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RELATIVISTIC QUANTUM FIELD THEORY OF FINITE NUCLEI*

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Abstract:

Finite nuclei are studied in a relativistic quantum field theory proposed by Walecka. It is approximated by the Thomas-Fermi approximation. We study Ca^{40}, Ca^{48}, Ni^{56}, Zr^{90} and Pb^{208}. We show that over all the theory agrees quite well with the available data and is consistent with conventional Hartree-Fock calculations. The Walecka model is an excellent phenomenological model of nuclear matter.

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A relativistic quantum field theory of nuclear matter was proposed by Walecka,[1)] in which the dominant properties of nuclear matter such as saturation and binding energy per particle are accounted for by the presence of scalar (σ) and vector (ω_μ) meson fields. This model conceptually is in sharp disagreement with conventional nuclear physics approach, where a static two body force together with a non-relativistic Schrödinger equation form the basis to describe a many body system such as the nucleus. The conventional approach, after 15 years of intense cultivation, has reached such a point of sophistication that only minor modifications are possible.[2)] Nontheless, it still fails to provide a reliable phenomenological tool by which normal nuclear properties can be tied together with higher density and temperature phenomenon in nuclear matter. Furthermore, the whole approach might be unduly too complicated. Ideally, one would like to have a complete description of nuclear physics, where the bulk properties of nuclei are directly related to the forces derived from low energy nucleon-nucleon scattering. It has been emphasized by Serber[3)] and Walecka[1)] and a long time ago by Teller and Johnson,[4)] that nuclear structure can perhaps be easier understood by a much simpler, single body central force. After all, this is the basis of the shell model. It has been shown by Miller[5)] that relativistic effects in such a mesonic central field approach are very important, in that they affect the single body kinetic energy. Thus the investigation of alternative nuclear matter models is desirable. In this note we shall explore the phenomenological consequences of the Walecka model.

Recently, Walecka and Serot[6] showed that the field equations of the Walecka model, truncated for numerical convenience, give a fair description of the charge densities in Ca^{40} and Pb^{208}. Serber has shown that a nuclear model based on a mesonic field σ, together with a hard core repulsion given by an excluded volume, gives rather good results for finite nuclei.[3] In this work we show that a careful analysis of the Walecka model, together with a better choice of parameters leads to charge distributions in Ca^{40}, Ca^{48}, Ni^{56}, Zr^{90}, Pb^{208} that are in rather good agreement with experimental data, where it is available, and in an overall agreement with the more refined non-relativistic Hartree-Fock calculations. The quality of our results in the Thomas-Fermi approximation leads us to believe[10] that a relativistic Hartree approximation to the Walecka model will give an excellent description of spherical, closed shell nuclei.

The Lagrangian describing the interaction of nucleons ψ with scalar field σ isoscalar vector field ω_μ and isovector meson field $\hat{R}_\mu$ and electromagnetic field A_μ is assumed to be

$$\mathcal{L} = -\bar{\psi}(\gamma_\mu \partial_\mu + m_n)\psi - \frac{1}{2}((\partial_\mu \sigma)^2 + m_\sigma^2 \sigma^2) - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}_{\mu\nu} \tag{1}$$

$$- \frac{1}{4} H_{\mu\nu} H_{\mu\nu} - \frac{1}{2} m_V^2 \omega_\lambda \omega_\lambda - \frac{1}{2} m_V^2 \hat{R}_\mu \cdot \hat{R}_\mu + i g_V \bar{\psi} \gamma_\lambda \psi \omega_\lambda$$

$$+ i g_r \bar{\psi} \gamma_\lambda \hat{\tau} \cdot \hat{R}_\lambda \psi + i e \bar{\psi} \gamma_\mu \frac{(1 + \tau_3)}{2} \psi A_\mu - g_S \bar{\psi} \psi \sigma$$

where

$$F_{\mu\nu} = \frac{\partial}{\partial X_\mu} \omega_\nu - \frac{\partial}{\partial X_\nu} \omega_\mu \tag{2a}$$

$$\vec{G}_{\mu\nu} = \frac{\partial}{\partial X_\mu} \hat{R}_\nu - \frac{\partial}{\partial X_\nu} \hat{R}_\mu \tag{2b}$$

$$H_{\mu\nu} = \frac{\partial}{\partial X_\mu} A_\nu - \frac{\partial}{\partial X_\nu} A_\mu \tag{2c}$$

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \tag{3}$$

The field ω_μ corresponds to omega meson with a mass of m_V = 780 MeV, the field $\hat{R}_\mu$ corresponds to rho meson with the same mass. In the Lagrangian m_s is the mass of the sigma meson, adjusted to give correct nuclear surface thickness, and m_n is the nucleon mass. The quantum field theory is approximated by the use of the mean field approximation, which amounts to replacement of the field operators by their expectation values. That is, $\sigma \to \langle\sigma\rangle = \sigma_o$, $\omega_\mu \to \langle\omega_\mu\rangle = \delta_{\mu o}\omega_o$, $R_\mu^{(k)} \to \langle R_\mu^{(k)}\rangle = \delta_{ko}\delta_{\mu o}R_o^{(o)}$. The mean field approximation is known to be good at high nuclear densities. We shall show that it is also a good phenomenological assumption for finite nuclei. The dimensionless coupling constants $C_s = g_s(m_n/m_s)$, $C_V = g_V(m_n/m_V)$ and $C_r = g_r(m_n/m_V)$ are adjusted to saturate infinite symmetric nuclear matter at a Fermi momentum of $k_F = 1.34\ \mathrm{fm}^{-1}$ with a binding energy of –15.75/particle. And a symmetry energy of 30 MeV. The choice of the saturating Fermi momentum is dictated by the desire to fit simultaneously the central density of Ca^{40} and Pb^{208}. Small variations in k_F will produce small variations in the central density of Pb^{208}, but will substantially affect that of Ca^{40}. The best choice of k_F = 1.34 fm^{-1} is in good agreement with that determined by quasielastic electron scattering.[7)] The corresponding coupling constants are C_s = 17.96, C_v = 15.6 and C_r = 3.5.

The time independent, spherically symmetric field equations are

$$\frac{d^2\sigma_o}{dr^2} + \frac{2}{r}\frac{d\sigma_o}{dr} = m_s^2\sigma + g_s\left(\rho_s^{(n)} + \rho_s^{(p)}\right) \tag{4a}$$

$$\frac{d^2\omega_o}{dr^2} + \frac{2}{r}\frac{d\omega_o}{dr} = m_v^2\omega_o - g_v\left(\rho_v^{(n)} + \rho_v^{(p)}\right) \tag{4b}$$

$$\frac{d^2R_o^{(o)}}{dr^2} + \frac{2}{r}\frac{dR_o^{(o)}}{dr} = m_v^2R_o^{(o)} - g_r\left(\rho_v^{(n)} - \rho_v^{(p)}\right) \tag{4c}$$

$$\frac{d^2A_o}{dr^2} + \frac{2}{r}\frac{dA_o}{dr} = e\rho_v^{(p)} \tag{4d}$$

where

$$\rho_s = \sum_\ell \bar{\psi}_\ell\psi_\ell \tag{4e}$$

$$\rho_v = \sum_\ell \psi_\ell^+\psi_\ell \tag{4f}$$

for neutrons and protons respectively. The summation extends over all occupied states. A Thomas-Fermi approximation means that the neutron or proton trial wave function is taken to be

$$\psi \sim \begin{pmatrix} \chi \\ \dfrac{\vec{\sigma}\cdot\vec{k}\chi}{\sqrt{k^2+m^{*2}} + m^*} \end{pmatrix} e^{i\vec{k}(r)\cdot\vec{r}} \tag{5}$$

where the local Fermi momentum for neutrons $k_F^{(n)}(r)$ and protons $k_F^{(p)}(r)$ is determined by the corresponding Fermi energies $E_F^{(n)}$ and $E_F^{(p)}$ through the following relationship

$$E_F^{(p)} = g_\omega \omega_o - g_r R_o^{(o)} + eA_o + \sqrt{k_F^{(p)}(r)^2 + m^*(r)^2} \tag{6a}$$

$$E_F^{(n)} = g_\omega \omega_o + g_r R_o^{(o)} + \sqrt{k_F^{(n)}(r)^2 + m^*(r)^2} \tag{6b}$$

$$m^* = m_n + g_s \sigma \tag{6c}$$

In the Thomas-Fermi approximation the nuclear source terms become spacially dependent through an easily calculable relationship

$$\rho_s = \frac{2}{(2\pi)^3} \int^{k_F(r)} d^3k \frac{m^*(r)}{\sqrt{k^2 + m^*(r)^2}} \tag{7a}$$

$$\rho_v = \frac{2}{(2\pi)^3} \int^{k_F(r)} d^3k = \frac{1}{3\pi^2} k_F^3(r) \tag{7b}$$

The field equatins given by Eqs. (4a-4d) together with those of Eqs. (7a-7b) constitute a set of non-linear differential equations. We solve them numerically as outlined in the work of Boguta and Rafelski.[8)]

In Fig. 1 we show the proton density in Ca^{40} and Pb^{208}, compared with recent experimental results. The sigma meson mass is m_s = 500 MeV. The surface behavior is quite good, with the usual fast roll off of the densities characteristic of the Thomas-Fermi approximation. The central density in Pb^{208} does not show the slight depression or the slight rise in Ca^{40}. These effects, we believe, will result from a complete Hartree calculation. Nontheless, quantitatively speaking even our Thomas-Fermi approximation gives good results. They are comparable with refined conventional Hartree-Fock calculations. In Fig. 2 we show the proton and neutron distributions in Ni^{56} and Zr^{90}. The trend is correct once again, excluding the fast roll of at the toe of the distributions. In Fig. 3 we show the expected deviation in the differential cross section of electron scattering from Ca^{40} as compared with electron scattering from Ca^{48}. It is obtained by computing the form factors for Ca^{40} and Ca^{48}. We see that the relativistic model does quite well (as compared to other Hartree-Fock calculations).[9)] In Table 1 we show the root mean square radii for neutrons and protons in various closed shell nuclei. As expected, the radii are slightly small (on account of the Thomas-Fermi approximation), but the differences are quite good.

In the Walecka model, the relativistic effects, roughly speaking, are measured by the difference in the scalar and vector densities. This difference is the square of the small component of the nucleon wave function (summed over momentum). In Pb^{208} the central density ratio is ρ_S/ρ_V = 0.93. A similar ratio hold for a light nucleus such as O^{16}. The effective mass of the nucleon in this model is

$m^* \approx 0.5\ m_n$ and thus there is a significant correction to the nucleon kinetic energy due to relativistic kinematics. By way of comparison, it is interesting to analyze the model of Boguta and Bodmer.[10)] In this model the strong vector meson repulsion is significantly reduced by the introduction of non-linear sigma interactions. The attraction given by scalar meson exchange is effectively balanced by the σ^4 repulsion. Consequently, the effective mass is $m^* \simeq 0.9\ m_n$ and the relativistic effects and spin-orbit splitting are considerably smaller. The spin-orbit splitting is a factor 4 too small.[11)] Since the relativistic aspect of the Walecka model is quite important, the use of the non-relativistic limit of the model in comparing it to conventional many body theories is unjustified.[12)] In fact, the non-relativistic limit will not saturate infinite nuclear matter. The phenomenological success of the relativistic mean field model should stimulate effort for a better theoretical understanding of its relation to conventional nuclear physics. Perhaps nature has so conspired that relativistic and mesonic effects, when properly analyzed in conventional nuclear physics language, will lead to the complex structure of the non-relativistic theory.

Table 1.

	r_n(fm)	r_p(fm)	$r_n - r_p$
Ca^{40}	3.291	3.340	−0.049
Ca^{48}	3.474	3.409	+0.065
Ni^{56}	3.605	3.643	−0.038
Zr^{90}	4.237	4.158	0.079
Pb^{208}	5.512	5.317	−0.195

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FIGURE CAPTIONS

Fig. 1a. Proton density distribution in Ca^{40}. The solid line is experimental data and the broken line is the Walecka model prediction.

Fig. 1b. Proton density distribution in Ca^{40}. The solid line is experimental data and the broken line is the Walecka model prediction.

Fig. 2a. Proton density distribution in Ni^{56} and Zr^{90} as predicted by the Walecka model.

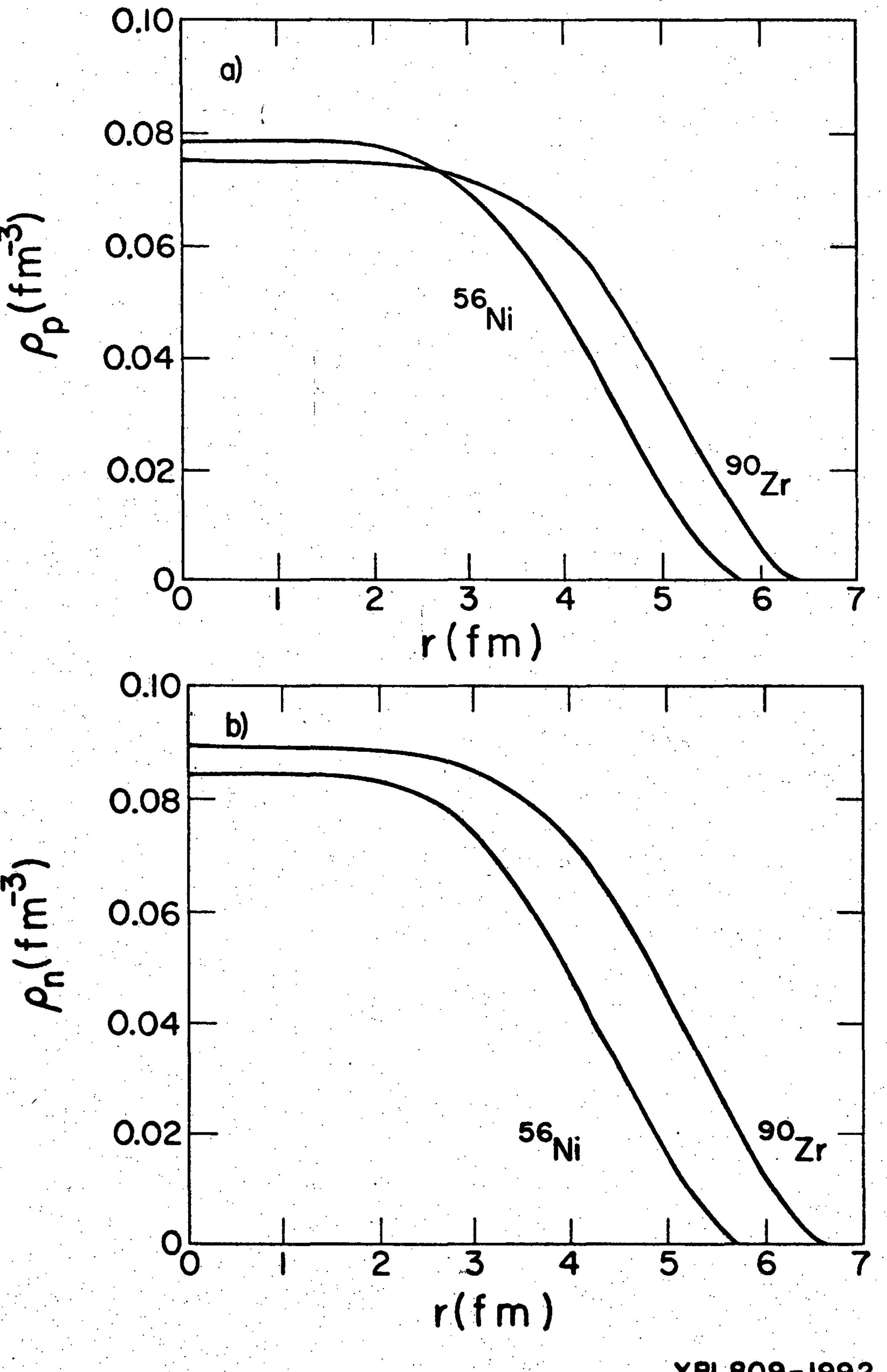
Fig. 2b. Neutron density distribution in Ni^{56} and Zr^{90} as predicted by the Walecka model.

Fig. 3. The deviation in the differential cross section for electron scattering from Ca^{40} as compared to scattering from Ca^{48}

$$2\left[\frac{d\sigma}{d\Omega}(40) - \frac{d\sigma}{d\Omega}(40)\right] / \left[\frac{d\sigma}{d\Omega}(40) + \frac{d\sigma}{d\Omega}(48)\right] .$$

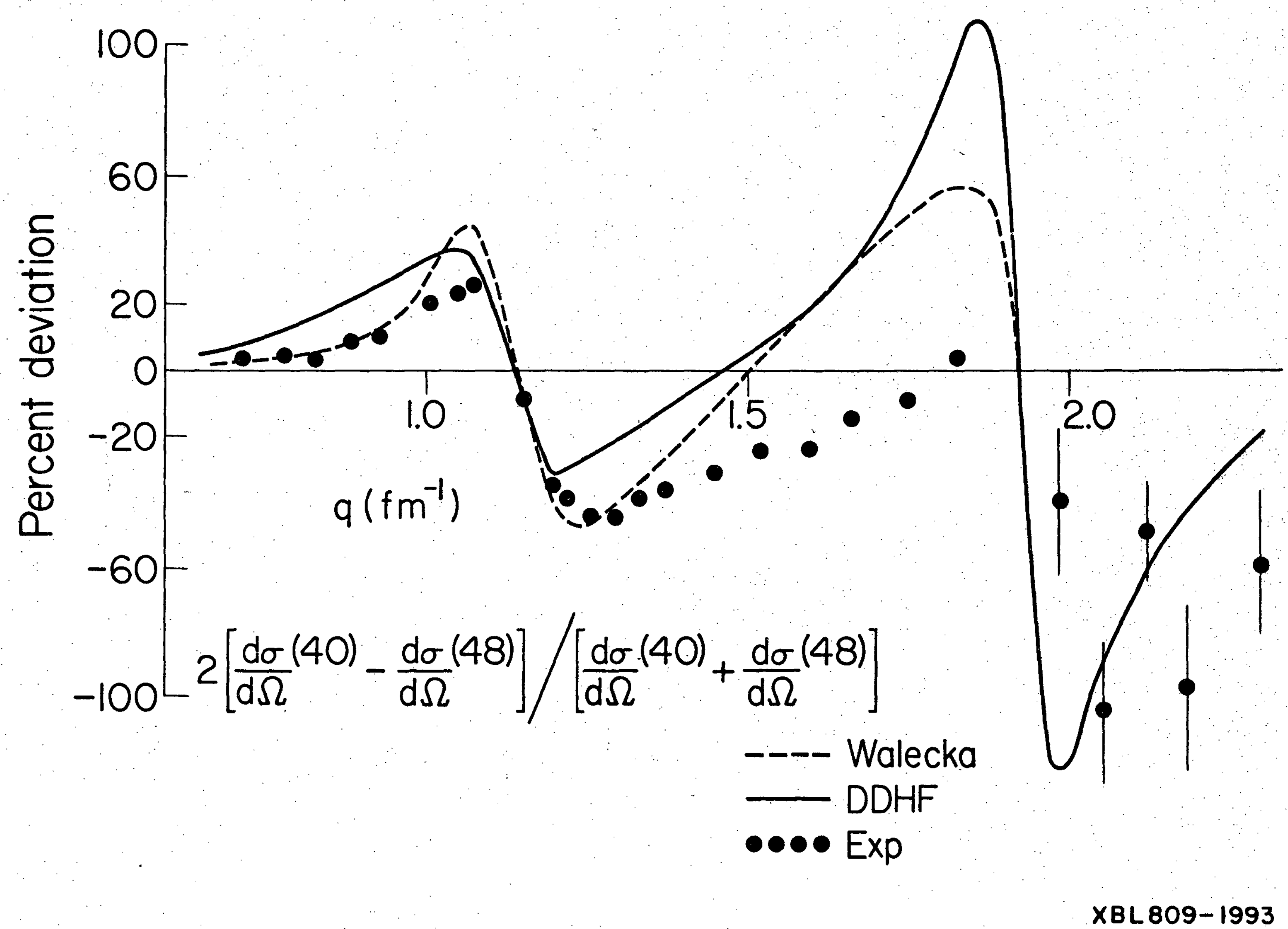
The broken line is the Walecka model prediction. The solid line is DDH prediction, and the dots are the data.

The figures were printed from originals provided by the author.



a)
b)
0.10
0.08
0.06
0.04
0.02
0
$\rho_p(fm^{-3})$
$\rho_n(fm^{-3})$
r (fm)
^{56}Ni
^{90}Zr
0 1 2 3 4 5 6 7
XBL809-1992

Fig. 1



Percent deviation
100
60
20
0
-20
-60
-100
1.0
1.5
2.0
q (fm^{-1})
$2\left[\frac{d\sigma}{d\Omega}(40) - \frac{d\sigma}{d\Omega}(48)\right] \Big/ \left[\frac{d\sigma}{d\Omega}(40) + \frac{d\sigma}{d\Omega}(48)\right]$
Walecka
DDHF
Exp
XBL 809-1993

Fig. 2

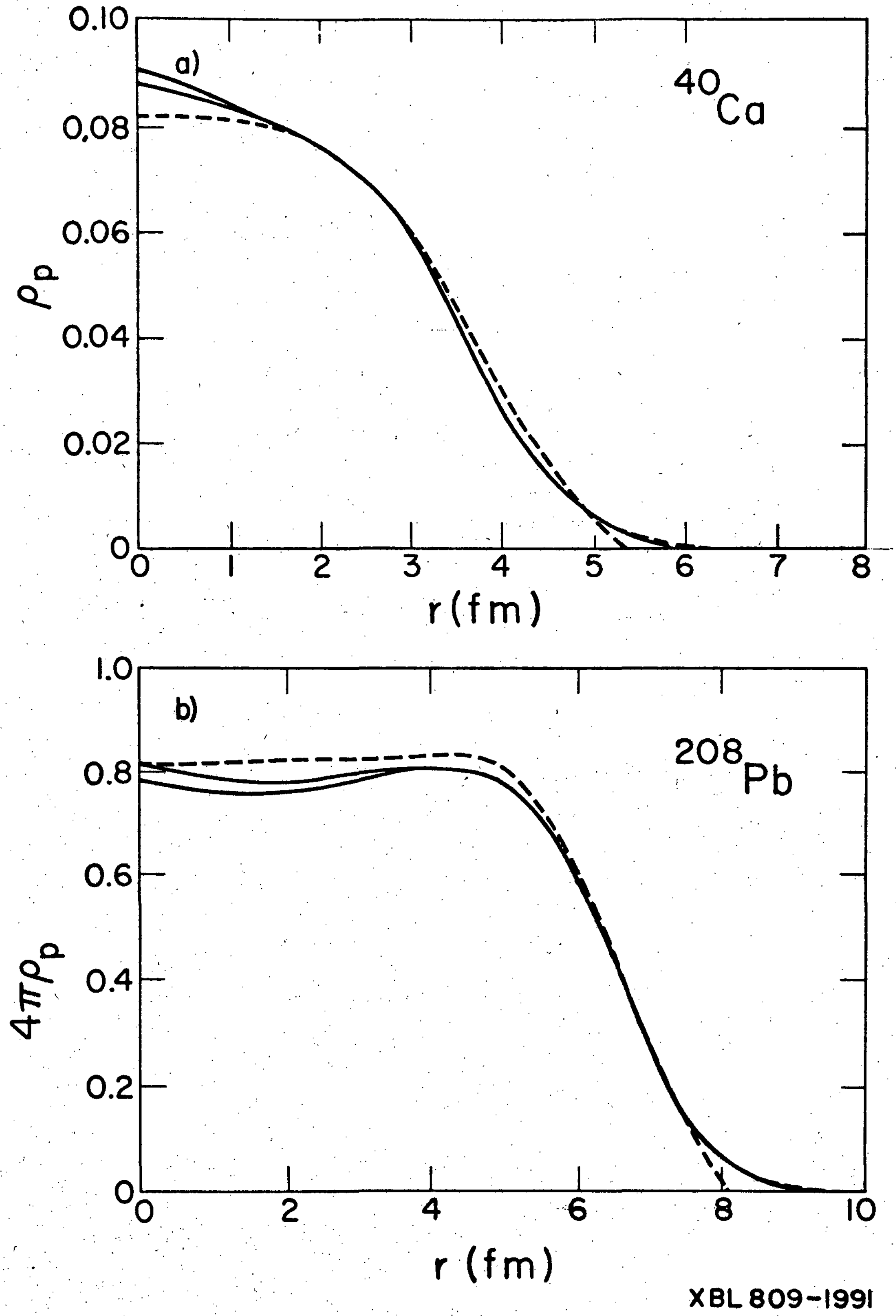

0.10
0.08
0.06
0.04
0.02
0
a)
40Ca
ρp
0 1 2 3 4 5 6 7 8
r (fm)
1.0
0.8
0.6
0.4
0.2
0
b)
208Pb
4πρp
0 2 4 6 8 10
r (fm)
XBL 809-1991

Fig. 3