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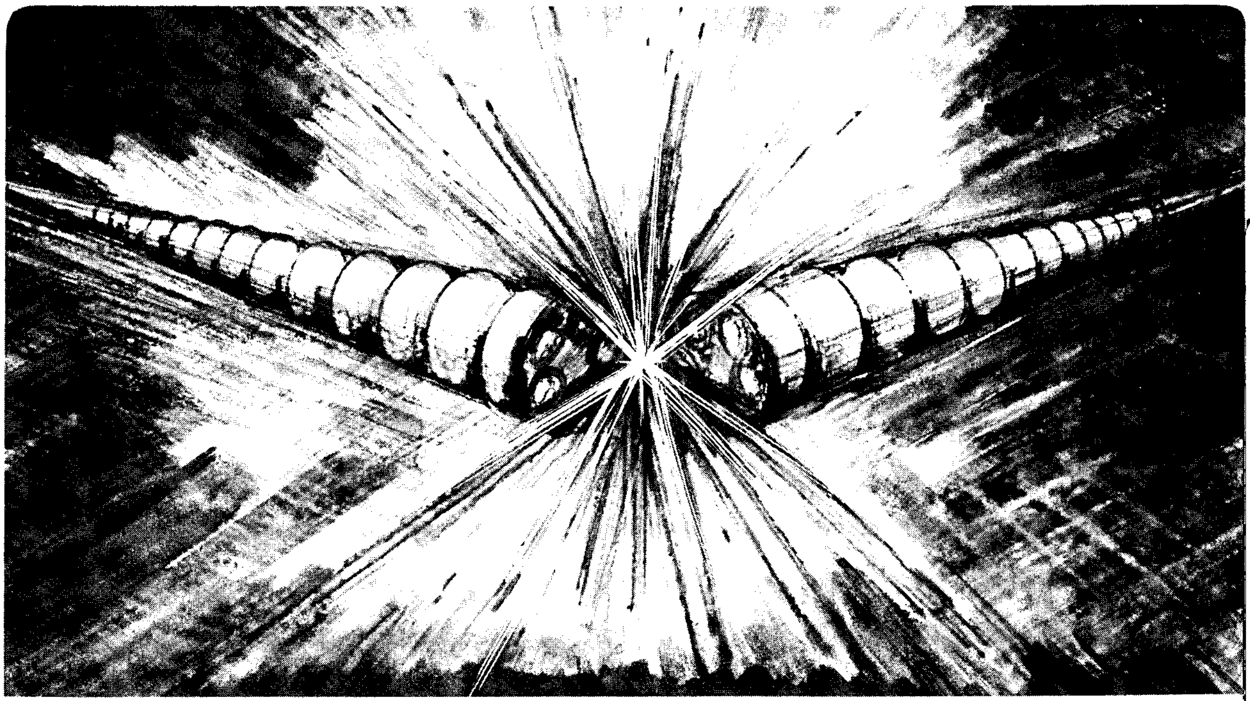

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A CONTINUOUS PLASMA FINAL FOCUS

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A CONTINUOUS PLASMA FINAL FOCUS

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Scaling laws are set down for a plasma cell used for transport, focussing and current neutralization of fine, intense, relativistic electron beams. It is found that there exists a minimum beam spot size, $\sigma_{min} \sim \varepsilon_n(I_A/\gamma I)^{1/2}$, in such a focussing system. Propagation issues, including channel formation, synchrotron radiation, beam ionization and instabilities, are discussed. Three numerical examples are considered.

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INTRODUCTION

A relativistic electron beam (REB) injected into a plasma less dense than the beam expels plasma electrons from the beam volume, producing an "ion-channel." The radial electric field due to the ions then focusses the beam. A plasma more dense than the beam will neutralize the beam charge, so that the REB is focussed by its own magnetic field. A still denser plasma may partially neutralize the current. Over the last twenty years, these and other features of REB propagation in plasmas have been studied extensively, theoretically and experimentally.[1] In recent years, ion-channel focussing has been successfully employed in the transport of high current beams for accelerator research.[2,3]

The "adiabatic focusser", proposed by Chen *et al.*[4] extends the underdense-plasma ion-focussing mechanism, for use in a TeV linear electron-positron collider. They propose to increase the plasma density along the direction of beam propagation, so as to focus the beam continuously to a spot size smaller than can be achieved through conventional magnetic optics. They observe that continuous focussing is a means of circumventing the Oide limit on the spot size in a discrete focussing system.[5]

At the same time, a subject of ongoing interest, in TeV linear electron-positron collider design, is the reduction of coherent beam-beam effects: beamstrahlung and disruption.[6,7] One method which has been proposed is current neutralization in an overdense plasma at the interaction point (IP).[8,9] Beamstrahlung and disruption are suppressed due to plasma return currents which reduce the magnetic pinch forces seen by the two colliding beams.

In this note, a plasma final focussing system, consisting of an underdense adiabatic focussing cell, as proposed by Chen *et al.*, followed by an overdense current neutralization cell at the IP, is considered (Fig.1). The parameter range of interest consists of electron bunches which are short (1-10 ps), fine (mm-μm radius), high current (100 A - 1000 A) and highly relativistic (100 MeV - 1 TeV), propagating through a plasma of density 10^{12} cm^{-3} - 10^{22} cm^{-3}. Parameters in this range have attracted growing interest in recent years, in connection with the plasma wakefield accelerator,[10,11] the plasma lens,[12] and the beat-wave accelerator.[13]

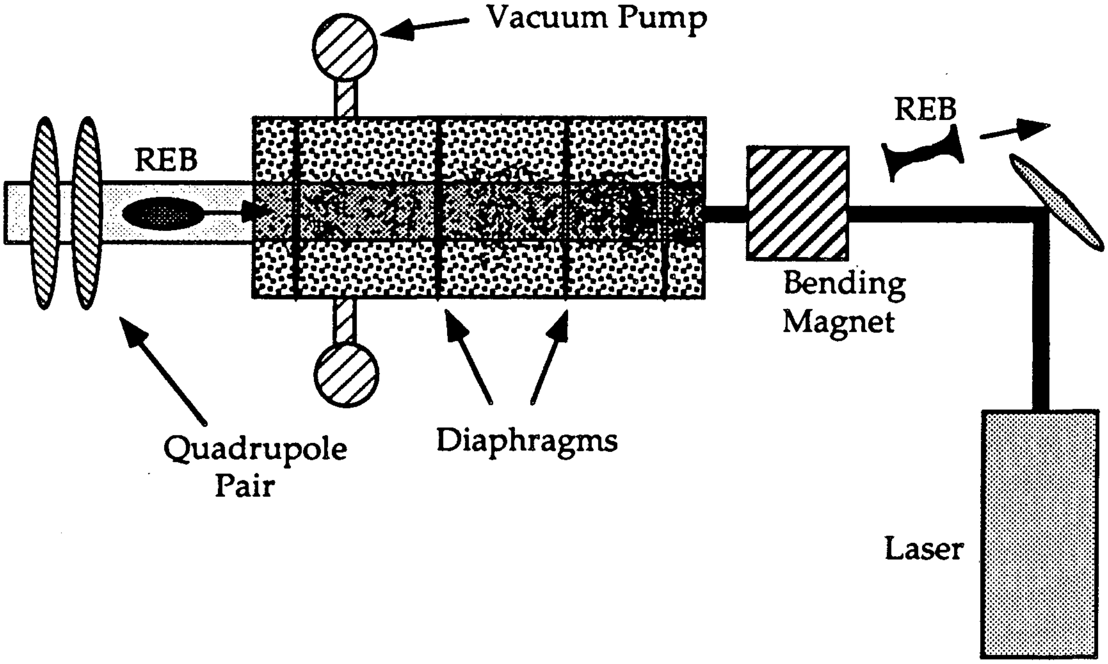

Figure 1. Set-up for a "proof-of-principle" continuous plasma focus experiment.

In the following sections, the basic scaling laws for such a "continuous plasma final focus" are set down. Issues discussed are scattering, ion channel formation, radiative losses, beam ionization and instabilities. Numerical examples are given and conclusions are offered.

SCALING LAWS

In the adiabatic focusser, an axial density gradient in a neutral gas is maintained prior to ionization, through differential pumping. An ionizing laser pulse then produces an axially increasing plasma density. Within less than a recombination time, an REB is injected. The gradient in plasma density results in an axially increasing electrostatic force, due to ion space-charge, on beam electrons as they traverse the cell. Consequently, the beam spot size is continuously reduced; the beam is "adiabatically focussed."

The continuous plasma final focus consists of such an adiabatic focussing cell, terminated with an abruptly increased plasma density extending through the IP. As the beam enters this overdense plasma, return currents are induced within the beam volume, reducing the azimuthal magnetic field that would otherwise disrupt the two beams in collision.

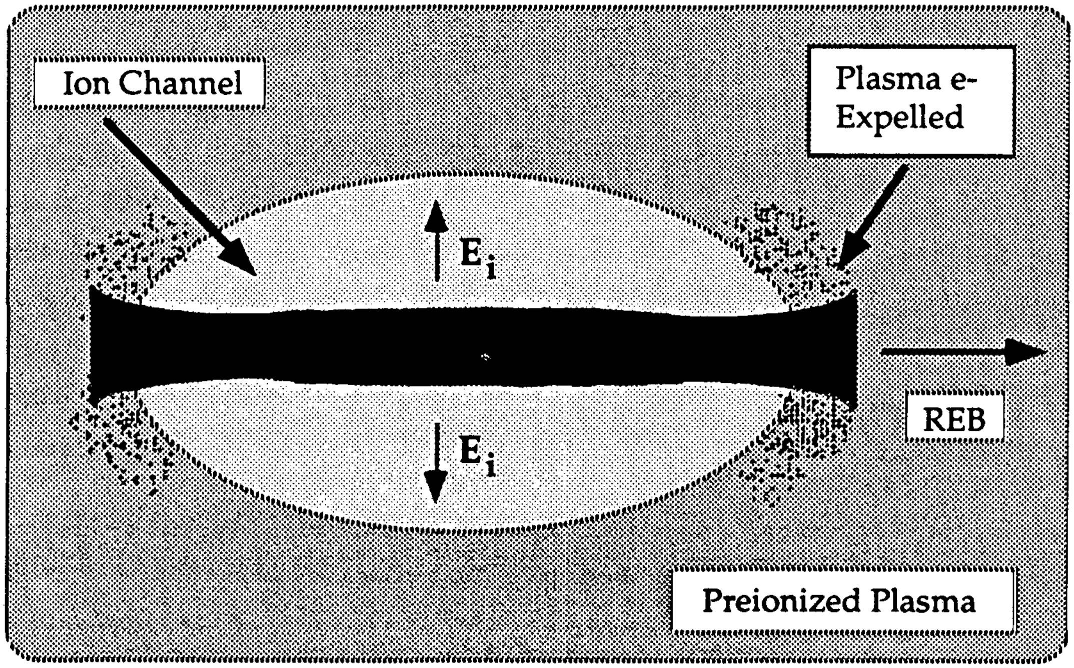

Figure 2. The radial electric field of the beam expels plasma electrons from a large volume, or "channel". Beam electrons are then focussed by the radial electric field of the relatively immobile ions.

In this section, focussing near the axial center of a long cigar-shaped beam, in a perfectly rigid channel, is considered (Fig. 2), and the relevant scaling laws are noted. Discussion of channel formation is taken up in the next section.

As the beam head propagates through the underdense plasma, it continuously expels plasma electrons from the beam volume, forming an "ion-channel", or volume from which plasma electrons have been completely ejected by the beam charge. For a very underdense plasma, the radius of this channel is given by $R \sim \sigma(2n_b/n_p)^{1/2}$, where σ is the rms beam radius, n_p is the plasma density prior to channel formation, and n_b is the beam density on axis.[14]

For effective focussing, the restoring force due to the ion charge should be much larger than the transverse Lorentz force on the beam due its self-fields. This requires $n_b>n_p>>n_b(1/\gamma^2+\beta_\perp^2)$, where $\beta_\perp=v_\perp/c$, with $v_\perp<<c$ the transverse velocity. The speed of light is c and γ is the Lorentz factor. The transverse Lorentz force seen by a beam electron in the channel is then, just the radial electric field due to the ion charge, $E_i \sim 2\pi e n_p r$, with r the radial coordinate. The electron charge is -e and its mass is m. In the potential well of the ion-charge, beam electrons oscillate transversely with wavenumber $k_\beta=k_p/(2\gamma)^{1/2}$, where $k_p=\omega_p/c$ and ω_p is the plasma frequency, $\omega_p^2=4\pi n_p e^2/m$.

To transport the beam into the plasma without excessive emittance growth, k_β should always vary adiabatically. This determines the initial plasma electron density, n_{pi}, in terms of the initial beam spot size, σ_i: $n_{pi}=\varepsilon_n^2/(2\pi\gamma r_e\sigma_i^4)$. Here, $\varepsilon_n=\gamma k_\beta\sigma^2$ is the normalized emittance, r_e is the classical electron radius and σ is the rms spot size. Neglecting radiation, scattering, and self-fields, ε_n is an adiabatic invariant. Thus an adiabatic increase in k_p, increases k_β, and decreases σ. This is the principle of the adiabatic focusser.

This adiabaticity requires that the plasma density be tapered over a length of order the initial betatron wavelength, and provides an estimate of the overall length of the plasma cell, $L_p \sim 2\pi\gamma\sigma^2/\varepsilon_n$.

As the beam is focussed to an ever smaller spot, the plasma density approaches the beam density and the character of the focussing changes. In the overdense regime, the ion space-charge is sufficient to neutralize the beam charge, so that the beam is focussed by its own magnetic field. This transition, from the underdense regime and ion space-charge focussing, to the overdense regime and beam self-pinching, occurs for a minimum beam radius, σ_{min}, determined by setting $n_p=n_b$: $\sigma_{min}=\varepsilon_n(I_A/\gamma I)^{1/2}$. The quantity $I_A=mc^3/e=17.05$ kA, is the Alfven current, and $I=Nec/(2\pi)^{1/2}\sigma_z$ is the peak beam current. The number of electrons per bunch is N. The density at this transition is $n_{pt}=\gamma(I/I_A)^2/(2\pi r_e\varepsilon_n^2)$, and the betatron wavenumber at this density is $k_{\beta max} \sim (I/I_A)\varepsilon_n^{-1}$.

In the overdense regime, the effective betatron wavenumber provided by the beam magnetic field is $k_\beta \sim (I_{net}/2\gamma I_A)^{1/2}/\sigma$, where the net current, I_{net}, is the sum of the beam current and the plasma return current within the beam volume.[15] Since $I_{net} \leq I$, the maximum focussing strength is bounded: $k_\beta \leq k_{\beta max}$.

Therefore, once the beam spot size is focussed to σ_{min}, the adiabatic focussing is complete, and the overdense ion-focussed regime relied on in conventional ion-focussing experments obtains. This establishes a limit on spot size in the adiabatic focusser, neglecting radiation damping. However, for the low emittance, high energy beams of a TeV collider, this limit is far smaller than the beam spot size required. Therefore, the design final spot size, σ_f, will usually be larger than the minimum possible spot size, σ_{min}. In this case, the final density in the focussing section, $n_{pf} = \varepsilon_n^2/(2\pi\gamma r_e \sigma_f^4)$, will usually be much less than n_{pt}.

Current neutralization requires a plasma skin depth short compared to the beam radial size, and a magnetic diffusion time long compared to the beam length. It is shown in Ref. 8 that the magnetic field reduction associated with an REB in a collisionless plasma scales as a function only of $k_p\sigma$. Taking a reduction of 70% as a figure of merit, $k_p\sigma_f \sim 1.4$ is required. To obtain partial current neutralization, without an increase in beam spot size, the adiabatic focussing cell should then be terminated within a distance $\lambda_{\beta f}$ of the IP with a nonadiabatic increase in plasma density to a value, n_{pc}, such that, $n_{pc} \sim 1/(2\pi r_e \sigma_f^2)$.

The length of this cell should be of order a few bunch lengths, and, to avoid defocussing due to plasma return currents, it should be less than the final betatron wavelength at the focusser exit. This implies, $\sigma_z < \lambda_{\beta f}$. If the adiabatic focusser is terminated with $n_{pf} < n_{pt}$, the beam may pinch as it enters the current neutralization cell. Pinching is ngeligible if the cell length is much less than $\lambda_{\beta min}$; this requires $\sigma_z < \lambda_{\beta min}$.

BEAM PROPAGATION

The simplified analysis of the last section considered focussing of a long cigar shaped bunch, neglecting the details of channel evolution at the bunch head and tail. However, these details are known to limit propagation in many beam-plasma applications and, in this section, their effect in the continuous plasma focus is considered.

Emittance growth due to scattering has been calculated by Montague and Schnell.[16] Applying their result, $\Theta_{rms}(z)$, the rms scattering angle after traversing a length, z, of gas, varies according to,

$$\frac{d}{dz}\Theta_{rms}^2 = 8\pi n_0 \frac{Z^2 r_e^2}{\gamma^2} \ln\left(\frac{\theta_{max}}{\theta_{min}}\right), \qquad (1)$$

where n_0 is the density of neutral atoms. For a partially ionized gas from which plasma electrons have been ejected, $\theta_{min} \sim \hbar/(Rmc\gamma)$, for scattering from ions, and $\theta_{min} \sim \hbar/(amc\gamma)$ for scattering from neutral atoms. However, it will be assumed that the ionization fraction, f, is sufficiently low that scattering with neutral atoms dominates. The atomic number is Z and $a \sim 1.4\ a_B\ Z^{-1/3}$, is the screening radius in the Thomas-Fermi model. Planck's constant is $\hbar$ and a_B is the Bohr radius. The maximum scattering angle is $\theta_{max} \sim \hbar/(r_n mc\gamma)$ where $r_n \sim 0.5\ r_e\ A^{1/3}$ is the nuclear radius and A is the atomic weight. This gives, $\theta_{max}/\theta_{min} \sim 5.26\ 10^4/(AZ)^{1/3}$.[17] Emittance growth is given by,

$$\frac{d\varepsilon_n}{dz} = \frac{\gamma}{2\,k_\beta}\frac{d}{dz}\Theta^2_{rms}\,, \tag{2}$$

The change in normalized emittance in passing through the cell is then

$$\Delta\varepsilon_n \approx \frac{r_e Z^2}{\alpha_0 f}\ln\left(\frac{\theta_{max}}{\theta_{min}}\right)\ln\left(\frac{\lambda_{\beta i}}{\lambda_{\beta f}}\right). \tag{3}$$

A linear variation in λ_β is assumed: $\lambda_\beta = \lambda_{\beta i} - 4\pi\alpha_0 z$. For the examples, $\alpha_0 \sim 1/4\pi$, corresponding to a length, $L_p \sim \lambda_{\beta i}$.

In the overdense regime, envelope expansion is qualitatively different because the quasistatic beam equilibrium is maintained by the beam magnetic field, rather than the (external) field of the ion charge. As the beam expands, the focussing is reduced, with the result that the beam envelope exponentiates, on the scale of the Nordsieck length,[18]

$$L_N = \frac{1}{4\pi\, n_0 r_e^2}\,\frac{\gamma I}{I_A}\,\frac{1}{Z^2 \ln\left(\dfrac{\theta_{max}}{\theta_{min}}\right)}\,, \tag{4}$$

where channel radiation has been neglected. The Nordsieck length is always much longer than the current neutralization cell.

As the beam moves through the plasma, the beam head must eject electrons from the channel. The ion charge thus exposed provides focussing for electrons to the rear. In the meantime, electrons at the front are not strongly focussed, and expand due to emittance. These and other issues have been discussed in connection with "beam head erosion" of long pulses injected into an unionized gas,[19] and for long pulses in a preionized plasma of radial extent comparable to the beam.[20] The regime of interest here has not been extensively studied, but it is expected that erosion should be negligible, since the plasma is preionized, the electron bunch is short, the emittance is low, the energy is high, and the propagation length is short.[21]

Specifically, regions at the beam head and tail will be less dense than the plasma and will be magnetically focussed by much less than the peak azimuthal magnetic field. Thus a realistic beam profile would appear flared at each end. For best focussing the beam current rise should be adiabatic on the ω_p^{-1} time scale, i.e.,[22] $\sigma_z > 1/(4\pi n_{pi} r_e)^{1/2}$.

RADIATION IN THE ION-CHANNEL

Radiation in the ion-channel is of interest as a diagnostic, and of possible concern for its effect on beam optics. Two types of radiation are considered: bremstrahlung and synchrotron radiation due to the betatron motion.

Bremstrahlung may be characterized by the radiation length λ_R,[23]

$$\lambda_R^{-1} = \frac{16}{3}\alpha n_0 r_e^2 Z^2 \ln\left(\frac{233}{Z^{1/3}}\right), \tag{5}$$

where α is the fine structure constant. The fractional energy loss is then

$$\left(\frac{\Delta\gamma}{\gamma}\right)_B \approx \int_0^{L_p} \frac{dz}{\lambda_R} \approx \frac{4\alpha}{3\pi\alpha_0} r_e^2 \lambda_{\beta i}\left(\sqrt{n_{0f} n_{0i}} - n_{0i}\right) Z^2 \ln\left(\frac{233}{Z^{1/3}}\right), \tag{6}$$

where n_{0i} and n_{0f} are the initial and final neutral densities, respectively. This loss is typically very small.

Radiation due to the betatron motion takes on the character of wiggler radiation, for strong focussing ($\gamma\beta_\perp \geq 1$).[24] In principle, the exact single particle trajectory should be used to compute the radiation fields; but for estimates here, it is enough to note the main features.

The spectrum on axis will extend through the frequency range $\omega \sim 2\gamma^2 c k_\beta / (1+\gamma^2 \beta_\perp^2)$ to $\omega \sim 2\gamma^2 c k_\beta$, due to the spread in $\beta_\perp$ within the beam. Integrated over all angles, the spectrum is characterized by the critical frequency, $\omega_c = 3\gamma^3 c/\rho$, where $\rho = 1/(k_\beta^2 \sigma)$, is the effective bending radius. The angular distribution extends to angles of order $\beta_\perp$. Quantum effects are small provided $\Upsilon < 0.2$, where, $\Upsilon = \gamma^2 ƛ_c / \rho$, and $ƛ_c$ is the Compton wavelength.[25]

As in a damping ring, synchrotron radiation can decrease the normalized emittance of the beam.[26] However, for the continuous plasma focus it is desirable to limit radiation losses to a small fraction of the beam energy. Fractional energy loss is computed in Ref. 4 and the result, for Υ small, is

$$\left(\frac{\Delta\gamma}{\gamma}\right)_s = -\frac{2\pi^2}{3} \gamma^2 \varepsilon_n r_e \frac{\left(1+\alpha_0^2\right)^2}{\alpha_0} \left(\frac{1}{\lambda_{\beta f}^2} - \frac{1}{\lambda_{\beta i}^2}\right). \tag{7}$$

Here, $\Delta\gamma$ is the change in γ.

BEAM IONIZATION

Ionization by the beam is of concern in determining the actual axial plasma density profile. Ionization is produced by the beam through electron impact, gas breakdown, and stripping of atoms and ions in the strong radial electric field at the beam "edge". To accurately compute the net volume rate of ionization requires numerical solution of detailed rate equations, and modelling of the chemistry of the particular gas used. To estimate the effect of impact ionization, a phenomenological estimate must be made for the effective area into which secondary electrons are ejected.[27] In this section, only a few simple estimates are made.

The time scale for ionization in the overdense regime via impact ionization of neutrals by beam electrons and secondaries is $\tau_b \sim 1/(n_0 \sigma_{bi} c)$, where σ_{bi} is an effective ionization cross-section of order 10^{-18} cm^2.[28] This ionization time is ~1 ps at a density of $3\ 10^{19}$ cm^{-3}.

The character of breakdown produced by long pulses is determined by the value of E/p, the ratio of radial electric field to pressure.[29] For very fine beams, E/p will be sufficiently large that secondary electrons are ejected far beyond the beam volume before they create additional ionization.

In addition, for short pulses, a key limitation is the formative time required for breakdown. This is roughly the time for one secondary electron accelerated in the beam field, to ionize one neutral, $\tau_e \sim 1/(n_0 \sigma_{ei} v_e)$, where σ_{ei} is the cross-section for ionization by secondaries and v_e is the secondary velocity. The quantity $\sigma_{ei} v_e$ peaks at secondary electron energies of order ~ 100 eV, with $\sigma_{e1} v_e \sim 10^{-7} - 10^{-8}$ cm^3/sec, depending on the gas.[30] The time scale τ_e is then of order ~1 ps at a density of $3\ 10^{19}\ cm^{-3}$. Based on this estimate, energetic secondaries, and significant ionization beyond the beam volume may be expected depending on the particular parameters.

The radial electric field at the beam edge will be adequate to strip an atomic electron with ionization potential, $\Delta\varepsilon$, for currents of order

$$I \approx \alpha^4 \frac{\sigma}{r_e} \left(\frac{\Delta\varepsilon}{e^2/a_B} \right) I_A . \tag{8}$$

For very fine beams, this mechanism may fully ionize a channel larger than the beam, with some multiple ionization.

When field stripping may be neglected, plasma electrons are also lost through recombination on a time scale $\tau_r \sim 1/(\alpha_r n_p)$, and through attachment on a time scale $\tau_r \sim 1/(\alpha_a n_0)$. Here, α_r and α_a are the recombination and attachment coefficients, respectively.[31] Taking recombination in N_2 as an example, $\alpha_r \sim 2\ 10^{-7}$ cm^3/sec, at electron energies ~1 eV.[31] At a density of $3\ 10^{19}\ cm^{-3}$, $\tau_r \sim 0.2$ ps and this is quite short. However, α_r will be lower for more energetic electrons. In addition, despite recombination and attachment, the beam volume will become depleted of plasma electrons, provided the impact ionization time scale is short enough. This occurs because, as electrons go through successive ionizations and recombinations, they diffuse away from the beam center.

Any realistic model of beam ionization, for TeV collider parameters, will have to incorporate all of these effects.

INSTABILITIES

A number of instabilities complicate the equilibrium outlined above, and in this section, the growth rates are noted.

In the focussing cell, the equilibrium discussed so far, consisting of a beam travelling down a static channel, is maintained only to the extent that ions are immobile. In fact, ions at radius R collapse inward, neutralizing the beam charge, on a time scale, τ_{ion}~$(m_i/m)^{1/2}(I_A/I)^{1/2}(R/4c)$, where m_i is the ion mass. For pulses longer than τ_{ion}, focussing is stronger for the beam tail than the head. This should be avoided since it may result in disruption and emittance growth.[32]

In addition, in the underdense regime, it has been suggested that a "transverse two-stream instability" may develop,[33] whereby a displacement of the beam centroid perturbs the channel wall, which then acts back on the beam. However, only preliminary work has been performed on this problem and a growth rate has not yet been derived.

In the current neutralization cell, significant current cancellation requires a low collision rate. However, in the collisionless limit, instabilities may replace collisions in dissipating the energy of the secondaries.[34] In particular, the two-stream (Buneman) instability will couple the electron motion to the ions on a time scale ν_{eiTS}^{-1}, where ν_{eiTS}/ω_p~$(3^{1/2}/2)(m/2m_i)^{1/3}$. This time scale can be quite short in the current neutralization cell. On the other hand, this instability convects away from the beam, and the carriers of the return current are constantly being replaced with an unperturbed flow of plasma electrons. A thorough analysis of the effect of this instability on current neutralization, including the effects of energy spread, has not been performed. However, numerical simulations performed in Ref. 8 show no evidence of return current disruption due to this effect, indicating that the plasma electron energy spread is probably sufficient to damp growth.

In addition, in the overdense regime, significant return currents flow within the beam volume and two adjacent plasma electron return current filaments attract. Filaments form and disrupt the intended current neutralization.[35] The growth rate for the Weibel or filamentation instability is ν_w~$\omega_p(n_b/\gamma n_p)^{1/2}$, and typically a few e-folds may develop.[36,37]

Finally, the ion-hose instability[20] requires numerical study; however, a simple estimate using the rigid beam model indicates that for the examples considered here, at most a few e-folds can be expected.

EXAMPLES

In this section, three applications of a continuous plasma final focus are considered: a proof-of-principle experiment at the TRISTAN injector at the National Laboratory for High Energy Physics (KEK), an application for luminosity enhancement at the Stanford Linear Collider (SLC), and a hypothetical TeV electron-positron collider. A summary of parameters is given in Table I.

The TRISTAN injector offers the possibility of doing single beam focussing and current neutralization experiments as a "proof-of-principle." A cell of length L_p ~3 m, with initial density, n_i ~1 10^{11} cm^{-3}, and final density, n_f ~ 2 10^{13} cm^{-3} would focus the spot size from, σ ~ 1 mm, to σ ~0.3 mm. An increase in plasma density up to n_c ~6 10^{14} cm^{-3} over a length of a few millimeters would produce partial current neutralization.

One complication with these parameters is that the adiabatic current rise condition is not satisfied at injection. Thus nonlinear plasma oscillations would be excited and may cause emittance growth.

Table I. Parameters for applications of a continuous plasma focus.

	TRISTAN	SLC	TeV-LC	(Units)
Beam Parameters				
$mc^2\gamma$	0.25	46	1000	(GeV)
I	0.4	0.25	1.0	(kA)
ε_n	10^{-3}	10^{-5}	10^{-5}	(m-rad)
N	$6\ 10^{10}$	10^{10}	$5\ 10^{10}$	(—)
σ_z/c	10	2.5	1.0	(ps)
Plasma Parameters				
n_{pi}	$1\ 10^{11}$	$1\ 10^{14}$	$3\ 10^{15}$	(cm^{-3})
n_{pf}	$2\ 10^{13}$	$1\ 10^{18}$	$3\ 10^{19}$	(cm^{-3})
n_{pc}	$6\ 10^{14}$	$2\ 10^{20}$	$6\ 10^{21}$	(cm^{-3})
Focussing Cell Parameters				
σ_i	1000	5.0	1.0	(μm)
σ_f	300	0.5	0.1	(μm)
σ_{min}	300	0.3	0.03	(μm)
$\lambda_{\beta f}$	27	1.4	1.2	(cm)
L_p	3.0	1.4	1.3	(m)
Radiation & Scattering Parameters				
$(\Delta\gamma/\gamma)_S$	$8\ 10^{-10}$	$1\ 10^{-4}$	0.1	(—)
Υ	$2\ 10^{-8}$	$3\ 10^{-4}$	$6\ 10^{-2}$	(—)
$\Delta\varepsilon_n$	$2\ 10^{-11} Z^2/f$	$8\ 10^{-13} Z^2/f$	$2\ 10^{-12} Z^2/f$	(m-rad)

At the SLC, a continuous plasma focus could be employed for a proof-of-principle experiment, and to significantly enhance the luminosity in a working collider.[38] The initial plasma electron density would be n_i ~1 10^{14} cm^{-3} and the length of the focusser would be L_p ~1.4 m. For a final density n_{pf} ~ 1 10^{18} cm^{-3}, the spot size would be σ ~0.5 μm. The density required for partial current neutralization would be ~2 10^{20} cm^{-3}. As a proof of principle, current neutralization experiments would be interesting; however, for the SLC, beamstrahlung and disruption are small and current neutralization is not required.

With A~100, the time scale for ion motion at the focusser exit is τ_{ion}~3 ps and this is probably acceptable. In the current neutralization section, the time scale for filamentation is τ_w ~2 ps. The time-scale for the electron-ion two-stream instability is τ_{eiTS} ~0.1 ps and this is short. However, plasma electron energy spread will likely damp growth.

At a density of 10^{18} cm^{-3} the formative time for breakdown will be τ_e ~10 ps, so that breakdown will be marginal. In the neutralization cell, τ_e ~ 0.03 - 0.3 ps depending on the gas, while the impact ionization time scale is τ_b ~0.1 ps. Field stripping of atoms will be significant for ionization potentials less than ~ 5 eV. Otherwise, recombination may be significant since a simple estimate gives τ_r ~10^{-2} ps.

The last example is a hypothetical TeV linear collider. The normalized emittance used for this example was an order of magnitude larger than in conventional TeV collider designs, and the charge per bunch was taken to be 5 10^{10}, which is a bit higher than is typical. The initial plasma electron density would be n_i ~ 3 10^{15} cm^{-3}. The length of the focusser would be L_p ~ 1.3 m. For a final density of 3 10^{19} cm^{-3}, the spot size would be σ ~ 0.1 μm. The betatron wavelength would be ~ 1.2 cm and the density required for partial current neutralization would be ~ 6 10^{21} cm^{-3}. Densities of 10^{22} cm^{-3} would be desirable.

Taking A ~ 100, the time scale for ion motion near the focusser exit is τ_{ion} ~0.5 ps and this is shorter than a bunch length. In the current neutralization section, the time scale for filamentation is τ_w ~1 ps. The time-scale for the electron-ion two-stream instability is τ_{eiTS} ~4 10^{-3} ps. Further work remains to assess the effect of these instabilities on current neutralization.

At a density of 3 10^{19} cm^{-3} the formative time for breakdown will be τ_e ~0.3 ps, while the impact ionization time is τ_b ~1 ps. The recombination time will be of order τ_r ~0.2 ps. Therefore it is likely that beam ionization will be significant at the focusser exit. In the neutralization cell, τ_e ~ 2 - 20 fs depending on the gas, while τ_b ~5 fs. Field stripping of atoms will be significant for ionization potentials less than

~ 30 eV, so that an annulus will be cleared around the beam edge, in which all atoms are at least singly ionized. It is evident from these simple estimates that copious ionization will be produced by the beam in the current neutralization section.

Fractional energy loss is ~10% and this is roughly the beamstrahlung energy loss in conventional TLC designs. However, energy loss of this size is not an intrinsic feature of the continuous plasma focus and it can be reduced by reducing the emittance.

CONCLUSIONS

The concept of a continuous plasma final focus, consisting of an adiabatic plasma focussing cell, followed by a short, dense current neutralization cell has been outlined. The scaling laws for such a device have been set down, together with three numerical examples.

Further work remains to assess the effect of ion-motion at the focusser exit in a TeV collider design, as well as the Weibel and Buneman instabilities in the current neutralization cell.

Much analytical and numerical work remains to be done for a practical experiment. Interesting problems include: (1) studies of the high energy products of beam-plasma collisions, (2) studies of continuous plasma focussing of positron beams, (3) design of the vacuum system, (4) studies of realistic beam ionization profiles and their effect on focussing, (5) matching of weakly focussed beams (nonlinear wakefield theory in a very underdense plasma), (6) the effects of ion motion at the focusser exit, and (7) numerical simulation of channel formation, including ion-motion, collisions, and dipole perturbations to the beam centroid.

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[38] Plasma focussing of positron beams is not addressed here. To estimate luminosity enhancement, positron beams focussed conventionally or by a discrete plasma lens must be considered.

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