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Robert Geske and Kuldeep Shastri

Abstract

It is thought that American options always gain value as the time to the option's expiration date increases. Merton (1973) proved this result using simple arbitrage arguments for options on non-dividend paying stocks. However, market prices reveal that (i) an American put can increase in value as calendar time passes diminishing the options time to expiration and (ii) two puts which differ only in expiration cycle can "sell" for the same price (i.e. a zero differential time premium). This paper demonstrates that dividend payments can cause this seemingly anomalous time premium behavior for American put options.

1. Introduction

Market makers and option traders observe that listed put options on stocks sometimes exhibit seemingly anomalous time premium behavior. As calendar time passes, ceteris paribus, the put price may be observed to increase. Furthermore, two puts which differ only by expiration cycle may be observed to sell for the same price. Such time premium behavior is thought to be anomalous because American put options are considered "wasting" assets, or assets whose option value diminishes with the passage of time. This paper explains that the time premium behavior for American put options can be attributed to dividends paid on the underlying stock.

It is well known that the value of an option is affected by time. Presumably, the longer one has the right to exercise, the more valuable the option. The Black-Scholes formula for a European option is dependent on the price of the underlying stock, the exercise price of the option, the expected volatility of the stock return over the life of the option, the risk-free interest rate relevant to the expiration date, and the time until the option expires. This European model assumes that the underlying stock return volatility and the risk-free interest rate are either constant or only change deterministically while the option is alive. Black-Scholes also assume that no dividends are paid to shareholders of the stock prior to the option's expiration.

Using dominance arguments, Merton (1973) proved that the time premium for either calls or puts decreases monotonically as time passess if the options are American and the stock pays no dividends. It is this property that gives rise to the description of an option as a "wasting" asset. However, for European puts Merton demonstrated that this time relation is ambiguous.[1]

The intuition underlying the monotonic behavior of the American put option's time premium is that an American option is a sequence of options to exercise at each instant if you have not exercised at the previous instant (i.e., compound options; see Geske-Johnson [1984]). Obviously, more time implies more sequential options and consequently more value. An alternate intuition is since options are insurance, the longer the coverage the higher the premium. The purpose of this paper is to analyze and explain how the behavior of the American put option time premium is effected by dividends paid on the underlying stock. We demonstrate that an American option is not always a "wasting" asset.

When analyzing the time premium behavior of American put options, we show that it is necessary to distinguish between differences in time to expiration arising because of passing calendar time or because of different expiration dates (i.e., cycles). This distinction is important because changing expiration dates keeps the amount of time to the next dividend constant, while changing calendar time simultaneously reduces the time to expiration and the time to the next dividend payment. Therefore, the impact of changing time to expiration on the value of American put options will vary depending on whether calendar time or the expiration cycle is changing.

2. The Time Premium

Time affects option values in several ways. When a riskless hedge is possible, time enters the valuation through the (i) interest rate, (ii) the

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[1] A put option-holder might prefer to exercise early but could not if the option is European. In this case an extension of time to expiration would further diminish the option value.

variance rate, and (iii) the dividend stream.[2] If a riskless hedge were not possible, time might also enter through (iv) the stock's expected return. The time premium is defined as the option's current market value less its parity value. This is also called the premium above parity. For a put (P) option, this time premium (TP) is

$$(1) \qquad TP_p = P - \max(0, X-S)$$

where P, S, and X represent the put, stock and exercise prices. At expiration, when the option has no time remaining, the time premium is zero.

Recall, Merton (1973) proved that when the underlying stock does not pay dividends, an American put option unambiguously decreases in value as time to expiration decreases. However, when the underlying stock pays dividends this result does not hold. Since time to expiration can be altered by changing calendar time or the expiration cycle, when the underlying stock pays dividends this distinction is important. The next two subsections illustrate this point.

2.1 Changes in Calendar Time

Figure 1 depicts the American put value versus time to expiration as calendar time changes. This would be equivalent to examining *Wall Street Journal* price quotations on "different days" when the only parameter that has changed is calendar time. The four curves represent four stock prices which range from \$20.00 to \$50.00 in increments of \$10.00. The exercise price, X =

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[2] Black-Scholes (1973) considered European options with no dividends, and Merton (1973) considered European and American options with no dividends. Roll (1977), Geske (1979), Whaley (1981), and Geske-Johnson (1984) derive values for American call and put options with dividends.

\$45.00, the volatility, $\sigma = 0.3$, the interest rate, $r = 5\%$ per annum, and a dividend of \$0.50 is paid quarterly at ½, 3½, and 6½ months prior to expiration. It is assumed that the dividend is suspended if the stock price is less than the dividend on the ex-dividend date. Figure 1 shows that an in-the-money American put may increase in value as calendar time passes, reducing time to expiration (and time to the ex-dividend date), while out-of-the-money American put is shown to decline in value.

Insert Figure 1

Figure 1 illustrates that if the stock price was \$20.00 an instant before the ex-dividend date (\$19.50 after), a put with exercise price of \$45.00 would sell for about \$25.50. As calendar time passes, ceteris paribus, in order to avoid riskless arbitrage opportunities, the put's value must rise by an amount equal to the next scheduled dividend payment by the next ex-dividend date. The reason for this rise in put value as passing calendar time reduces both time to the option's expiration date and time to the next ex-dividend date is the near ex-dividend stock price decline has a more positive effect on put value than the negative effect of diminished time to expiration.

2.2 Changes in Expiration Cycles

Figure 2 depicts the American put value versus time to expiration as the expiration cycle changes, holding calendar time constant. This is equivalent to examining Wall Street Journal price quotations on the "same day" for two put options which differ only by expiration cycle. All the parameters except the expiration date are identical to Figure 1. This figure demonstrates why some in-the-money puts may "sell" from the same price, i.e., have a negligible time premium within the \$1/16 discrete trading unit).

Insert Figure 2

Consider the case where the stock price is \$20.00. Two puts with different expiration cycles but with the same time until the next ex-dividend date will "sell" for the same price. The reason for this is both puts have a high probability of being exercised at the next ex-dividend date, and therefore have the same effective maturity. Traders who empirically observe this effective zero time premium for American put options with different expiration cycles are cautioned that this is not necessarily a time (or horizontal) spread opportunity.

Figures 1 and 2 imply the following partial derivatives.

$$(2) \qquad \frac{\partial P(S_t, X, T - t)}{\partial t} \gtreqless 0$$

and

$$(3) \qquad \frac{\partial P(S_t, X, T - t)}{\partial T} \geqq 0$$

where t is calendar time and T is the expiration date. The partial derivative of the American put value with respect to calendar time (e.g. (2)) is ambiguous, while the partial derivative with respect to the expiration cycle (e.g. (3)) is non-negative, but may be effectively zero. Since most put options are listed on dividend paying stock, it is important to distinguish between changing calendar time and the expiration cycle when discussing the time premium behavior of the American put.

In the above discussion, we focus on the behavior of the time premium for American put options assuming that the only parameter varying is the time to expiration by changing either calendar time or the expiration cycle. Variation in other parameters could also explain this phenomena. For example, since put option values are an increasing function of volatility and a decreasing function of interest rates, the time premium would also be effected by changes in expected volatility or interest rates.[3] However, if the seemingly anomalous behavior persists on a regular basis, it is more likely attributable to the steady passage of time to scheduled ex-dividend and expiration dates.

4. Summary

This paper establishes conditions when an American put is not a "wasting asset." The paper provides an explanation for the occasionally observed put price increases as passing calendar time reduces time to expiration. It is shown that this time premia behavior is necessary to reflect the change in the present value of an upcoming dividend. In addition, we show that puts with different expiration cycles may trade at the same effective price because they have the same effective maturity.

[3]As demonstrated, put options increase in value due to an approaching stock dividend payment only if they have a high probability of being exercised at the next ex-dividend date. Thus a similarly observed increase in the price of an out-of-the-money put may be more likely induced by a change (i.e. an increase) in expected volatility (assuming the stock price remains constant). On the other hand, if the put is in-the-money, then the increase could be caused by either factor. More specifically, if the change in put price is greater than the change in the present value of the upcoming dividend, then at least a portion of the price increase is likely to be due to a volatility change.

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Figure 1

PUT VALUE VS. TIME TO EXPIRATION

(Δ CURRENT TIME)

$X=\$45.00$, $r=5\%$, $\sigma=0.3$, $D=\$0.50$,
$t_D^*=0.5, 3.5, 6.5$ months, dividend suspended if $S<D$

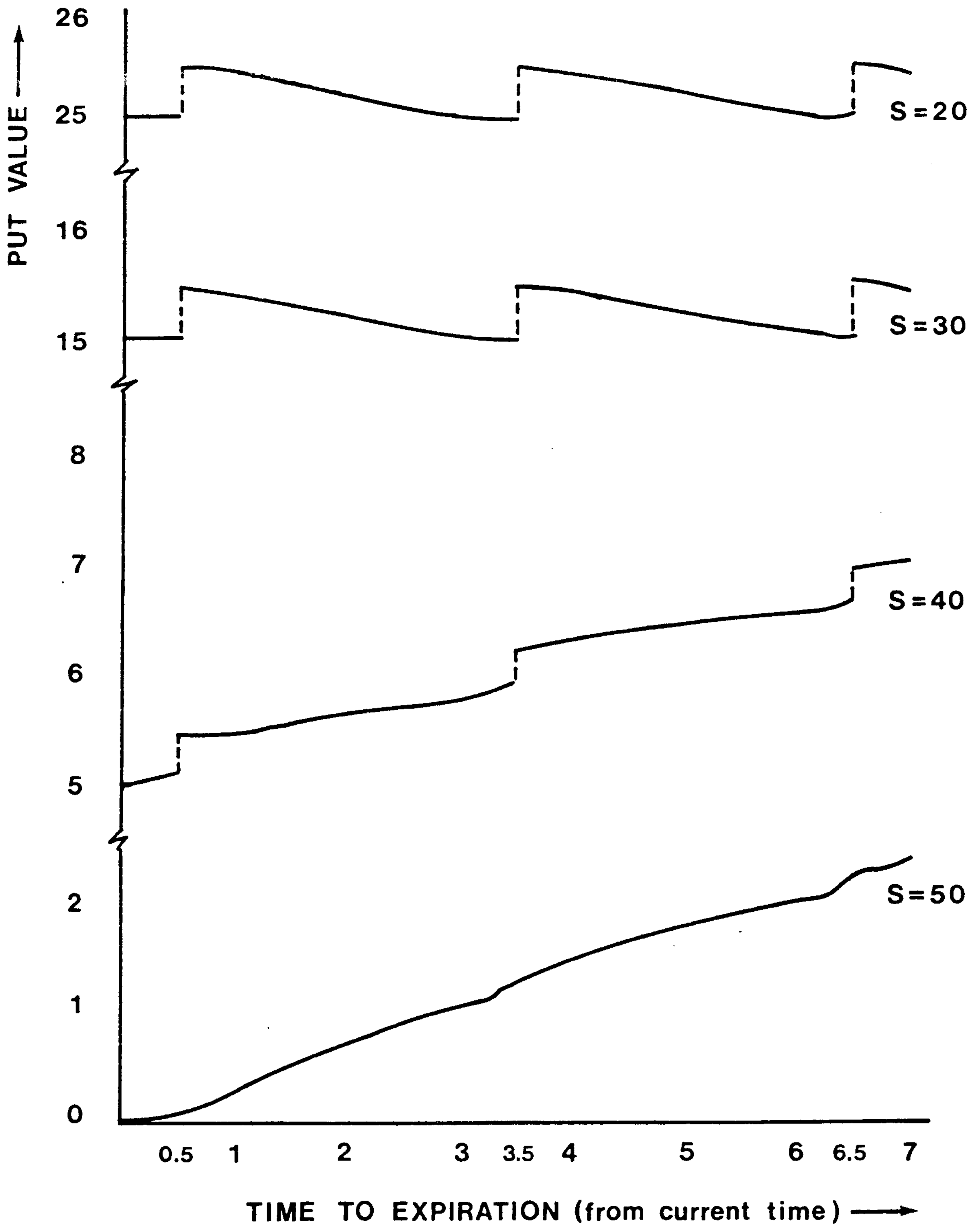

Figure 2

PUT VALUE VS. TIME TO EXPIRATION

(Δ EXPIRATION DATE)

X=\$45.00, r=5%, σ=0.3, D=\$0.50,
t_D=0.5, 3.5, 6.5 months, dividend suspended if S<D