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Laser cooling of electron beams via Thomson scattering

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Abstract

The laser radiative cooling of electron beams via Thomson scattering is discussed, analyzed, and simulated. As a beam radiates via Thomson scattering, it is subsequently cooled, i.e., the mean energy, normalized energy spread, and normalized emittance are reduced in a similar manner. An analytical expression is derived for the damping distance, and this is found to agree well with electron beam transport simulations. Although laser cooling can occur rapidly (on ps time scales), it does not occur indefinitely, due to the effects of quantum excitation from discrete photon emission. Quantum excitation places serious limitations on the minimum energy spread and normalized emittance that can be obtain by laser cooling.

I. INTRODUCTION

The interaction of intense laser pulses with electron beams has received considerable attention due to the possibility of producing short pulse x-rays by Thomson scattering [1-8]. In the laser synchrotron source [1,2], an intense laser pulse is backscattered from a relativistic electron beam to produce x-rays of frequency $\omega \simeq 4\gamma_0^2\omega_0$, where ω_0 is the laser frequency and γ_0 is the relativistic factor of the electrons. In the backscatter geometry, the x-ray pulse duration is approximately the electron bunch duration, which can be on the order of a ps if obtained from a photocathode RF gun. In the 90° scattering geometry [3-5], the x-ray pulse duration can be very short, approximately $\tau_L + 2r_b/c$, where τ_L is the laser pulse duration and r_b the electron beam radius. In the 90° geometry, the x-ray frequency is reduced $\omega \simeq 2\gamma^2\omega_0$ as is the x-ray flux due to a decreased interaction time. The generation of sub-ps x-ray pulses from 90° Thomson scattering has been demonstrated by W.P. Leemans et al. [6,7]. More recently, Thomson backscattered x-rays have been observed by several groups [8].

One consequence of Thomson scattering is the laser radiative cooling of the electron beam. Laser radiative cooling was first proposed and analyzed by Sprangle and Esarey [9-11], and more recently by Telnov [12], as well as and Huang and Ruth [13]. As an electron radiates, it loses energy. The total energy radiated by a single electron scales as the square of the electron energy $(\gamma mc^2)^2$. Hence, an electron with a higher energy radiates more than one with a lower energy. Consequently the energy spread of an electron beam is reduced as it radiates (see Fig. 1). In fact, the electron beam mean energy $\langle\gamma\rangle$, the normalized energy spread $\sigma_\gamma/\langle\gamma\rangle$, and the normalized emittance ϵ_n are all reduced in a similar manner. This cooling can not occur indefinitely, however, due to quantum fluctuations. Since photons are emitted in discrete quantum, the electrons receive discrete momentum kicks. This leads to a finite growth in both the electron beam energy spread and normalized emittance and ultimately limits the cooling process. The limits imposed by quantum fluctuations can be significant, since the energy of the emitted photons, and consequently the momentum recoil, can be relatively large.

In the following, laser cooling of electron beams is analyzed for the case of Thomson backscattering. The incident laser field is assumed to be a circularly polarized, 1D plane wave, i.e., the laser radius is assumed to be large compared to the electron beam radius. The cooling process is analyzed in the classical limit of Thomson scattering [2], which assumes that the scattered photon energy is small compared to the electron energy, i.e., $\hbar\omega \ll \gamma_0 mc^2$ or $\gamma_0 < \lambda_0/4\lambda_C$, where $\lambda_0 = 2\pi c/\omega_0$ is the laser wavelength and $\lambda_C = h/mc = 2.43 \times 10^{-10}$ cm is the Compton wavelength. For an laser wavelength on the order $\lambda_0 \simeq 1$ μm, this implies $\gamma_0 < 10^5$, i.e., electron energies less than 50 GeV. Quantum mechanical arguments will be invoked to determine limitations imposed by quantum fluctuations.

II. ELECTRON MOTION IN INTENSE LASER FIELDS

The laser field and space charge field of the electrons can be represented using the normalized vector and scalar potentials, $\mathbf{a} = e\mathbf{A}/mc^2$ and $\hat{\Phi} = e\Phi/mc^2$, respectively. In the Coulomb gauge, $\nabla \cdot \mathbf{a} = 0$ implies $a_z = 0$ in 1D. Then, $a_\perp$ represents the laser field and $\hat{\Phi}$ represents the space-charge field. The normalized vector potential of a circularly polarized laser is

$$\mathrm{a} = a_0(\cos k_0\eta \; \mathrm{e}_x + \sin k_0\eta \; \mathrm{e}_y), \tag{1}$$

where $k_0 = 2\pi/\lambda_0$ is the wavenumber of the laser field and $\eta = z + ct$. Here, the laser strength parameter a_0 is related to the laser intensity I by

$$a_0^2 \simeq 3.66 \times 10^{-19} \lambda_0^2[\mu\mathrm{m}] I[\mathrm{W/cm^2}]. \tag{2}$$

In the following, the laser field is assumed to be moving to the left ($-z$ direction) and the electrons are initially (prior to the interaction with the laser field) moving to the right ($+z$ direction) with an initial axial velocity $v_z = v_0 = c\beta_0$.

The electron motion in the fields $\mathbf{a}$ and $\hat{\Phi}$ is governed by the relativistic Lorentz equation, which may be written in the form

$$\frac{1}{c}\frac{d}{dt}\mathbf{u} = \nabla\hat{\Phi} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{a} - \boldsymbol{\beta} \times (\nabla \times \mathbf{a}), \tag{3}$$

where $\boldsymbol{\beta} = \mathbf{v}/c$ is the normalized electron velocity, $\mathbf{u} = \mathbf{p}/mc = \gamma\boldsymbol{\beta}$ is the normalized electron momentum, and $\gamma = (1+u^2)^{1/2} = (1-\beta^2)^{-1/2}$ is the relativistic factor. Assuming that the laser field, $\mathbf{a}_\perp$, and hence the quantities $\hat{\Phi}$, $\boldsymbol{\beta}$, $\mathbf{u}$, and γ, are functions only of the variable $\eta = z + ct$, Eq. (2) implies the existence of two constants of the motion [2]: $d(\mathbf{u}_\perp - \mathbf{a}_\perp)/d\eta = 0$ and $d(\gamma + u_z - \hat{\Phi})/d\eta = 0$. These can be integrated to give

$$\mathbf{u}_\perp = \mathbf{a}_\perp, \tag{4}$$

$$\gamma + u_z - \hat{\Phi} = \gamma_0(1 + \beta_0), \tag{5}$$

where, prior to the laser interaction ($\mathbf{a}_\perp = 0$), $\mathbf{u}_\perp = \hat{\Phi} = 0$, $\gamma = \gamma_0$ and $u_z = \gamma_0\beta_0$ have been assumed. Equation (4) is conservation of canonical transverse momentum in 1D, and Eq. (5) can be interpreted as conservation of energy in the wave frame. Theses two constants of the motion completely describe the nonlinear motion of electrons in the potentials $\mathbf{a}$ and $\hat{\Phi}$. They allow the electron motion to be specified solely in terms of the fields [2],

$$\beta_z = \frac{h_0^2 - (1+a^2)}{h_0^2 + (1+a^2)}, \tag{6}$$

$$\gamma = (h_0^2 + 1 + a^2)/2h_0, \tag{7}$$

$$\beta_\perp = \mathbf{a}_\perp/\gamma, \tag{8}$$

where $h_0 = \gamma_0(1+\beta_0) + \hat{\Phi}$. For an electron beam interacting with a laser with pulse length τ_L, it can be shown [2] that the space charge potential $\hat{\Phi}$ can be neglected provided $\tau_L \ll (c\hat{k}_p)^{-1}$, where $\hat{k}_p = k_p/\gamma_0^{3/2}(1+\beta_0)$, $k_p = (4\pi n_b e^2/mc^2)^{1/2}$, and n_b is the electron beam density. For laser pulse lengths $\tau_L \sim 1$ ps, this implies $n_b/\gamma_0^3 \ll 10^{16}$ cm^{-3}.

III. SCATTERED RADIATION

The power radiated by a single electron, P_s, undergoing relativistic quiver motion in an intense laser field can be calculated from the relativistic Larmor formula [14]

$$P_s = \frac{2e^2}{3c}\gamma^2 \left[\left(\frac{d\mathbf{u}}{dt}\right)^2 - \left(\frac{d\gamma}{dt}\right)^2 \right]. \tag{9}$$

Assuming the electron orbit is a function of only the variable $\eta = z + ct$ implies

$$P_s = \frac{2}{3}e^2 c(\gamma + u_z)^2 \left[\left(\frac{d\mathbf{u}}{d\eta}\right)^2 - \left(\frac{d\gamma}{d\eta}\right)^2 \right]. \tag{10}$$

Using the orbits described in Sec. II, the power radiated by an electron in the presence of a circularly polarized laser field is given by [2]

$$P_s \simeq \frac{2}{3}e^2 c(1+\beta_0)^2 \gamma_0^2 k_0^2 a_0^2. \tag{11}$$

This expression is valid for arbitrary values of a_0. Notice that $P_s \sim \gamma_0^2$, i.e., electrons with a higher energy radiate more power.

The frequency of the scattered photons is given by [2]

$$\omega = \frac{n_n 4\gamma_0^2\omega_0}{(1 + a_0^2 + \gamma_0^2\theta^2)}, \tag{12}$$

assuming $\gamma_0^2 \gg (1 + a_0^2)$ and $\theta^2 \ll 1$, where θ is the scattering angle with respect to the electron beam axis. Here, n_n is the harmonic number [2]. In the low intensity limit $a_0^2 \ll 1$, radiation is scattered only at the fundamental $n_n = 1$. In the nonlinear limit $a_0^2 \gg 1$, numerous harmonics are generated. This results in a near continuum of scattered radiation with harmonics extending out to a critical harmonic number $n_{cr} = 3a_0^3$, beyond which the intensity of the scattered radiation rapidly decreases. Moreover, the intensity of the scattered radiation is peaked about a cone angle of $\theta = \theta_0 \simeq \gamma_\perp/\gamma_0$, where $\gamma_\perp = (1+a_0^2)^{1/2}$. It useful to define the critical frequency ω_{cr} given by Eq. (12) evaluated at $n_n = n_{cr}$ and $\theta = \theta_0$. Hence,

$$\omega_{cr} = \alpha_{cr} 4\gamma_0^2\omega_0, \tag{13}$$

where $\alpha_{cr} \simeq 1$ for $a_0^2 \ll 1$ and $\alpha_{cr} \simeq 3a_0/2$ for $a_0^2 \gg 1$. When $a_0^2 \gg 1$, half the energy is radiated at frequencies $\omega < \omega_{cr}/2$ and half at $\omega > \omega_{cr}/2$ with the peak intensity occurring at $\omega \simeq \omega_{cr}$.

IV. RADIATION REACTION FORCE

As an electron radiates, it's motion will be altered, which in the classical limit is described by the radiation reaction force $\mathbf{F}_R$. The relativistic equation of motion is given by $mc d\mathbf{u}/dt = \mathbf{F}_L + \mathbf{F}_R$, where $\mathbf{F}_L/mc^2$ is given by the right side of Eq. (3) and [14]

$$\mathbf{F}_R = mc\tau_R \left\{ \frac{d}{dt}\left(\gamma \frac{d\mathbf{u}}{dt}\right) + \gamma \mathbf{u} \left[\left(\frac{d\gamma}{dt}\right)^2 - \left(\frac{d\mathbf{u}}{dt}\right)^2 \right] \right\}, \tag{14}$$

with $\tau_R = 2e^2/3mc^3 = 2r_e/3c$, where $r_e = e^2/mc^2 = 2.82 \times 10^{-13}$ cm is the classical electron radius. Using the independent variable $\eta = z + ct$ along with the approximation $(d\gamma/dt)^2 \simeq (du_z/dt)^2$ gives

$$\mathbf{F}_R \simeq mc^3\tau_R(1+\beta_z)^2\gamma \left[\frac{d^2\mathbf{u}}{d\eta^2} - \mathbf{u}\left(\frac{d\mathbf{u}_\perp}{d\eta}\right)^2 \right], \tag{15}$$

where terms of order $\gamma_0(1+a_0^2)r_e/c\tau_L \ll 1$ and of order $\gamma_0 k_0 r_e \ll 1$ have been neglected and $\gamma \gg (1+a_0^2)^{1/2}$ has been assumed. Noting that $\mathbf{u}_\perp \simeq \mathbf{a}_\perp$, the fast (on the laser frequency time scale, denoted by the subscript f) and slow (averaged over the fast laser frequency, denoted by the subscript s) components of the radiation reaction force are given by

$$\mathbf{F}_{Rf} \simeq -mc^3\tau_R(1+\beta_z)^2\gamma k_0^2(1+a_0^2)\mathbf{a}_\perp. \tag{16}$$

$$\mathbf{F}_{Rs} \simeq -mc^3\tau_R(1+\beta_z)^2\gamma k_0^2 a_0^2 \mathbf{u}_s, \tag{17}$$

assuming $k_0^2 a_0^2 \gg 1/c^2\tau_L^2$.

Adding $\mathbf{F}_R/mc^2$ onto the right of Eq. (3) allows the electron motion to be studied in the presence of radiation damping. For example, on the fast time scale the transverse momentum is given by $\mathbf{u}_\perp = \mathbf{a}_\perp + \delta\mathbf{u}_\perp$, where

$$\delta\mathbf{u}_\perp \simeq ic\tau_R k_0 \gamma(1+\beta_z)(1+a_0^2)\mathbf{a}_\perp. \tag{18}$$

Hence $|\delta u_\perp/a_\perp| \ll 1$ provided $(4/3)k_0 r_e \gamma(1+a_0^2) \ll 1$ and the approximation $\mathbf{u}_\perp \simeq \mathbf{a}_\perp$ holds on the fast time scale in the presence of radiation damping.

V. LASER COOLING

Using Eq. (3) including the radiation reaction force, the quantity $\gamma + u_z$ evolves via

$$d(\gamma + u_z)/d\eta \simeq -c\tau_R k_0^2 a_0^2 \beta_z (\gamma + u_z)^2. \tag{19}$$

In the absence of radiation damping $\gamma + u_z$ is a constant. For highly relativistic beams, $\gamma \gg (1 + a_0^2)^{1/2}$, the electron energy decreases according to [9,10]

$$\gamma \simeq \frac{\gamma_0}{(1 + z/L_R)}, \tag{20}$$

where the radiation damping length is given by

$$L_R = (4c\tau_R \gamma_0 k_0^2 a_0^2)^{-1}, \tag{21}$$

which is valid for arbitrary values of a_0. This result can also be obtained by setting the rate of energy loss equal to the total power radiated via Thomson scattering, i.e., $mc^2 d\gamma/dt = -P_s$, where P_s is given by Eq. (11). In practical units,

$$L_R[\text{cm}] \simeq \frac{337\lambda_0^2[\mu\text{m}]}{a_0^2 \gamma_0} \simeq \frac{4.71 \times 10^{20}}{I[\text{W/cm}^2] E_{b0}[\text{MeV}]}, \tag{22}$$

where $E_{b0} \simeq \gamma_0 mc^2$ is the initial electron beam energy. The radiation damping length can be very short, e.g., $L_R \simeq 300$ μm ($L_R/c \simeq 1$ ps) for an initial beam energy of 200 MeV ($\gamma_0 = 400$) and a $\lambda_0 = 1$ μm laser of intensity $I = 7.7 \times 10^{19}$ W/cm^2 ($a_0 = 5.3$). In practice, for a single laser pulse, the interaction length will be limited to the smaller of Z_R or $c\tau_L/2$, where $Z_R = \pi r_0^2/\lambda_0$ is the Rayleigh (diffraction) length and r_0 is the spot size at the laser focus.

The electron beam energy spread can be estimated by assuming Eq. (20) holds for each electron in the beam, i.e., $\gamma = \gamma_0/(1 + c_0\gamma_0 z)$, where $c_0 = (\gamma_0 L_R)^{-1}$ and γ_0 is the initial energy for a particular electron. Letting $\gamma_0 = \langle\gamma_0\rangle + \delta\gamma_0$, expanding about $\langle\gamma_0\rangle$, and averaging over an ensemble of electrons (the angular brackets denotes an ensemble average) indicates that

$$\langle\gamma\rangle \simeq \frac{\langle\gamma_0\rangle}{(1+c_0\langle\gamma_0\rangle z)}\left[1 - \frac{c_0\langle\gamma_0\rangle z}{(1+c_0\langle\gamma_0\rangle z)^2}\frac{\langle\delta\gamma_0^2\rangle}{\langle\gamma_0\rangle^2}\right], \tag{23}$$

i.e., the mean energy decreases as $\langle\gamma\rangle \simeq \langle\gamma_0\rangle/(1+z/L_R)$, assuming the initial energy spread is small, $\langle\delta\gamma_0^2\rangle/\langle\gamma_0\rangle^2 \ll 1$. Furthermore, the energy spread of the beam is given by

$$\sigma_\gamma = \left(\langle\gamma^2\rangle - \langle\gamma\rangle^2\right)^{1/2} \simeq \frac{\langle\delta\gamma_0^2\rangle^{1/2}}{(1+c_0\langle\gamma_0\rangle z)^2}. \tag{24}$$

Hence, the normalized beam energy spread decreases as [9,10]

$$\sigma_\gamma/\langle\gamma\rangle \simeq (\sigma_{\gamma 0}/\langle\gamma_0\rangle)/(1+z/L_R). \tag{25}$$

The electron beam envelope behavior can be studied via the equation of motion for the time-averaged (averaged over the fast laser frequency) transverse electron orbit $\tilde{x}$, which in the highly relativistic regime, $\gamma \gg (1+a_0^2)^{1/2}$, is given by

$$\tilde{x}'' + K_B^2\tilde{x} = -(\gamma'/\gamma + \nu_\perp)\tilde{x}', \tag{26}$$

where $d\tilde{x}/dct = \beta_z d\tilde{x}/dz = \beta_x$ is the transverse electron velocity, the prime denotes d/dz, and K_B is the betatron wavenumber due to any external linear focusing forces. Also, $\nu_\perp$ is from the time-averaged (slow) radiation reaction force $F_{Rs\perp} = -\nu_\perp mc^2 u_\perp$, i.e.,

$$\nu_\perp \simeq 4c\tau_R k_0^2 a_0^2 \gamma. \tag{27}$$

A similar equation applies for the orbit $\tilde{y}$. An equation for evolution of the electron beam radius $r_b = \langle\tilde{x}^2 + \tilde{y}^2\rangle^{1/2}$ can be obtained by averaging over an ensemble of particles [15]

$$r_b'' + (\gamma'/\gamma + \nu_\perp)r_b' + K_B^2 r_b = (\epsilon_n^2 + \gamma^2 L_e^2)\gamma^{-2} r_b^{-3}, \tag{28}$$

where $\epsilon_n^2 = \gamma^2 r_b^2 \langle \delta\beta_\perp^2 \rangle$ is the normalized beam emittance, $L_e = \langle \tilde{x}'\tilde{y} - \tilde{x}\tilde{y}' \rangle$ is the average angular momentum, and $\delta\boldsymbol{\beta}_\perp = \boldsymbol{\beta}_\perp - \langle \boldsymbol{\beta}_\perp \rangle$, i.e., $\boldsymbol{\beta}_\perp = (r_b'/r_b)\mathbf{e}_r + (L_e r/r_b^2)\mathbf{e}_\theta + \delta\boldsymbol{\beta}_\perp$. Furthermore, the normalized emittance and average angular momentum evolve via [15]

$$\epsilon_n = \epsilon_{n0} \exp\left(- \int_0^z dz \nu_\perp \right), \tag{29}$$

$$L_e = (\gamma_0/\gamma) L_{e0} \exp\left(- \int_0^z dz \nu_\perp \right), \tag{30}$$

where the subscript zero refers to the initial ($z = 0$) value. Using the definition of $\nu_\perp$ and the result $\gamma = \gamma_0(1 + z/L_R)$ where $L_R^{-1} \simeq \nu_\perp \gamma_0/\gamma$ indicates [9,10]

$$\epsilon_n \simeq \frac{\epsilon_{n0}}{(1 + z/L_R)}, \tag{31}$$

and $L \simeq L_{e0}$. Note that in the absence of focusing ($K_B = 0$) and for an emittance dominated beam with $\epsilon_n^2 \gg \gamma^2 L_e^2$, the electron beam radius increases $r_b = r_{b0}(1 + z^2\gamma_0^2/\epsilon_{n0}^2)^{1/2}$.

VI. QUANTUM EXCITATION

The above results for laser cooling are based on a purely classical analysis and neglect effects arising from the discrete nature of the photon emission process. When a photon is emitted, an electron receives a discrete change in momentum and energy. The effect of quantum fluctuations on the beam energy spread can be estimated as follows [10,12,13,16]. The change in the electron energy $\delta\gamma$ after emitting a photon is $\delta\gamma \simeq (\hbar\omega/\gamma mc^2)\gamma$. After emitting N_p photons, the total change in the electron energy spread σ_γ is given by $\Delta\sigma_\gamma^2 \simeq N_p\langle \delta\gamma^2 \rangle$, where $\langle \delta\gamma^2 \rangle \simeq (\hbar\omega_{cr}/mc^2)^2$ and $\hbar\omega_{cr} = \alpha_{cr}(4\gamma^2\hbar\omega_0)$ is the critical photon energy. For a radiative electron energy loss of $\Delta\gamma mc^2$, the number of photons emitted is $N_p \simeq \Delta\gamma mc^2/\hbar\omega_{cr}$. Hence,

$$\Delta\sigma_\gamma^2 \simeq \hbar\omega_{cr}\Delta\gamma/mc^2. \tag{32}$$

On the other hand, the rate of energy spread reduction by radiative cooling is given by Eq. (24), i.e., $\Delta\sigma_\gamma^2 \simeq 4\sigma_\gamma^2\Delta\gamma/\gamma$. Balancing the increase due to quantum excitation with the decrease due to radiative cooling gives the minimum energy spread

$$(\sigma_\gamma/\gamma)_{min} \simeq (\alpha_{cr}\lambda_C\gamma/\lambda_0)^{1/2}, \tag{33}$$

where $\lambda_C = h/mc$.

The effect of quantum excitation on the normalized beam emittance can be estimated as follows [10,12,13,16]. The change in the electron angle $\delta\psi$ is related to the photon scattering angle θ by $\delta\psi \simeq \hbar\omega\theta/\gamma mc^2$. After N_p photon emissions, the total change in the electron angle is $\Delta\langle\psi^2\rangle \simeq N_p\langle\delta\psi^2\rangle$, where $\langle\delta\psi^2\rangle \simeq (\hbar\omega_{cr}/\gamma mc^2)^2\langle\theta^2\rangle$, $N_p \simeq \Delta\gamma mc^2/\hbar\omega_{cr}$, and the average scattering angle is $\langle\theta^2\rangle \simeq (\gamma_\perp/\gamma)^2/2$. Using the definition of normalized emittance $\epsilon_n \simeq \gamma\beta^*\langle\psi^2\rangle/4$, where β^* is the beta-function of the beam, the change in normalized emittance due quantum excitation from an emission of $\Delta\gamma$ in energy is

$$\Delta\epsilon_n \simeq \frac{\alpha_{cr}\lambda_C\Delta\gamma}{2\lambda_0\gamma}\gamma_\perp^2\beta^*. \tag{34}$$

Balancing this against the change in normalized emittance due to radiative losses, Eq. (31), i.e., $\Delta\epsilon_n \simeq (\epsilon_n/\gamma)\Delta\gamma$, gives a minimum normalized emittance of

$$\epsilon_{n,min} \simeq \alpha_{cr}\gamma_\perp^2\beta^*\lambda_C/2\lambda_0. \tag{35}$$

For a rapid laser cooling time of $L_R/c = 1$ ps with $\gamma_0 = 400$, $\lambda_0 = 1$ μm, and $a_0 = 5.3$, the limits imposed by quantum fluctuations are $(\sigma_\gamma/\gamma)_{min} \simeq 0.09$ and $\epsilon_{n,min} \simeq 3$ mm-mrad (assuming $\beta^* = 1$ cm).

VII. SIMULATION

To simulate the classical effects of laser cooling, a particle-in-cell electron beam transport code was used [10,11]. In this simulation, the laser fields were specified analytically and the radiation reaction force was included via Eq. (14). The parameters used in the simulation were an initial beam energy of 200 MeV ($\gamma_0 = 400$), a beam current of 200 A, an initial beam radius of 50 μm, an initial normalized emittance of $\epsilon_{n0} = 6$ mm-mrad, a laser wavelength of $\lambda_0 = 1$ μm, a laser pulse duration of $\tau_L = 2$ ps, and a peak intensity of $I = 7.7 \times 10^{19}$ W/cm^2 ($a_0 = 5.3$). For these parameters the theoretical damping length is $L_R \simeq 300$ μm ($L_R/c \simeq 1$ ps). Figure 2 shows the transverse phase space (p_x, x) of the electrons, (a) initially at $z = 0$ and (b) after propagating $z = 300$ μm. Note that the momentum p_x width of the distribution has diminished (due to γ decreasing), whereas the x width has remained unchanged. Figure 3 shows (a) the mean electron energy $\bar{\gamma}$ and (b) the normalized emittance as a function of propagation distance. These curves are in excellent agreement with the theoretically predicted decrease $(1 + z/L_R)^{-1}$.

VIII. DISCUSSION

As a beam radiates via Thomson scattering, it is subsequently cooled, i.e., the mean energy, energy spread, and normalized emittance are reduced. Physically, cooling is the result of the fact that the total power radiated by a single electron scales as γ^2. Hence, higher energy electrons radiate more than lower energy electrons and the energy distribution is narrowed. In the classical limit, these effects can be analyzed by including the radiation reaction force into the relativistic equation of motion. It is found that the mean energy $\bar{\gamma}$, normalized energy spread $\sigma_\gamma/\bar{\gamma}$, and normalized emittance ϵ_n all decrease algebraically via $(1+z/L_R)^{-1}$, where $L_R \simeq (\gamma_0 a_0^2)^{-1}$ is given by Eqs. (21) and (22). For high beam energies γ_0 and/or high laser intensities a_0^2, this cooling can be very rapid, i.e., on the order of picoseconds for $a_0 \simeq 5$ and $\gamma_0 \simeq 400$.

Quantum fluctuation impose serious limitations to the laser cooling process. The fact that photons are emitted in discrete quanta limit the minimum energy spread and normalized emittance that can be achieved, the values of which are given by Eqs. (33) and (35), respectively. Notice that $(\sigma_\gamma/\bar{\gamma})_{min}$ and $\epsilon_{n,min}$ scale as $(\alpha_{cr}\gamma/\lambda_0)^{1/2}$ and $(\alpha_{cr}\gamma_\perp/\lambda_0)$, respectively. Hence, these limits are greater at high intensity $a_0^2 \gg 1$ where $\alpha_{cr} \simeq 3a_0/2$ and $\gamma_\perp \simeq a_0$. More importantly, these limits are greater at shorter laser wavelengths λ_0. Whereas short wavelengths are beneficial to generating short wavelength x-rays via Thomson scattering, i.e., $\lambda = \lambda_0/4\gamma^2$, they are detrimental to laser cooling. Since short laser wavelengths lead to very high energy x-ray generation, the momentum kick given to the electron upon photon emission is also relatively high, which leads to correspondingly high limits on the minimum obtainable beam energy spread and normalized emittance. For a rapid laser cooling time of $L_R/c = 1$ ps with $\gamma_0 = 400$, $\lambda_0 = 1$ μm, and $a_0 = 5.3$, the limits imposed by quantum fluctuations are $(\sigma_\gamma/\gamma)_{min} \simeq 0.09$ and $\epsilon_{n,min} \simeq 3$ mm-mrad (assuming $\beta^* = 1$ cm).

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REFERENCES

[1] P. Sprangle, A. Ting, E. Esarey and A. Fisher, J. Appl. Phys. 72 (1992) 5032.

[2] E. Esarey, S.K. Ride and P. Sprangle, Phys. Rev. E 48 (1993) 3003.

[3] K.J. Kim, S. Chattopadhyay and C.V. Shank, Nucl. Instr. and Meth. A 341 (1994) 351.

[4] S.K. Ride, E Esarey and M. Baine. Phys. Rev. E 52 (1995) 5425.

[5] E. Esarey, S.K. Ride, M. Baine, A. Ting and P. Sprangle, Nucl. Instr. and Meth. A 375 (1996) 545.

[6] R.W. Schoenlein, W.P. Leemans, A.H. Chin, P. Volfbeyn, T.E. Glover, P. Balling, M. Zolotorev, K.J. Kim, S. Chattopadhyay and C.V. Shank, Science 274 (1996) 236.

[7] W.P. Leemans, R.W. Schoenlein, P. Volfbeyn, A.H. Chin, T.E. Glover, P. Balling, M. Zolotorev, K.J. Kim, S. Chattopadhyay and C.V. Shank, Phys. Rev. Lett. 77 (1996) 4182; IEEE J. Quantum Electron. 33 (1997) 1925.

[8] See, e.g., other articles in this issue.

[9] P. Sprangle and E. Esarey, Phys. Fluids B 4 (1992) 2241.

[10] E. Esarey, P. Sprangle and G. Joyce, Bull. Amer. Phys. Soc. 37 (1992) 991.

[11] G. Joyce, E. Esarey and P. Sprangle, unpublished.

[12] V. Telnov, Phys. Rev. Lett. 78 (1997) 4757.

[13] Z. Huang and R.D. Ruth, Phys. Rev. Lett. 80 (1998) 976.

[14] J.D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975), Chaps. 14 and 17.

[15] A.C. Ting and P. Sprangle, Particle Accelerators 22 (1987) 149

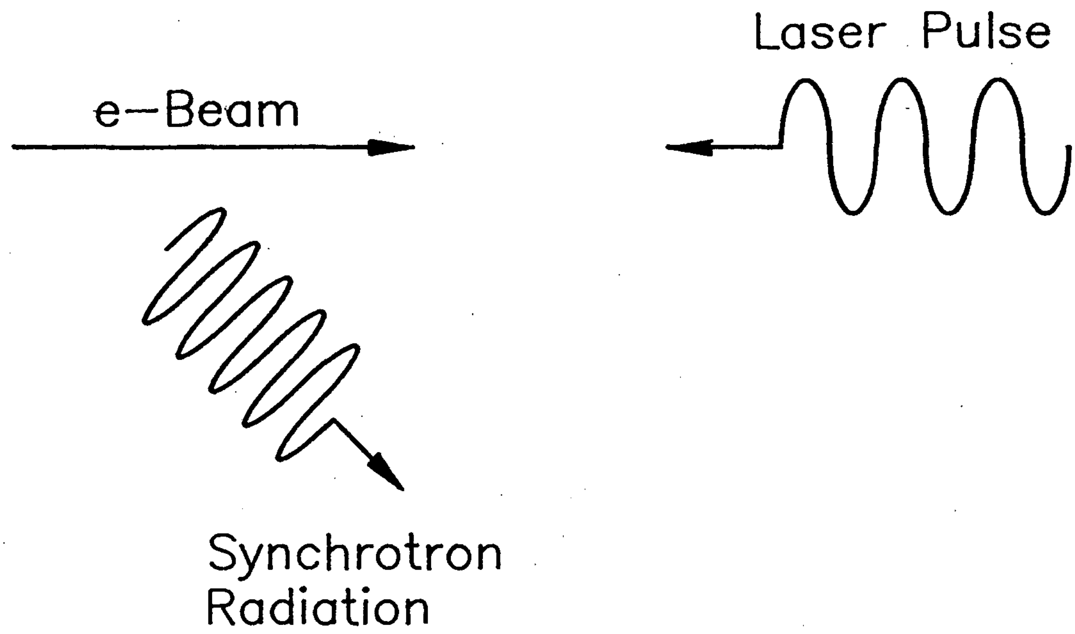
[16] M. Sands, Stanford Linear Accelerator Center Report No. 121, 1970.

FIGURE CAPTIONS

FIG. 1: Schematics of (a) Thomson scattering, (b) laser radiative cooling of the energy distribution, and (c) reduction of transverse phase space (the outer circle is the initial phase space, the middle ellipse is final phase space, and the inner Elise is the limit due to quantum fluctuations).

FIG. 2: Transverse phase space, p_x (normalized to mc) versus x (cm), of the electrons (a) initially at $z = 0$ and (b) after propagating $z = 300$ μm from a simulation with $\gamma_0 = 400$, a beam current of 200 A, a beam radius of 50 μm, $\epsilon_{n0} = 6$ mm-mrad, $\lambda_0 = 1$ μm, $\tau_L = 2$ ps, and $a_0 = 5.3$. The theoretical damping length is $L_R \simeq 300$ μm ($L_R/c \simeq 1$ ps).

FIG. 3: (a) Mean electron energy $\bar{\gamma}$ and (b) normalized emittance as a function of propagation distance for the parameters of Fig. 2. These curves are in excellent agreement with the theoretically predicted decrease $(1 + z/L_R)^{-1}$.


Laser Pulse
e—Beam
Synchrotron
Radiation

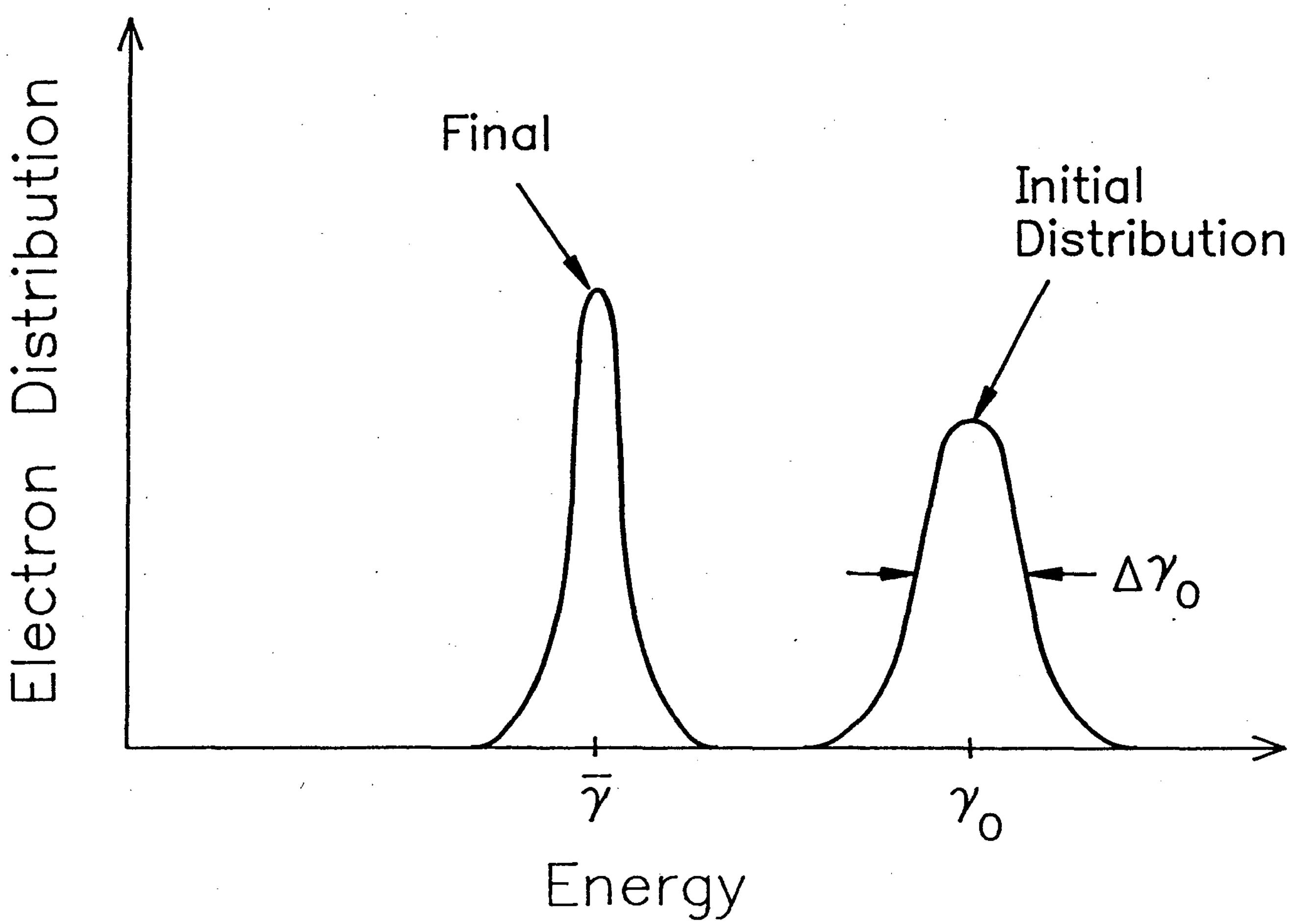

Electron Distribution
Final
Initial
Distribution
$\Delta\gamma_0$
$\bar{\gamma}$
γ_0
Energy

FIG. 1(a),(b)

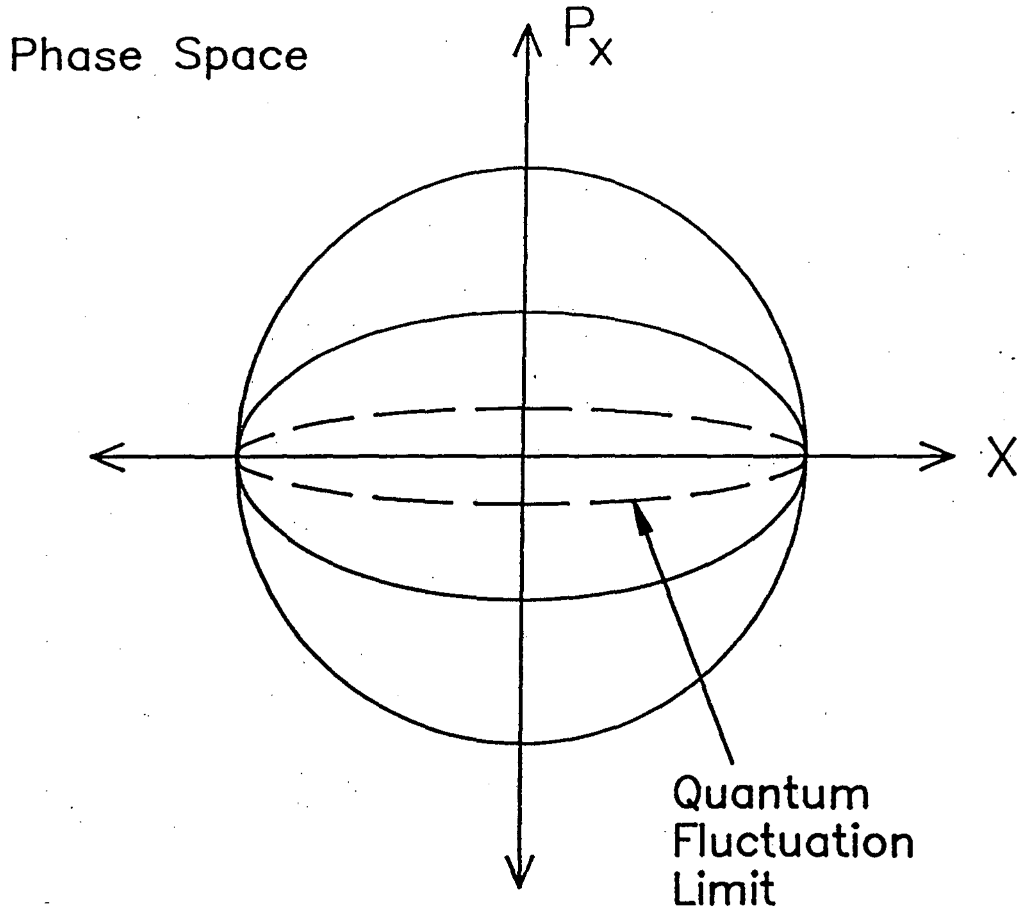


FIG. 1(c)

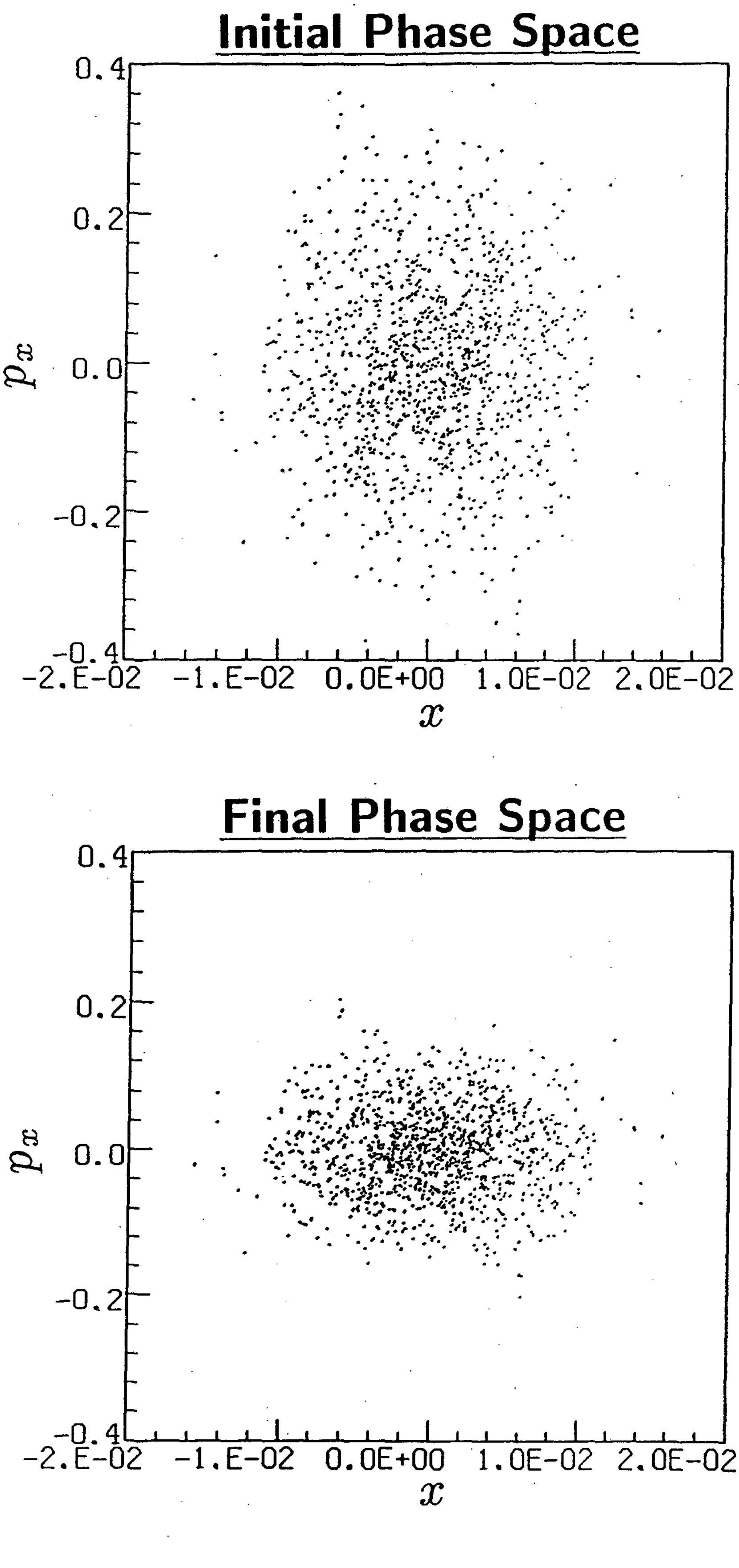

Initial Phase Space
0.4
0.2
0.0
-0.2
-0.4
p_x
-2.E-02
-1.E-02
0.0E+00
1.0E-02
2.0E-02
x
Final Phase Space
0.4
0.2
0.0
-0.2
-0.4
p_x
-2.E-02
-1.E-02
0.0E+00
1.0E-02
2.0E-02
x

FIG. 2(a),(b)

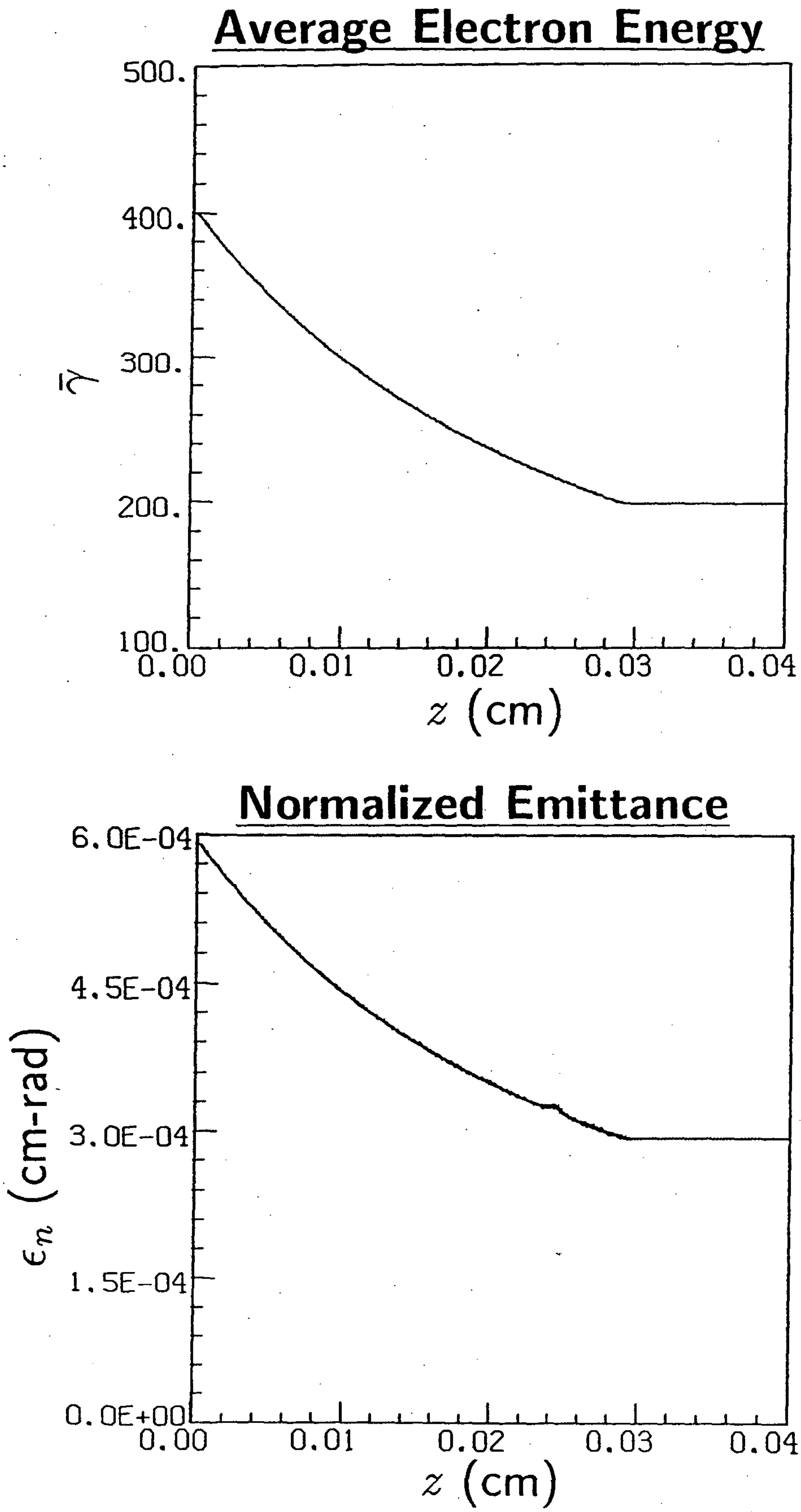

Average Electron Energy
500.
400.
300.
200.
100.
$\bar{\gamma}$
0.00
0.01
0.02
0.03
0.04
z (cm)
Normalized Emittance
6.0E-04
4.5E-04
3.0E-04
1.5E-04
0.0E+00
ϵ_n (cm-rad)
0.00
0.01
0.02
0.03
0.04
z (cm)

FIG. 3(a),(b)

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