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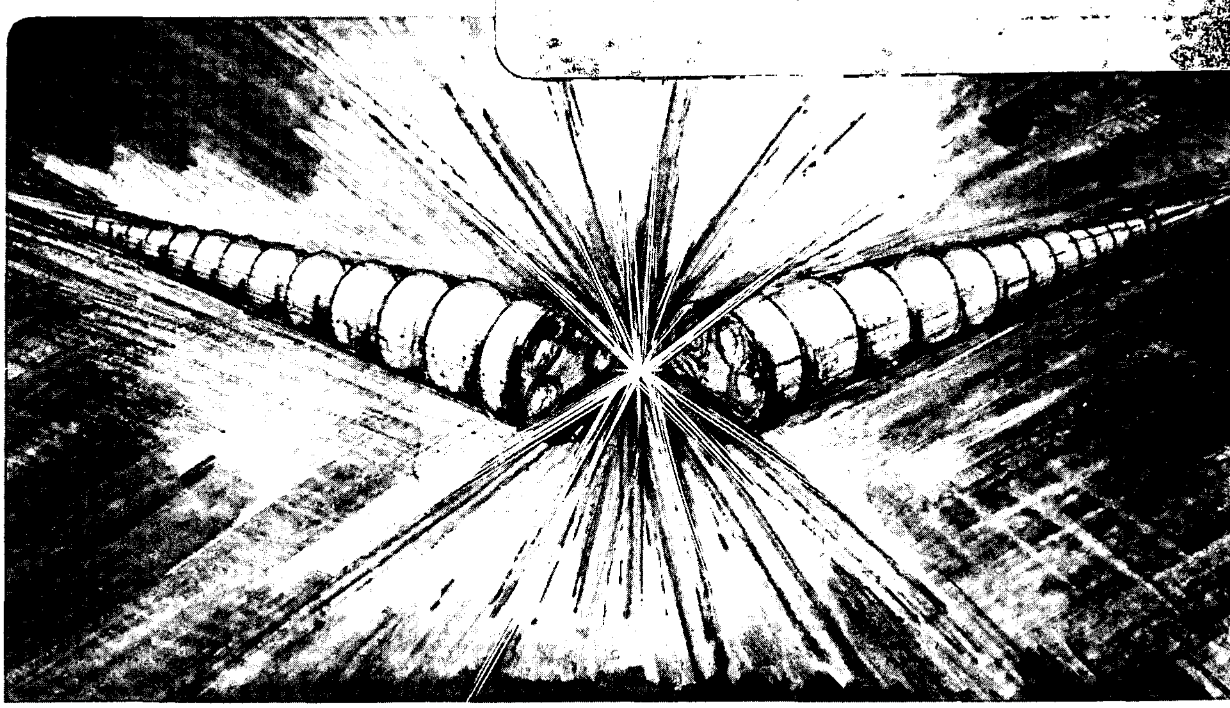

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Poisson-Type Multiplicity Distribution and the Giovannini-Van Hove Model*

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Properties of a Poisson-type distribution satisfying the same recurrence relation as the negative binomial distribution are investigated using the charged multiplicity data of $\bar{p}p$, pp, π^+p, K^+p and e^+e^- interactions. Its two parameters are found to deviate significantly from the predictions of the Giovannini-Van Hove model. Remarks are made on the property of quark content of the multiplicity due to fragmentation of colliding particles such as p, π/K and e under investigation.

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I. Introduction

It is now well established that the charged multiplicity of both hadron-hadron (hh) and e^+e^- interactions follow the negative binominal (NB) distribution

$$P_n = \frac{(n+k-1)!}{n!\,(k-1)!}\left(\frac{\bar{n}}{\bar{n}+k}\right)^n \frac{1}{(1+\bar{n}/k)^k} . \qquad (1)$$

The two parameters, the average multiplicity $\bar{n}$ and the number of cells in the phase-space k, are constrained by the width parameter, characteristic of the distribution:

$$f_2 \equiv \overline{n^2} - \bar{n}^2 - \bar{n} = \bar{n}^2/k \qquad (2)$$

so that for $k = 1$, $f_2 = \bar{n}^2$, the NB distribution (1) reduces to the Bose-Einstein distribution; whereas it becomes the Poisson distribution for $k = \infty$, i.e. $f_2 = 0$.

Recently, Giovannini and Van Hove[1] have pointed out an interesting property of (1), namely its recurrence relation (see Eq. (6) below); is also satisfied by a Poisson-type distribution with two parameters a and b see Eq. (5); the meaning of the second parameter b in (5) is to account for the interaction among the particles produced in the primary interaction. Clearly, a and b are related to $\bar{n}$ and k of (1), see Eqs. (7) and (8) below. This property is used by Giovannini and Van Hove to formulate a novel cluster model of particle production including stimulated emission.

It is a remarkable case that two different distributions (1) and (5) describe the same Markov process governed by the recurrence relation (6). We propose to investigate the behavior of the relations, Eqs. (7) and (8), derived by Giovannini and Van Hove and the validity of the Poisson-type distribution, Eq. (5), which has been introduced previously to investigate the negative multiplicity of pp interactions at P_{lab} = 12 to 300 GeV/c,[2] Sec. 2.

We shall analyze the same $\bar{p}p$ data at $\sqrt{s}$ = 540 GeV of the UA5 collaboration,[3] Sec. 3, as in their work.[1] In addition, we will investigate the pp, π^+p and K^+p data at P_{lab} = 250 GeV/c of the NA22 Collaboration,[4] Sec. 4 and the e^+e^- data at $\sqrt{s}$ = 29 GeV of the HRS Collaboration,[5] Sec. 5. We will discuss the deviations of the fitted parameters of the distribution (5) from the predictions by Giovannini and Van Hove for the $\bar{p}p$ case, Fig. 3, Sec. 3.

Some remarks will be made on the Poissonian distribution of the charged multiplicity of e^+e^- interaction, Sec. 5, and the stimulation emission of the Giovannini and Van Hove Model, Sec. 6, as well as the quark property of the average multiplicity of fragmentation of colliding particles such as p, π^+/K^+ and e analyzed in the present work, Sec. 6.

2. The Poisson-type Distribution

It is well-known that the characteristic feature of the multiplicity distribution of high energy interactions is its resemblance to the Poisson distribution

$$p_n = N \frac{\alpha^n}{n!} \qquad (3)$$

where $\alpha = \bar{n}$ and $N = e^{-\alpha}$ is the normalization coefficient. As has been reported that if the log of $n!\, p_n$ of the data is plotted vs. n and compared with the Poisson distribution (3), we notice deviations increasing with n.[2] Thus, (3) may be used to describe the data, provided that α is a function of n. As a first approximation, we may write

$$\alpha(n) = a + bn, \tag{4}$$

leading to the following Poisson-type distribution

$$p_n \sim \frac{(a+bn)^n}{n!} \tag{5}$$

with two parametes a and b. For $b < 1$, cf. infra Eq. (12). the distribution (5) satisfies the following recurrence relation:

$$(n+1)\, p_{n+1} = (a+bn)\, p_n \,. \tag{6}$$

We recall that this Poisson-type distribution is adequate to account for the negative multiplicity of pp interactions at $P_{lab} = 12$ to 303 GeV,[2] and that the properties of this distribution have been investigated by Blankenbecler.[6]

Now, as mentioned above, Giovannini and Van Hove have pointed out that the NB distribution (1) and the Poisson-type distribution (5) satisfy the same recurrence relation (6) and that these parameters are related as follows

$$a = \bar{n}k/(\bar{n}+k), \quad b = \bar{n}/(\bar{n}+k)\,, \tag{7}$$

or conversely

$$\bar{n} = a/(1-b), \quad k = a/b\,. \tag{8}$$

The question arises: are all the mathematical properties of these two distribution (1) and (5) known from the recurrence relation (6)?

In this regard, we note that if a = 0, then $\bar{n} = 0$ according to (8) and $P_n \equiv 0$. But according to (6), a = 0 defines a certain distribution by (5), which reduces to the Bose-Einstein distribution for large n, a property not shared by (1).

More specifically, we note that the relation (8), which gives $\bar{n}$ in terms of a and b, seems too simple in comparison with the average n derived from (5) by Blankenbecler,[6] namely

$$\bar{n} = \frac{y_1}{(1 - by_1)^2} [a(1 - by_1) + b] \quad (9)$$

where y_1 is a parameter defined by $1 = y_1 e^{-by_1}$. We see that (8) is a special case corresponding to negligible b, so that $y_1 \simeq 1$ and (9) reduces to $\bar{n} = a/(1 - b)$ as (8), suggesting that formula (8) holds better for small b, i.e., distributions of narrow width.

An attempt is therefore made to analyze in what follows properties of charged multiplicity distributions of high energy hh and e^+e^- interactions using the Poisson-type distribution (5), so that we may compare the parameters a and b with the predictions by the relations (7) due to Giovannini and Van Hove.[1]

3. $\bar{p}p$ interaction at $\sqrt{s}$ = 540 GeV

Consider the charged multiplicities of $\bar{p}p$ interactions at $\sqrt{s}$ = 540 GeV for rapidity intervals $|\eta|$ = 5, 3, 1.5 and 0.5 of the UA5 Collaboration.[3] Their parameters $\bar{n}$ and k of the NB distribution (1) are listed in Table I. Note that henceforth, unless otherwise stated, we denote the charged multiplicity by n.

Knowing $\bar{n}$ and k, we compute the parameter a and b of (4) by means of (7). The results are listed in Table I, to be compared with those estimated by fits to the data.

But before proceeding to the analysis, we have to test the validity of the distribution (5). For this purpose we plot $\alpha(n) = (n!\, p_n/N)^{1/n}$ vs n using the UA5 data for $|\eta| = 5$, N being the normalization coefficient of the experimental distribution estimated from the NB distribution fit of UA5. We find

$$I/N = (1 + \bar{n}/k)^k = 1.1182 \times 10^3 . \qquad (10)$$

The result is shown in Fig. 1. According to (4), the plot should be linear. The straight line represents the least-squares fit

$$\alpha(n) = (2.60 \pm 0.23) + (0.35 \pm 0.01)\, n$$

and the dashed lines are its extrapolations. We note that the test is very satisfactory, except the points at the ends of the distribution, and that the uncertainty entailed on the normalization coefficient N may shift the points near the origin but not those of the other end; consequently the slope, therefore b, is well determined.

We now proceed to fit the UA5 data using the Poisson - type distribution (5). The fits are shown by the solid curve in Fig. 2 for $|\eta| = 5$ and 1.5. They are comparable to the fits of UA5 using the NB distribution shown by the dashed lines except near the end of the distribution.

The parameters a and b obtained from the fits are listed in Table I. A comparison with the predictions according to Giovannini and Van Hove is shown in Fig. 3, the solid curves being taken from their paper.[1] We find the estimates of a are systemtically larger than the predictions, whereas contrary is the case of b, which has a bound equal to l/e = 0.368 indicated by the dashed line.

Indeed, for large n, we may replace the factorial n! in (5) by Stirling's formula and obtain

$$p_n/p_{n-1} \simeq e\,b \tag{11}$$

where e = 2.718. The condition $\Sigma\, p_n = 1$ as $n \to \infty$ requires $p_n/p_{n-1} < 1$, thus

$$b < 1/e\,. \tag{12}$$

In contradiction with the prediction $b \to 1$ according to (7), this property is also expected from the average multiplicity of (5), see Eq. (9) for $\bar{n} \to \infty$; it indicates that the interaction of particles in the final state is saturated.

4. Properties of p, π and K Fragmentations

Consider next a recent experiment of pp, π^+p and K^+p interactions at P_{lab} = 250 GeV/c by the NA22 Collaboration.[4] The parameters $\bar{n}$ and k of their fits with the NB distribution (1) are listed in Table I together with a and b computed according to (7). We have analyzed their data using the Poisson-type distribution (5); the parameters are listed in Table I. The fit for pp is shown by the solid curve in Fig. 4; similar fits are obtained for π^+p and K^+p, they are not shown for brevity.

Here also, the estimates of a and b are different from the predictions by (7) as in the case of $\bar{p}p$ discussed in Sec. 3. We find b less than the upper limit l/e, because the energy is lower, $\sqrt{s}$ = 21.65 GeV.

Referring to the average charged multiplicity $\bar{n}$ of these interactions, Table I, we note $\bar{n}_\pi \simeq \bar{n}_k$, whereas $\bar{n}_p < \bar{n}_\pi$, indicating that in the π^+p interaction, as well as in K^+p, the multiplicity of particles arising from the incident π^+ or K^+ is higher than that of the target proton, a well-known property which has been investigated previously.[7] We now analyze the fragmentation of π^+/K^+ in terms of the Poisson- type distribution (5).

For this purpose, we have to estimate, e.g., the multiplicity due to the projectile fragmentation of the incident π^+ denoted by $p_n(\pi)$, from those $p_n(\pi p)$ and $p_n(pp)$ of the experimental data of multiplicities of π^+p and pp interactions. We assume, as in Ref. [7],

$$p_n(\pi) = p_n(\pi p) - \frac{1}{2}\, p_n(pp)\,, \tag{13}$$

likewise for the incident K^+. The results are shown in Fig. 4 by full and open circles for π and K, respectively. Note that the normalization of these two distributions is about one half that of pp, and that they are practically the same within large errors of uncertainty in the extraction of data:

$$p_n(\pi) = p_n(K). \tag{14}$$

which follows from the equality of their parameters a and b listed in Table I. We therefore plot in Fig. 4 the average of the two fits shown by the dashed line.

Note that (14) is the similarity property of π and K fragmentation as reported previously,[7(b)] and that $\bar{n}_\pi = \bar{n}_k = 8.21 \pm 0.12$ is larger than $\bar{n}_p = 7.85 \pm 0.08$ of the target proton fragmentation. Furthermore, we note that its width is broader than that of pp, namely $f_2 = 14.7 \pm 2.4$ compared to 7.9 ± 0.5 in the case of pp interaction. We shall further discuss their property in Sec. 6.

5. e^+e^- interaction at $\sqrt{s} = 29$ GeV

Finally, we analyze the multiplicity of e^+e^- interaction at cm energy 29 GeV of the HRS Collaboration.[5] Their data are shown in Fig. 5, their parameters $\bar{n}$ and k are listed in Table I. Note that k is very large due to the narrowness of the width of the distribution: $f_2 = 0.60 \pm 0.18$.[5] Experimentally, f_2 is found to be almost the same for $\sqrt{s} = 7$ to 34 GeV according to the TASSO Collaboration.[8]

We have fitted the HRS data using the Poisson-type distribution (5), the result is shown by the solid curve in Fig. 5. The parameters are listed in Table I, they are comparable to the predictions by Eq. (7), as discussed in Sec. 2.

It should be noted that the estimate b is different from zero in spite of large fitting errors. The reasons are two fold: first because b is related to f_2 as has been noted in Ref. [2], so that $b = 0$ implies $f_2 = 0$, contrary to the experimental value $f_2 = 0.60 \pm 0.18$. Next if we assume $b = 0$ and try to fit the data with (3), namely the Poisson distribution, instead of (5), we get for the parameter $\alpha = 13.13 \pm 0.07$ in disagreement with the average $\bar{n} = 12.87 \pm 0.03$ of the experiment as is listed in Table I.

The fact that b is very small indicates that the HRS distribution is indeed almost Poissonian. The question arises: How much does it actually differ from the Poisson distribution? In this regard, we note that if we assume this distribution to be a mixture of the Poisson and the Bose-Einstein distributions of the same average multiplicity, say α, and if the percentage of the latter is x, then

$$p_n = (1-x)\,\frac{\alpha^n}{n!}\,e^{-\alpha} + x\,\frac{\alpha^n}{(1+\alpha)^{n+1}}\,. \qquad (15)$$

It is clear that

$$\bar{n} = \alpha, \qquad \overline{n(n-1)} = (1+x)\bar{n}^2 . \tag{16}$$

We find in view of (2)

$$x = 1/k . \tag{17}$$

For the HRS data, k = 253, the multiplicity distribution in Fig. 5 differs only ~ 0.4% from the Poissonian distribution.

However, this property does not imply that the source emitting the hadrons is almost coherent. It is a case analoguous to the classical Young holes experiment of interference with an incandescent light bulb.[9] The Poisson-like feature of the HRS multiplicity distribution reflects rather the property of the independent particle model as has been noted by Giovannini and Van Hove. [1]

6. Remarks

We have analyzed the charged multiplicity distributions of hh and e^+e^- interactions using the Poisson-type distribution (5) of the Giovannini-Van Hove mdoel.[1] We find the parameters a and b different from the predictions, Eqs. (7); especially b is much smaller and limited by b < 1/e, Eq. (12). As the width of the distribution depends critically on b,[2] the predicted shape, namely the distribution (5) with a and b in terms of $\bar{n}$ and k, Eqs. (7), is much broader than the experimental distribution, so that the maximum is no longer at the place near $\bar{n}$.

There remains to examine if the linear expansion of $\alpha(n)$, Eq. (4), is adequate. For this purpose, consider for instance

$$\alpha(n) = a + bn + cn^2 \qquad (18)$$

such a tern in n^2 is also needed to describe the stimulated emission in the Giovannini-Van Hove model.[1] We have reanalyzed the UA5 data of $\bar{p}p$ at 540 GeV for $|\eta| > 5$ of Sec. 3 and found

$$a = 3.52 \pm 0.10, \quad b = 0.35 \pm 0.02, \quad c = -(5.10 \pm 4.15)\cdot 10^{-4}$$

Note that a and b remain practically the same as before and that c is negative and extremely small. Now, $c < 0$ corresponds to the downward curvature of the plot in Fig. 1, due mostly to large errors of the end points. On the other hand $c > 0$, according to the model.[1] Therefore, c is consistent with zero, indicating that the linear approximation of $\alpha(n)$, Eq. (4), holds for the UA5 data.[3] It would be interesting to investigate its effect at higher energy.

In the analysis of the charged multiplicity distributions of pp, π^+p and K^+p of the NA22 data,[4] we have estimated $\bar{n}$ of fragmentation of the incident p, π^+ and K^+, see Table I. As the observed multiplicity is proportional to the energy density, therefore inversely proportional to the volume containing the energy available for particle production with Lorentz contraction in the direction of collision. Assuming the volume proportional to the number of valence quarks n_q of the particle in collision, we expect at a given energy, [10]

$$n_{\pm 0} \sim 1/n_q^{1/3} . \qquad (19)$$

As the neutral to charged ratio for $\bar{n}$ is ~1/2 for both hh and e^+e^- interactions, we may check (19) with the charged multiplicity for p and π^+/K^+ assuming $n_q = 3$ and 2, respectively. In addition, as $\bar{n} \sim E_{am}^{1/2}$ for e^+e^- interaction, we may also include the HRS data by multiplying its $\bar{n}$ by $(21.9/29)^{1/2}$. to scale down to the same energy as NA22, we find

$$\bar{n}_{\pm}\, n_q^{1/3} = \begin{array}{ll} 11.43 \pm 0.11 & \text{for p,} \\ 11.89 \pm 0.15 & \text{for } \pi/K \\ 11.15 \pm 0.03 & \text{for e,} \end{array}$$

constant within experimental errors, in agreement with (19). It would be interesting to further investigate this important property when data of the forthcomming e-p colliders will be available.

Acknowledgments

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Added Note - We have analyzed the forward charged multiplicity of pp at 200 GeV for $\eta = 3.5$ of a recent experiment by the Munich-Cracow Collaboration,[11] their fit with the NB distribution gives $\bar{n} = 4.11 \pm 0.06$ and $1/k = 0.11 \pm 0.01$ leading to the prediction $b = 0.31 \pm 03$. Our fit with (5) yields $a = 2.95 \pm 0.14$ and $b = 0.11 \pm 0.01$, comparable to those of the negative multiplicity of pp at 205 GeV reported in Ref. [2].

References and Footnotes

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9. See, e.g. R. Loudon, The Quantum theory of light, Clarendon Press, Oxford, (1983), p. 233.
10. A similar property is observed in the case of the radius parameter of high energy p and π/K nuclear cross-sections; T. F. Hoang, Bruce Cork and H. Crawford, Z. Phys. C29, 611 (1985).
11. Munich-Cracow Collaboration, F. Dangler et al., Z. Phys. C33, 187 (1986).

Table I - Parameters a and b of the Poisson-type distribution, Eq. (5) for pp at $\sqrt{s}$ = 540 GeV, Ref. [2]; pp, π^+p and K^+p at P_{lab} = .250 GeV, Ref. [4]; and e^+e^- at $\sqrt{s}$ = 29 GeV, Ref. [5]. Predicted parameters by Giovannini-Van Hove model using n and k of the negative binominal fits of the experiments. π and $k \to h^+h^-$ are charged multiplicity of π^+ and K^+ in fragmentations, see Sec. 4.

Experimental	n	k	Prediction a	Prediction b	Present work a	Present work b
pp $\sqrt{s}$ = 540 GeV $\|\eta\|$ = 5	26.4 ± 0.2	3.17 ± 0.03	2.72	0.89	3.51 ± 0.3l	0.34 ± 0.02
3	18.9 ± 0.2	2.44 ± 0.02	2.16	0.88	2.91 ± 0.31	0.33 ± 0.02
1.5	9.5 ± 0.1	1.99 ± 0.02	1.69	0.85	2.27 ± 0.19	0.31 ± 0.03
0.5	3.01 ± 0.03	1.69 ± 0.02	1.08	0.65	1.36 ± 0.14	0.24 ± 0.05
pp P_{lab} = 250 GeV/c	7.85 ± 0.08	6.45 ± 0.35	3.54	0.55	3.47 ± 0.41	0.23 ± 0.04
π^+p	8.43 ± 0.04	10.00 ± 0.33	4.53	0.39	3.95 ± 0.46	0.22 ± 0.04
K^+p	8.38 ± 0.04	10.53 ± 0.37	4.78	0.46	3.97 ± 0.49	0.22 ± 0.05
$\pi \to h^+h^-$	8.05 ± 0.15				4.45 ± 0.50	0.20 ± 0.05
$K \to h^+h^-$	8.39 ± 0.14				4.51 ± 0.55	0.20 ± 0.05
e^+e^- $\sqrt{s}$ = 29 GeV	12.87 ± 0.03	253	12.84	0.048	12.29 ± 0.75	0.02 ± 0.02

Figure Captions

1. Validity test of the Poisson-type distribution Eq. (5). $\bar{p}p$ data at $\sqrt{s} = 540$ GeV for for $\eta = 5$ of UA5, Ref. [3]. The straight line is the least-squares fit required by the distribution (5), see text.
2. Poisson-type distribution fits to $\bar{p}p$ data at $\sqrt{s} = 540$ GeV of UA5 Collaboration, Ref. [3]. The parameters a and b are listed in Table I. The dashed lines are fits using the negative binomonial distribution, (1) of UA5 Collaboration, Ref. 3.
3. Comparision of parameters a and b with predictions of Giovannini and Van Hove, the solid curve, Ref. [1]. The dashed line is the upper limit for $b < l/e$, see text.
4. Poisson-type distribution for pp, π^+p, K^+p at $P_{lab} = 250$ GeV of NA22 Collaboration, Ref. [4]. The solid curve is for pp. The full and open circles present π and K fragmentations, respectively, the fit is shown by the dashed line, see text. Parameters a and b are in Table I.
5. Poisson-type distribution for e^+e^- interaction at $\sqrt{s} = 29$ GeV, HRS data, Ref. [5]. Parameters a and b are in Table I.

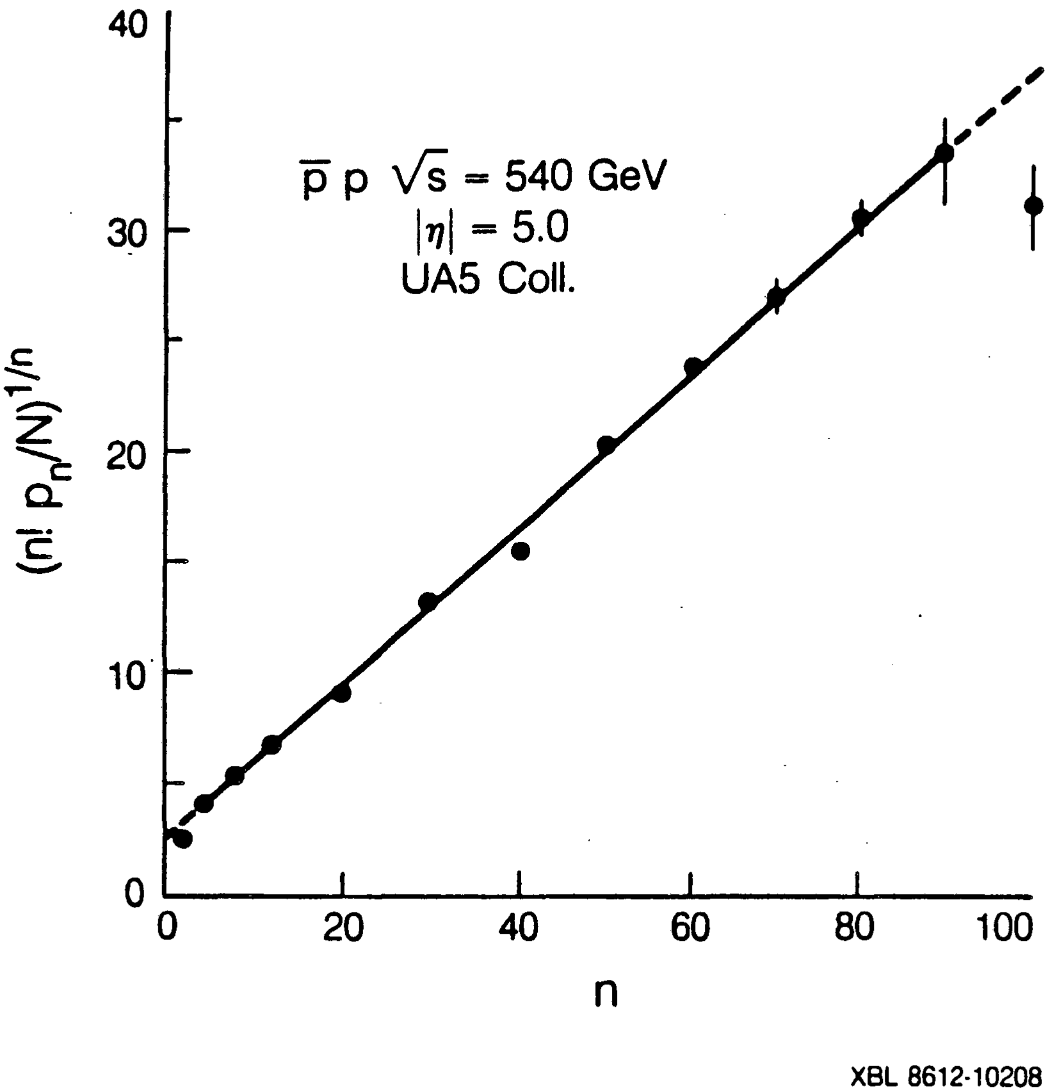
$\bar{p}$ p $\sqrt{s}$ = 540 GeV
$|\eta|$ = 5.0
UA5 Coll.
$(n!\, p_n/N)^{1/n}$
40
30
20
10
0
0
20
40
60
80
100
n
XBL 8612-10208

Fig. 1

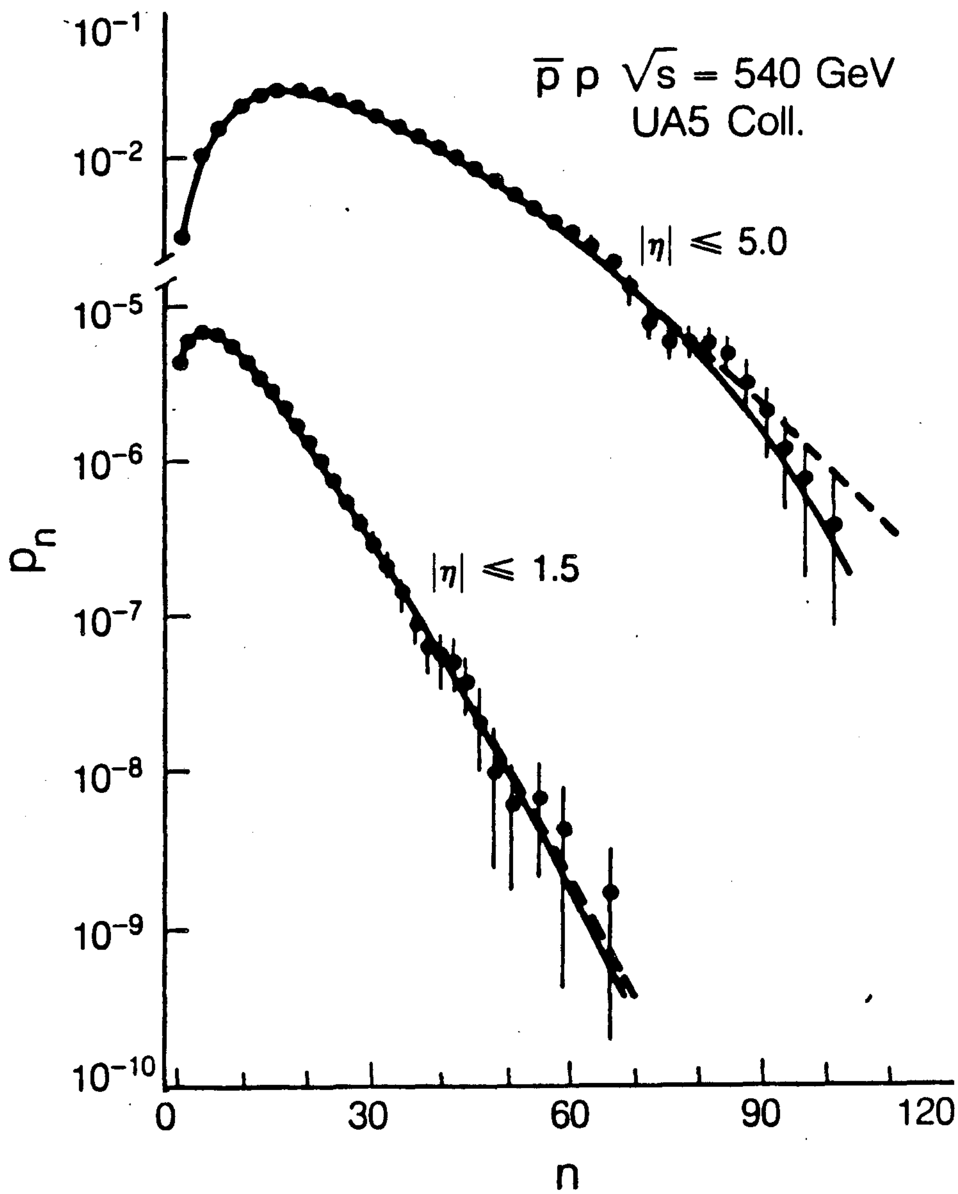

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Fig. 2

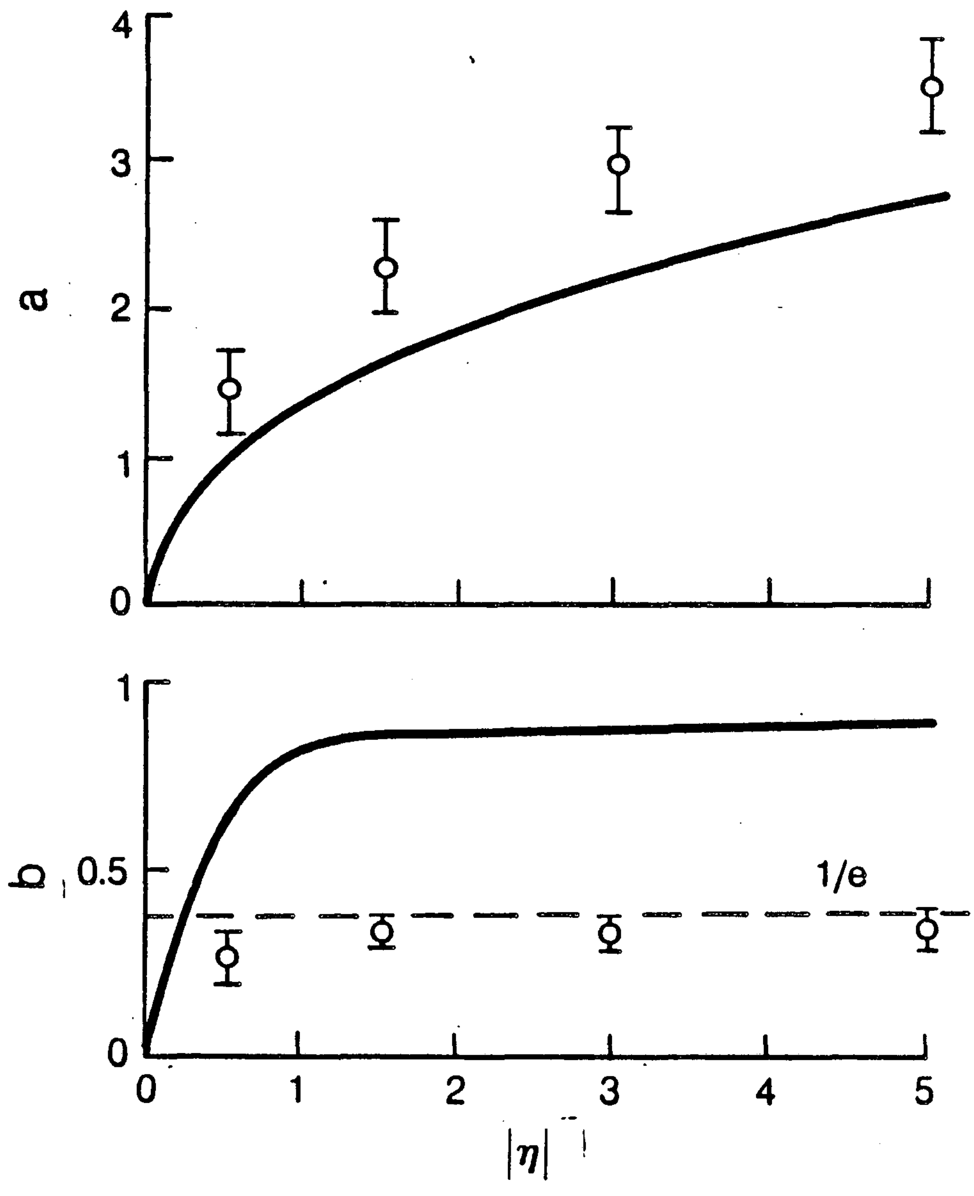

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Fig. 3

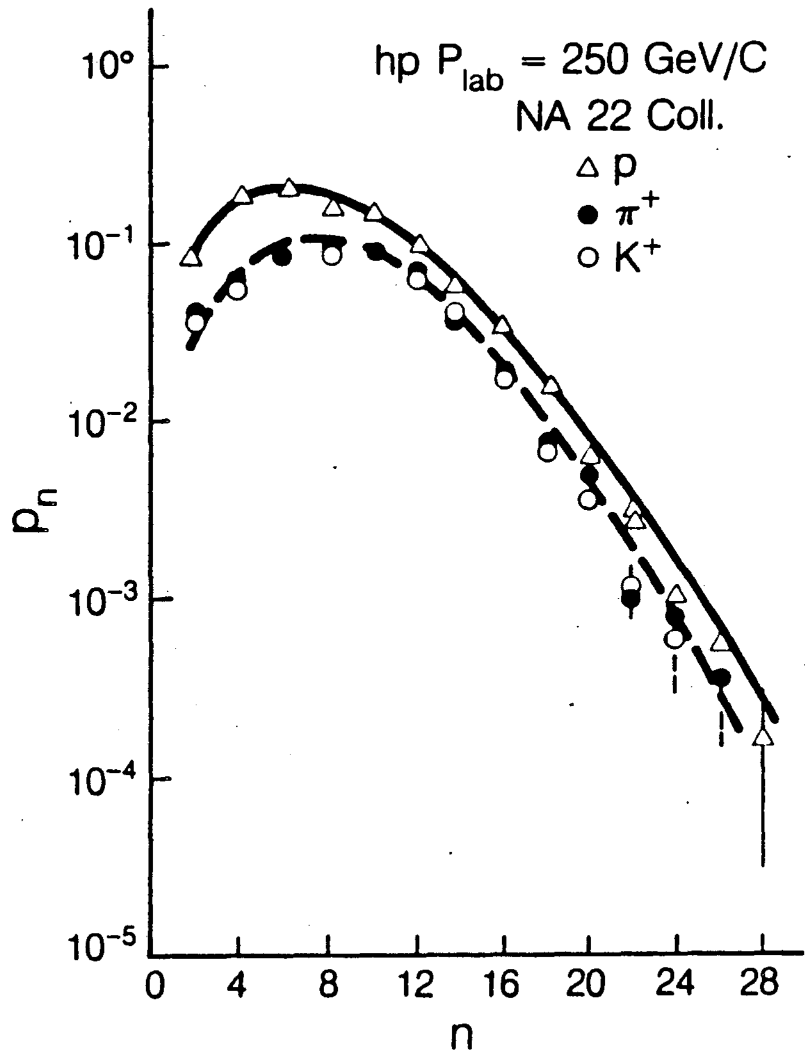

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Fig. 4

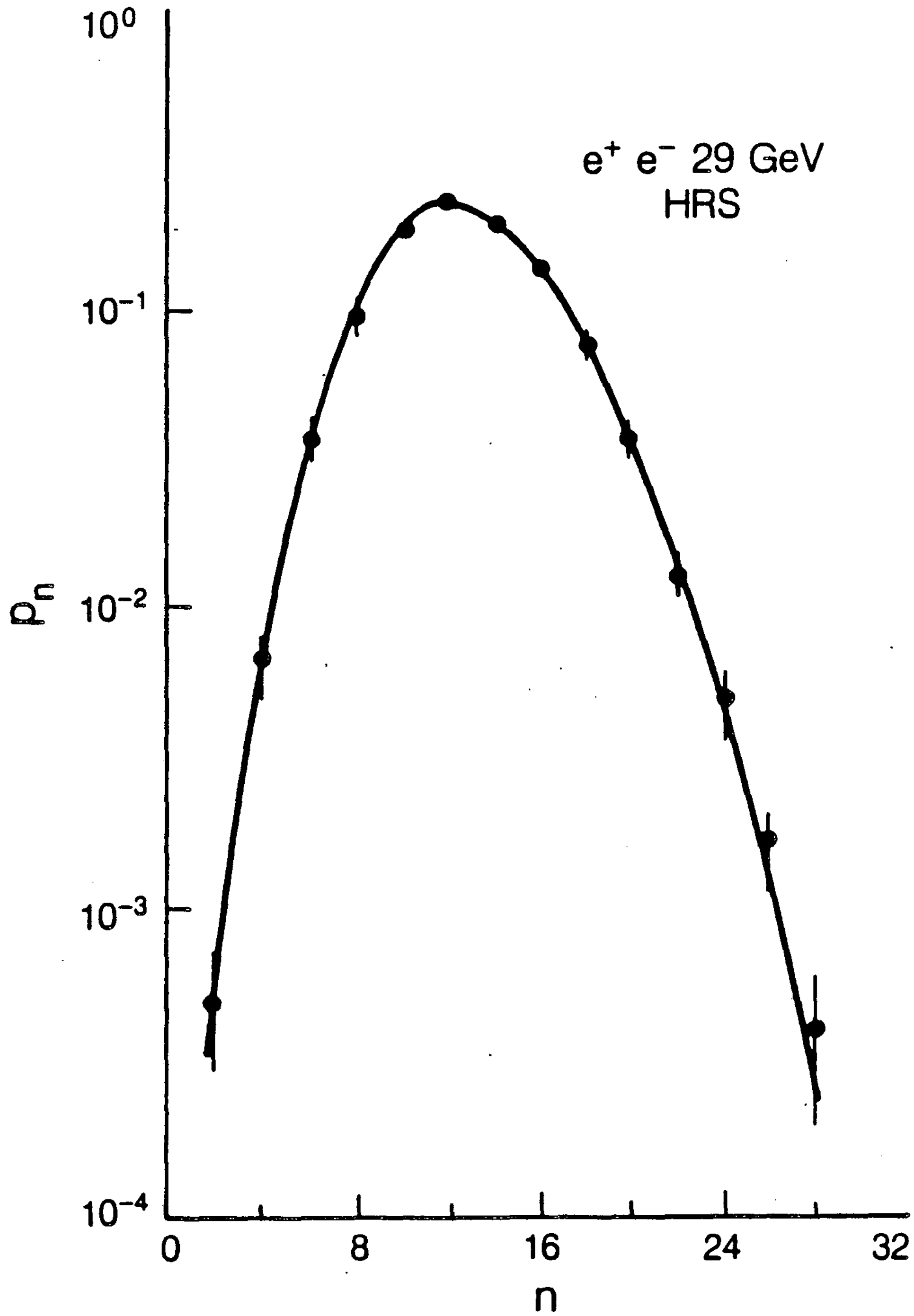

Fig. 5

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