Submitted to Nuclear Physics B

LBL-2109
Preprint

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K. Bardakci

August 7, 1973

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

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K. Bardakci

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

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ABSTRACT

The question of spontaneous symmetry breaking in dual models is investigated. In the context of a particular model with a conserved "charge", two different approaches to the problem, spurion emission and the effective potential methods, are developed. A method is described for the calculation of the effective potential, and it is applied to determine the first few terms of the potential.

* This work was supported in part by the U. S. Atomic Energy Commission.

1. Introduction

It has been realized for some time that dual models suffer from too much symmetry. For example, when an SU(3) symmetry is introduced through Chan-Paton [1] factors, it is difficult to break the symmetry in a satisfactory fashion. The Neveu-Schwartz [2] model has a kind of (unwanted) "G" parity that is hard to get rid of. Finally, the orbital model with intercept (-1) and seemingly with no symmetry, has a gauge invariance of the second kind which constrains the mass of the lowest lying vector meson to be zero [3].

The symmetry in this last case results from invariance under the Virasoro [4] algebra, which is of course needed if the model has to be ghost free [5]. The situation is similar to quantum electrodynamics or Yang-Mills type theories, the local symmetry (gauge group), needed to make the vector particle transverse (and ghost free), results in also making its mass zero.

Of course, in a Lagrangian theory, one can always add a mass term by hand, and break the gauge invariance explicitly. The only thing that is lost in the process is renormalizability for non-abelian gauge theories. In the case of dual models, however, we are unable to change the intercept of the Regge trajectory without introducing ghosts or violating duality and in the process ruining the model.

There is an alternative approach to the problem of generating a gauge invariant Lagrangian with a massive vector field. In this approach, due to Goldstone [6] and Higgs [7], the gauge invariance is broken by the vacuum, although it is formally preserved in the equations motion. The original symmetry of the theory is therefore broken spontaneously, and the vector meson acquires a finite mass by absorbing the massless scalar Goldstone boson.

In what follows, we shall try to follow the path suggested by field theory as closely as possible in the case of dual models. As a specific example, we shall consider the orbital model, although our method can be extended to more complicated models like the Neveu-Schwartz model or the quark model [8]. We wish to construct a model in which the lowest lying vector meson mass is moved away from its canonical value of zero by virtue of a spontaneous breakdown of the gauge symmetry.

Clearly, the mechanism for spontaneous breakdown is already present in the orbital model with intercept (-1), since the "bare" vacuum is unstable under decay into pairs of tachyons. From this point of view, the existence of a scalar tachyon in the model turns out to be an advantage, rather than a defect! If one could then solve the theory exactly by adding up the whole perturbation series, then necessarily one or the other of the following two alternatives would be true. Either there would emerge a new stable vacuum, or the theory would simply collapse without a stable vacuum. Since we are unable to sum the perturbation series, we have to treat the problem in the semiclassical or the tree approximation, again in full analogy to the Lagrangian approach.

At this point we have to face a technical problem. Consider, for example, the spontaneous breakdown of $SU(2) \otimes SU(2)$ in the sigma model [9]. At the level of tree approximation, one can treat fields in the sigma model Lagrangian as classical fields and search for the minimum of the Lagrangian. This minimum corresponds to a nonzero vacuum expectation value for the sigma field. We do not have any reliable Lagrangian formulation for dual models, so we are forced to approach the problem from an S-matrix point of view. Such an approach

was already used in connection with the sigma model [10]; it involves emitting zero four momentum sigma particles (spurions) into the vacuum. This new approach is completely equivalent to shifting the vacuum expectation value of the sigma field; however, it is easier to cast it into an S-matrix language. For practical calculations, it turns out that one can always go back to an <u>effective</u> Lagrangian, which usually simplifies the calculations.

A more serious difficulty is connected with the off mass-shell continuation. If we try to induce the spontaneous breakdown by emitting tachyons into the vacuum, in analogy with sigma model, we face the problem of continuing the vertex for the tachyon emission to an off shell point corresponding to zero four momentum. Since as yet no satisfactory off mass-shell continuation of the dual model exists, we "promote" the tachyon into a zero mass particle by introducing a conserved "fifth momentum" of unit magnitude into the model. In this modified model, there are two states which consist of the original tachyon state plus a unit of fifth momentum. These states can be emitted into vacuum at zero four momentum on the mass shell, since they have zero mass.

This method looks somewhat provisional; however, it enables us to stay on the mass shell and retain all the nice features of the original dual model, such as duality in the form of cyclic symmetry, absence of negative norm states, etc. It is also easy to understand it in a simple manner. The fifth momentum can be thought of as a conserved "charge", and the states of zero mass and plus or minus one unit of charge can be represented by a complex scalar field of (bare) mass zero. The emission of charged spurions into the vacuum breaks charge conservation and hopefully gives rise to a new nonsymmetrical solution. In this respect, the situation is the same as in the original models of Goldstone and Higgs, where a similar breakdown of charge conservation occurs. The only difference is that the bare mass of the scalar field here is constrained to be zero, for reasons given earlier, as opposed to the situation in field theory, where the mass is arbitrary.

In what follows, we first define and describe the model in the symmetric (normal) case. We then develop the theory of the vacuum emission of the spurions in parallel to the field-theoretical case. We then show that the spurion amplitudes can be summed by means of an effective Lagrangian, which itself contains an infinite number of contact terms. The first few terms are explicitly calculated, and some tentative suggestions are made. The difficult problem of the calculation of all the terms of the effective Lagrangian is not attempted in this paper.

2. Symmetric Orbital Model

We consider an "N" point amplitude of charged scalars of mass zero. The charge takes on the values ± 1 and is identified with the fifth momentum. By charge conservation, N is necessarily even. Up to a constant of proportionality, the amplitude is given by the following formula,

$$B_N = \int du \times J \times \prod_{ij} (u_{ij})^{-s_{ij}+Q_{ij}^2-2} , \qquad (2.1)$$

where (du) stands for the volume element in any $N-3$ nonoverlapping u_{ij}'s, J is an appropriate weight factor, u_{ij}'s are the Koba-Nielsen variables [11] for the channel defined by indices i and j

(see Fig. 1), s_{ij} is the channel center of mass energy squared and Q_{ij} is the total charge of the channel. If one chooses a cyclic configuration for the $N-3$ nonoverlapping channels, they can be taken, say, to be $u_{1,i}$, $i = 2,3,\cdots,N-2$, as in Fig. 1, in which case $u_{i,j}$ are given by the following,

$$u_{2,k} = \frac{1 - u_{1,2}\cdots u_{1,k-1}}{1 - u_{1,2}\cdots u_{1,k}}$$

$$u_{k,\ell} = \frac{(1 - u_{1,k}\cdots u_{1,\ell-1})(1 - u_{1,k-1}\cdots u_{1,\ell})}{(1 - u_{1,k}\cdots u_{1,\ell})(1 - u_{1,k-1}\cdots u_{1,\ell-1})} \qquad (2.2)$$

$$(k < \ell, \quad k \neq 2, \quad \ell \neq N-1) .$$

Of course, one need not choose a cyclic set for the independent variables, in which case the set independent channels suggested by any planar Feynman graph with cubic coupling will serve just as well. One can then express all the u's in terms of a noncyclic set of independent variables; however, the expressions are complicated and are not needed in what follows.

It remains to define J. For a cyclic choice of variables, as in Fig. 1, J is given by

$$J = \frac{1}{1 - u_{1,2}u_{1,3}} \times \frac{1}{1 - u_{1,3}u_{1,4}} \times \cdots \times \frac{1}{1 - u_{1,N-3}u_{1,N-2}} . \qquad (2.3)$$

For further details, see, for example, reference 11.

We have now to specify the ordering of the external charged lines. At first, it seems that one can construct an amplitude of the form given by Eq. (2.1) by ordering the positive and negative charges in any pattern whatsoever, subject solely to charge conservation. In fact, one can further form superpositions of amplitudes with different charge patterns. However, it turns out that there are only three schemes consistent with factorization. The scheme we are going to use involves an alternation of the sign of the charge in the cyclic confiugration and is well known to be factorizable (see Fig. 2). The second scheme involves a coherent superposition of positive and negative charge states, and therefore ends up as model with no charge. This is clearly uninteresting. The third scheme is more complicated, and although there seems to be nothing wrong with it, will not be considered any further. The schemes mentioned above are more fully discussed in Appendix 1.

Specializing Eq.(2.1) to the case of alternating charge assignment, $(Q_{ij})^2 = 0$ or 1 according to whether the channel labeled by i and j carries an even number of charged lines with net charge zero (even channel), or an odd number of lines with net charge ± 1 (odd channel). We therefore have,

$$B_N = \int du \times \underline{J} \times \prod_{ij} (u_{ij})^{-s_{ij}+\epsilon_{ij}-2} \qquad (2.4)$$

where $\epsilon_{ij} = 0$ for even channels and $+1$ for odd channels.

It is of some interest to write B_N in factorized form, using the well-known harmonic oscillator operators. The factorized form is as follows:

$$B_N(k_1,k_2,\cdots,k_N) = \langle 0|V^+(k_2)\frac{1}{R - s_{12} - 1} V^-(k_3) \times \frac{1}{R - s_{13}}$$

$$\times\ V^+(k_4)\frac{1}{R - s_{14} - 1} \times \cdots \times \frac{1}{R - s_{1,N-2} - 1} V^-(k_{N-1})|0\rangle \ ,(2.5)$$

where $k_1,k_2,\cdots,k_N$ are the external momenta, $s_{ij} = (k_i + k_{i+1} + \cdots + k_j)^2$, and R and $V^{\pm}$ are given by the usual expressions:

$$R = \sum_{n=1}^{\infty} n[(a_n{}^{\mu})^+(a_{n,\mu}) + a_n^+ a_n] \ ,$$

$$V^{\pm}(k) = \exp\left\{\sqrt{2}\, k_{\mu} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left(a_n{}^{\mu} + (a_n{}^{\mu})^+\right) \pm i\sqrt{2} \times \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} (a_n + a_n{}^+)\right\} .$$

The a's satisfy the following commutation relations

$$[a_n{}^{\mu},(a_m{}^{\nu})^+] = g^{\mu\nu}\, \delta_{n,m}$$

whereas the "fifth" operator "a" satisfies the usual commutation relations:

$$(a_n,a_m{}^+) = \delta_{n,m} \ .$$

Note that, compared to the standard convention, we have changed the sign of the commutator of the "fifth" oscillator, and compensated for it by an extra "i" in the expression for $V^{\pm}$.

We conclude the section with a brief discussion of the spectrum. One has to distinguish between odd and even channels, which correspond to intercepts zero and one respectively. In the odd sector, the lowest lying state is the vacuum $|0\rangle$, and it corresponds to a charged scalar of zero mass. In fact, this is nothing but the charged scalar of the external lines. In the even sector, the vacuum corresponds to a scalar with the mass squared minus one (the tachyon). Further, there is the state $a_1^+|0\rangle$, corresponding to a neutral scalar of mass zero, and the state $(a_1^\mu)^+|0\rangle$, corresponding to a vector of mass zero (the photon). These states will play a special role in what follows, whereas states of higher mass will not be of any particular interest.

3. Spurion Emission

The spurion is defined to be the charged scalar at zero four momentum. We have to define auxiliary amplitudes where a certain number of spurions disappear into the vacuum, and in our treatment, we follow a similar treatment of the sigma model by Lee [10]. Let us begin by defining an amplitude B_{N+m} to have $N + m$ external lines, N of them belonging to charged scalars of nonzero four momenta $k_1, k_2, \cdots, k_N$, and the remaining "m" belonging to spurions. The only difference between spurions and the rest is that spurions carry zero four momenta.

As usual, the charge alternates in the cyclic configuration. Since the dual amplitude has only cyclic symmetry, the full Bose symmetry must be put in by hand, and so we symmetrize the external of positive and negative charges separately with respect to all the elements of the full symmetry group, excluding the cyclic elements.

This operation includes symmetrization between spurions and the other external lines; however, since the amplitude is already symmetric with respect to the spurions themselves (they all carry the same momentum), external spurion lines of like charge are not symmetrized any further to avoid multiple counting. The resulting amplitude is now unambiguous. The final step consists of multiplying the above amplitude by an appropriate power of spurion to vacuum transition constant and summing over the number of emitted spurions:

$$\bar{B}_N(k_1,k_2,\cdots,k_N;\, c) \;\equiv\; \sum_{m=0}^{\infty} c^m\, S\, B_{N+m}(k_1,k_2,\cdots,k_N,\underbrace{0,0,0,0}_{m\ \text{times}}) \,. \tag{3.1}$$

Here c is the spurion to vacuum transition constant, S stands for the Bose symmetrization described above, and the zeros symbolically stand for the momenta of the spurion lines. In defining $\bar{B}_N$, what we have done so far is similar to adding a term "$c6$" to the sigma model Lagrangian, and then summing over all sigma to vacuum transitions. Note that $\bar{B}_N$ in general need not conserve charge and N can be odd; the deficit is taken up by the spurions.

We are ultimately interested in the limit $c \to 0$. It is easy to recover the amplitude we started with by setting $c = 0$ in Eq. (3.1), and this is the uninteresting "normal" solution.

To get the Goldstone solution, we have to assume that there is a branch cut singularity somewhere in the complex c plane, and that one can come back to the point $c = 0$ in a different sheet after going through the branch cut. To demonstrate the assumed analyticity in the complex c plane is the standard problem of phase transition in statistical mechanics. This is a difficult problem which will occupy us for the rest of the paper; however, the following point should be made clear before we plunge into the technical complications. If the amplitude has the required analyticity properties in the c-plane and the Goldstone solution exists, then the solution is a dual amplitude with all the nice properties one has been looking for, so far unsuccessfully. Cyclic duality easily follows since each term in the sum (3.1) has this property, and factorization is also satisfied, as will be shown explicitly later. Finally, there can be no ghosts since the amplitude B_N we started with had no ghosts. Indeed, the situation here is again quite similar to what happens in field theory: the Goldstone solution shares all the nice properties of the normal solution (absence of ghosts, factorization, crossing symmetry etc.), except for the original symmetry of the Lagrangian that gets broken. The symmetry in our case is charge conservation and the associated gauge invariance, and the new "goldstone" model which breaks this symmetry spontaneously should preserve all the good features of the normal model. The spectrum of the new model will be different from the old one, and in particular the "photon" is expected to acquire a finite mass, eliminating an undesirable feature of the old model. This should be compared with the "hit and miss" approaches to constructing new dual models on the basis of guess work, when it is not initially clear whether the model will enjoy the various desirable properties mentioned earlier. In contrast, apart from the admittedly difficult and crucial problem of demonstrating the existence of the Goldstone solution, we are guaranteed of all the correct features for the model from the start.

We close this section by presenting a formal summation of the spurion lines in the operator formalism. Consider the amplitude with N ordinary external lines and m spurions defined previously; this amplitude can be split into a vertex of N_1 particles and m_1 spruions, another vertex of N_2 particles and m_2 spurions, joined by a propagator which emits ℓ spurions, as shown in Fig. 3b, with $N_1 + N_2 = N$, and $m_1 + m_2 + \ell = m$. The wavy lines in the figure indicate spurions, the solid ones indicate the particles, and we have chosen a particular pattern of alternation between particles and spurions. (We have, of course, to sum over all distinct patterns.) The factorized form described above is represented by the following formula:

$$B_{N+m} = \langle 0| \Gamma_{(\text{left})} \Delta(s) \Gamma_{(\text{right})} |0\rangle , \tag{3.2a}$$

where $\Gamma_{(\text{left})}$ and $\Gamma_{(\text{right})}$ are the vertices that go with the bunch of lines on the left and right, respectively, in Fig. 3, and $\Delta(s)$ is the propagator, with $s = (k_1 + k_2 + \cdots + k_{N_1})^2$. Γ and Δ are easily expressed in terms of the operators of Eq. (2.5). In order to treat the odd and even (charged and neutral) channels uniformly, we take Γ's as two component spinors and Δ as a two by two matrix in the odd-even channel space.

We have,

$$\Delta(s) = \Delta^{(+)}(s)\,\Delta^{(-)}(s)\ ,$$

$$\Delta^{(\pm)} = c^{\ell(\pm)}\left(\tilde{\Delta}^{(\pm)}\right)^{\ell(\pm)}\Delta_0,\quad \Delta_0 = \begin{pmatrix} \frac{1}{R-s-1} & 0 \\ 0 & \frac{1}{R-s-2} \end{pmatrix},$$

$$\tilde{\Delta}^{(\pm)} = \begin{pmatrix} 0 & \tilde{\Delta}^{(\pm)}_{12} \\ \tilde{\Delta}^{(\pm)}_{21} & 0 \end{pmatrix},$$

$$\tilde{\Delta}^{(\pm)}_{12} = \frac{1}{R-s-1}\exp\left(\sqrt{2}\,i\,Q^{(\pm)}\right),\qquad (3.2b)$$

$$\tilde{\Delta}^{(\pm)}_{21} = \frac{1}{R-s-2}\exp\left(-\sqrt{2}\,i\,Q^{(\pm)}\right),$$

$$Q^{(+)} = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}a_n\ ,\qquad Q^{(-)} = \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}a_n\ .$$

The operators R and a_n above are defined in the last section. The expression for Γ is not needed here and will not be written down. We note that $\Delta^{(+)}$ corresponds to spurions emitted from the propagator in the up direction, and $\Delta^{(-)}$ corresponds to spurions emitted in the down direction, so that the product is symmetric under the "twist" operation [11]. One can sum over the number of up and down spurions $\ell_{(\pm)}$, and arrive at the following propagator:

$$D = D^{(+)}\,D^{(-)}\,\Delta_0\ ,\qquad (3.2c)$$

$$D^{(\pm)} = \frac{1}{1-c\,\tilde{\Delta}^{(\pm)}}\ .$$

D is the propagator of the amplitude given by Eq. (3.1), and therefore it determines the mass spectrum. An amusing fact, which we note in passing, is that D^{-1} is a generator of a new conformal group. We know, for the normal orbital model, the following properties are valid;

$$(L_n,L_m) = (n-m)\,L_{n+m} + \tfrac{1}{3}n(n^2-1)\,\delta_{n,-m}$$
$$L_0 = R - s\ .\qquad (3.3a)$$

The expressions for L_n's are well known and will not be reproduced here [11]. This is, of course, the algebra that is responsible for the elimination of negative norm states. The propagator defined above also turns out to be part of a similar algebra:

$$\left(J^{(+)}_n, J^{(+)}_m\right) = (n-m)\,J^{(+)}_{n+m} + \tfrac{1}{3}n(n^2-1)\,\delta_{n,-m}\ ,$$
$$J^{(+)}_n = L_n + c\,\exp\{\sqrt{2}\,i\,Q^{(+)}\},\qquad (3.3b)$$

with a similar expression for the $(-)$ operators. To verify the algebra, we note that the spurion emission vertex carries conformal spin one, since the spurion has zero mass:

$$[(L_n - L_m),\ \exp\{\pm\sqrt{2}\,i\,Q^{(+)}\}] = (n-m)\,\exp\{\pm\sqrt{2}\,i\,Q^{(+)}\}\ .\qquad (3.3c)$$

Using (3.3c), one can easily verify (3.3b), so the conformal algebra is formally intact! It then seems reasonable to try to build a model based on this new representation of the conformal algebra.

Unfortunately, the operator manipulations we have been indulging in are purely formal and cannot be taken too seriously. For example, if we let $c \to 0$ in (3.2c), we get back to the normal model, completely missing the possible Goldstone solution.

The trouble is easy to identify. Because of duality, the sum that appears in Eq. (3.2) contains not only the poles in the s channel which are explicit in the formula, but also poles in crossed dual channels as shown in Fig. 4. To produce crossed channel poles from a direct channel sum, the sum must diverge at the position of the pole. The invariant energy of a collection of spurions is zero, which is right on top of the pole generated by the exchange of zero mass particles. We therefore conclude that the sum in Eq. (3.2c) is divergent as it stands! To derive a meaningful expression, we have to do two things. Firstly, we have to start with finite spurion momenta so that all spurion subenergies are large and negative. This is the region, free of poles, in which the integral for the dual amplitude converges as it stands. When we reach the tachyon pole in any subenergy the integral diverges, so we have to solve the problem of analytic continuation around the tachyon pole. The second problem is the one mentioned before; even after the analytic continuation, we cannot directly reach the point at which all spurion invariant subenergies vanish, since we are right on top of zero mass particle poles. This is a spurious infrared problem which goes away when the infinite sum is done, since the zero mass particles will then acquire finite masses. However, to treat the finite order terms correctly and unambiguously, we have to separate the amplitude into pole term plus a finite part, and sum them separately. To do this using the operator formalism seems to be a formidable task, so that we will not here try to give a meaning to the manipulations of Eq. (3.3). Instead, in the next section, we will approach the problem using an effective Lagrangian.

4. Effective Lagrangian

Let us consider, once more, the sigma model. The "spurion" in this case is the sigma field at zero four momentum, and the sum over all spurion emissions is carried out by translating the sigma field to the minimum point of the Lagrangian. To verify that such a minimum exists, we have only to know the self couplings of the sigma field; the couplings with other fields are not needed. Similarly, if we know the Lagrangian for only the spurion self-couplings, and if that Lagrangian has a nontrivial minimum, that is sufficient for the existence of a Goldstone solution. Of course, to learn more about the model and, for example, to determine the mass spectrum etc., one has to take into account the other terms in the Lagrangian. In what follows, we shall mainly concentrate on the problem of showing the existence of the Goldstone phenomenon; we will not attempt to determine various other properties of the model. Hence, we shall only need the effective Lagrangian for spurion self couplings.

The method for determining the effective Lagrangian from any given S matrix is well known [12]. Given the amplitude B_N, one defines a one particle irreducible $\tilde{B}_N$ from which all zero mass one particle poles have been subtracted. (In our case, the zero mass particles to be eliminated are the charged and neutral scalars and the "photon", discussed in Sec. 2.) One then sets all the external momenta equal to zero to define $\tilde{B}_N(0)$. The effective Lagrangian or potential contains (in general an infinite number of) contact interactions of the fundamental zero mass fields. The coefficients of the interaction terms are determined by the requirement that in the tree approximation they reproduce $\bar{B}_N(0)$ for all N. There should be no confusion about one point: The effective Lagrangian reproduces the dual amplitude only when all external legs are at zero momentum, and in general it fails to do so for other values of the momenta. For one thing, the Lagrangian does not contain the full spectrum of the dual model; it only contains zero mass fields. Even if one tried to include more of the spectrum of the dual amplitude by introducing many more fields, it is well known that a Feynman graph expansion where dual channel poles are simply additive runs into serious difficulties with duality. However, in the semiclassical (tree) approximation, the only thing that is needed to work out the Goldstone solution is the value of the field at zero four momentum, and the effective Lagrangian is quite adequate for that purpose.

At this point, one may wonder why, of the four zero mass particles at our disposal, we are using two only as spurions. We cannot use the vector particle (photon) for fear of breaking Lorentz invariance, but there is no reason why, in addition to the charged scalars, we should not use the neutral scalar. In fact, we shall enlarge our "collection" of spurions to contain the neutral zero mass scalar, and so we shall consider amplitudes where external lines are either charged or neutral scalars. The order in which various external lines appear in the cyclic configuration again becomes a problem. The solution is provided by the requirement of factorization. The new amplitudes are obtained from the old ones by factorizing at the zero mass poles; charged scalars appear in the odd channels and the

neutral ones in the even channels. The final rule is the following: In the cyclic configuration, charged lines alternate in the sign of the charge same as before, and the neutral lines are inserted between charged ones in all possible distinct ways; the result being summed over all distinct insertions. Figure 5 is a simple example of this rule. Of course, everything must be Bose symmetrized among like charge states at the end.

Let us denote the charged and nuetral scalars by a complex field ϕ and a hermitian field χ respectively, and the "photon" by A_μ. The effective potential is of the following form:

$$V = i e_1 A^\mu\Big(\phi^+(\partial_\mu\phi) - \phi(\partial_\mu\phi^+)\Big) - e_1{}^2 (A^\mu)^2 \phi^+\phi + \cdots + \frac{\lambda_1}{4}(\phi^+\phi)^2 + \lambda_2\,\chi\,\phi^+\phi + \frac{1}{2}\lambda_3\,\chi^2\,\phi^+\phi + \frac{\lambda_4}{4}\chi(\phi^+\phi)^2 + \frac{\lambda_5}{6}\chi^3\,\phi^+\phi + \cdots\,, \tag{4.1}$$

$$\mathcal{L} = (\partial_\mu\phi^+)(\partial^\mu\phi) + \tfrac{1}{2}(\partial_\mu\chi)^2 - \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - V\,.$$

The effective Lagrangian in general contains an infinite series of terms, of which a few typical ones are exhibited above. As explained earlier, except for the couplings of A_μ, terms that contain gradients of fundamental fields are neglected.

The problem now is how to determine the various coefficients in Eq. (4.1). The first constant, e_1, is arbitrary, and all other constants are then determined in terms of e_1. The coefficient of the second term follows from gauge invariance, and can easily be checked directly. To determine the other coefficients, we have to calculate the dual amplitude at zero four momentum. For convenience, let us introduce the following notation: let

$$B_{N_1N_2N_3}(k_1,k_2,\ldots,k_{N_1};\ k_{N_1+1},\ldots,k_{N_1+N_2};\ k_{N_1+N_2+1},\ldots,k_{N_1+N_2+N_3})$$

denote the N point dual amplitude where the first set of N_1 momenta go with external particles of positive charge, the second set of N_2 momenta go with particles of negative charge, and the third set of N_3 momenta with neutral particles, where $N_1 + N_2 + N_3 = N$. Also let $\tilde{B}_{N_1,N_2,N_3}(0)$ denote the one particle irreducible part at zero momentum as before. If $N = 4$, $\tilde{B}$ can be calculated directly from the expression for the beta function in terms of gamma functions. For $N > 4$ however, the calculation is not so easy. Instead, we use the integral representation for B_N, and what we call the method of subtraction to carry out the analytic continuation to zero momenta. Let us illustrate this method for the case of $N = 4$, where it can also be checked directly. We first give a list of the integral representations for various four point functions:

$$B_{0,0,4}(k_1,k_2,k_3,k_4) = i\left\{\alpha_1\int_0^1 dx\, x^{-s-2}(1-x)^{-t-2} \times\left(x^2 + (1-x)^2 + x^2(1-x)^2\right) + (s\leftrightarrow u) + (t\leftrightarrow u)\right\},$$

Equation (4.2) continued next page

Equation (4.2) continued

$$B_{1,1,2}(k_1;\ k_2;\ k_3,k_4) = i\,\alpha_2\int_0^1 dx\, x^{-u-1}(1-x)^{-t-1} \times (2 + x^2 - x) + i\alpha_3\left\{\int_0^1 dx\, x^{-t-1}(1-x)^{-s-2} \times (2 + 2x^2 - 3x) + (t\leftrightarrow u)\right\},\tag{4.2}$$

$$B_{2,2,0}(k_1,k_2;\ k_3,k_4) = i\,\alpha_4\int_0^1 dx\, x^{-u-2}(1-x)^{-t-2}$$

$$s = (k_1+k_2)^2,\quad t = (k_2+k_3)^2,\quad u = (k_2+k_4)^2$$

$$s + t + u = 0\ .$$

The first amplitude corresponds to four external neutrals, the second to two neutrals and two charged lines, and the third to four charged lines. The terms indicated by $(s\leftrightarrow u)$ etc. are needed for Bose symmetry. Figure 6 depicts the way various momenta are ordered in the planar dual amplitudes; for example, in the expression for $B_{1,1,2}$, the first term corresponds to a configuration where the neutral and charged lines alternate, in the second term the two neutrals and the two charged lines are adjacent. Factorization can be used to determine relations between the coefficients as follows:

$$\alpha_4 = 2e_1^{\ 2} = \tfrac{1}{2}(\lambda_2)^2 ,$$

$$2(\alpha_2 + \alpha_3) = \lambda_2^{\ 2}, \qquad \alpha_2 = \alpha_3 , \tag{4.3a}$$

$$\alpha_1\alpha_4 = 2\alpha_3^{\ 2} .$$

Expressing everything in terms of e_1, we have

$$\alpha_4 = 2e_1^{\ 2}, \qquad \lambda_2 = 2e_1 ,$$

$$\alpha_2 = \alpha_3 = e_1^{\ 2}, \qquad \alpha_1 = e_1^{\ 2} . \tag{4.3b}$$

We now face the problem of computing these amplitudes at zero four momentum. The subtraction procedure we employ is simple: Since the tachyon and the various zero mass particles are responsible for the divergence of the integral representation, we subtract them out in order to get a convergent formula. We illustrate this procedure by exhibiting the calculation of $\tilde{B}_{2,2,0}(0)$. By definition, we have,

$$\frac{\tilde{B}_{2,2,0}(0)}{i\,\alpha_4} = -2 + \lim_{\substack{t\to 0\\ u\to 0}} \left\{ \frac{1}{t+1} + \frac{1}{u+1} + \frac{2}{u} + \frac{2}{t} + \frac{1}{2}\,\frac{t-s}{u} + \frac{1}{2}\,\frac{u-s}{t} + \int_0^1 dx\, x^{-u-2}(1-x)^{-t-2} \right\} . \tag{4.4}$$

The poles that appear after the limit symbol are the poles to be subtracted by the foregoing argument. Note that the residue of the photon pole is calculated without ambiguity from the gauge

invariant coupling given by (4.1). The definition of $\tilde{B}(0)$ calls only for the elimination of the zero mass poles; the first term on the right, (-2), compensates for the extra subtraction of the tachyon poles. Setting

$$-\frac{1}{t+1} = \int_0^1 dx(1-x)^{-t-2} , \qquad -\frac{1}{u+1} = \int_0^1 dx\, x^{-u-2} ,$$

$$-\frac{1}{t} = \int_0^1 dx(1-x)^{-t-1} , \qquad -\frac{1}{u} = \int_0^1 dx\, x^{-u-1} ,$$

$$-\frac{1}{2}\,\frac{u-s}{t} = -\frac{1}{2} + u\int_0^1 dx(1-x)^{-t-1} ,$$

etc., we have the following:

$$\frac{\tilde{B}_{2,2,0}(0)}{i\,\alpha_4} = -1 + \lim_{u,t\to 0}\int_0^1 dx \Big\{ x^{-u-2}(1-x)^{-t-2} - x^{-u-2} - (1-x)^{-t-2} - 2\,x^{-u-1} - 2(1-x)^{-t-1} - t x^{-u-1} - (1-x)^{-t-1} \Big\} . \tag{4.5}$$

In the above formula, the integrand can easily be shown to be nonsingular in the range of integration when u and t are near zero, as a result of the subtraction procedure. We can, therefore, set $u = 0$, $t = 0$ in the integrand itself, and since,

$$\frac{1}{x^2}\,\frac{1}{(1-x)^2} - \frac{1}{x^2} - \frac{1}{(1-x)^2} - \frac{2}{x} - \frac{2}{1-x} = 0 ,$$

The integrand and hence the integral goes to zero. The yields us the result

$$\tilde{B}_{2,2,0}(0) = - i \alpha_4 = -2 i e_1^2 . \qquad (4.6a)$$

The other $\tilde{B}$'s can be computed similarly, with the following result:

$$\tilde{B}_{0,0,4}(0) = 0$$

$$\tilde{B}_{1,1,2}(0) = - i(\alpha_2 + \alpha_3) = - 2 i e_1^2 . \qquad (4.6b)$$

This enables us to determine some further coupling constants in the Lagrangian of Eq. (4.1):

$$\lambda_1 = 2 e_1^2 , \qquad \lambda_3 = 2 e_1^2 . \qquad (4.6c)$$

It is an amusing exercise at this point to truncate the effective potential; i.e., set all the higher terms equal to zero. The truncated Lagrangian, L_t, is given by the following expression:

$$L_t = (\partial_\mu \phi^+)(\partial^\mu \phi) + \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

$$- ie_1 A^\mu[\phi^+ \partial_\mu \phi - \phi \partial_\mu \phi^+] + e_1^2 A_\mu^2 \phi^+ \phi$$

$$- \frac{e_1^2}{2} (\phi^+ \phi)^2 - 2e_1 \chi \phi^+ \phi - e_1^2 \chi^2 \phi^+ \phi . \qquad (4.7)$$

This Lagrangian exhibits the Goldstone and Higgs mechanisms. The minimum of the potential occurs at $\chi = - \frac{1}{e_1}$, $\phi = \pm \frac{1}{e_1}$,

and when the fields χ and ϕ are translated so that the minimum is at the origin, in the new Lagrangian the "photon" acquires a mass

of $m_A^2 = 2$, the neutral scalar field χ acquires a mass $m_\chi^2 = 2$, the field $\phi_1 = \frac{1}{\sqrt{2}}[\phi + \phi^+]$ acquires a mass $m_1^2 = 1$, and the field $\phi_2 = \frac{i}{\sqrt{2}}(\phi - \phi^+)$ decouples. In an exact treatment, the numbers will, of course, be different, but the general features of this simple solution are expected to persist. For example, the zero mass particles will always acquire mass, and one of the zero mass scalars will decouple. Another important feature is that there is only one free coupling constant, e_1, and consequently all the masses are <u>independent</u> of e_1. They are given by numerical constants times the slope parameter which provides the scale. It is of some interest to note that the mass spectrum of the dual model is completely fixed even in the presence of the Goldstone phenomenon.

Are the results obtained from the truncated Lagrangian reliable? Unfortunately, the answer seems to be no. One may be able to justify the truncated Lagrangian as a zero slope limit of the full Lagrangian, although the significance of this is not clear. On the other hand, the naive hope that the neglected terms may be small is easily dashed by the calculation of, for example, λ_4 and λ_5. The calculations, although they are somewhat tedious, can be carried out using essentially the method outlined in this section, and some of the details are given in Appendix B. The constants are given by the following:

$$\lambda_4 = 8 e_1^3 I, \qquad \lambda_5 = 6 e_1^3 I' ,$$

where

$$I = \int_0^1 \int_0^1 \frac{dx\,dy}{1 - xy} [5 + x - xy] = 2 + \frac{2}{3}\pi^2 ,$$

$$I' = \int_0^1 \int_0^1 \frac{dx\,dy}{1 - xy} \left[4 + 2\frac{1 - x}{1 - xy} - 2xy\right] = 4 + \frac{1}{3}\pi^2 . \tag{4.8}$$

Clearly, the coefficients of the fifth order terms are quite sizable. In fact, their inclusion in the truncated Lagrangian would completely upset our tentative solution.

The lesson we learn from this calculation is that higher order contact terms in the effective potential are important. One either has to do an exact calculation or develop a reliable approximation scheme. The direct subtraction method we have used so far works pretty well for the lower order amplitudes, but it is too cumbersome to be of much use for the higher point functions. Instead, an approach based on factorization seems to be more promising. In contrast to the usual multiperipheral factorization of the dual model, this new factorization scheme is of the Feynman type and involves all the channels additively. Such an approach also suggests a natural approximation scheme, hopefully much better than the naive truncation. One could do approximate calculations by keeping a few low lying states only, and neglecting the higher mass states. Work on this new approach is under way.

5. Conclusions

In the preceding sections, we have presented a simple dual model with a natural gauge symmetry connected with charge conservation. We have also shown how one could proceed to break this symmetry spontaneously by emitting charged spurions into the vacuum. The problem was then converted into one of constructing an effective Lagrangian, whose nontrivial minimum (if it exists) corresponds to the Goldstone solution. A procedure for the calculation of the effective Lagrangian was outlined, and a few lowest order terms were exhibited. The fundamental problem is then, to develop a technique for either an exact or failing that, an approximate calculation of the higher order terms in the Lagrangian. Although this is a difficult problem, at this point it does not look hopelessly so, and work is under progress along directions suggested here.

Finally, we would like to point out that, once the technical problem stated in the last paragraph is solved, there would be no difficulty in applying the ideas of this paper to more sophisticated and physically more interesting models. For example, in the Neveu-Schwartz model, the tachyon at $m^2 = -\frac{1}{2}$ can be promoted to zero mass by attaching a "fifth" momentum of magnitude $\pm\frac{1}{\sqrt{2}}$ to it, and this new state can be used as a spurion. It would be most interesting to see what kind of a theory emerges as a consequence.

One point which we have not touched upon is what happens beyond the tree approximation. Our whole discussion so far has been based on the tree approximation; the way in which, for example, the one loop calculation is affected by the foregoing remains as an interesting question.

Acknowledgments

Parts of this work were carried out when I was visiting at the Department of Applied Mathematics and Theoretical Physics in Cambridge and at CERN, Geneva. I would like to thank Professors Polkinghorne and Zumino for the hospitality at their institutions. I would also like to thank Dr. I. Drummond for his collaboration at the beginning of this project.

Appendix A

We will here outline the arguments which lead to the three distinct factorizable dual amplitudes of Sec. 2. As pointed out earlier, the most general amplitude can be written as a superposition over all different cyclic orderings of charge. For example, $B_{2,2,0}$, the four point amplitude of external charged particles, can in general be written as follows:

$$B_{2,2,0} = i\,\beta_1 \int_0^1 dx\, x^{-u-2}(1-x)^{-t-2} + i\,\beta_2 \left\{ \int_0^1 dx\, x^{-s+2}(1-x)^{-t-2} + (u \leftrightarrow t) \right\} . \qquad (A.1)$$

As shown in Fig. 7, the first term corresponds to an alternation of charge, and in the second term, two positive and two negative charges are adjacent. Similarly, the six point function is the sum of three different terms, as shown in Fig. 7. Higher point functions can also be written as a general superposition of increasing number of terms.

Factorization imposes a stringent constraint on the arbitrary constants that appear in a formula like (A.1). For example, factorizing a six point function in a three particle channel, we must recover the four point function. There are clearly a large number of such consistency requirements, and we have explicitly solved the consistency relations involving four, six, and eight point functions. We expect that the inclusion of the conditions involving higher point functions will not change our results. As noted in Sec. 2, there emerges three distinct solutions, which we exhibit below in terms of

β_1 and β_2,

$$\text{Solution 1:} \quad \beta_1 = \beta_2$$

$$\text{Solution 2:} \quad \beta_2 = 0 \qquad \text{(A.2)}$$

$$\text{Solution 3:} \quad \beta_2 = -\frac{3}{2}\beta_1 \; .$$

The second solution is the one used throughout this paper. The first solution gives a "neutral" amplitude, i.e., the photon decouples. It is, therefore, uninteresting. Note that, in any case, only one free coupling constant is allowed.

Appendix B

Here, very briefly, we sketch the calculation of $\tilde{B}_{2,2,1}(0)$. The calculation of the other reduced five point functions is similar and will be skipped. We start with the following integral representation:

$$B_{2,2,1} = 8\, e_1^3 \int_0^1 \int_0^1 \frac{dx_{12}\, dx_{45}}{1 - x_{12}x_{45}}\, x_{12}^{-s_{12}-1}$$

$$x_{45}^{-s_{45}-2}\; x_{23}^{-s_{23}-2}\; x_{34}^{-s_{34}-2}\; x_{15}^{-s_{15}-1}(2 - x_{12} - x_{15}) \, , \qquad \text{(B.1a)}$$

where s_{ij} stands for the appropriate Mandelstam variables, with the legs numbered from 1 to 5 in the cyclic order, and the variables of integration satisfy the following relations:

$$x_{23} = \frac{1 - x_{12}}{1 - x_{12}x_{45}} \, , \qquad x_{34} = \frac{1 - x_{45}}{1 - x_{12}x_{45}} \, , \qquad x_{15} = 1 - x_{12}x_{45} \; .$$

(B.1b)

We have again to subtract all the dangerous poles to get an expression that converges as all the s tend to zero. In addition to poles in single variables, we also have poles in two nonoverlapping variables to subtract. The residues of these poles can be computed either in terms of four point functions through factorization, or directly from (B.1). We again skip the details, which are very tedious and also very straightforward. The poles can then be cast in a form similar to (4.5). i.e.,

$$\frac{1}{s_{12}(1 + s_{45})} = \int_0^1 \int_0^1 dx_{12}\, dx_{45}\, x_{12}^{-s_{12}-1}\; x_{45}^{-s_{45}-2} \, , \qquad \text{(B.2)}$$

etc. After the subtraction procedure, the integrand becomes non-singular, so that we can let all s's go to zero. After some algebra, that yields the following result,

$$\tilde{B}_{2,2,1}(0) = 8\, e_1^3 \int_0^1 \int_0^1 \frac{dx\, dy}{1 - xy}(5 + x - xy) \, , \qquad \text{(B.3)}$$

which is the first equation in (4.8).

References

[1] Chan Hong-Mo and J. Paton, Nucl. Phys. B10 (1969) 519.

[2] A. Neveu and J. H. Schwarz, Nucl. Phys. B31 (1971) 86.

[3] M. B. Halpern, Phys. Rev. D3 (1971) 3068.

[4] M. A. Virasoro, Phys. Rev. D1 (1970) 2933.

[5] R. Brower, Phys. Rev. D6 (1972) 1655;
P. Goddard and C. B. Thorn, Phys. Letters 40B (1971) 235.

[6] J. Goldstone, Nuovo Cimento 19 (1961) 154.

[7] P. Higgs, Phys. Letters 12 (1964) 132;
F. Englert and R. Brout, Phys. Rev. Letters 13 (1964) 321.

[8] K. Bardakci and M. B. Halpern, Phys. Rev. D3 (1971) 2493.

[9] M. Gell-Mann and M. Levy, Nuovo Cimento 16 (1960) 705.

[10] B. W. Lee, Nucl. Phys. B9 (1969) 649.

[11] For a review of dual models, see, for example; S. Mandelstam, Proceedings of Brandeis Summer School, or J. H. Schwartz, to be published.

[12] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127 (1962) 965.

FIGURE CAPTIONS

Fig. 1. The N-point amplitude.

Fig. 2. The pattern of alternation of charge.

Fig. 3. Spruion emission and factorization. Solid lines stand for particles, wavy lines for spurions, and the dotted line stands for the propagator.

Fig. 4. Direct and dual channel poles.

Fig. 5. The pattern of charged and neutral lines for a five-point amplitude. The five different cyclic arrangements have to be added.

Fig. 6. Various four-point amplitudes. The signs (+,-,0) refer to the charges.

Fig. 7. Various patterns of charge in the four- and six-point amplitudes.

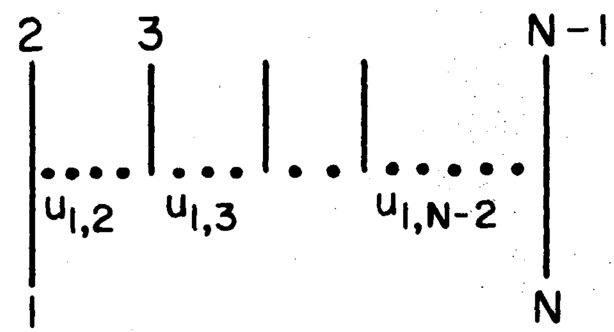

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Fig. 1

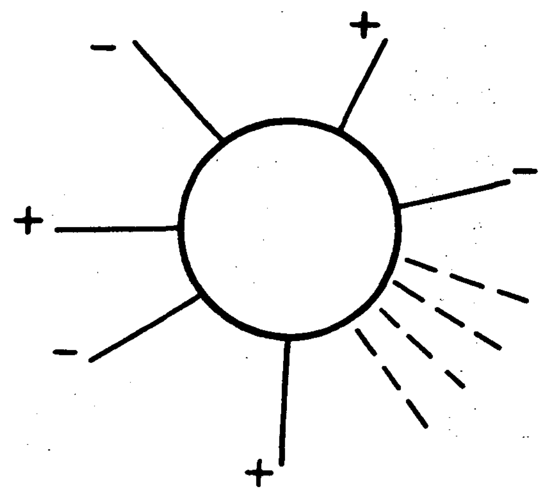

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Fig. 2

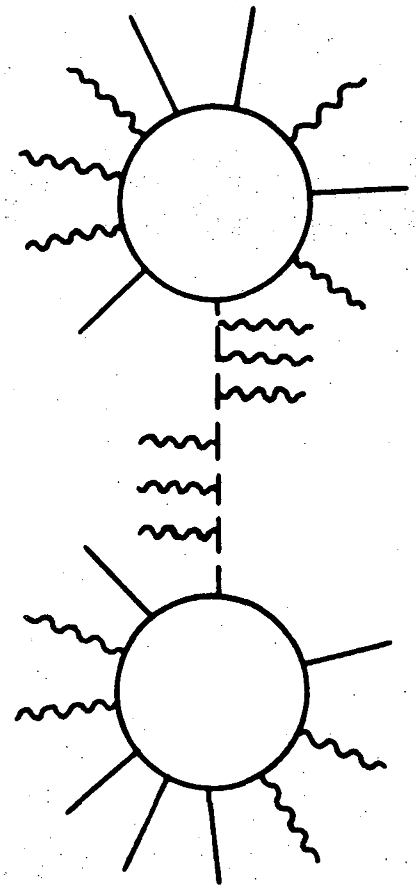

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Fig. 3

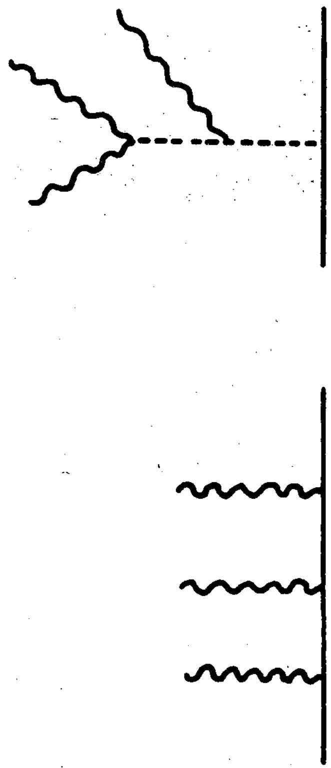

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Fig. 4

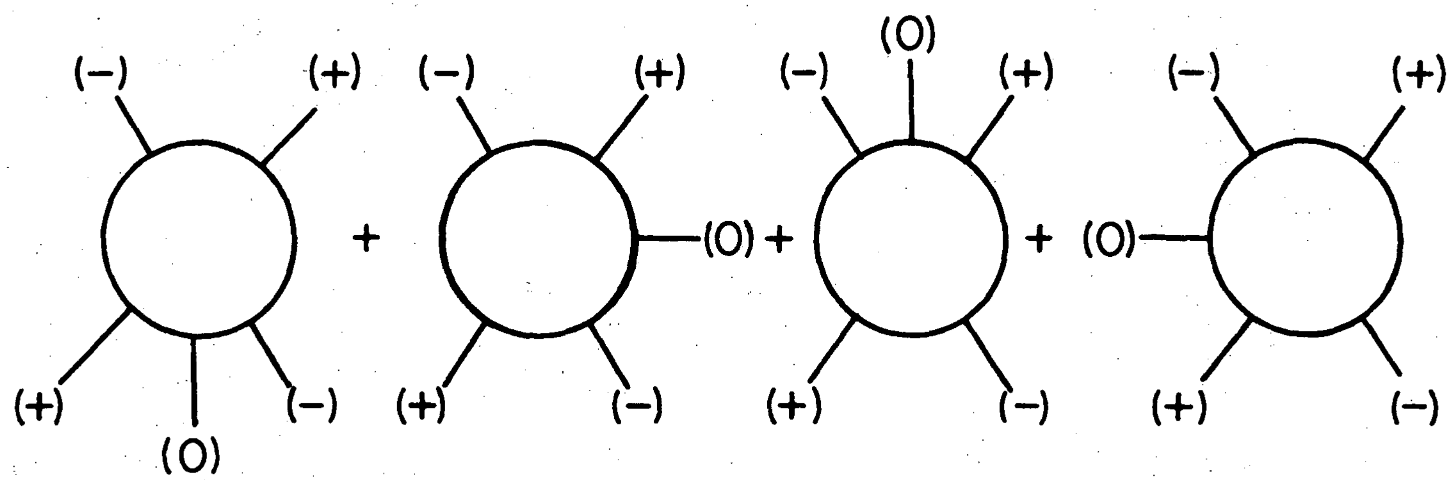

Fig. 5

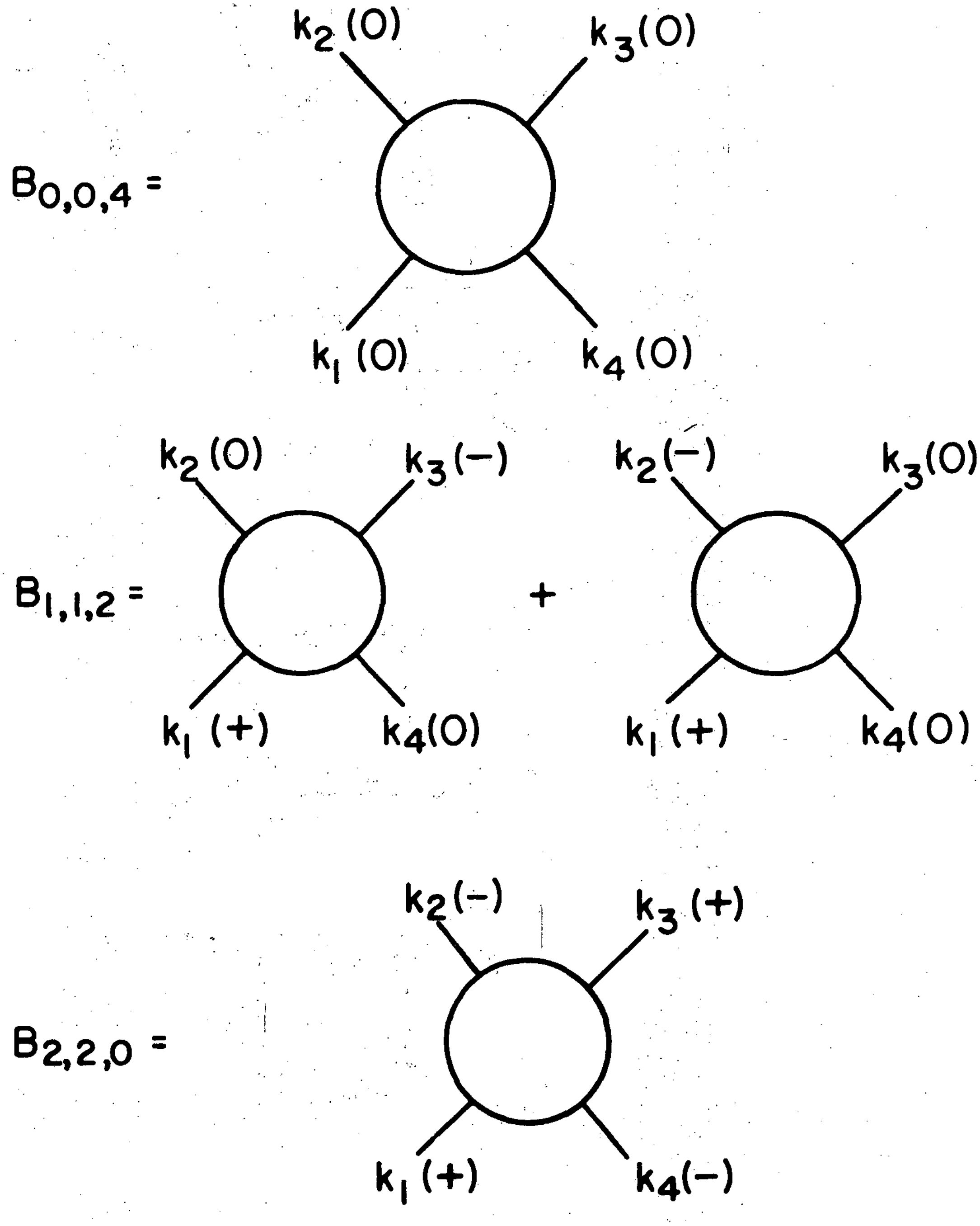

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Fig. 6

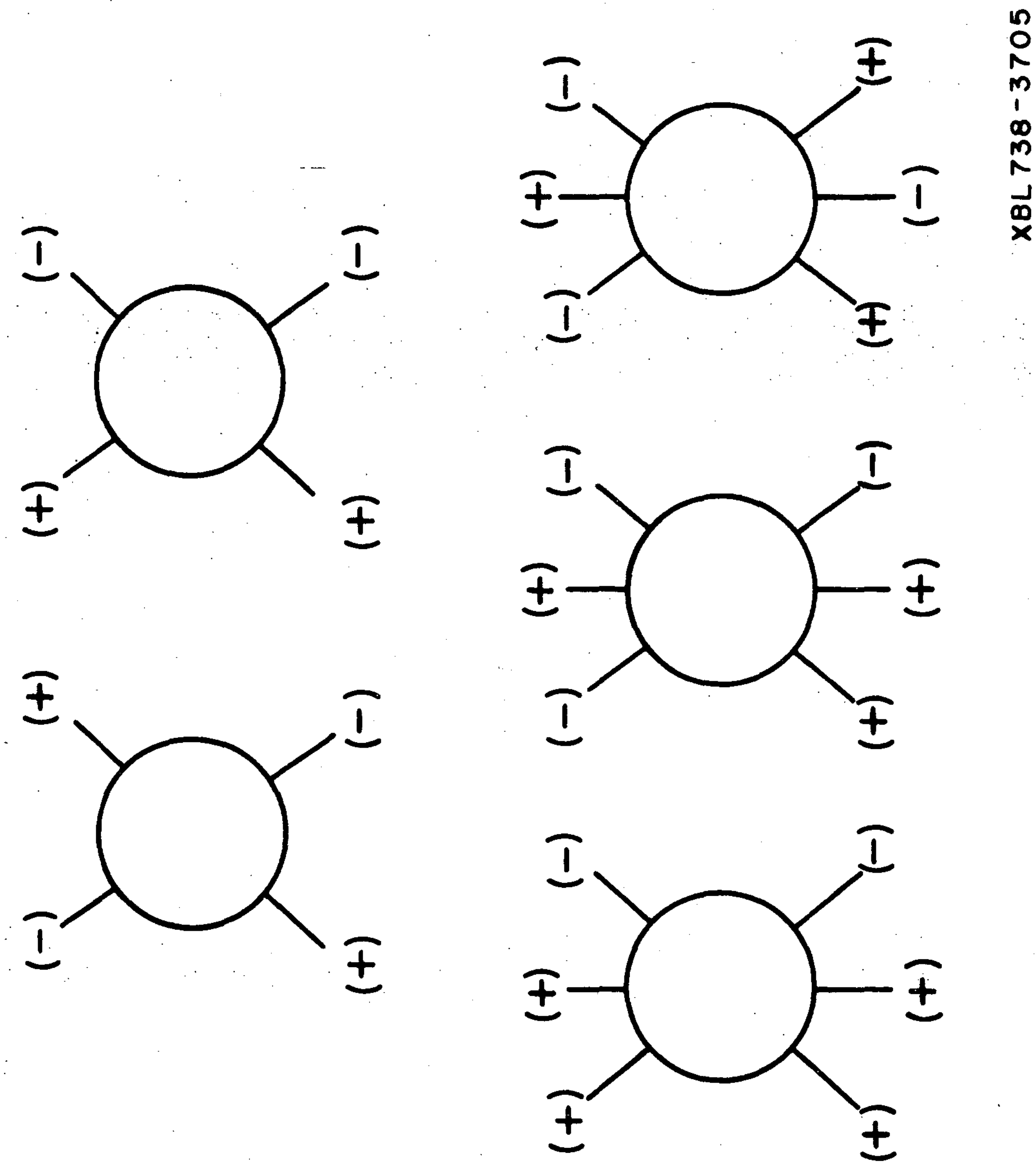

Fig. 7

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UNIVERSITY OF CALIFORNIA
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