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THE TWO-NUCLEON STRIPPING REACTION

Norman K. Glendenning

November 1960

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ABSTRACT

Expressions for the angular distribution of the (α, d) reaction are derived. Explicit coupling schemes for even-even, even-odd, and odd-odd target nuclei in the j - j limit are used to define the nuclear structure factors.

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I. INTRODUCTION

A considerable body of experimental data is accumulating on two-particle stripping reactions $[(\alpha, d), (\text{He}^3, p), \text{etc.}]$. The simplest way of treating such reactions from the theoretical point of view is to regard the incident particle as two structureless particles, one of which is stripped as a unit. The angular distribution could be calculated in complete analogy with the (d, p) reaction. Such a treatment would be appropriate if one believed that the residual state formed was a cluster state with the parentage of the target nucleus plus the stripped nuclide. If, however, the residual nucleus is better described within the framework of the conventional shell model, then the stripped pair is captured into single-particle states of the residual nucleus and do not retain their correlation.

The two-nucleon stripping reaction differs in several respects from the (d, p) reaction. In the latter, the angular distribution is characterized by the single orbital angular momentum of the state into which the neutron is stripped. The type of level formed is a single-particle excitation of the final nucleus. However, in the two-nucleon stripping reactions, the angular distribution is characterized by the total orbital angular momentum $L = l_n + l_p$ and in $j-j$ coupling the wave function of the residual state in general has several L components. Moreover, the types of levels that can be excited include levels of two-nucleon excitations. Thus, in principle, these reactions should

provide a means of investigating levels of a more complicated nature than is provided by the (d,p) reaction. The two-nucleon stripping reaction was first considered by El Nadi,¹ and we shall take advantage of some of his work.

There are several added refinements, however, that make the model more realistic.² More recently the same problem was treated by News.³ However, in that work the nuclear structure factors were left in a general undefined form. Here we consider explicit coupling schemes for even-even, even-odd, and odd-odd target nuclei in the j-j coupling limit.

The principal assumptions used to calculate the angular distribution are: (a) the center-of-mass motion of incident and outgoing particles is described by plane waves; (b) the target nucleus forms the core of the residual nucleus and is not excited by the reaction; and (c) the neutron and proton are captured into spin-orbit states in the nucleus.

In the next section, expressions for the angular distributions are derived for the (α,d) reaction. Selection rules are discussed. In a paper following this one, Cerny, Harvey, and Pehl have applied the theory to a number of their experimental results.⁴

II. CALCULATION

A. Even-Even Target Nucleus

We derive in this section the angular distribution for the (α,d) reaction on an even-even target nucleus. The target nucleus has $J_i = 0$. However, for comparison with previous work we do not explicitly set $J_i = 0$. The neutron and proton are captured into spin-orbit states $j_p = l_p + 1/2$ and $j_n = l_n + 1/2$, which are coupled to J_i to form the spin of the residual nucleus J_f . The wave function of the final nucleus is written

$$\Psi_{J_f}^{M_f}(A+2) = \sum C(J_i J J_f; M_i M M_f) \Psi_{J_i}^{M_i}(A) \Phi_J^M \quad (1)$$

where in an obvious notation, Φ_J^M denotes the following angular momentum

couplings:

$$\Phi_J^M(r_n, r_p) \equiv |(l_n 1/2) j_n, (l_p 1/2) j_p; JM\rangle \quad (2)$$

For the internal wave function of the α particle we use

$$\chi_0(\alpha) \propto X_0^0(n, n') X_0^0(p, p') \exp[-\gamma^2 \sum r_{ij}^2] \quad (3)$$

where X_0^0 is the singlet spin function. This wave function is antisymmetric upon interchange of two identical nucleons. When a matrix element of a symmetric operator is taken, if the wave function on the right (or left) has the correct symmetry under interchange of identical nucleons, it is not necessary to antisymmetrize the left (or right) side. Therefore we do not explicitly antisymmetrize the final state. We note now that we can rewrite the product

$$X_0^0(n, n') X_0^0(p, p') = \frac{1}{2} \sum_{S=0}^1 \sum_{M=-S}^S \sqrt{2S+1} C(SS0; M-M0) \\ \times X_S^M(n', p') X_S^{-M}(n, p) \quad (4)$$

Let the nucleons in the outgoing deuteron be n' and p' . Since they are in a triplet state, only the triplet part of $\chi_0(\alpha)$ contributes to the reactions.

We shall use the following notation:

$$\underline{R} = 1/2 (\underline{r}_n + \underline{r}_p) = \text{center of mass (c.m.) of captured pair,}$$

$$\underline{R}_d = 1/2 (\underline{r}_{n'} + \underline{r}_{p'}) = \text{c.m. of outgoing deuteron,}$$

$$\underline{r}_{\alpha d} = \underline{R}_d - \frac{2m}{M_f} \underline{R} = \text{deuteron c.m. relative to c.m. of the residual nucleus,}$$

$$\underline{k}_\alpha = \text{alpha-particle wave number,}$$

$$\underline{k}_d = \text{deuteron wave number.}$$

The matrix element for the reaction leading to the magnetic substates

indicated is

$$III(M_i \rightarrow M_f, M) \propto \int d\underline{r}_n d\underline{r}_p d\underline{R}_d d\underline{A} X_1^{M*}(n', p') \\ \times \exp[-i \underline{k}_d \cdot \underline{r}_{\alpha d}] \Psi_{J_f}^{M_f*}(A) V \Psi_{J_i}^{M_i}(A) \chi_0(\alpha) \exp[i \underline{k}_\alpha \cdot (\underline{R} + \underline{R}_d)/2] \quad (5)$$

where μ is the Z component of the outgoing deuteron spin. We call

$$\int \Psi_{J_i}^{M_i^*}(A) V \Psi_{J_i}^{M_i}(A) dA = V_{eff} \quad (6)$$

the effective interaction between the target and the incident α particle responsible for the reaction. It is assumed to depend only on the radial coordinates of the two captured nucleons. Introducing Eqs. (1), (3), and (4) into (5),

we find

$$\begin{aligned} \mathcal{M} &\propto C(110; \mu - \mu_0) C(J_i J_f; M_i M_f) \\ &\times \int d\underline{r}_n d\underline{r}_p d\underline{R}_d \exp[-i \underline{k}_d \cdot \underline{r}_d] \Phi_J^{M^*}(\underline{r}_n, \underline{r}_p) V_{eff} \\ &\times \chi_1^{\mu}(\underline{r}_n, \underline{r}_p) \exp[-\gamma^2 \sum r_{ij}^2] \exp[i \underline{k}_\alpha \cdot (\underline{R} + \underline{R}_d)/2]. \quad (7) \end{aligned}$$

To carry out the integration on the spin functions we transform Eq. (2) to an L-S basis:

$$\Phi_J^M(\underline{r}_n, \underline{r}_p) = \sum_{L,S} a_{LSJ}(\underline{r}_n, \underline{r}_p) \langle (l_n l_p) L, (\frac{1}{2} \frac{1}{2}) S; JM \rangle \quad (8)$$

where a_{LSJ} are the transformation coefficients (Cf. Racah)⁵ from LS to j-j coupling,

$$a_{LSJ}(\underline{r}_n, \underline{r}_p) = \langle (l_n l_p) L, (\frac{1}{2} \frac{1}{2}) S; J | (l_n \frac{1}{2}) j_n, (l_p \frac{1}{2}) j_p; J \rangle \quad (9)$$

The integration is on the spin projects triplet states only, and we can write

$$\begin{aligned} \mathcal{M} &\propto C(110; \mu - \mu_0) C(J_i J_f; M_i M_f) \sum_L a_{L1J} \\ &\times C(L1J; M+\mu, -\mu M) F(l_n l_p L; M+\mu) \quad (10) \end{aligned}$$

The symbol F stands for

$$\begin{aligned} F(l_n l_p L; M) &= \sum_{m_n m_p} C(l_n l_p L; m_n m_p M) \\ &\times \int d\underline{r}_n d\underline{r}_p d\underline{R}_d \exp[-i \underline{k}_d \cdot \underline{r}_d] \phi_{l_n m_n}^*(\underline{r}_n) \phi_{l_p m_p}^*(\underline{r}_p) \\ &\times V_{eff} \exp[-\gamma^2 \sum r_{ij}^2] \exp[i \underline{k}_\alpha \cdot (\underline{R} + \underline{R}_d)/2] \quad (11) \end{aligned}$$

where the single-particle wave functions are of the form

$$\phi_{\ell m}(r) = R_{\ell}(r) Y_{\ell m}(\theta, \phi) \quad (12)$$

This quantity is common to all the coupling schemes in the type of treatment envisioned here. The particular coupling scheme affects only the geometrical factors by which F is multiplied. We now evaluate F . First transform to a new coordinate system,

$$\underline{r} = \underline{R}_d - \underline{R}, \quad (13)$$

and introduce the linear momentum transfer vectors

$$\underline{K} = \underline{k}_d - \underline{k}_{\alpha}/2, \quad \underline{Q} = \underline{k}_{\alpha} - (M_i/M_f) \underline{k}_d \quad (14)$$

The physical meaning of these vectors is that \underline{K} is the momentum transferred to the outgoing deuteron, while \underline{Q} is the momentum carried into the nucleus by the stripped pair. Then

$$\begin{aligned} F(l_n l_p L; M) &= \sum_{m_n m_p} C(l_n l_p L; m_n m_p M) \\ &\times \int d\underline{r}_n d\underline{r}_p d\underline{r} \exp[i \underline{Q} \cdot \underline{R} - i \underline{K} \cdot \underline{r}] \\ &\times \phi_{l_n m_n}^*(\underline{r}_n) \phi_{l_p m_p}^*(\underline{r}_p) V_{\text{eff}} \exp[-\gamma^2 \sum r_j^2] \end{aligned} \quad (15)$$

This integral would be trivial if we were to assume that the α particle is a point cluster of nucleons. Then $\gamma \rightarrow \infty$, and we would find, for a surface interaction,

$$\begin{aligned} F(l_n l_p L; M) &\propto \delta_{M0} \sqrt{(2l_n+1)(2l_p+1)} C(l_n l_p L; 000) \\ &\times j_L(Q R_0) \end{aligned} \quad (16)$$

We shall, however, evaluate F for the finite-size α particle. First expand $\exp(i \underline{Q} \cdot \underline{R}) = \exp(i \underline{Q} \cdot \underline{r}_p/2) \exp(i \underline{Q} \cdot \underline{r}_n/2)$ as the product of two multipole expansions with \underline{Q} defining the Z axis. The α particle radial function can be rewritten in terms of coordinates r, r_{np}, r_{mp} (where

$$r_{np} = r_n - r_p):$$

$$\begin{aligned} \exp[-\gamma^2 \sum r_{ij}^2] &= \exp[-2\gamma^2 (2r^2 + r_{np}^2 + r_p^2)] \\ &= \exp[-2\gamma^2 (2r^2 + r_{np}^2 + r_n^2 + r_p^2 - 2r_n r_p \cos \omega_{np})] \end{aligned} \quad (17)$$

where ω_{np} is the angle between \underline{r}_n and \underline{r}_p . We use the expansion

$$\begin{aligned} \exp[4\gamma^2 r_n r_p \cos \omega_{np}] &= \left(\frac{\pi}{2\gamma^2 r_n r_p} \right)^{1/2} \sum_{n=0}^{\infty} (n+1/2) \\ &\times I_{n+1/2}(4\gamma^2 r_n r_p) P_n(\cos \omega_{np}) \end{aligned} \quad (18)$$

where

$$\begin{aligned} I_{n+1/2}(\rho) &= i^{n+1/2} J_{n+1/2}(-i\rho) \\ &= \left(\frac{\rho}{2} \right)^{n+1/2} \sum_{m=0}^{\infty} \frac{(\rho/2)^m}{m! \Gamma(n+n+3/2)} \end{aligned} \quad (19)$$

Hence for F we have

$$\begin{aligned} F &\propto \sum_{m_n m_p} C(l_n l_p L; m_n m_p M) \sum_{\lambda_n \lambda_p} \sqrt{(2\lambda_n+1)(2\lambda_p+1)} \\ &\times i^{\lambda_n+\lambda_p} \sum_{n m} (l_n m_n | Y_n^{m*} | \lambda_n 0) (l_p m_p | Y_n^m | \lambda_p 0) \\ &\times \int d\underline{r} \exp[-i \underline{K} \cdot \underline{r} - 4\gamma^2 r^2] \int d r_n d r_p r_n^2 r_p^2 \\ &\times R_{l_n}^*(r_n) R_{l_p}^*(r_p) j_{\lambda_n}(Q r_n/2) j_{\lambda_p}(Q r_p/2) \\ &\times (\gamma^2 r_n r_p)^{-1/2} I_{n+1/2}(4\gamma^2 r_n r_p) V_{\text{eff}} \exp[-2\gamma^2 (r_n^2 + r_p^2)] \end{aligned} \quad (20)$$

The r integration can be done under the already stated assumption that V_{eff} depends only on r_n and r_p :

$$\int dr \exp[-iK \cdot r - 4\gamma^2 r^2] = \left(\frac{\pi}{4\gamma^2}\right)^{3/2} \exp\left[-\frac{K^2}{16\gamma^2}\right] \quad (21)$$

Thus the angular distribution is multiplied by the factor $\exp[-K^2/8\gamma^2]$, which damps the cross section at larger angles. The larger the α radius (γ^{-1} increasing), the fewer high-momentum components are contained in the α particle; hence the more strongly the cross section is damped for large momentum transfer (large angle).

The angle integrals $(l'm' | Y_n^m | lm)$ can be done in the usual way (cf Rose).⁶ The sum on magnetic quantum numbers then becomes

$$\sum_{m_n, m_p, m} (-)^m C(l_n l_p L; m_n m_p M) C(l_n n \lambda_n; m_n - m 0) \times C(l_p n \lambda_p; m_p m 0) = \delta_{M0} (-)^n \sqrt{(2\lambda_n+1)(2\lambda_p+1)} \times W(l_n l_p \lambda_n \lambda_p; L n) C(\lambda_n \lambda_p L; 000) \quad (22)$$

Hence we find

$$F(l_n l_p L; M) \propto \delta_{M0} \sqrt{(2\lambda_n+1)(2\lambda_p+1)} \times \exp\left[-\frac{K^2}{16\gamma^2}\right] B(l_n l_p L; Q) \quad (23)$$

where

$$B(l_n l_p L; Q) = \sum_{\lambda_n \lambda_p n} (-)^n i^{\lambda_n + \lambda_p} \sqrt{(2\lambda_n+1)(2\lambda_p+1)} \times C(l_n n \lambda_n; 000) C(l_p n \lambda_p; 000) C(\lambda_n \lambda_p L; 000) \times W(l_n l_p \lambda_n \lambda_p; L n) R_{\lambda_n \lambda_p n} \quad (24)$$

(Note that the product of the three C-coefficients implies $(-)^L = (-)^{\lambda_n + \lambda_p}$.)

and

$$R_{\lambda_n \lambda_p n} = (2n+1) \int dr_n dr_p r_n^2 r_p^2 R_{\lambda_n}^*(r_n) R_{\lambda_p}^*(r_p) j_{\lambda_n}(QR_n/2) j_{\lambda_p}(QR_p/2) (\gamma^2 r_n r_p)^{-1/2} V_{\text{eff}} \exp[-2\gamma^2(r_n^2 + r_p^2)] I_{n+1/2}(4\gamma^2 r_n r_p) \quad (25)$$

Inserting the result of Eq. (23) into Eq. (10), we have

$$\begin{aligned} |H| &\propto \sqrt{(2l_n+1)(2l_p+1)} \exp\left[-\frac{k^2}{16\gamma^2}\right] C(110; \mu-\mu_0) \\ &\times C(J_i J_f; M_i, -\mu, M_f) \sum_L a_{Lij} C(Lij; 0, -\mu, \mu) \\ &\times B(l_n l_p L; Q) \end{aligned} \quad (26)$$

The angular distribution is

$$\begin{aligned} \frac{d\sigma}{d\Omega} &\propto \frac{1}{2J_i+1} \sum_{M_i M_f} |H|^2 = (2l_n+1)(2l_p+1) \frac{2J_f+1}{2J_i+1} \\ &\times \exp\left[-\frac{k^2}{8\gamma^2}\right] \sum_L \frac{1}{2L+1} C_L(e-e) |B(l_n l_p L; Q)|^2 \end{aligned} \quad (27)$$

$$C_L(e-e) = a_{Lij}^2 (J_n, J_p) \quad (28)$$

In practice it is convenient and reasonable to assume a surface interaction for the stripping process. Then the integral Eq. (25) becomes

$$R_{\lambda_n \lambda_p n} \propto (2n+1) I_{n+1/2}(4\gamma^2 R_0^2) j_{\lambda_n}(QR_0/2) j_{\lambda_p}(QR_0/2) \quad (29)$$

If, instead of a finite-size α particle we had assumed a point cluster, then according to Eq. (16) we would drop the dampings factor and replace

$$B(l_n l_p L; Q) \longrightarrow C(l_n l_p L; 000) j_L(QR_0) \quad (30)$$

and obtain

$$\frac{d\sigma}{d\Omega} \propto (2l_n+1)(2l_p+1) \frac{2J_f+1}{2J_i+1} \times \sum_L \frac{1}{2L+1} |a_{L1J}(j_n, j_p) C(l_n l_p L; 000) f_L(QR_0)|^2 \quad (31)$$

The selection rules on L are

$$l_n + l_p = L = J + 1$$

$$\pi_i \pi_f = (-)^{l_n + l_p} = (-)^L, \quad (32)$$

where π_i and π_f are the parities of the initial and final nucleus, while the selection rules on J are

$$j_n + j_p = J \quad \text{and} \quad J_i + J = J_f \quad (33)$$

B. Odd-odd Target Nucleus

We adopt the following model for (α, d) reactions from an odd-odd target nucleus. The target consists of a core of spin $J_C = 0$ which remains unchanged by the reaction. The odd neutron and proton are in spin-orbit states j_p and j_n and are coupled to form the spin of the target J_i :

$$J_C = 0, \quad j_n + j_p = J_i \quad (34)$$

The final nucleus is assumed to have one pair of like nucleons coupled to zero angular momentum, while the other pair is coupled to the final spin J_f . This suffices to describe the low-lying states and makes development easier. Whether it is the proton pair that is coupled to zero, or the neutron pair (or a superposition) will of course depend on the particular situation.

Thus the final nucleus is characterized by

$$\left. \begin{aligned} \underline{j}_p + \underline{j}_{p'} &= \underline{J}_f \\ \underline{j}_p + \underline{j}_n &= 0 \end{aligned} \right\} \text{ or } \left\{ \begin{aligned} \underline{j}_p + \underline{j}_{p'} &= 0 \\ \underline{j}_n + \underline{j}_{n'} &= \underline{J}_f \end{aligned} \right. \quad (35)$$

For $j_p = j_{p'}$, then $J_f = \text{even}$ only are allowed. Since the one scheme can be obtained from the other upon interchanging the indices $n \longleftrightarrow p$, we consider explicitly the left-hand one.

Denote the wave function for the core by Ψ_0 (A-2). Then the target nucleus wave function is written

$$\Psi_{J_i}^{M_i}(A) = \Psi_0(A-2) \Phi_{J_i}^{M_i}(r_n, r_{p'}) \quad (36)$$

where $\Phi_{J_i}^{M_i}$ denotes the following angular momentum coupling:

$$\Phi_{J_i}^{M_i} \equiv | j_n j_p ; J_i M_i \rangle \quad (37)$$

The residual nucleus wave function is written

$$\Psi_{J_f}^{M_f}(A+2) = \Psi_0(A-2) \Phi_0(r_n, r_{n'}) \Phi_{J_f}^{M_f}(r_p, r_{p'}) \quad (38)$$

Before we write down the matrix element, it is convenient to recouple the nucleons in the wave function for the residual nucleons, so that the pair n'p' are coupled together, as they are in the target (Eq. (36)). The recoupling is done by techniques due to Racah (cf. Edmonds).⁷ First we note

the identity

$$\begin{aligned} & \Phi_0^o(r_n, r_n') \Phi_{J_f}^{M_f}(r_p, r_p') \\ & \equiv \left| \left[j_n(r_n) j_n(r_n') \right] 0, \left[j_p'(r_p) j_p(r_p') \right] J_f; J_f M_f \right\rangle. \end{aligned} \quad (39)$$

We now use the recoupling coefficients to rewrite this

$$\begin{aligned} & \Phi_0^o(r_n, r_n') \Phi_{J_f}^{M_f}(r_p, r_p') = \sum_{I, I'} \sqrt{(2I+1)(2I'+1)(2J_f+1)} \\ & \times \left\{ \begin{matrix} j_n & j_p' & I' \\ j_n & j_p & I \\ 0 & J_f & J_f \end{matrix} \right\} \left| \left[j_n(r_n) j_p'(r_p) \right] I', \left[j_n(r_n') j_p(r_p') \right] I; J_f M_f \right\rangle \end{aligned} \quad (40)$$

The bracket { } denotes a 9-j symbol⁷ and reduces to a Racah coefficient

because of the 0:

$$\left\{ \begin{matrix} j_n & j_p' & I' \\ j_n & j_p & I \\ 0 & J_f & J_f \end{matrix} \right\} = \frac{(-)^{j_n + J_f - j_p' - I}}{\sqrt{(2j_n+1)(2J_f+1)}} W(I j_p I' j_p'; j_n J_f) \quad (41)$$

Using the above wave functions and the notation of the preceding section, we find after several steps that the matrix element for the transition can be written

$$\begin{aligned} & \mathcal{M}(M_i \rightarrow M_f, \mu) \propto \sum_{I'} \sqrt{(2J_i+1)(2I'+1)/(2j_n+1)} \\ & \times W(J_i j_p I j_p'; j_n J_f) C(I' J_i J_f; M_f - M_i, M_i) C(1 1 0; \mu, -\mu) \\ & \times \sum_L a_L I I' (j_n j_p') \sum_{M_L} C(L 1 I'; M_L, -\mu, M_f - M_i) \\ & \times F(l_n l_p' L M_L) \end{aligned} \quad (42)$$

where F was defined in the preceding section. The angular distribution can

readily be obtained:

$$\frac{d\sigma}{d\Omega} \propto \exp\left[-\frac{K^2}{8\gamma^2}\right] \frac{(2J_T+1)(2l_{n+1})(2l'_p+1)}{2j_{n+1}} \times \sum_L \frac{1}{2L+1} C_L(0-0) |B(l_n l'_p L; Q)|^2 \quad (43)$$

$$C_L(0-0) = \sum_{I=|L-1|}^{L+1} (2I+1) W^2(j_i j_p I j'_p; j_n j_T) a_{L I}^2(j_n j'_p) \quad (44)$$

where B has the same structure as before.

The selection rules on L are

$$l_n + l'_p = L = j_i + j_T + 1 = j_n + j'_p + 1$$

$$\pi_i \pi_T = (-)^{l_n + l'_p} = (-)^L \quad (45)$$

Because of the sum on I over the values allowed by the selection rules implied by the Racah coefficient, there can be more than the maximum of two allowed L's of the preceding section.

C. Even-Odd Target Nucleus

Consider, as an example, an odd-proton nucleus with an even-even core $J_C = 0$. Let the proton occupy the spin orbit state j_p' , so that the spin of the target is $J_i = j_p'$. The wave function is written

$$\Psi_{j_p'}^{m_p'}(A) = \Psi_0(A-1) \Phi_{j_p'}^{m_p'}(r_p') \quad (46)$$

In the residual nucleus the odd proton and the stripped proton couple to spin J , which is coupled to the spin j_n of the stripped neutron to form J_f .

$$j_p' + j_p = J, \quad J + j_n = J_f \quad (47)$$

If $j_p = j_p'$ only $J = \text{even}$ are allowed.

The wave function for the final nucleus is written

$$\Psi_{J_f}^{M_f}(A+2) = \Psi_0(A-1) [\chi_p(r_p) \chi_p'(r_p')]_{J, j_n; J_f M_f} \quad (48)$$

It is again convenient to recouple the particles so that the stripped pair of nucleons stands together. To do this we use the recoupling coefficients and obtain

$$| (j_p j_p') J, j_n; J_f M_f \rangle = \sum_{J'} \sqrt{(2J'+1)(2J+1)} \\ \times W(J' j_p' j_n J; J_f j_p) | (j_p j_n) J', j_p'; J_f M_f \rangle \quad (49)$$

We proceed in much the same manner as before and obtain finally

$$\frac{d\sigma}{d\Omega} \approx (2l_n+1)(2l_p+1) \frac{2J_f+1}{2j_p'+1} \exp\left[-\frac{k^2}{8\gamma^2}\right] \\ \times \sum_L \frac{1}{2L+1} C_L(e-0) | B(l_n l_p L; Q) |^2 \quad (50)$$

$$C_L(e-0) = (2J_f+1) \sum_{J'=L-1}^{L+1} (2J'+1) W(J' j_p' j_n J; J_f j_p)^2 a_{L J_f}^2(l_n j_p) \quad (51)$$

Discussion of the selection rules follows exactly as in the preceding section.

III. SELECTION RULES

The selection rules on the angular momentum L which characterizes the angular distribution for two-nucleon stripping reactions can be stated quite generally as

$$\pi_i \pi_f = (-)^{l_n + l_p} = (-)^L, \quad L = l_n + l_p,$$

$$\underline{J}_f = \underline{J}_i + \underline{L} + \underline{S}, \quad \underline{S} = \underline{S}_n + \underline{S}_p. \quad (52)$$

In addition, if the level that is formed is described in terms of the spin-orbit quantum numbers, then

$$\underline{j}_n + \underline{j}_p = \underline{L} + \underline{S}. \quad (53)$$

Here π_i and π_f are the parities of the initial and final nucleus, and l_n and l_p are the orbital angular momentum quantum numbers of the states into which the neutron and proton are captured.

The parity-selection rule on L is not an obvious one, because with the two angular momenta l_n and l_p one can form all states of total angular momentum $|l_n - l_p|$ to $l_n + l_p$. However, in the stripping reaction, only those components of a state that obey the stated parity selection rule can be excited. The reason for this is the following. Out of the plane wave describing the incident α particle, only that part contributes to the excitation of a level with given $L = l_n + l_p$ which has the two nucleons in the total orbital angular momentum state L . Now we may resolve L into the center-of-mass angular momentum Λ and relative angular momentum λ . Therefore the parity of the state describing the pair is $(-)^{\Lambda + \lambda}$ and has angular momentum $\underline{L} = \underline{\Lambda} + \underline{\lambda}$. However, in the incident nuclide the nucleons are in their lowest state, i.e., in s states. Hence $\lambda = 0$ and the connection of L with the parity is now clear.

The total spin S obeys the following selection rules for the various two-nucleon stripping reactions listed:

$$\begin{aligned}
 (\alpha, d) &\longrightarrow S = 1, \\
 (\text{He}^3, p) &\longrightarrow S = 0, 1, \\
 (t, p) &\longrightarrow S = 0.
 \end{aligned}
 \tag{54}$$

The first of these selection rules has been explained in section II-A. As for the (He^3, p) reaction, we note that since the He^3 has a spatially symmetric wave function, the two identical particles must be in a spin antisymmetric state (i.e., $S = 0$). Therefore the spin function for He^3 is $X_0^0(p, p') X_{1/2}^\mu(n)$ and the proton and neutron that are captured clearly are part of the time in a singlet state and part of the time in a triplet state in the He^3 . These selection rules are rigorous as long as there is no mechanism for flipping the spin of one of the captured nucleons alone.

IV. SUMMARY

We have derived expressions for the angular distribution of the (α, d) reaction in the plane-wave Born approximation. It is assumed that the α particle is stripped of a neutron and proton at the nuclear surface, since if the stripping occurred in the interior, the outgoing deuteron would likely be absorbed. Coupling schemes appropriate to the even-even, odd-odd, and even-odd target nuclei have been used in the extreme j - j limit. In all cases, the angular distribution is of the form

$$\frac{d\sigma}{d\Omega} \propto \exp\left[-\frac{k^2}{8\gamma^2}\right] \sum_L \frac{1}{2L+1} C_L |B_L(Q)|^2$$

The quantity B is given by Eqs. (24) and (29) for a finite-size α particle. In practice, however, B can be approximated by a much simpler expression:⁸

$$B_L(Q) \approx C(l_n l_p L; 000) j_L(QR_0). \quad (56)$$

This is exactly the form B would take for a point α particle. The damping factor in Eq. (55), which would be absent for a point α particle, should be retained, however, since we regard Eq. (56) as an approximation not as replacement of our assumption on the α structure.

The coefficients C_L in Eq. (55) have been explicitly calculated in the j-j limit for the three different coupling schemes used to describe even-even, odd-odd, and even-odd targets, and are given in Eqs. (28), (44), and (51). Departure from these coupling schemes would be reflected in changes in C_L and not in B. Thus for the extreme L-S scheme only one L would be allowed, and since we have not considered the possibility of spin flip, only triplet states could be excited.

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