let  $h_t$  denote the history of the game at the beginning of period  $t$ , and set  $h_0 = \emptyset$ . A bargaining protocol  $x(h_t) = (x_1(h_t), x_2(h_t)) \in [0, S(t)]^2$  determines a proposal in the bargaining round of period  $t$  after every history  $h_t$ , where any  $x_1(h_t) + x_2(h_t) > S(t)$  is not feasible, therefore rejected automatically.<sup>3</sup> This formalization allows for any bargaining protocol that distributes all available resources in finite time in each bargaining round. Let

$k_i(h_t)$  denote player  $i$ 's capital stock at the beginning of period  $t$  after history  $h_t$ , so  $k_i(h_1) = K_i$ .

A strategy,  $\sigma_i$ , for player  $i$  determines  $i$ 's actions in the bargaining and fighting rounds after every possible history. Formally, given a history  $h_t$ , players decide in the bargaining round of period  $t$  whether to agree to the division  $x(h_t)$ . Let  $a_i^t \in \{\text{accept, reject}\}$  denote  $i$ 's decision to agree to the division  $x(h_t)$ . Given decisions  $a^t = (a_1^t, a_2^t)$ , let  $k_i^t(h_t, a^t)$  denote player  $i$ 's capital stock at the beginning of the fighting round in period  $t$ . Then  $k_i^t(h_t, a^t) = x_i(h_t)$  if  $a^t = (\text{accept, accept})$ , and  $k_i^t(h_t, a^t) = k_i(h_t)$  otherwise. That is, if players agree on a distribution  $x(h_t)$ , it is implemented and resources become immediately available for fighting, and if at least one player disagrees, they keep the resources they had at the beginning of the period. After observing  $h_t, x(h_t)$ , and  $a^t$ , players simultaneously decide whether to fight or not. Let  $f_i^t \in \{\text{fight, do not fight}\}$  denote  $i$ 's decision in the fighting round. If players play  $a^t$  and  $f^t$  in period  $t$  and the game transits to period  $t + 1$ , the history at the beginning of period  $t + 1$  becomes  $h_{t+1} = (h_t, a^t, f^t)$ . In this case, the capital stocks at the beginning of period  $t + 1$  are given by  $k_i(h_{t+1}) = x_i(h_t) - c_i$  if  $a^t = (\text{accept, accept})$  and  $k_i(h_{t+1}) = k_i(h_t) - c_i$  otherwise.

The game ends when peace is reached or one of the players collapses, either militarily or through attrition. The terminal histories comprise outcomes where (i) players agree to peace—any history such that  $f^t = (\text{do not fight, do not fight})$ ; (ii) player  $i$  collapses from attrition—any history such that  $k_i(h_t) < c_i$  for some player  $i$ ; or (iii) a player achieves military battlefield victory—any history with  $f_i^t = \text{fight}$  for at least one player and in which the realization of the costly fighting lottery is the collapse of a player.

*Equilibrium Solution Concept.* Given a bargaining protocol, a strategy profile  $\sigma = (\sigma_1, \sigma_2)$  is a Nash equilibrium if  $\sigma_i$  is a best response to  $\sigma_j$  for  $i, j \in \{1, 2\}$  and  $i \neq j$ . Given a bargaining protocol, a Nash equilibrium is subgame-perfect (SPE) if the subgame strategies induced by  $\sigma$  constitute a Nash equilibrium in every subgame.

Subgame perfection eliminates incredible threats in the sense that no player would unilaterally deviate from the actions prescribed by the strategy when he is called to carry out a threat. However, this notion of credibility is too weak. For example, given a bargaining protocol, players may look for an alternative SPE that Pareto-dominates the one induced by the original strategies. They may even look for a different bargaining protocol that allows them to achieve another such SPE. In any case, we should expect players to renegotiate their original positions and switch to the new equilibrium. But if such alternatives exist, then the credibility of the original SPE is undermined and the

<sup>3</sup>A more elaborate notation for a proposal calls for a time index on  $x_i$ , e.g.,  $(x_1^t(h_t), x_2^t(h_t))$ . In order to simplify the exposition, we drop the time index on our functions when the time index in its argument suffices.

threats that sustain it become incredible: there is no reason players should expect to remain in this SPE if they can always go back to the negotiation table and find a better one. We should look for SPE that are immune to such renegotiations because these are the only ones that actors can reasonably expect to play. Hence, we focus on SPE that are *renegotiation-proof* in the sense of Farrell and Maskin (1989) as the only plausible candidates for solutions to our model.

Formally, a bargaining protocol and an induced SPE pair  $(x, \sigma)$  is *renegotiable* if there exists a subgame, a bargaining protocol  $x'$  starting at that subgame, and an SPE  $\sigma'$  induced by  $x'$  in that subgame that is weakly preferred by each player and strictly preferred by at least one player to the SPE induced by  $(x, \sigma)$  in that subgame. We say that a bargaining protocol and an induced SPE pair,  $(x, \sigma)$ , is renegotiation-proof (RPSPE) if it is not renegotiable. For the rest of the article, we will explicitly refer to SPE when we rely on SPE; otherwise, all references to an equilibrium will implicitly refer to RPSPE.

*Discussion.* It is worth discussing briefly some of the assumptions in this model and how they relate to the requirements we outlined in the previous section. First, war is a sequence of engagements rather than a one-shot event. We shall refer to these engagements as battles with the understanding that we mean any period of time during which fighting occurs and actors do not negotiate (e.g., campaigns). Military victory can be achieved in two ways: either by causing the military collapse of the opponent during a campaign or by exhausting the opponent's resource base. However, the war can end as soon as both actors agree to a division of the benefits and as long as neither then engages in additional fighting after the distribution. The model thus allows for negotiated outcomes without assuming away the commitment to uphold the resulting distribution in peace. In addition, we have assumed that when actors bargain, they can use their resources for side-payments too. We allow for a general bargaining protocol as long as it ends in finite time and does not artificially preclude settlement (we explain later what we mean by that). Furthermore, by allowing actors to turn around and use the resources obtained by negotiation for further war, we have effectively endogenized peace: any equilibrium that involves a successful negotiated settlement will necessarily incorporate disincentives to renege from it.

Second, we have created an environment where the original CCP can arise by allowing for large, rapid shifts of power when an actor surprises its opponent by attacking when the other is not. The shift of power arises from the increased probability of military collapse of the actor caught by surprise. This is very similar to the first-strike advantage notion that Fearon (1995) uses. The difference

is that instead of conferring an advantage for the entire war, it only does so temporarily on the tactical level. We find this assumption much more tenable for two reasons. Strategic surprise, although possible to achieve, is often indecisive for the entire war (the two most famous examples are the Japanese attack on Pearl Harbor in 1941 and the Egyptian/Syrian attack on Israel in 1973). On the other hand, tactical surprise often is decisive for the particular engagement, and such an engagement could end the war. For example, the surprise Spartan destruction of the Athenian fleet at Aegospotami ended the Peloponnesian War, the surprise Soviet invasion of Manchuria coupled with the dropping of the American atomic bombs ended the War in the Pacific, and the surprise Israeli crossing of the Suez Canal with the resulting encirclement of the Egyptian army in the Sinai ended the Yom Kippur War. The assumption that the side which achieves surprise has zero probability of collapse in that engagement is made for simplicity and does not affect the results as long as that probability is well below the one for the surprised opponent. Finally, it is worth stressing that the costs are per engagement, not for the entire war. This makes the model a bit more attractive on substantive grounds: we do not have to assume, like most models do, that war costs are relatively small. Indeed, as we shall find, cumulative war costs here can be gargantuan, far exceeding the value of the prize actors are fighting over.

## Total War

Total war occurs when players fight until one of them collapses from exhaustion. In this section, we establish the conditions for such an equilibrium. (All proofs are in the appendix.) We first show that if there exists a period from which players will fight at least one battle, then they will also fight in all previous periods provided  $S$  is large enough. We then prove that players will fight to the end if at least one of them is sure to collapse after one battle. Together, these results imply that if  $S$  is large enough, total war will be inevitable: in any SPE, players will fight without redistributing resources until one of them collapses from exhaustion.

**Lemma 1.** *If players fight for  $T \geq 1$  periods when the resources are  $(k_1, k_2; S)$ , then they fight  $T + 1$  periods when the resources are  $(k_1 + c_1, k_2 + c_2, S + C)$  provided that  $S$  and  $p$  are large enough.*

It is worth elaborating what this lemma means. We are not claiming that if players fight for some  $(k_1, k_2; S)$ , then they would fight for an arbitrary distribution  $(\hat{k}_1, \hat{k}_2; S + C)$  where  $\hat{k}_i \neq k_i + c_i$ . Rather, we prove that

if the resource distribution  $(k_1, k_2; S)$  is such that players fight, then players will also fight when the resource distribution is exactly  $(k_1 + c_1, k_2 + c_2; S + C)$  as long as  $S$  and  $p$  are large enough (observe that a large  $v$  will be sufficient to ensure that this will happen). The next step is to locate a period in which players will fight for sure.

**Lemma 2.** *Consider a period with  $(k_1, k_2; S + C)$ . If  $k_i \in [c_i, 2c_i)$  and  $k_j \geq 2c_j$ , then players fight in this period provided that  $p_j S > C$ .*

In words, if one of the players has just enough resources for exactly one additional battle, then no peace is possible if the stakes are sufficiently high. If players have not redistributed resources until the period prior to  $i$ 's imminent collapse, then  $S > v$ , and hence  $p_j v > C$  will be sufficient to ensure war in that period. This means that if  $v$  is large enough and players do not redistribute, then the last battle is inevitable. We now show that if the condition of Lemma 1 is satisfied, then players would not, in fact, redistribute before the penultimate period, which implies that they will fight a total war.

**Proposition 1 (Total War).** *If the conditions of Lemma 1 and Lemma 2 are satisfied, then players never redistribute resources and fight until the weaker one collapses from exhaustion or someone is decisively defeated in battle.*

This result gives us the first cut at a solution to the commitment problem: war resolves it by eliminating one of the opponents militarily. As Stalin famously (reportedly) quipped, "Death solves all problems—no man, no problem." In this total war equilibrium, the war may end short of one player getting exhausted if some player collapses in a battle. In this, the result is similar to the long civil wars one can observe in the equilibrium of Fearon's (2004) model, where this outcome is likewise probabilistic. From our bargaining perspective, this solution does not help answer the puzzle because players do not *choose* to end the war but are rather forced to by military exigencies. A nontrivial explanation must involve them choosing actions such that they fight and then settle on the equilibrium path, all with complete information.

## Limited War

Limited war occurs when players fight for some length of time and then settle on a negotiated redistribution. This is the most interesting case because it involves inefficient use of power under complete information: the opponents waste resources and then manage to negotiate the peace

even though they agree in expectation on how the war will evolve from the very beginning. The results in this section demonstrate that peace is crucially dependent on the ability to threaten war, and in particular, the ability to threaten to impose sufficient costs by prolonging the fight. The problem actors face when negotiating peace is that these threats may not be credible: when it is common knowledge that negotiations will be available in the future, and actors will be tempted to reach a peaceful agreement then, the incentive to prolong the war in the future is undermined, and hence the threat to impose costs today becomes unbelievable.

We now proceed in several steps. First, we show that for peace to occur, actors must credibly threaten to fight if it is violated. This suggests that the most permissive conditions for peace are those where players can credibly make the most deterrent threats, that is, threats that would impose the highest costs on the opponent. Since the most deterrent threats are those where players fight until the bitter end (when one of them collapses from exhaustion), the second step is to derive the equivalent to Powell's (2004) sufficiency condition for war: if peace cannot be attained when even the most deterrent threats are credible, then peace cannot be attained in any SPE. We demonstrate that fighting can lead to violation of this condition, opening up the road to peace. The next step is to show that peace can be attained once this condition fails. We construct an SPE in which peace can be sustained by threats to fight to the finish. We then demonstrate that even though these threats are subgame-perfect, they are not credible because players have incentives to renegotiate rather than fight a total war. We then construct an RPSPE that is immune to such renegotiations and investigate the conditions under which peace can be sustained.

## Strongest Deterrent Threats

We begin with the following lemma, which makes it very clear that peace is sustained by the threat of fighting off the equilibrium path.

**Lemma 3.** *Suppose  $(x_1, x_2)$  is a peaceful bargain when resources are  $(k_1, k_2; S)$ . Then there is no peaceful bargain when resources are  $(x_1 - c_1, x_2 - c_2; S - C)$  and  $S$  is large enough.*

Lemma 3 shows that if players are able to conclude a bargain that is peaceful in equilibrium and one of them tries to deviate and attack after the redistribution of resources according to that agreement, then the next period surely involves fighting as well. The longer the fighting one

can threaten with, the higher the expected costs for both players, and hence the less each would be willing to accept at the negotiating table, and the better the prospects for peace. The most deterrent threat is to fight to the bitter end. The minmax strategies to fight to the end do form a Nash equilibrium. Furthermore, these strategies can also be subgame-perfect depending on the bargaining protocol.<sup>4</sup> In other words, *if we allow players to commit to playing the Nash/SPE equilibrium that involves fighting to the end, then we are allowing them to make the most severe threats possible, which creates the strongest incentive to make peace today.* If for some period peace is impossible with the commitment to fight to the end should they fail to reach an agreement, peace certainly will not be possible in any subgame perfect equilibrium either.

We now turn to investigation of the conditions for war provided players can commit to their most deterrent threats. Suppose that given the current stocks  $(k_1, k_2; S)$ , the war can last at most  $T$  more periods. That is, if both players fight in each of the following periods without reallocating, then at least one of them will collapse after  $T = \min\{T_1, T_2\}$  fights, where  $T_i$  is the integer part of  $k_i/c_i$ . We refer  $T_i$  as  $i$ 's resolve.

Denote the current period as the first and let  $F_i(t | T)$  denote player  $i$ 's expected payoff from rejecting all offers and fighting to the end starting in period  $t$  and fighting up to, and including, period  $T$ . For example, suppose that one of the players would collapse after  $T = 7$  battles (so if they start fighting now, he would collapse in period  $T + 1 = 8$ ). Then  $F_1(3 | 7)$  would denote player 1's expected payoff in period 3 from fighting to this end (that is, fighting five more battles). When  $T = T_i < T_j$ , if players never reallocate and fight in each period, eventually player  $i$  will collapse in period  $T + 1$ . Hence,  $F_i(T | T) = p_i(v + k_1 + k_2 - TC) = p_i(v + k_j - Tc_j)$  and  $F_j(T | T) = (1 - p_i)(v + k_j - Tc_j)$ . For  $t \in \{1, 2, \dots, T - 1\}$ , define the following recursive equation:

$$\begin{aligned} F_i(t | T) &= p_i(S - tC) + (1 - p)F_i(t + 1) \\ &= p_i \sum_{n=0}^{T-t-1} (1 - p)^n [S - (n + t)C] \\ &\quad + (1 - p)^{T-t} F_i(T | T). \end{aligned} \tag{1}$$

<sup>4</sup>For example, players use a bargaining protocol that requires them to submit simultaneous demands, with agreement obtaining if, and only if, they sum up to  $S(t)$ . Then, the strategies "demand twice the entire surplus and fight regardless of outcome in the bargaining round" are subgame-perfect: given the strategy of  $j$ , agreement can never be reached regardless of the proposal  $i$  makes, and given  $j$ 's unconditional attack,  $i$ 's best response is to fight as well. We thank Bob Powell for suggesting this.

This is player  $i$ 's expected payoff in period  $t$  if both players fight without redistribution until the weaker player collapses. It is also player  $i$ 's reservation value: the payoff that can be unilaterally guaranteed. The lower this payoff for player  $i$ , the more deterrent  $j$ 's threat. Since fighting is inefficient,  $\sum_i F_i(1 | T) < S$ , so bargains that improve on the minmax payoffs always exist. However, as we shall now see, this is not enough for peace to occur. Since we want to find the most permissive condition for peace, we need to derive the most deterrent threats for both players. To do this, we show that the joint expected payoff from fighting until one player collapses from exhaustion is strictly decreasing in the number of potential fights:

**Lemma 4.** *When resources are redistributed to increase the number of potential fights, the sum of the players' payoffs from fighting to the end decreases.*

In other words, if the distribution  $(x_1, x_2)$  enables players to fight  $T$  periods at most, and the distribution  $(y_1, y_2)$  enables them to fight one more period, then the sum of their expected fighting payoffs under  $(y_1, y_2)$  is strictly worse, and hence  $(y_1, y_2)$  involves more deterrent threats and is more conducive to peace. As the proof shows, the joint loss if peace fails is precisely  $(1 - p)^{T-t+1}C$ , which is the cost of the additional battle times the probability of having to fight it.

The result in Lemma 4 is intuitive: the longer players expect the war to last, the more resources they expect to waste waging it. Hence, their joint payoffs must necessarily decrease in expected duration. The most deterrent threat a player can make is to fight to the end. Given total resources  $S$ , the maximum number of battles players can fight after some distribution is  $\bar{T} = S/C$ . The largest number of battles under  $(x_1, x_2; S)$  is  $T_1 = x_1/c_1 = x_2/c_2 = T_2 \Rightarrow x_i = c_i S/C$ , where we used  $x_1 + x_2 = S$ . Therefore, if players redistribute such that they have the same resolve,  $T_1 = T_2 = \bar{T}$ , they can make the most mutually deterrent threats possible, and the resulting environment would be most conducive to peace because deviations would be punished most severely.

At this point it is worth emphasizing that it is the structure of the model that requires longer fighting to increase the costs. Although this is intuitively sound, one can think of alternative ways to make warfare costlier to the opponent. Nuclear strikes are an extreme example of an instantaneous escalation that would cause very significant destruction, but it is also possible to envision more plausible scenarios that involve altering the targeting strategy or increasing the intensity of effort. Many of these may also make fighting very costly to the opponent without



necessarily increasing one's own costs in proportion to the effort as would be the case in the imposition mechanism that requires longer fighting. When such alternatives are available, it should be easier to make more deterrent threats, and the chances for peace should be better. When we talk of most deterrent threats in the model, we necessarily mean threats that involve the longest fighting spells, but one should keep in mind that the implications would hold for other mechanisms with suitable revisions.

### The Sufficient Condition for War

Simply offering a player his minmax payoff is not enough to induce him to agree to peace. Peace requires that *both players forego the advantages of surprise attack*. To see this, suppose a player accepted a division that exactly matched his minmax payoff,  $x_i = F_i(1 | T)$ , and suppose in equilibrium players achieve peace immediately. Since they achieve peace,  $j$ 's strategy after this distribution must be not to attack. But then surprise attack yields  $i$  at least  $p(S - C) + (1 - p)F_i(2 | T) > p_i(S - C) + (1 - p)F_i(2 | T) = x_i$ . Therefore, attacking is a best response for  $i$ , which contradicts the assumption that  $x_i$  is accepted in a peaceful equilibrium. Because a peaceful bargain must deter surprise attacks, it must exceed the minmax payoffs.

To see the minimum demands that players would make, suppose they have divided everything such that  $(x_1, x_2; S)$  with  $x_1 + x_2 = S$ . Let  $T = \min\{x_1/c_1, x_2/c_2\}$ , the resolve of the least resolved player, denote the largest number of battles they can fight under the new distribution without reallocation until one of them collapses from exhaustion. Player  $i$ 's payoff from sneak attack after the negotiated division is *at least*  $A_i(x_1, x_2) = p(x_1 + x_2 - C) + (1 - p)F_i(2 | T)$ . As we have established, peace requires that surprise attacks are not profitable. Therefore, fighting a battle is going to be unavoidable if there are not enough resources to satisfy minimum deterrent demands. That is, players are sure to fight if

$$A_1(x_1, x_2) + A_2(x_1, x_2) > S \quad \text{for all feasible } (x_1, x_2; S). \quad (2)$$

This is the logic of Powell's (2004) sufficiency condition which guarantees that complete-information bargaining will break down in any stochastic game, a general category that encompasses our model. To see that this is the case, note that  $F_i$  denotes player  $i$ 's minmax payoff in any period  $t$  because  $i$  can always guarantee himself this payoff by rejecting all offers and fighting in each period. Further, since  $A_i = p_j(S - C) + F_i(1 | T)$ ,  $p_j(S - C)$  reflects the increase in  $i$ 's payoff if he catches his opponent by surprise and results from the temporary "power shift" in

$i$ 's favor whenever his opponent is expected not to fight. Hence, in any peaceful equilibrium,  $i$  must obtain at least  $A_i$ , leaving at most  $S - A_i$  to meet the minimal demand of the other player. In a peaceful equilibrium both players must have their minimal demands satisfied, yielding the condition in (2).

To ensure that peace will not be possible, we have to establish that no distribution can violate (2). By Lemma 4,  $\sum_i F_i(2 | T)$  is minimized by taking the largest number of battles, which implies that  $\sum_i A_i \geq 2p(S - C) + (1 - p)\sum_i F_i(2 | \bar{T})$ . Since the right-hand side of this inequality is the *worst* players can jointly expect, if this amount exceeds the available surplus, then fighting is guaranteed. In other words, we obtain a sufficient condition of war in the current period:  $2p(S - C) + (1 - p)\sum_i F_i(2 | \bar{T}) > S$ , which reduces to

$$p^2(\bar{T} - 1) + (1 - p)^{\bar{T}} > 1. \quad (P)$$

Condition (P) is sufficient to guarantee that peace will not be possible in period  $t = 1$  because it means that a sneak attack is profitable for at least one player even if they redistribute resources to equalize resolve. This is because with equal resolve, deviation would lead to the longest possible war,  $\bar{T}$ , and hence the most deterrent punishment. If this punishment is not enough to deter sneak attack, then certainly no other redistribution would be able to. This condition is analogous to Powell's (2004), and we have emphasized this with our labeling choice.

### Peace with Threats of Total War

Whereas condition (P) shows that limited war is possible in principle, it does not prove that it can happen. With a sufficient condition for fighting, we only have a *necessary condition* for peace in its converse. Although, as we shall see, this condition for peace can be achieved through fighting, we do not know whether players will be able to commit to peace once it is satisfied. Perhaps whenever they fight in equilibrium, they always end up in a total war? If this is the case, then our arguments do not take us very far. Therefore, it is imperative to demonstrate that limited war can happen in equilibrium. That is, that players fight and then settle, all with complete information.

Condition (P) is defined entirely in terms of the fixed exogenous parameters and the total resources available at the beginning of the period. This means that we can apply this condition to each period of the game by taking  $S(t) = S - (t - 1)C$  to be the surplus in period  $t$ , and  $\bar{T}(t) = S(t)/C$  to be the maximum number of battles that can be fought until some player collapses from this period on provided players redistribute to equalize resolve. The

following lemma shows that fighting can lead to violation of that condition.

**Lemma 5.** *Condition (P) is satisfied if, and only if,  $\bar{T}$  is sufficiently high.*

Since  $\bar{T}$  depends on the amount of available resources,  $S(t)$ , which is decreasing as fighting continues, Lemma 5 states that even if war is certain under the initial resource distribution, the squandering of resources fighting entails will eventually make settlement possible. In other words, destruction opens up the road to peace by making surprise deviations from peace less profitable.

We now wish to see whether this implies that players can, in fact, achieve peace in SPE provided (P) fails. Since our model allows players to bargain over the protocol they use in addition to the distribution they achieve, all we have to do is show that peace can be sustained in SPE for some protocol that makes threats of total war upon deviation subgame-perfect. As we noted before, these threats can be SPE, and we now use this result to support a limited war equilibrium. Recall that (P) depends on  $\bar{T}$ , that is, threats of the longest possible total war (which can be fought only if players have equal resolve), whereas (2) depends on  $T$ , the longest total war that can be fought under the current distribution. If  $T_i \ll T_j$ , so one of the players is significantly less resolved than the other either because his resource endowment is smaller or costs of fighting larger, then  $T \ll \bar{T}$ , and the deterrent threat players can actually make without equalizing resolve will not be sufficiently strong to prevent fighting. In other words, even if (P) fails, (2) will still hold, and peace will be impossible. However, when  $T_1$  and  $T_2$  are not too dissimilar, then (2) will approximate (P), and peace will be possible when (P) fails.

**Proposition 2.** *If players are similarly resolved, they can achieve peace with threats of total war in SPE in any period in which condition (P) is not satisfied.*

This result gives us a first cut at a real solution to the commitment problem through fighting. Suppose the initial distribution of resources is such that players have similar resolve and condition (P) is satisfied. Players are guaranteed to fight and by Lemma 5 the destruction of resources eventually leads to violation of that condition. By Proposition 2, players then end the war immediately. This peace is sustained by threats to fight to the end if any player violates it with a sneak attack. Whereas such a threat is not capable enough to deter players while the resource base is very large, it does become sufficient as fighting shrinks that base. This happens because the power shift resulting from a sneak attack gets smaller and hence the

temptation to benefit from a deviation decreases. Since the original commitment problem depends on large, rapid shifts of power, this result shows that *fighting resolves the commitment problem by reducing the size of the power shift.*

It is natural to ask whether the mechanism through which this is achieved is reasonable. In some sense, it appears to be: after all, the threats are subgame-perfect, and hence no actor has an incentive to change strategy unilaterally. However, this is not enough to make them credible in a more intuitive sense.

### Credibility and Renegotiation Incentives

The result in Proposition 2 depends on finding an SPE in which fighting to the end is subgame-perfect. The key to constructing such an equilibrium is the fact that players make their attack decisions simultaneously and in effect cannot reciprocate unexpected peace feelers. To see what this means, suppose that the bargain protocol is such that players redistribute resources such that one of them, say player 1, obtains a share that is strictly better than what he is getting in the SPE they are playing but player 2's strategy is to fight at such an allocation. Suppose further that given player 2's strategy, player 1's best response is to attack as well. Since the bargaining outcome represents a deviation, players fight and the game continues. Now suppose that player 2 deviates and does not attack. Player 1's optimal course of action would be to reciprocate because peace then prevails, and his payoff under the new allocation is strictly better. If he could condition his attack decision on what player 2 does, then subgame-perfection would indicate that fighting is not credible. However, the extensive form does not allow the player to condition in that way and since he is forced to make the attack decision "in the dark," subgame-perfection does not have a bite.

On one hand, this may appear to be an artifact of the extensive form which we can alleviate by making fighting decisions sequential. On the other hand, the essence of a sneak attack seems to be that an actor is making the relevant decision in the dark. The modeling choice seems appropriate. However, there is still something unsatisfying about the threats that support the limited war SPE in Proposition 2.

Observe that the strategy to deter deviations requires punishments that hurt both players. In the example SPE, the punishment is extreme: total war. Suppose now that the players find themselves off the equilibrium path because someone has deviated, and they are doomed by their SPE strategies to fight to the bitter end. The question then arises: given that players have found themselves in an inefficient situation, would they renegotiate to get out of it?

That is, if there exists an SPE such that each player gets at least his total war payoff and at least one gets a strictly better payoff, it is reasonable to expect that players will then renegotiate their original “agreement” that has now led them to this mutually hurtful situation.

Given that players should be expected to search for a better SPE if they ever find themselves in an inefficient equilibrium and bearing in mind the fact that peace requires deterrent threats, the strongest threats they can make and still be believed are the ones that can be sustained in a renegotiation-proof SPE. We refer to threats that can be sustained in an RPSPE as *credible*, keeping in mind that such a notion of credibility is more demanding than the one required by subgame-perfection.

### Peace with Credible Threats

Intuitively, the strongest credible threat is the one that involves the most periods of fighting, provided players have no incentives to renegotiate during any of these periods. In other words, suppose players can achieve peace in some period  $T$ . Then, the strongest credible threat they can make in  $T - 1$  is to fight one battle. If they have no incentive to renegotiate in that period either, the strongest credible threat they can make in  $T - 2$  is to fight two battles, and so on. This immediately suggests that threats of total war may be incredible, which in turn means that condition (P) may not be able to pin down whether peace can be obtained in a limited war RPSPE. If players cannot commit to total war but to some (much smaller) number of battles, the total costs they can credibly impose on each other are also smaller. Since peace depends on the severity of punishment of sneak attacks, if this punishment is not that costly, the deterrent effect is so much weaker, and prospects for peace so much gloomier. The question then becomes this: can players achieve peace if they can use only credible threats?

Letting  $V_i(x_1, x_2; S)$  denote player  $i$ 's RPSPE payoff in the continuation game given a feasible distribution  $(x_1, x_2; S)$ , the necessary and sufficient condition for fighting at this distribution is  $\hat{A}_i(x_1, x_2) = p(S - C) + (1 - p)V_i(x_1 - c_1, x_2 - c_2; S - C) > x_i$  for some  $i$ . Hence, the sufficient condition for fighting using only credible threats is

$$\sum_i \hat{A}_i(x_1, x_2) = 2p(S - C) + \sum_i (1 - p) \times V_i(x_1 - c_1, x_2 - c_2; S - C) > S. \tag{W}$$

Because  $\sum V_i(x_1 - c_1, x_2 - c_2; S - C) \geq 0$  and  $S > C$ , it follows that for  $p$  and  $S$  large enough, (W) will be satisfied.

Bargaining in this period will break down and at least one battle will be guaranteed in any RPSPE. However, whereas (P) implies (W), there will be instances where (W) is satisfied but (P) is not. In other words, if condition (P) fails, we are not guaranteed that peace will occur because (W) may still be satisfied. If this is the case, then (P) does not provide a compelling resolution to the commitment puzzle because threats of total war are not credible. If, however, we can find an equilibrium in which fighting leads to violation of (W), then we do have such a mechanism.

We now use condition (W) to construct an example SPE that uses only credible threats and that demonstrates the main theoretical results that we use for our substantive discussion. Assume that players are *symmetric*, that is,  $K_1 = K_2 = K$ ,  $c_1 = c_2 = c$ , and  $p_1 = p_2 = p/2$ . Note that this immediately implies that they have the same resolve, and therefore should be in a position where peace is easiest to achieve. If we find limited fighting even in this environment, then we can certainly find it when resolve is asymmetric. Suppose the game begins with  $v > 0$ . After  $T$  battles without redistributing, each player has  $k_i = K - Tc$  resources left, so the total is  $S = 2k_i + v$ . If they distribute now, players face a situation with surplus  $S$  and  $v = 0$ , and therefore in any SPE the relevant behavior is that in these continuation games. Consequently, we now explore games with symmetric players and  $v = 0$ . We first show that in this setup, subgames with equalization of resources are particularly helpful because they capture all relevant aspects and are easy to analyze. We then use these results to construct our numerical example.

The following lemma proves that in games with symmetric players who have equal resources, we do not lose any generality by restricting analysis to subgames in which players do not redistribute along the equilibrium path. Using this result, the lemma further derives the necessary and sufficient condition for fighting in any arbitrary period.

**Lemma 6.** *Assume  $v = 0$  and symmetric players with  $k_i = nc$ , where  $n \geq 1$ . Then, without loss of generality, the players do not redistribute in equilibrium. Suppose that they find a peaceful settlement when  $k_i = nc$ . Then*

- (A) *they fight with  $k_i = (n + 1)c$  if, and only if,  $pn > 1$ ;*
- (B) *if they fight with  $k_i = (n + t)c$ ,  $t = 1, \dots, T$ , then they fight with  $k_i = (n + T + 1)c$  if, and only if,*

$$p^2(n + T) + (1 - p)^{T+1} > 1. \tag{3}$$

Observe now that by Lemma 6, if players are symmetric and have equal resources, there is no loss of generality if we consider only SPE where players do not redistribute. This now means that if they cannot achieve peace in equilibrium with  $(nc, nc)$ , they cannot achieve it under any

alternative allocation. This yields the following helpful result:

**Corollary 1.** *Suppose players are symmetric and cannot achieve peace if they redistribute such that they maintain equal resolve:  $k_i = nc$  and  $S = 2k_i$ . Then they cannot achieve peace under any alternative distribution.*

This is a powerful result: for any distribution  $(x_1, x_2)$  with  $v = 0$  and  $S = x_1 + x_2$ , we only need to check if players can achieve peace by equalizing their shares (because with symmetric players equalizing shares implies equalizing resolve). That is, we use the conditions in Lemma 6 to check if players can achieve peace by sharing  $(S/2, S/2)$ . This saves us a lot of work because otherwise we would have had to compute continuation values for any subgame with shares  $x_i - c$ , rather than just with  $S/2 - c$ . But since in any period with  $v > 0$ , the credibility of peace will depend on what happens after they redistribute, this result provides the key to unraveling the SPE in the entire game. The algorithm we use is to take any period, suppose they equalize resources, and check if they will still fight using Lemma 6. If they do, then no alternative distribution can produce peace (by Corollary 1), and we know a battle is inevitable in any SPE. If they do not, then they will certainly achieve peace in this period (because we have identified at least one distribution that can do it and because we assumed that players will be able to utilize the opportunity).

**Example 1 (Limited War).** Assume  $v = 12$  and  $p = .18$ , and symmetric players with  $K_i = 30$ ,  $c_i = 1$ , and  $p_i = p/2$ . The players fight 12 battles without distributing resources and achieve peace in  $t = 13$  provided neither collapses in the interim.

Since the construction is illustrative, we show it here. Letting  $S$  denote the surplus in period  $t$  and noting that since players do not redistribute in any period in which they fight, we can use the time index to denote the continuation values, and so  $V_i(t)$  is player  $i$ 's expected equilibrium payoff in period  $t$ . Condition (W) then is  $S < .36(S - 2) + .82 \sum_i V_i(t + 1)$ . If (W) fails in some period  $t$ , then  $\sum_i V_i(t) = S$  because the entire surplus is peacefully distributed by our requirement that fighting cannot continue beyond periods in which peace is possible under some bargaining protocol and induced SPE. This effectively limits the duration of war to which players can credibly commit to that period. If, on the other hand, (W) is satisfied, then peace is impossible in this period and  $\sum_i V_i(t) = p(S - C) + (1 - p) \sum_i V_i(t + 1)$ .

Recall from the proof of Proposition 1 that if players are sure to fight in some period  $t$ , they cannot improve

matters by redistributing, and therefore there is no strict incentive to do so in equilibrium. In other words, we can restrict attention to SPE where players do not redistribute in any period in which they expect to fight for sure. This now allows us to backward-induct along the no-distribution path using condition (W) and Lemma 6.

Consider now the no-redistribution path at  $t = 30$ , where players have fought 29 battles already, and so  $S = 14$ . If they distribute and equalize resources now, they could fight  $n = 7$  more battles. By part (A) of Lemma 6, players will not fight with any  $n \leq 6$  but will fight with  $n = 7$  (because not fighting with  $n = 6$  results in  $p = .18 > .17 = 1/n$  in the current period). Therefore, players will surely fight at  $t = 30$  and settle in the next period (by collapsing simultaneously if they do not redistribute now or by redistributing if they do), and so  $\sum_i V_i(30) = p(14 - 2) + (1 - p)(12) = 12$ .

Going to the previous period along the no-distribution path, we have  $t = 29$  with  $S = 16$ . Equalization would yield  $n = 8$ , and since players fight with  $n = 7$  but settle with  $n = 6$ , part (B) of Lemma 6 applies with  $T = 1$  and  $n = 6$ . Solving (3) shows that players can achieve peace in this period. Therefore,  $\sum_i V_i(29) = 16$ . Continuing in this way, we construct Table 1, which shows the SPE outcomes for all periods of the game along the no-distribution path.

Observe that (P) is satisfied for all  $t \leq 5$ . This implies that peace is impossible in the first four periods even according to the stricter criterion. However, even though (P) fails for all  $t > 5$ , there are many periods where peace cannot be achieved because (W) still holds. If, for example, this game started out with  $K_i = 25$ , then (P) would have no bite at all but we will still get seven battles in equilibrium. This illustrates our claim that with its reliance on incredible threats, Proposition 2 does not provide a compelling reason to expect players to achieve peace, and therefore is not a persuasive solution of the commitment problem.

## Discussion

We now turn to the substantive implications from the analysis of the Limited War RPSPE. One very general result is a vindication for Vegetius' dictum "if you want peace, prepare for war." As Lemma 3 shows, a peaceful settlement must be sustained by the threat to punish attempts to exploit it; that is, peace necessarily involves a credible deterrent to surprise attacks. This is in keeping with results in Powell (1993) and Slantchev (2005), who show that peace may require substantial military investments

**TABLE 1 Limited War SPE (No-distribution Path)**

$t$	$(k_1, k_2; S)$	$\sum_i \hat{A}_i$	Lemma 6	(P)	Outcome	$\sum_i V_i(t)$
1	(30, 30; 72)	74.516	1.226	1.135	fight	61.92
2	(29, 29; 70)	72.381	1.214	1.103	fight	60.14
3	(28, 28; 68)	70.296	1.207	1.070	fight	58.42
4	(27, 27; 66)	68.271	1.204	1.038	fight	56.75
5	(26, 26; 64)	66.320	1.209	1.006	fight	55.16
6	(25, 25; 62)	64.459	1.221	0.974	fight	53.66
7	(24, 24; 60)	62.707	1.244	0.942	fight	52.27
8	(23, 23; 58)	61.088	1.278	0.910	fight	51.01
9	(22, 22; 56)	59.632	1.327	0.879	fight	49.91
10	(21, 21; 54)	58.375	1.394	0.847	fight	49.02
11	(20, 20; 52)	57.360	1.482	0.816	fight	48.36
12	(19, 19; 50)	56.640	4.320	0.785	fight	48.00
13	(18, 18; 48)	47.939	0.994	0.754	peace	48.00
14	(17, 17; 46)	46.187	1.017	0.723	fight	38.27
15	(16, 16; 44)	44.568	1.051	0.693	fight	37.01
16	(15, 15; 42)	43.112	1.100	0.663	fight	35.91
17	(14, 14; 40)	41.855	1.167	0.634	fight	35.02
18	(13, 13; 38)	40.840	1.256	0.606	fight	34.36
19	(12, 12; 36)	40.120	3.060	0.579	fight	34.00
20	(11, 11; 34)	33.672	0.971	0.553	peace	34.00
21	(10, 10; 32)	32.415	1.037	0.528	fight	27.02
22	(9, 9; 30)	31.400	1.126	0.505	fight	26.36
23	(8, 8; 28)	30.680	2.340	0.483	fight	26.00
24	(7, 7; 26)	25.335	0.940	0.465	peace	26.00
25	(6, 6; 24)	24.320	1.029	0.449	fight	20.36
26	(5, 5; 22)	23.600	1.800	0.437	fight	20.00
27	(4, 4; 20)	19.600	0.964	0.429	peace	20.00
28	(3, 3; 18)	18.880	1.440	0.427	fight	16.00
29	(2, 2; 16)	14.880	0.899	0.431	peace	16.00
30	(1, 1; 14)	14.160	1.080	0.444	fight	12.00

to produce such mutual deterrence. That the notion of the *armed peace* has emerged in three rather different stylized conflict environments and that the logic behind it is essentially equivalent across them signify that it may be a very general phenomenon.

### How War Resolves the Commitment Problem

Wagner observed that any agreement that avoids further fighting must be self-enforcing: “implementing an agreement cannot be expected to enable one of the parties to overturn it and enforce a still more favorable agreement” (1994, 603). The model illustrates precisely why this must be so, for it is the inability to commit credibly not to seek the advantage of a sneak attack that prolongs fighting.

Furthermore, our results show that this problem arises from the inability of actors to commit credibly to punish such deviations with the required ferocity. In other words, peace is undermined because at least one actor has an incentive to exploit the other’s expectation to stop fighting, and the incentive arises from lack of credibility of the deterrent threat, which in turn arises from the very desirability of peace.

Recall Fearon’s (1995, 402–404) discussion of how first-strike advantages can close the bargaining range and cause war. As Powell (2006) has shown, the mechanism that causes inefficiency in that model is equivalent to the general commitment problem resulting from large, rapid shifts of power. Intuitively, foregoing the advantages of a first strike produces a power shift in favor of one’s opponent. The “declining” actor needs to be compensated for not striking first but the “rising” actor cannot credibly promise to deliver the rest of the compensation tomorrow when he finds himself in a strong position. As a result, the declining actor wages preventive war today. The logic of the general power shift mechanism implicitly relies on a stylized representation of war that abstracts from the effect of fighting on the shift itself. If states go to war because of a commitment problem caused by first-strike advantage, then how do they agree to end the war while this technology is still present?

Observe that the power shift reflects the change in continuation payoffs, and these depend not just on the first-strike advantage that comes from surprise but also on the subsequent behavior of the actors. Whereas the advantage expressed as an increased probability of military victory in a battle remains constant throughout, the continuation payoffs change as the resources shrink with the duration of fighting. In other words, the *size* of the power shift varies even though the technology that makes it possible does not. Hence, *in general the solution to the commitment problem involves playing strategies that minimize the power shift to the point where the incentives for surprise attack disappear.*

The question then becomes this: under what conditions would players choose strategies that undermine these incentives? Condition (P) shows that there are situations in which no such strategies exist, and so players cannot avoid fighting. However, as Proposition 2 makes clear, the resulting destruction eventually opens up the road to peace and players can achieve a peaceful settlement provided they can threaten to punish attempts to undermine it by total war. We argued, however, that such threats are not credible because they commit players to a painful equilibrium even though both have incentives to renegotiate and find a better one. Since our model provides opportunities for such renegotiations, it is reasonable to expect players to utilize them.

Unfortunately, this ability to find mutually better solutions reduces the severity of the threats players can credibly make, which in turn undermines the deterrent effect that is supposed to maintain the peace settlement. If players expect to be able to renegotiate and end the war as soon as possible, then they cannot threaten to fight to the bitter end. At best, they can threaten to prolong fighting until the next such opportunity presents itself. This now implies that they may not be able to reduce the size of the power shift sufficiently to avoid fighting. In other words, *war is caused by the inability of players to commit credibly to punish an attempt to exploit the peace disposition of the opponent with sufficient severity to deter it. This inability arises from their incentives to seek peace at first opportunity.*

The example in Table 1 helps follow the logic. At  $t = 1$ , players could fight 30 battles without redistributing. Unfortunately, even threatening to fight all of them until they collapse from exhaustion cannot prevent fighting in the first five periods where condition (P) holds. A rather dark implication of this analysis is that there may exist situations where even a credible threat to fight to the end may not avert war. Sometimes the stakes can be so high that neither player can impose enough costs on its opponent to deter him from risking a few battles to win them.

From  $t = 6$  onward, however, players could achieve peace in any period if they only could threaten to fight to the bitter end. These threats are not credible because players know that if the war does not end by  $t = 13$ , they will renegotiate and achieve peace then. To wit, surprise attack at, say  $t = 7$ , does not risk a total war but a limited one, and both players know this. Because of this, neither player can impose sufficient costs on the opponent to deter him from sneak attack, and the incentives to strike a bargain in all prior periods dissipate. Players are not credibly prepared for war, and therefore cannot obtain peace.

The difference between the two conditions is intuitive: whereas (P) uses the largest number of battles, (3) only uses the number that players are actually expected to fight *in an equilibrium with credible threats*. In the latter,  $T + 1$  is the largest number of battles players expect to fight until they can achieve peace. As such, it is equivalent to  $\bar{T}$  in (P). Hence, the crucial difference between the two conditions is in the  $n$  term, which one can interpret as the number of fights players would have been able to fight if they could commit credibly to doing so. Since they cannot, increasing this term leads to (3) being satisfied even under conditions where (P) would fail. The larger this discrepancy, the worse the prospects for peace.

This highlights our main conclusion: the ability to achieve peace critically depends on the credibility of the

mutually deterrent threats that players can make. Even if (P) could be satisfied with Nash/SPE threats, players have no reason to believe them but will instead only take into account how many battles the opponent is actually prepared to fight before renegotiating. As we have seen, this unhappy calculation can undermine the incentives for peace today. Conversely, if one can credibly threaten to punish an opportunistic move by imposing very large costs on the attacker, then peace can be sustained.

How does fighting resolve this war commitment problem then? Unhappily, it does so by war's very nature, its sheer destructiveness. Initially, both players are rich in resources and the stakes are high. What are a few battles compared to the possibility of obtaining these riches should a military operation prove successful? As war progresses, the pie shrinks and continuation becomes less and less tempting, which in turn means that it takes weaker threats to deter participants from surprise aggression.<sup>5</sup> Eventually, players can credibly commit to fighting a handful of battles and this minimizes the power shift to the point that peace can be maintained. Every war carries the seeds of its own peace.<sup>6</sup>

## War Costs and Rational Escalation

Can two players rationally escalate war and end up agreeing to peace terms that are worse than what each originally started the war with? That is, can rational players pay war costs that exceed the value of the prize?

In the Dollar Auction game (Shubik 1971), two players alternate in bidding for a prize of one dollar. The highest bidder wins the prize but both have to pay their bids. O'Neill (1986) analyzes a discrete complete-information version of the Dollar Auction with budget constraints. He finds a unique subgame perfect equilibrium in which no escalation occurs—player 1's initial bid forces player 2 out of the game.<sup>7</sup> This result differs markedly from experimental studies of the Dollar Auction in which players often escalate, sometimes bidding more than the value of

<sup>5</sup>In our model the pie consists of all resources which are assumed to be fully fungible; that is, they can all be used for fighting and for consumption when war ends. In other words, there is no guns/butter trade-off. Allowing for such a trade-off would make it even more difficult to achieve peace because players would have an additional incentive to reduce military spending, which in turn would imply an incentive to fight shorter wars, which in turn would imply more difficulty in making credible threats to support peace.

<sup>6</sup>Unless, of course, one's goal is to exterminate the opponent. We have in mind here the typical bargaining situation in which one is willing to accept less than the entire pie to avoid costly fighting (which implies that one is willing to let the opponent live).

<sup>7</sup>See Leininger (1989) for a more comprehensive analysis.

the prize itself (Teger 1980). O'Neill concludes that such escalatory behavior must be irrational.

The Dollar Auction has been used extensively to shoe-horn interstate crises and wars, among other events, into its interpretive framework of loss-avoidance, a case of "we have invested too much to quit now." For many analysts, the game provides an especially apt analogy that illustrates the pernicious consequences of various forms of irrationality. They see escalation as pathological, and the policy prescriptions derived usually take the form of a wish list: if only players could foresee . . . , if only they knew . . . , if only they could admit their mistakes . . . .<sup>8</sup> The general implication is clear: if only players were fully rational and knew everything about each other, then we would never see "senseless" escalation that ends with both of them paying more than the prize itself is worth.

This conclusion is implicitly endorsed by a great many formal models of conflict which assume that the costs of war are less than the value of the prize. In fact, even the canonical bargaining model of war is forced to make this assumption if conflict is ever to occur with positive probability in equilibrium. After all, if the costs of war are expected to exceed the value of victory, then a rational player would never go to war. In other words, most of our rationalist theories of war tacitly agree with the view that escalation of the sort that happens in experimental plays of the Dollar Auction is inherently irrational.

Leaving aside the question of whether the Dollar Auction is actually a good model of crises and wars (after all, it admits no bargaining and no negotiated outcomes), we argue that excessive escalation can happen with fully informed rational players even when they are virtually unconstrained by the bargaining protocol. While we do not mean to discount psychological explanations, we want to emphasize that there is no need to resort to irrationality to understand situations in which both players end up paying more than the prize itself is worth.

When we replace the heroic assumption that war costs are low relative to the value of the stakes with the milder one that the stakes are worth at least one battle, the tragedy of war reveals itself: *If players survive to make a peaceful settlement, war is sure to have become a net loss for both!* When players settle at  $t = 13$  in the Limited War SPE, the amount to be divided is  $S = 48$ , whereas it started out at 72. They have collectively paid war costs of 24 for a prize ostensibly worth only 12. Of course, since players actually aim at total military victory which would give them more than the nominal prize, this comparison is

misleading. However, observe now that *at the time of peace*, players accept shares that leave them worse off relative to just conceding the prize from the outset. For example, if players share equally at  $t = 13$  (not unlikely given their symmetry), each would obtain a payoff of 24. This is worse than immediate concession at the outset and it is less than the initial resources the player had. Each has paid war costs of 12 for the dubious privilege of obtaining a benefit of 6. Furthermore, since (P) would hold if we increase the size of initial stocks, richer players would fight longer wars. For example, letting  $K = 35$  results in five additional battles, and total individual costs of 17 at the time of negotiated peace.

War becomes unprofitable very quickly: from  $t = 7$  on, at least one of the players cannot recover the resources he started the war with. In our example, negotiated peace can occur only after war has lasted nearly twice the duration that could be potentially profitable for one of the players. However, the piling costs do not deter players from fighting because they are sunk, and hence all that matters are the expected future gains and losses. It is that forward-looking aspect of war that may make it a dead loss to both players if they fight into peace. Pillar argues that it can be rational to continue fighting even after the war "has already escalated well out of proportion to the value of the objectives at stake" (1983, 173) because one cannot manipulate past costs, only future ones. The model vindicates this logic even in an environment where all cost manipulation is only implicit in the threat to continue to fight. It also provides rationalist theoretical foundations for the empirical findings by Orme (2004) and what he terms the "paradox of peace": peace is most likely when the threat of costly conflict is greatest.

These results show that it is rational to risk small escalatory steps that eventually may accumulate enough costs to exceed the value of the issue at stake. This highlights the problematic assumption in traditional models of war and suggests that we may need to rethink some of the causal mechanisms derived from such theories.

## War as Punctuated Equilibrium

Our analysis suggests that one can usefully view war as a mutually coercive process that involves continuous fighting punctured by occasional opportunities for peace. Even though peace negotiations are available throughout the war, a credible commitment to a settlement is only possible at specific junctures. There are specific *windows of opportunity* to end the conflict, and if such a window closes, players are stuck fighting until the next one comes along. In a sense, it seems true that conflicts have to be

<sup>8</sup>See, for example, Poundstone (1992, 261–65) and Teger (1980) for some examples. Deutsch's (1973, 356–57) analysis of U.S. involvement in Vietnam essentially follows the same logic.

“ripe for resolution” (Zartman 1985). However, in our formulation ripeness is not a battlefield property of conflict that appears when players reach a “mutually hurting stalemate.” Rather, it is a function of the credibility of threats to punish attempts to take advantage of the peace negotiations. If players fail to cease the fleeting opportunity to end the war, they will be condemned to fight it out until another window presents itself.

In our example, should players for some reason be unable to negotiate at  $t = 13$ , they have to carry on the war for seven more periods until  $t = 20$  opens up the possibility for peace again. At this puncture in fighting, the terms of peace are much worse for both: players would have jointly paid costs of 40 to divide the prize, and each can hope to live with a little more than a third of his original resources. The desire to avoid this additional fighting and worse outcome in turn induces players to agree to peace at  $t = 13$ . Observe, however, that nowhere in our model are players stalemated (they can always risk a battle that gives a chance of outright victory) and neither does inability to negotiate a termination of war hinge on problems with perception.

These windows of opportunities are rarer when the stakes are higher. The more resource-rich the warring parties, the longer the fighting spells between these windows. Their frequency, however, increases the longer the war lasts, and their closure gets ever shorter as opponents approach exhaustion. That is, the more weakened the actors are from fighting, the more willing to negotiate they become. As they approach collapse, the terms of peace begin to approximate the expected payoff from continued fighting, obviating the incentive to risk it. As the terms of peace deteriorate, so does the expected payoff from prolonging the war, and hence the prospects for war termination improve. Ironically, the better the expected terms of the settlement, the worse the prospects for immediate peace. This is because the peace settlement itself is a function of the available benefits to be divided and since fighting may secure these benefits completely, the stronger the incentive to risk it.

It is this pattern of windows of opportunities for peace, which cluster toward the military end of war, that leads us to view war as a *punctuated equilibrium*.

## Conclusion

One of the canonical rationalist explanations of war is that opponents cannot credibly commit themselves to follow through on the terms of agreement because a change in relative power renders such promises against their inter-

ests. Actors then may prefer to start a war today rather than face the unpalatable consequences of peace tomorrow. However fundamental and intuitive, this mechanism is incomplete and incoherent because it does not explain how fighting alleviates that commitment problem. We argued that unless we view war as a process that traces the effect of fighting, we will not be able to resolve this puzzle.

The analysis uncovered a subtlety that essentially turns the original commitment problem on its head. In our account, an actor’s inability to promise credibly to fight for long lowers the costs of war and causes his opponent to demand so much today that he prefers to continue fighting rather than concede. The credibility problem arises from the opportunities for peace in the future: when both actors know that they want to settle the costly conflict as soon as possible, threats to extend fighting beyond such an opportunity for peace become unbelievable. Actors are tempted to risk some more fighting because they cannot deter each other by threatening not to negotiate in the future. In showing how fighting resolves that problem, we provide a complete and coherent rationalist explanation of war that does not require asymmetrically informed players.

Ironically, the very desirability and possibility of peace make war more likely because they decrease its expected duration and costs. An obvious tactic then suggests itself: if one could conceal such temptation to negotiate and somehow commit not to seek peace until a military resolution of the conflict, the likelihood of being able to negotiate an early termination will increase. Of course, it should also be obvious how difficult it will be to pull such a trick: one must simultaneously demonstrate complete resolve to fight to the bitter end and willingness to negotiate peace. The problem becomes worse in societies where leaders might be constrained by the public to fight short wars: all else equal, democracies may be unable to mount credible threats to fight to the end, and this may embolden their opponents and needlessly prolong the wars they fight. After all, it is not at all clear that democratic leaders cannot mobilize for a long haul in spite of widespread opposition. But then again, neither is it clear that these leaders will persevere against popular opinion for too long.

## Appendix: Proofs

Complete proofs are available at <http://polisci.ucsd.edu/slantchev> and <http://www.duke.edu/~bl38/>.



**Proof of Lemma 1.** Let  $V_i(k_1, k_2; S)$  denote  $i$ 's equilibrium payoff in the game that begins with total surplus  $S$  and capital stocks  $(k_1, k_2)$ . Suppose that when the resources are  $(k_1, k_2; S)$ , players will fight for  $T \geq 1$  more periods. Consider now the distribution  $(k_1 + c_1, k_2 + c_2; S + C)$  and any feasible  $(x_1, x_2)$  such that  $x_1 + x_2 = S + C$  and  $x_i \geq 0$ . If  $i$  rejects this distribution and fights this period, then they continue with  $(k_1, k_2; S)$  and fight  $T$  periods by our supposition, so  $i$  can expect  $W_i(1 | T + 1) = p_i \sum_{n=0}^T (1 - p)^n [S - nC] + (1 - p)^{T+1} V_i(T + 1)$ . Hence, a necessary condition for  $(x_1, x_2)$  to be a peaceful bargain is  $x_i \geq W_i(1 | T + 1)$  for  $i \in \{1, 2\}$ . Another necessary condition for  $(x_1, x_2)$  to be a peaceful bargain is that each player  $i$  prefers  $x_i$  to the payoff he would get from accepting the division of  $(x_1, x_2)$  in the bargaining round then sneak attacking in the fighting round. That is,  $x_i \geq pS + (1 - p)V_i(x_1 - c_1, x_2 - c_2; S)$ . We now show that for any feasible distribution  $(x_1, x_2)$  that satisfies the first necessary condition, there exists  $i$  that violates the second, that is:

$$x_i < pS + (1 - p)V_i(x_1 - c_1, x_2 - c_2; S). \quad (4)$$

Such a distribution is not peaceful because at least one player expects to do better by deviating and fighting at least one battle.

We proceed by contradiction. Suppose that there exists  $(x_1, x_2)$  such that  $x_1 + x_2 = S + C$ , and both necessary conditions are satisfied. Suppose further that  $(x_1 - c_1, x_2 - c_2; S)$  is a peaceful bargain, so  $V_i(x_1 - c_1, x_2 - c_2; S) = x_i - c_i$ . By our first supposition, this implies that  $x_i \geq pS + (1 - p)(x_i - c_i)$ . We now obtain:  $\sum_i x_i = S + C \geq 2pS + (1 - p)(x_1 - c_1 + x_2 - c_2) = 2pS + (1 - p)S \Rightarrow C \geq pS$ , which is violated for  $v$  large enough. Therefore,  $(x_1 - c_1, x_2 - c_2)$  cannot be a peaceful bargain when resources are  $(x_1 - c_1, x_2 - c_2; S)$ . That is, it must be the case that for some  $i$ ,

$$x_i - c_i < p(S - C) + (1 - p)V_i(x_1 - 2c_1, x_2 - 2c_2; S - C). \quad (5)$$

The rest of the proof boils down to algebraic manipulation which shows that if  $S$  and  $p$  are large enough, then (5) implies (4) and therefore  $(x_1, x_2)$  cannot be a peaceful bargain.  $\square$

**Proof of Lemma 2.** Let the distribution be  $(k_1, k_2; S + C)$ . Without loss of generality, let  $k_1 \in [c_1, 2c_1)$ . Assume  $p_2S > C$  and  $k_2 \geq 2c_2$ , so that player 2 will outlast player 1 if they fight. Consider an arbitrary  $(x_1, x_2)$  such that  $x_1 + x_2 = S + C$ . Since each player can reject and fight, any feasible equilibrium division must satisfy  $x_1 \in [p_1S,$

$p_1S + C]$ . If  $(x_1, x_2)$  is peaceful, then  $x_1 \geq pS + (1 - p)V_1(x_1 - c_1, x_2 - c_2; S) \geq pS$ . Since  $x_1 \leq p_1S + C$ , it follows that  $p_1S + C \geq pS \Leftrightarrow C \geq p_2S$ , a contradiction. Hence, player 1's life span must remain unchanged:  $(\hat{k}_1, \hat{k}_2)$  with  $\hat{k}_1 \in [c_1, 2c_1)$ . If this is peaceful, then  $\hat{k}_1 \geq pS$  and  $\hat{k}_2 \geq pS + (1 - p)S = S$ . Adding these yields  $S + C \geq (1 + p)S \Rightarrow C \geq pS$ , a contradiction. The proof for the case where both players are about to collapse simultaneously is analogous.  $\square$

**Proof of Proposition 1.** Consider the extensive form of the game and take the path where players fight in each period without redistributing resources. Its terminal node is the collapse from exhaustion of one of the players, say player 1. Hence, at the penultimate node resources are  $(k_1, k_2; S + C)$  such that  $k_1 \in [c_1, 2c_1)$  and  $k_2 \geq 2c_2$ , with  $S + C = v + k_1 + k_2$ . By Lemma 2, players will fight at that node and will not redistribute resources. Consider now the node prior to that, with resources  $(k_1 + c_1, k_2 + c_2; S + 2C)$ . Since players fight one battle with  $(k_1, k_2; S + C)$ , Lemma 1 implies that they will fight at that node as well. The rest of the proof establishes the players will not redistribute at that node, and so a repeated application of the lemma unravels the game.  $\square$

**Proof of Lemma 3.** Seeking a contradiction, suppose  $(x_1 - c_1, x_2 - c_2; S - C)$  admits a peaceful bargain. Since  $(x_1, x_2)$  is peaceful  $x_i \geq p(S - C) + (1 - p)V_i(x_1 - c_1, x_2 - c_2; S - C)$  for  $i = 1, 2$ . This implies  $x_1 + x_2 = S \geq 2p(S - C) + (1 - p)\sum_i V_i(x_1 - c_1, x_2 - c_2; S - C) = 2p(S - C) + (1 - p)(S - C) = S - C + p(S - C)$ . This now implies  $C \geq p(S - C) \geq pv$ , which fails for  $p$  or  $v$  large enough.  $\square$

**Proof of Lemma 4.** Note that  $\sum_i F_i(t | T) = p \sum_{\tau=0}^{T-t-1} (1 - p)^\tau [S - (\tau + t)C] + (1 - p)^{T-t} (S - TC)$ . Take an arbitrary  $T \geq 2$ , so  $\sum_i F_i(t | T + 1) = p \sum_{\tau=0}^{T-t} (1 - p)^\tau [S - (\tau + t)C] + (1 - p)^{T-t+1} [S - (T + 1)C]$ . Then  $\sum_i F_i(t | T + 1) - \sum_i F_i(t | T) = -(1 - p)^{T-t+1} C < 0$ .  $\square$

**Proof of Lemma 5.** We want to show that  $h(x) = p^2(x - 1) + (1 - p)^x > 1$  holds iff  $x$  is sufficiently high. Note that  $h''(x) = (1 - p)^x (\ln(1 - p))^2 > 0$  for all  $x$ ,  $h(0) = 1 - p^2 < 1$ , and  $\lim_{x \rightarrow \infty} h(x) = \infty$ . So there exists a unique  $\bar{x}$  such that  $h(\bar{x}) = 1$  and  $h(x)$  is increasing on  $[x^*, \infty)$  for some  $x^* < \bar{x}$ . Therefore,  $h(x) > 1$  holds if, and only if,  $x > \bar{x}$ .  $\square$

**Proof of Proposition 2.** Assume  $T_1 = T_2$  so (2) is equivalent to (P). Let  $\sigma_f$  denote an SPE in which players never redistribute and fight to the end. Consider any

period  $t$  in which (P) does not hold. Let  $(\tilde{x}_1(t), \tilde{x}_2(t))$  denote demands such that  $\tilde{x}_1(t) + \tilde{x}_2(t) = S(t)$ ,  $\tilde{x}_i(t) \geq A_i(\tilde{x}_1(t), \tilde{x}_2(t))$ , and  $\tilde{x}_i(t) \geq F_i(t | T)$ , where  $T = \lfloor \min\{k_1(t)/c_1, k_2(t)/c_2\} \rfloor$ . The following strategies are SPE: in period  $t$ , demand  $\tilde{x}_i(t)$  and do not fight if the negotiated distribution is  $(\tilde{x}_1(t), \tilde{x}_2(t))$ ; otherwise fight in the current period and then play the strategies prescribed by  $\sigma_f$  in all future periods.  $\square$

**Proof of Lemma 6.** The first step (omitted here) shows that there is no loss of generality if players do not reallocate in equilibrium. This allows construction of SPE by backward induction. When  $n = 1$ , fighting destroys all resources and both get 0, so they agree on peace with current resources. If players achieve peace in  $k_i = nc$ , then player  $i$  would sneak attack at  $k_i = (n + 1)c$  if, and only if,  $p(2(n + 1)c - 2c) + (1 - p)nc = (p + 1)nc > (n + 1)c$ , that is,  $pn > 1$ . To prove part (B), label the period where we want to see if players would fight as  $t = 1$ . If they fight here, they fight  $T$  battles and settle in  $T + 2$  on  $k_i = nc$ . We now have  $S = 2(n + T + 1)c$ , and so  $S - C = 2(n + T)c$ . Further,  $F_i(T + 1 | T + 1) = p_i(2(n + 1)c - 2c) + (1 - p)nc = nc$ . Player  $i$  will sneak attack at  $t = 1$  if, and only if,  $2p(n + T)c + (1 - p)F_i(2 | T + 1) = 2p(n + T)c + 2cp_i \sum_{s=1}^{T-1} (1 - p)^s (n + T - s) + (1 - p)^T nc > (n + T + 1)c = k_i$ , which simplifies to condition (3) stated in the lemma.

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