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Custodial Symmetry and the Triviality Bound on the Higgs Mass

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Abstract

The triviality of the scalar sector of the standard one-doublet Higgs model implies that it is only an effective low-energy theory valid below some cut-off scale Λ . In this note we show that the experimental constraint on the amount of custodial symmetry violation, $|\Delta\rho_*| = \alpha|T| \leq 0.4\%$, implies that the scale Λ must be greater than of order 7.5 TeV. For theories defined about the infrared-stable Gaussian fixed-point, we estimate that this lower bound on Λ yields an upper bound of approximately 550 GeV on the Higgs boson's mass, independent of the regulator chosen to define the theory. We also show that some regulator schemes, such as higher-derivative regulators, used to define the theory about a different fixed-point are particularly dangerous because an infinite number of custodial-isospin-violating operators become relevant.

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1 Introduction

The triviality [1] of the scalar sector of the standard one-doublet Higgs model implies that this theory is only an effective low-energy theory valid below some cut-off scale Λ . Physically this scale marks the appearance of new strongly-interacting symmetry-breaking dynamics. As the Higgs mass increases, the upper bound on the scale Λ decreases. If we require that M_H/Λ be small enough to afford the effective Higgs theory some range of validity (or to minimize the effects of regularization in the context of a calculation in the scalar theory), one arrives at an upper bound on the Higgs boson's mass [2, 3].

Quantitative studies on the lattice using analytic [4] and Monte Carlo [5, 6, 7, 8, 9] techniques result in an upper bound of approximately 700 GeV. However, these lattice results are potentially ambiguous because the precise value of the bound on the Higgs boson's mass depends on the restriction placed on M_H/Λ . The “cut-off” effects arising from the regulator are not universal: different schemes can give rise to different effects of varying sizes and can change the resulting Higgs mass bound.

In this note we show that, for models that reproduce the standard one-doublet Higgs model at low energies, electroweak phenomenology provides a lower bound on the scale Λ that is regularization-independent (i.e. independent of the details of the underlying physics). Recall that the standard one-doublet Higgs model has an accidental custodial isospin symmetry [10], which naturally implies that the weak-interaction ρ -parameter is approximately one. While all $SU(2) \times U(1)$ invariant operators made of the scalar-doublet field that have dimension less than or equal to four automatically respect custodial symmetry, terms of higher dimension that arise from the underlying physics at scale Λ in general will not. We show that current results from precision electroweak tests [11], which provide the constraint

$$|\Delta\rho_*| \leq 0.4\% \tag{1.1}$$

on $\Delta\rho_*$ ($= \alpha T$) [12, 13] at the 95% confidence level, imply that the scale Λ must be greater than approximately 7.5 TeV. This lower bound on Λ implies that the Higgs boson's mass must be less than approximately 550 GeV, independent of the cut-off method chosen to define the theory.

Implicitly assumed in these bounds is the naive scaling that one expects near the infrared-stable Gaussian fixed point of scalar field theory. Other fixed points with very different scaling behavior may also exist. Typically, these new fixed points correspond to field theories with an infinite number of relevant operators [14]. A nice example of this possibility has been explored by Jansen, Kuti, and Liu [15], who performed an analytic (large-N) analysis of the Higgs model in the presence of a pair of complex-conjugate Pauli-Villars regulator fields. Their calculations show the possibility of defining the theory with a Higgs mass of 2 TeV (!) while forcing the ghost (Pauli-Villars) states to have masses greater than 4 TeV. However, we will show that in this theory there are an infinite number of relevant custodial-isospin-violating operators. Therefore, given the degree of custodial isospin violation present

in the splitting between the masses of the top and bottom quarks, these theories cannot give rise to phenomenologically viable theories of a heavy Higgs boson. We expect that our results will generalize to other potentially “non-trivial” scalar field theories as well.

2 Triviality and custodial symmetry

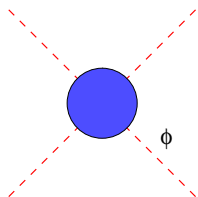
We begin by considering an underlying theory which is arbitrary and does not respect custodial symmetry. We are interested in cases which reproduce the standard one-doublet Higgs model at low energies. The low-energy theory should respect $SU(2)_W \times U(1)_Y$ and the only low-energy state resulting from the underlying dynamics should be the Higgs doublet. Since we are considering theories with a heavy Higgs field, we expect that the underlying high-energy theory will be strongly interacting.

To estimate the sizes of various effects of the underlying physics, we will rely on dimensional analysis. As noted by Georgi [16], a theory¹ with light scalar particles belonging to a single symmetry-group representation depends on two parameters: Λ , the scale of the underlying physics, and f (the analog of f_π in QCD), which measures the amplitude for producing the scalar particles from the vacuum. Our estimates will depend on the ratio $\kappa = \Lambda/f$, which is expected to fall between 1 and 4π . For example, in a QCD-like theory with N_c colors and N_f flavors one expects [17] that

$$\kappa \approx \min \left(\frac{4\pi a}{N_c^{1/2}}, \frac{4\pi b}{N_f^{1/2}} \right), \quad (2.2)$$

where a and b are constants of order 1. In the results that follow, we will display the dependence on κ explicitly; when giving numerical examples, we set κ equal to the geometric mean of 1 and 4π , *i.e.* $\kappa \approx 3.5$.

Because of the $SU(2)_W \times U(1)_Y$ symmetry of the low-energy theory, all terms of dimension less than or equal to four respect custodial symmetry [10]. The leading custodial-symmetry violating operator is of dimension six [18, 19] and involves four Higgs doublet fields ϕ . According to the rules of dimensional analysis, the operator



$$\Rightarrow \frac{\kappa^2}{\Lambda^2} (\phi^\dagger D^\mu \phi) (\phi^\dagger D_\mu \phi), \quad (2.3)$$

¹These dimensional estimates only apply if the low-energy theory, when viewed as a scalar field theory, is defined about the infrared-stable Gaussian fixed-point. We return to potentially “non-trivial” theories below.

should appear in the low-energy effective theory with a coefficient of order one [19]. Such an operator will give rise to a deviation

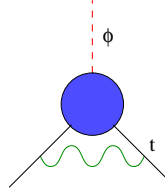
$$\Delta\rho_* = -\mathcal{O}\left(\kappa^2\frac{v^2}{\Lambda^2}\right), \quad (2.4)$$

where $v \approx 246$ GeV is the expectation value of the Higgs field. Imposing the constraint [11] that $|\Delta\rho_*| \leq 0.4\%$, we find the lower bound

$$\Lambda \gtrsim 4 \text{ TeV} \cdot \kappa. \quad (2.5)$$

For $\kappa \approx 3.5$, we find $\Lambda \gtrsim 14$ TeV.

Alternatively, it is possible that the underlying strongly-interacting dynamics respects custodial symmetry. Even in this case, however, there must be custodial-isospin-violating physics (analogous to extended-technicolor interactions [20]) which couples the $\psi_L = (t, b)_L$ doublet and t_R to the strongly-interacting “preon” constituents of the Higgs doublet in order to produce a top quark Yukawa coupling at low energies and generate the top quark mass. If, for simplicity, we assume that these new weakly-coupled custodial-isospin-violating interactions are gauge interactions with coupling g and mass M , dimensional analysis allows us to estimate the size of the resulting top quark Yukawa coupling

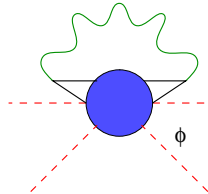


$$\Rightarrow \frac{g^2}{M^2} \frac{\Lambda^2}{\kappa} \bar{t}_R \phi \psi_L. \quad (2.6)$$

In order to give rise to a suitably large top-quark mass the Yukawa coupling must be greater than or of order one, implying that

$$\Lambda \gtrsim \frac{M}{g} \sqrt{\kappa}. \quad (2.7)$$

These new gauge interactions will typically also give rise to custodial-isospin-violating 4-preon interactions² which, at low energies, will give rise to an operator of the same form as the one in eqn. 2.3. Using dimensional analysis, we find



$$\Rightarrow \frac{g^2}{M^2} (\phi^\dagger D^\mu \phi) (\phi^\dagger D_\mu \phi), \quad (2.8)$$

²These interactions have previously been considered in the context of technicolor theories[21].

which results in the bound $M/g \gtrsim 4$ TeV. From eqn. 2.7 we then derive the limit

$$\Lambda \gtrsim 4 \text{ TeV} \cdot \sqrt{\kappa} . \quad (2.9)$$

For $\kappa \approx 3.5$, we find $\Lambda \gtrsim 7.5$ TeV.

Because of triviality, a lower bound on the scale Λ yields an upper-bound on the Higgs boson mass. A rigorous result would require a nonperturbative calculation of the Higgs mass in an $O(4)$ -symmetric theory subject to the constraint that $\Lambda/v \gtrsim 30$. Here we provide an estimate of this upper bound by naive extrapolation of the lowest-order perturbative result³. Integrating the lowest-order beta function for the Higgs self-coupling λ

$$\beta(\lambda) = \mu \frac{d\lambda}{d\mu} = \frac{3}{2\pi^2} \lambda^2 + \dots , \quad (2.10)$$

we find

$$\frac{1}{\lambda(\mu)} - \frac{1}{\lambda(\Lambda)} = \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu} . \quad (2.11)$$

Using the relation $m_H^2 = 2\lambda(m_H)v^2$ we find the relation

$$m_H^2 \log \left(\frac{\Lambda}{m_H} \right) \leq \frac{4\pi^2 v^2}{3} . \quad (2.12)$$

For $\Lambda \gtrsim 7.5$ TeV, this results in the bound⁴ $m_H \lesssim 550$ GeV.

3 Non-trivial scaling behavior

Dimensional analysis was crucial to the discussion given above. If the low-energy Higgs theory does not flow toward the trivial Gaussian fixed-point in the infrared limit, the scaling dimensions of the fields and operators can be very different than naively expected. In this case the bounds given above do not apply.

A nice example of a scalar theory with non-trivial behavior has been given by Jansen, Kuti, and Liu [15]. They consider a theory defined by an $O(4)$ -symmetric Lagrange-density with a modified kinetic-energy

$$\mathcal{L}_{kin} = -\frac{1}{2} \phi^\dagger \left(\square + \frac{\square^3}{\mathcal{M}^4} \right) \phi . \quad (3.13)$$

In the large- N limit, this higher-derivative kinetic term is sufficient to eliminate all divergences. A lattice simulation of this theory [22] indicates that this approach

³Though not justified, the naive perturbative bound has been remarkably close to the non-perturbative estimates derived from lattice Monte Carlo calculations [5, 6, 7, 8, 9].

⁴If $\kappa \approx 4\pi$, Λ would have to be greater than 14 TeV, yielding an upper bound on the Higgs boson's mass of 490 GeV. If $\kappa \approx 1$, Λ would be greater than 4 TeV, yielding the upper bound $m_H \lesssim 670$ GeV.

can be used to define a non-trivial Higgs theory with a Higgs boson mass as high as 2 TeV, while avoiding any noticeable effects from the (complex-conjugate) pair of ghosts which are present because of the higher derivative kinetic-energy term.

As shown by Kuti [14], in the infrared this higher-derivative theory flows to a non-trivial fixed point on an infinite dimensional critical surface, which corresponds to a continuum field theory with an infinite number of relevant operators. The reason there are an infinite number of relevant operators is that, if the continuum limit is taken so that the scale \mathcal{M} remains finite as required in order to flow to a non-trivial theory, the scaling dimension [14] of the Higgs doublet field ϕ is -1 instead of the canonical value of +1!

If one could impose an exact $O(4)$ symmetry on the symmetry breaking sector, this would lead to a strongly-interacting electroweak symmetry-breaking sector without technicolor [22]. However, as argued above, custodial isospin violation in the flavor sector must couple to the symmetry-breaking sector to give rise to the different top- and bottom-quark masses. Furthermore, if the scaling dimension of the Higgs field is -1, there is an infinite class of custodial-isospin-violating operators (including the operator in eqn. 2.3) which are relevant. Since these operators are relevant, even a small amount of custodial isospin violation coming from high-energy flavor dynamics will be amplified as one scales to low energies, ultimately contradicting the bound on $\Delta\rho_*$. We therefore conclude that these non-trivial scalar theories cannot provide a phenomenologically viable theory of electroweak symmetry breaking.

To construct a phenomenologically viable theory of a strongly-interacting Higgs sector it is not sufficient to simply construct a theory with a heavy Higgs boson, one must also ensure that all potentially custodial-isospin-violating operators remain irrelevant⁵.

4 Conclusions

We have shown that theories with a heavy Higgs boson which reproduce the standard model at low energies are caught between the rock of the ρ parameter and the hard place of the top-bottom mass splitting. In theories which flow to the infrared-stable Gaussian fixed point, the scale of the new strongly-interacting dynamics must be greater than of order 7.5 TeV, and therefore the Higgs boson must weigh less than approximately 550 GeV. This result applies whether the strongly-interacting preon dynamics that underlies the Higgs state conserves or violates custodial isospin. In theories with non-trivial scaling behavior, the presence of an infinite class of relevant custodial-isospin-violating operators makes it impossible to both provide a top quark mass and obey the bound on $\Delta\rho_*$. Such theories cannot, therefore, provide a phenomenologically acceptable description of electroweak symmetry breaking.

⁵This is also a concern in walking technicolor [23].

Authors' Note After the completion of this work, it was brought to our attention that the possibility of using the ρ parameter as an additional handle on limiting the Higgs self-coupling had been suggested, but not pursued, in [24].

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