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Interleaving Worked Examples and Cognitive Tutor Support for Algebraic Modeling of Problem Situations.

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Abstract

Integrating worked examples with problem solving yields more effective and efficient learning, as does intelligent tutoring support for problem solving. This study examines the impact of integrating worked examples and intelligent tutor support for algebra modeling problems. Students in three conditions alternately studied worked examples (either static graphics, interactive graphics or static tables) and solved Algebra Cognitive Tutor problems. A control group solved all the problems with the Cognitive Tutor. Students in the four groups developed equivalent problem-solving skills, but students learned more efficiently in the interleaved worked example conditions, requiring 26% less time to complete the problem set. There were no differences among the four groups in two measures of robust learning – a retention test and a transfer test. But students in the static table condition could more accurately describe what algebraic model components represent in problem situations than could students in the other three conditions.

Keywords: Education; Problem solving; Learning; Classroom Study; Intelligent Tutors; Worked Examples.

Introduction

Extensive research has documented the beneficial impact on learning of interleaving worked examples with problem solving (Kalyuga, et al 2001; Pashler, et al, 2007; Sweller & Cooper, 1985; von Gog, Paas, & Van Merrienboer, 2004). Novices learn more quickly and deeply from a sequence of problems if they are asked to alternate between explaining worked-out examples of problem solutions and solving problems than if they are asked to solve all the problems in the sequence.

Typically in this research problem solving is supported by whole-answer feedback. After students complete a problem solution, whether successfully or not, they are given an example of a correct solution. This comparison condition is relatively weak, since step-by-step assistance in problem solving has been shown to be both more effective (improved learning outcomes) and more efficient (less learning time to achieve the same learning outcome) than whole answer feedback. For instance, Corbett & Anderson (2001) compared step-by-step feedback and whole-answer feedback in the Lisp Programming Cognitive Tutor and found that students in the former condition finished a fixed set of problems in one-third the time required by those in the latter condition, and made 40% fewer errors on posttests.

As a result, the question arises whether interleaving worked examples with problem solving scaffolded by intelligent tutoring systems might also yield improved learning outcomes and/or improved learning efficiency. McLaren, Lim and Koedinger (2008) examined this question in an intelligent tutor for chemistry problem solving and found that interleaving worked examples with problem solving yielded the same learning outcome as the baseline problem-solving condition, but in less time, thereby increasing learning efficiency.

Several studies have examined the impact of incorporating "faded" worked examples into Geometry Cognitive Tutor (GCT) modules in which students solve geometry problems and justify each step with a problem-solving principle (Aleven & Koedinger, 2002). In example fading (Renkl & Atkinson, 2003) the first problem is presented as a complete worked example, and in successive

problems students complete progressively more steps themselves until students are finally solving complete problems. When faded worked examples were incorporated into GCT, learning was more efficient (students spent less time to reach the same level of skill) and some evidence was obtained that the worked-example condition yielded deeper understanding (Salden, et al, 2008; Schwonke, et al, 2009).

The present study examines the impact of interleaved worked examples in a Cognitive Tutor (CT) module for Algebra problem solving. The study has two purposes. First, the study examines the impact of interleaving worked examples on students' learning time, their problem-solving skill and their depth of understanding. Second, the study evaluates three alternative types of worked examples: (1) Static Graphics in which problem components are represented graphically; (2) Interactive Graphics in which students participate in constructing the graphical problem representation; and (3) Static Tables in which problem components are represented symbolically in a table, analogous to the problem-solving interface.

This study compares four learning conditions; three conditions in which each type of worked example is interleaved with Cognitive Tutor problem solving and a fourth, Cognitive Tutor problem-solving baseline condition.

The following sections describe the problem solving domain, the Cognitive Tutor problem-solving environment and the three types of worked examples.

The Domain: Algebraic Modeling

In this study students are asked to solve "mixture problems," for example:

You have an American Express credit card with a balance of \$715 at an 11% interest rate and a Visa credit card with a 15% interest rate. If you pay a total of \$165 in annual interest, what is the balance on your Visa card?

The problem-solving goal is to construct a symbolic model of the situation that can be used to solve the problem, e.g.:

$$(.11 \times \$715) + (.15 \times V) = \$165$$

The problem-solving curriculum consists of four problem types: Two types of "arithmetic problems," in which the unknown value is naturally represented as an isolated variable on one side of the equation, and two types of "algebra problems" in which the unknown quantity is more naturally represented as a variable that is embedded in one or in two expressions in the equation. See Figure 1 for an example of each type.

Cognitive Tutor Problem Solving

Figure 2 displays the interface for the Cognitive Tutor at the end of a problem. Each problem describes a mixture scenario and provides a table to scaffold the relationship between the scenario components and the mathematical representations of the components. Students enter a number, variable or operation into each cell. After completing the

[Arithmetic Type 1] You have a MasterCard with a balance of \$532 at a 21% interest rate. You also have a Visa credit card with a balance of \$841 at a 16% interest rate. How much money are you paying in total interest?

$$(.21 \times \$532) + (.16 \times \$841) = T$$

[Arithmetic Type 2] Shelly owed \$475 in total interest on her MasterCard and Visa accounts. Her MasterCard charges 19% interest and her Visa Card charges 22% interest. She paid the interest on her Visa Card debt of \$1100. How much interest does she still owe on her MasterCard?

$$$475 - (.22 \times $1100) = M$$

[Algebra Type 1] You have an American Express credit card with a balance of \$715 at an 11% interest rate and a Visa credit card with a 15% interest rate. If you pay a total of \$165 in annual interest, what is the balance on your Visa card?

$$(.11 \times $715) + (.15 \times V) = $165$$

[Algebra Type 2] You have a total balance of \$1405 on two different credit cards— an American Express credit card with a 12% interest rate and a Discover credit card with a 24% interest rate. If you owe a total of \$224 in annual interest, what is your balance on the Discover card?

$$(.24 \times D)+(.12 \times [\$1405 - D]) = \$224$$

Figure 1: An example problem situation and symbolic model for each of the four problem types.

table, the student enters an equation to model the situation in the text cell at the bottom of the screen. The activities were created with the Cognitive Tutors Authoring Tools (CTAT) environment (Aleven, et al, in 2009). As in all cognitive tutors, students received accuracy feedback on each step, could request advice on any step, and were required to complete a correct solution to each problem.

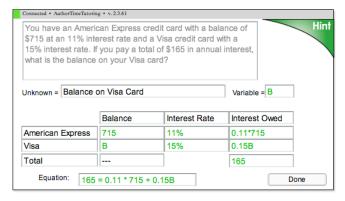


Figure 2: The Cognitive Tutor interface at the completion of a problem.

Worked Examples

Three types of worked examples were developed, in the Animation Tutor environment (Reed, 2005), each consisting

of multiple successive screens. In each case the first screen presented a problem statement alone. Successive screens developed an analysis of the problem's component structure in graphical or tabular form.

(1) Static Graphics (SG). Figure 3 shows the final screen of a static graphics worked example. The first screen displayed just the problem statement at the top. Students successively press the Continue arrow to see (1) the first stack of money which represents an account balance and interest owed, (2) the second stack of money which represents the second account balance and interest owed, and (3) both the third stack, which represents the total interest, and the symbolic model at the bottom of the screen.

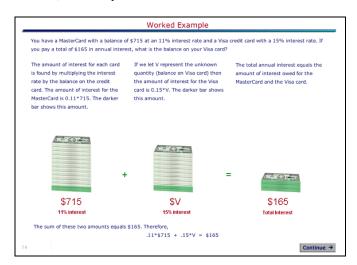


Figure 3: A static graphics worked example at the completion of the example.

(2) Interactive Graphics (IG). Interactive graphics worked examples are the same as the SG worked examples, except that students construct the total interest stack. Students click on the interest component at the bottom of each of the other two stacks and drag that component over to the total interest stack to add up the total interest. Interactive worked examples were developed for all the algebra problems and introduced with a single arithmetic problem. Students in the IG condition viewed static graphic examples for the other arithmetic problems.

(3) Static Table (ST). Figure 4 displays the final screen of a static table worked example. As with the graphics examples, the first screen displays the problem statement alone. Students successively click the Continue arrow to see (1) the column labels and first row of the table, which represents an account balance and interest owed, (2) the second row of the table which represents the second account balance and interest owed, and (3) the symbolic model of the situation beneath the table.

Design Principles. The three types of worked examples all follow two principles of multimedia design (Sweller, 2003; Mayer 2001; Moreno & Mayer, 2007). The first is the proximity principle that different media be closely integrated in space. Verbal explanations are therefore placed immediately above, and the equation immediately below,

either the bars or the table in the worked examples. The second principle, minimize cognitive load, is achieved by presenting the solution in successive segments.

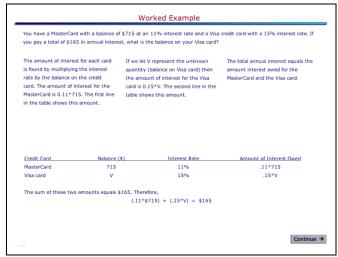


Figure 4: A static table worked example at the completion of the example.

Predictions

Time and Learning efficiency. Time-on-task in learning is expected to be less in the worked example conditions than in the problem-solving condition. Students typically study worked examples in less time than they can generate problem solutions, even with intelligent tutoring support (McLaren, et al, 2008; Salden, et al, 2008; Schwonke, et al, 2009). However, interleaved worked examples are only more efficient if students in those conditions acquire as good, or better, problem-solving skills as students in the problem solving condition.

Robust Learning. There are several reasons to expect that students may acquire a deeper understanding of problem solving in the interleaved worked example conditions. Cognitive Load theory (Sweller, 2003) suggests that worked examples can eliminate the cognitive load associated with generating problem solutions, and free up capacity that students can devote to understanding the solutions. In this study, all the worked-example conditions describe the mapping between the mathematical representations and the problem situations, so students may acquire a better understanding of the underlying semantics. understanding that should support better retention and transfer to novel problem situations. In addition, the two graphics conditions may promote better retention than the other two conditions, since they encourage visual thinking (Reed, 2010), thereby creating multiple memory codes, both graphical and symbolic (Mayer, 2001; Paivio, 1986). Finally, interactive graphics may foster still better retention than static graphics, since interactively constructing key quantities in the graphics representation, (Moreno & Meyer, 2007), creates a third, motor code (Engelkamp, 1998; Glenberg, et al, 2004; Reed, 2006, 2008).

Robust Learning Measures

A problem-solving pretest and posttest were employed to measure gains in students' algebra problem-solving skills. In addition, three "robust learning" tests were employed to measures students' depth of understanding.

- (1) Retention. A retention test examined students' arithmetic and algebra problem solving skills after a one-week interval. (2) Transfer. A transfer test described "mixture" situations with novel quantitative structures and asked students to generate mathematical models of the situations, which also had novel structures.
- (3) Model Description. The Cognitive Tutor Model Analysis Tool (Corbett, et al, 2000, 2007; Corbett, Wagner & Raspat, 2003) was employed to ask students to explain the structure of arithmetic and algebraic models. As displayed in Figure 5, each problem presents a problem description and a mathematical model of the situation. Students select entries from menus to describe what each hierarchical component of the symbolic model represents in the problem situation. As in all Cognitive Tutors, students receive feedback on each problem step, can request advice on each step, and are required to complete a correct solution to the problem.

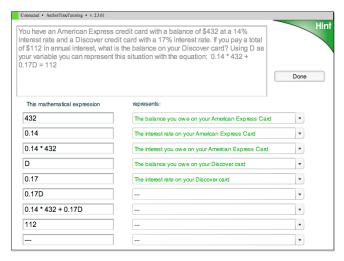


Figure 5: The Model Analysis tool partway through a problem.

Method

Participants

128 students enrolled in Cognitive Tutor Algebra courses in three Pittsburgh-area high schools participated in the study.

Design

The study was completed over the course of three computer sessions in the students' Algebra Cognitive Tutor courses. In the first two sessions, students completed 16 mixture problems, eight problems per day. The students in each of the three courses were randomly assigned to one of four learning conditions. Students in the three worked example conditions studied example solutions for the odd numbered

problems and solved the even numbered problems with the Cognitive Tutor each day. Students in the fourth condition solved all the problems each day with the Cognitive Tutor.

Learning Materials

Four types of mixture problems were developed, two "arithmetic" types and two "algebraic" types, as displayed in Figure 1. Four problems of each type were developed, for a total of 16 problems. Two problems of each type involved interest payments on two credit cards, as displayed in the figures. The other two were mining problems, about extracting metals from two ores of different quality. The four problems of each kind were presented in succession, with the two equivalent interest problems first, followed by the two equivalent ore problems.

Test Materials

Four test measures of student learning were developed.

Day-2 Problem-Solving Test. Paper-and-pencil tests were developed consisting of two problems, equivalent to the two types of algebra problems students solved with the online tutor that day. Each problem presented a mixture problem situation and students were asked to generate an equation to model the situation. Two test forms were developed and within each condition, each form served as the pretest for half the students, who then switched to the other form for the posttest, so that the pretests and posttests were matched across the full set of students, but for each student the pretest and posttest were different.

Day-3 Retention Test. This test consisted of four problems, equivalent to the four types of problems students had solved with the online tutor. Again, each problem presented a mixture problem situation and students were asked to generate an equation to model the situation.

Day-3 Transfer Test. The Day-3 transfer test consisted of an arithmetic problem and an algebra problem in which students were asked to generate symbolic models of situations with novel structures.

Day-3 Model Component Descriptions. Four Model Analysis problems were developed. Each problem corresponded to one of the four problem types students had solved on the prior two days of the study. Each problem presented a mixture scenario and presented a symbolic model of the scenario. Students were asked to describe what each hierarchical component of the equation represents in the real-world situation, by selecting entries from menus.

Procedure

In the first session, the online problem solving and worked example activities were introduced, then students worked through the eight arithmetic mixture problems. In the second session, students completed a two-problem paper pretest, worked through eight algebraic problems, then completed a two-problem paper posttest. In the third session, which followed a week later, students completed the four-problem paper retention test, followed by the two-problem paper transfer test and finally the four Model Analysis problems.

Results and Discussion

Four students were excluded from the analyses because they missed the second session and seven others were excluded for talking to others as they worked on the problems.

Day-2 Pretest-Posttest Learning Gains

As displayed in Table 1, there were substantial pretest-posttest learning gains in all four learning conditions, averaging 26 percentage points. In an analysis of variance, this main effect of test type was significant F(1,105) = 52.14, p < .001. There was no significant difference of learning condition F(3,105) < 1, and no significant interaction of test type and learning condition F(3,105) < 1.

Table 1: Learning Time per problem for Day 1 and Day 2 (minutes) and Day-2 pretest and posttest accuracy (percent correct).

Learning	Day 1	Day 2	Pretest	Posttest
Conditions	Time	Time	%correct	% correct
CT	2.30	2.15	7	37
IG	1.52	1.68	7	28
SG	1.68	1.52	4	34
ST	1.75	1.72	8	28
Mean	1.81	1.77	6	32

Learning Efficiency

Table 1 displays average learning time per problem for the first two sessions. Elapsed time was not measured for the first worked example in each session (since the environment did not directly record time), so the first pair of equivalent problems in each session is excluded from this analysis for all four groups. In addition, 13 students were excluded from the Day-1 analysis and 16 students from the Day-2 analysis because of missing data. While there were no differences in skill acquisition outcomes among the four conditions, students in the three interleaved worked example conditions spent less time in learning, and so learned more efficiently.

Students in the three worked example conditions averaged 28% less time per problem on Day 1 than students in the problem solving condition (1.65 vs 2.30) and 24% less time per problem on Day 2 (1.64 vs. 2.15). The main effect of condition is significant for Day 1, F(3,100) = 6.88, p < .001 and for Day 2, F(3,97) = 6.33, p < .001. Bonferroni comparisons revealed that the CT group differed from each one of the three worked example groups both on Day 1 and on Day 2, p < .02 in each case. The three worked example groups did not differ from each other.

These average times mask a highly significant Group x Problem interaction on Day 1, F(3,100) = 93.12, p < .001, and on Day 2, F(3,97) = 90.19, p < .001. On Day 1 the three worked example (WE) groups averaged 0.78 min. on the worked examples, while the CT group averaged 2.98 min. solving the corresponding problems. The WE groups averaged 2.53 min. on solving the subsequent equivalent problems, while the CT group averaged 1.63 min. on those problems. On Day 2, the WE groups averaged 0.62 min. on

the worked examples and the CT group averaged 2.82 min. solving those problems. The WE group averaged 2.67 min. solving the subsequent problems and the CT group averaged 1.50 min. on those problems.

Robust Learning

Of the 117 students included in the study, 102 completed the day 3 robust learning activities. Table 2 displays results of the three robust learning measures included in the study: (1) retention of problem-solving skill; (2) transfer of problem-solving skill; and (3) explanations of symbolic model components.

Retention Test. Table 2 displays students' test accuracy on the one-week retention test of problem-solving skill. Retention test accuracy did not vary significantly across the four learning conditions, F(3,90) < 1.

Transfer Test. As can be seen in Table 2, students in the four learning conditions averaged 17% correct on the transfer test of problem-solving skill. The main effect of learning condition was not significant F(3,90) < 1.

Model Component Descriptions. The model analysis task required students to describe what a total of 31 hierarchical equation components represented in the four real-world problem situations. Table 2 displays the average percentage of these 31 descriptions on which students' first menu selection was correct. There was no significant difference among the groups in an ANOVA, F(3,97) < 1. But the ST group performed consistently best in describing the model components, achieving the highest accuracy for 18 of the 31 components (vs. 5 for the IG and SG groups and 3 for the CT group). This difference is significant in a Friedman two-way ANOVA of rank ordering, $\chi^2(3) = 20.00$, p < .001.

Table 2: Day-3 Robust learning measures: Retention, transfer and model analysis accuracy (percent correct).

Learning Conditions	Retention %correct	Transfer % correct	Model Analysis
			% correct
СТ	32	15	52
IG	29	18	52
SG	29	21	53
ST	26	13	58
Mean	29	17	54

Conclusion

The main results confirm earlier conclusions in chemistry and geometry that incorporating worked examples into intelligent tutor-supported problem solving can improve learning efficiency. While students developed similar problem-solving skills across the four conditions, students spent 26% less time completing the sixteen problems in the three interleaved worked-example conditions than in the problem-solving comparison condition.

However, there is relatively thin evidence that incorporating worked examples yielded a deeper understanding of problems solving, as expected by Cognitive Load theory. Students in the static table worked example condition demonstrated a better understanding of the referential semantics that link the mathematical representations and real-world problem situations than students in the problem solving condition. However, this deeper knowledge did not support greater problem solving accuracy, retention or transfer. Students in the two graphics worked example conditions also did not show more robust learning than students in the problem solving condition.

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