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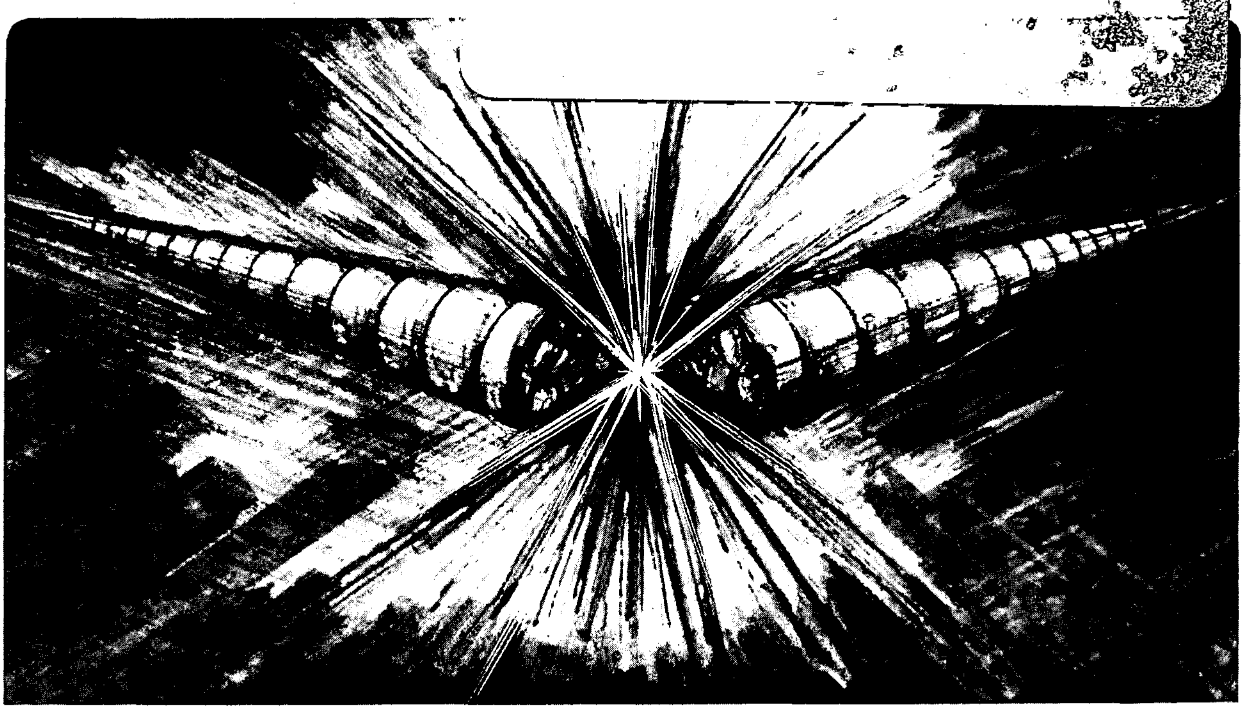

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ABSTRACT

In the earlier stages of acceleration in a heavy-ion-induction linac, acceleration and bunching rates are constrained by the allowable value of head-to-tail velocity tilt at a given location. If focusing parameters at a given location are fixed, the velocity tilt should be less than a certain upper bound to avoid too much envelope variation and consequent beam losses. For space charge dominated beams, we found some favorable particle distributions in longitudinal phase space for which the maximum-matched-beam envelope at a given location is almost constant with respect to time, in spite of the presence of a large velocity tilt. Mismatch oscillations can be reduced by slow variation of the velocity tilt and slow current amplification. Under these circumstances, the velocity tilt can be as large as allowed by the usable range of σ_0. Behavior of Cs ion beams with very large velocity tilts (up to 40%) are studied experimentally in MBE-4 and the results are presented.

I. INTRODUCTION

In cost optimizing the design of an induction-linac-based HIF driver, design tradeoffs are necessary between the required core material and the cost of focusing structures; a shorter pulse requires less core material but the resulting higher current requires greater focusing strength. As a result, specific acceleration and current-amplification schedules are imposed along the accelerator.[1] The acceleration rate and the current-amplification rate are kinematically interrelated with each other through the head-to-tail velocity tilt; for instance, a larger velocity tilt permits a higher acceleration rate for a required current-amplification rate.

In the earlier stages of acceleration when the kinetic energy is low and the bunch length is large, the acceleration rate is severely limited by this constraint. Since the focusing strength at a given location is fixed in time, any variation of the kinetic energy for different longitudinal positions along the bunch may result in a variation of the beam radius and possibly in beam losses. A large velocity tilt and the consequent brisk acceleration and bunching can cause mismatch oscillations. The purpose of the present investigation is to find a practical upper bound for the permissible values of the velocity tilt.

A kinematic relationship between velocity tilt, acceleration rate and the bunching rate is derived in Section II. Some favorable longitudinal particle distributions are discussed in Section III for electric and magnetic focusing systems. These distributions can tolerate higher velocity tilts because the maximum-matched-beam radius at a given location is quite

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insensitive to the kinetic energy. An experimental demonstration of a high velocity tilt in an electric focusing system is described in Section IV. A discussion on mismatch oscillations is included in Section V.

II. A KINEMATIC RELATION

A bunch of ions moves along an induction linac under the influence of a series of accelerating kicks. Its progress may be approximately represented by a set of smooth curves in z-t space traversed by particles at the bunch head (H), tail (T), and center (C) as sketched in Fig. 1. Also assume that the bunch center is located at z where the pulse duration is Δt and at time t when the bunch length is L. The slope of a particle's curve is its velocity (v) and the rate of change of the slope is its acceleration (A). The velocity tilt is defined as:

$$\Delta\beta/\beta = \left(v_T - v_H\right)/v_o \tag{1}$$

We expand v_{tail} and v_{head} in a Taylor series about the respective points T_o and H_o in terms of $(t_T - t_{To})$ and $(t_H - t_{Ho})$, and keep only the first order terms. Using the relations,

$$v_{Ho} - v_{To} = dL/dt \;,$$

$$t_T - t_H \cong L/v_o$$

$$d/dt = v_o \, d/d_z \;,$$

and

$$A_o = (qe/m) \; dV_o/dz \;,$$

one can rewrite equation (1) in a more convenient form:

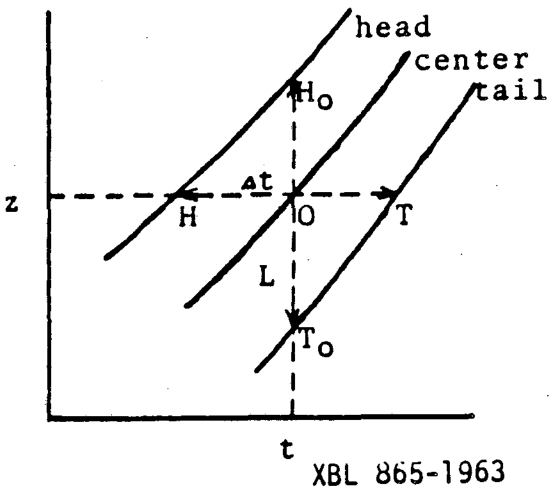

Fig. 1. Trajectories of particles at the beam head (H), center (C), and tail (T).

$$(1/L)\,(\Delta\beta/\beta) = (1/2V_o)\,(dV_o/dz) - (1/L)\,(dL/dz) \;, \tag{2}$$

where V_o is the kinetic energy (E) for particles at the bunch center divided by the charge state (q).

Accordingly, if it is desirable to impose a restriction on the magnitude of $\Delta\beta/\beta$ for reasons which will be discussed in the following sections, then a limitation is necessarily imposed on the acceleration rate, dV/dz. This restriction, unfortunately, acts to lengthen the accelerator and is particularly restrictive if the bunch length is large and the kinetic energy is low.

The average current is defined as $I(z) = Qv_o/L$ where Q is the total charge. Differentiating I(z) with respect to z and using equation (2), we have an expression for the rate of current amplification:

$$\left(1/I(z)\right)\, dI(z)/dz = (1/L)\ \Delta\beta/\beta\ . \qquad (3)$$

III. MATCHED BEAM CONSIDERATIONS

The presence of a velocity tilt can have several important consequences. With the focusing field gradient of an individual quadrupole lens constant with respect to time, the focusing spring constant, K, becomes significantly different for ions of different kinetic energy. ($K \propto 1/V$ for electrostatic focusing and $K \propto 1/v$ for magnetic focusing). The value of σ_0 thus becomes quite different for particles at various locations within the bunch. The presence of a velocity tilt also will lead to differences in the size of the matched envelope at various portions of the beam.

We are interested in finding longitudinal particle distributions for which the maximum-matched-beam-radius at a given location is constant with respect to time in spite of the presence of the velocity tilt. A distribution which closely satisfies this condition in the space charge dominated regime for electric focusing systems is the constant-current distribution; by this we mean that the beam current at a given location is constant with respect to time. We illustrate this point with an example below.

We consider a matched Cs^{+1} beam of a constant current of 24 mA passing through one of the electric quadrupole lenses of MBE-4[2] with pole-tip voltages ± 12.5 kV. The matched beam parameters for different kinetic energies corresponding to different longitudinal segments of the bunch were calculated by numerically solving the envelope equations. The results are summarized in Table I.

Table I - Matched Beam parameters at a given lens (MBE-4)

$V_q = 12.5$ kV, $I = 24$ mA, and $\epsilon_N = 1.5\times10^{-7}$ π.m.rad

K.E.	(keV)	200	250	300	350	400	450
A_{max}	(mm)	21.1	21.2	21.4	21.7	22.0	22.3
A_{avr}	(mm)	15.4	16.4	17.4	18.2	18.9	19.5
σ_0	(deg)	80.0	61.5	50.7	43.1	38.4	35.0
σ	(deg)	8.8	6.8	5.4	4.5	3.8	3.4

From the tabular material, one notes the very marked variation of the energy and σ_0. In contrast, one notes the virtually negligible variation of the maximum-matched-beam radius (A_{max}). The constant-current distribution can be obtained (at least for the flat part of the bunch) by designing the accelerating voltage wave-forms in a current-self-replicating fashion.[3]

For magnetic focusing, the focusing strength diminishes less rapidly with higher beam velocity than electric focusing; thus the spread of σ_0 is much narrower for the same range of energy. We found that, for the space charge dominated regime, the desirable distribution for which the maximum-matched-beam radius varies the least is the constant-density distribution; by this we mean that the line-charge density at a given location is constant with respect to time. Notice that this distribution is somewhat different from the one considered in a constant-density acceleration scenario[4] where the density at a given time is assumed to be constant with respect to the longitudinal position along the bunch.

Significantly higher velocity tilts can be tolerated in magnetic focusing systems than in electric focusing systems.

IV. THE EXPERIMENT

An experiment was performed in MBE-4 to see whether beams with very high velocity tilt would pass through the transport channel. Four individually focused Cs^{+1} beams, 200 keV, 10 mA each with 3 μsec pulse durations are injected into the multiple-beam induction linac which was operated with 8 accelerating gaps. These experimental conditions are similar to the ones described in reference (2) except that the last four accelerating wave-forms were modified to triangular shapes by rearranging the timing sequence of the individual pulsers. These wave-forms produced a final velocity tilt of up to 29% over the flat-current portion and up to 38% over the total length of the bunch.

Injected current, I(t), and injected kinetic energy, E(t), were measured as shown in Fig. 2a. These data and the measured accelerating voltage wave forms were used in digitized forms to calculate the expected values of E(t) and I(t) at the end of the linac (Fig. 2b). The

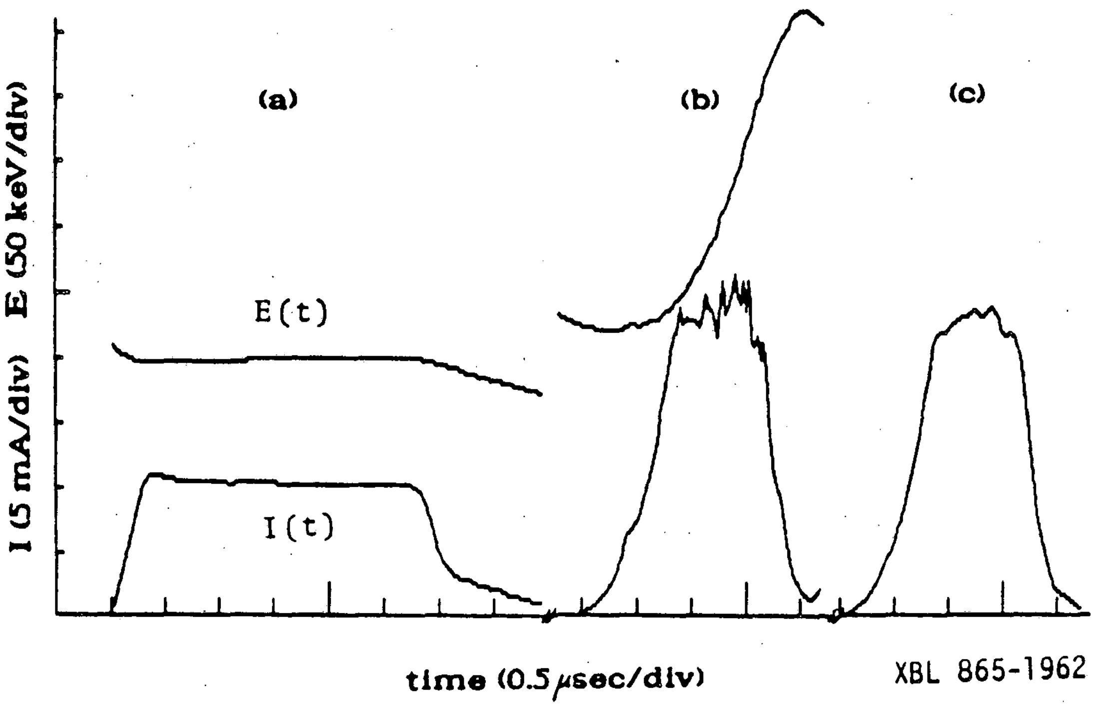

Fig. 2. (a) Measured beam energy, E(t), and beam current, I(t), at injection. (b) Calculated E(t) and I(t) at the end of MBE-4. (c) Measured I(t) at the end of MBE-4.

calculation was done numerically with a 1-dimensional code including the space charge effects (SLID).[3] Measured current at the end of the linac (Fig. 2c) shows a good agreement with the calculation. No beam losses were observed within the experimental accuracy.

The calculated $I(t)$ showed more high frequency noise than the measured $I(t)$ because of the presence of high frequency noise in the accelerating voltages and of the fact that infinitely narrow acceleration gaps were assumed in the calculation. For a finite ion-transit time (τ), a fluid analysis showed that the frequency response decreases by the factor, $\sin(\omega\tau/2)/(\omega\tau/2)$.

Beam emittances were measured at the end of the linac for two representative longitudinal locations of the bunch. No emittance growth was observed within the experimental accuracy.

Beam envelopes for about a dozen representative longitudinal segments of the beam were calculated by numerically integrating the envelope equations. Calculated values of beam currents and kinetic energies for each lattice period were used. These calculations agree very well with the measured values at the end of the fourth gap and at the end of the linac. For the front and central portion of the beam, mismatch oscillations are small as shown in Fig. 3a. The amplitude of mismatch

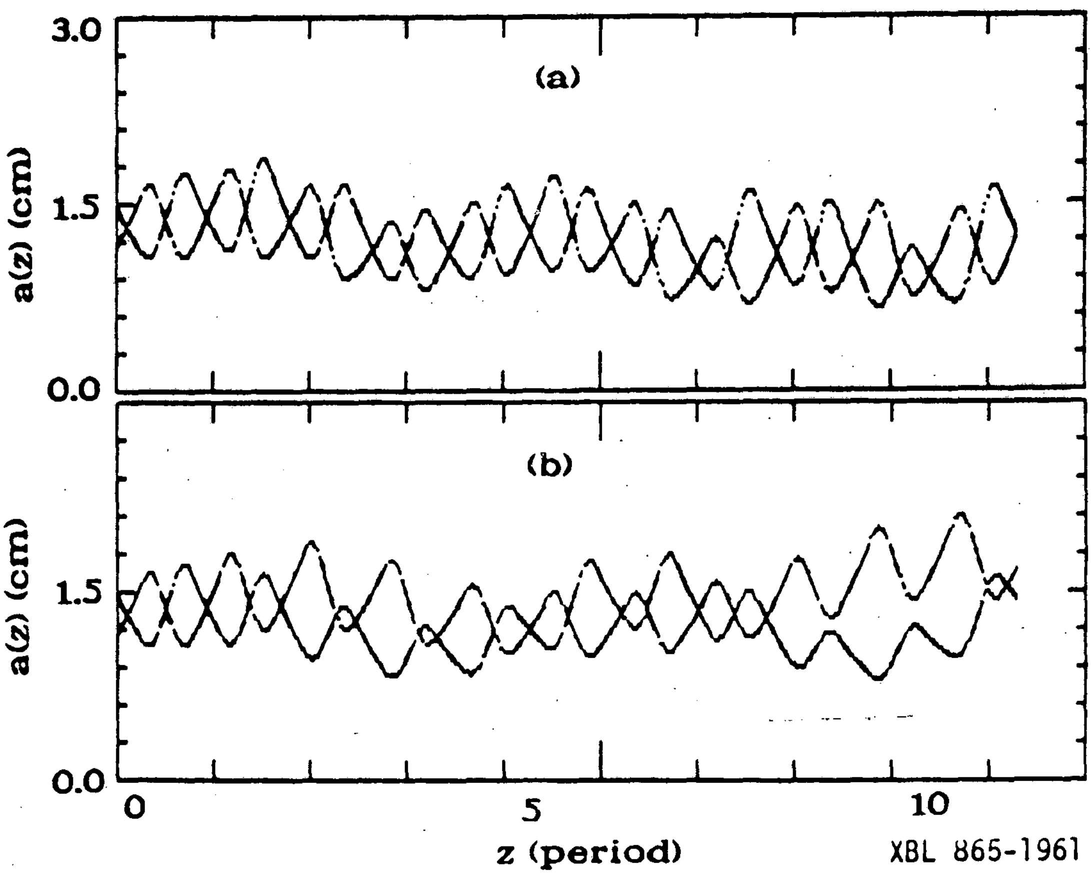

Fig. 3. Calculated beam envelopes (vertical and horizontal) typical for the beam head and center regions (a) and for the beam tail region (b).

oscillations is larger for the tail end of the beam as shown in Fig. 3b, but always well within the quadrupole-bore radius.

The high velocity tilt produced in this experiment may not be sustained in the subsequent acceleration gaps (which will be added later) because of too much current amplification as given in equation (3), unless the acceleration rate is increased from the value we are currently planning to implement.

V. MISMATCH OSCILLATIONS

We wish to study the qualitative behavior of mismatch oscillations, to which end it is sufficient to consider a round beam in smooth approximation with zero emittance. The envelope equation, taking acceleration into account is :

$$\frac{d}{dz}\beta\frac{da}{dz} = -K\beta a + \frac{qeIZ_o}{4\pi E_o a} \tag{4}$$

where K is the external force constant, E is kinetic energy in eV, and Z_o is 120 πohms. The equation applies separately to each longitudinal segment of the beam; if we adopt a constant current scenario, then I at fixed z is the same for all segments. For a matched solution (at the bunch center, for instance), take a independent of z. This condition determines K_o at the reference energy, E_o:

$$K_o = \frac{qeIZ_o}{4\pi E_o \beta_o a_o^2} \tag{5}$$

Then let $a = a_o(1+\alpha)$ for other segments of the beam at different energies and expand equation (4) to first order in α and $\Delta\beta/\beta$:

$$\frac{d^2\alpha}{d\tau^2} + \omega^2\alpha = -\epsilon K_o \beta_o^2 \frac{\Delta\beta}{\beta} \tag{6}$$

where $\omega^2 = 2K_o\beta_o^2$ is the free envelope oscillation frequency in the variable, $\tau = \int dz/\beta_o$ = (c times time of progress of the beam center), and $\epsilon = 2$ for magnetic focusing and 1 for electric focusing. To keep things simple, consider a scenario in which I is proportional to β_o so that ω is a constant. The solution of Eq. (3) for $\alpha = \dot{\alpha} = 0$ at $\tau = 0$ is then :

$$\alpha = -\frac{\epsilon}{2}\omega\int_0^\tau d\tau'\,\frac{\Delta\beta}{\beta}\sin\omega(\tau-\tau') \tag{7}$$

As an example, take

$$\frac{\Delta\beta}{\beta} = \Delta_{max}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right] \tag{8}$$

with $\lambda_2 >> \lambda_1$. This represents a rapid rise at the rate, λ_2, to a maximum of Δ_{max} followed by a slow decay at the rate of λ_1. For $\lambda_2\tau >> 1$ and $\lambda_1 << \omega$, the solution is:

$$\alpha = \frac{\varepsilon}{2}\Delta_{max}\left[\frac{\lambda_2}{\sqrt{\lambda_2^2+\omega^2}}\sin\left(\omega\tau+\tan^{-1}\frac{\lambda_2}{\omega}\right) - e^{-\lambda_1\tau}\right] \tag{9}$$

Other examples show similar behavior. Notice that the mismatch does not accumulate as in the case of misalignment errors. Rather, a ringing is set up, the magnitude of which is determined by the maximum value of $\Delta\beta/\beta$ and the rate of increase of $\Delta\beta/\beta$, λ_2, compared to the natural frequency, ω.

If I increases as a higher power of β, the natural oscillations are damped as $(I/\beta)^{1/4}$, but later contributions in (7) are increased. The net effect is a slight decrease in (9).

VI. SUMMARY

We have shown that the acceleration and current amplification rates in a heavy ion induction linac are constrained by the restrictions imposed on the value of velocity tilt at a given location; this restriction is particularly severe when the kinetic energy is low and the bunch is long.

We also have shown that a large velocity tilt can be tolerated if certain particle distributions are used: a constant current distribution for electric focusing and constant-density distribution for magnetic focusing. We have demonstrated experimentally in MBE-4 the electric focusing case.

Mismatch oscillations do not accumulate as the misalignment errors do; rather, a ringing is set up, the magnitude of which is determined by the magnitude and the rate of increase of the velocity tilt.

If any quadrupole misalignments are present, the velocity tilt will cause the frequency of coherent betatron oscillations to be different for different segments of the beam. These effects require further investigation.

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