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Author

Heydecker, Benjamin

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CALIFORNIA PATH PROGRAM
INSTITUTE OF TRANSPORTATION STUDIES
UNIVERSITY OF CALIFORNIA, BERKELEY

Incidents and Intervention on Freeways

Benjamin Heydecker

California PATH Research Paper

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Foreword

The occurrence of incidents on freeways causes disruption to the flow of traffic through the neighborhood of the incident for a period that lasts longer than the incident itself does. The practical measures that traffic managers can implement to mitigate this disruption include diversion of traffic as it approaches the incident. Evaluation of the likely effects on traffic flow of an incident and the benefits of implementing a diversion calls for modelling that respects the spatio-temporal nature of the formation and dissipation of congestion. In this report, the kinematic wave model of traffic is applied to investigate these issues. Expressions are derived in closed form for a number of quantities of interest, including the maximum extent of the congested region, the time at which that occurs, relevant times for an intervention, and the effects of that on the size of the congested region. Special forms of these expressions are established for the case of the linear speed-density relationship, and calculations, are performed to illustrate the application of the resulting analysis to examples of incidents both with and without intervention.

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CHAPTER 1

INTRODUCTION

An incident that occurs on a freeway which reduces capacity to below the rate at which traffic is arriving will cause queueing and disruption to the flow of traffic. This will have consequences for both individuals travelling on the freeway and for management of the road system, both in terms of delay and safety. From the point of view of incident management, estimates of the furthest upstream extent of the queue, the time at which this occurs and the duration of queueing at the location of the incident are of considerable importance. A possible response to the occurrence of incidents of this kind is to intervene by diverting some traffic from the freeway at an interchange upstream of the incident: this will reduce the rate at which the traffic queue grows on the freeway and hence will reduce the furthest upstream extent of the queue and also hasten the clearance of the queue.

Important issues that arise in management of this kind include the relevant timing and extent of any diversion, and its effect on size and timing of the main features of the incident. Thus, for example, a manager might wish to know the last time at which a diversion of a certain size could influence either the maximum upstream extent of the queue or the time at which the queue clears. Of similar interest is knowledge of an appropriate time at which to cancel a diversion. Several alternative diversions might be possible, affecting differing amounts of traffic: choice between them can be informed by estimates of their respective effects.

Evaluation of the behaviour of **traffic** at incidents such as this calls for modelling of the spatial dynamics of **traffic**. In the present paper, the hydrodynamic approach of traffic flow modelling is applied to freeway incidents and interventions. The approach that is pursued is based on that of Lighthill and **Whitham** (1955) as developed by, for example, **Haberman** (1977). The reason for pursuing this approach is that it offers the advantage of spatial dynamics of congestion in a traffic model that is relatively easy to **analyse**. **Michalopoulos** and Pisharody (1981) have applied this approach to the estimation of delays on **signal**-controlled arterial roads, and provided multiple formulae to be evaluated in combination to estimate delays and queue sizes. A point that is worthy of note is, however, that this approach is not the most direct available for the estimation of delays, either incurred by **all** traffic affected by the incident or by individual vehicles. As Daganzo (1983) observed, if delay is the quantity of prime interest, then relatively simple alternative methods of estimation are available based on accounting for cumulative arrivals and departures. The analysis presented here focuses on the spatio-temporal aspects of congestion caused by an incident and the way in which this can be alleviated by a traffic management intervention.

In this report some general analysis of incidents is developed first. The effects on the formation and decay of queues of intervening by diverting some traffic from a point upstream of the incident are then investigated. Special forms of the results of this analysis are established for Greenshields's (1934) linear speed-density relationship: these are in closed form and can be applied directly to estimate several **quantities** of interest in the management of incidents. Finally, two related numerical examples are investigated to illustrate the use of these formulae.

CHAPTER 2

TRAFFIC FLOW MODELLING

In this paper, we develop Lighthill's and Whitham's (1955) fluid model of traffic in which vehicles are not represented individually, but rather traffic is treated as a continuous, compressible medium. We assume that the speed of **traffic** at each point is determined completely by the traffic density there, so that the dynamic behaviour of traffic can be **characterised** completely by instantaneous knowledge of the spatial density distribution. Models of this kind treat traffic primarily as a bulk medium, placing emphasis on gross rather than individual behaviour. However, the traffic speed is well defined at each point and instant as a field so that vehicular trajectories can be recovered as the solutions to differential equations.

We assume that the functional relationship between space-mean speed and density is deterministic, continuous and decreasing in density. We use the notation:

- k spatial density of traffic (vehicles/mile),
- v space-mean speed (miles per hour),
- q traffic flow rate (vehicles/hour)

Then we assume the existence of a functional relationship between speed and density of the form

$$v = v(k) . \quad (1)$$

An immediate consequence of this is (Wardrop, 1952) that the flow at each point is the product of the space-mean speed and the density there, and hence is determined entirely by the density as

$$q(k) = k v(k) . \quad (2)$$

We denote the free-flow speed by

$$v_0 = v(0) \quad (3)$$

and the assumption of decreasing speeds with increasing density can be stated as:

$$\frac{\partial v}{\partial k} < 0 . \quad (4)$$

Furthermore, we assume that $N(q_p)$, the number of values of k for which $q(k) = q_p$, is exactly

$$N(q_p) = \begin{cases} 2 & (0 \leq q_p < q_x) \\ 1 & (q_p = q_x) \\ 0 & (q_p > q_x) \end{cases} \quad (5)$$

where q_x is the maximum **flow**, or **capacity** of the road. Then for $(0 \leq q_p < q_x)$ let $k^+(q_p)$ be the greater of the two solutions, and let $k^-(q_p)$ be the lesser. According to (5), $k^+(\cdot)$ and $k^-(\cdot)$ are single-valued functions on $[0, q_x]$: k^+ corresponds to congested traffic and k^- to freely flowing traffic. These conditions give that $k^-(0) = 0$ and we suppose for some value $k = k_j$ that $q(k_j) = 0$: this is called the jam **density**. We confine our attention to densities k in the range $(0 \leq k \leq k_j)$.

The uniqueness conditions are clearly satisfied if $q(k)$ is a concave function, and this is equivalent to

$$\begin{aligned} \frac{\partial^2 q}{\partial k^2} &< 0 \quad , \quad \text{or} \\ 2\frac{\partial v}{\partial k} + k \frac{\partial^2 v}{\partial k^2} &< 0 \quad . \end{aligned} \quad (6)$$

This means that the wave speed, $w(k)$, given by

$$\begin{aligned} w(k) &= \frac{\partial q}{\partial k} \\ &= v(k) + k \frac{\partial v}{\partial k} \end{aligned} \quad (7)$$

is a strictly decreasing function of k . The wave speed $w(k)$ specifies the rate at which regions of constant density k propagate and hence describes the way in which traffic conditions propagate through space. Waves corresponding to densities in the **uncongested** range $(0 \leq k \leq k_x)$, where $k_x = k^+(q_x)$, have positive speeds, whilst those corresponding to congested conditions with densities in the range $(k_x \leq k \leq k_j)$ have negative ones: according to (6) and (7), stationary waves correspond to the density at which maximum flow $q = q_x$ occurs. We assume further that the wave speed is a continuous function of density, so that the equation $w(k) = v_p$ has a unique solution k_p whenever $w_j \leq v_p \leq v_0$, where $w_j = w(k_j)$.

According to this model, the instantaneous traffic conditions at a point can be specified completely by the triple (v, k, q) of **speed**, **density** and **flow**. This is a convenience which is adopted throughout the present paper.

CHAPTER 3

DYNAMIC TRAFFIC MODELLING

We consider a long uniform section of road on which the traffic is initially uniform and is described by the (v, k, q) triple (v_a, k_a, q_a) , and suppose that an incident occurs at position $x = 0$, starting at time $t = 0$ and ending at time $t = t_0$. Throughout the duration of the incident, the flow past the position $x = 0$ is reduced to $q = q_f < q_a$, and that after time $t = t_0$, the capacity at position $x = 0$ returns to normal. In order to investigate the consequences of this incident for traffic flow in the vicinity of its occurrence, we appeal to a dynamic traffic model that indicates how the congestion caused will form, propagate and dissipate. The requirements on a traffic model for this include a satisfactory representation of spatial dynamics, and for that reason the Lighthill and Whitham wave model is adopted and developed here.

First of all, we develop a wave model of the traffic dynamics of this incident in the absence of an intervention. We consider the upstream congestion and queueing, and also the downstream conditions resulting from flow metering past the position of the incident. We then consider the effects on these dynamics of reducing the traffic flow arriving from upstream, for example by posting an advisory diversion notice, broadcasting traffic information, providing route guidance or closing an on-ramp that will be relevant to some drivers. This will normally have the effect of reducing the rate at which traffic approaches the incident from some fixed point upstream of it. The analysis presented in this section is independent of any particular speed-density relationship.

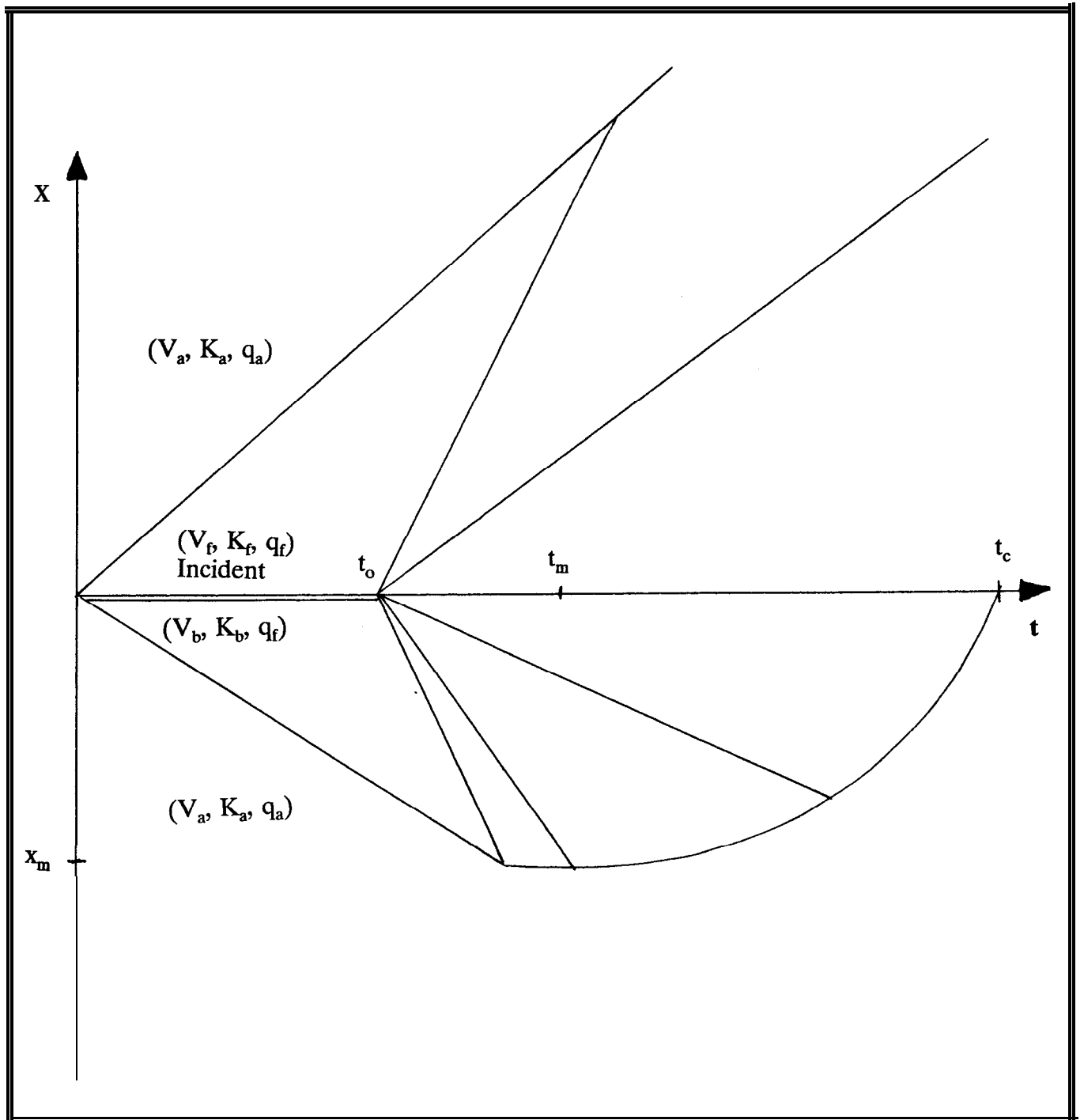
3.1 Modelling an incident

The main features of this model of an incident and its consequences are illustrated in Figure 1.

The traffic immediately upstream of the incident is congested because the arrival rate q_a exceeds the capacity q_f at the location of the incident. It is therefore characterised by the triple (v_b, k_b, q_f) , where

$$\begin{aligned} k_b &= k^+(q_f), \quad \text{and} \\ v_b &= v(k_b). \end{aligned} \tag{8}$$

FIGURE 1. HYDRODYNAMIC MODEL OF A FREEWAY INCIDENT



A shock-wave arises upstream of the incident corresponding to the back of the region of congested traffic. At this shock-wave, there is an abrupt increase in density from arrival density $k = k_a$ to the density of the congested region, $k = k_b$. According to Lighthill and Whitham, this shock-wave travels initially with speed

$$u_b = \frac{q_f - q_a}{k_b - k_a} < 0 \quad (0 < t < t_b) . \quad (9)$$

The last time $t = t_b$ at which this shock-wave propagates with speed u_b is determined by the time $t = t_0$ at which the incident ends. When the incident ends and the consequent restriction to traffic flow at $x = 0$ is removed, waves with density k throughout the range $(k_a \leq k \leq k_b)$ emanate from position $x = 0$, each at its associated wave speed $w(k)$: this dissipative behaviour of falling density is in marked contrast to the stable behaviour of shock-waves caused by increasing density. The time $t = t_b$ is therefore specified by the intersection of the shock-wave specified by (9) with the wave that travels backwards with most negative speed: because wave speed decreases with increasing density, this wave is associated with the greatest density, $k = k_b$. Thus

$$t_b = \left(\frac{w_b}{w_b - u_b} \right) t_0 \quad (10)$$

where $w_b = w(k_b)$ is the wave speed associated according to (7) with density $k = k_b$. At this instant, the shock-wave is at position

$$x_b(t_b) = \left(\frac{w_b u_b}{w_b - u_b} \right) t_0 . \quad (11)$$

After time $t = t_b$, the density immediately in front of this shock-wave is determined by waves of lesser density emanating from position $x = 0$ at time $t = t_0$. **The density in front** of the shock-wave therefore starts to fall: the time at which it is equal to k , ($k_a \leq k \leq k_b$) can be found, following Lighthill and Whitham, to be

$$t_k = \left(\frac{q_k - q_f - (k - k_a) w(k)}{q_k - q_a - (k - k_a) w(k)} \right) t_0 \quad (12)$$

when the position is

$$\begin{aligned} x_k &= w(k)(t_k - t_0) \\ &= \frac{(q_a - q_f) w(k) t_0}{q_k - q_a - (k - k_a) w(k)} \end{aligned} \quad (13)$$

The two equations (12) and (13) describe the trajectory of the shock-wave at the back of the disturbance caused by the incident parametrically in k , the density immediately downstream of it.

The furthest upstream position of the shock-wave corresponding to the back of the queue is reached when the shock-wave is stationary: this occurs when $q(k_m) = q_a$, so that $k_m = k^+(q_a)$. The position and time, (x_m, t_m) , of this event are given by (13) and (12) with $k = k_m$. After time $t = t_m$, this shock-wave travels forwards and represents the position at which traffic conditions return to normal after the last effects of the incident have ceased: for this reason, it is called the **clearing wave**.

The congestion clears from the position at which the incident occurred at time t_c , where according to (12) and (13) with $x_k = 0$,

$$t_c = \left(\frac{q_x - q_f}{q_x - q_a} \right) t_0 \quad (14)$$

Traffic immediately downstream of the incident is **uncongested**, being metered past the point $x = 0$ at flow rate $q = q_f$. It is therefore **characterised** by the triple (v_f, k_f, q_f) , where

$$\begin{aligned} k_f &= k^-(q_f) \quad , \quad \text{and} \\ v_f &= v(k_f) \end{aligned} \quad (15)$$

The boundary between the traffic immediately downstream of the incident and that further downstream which is unaffected by the incident forms a shock-wave because of the abrupt increase in density from $k = k_f$ to $k = k_a$. This shock-wave is stable and travels initially with constant speed given by

$$u_f = \frac{q_a - q_f}{k_a - k_f} > 0 \quad (16)$$

Behaviour of traffic downstream of the incident can be determined by similar considerations to those used for upstream conditions. The instantaneous position at time t , $x_f(t)$, of the shock-wave that corresponds to front of the disturbance caused by the incident is given by:

$$x_f = u_f t \quad (0 \leq t \leq t_f), \quad (17)$$

where

$$t_f = \left(\frac{w_f}{w_f - u_f} \right) t_0, \quad (18)$$

and $w_f = w(k_f)$ is the wave speed associated with density $k = k_f$.

At time $t = t_f$, the position of the downstream shock-wave is given by

$$x_f = \left(\frac{w_f u_f}{w_f - u_f} \right) t_0 \quad (19)$$

and after that time it is given parametrically in k , here representing the density immediately upstream of it, by (12) and (13) with k in the range $(k_f \leq k \leq k_a)$.

Throughout the region in which traffic behaviour is determined by the incident, the instantaneous point traffic density $k(x, t)$ can be calculated from consideration of waves. Thus for the duration of the incident itself, we have

$$k(x, t) = \begin{cases} k_b & (u_b t < x < 0) \\ k_f & (0 < x < u_f t) \\ k_a & (\text{otherwise}) \end{cases} \quad (0 \leq t \leq t_0) \quad (20)$$

After time $t = t_0$, when the incident ends, the traffic density in the neighbourhood of $x = 0$, the location of the incident, is determined by waves that emanate from that position at time $t = t_0$, whilst that further away is determined for some further period by conditions during the incident. Thus upstream of the incident,

$$k(x, t) = k_b \quad (u_b t < x < w_b t) \quad (t_0 \leq t \leq t_b), \quad (21)$$

and downstream of the incident

$$k(x, t) = k_f \quad (w_f t < x < u_f t) \quad (t_0 \leq t \leq t_f). \quad (22)$$

The traffic density elsewhere in the region between the upstream and downstream shock-waves caused by the incident can be determined from the linear trajectories of waves: the position $x(k, t)$ at which the density is k at time t is given by

$$x(k, t) = (t - t_0) w(k) . \quad (23)$$

3.2 Modelling an intervention

We now consider the effects of diverting an amount q_e of traffic from the freeway at a point $x = x_e$ upstream of the location of the incident, starting at time $t = t_e$. The main features of this model of an incident together with an upstream intervention are illustrated in Figure 2. The traffic conditions immediately downstream of the point at which the intervention is made are then described by the triple (v_r, k_r, q_r) , where

$$\begin{aligned} q_r &= q_a - q_e \\ k_r &= k^-(q_r) , \text{ and} \\ v_r &= v(k_r) . \end{aligned} \quad (24)$$

This intervention will cause the formation of a shock-wave. Because $q_r < q_a$, this shock-wave will move downstream at speed u_e given by

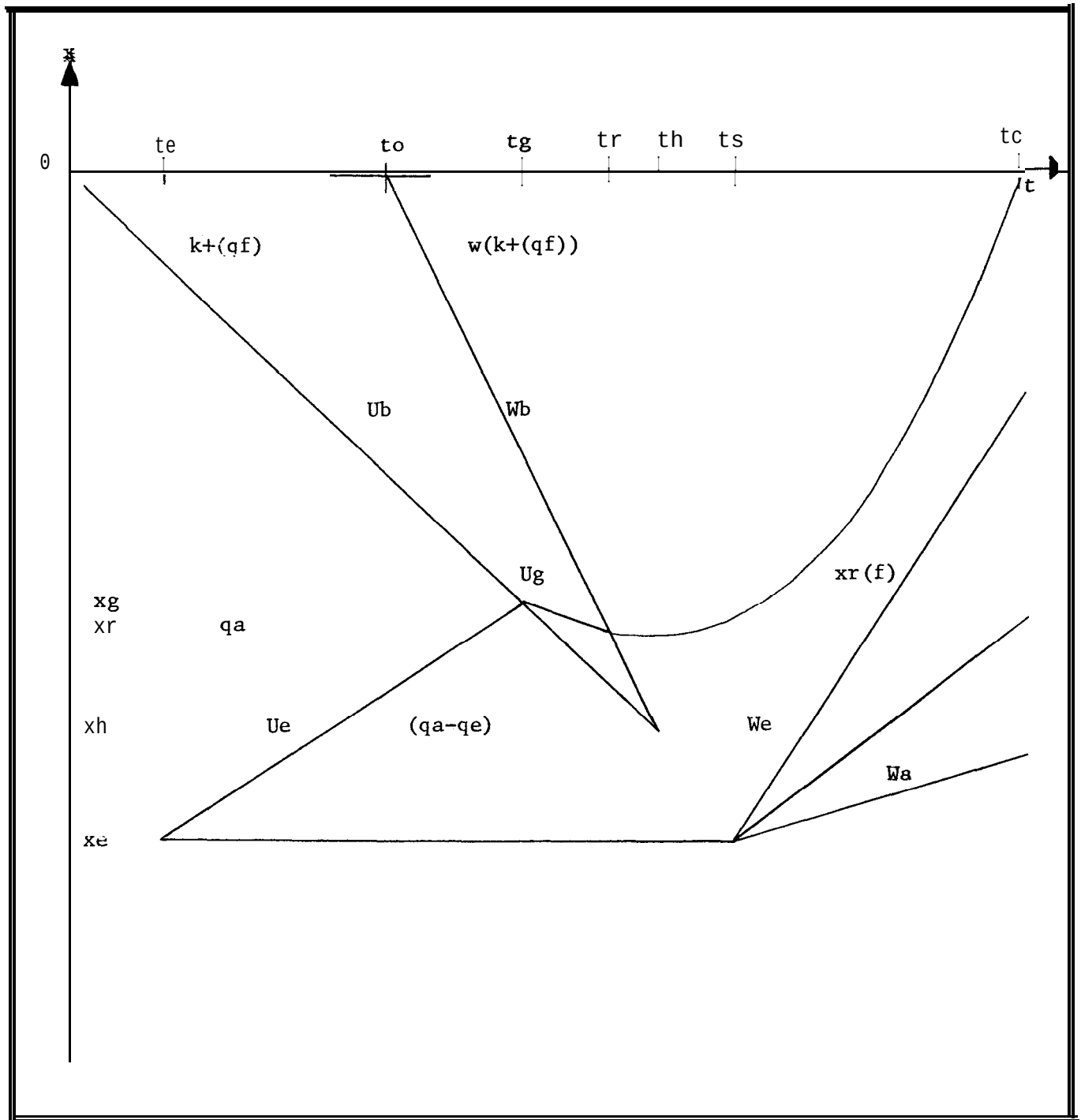
$$u_e = \frac{q_e}{k_a - k_r} . \quad (25)$$

If the incident can be anticipated, then the possibility arises that $t_e < 0$. In the case that

$$t_e < \frac{x_e(k_a - k_e)}{q_e} < 0 ,$$

the intervention has an effect at the location of the incident before the incident occurs. If we also have that $q_r \leq q_f$, then the diversion is **sufficiently** effective that it avoids any congestion and consequent queue formation at the location of the incident. In the remainder of the analysis presented here, we consider the more usual case where intervention has effect at the site of the incident after time $t = 0$, and typically is made in response to the occurrence of the incident, so that $t_e \geq 0$.

FIGURE 2. MODEL OF A FREEWAY INCIDENT WITH AN UPSTREAM INTERVENTION



The intervention will influence the traffic and queueing behaviour in the region of the incident. Initially this influence will be upstream of the location of the incident, but after time $t = t_c$ it will pass downstream of that location.

There are two distinct cases to consider for the interaction of the downstream-travelling shock-wave caused by the intervention with the upstream-travelling one at the back of the queue caused by the incident. In the first case, the time $t = t_g$ at which the two shock-waves coincide is either before the end of the incident or before the effect of its end reaches the back of the queue caused by it, so that $t_g < t_b$, where t_b is given by (10). In this case, the back of the queue grows at a constant rate until the intervention first has an effect on it. In the second case, $t_g > t_b$, so that the end of the incident has already had some effect on the shock-wave at the back of the queue. The shock-wave could still be travelling upstream (if $t_g < t_m$) or could have started travelling downstream. An expression is developed here for the bounding condition between these two sub-cases, corresponding to the latest time of intervention that will influence the position and timing of the maximum queue length. A further expression is developed to indicate the latest time of intervention that will influence the time at which congestion clears at the location of the incident.

The first case to consider is that in which the two shock-waves coincide before time $t = t_b$. To find the time at which coincidence occurs, we need to solve for t_g the equation

$$x_e + u_e(t_g - t_e) = u_b t_g$$

which yields the value

$$t_g = \frac{u_e t_e - x_e}{u_e - u_b}, \quad (26)$$

corresponding to intersection at position $x = x_g$, where

$$x_g = \left(\frac{u_b}{u_e - u_b} \right) (u_e t_e - x_e). \quad (27)$$

In order for this case to arise, we require that $t_g \leq t_b$, and hence that the time t_e of intervention satisfies

$$t_e \leq \frac{x_e}{u_e} + \frac{(u_e - u_b)w_b}{(w_b - u_b)u_e} t_0. \quad (28)$$

Immediately after the shock-wave caused by the diversion of traffic reaches the back of the queue caused by the incident, the back of the queue will travel with speed u_g which is given in this case by

$$u_g = \frac{q_f - q_a + q_e}{k_a - k} . \quad (29)$$

The start wave, which has density $k = k_b$, will then reach the back of the queue at time $t = t_r$, when

$$x_g + (t_r - t_g)u_g = (t_r - t_0)w_b$$

so that

$$t_r = \frac{t_g u_g - t_0 w_b - x_g}{u_g - w_b} . \quad (30)$$

The position $x = x_r$ of the shock-wave at that time is given by

$$x_r = \left(\frac{w_b}{u_g - w_b} \right) (t_g u_g - t_0 u_g - x_g) . \quad (31)$$

After time $t = t_r$ the position of the shock-wave at the back of the queue is given parametrically in the density k immediately downstream of it by

$$t_k = \frac{[q_k - q_f - (k - k_r)w(k)]t_0 + q_e t_e - (k_a - k_r)x_e}{q_k - q_r - (k - k_r)w(k)} , \quad (32)$$

and

$$x_k = \left[\frac{(q_k - q_f)t_0 + q_e t_e - (k_a - k_r)x_e}{q_k - q_r - (k - k_r)w(k)} \right] w(k) , \quad (33)$$

Equating the expression (33) for x_k to 0 yields $w(k) = 0$, and using this in (32) gives the time $t = t_c$ when the clearing wave passes the position at which the incident occurred as

$$t_c = \frac{(q_x - q_f)t_0 + q_e t_e - (k_e - k_r)x_e}{(q_x - q_r)}, \quad (34)$$

where q_x is the capacity of the road after the end of the incident, corresponding to the flow rate when the wave speed is 0.

To find the furthest upstream position of the back of the queue, first **notice** that if $q_e \geq q_a - q_f$, then from (29) $u_g \geq 0$ so that clearing starts immediately the shockwave caused by the intervention reaches the back of the queue caused by the incident. The maximum position of the back of the queue is given in this case by $x = x_g$ from (27) at time $t = t_g$ from (26). However, if $q_e < q_a - q_f$, then the maximum position of the back of the queue occurs at $x = x_m$, where

$$x_m = x_k(k^+(q_r)) \quad (35)$$

and $x_k(\cdot)$ is given by (33), and at time $t = t_m$, where

$$t_m = t_k(k^+(q_r)) \quad (36)$$

and $t_k(\cdot)$ is given by (32).

If the intervention occurs after the critical time given by (28), then the shock-wave caused by the diversion will coincide with the one at the back of the queue caused by the incident after it is first affected by the end of the incident. In this case, the queue is already dissipating before the diversion has any effect on it: the time t_r , location x_r and density k_r at the point of coincidence satisfy simultaneously the three equations (12) with $t_k = t_r$, (13) with $x_k = x_r$, and

$$x_r = x_e + u_e(t_r - t_e) \quad (37)$$

After time $t = t_r$, the trajectory of the clearing wave is given parametrically in density k by (32) and (33). The time $t = t_c$ at which the clearing wave passes the position at which the incident occurred is then given by (34), provided that $x_r < 0$ so that the intervention has an effect on the queue while it persists upstream of the position of the incident.

Because $u_b(t) < w_a < u_e$, the intervention will affect the clearing of the queue at some time. However, if $x_r \geq 0$, then this occurs after the queue has cleared at the position of the incident. This leads to the requirement on the time of intervention to ensure that congestion will be relieved upstream of the incident as

$$t_e \leq \frac{x_e(k_a - k_r)}{q_e} + \left(\frac{q_x - q_f}{q_x - q_a} \right) t_0 \quad (38)$$

In this case, the position and time of the maximum upstream extent of the queueing caused by the incident can be determined according to which one of three distinct sub-cases applies. In increasing order of t_e these are:

1. The queue increases after time $t = t_r$ as given in the simultaneous solution to (12), (13) and (37). This sub-case occurs if the shock-wave caused by the intervention coincides with that at the back of the queue before the wave of density $k^+(q_r)$ does. This will occur if

$$t_e < t_b(k^+(q_r)) + \frac{x_e - x_b(k^+(q_r))}{u_e} \quad (39)$$

where $t_b(\cdot)$ and $x_b(\cdot)$ are given by (12) and (13) respectively.

In this sub-case, the position of the maximum queue $x = x_m$ and time of occurrence $t = t_m$ are given by (33) and (32) respectively with $k = k^+(e)$.

2. The queue increases up to time $t = t_r$ as given in the simultaneous solution to (12), (13) and (37), but decreases immediately afterwards. This sub-case occurs if the shock-wave caused by the intervention coincides with that at the back of the queue after the wave of density $k^+(q_r)$ does but before the wave of density $k^+(q_a)$ does. This will occur if

$$t_b(k^+(q_r)) + \frac{x_e - x_b(k^+(q_r))}{u_e} \leq t_e \leq t_b(k^+(q_a)) + \frac{x_e - x_b(k^+(q_a))}{u_a} \quad (40)$$

where $t_b(\cdot)$ and $x_b(\cdot)$ are given by (12) and (13) respectively with the indicated values of density.

In this sub-case, the position of the maximum queue $x = x_m$ and time of occurrence $t = t_m$ are given by $x_m = x_r$ and $t_m = t_r$ as the simultaneous solution to (12), (13) and (37).

3. The queue starts to decrease before time $t = t_r$ as given in the simultaneous solution to (12), (13) and (37). This sub-case occurs if the shock-wave caused by the intervention coincides with that at the back of the queue after the wave of density $k^+(q_a)$ does. This will occur if

$$t_e > t_b(k^+(q_a)) + \frac{x_e - x_b(k^+(q_a))}{u_s} \quad (41)$$

where $t_b(\cdot)$ and $x_b(\cdot)$ are given by (12) and (13) respectively.

In this sub-case, the position of the maximum queue $x = x_m$ and time of occurrence $t = t_m$ are unaffected by the intervention, and are given by (12) and (13) with $k = k^+(q_a)$.

3.3 Removal of the intervention

When the intervention is removed and diversion of traffic ceases, at time $t = t_s$, say, waves with densities k throughout the range $[k^-(q_r), k_a]$ will emanate from the point $x = x_e$. The density downstream of that point will then increase continuously from $k = k^-(q_r)$ to $k = k_a$. The first effects of this will reach the position $x = 0$ after the clearing wave has passed there provided that

$$t_s \geq t_c + \frac{x_e}{w_r} \quad (42)$$

where t_c is given by (34) and $w_r = w(k^-(q_r))$ is the wave speed associated with density $k^-(q_r)$. The effects of the intervention at the position of the incident will cease completely at time $t = t_z$ when the last of these waves (which has density $k = k_a$) passes the position of the incident. This time is given by

$$t_z = t_s - \frac{x_e}{w_a} \quad (43)$$

where $w_a = w(k_a)$ is the wave speed associated with density $k = k_a$.

CHAPTER 4

LINEAR MODELLING

4.1 Introduction

A wide range of choices are available for the relationship (1) between speed and density: depending upon which is adopted, different forms of equations (2)-(43) will arise. The choice of relationship can be motivated by empirical considerations or by analytic convenience. Here, the latter consideration is overriding, so we adopt Greenshields's (1934) linear relationship between speed and density: this leads to closed-form expressions for the quantities of interest.

In this case (1) and (2) take the specific forms

$$v(k) = v_0 \left(1 - \frac{k}{k_j} \right), \quad (44)$$

and

$$q(k) = v_0 \left(k - \frac{k^2}{k_j} \right). \quad (45)$$

According to this model, the capacity of the road is related to the jam density k_j and free-flow speed v_0 by

$$q_x = \frac{v_0 k_j}{4}. \quad (46)$$

The second partial derivative of flow with respect to density is given by

$$\frac{\partial^2 q}{\partial k^2} = \frac{-2v_0}{k_j} \quad (47)$$

so (6) is satisfied and the various consequent uniqueness properties obtain in this case. In view of the particularly simple forms of these relationships, the functions $k^+(q)$ and $k^-(q)$ can be expressed in closed form as

$$k^+(q) = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4q}{v_0 k_j}} \right) k_j \quad (48)$$

and

$$k^-(q) = \frac{1}{2} \left(1 - \sqrt{1 - \frac{4q}{v_0 k_j}} \right) k_j \quad (49)$$

The wave speed associated with density k is given in this linear case by

$$w(k) = \left(1 - \frac{2k}{k_j} \right) v_0 \quad (50)$$

and in particular

$$w_j = -v_0. \quad (51)$$

The general expression for the speed of a shock-wave can be expressed conveniently for Greenshields's linear speed-density relationship as

$$u(k_1, k_2) = \frac{v_0}{k_j} (k_j - k_1 - k_2) \quad (k_2 > k_1). \quad (52)$$

4.2 Linear modelling of an incident

We consider first the development according to Greenshields's model of the **traffic** congestion caused by an incident in the absence of any intervention. We can apply the particular formulae for linear models given in section 4.1 in the general formulae of section 3.1 describing the dynamics of **traffic** in the vicinity of the incident. The traffic immediately downstream of the incident is **characterised** by the tiple (v_f, k_f, q_f) where the particular forms of (15) are

$$k_f = \frac{1}{2} \left(1 - \sqrt{1 - \frac{4q_f}{v_0 k_j}} \right) k_j ,$$

and

$$v_f = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4q_f}{v_0 k_j}} \right) v_0 ,$$

These can be expressed more compactly as

$$k_f = \frac{1}{2} (1 - S_f) k_j \tag{53}$$

and

$$v_f = \frac{1}{2} (1 + S_f) v_0 \tag{54}$$

where the dimensionless quantity S_f is defined as

$$S_f = \sqrt{1 - \frac{4q_f}{v_0 k_j}} \tag{55}$$

The downstream boundary of the region that is influenced by the incident moves initially with speed u_f given by (16). The specific form of this in the linear case is

$$u_f = \frac{1}{2} (S_a + s_A) v_0 \quad (56)$$

where

$$S_a = \sqrt{1 - \frac{4q_a}{v_0 k_j}} \quad (57)$$

Similarly, the traffic immediately upstream of the incident is characterised by the triple (v, k, q) where the particular forms of (8) are

$$k_b = \frac{1}{2} (1 + S_f) k_j \quad (58)$$

and

$$v_b = \frac{1}{2} (1 - S_f) v_0 , \quad (59)$$

The upstream boundary of the region that is influenced by the incident moves initially with speed u_b given by (9) which has specific form

$$u_b = \frac{1}{2} (S_a - S_f) v_0 . \quad (60)$$

The start wave, initiated by the end to the incident at time t_0 and position $x = 0$, travels with speed w_b which, according to (7), (21) and (23) have **specific** forms

$$\begin{aligned} w_b &= \left(1 - \frac{2k^+(q)}{k_j} \right) v_0 \\ &= -S_f v_0. \end{aligned} \quad (61)$$

The start wave reaches the back of the queue at time $t = t_b$, when

$$u_b t_b = w_b (t_b - t_0)$$

so that

$$t_b = \frac{2S_f}{(S_a + S_f)} t_0. \quad (62)$$

The position of the back of the queue at that time is

$$x_b = \left(\frac{S_a - S_f}{S_a + S_f} \right) S_f v_0 t_0. \quad (63)$$

After time $t = t_b$, the trajectory of the back of the queue is given according to (12) and (13) parametrically in k by

$$\begin{aligned} t_k &= \left(1 + \frac{(S_f^2 - S_a^2)k_j^2}{4(k - k_a)^2} \right) t_0, \text{ and} \\ x_k &= \frac{k_j(k_j - 2k)(S_f^2 - S_a^2)}{4(k - k_a)^2} v_0 t_0 \quad (k_a < k \leq k_b). \end{aligned} \quad (64)$$

From the former of these, we have

$$k = k_a - \frac{k_j}{2} \sqrt{\frac{(S_f^2 - S_a^2)t_0}{t_k - t_0}} \quad (65)$$

Using this in the latter gives

$$x_b(t) = \left[S_a(t-t_0) - \sqrt{(S_f^2 - S_a^2)(t-t_0)t_0} \right] v_0 \quad (t \geq t_b) \quad (66)$$

and similarly

$$x_f(t) = \left[S_a(t-t_0) + \sqrt{(S_f^2 - S_a^2)(t-t_0)t_0} \right] v_0 \quad (t \geq t_f) . \quad (67)$$

The density of traffic in the region between these shock-waves is given, in this linear case, by

$$k(x,t) = \frac{k_j}{2} \left(1 - \frac{x}{v_0(t-t_0)} \right) \quad (x_b(t) \leq x \leq x_f(t)) , \quad (t \geq t_0) . \quad (68)$$

The furthest position upstream that the back of the queue reaches is found from (64) with $k = k+(g)$ to be

$$x_m = \frac{(S_a^2 - S_f^2)}{4S_a} v_0 t_0 \quad (69)$$

at time

$$t_m = \left(1 + \frac{S_f^2 - S_a^2}{4S_a^2} \right) t_0 \quad (70)$$

Finally, the queue clears the position $x = 0$ at which the incident occurred at time

$$t_c = \left(\frac{S_f}{S_a} \right)^2 t_0 \quad (71)$$

which is consistent with the general form (14). After this time, the traffic conditions upstream of that position are no longer affected by the incident.

4.3 Linear modelling of an intervention

According to (52), the shock-wave caused by the intervention will move at speed

$$u_e = \frac{v_0}{k_j} (k_j - k^-(q_a) - k^-(q_a - q_e))$$

so that in view of (49),

$$u_e = \frac{1}{2} (S_a + S_e) v_0, \quad (72)$$

where

$$S_e = \sqrt{1 - \frac{4(q_a - q_e)}{v_0 k_j}}. \quad (73)$$

We now consider in turn each of the two distinct cases for the intersection of this shock-wave with the queue caused by the incident. The first of these corresponds to intersection when the queue is still growing at constant speed in a manner that is unaffected by the end of the incident. The second corresponds to intersection when clearing of the incident has started to affect the back of the queue, although it could still be growing upstream. An explicit form is derived for the boundary conditions between these cases, as is a condition for the intervention having no effect on the queue length before clearance at the position of the incident

Suppose that the shock wave caused by the intervention reaches the back of the queue caused by the incident at time $t = t_g$. If the intervention is made sufficiently early, this will occur before the growth of the back of the queue is affected by the end of the incident. In this case, we can identify the time and location of the intersection by solving

$$x_e + u_e(t_g - t_e) = u_b t_g.$$

Using (60) and (72) and rearranging gives the time $t = t_g$ of the intersection as

$$t_g = \frac{(S_a + S_e)t_e - 2x_e/v_0}{(S_e + S_f)} \quad (74)$$

and its position $x = x_g$ as

$$x_g = \left(\frac{S_a - S_f}{S_e + S_f} \right) \left[\frac{1}{2}(S_a + S_e)v_0 t_e - x_e \right]. \quad (75)$$

In order for the intervention to have an effect on the position of the back of the queue before clearing does, we require that it take place sufficiently early that

$$t_g \leq t_b.$$

Hence we require that t_e satisfies

$$t_e \leq \frac{2x_e}{(S_e + S_e)v_0} + \frac{2(S_e + S_f)S_f t_0}{(S_e + S_f)(S_e + S_e)}. \quad (76)$$

After the shock-wave caused by the diversion of traffic reaches the back of the queue, it will move initially with speed given according to (29), (57) and (73) by

$$u_g = \frac{1}{2}(S_e - S_f)v_0. \quad (77)$$

The time $t = t_r$ at which the start wave caused by the end of the incident reaches the back of the queue is given by (30), (50) and (74) as

$$t_r = \frac{2S_f}{(S_e + S_f)} t_0 + \frac{(S_e^2 - S_a^2)}{(S_e + S_f)^2} t_e - \frac{x_e}{(S_e + S_f)^2 v_0}, \quad (78)$$

at which time the position will be

$$x_r = S_f \left(\frac{S_e - S_f}{S_e + S_f} \right) v_0 t_0 - \frac{S_f (S_e^2 - S_a^2)}{(S_e + S_f)^2} v_0 t_e + \frac{2S_f (S_e - S_a)}{(S_e + S_f)^2} x_e. \quad (15)$$

After time $t = t_r$, the instantaneous speed of the back of the queue is given from (30), (31) and (50) by

$$u_r(t) = \frac{1}{2} \left(S_e + \frac{x}{v_0(t-t_0)} \right) v_0. \quad (80)$$

This can be integrated, using the initial condition $x = x_r$ at time $t = t_r$, to give the trajectory

$$x_b(t) = S_e v_0 [(t-t_0) - \sqrt{(t-t_0)(t_r-t_0)}] + x_r \sqrt{\frac{t-t_0}{t_r-t_0}} \quad (t \geq t_r) \quad (81)$$

Equating this expression for $x_b(t)$ to 0 and solving for t , we find that the clearing wave passes the position of the incident, $x = 0$, at time $t = t_c$ given by

$$t_c = \frac{(S_e^2 - S_a^2)}{S_e^2} t_r + \left(\frac{S_f}{S_e} \right)^2 t_0 - \frac{2(S_e - S_a)x_e}{S_e^2 v_0}. \quad (82)$$

To find the furthest upstream position of the back of the queue, first notice that if $q_e \geq q_a - q_f$, then from (57), (73) and (77) $u_g \geq 0$ so that clearing starts immediately. The maximum position of the back of the queue is then given by $x = x_g$ from (75) at time $t = t_g$ from (74). However, if $q_e < q_a - q_f$, then the maximum position of the back of the queue occurs at position $x = x_m$, where (from (35), (36) and (61))

$$x_m = \frac{-1}{4S_e^2} [(1 + S_f^2 - 2S_e^2)v_0 t_e - 2(S_e - S_a)x_e] \quad (83)$$

and at time $t = t_m$, where

$$t_m = t_0 + \frac{1}{4S_e^2} \left[(1 + S_f^2 - 2S_e^2)t_e - 2(S_e - S_a) \frac{x_e}{v_0} \right] \quad (84)$$

If the intervention occurs after the critical tune given by (76), then the shock-wave caused by the diversion will reach the back of the queue caused by the incident after the first effects of the end of the incident do. In this case, the queue is already dissipating before the diversion has any effect on it. Then from (37) and (7), the time t_r at which the diversion shock-wave and the back of the queue intersect satisfies

$$x_e + \frac{1}{2}(S_a + S_e)(t_r - t_e)v_0 = [S_a(t - t_0) - \sqrt{(S_f^2 - S_a^2)(t - t_0)t_0}] v_0$$

The solution of this is given by

$$\tau_r = \frac{\sqrt{(S_f^2 - S_a^2)t_0}}{(S_e - S_a)} \left[\sqrt{1 + \frac{(S_a - S_e)[2x_e - (S_a + S_e)v_0(t_0 - t_e)]}{(S_f^2 - S_a^2)v_0 t_0}} - 1 \right] \quad (85)$$

where $\tau_r = \sqrt{t_r - t_0}$ so that

$$t_r = t_0 + \tau_r^2 \quad (86)$$

This occurs at position $x = x_r$, where

$$x_r = x_e + u_e(t_r - t_e) \quad (87)$$

After that time, the instantaneous speed of the clearing wave is given as $y(t)$ in (80) and the position by $x_{,,}(t)$ in (81). As in the case of interaction with a growing queue, the time $t = t_c$ at which the clearing wave passes the position of the incident, $x = 0$, is given by (82).

The condition (38) for the intervention to have an effect on congestion at the position of the incident can be expressed in this linear case as the requirement on the time $t = t_e$ of the intervention that

$$t_e \leq \frac{2x_e}{(S_e + S_a)v_0} + \left(\frac{S_f}{S_a}\right)^2 t_0. \quad (88)$$

The position and time of the maximum upstream extent of the queueing caused by the incident can be determined according to which one of the three distinct sub-cases described in section 3.2 applies. In increasing order of t_e these are:

1. The queue continues to grow upstream after the effect of the intervention reaches it. This case occurs if

$$t_e < \left[1 + \frac{(S_f^2 - S_a^2)}{(S_e + S_a)^2} \left(1 + \frac{2S_e}{S_a + S_e} \right) \right] t_0 + \frac{2x_e}{(S_e + S_a)v_0} \quad (89)$$

In this sub-case the maximum upstream position of the queue is found from (83) to be

$$x_m = \frac{-1}{4S_e} \left[(1 + S_f^2 - 2S_e^2)v_0 t_e - 2(S_e - S_a)x_e \right] \quad (90)$$

The time at which this occurs is found from (84) to be

$$t_m = t_0 + \frac{1}{4S_e^2} \left[(1 + S_f^2 - 2S_e^2)t_e - 2(S_e - S_a)\frac{x_e}{v_0} \right] \quad (91)$$

2. The queue increases up to the time at which the effect of the intervention reaches it but decrease but decreases immediately afterwards. This case occurs if

$$t_e \geq \left[1 + \frac{(S_f^2 - S_a^2)}{(S_e + S_a)^2} \left(1 + \frac{2S_e}{S_a + S_e} \right) \right] t_0 + \frac{2x_e}{(S_e + S_a)v_0}$$

and

$$t_e \leq \left[1 + \frac{(S_f^2 - S_a^2)}{4S_a^2} \left(1 + \frac{2S_a}{S_a + S_e} \right) \right] t_0 + \frac{2x_e}{(S_e + S_a)v_0} \quad (92)$$

In this sub-case the maximum upstream position of the queue occurs when the shock-wave from the intervention reaches the back of the congested region. The location of this event is given in (79) as

$$x_m = S_f \left(\frac{S_e - S_f}{S_e + S_f} \right) v_0 t_0 - \frac{S_f (S_e^2 - S_a^2)}{(S_e + S_f)^2} v_0 t_e + \frac{2 S_f (S_e - S_a)}{(S_e + S_f)^2} x_e. \quad (93)$$

The time at which this occurs is given in (76) as

$$t_m = \frac{2 S_f}{(S_e + S_f)} t_0 + \frac{(S_e^2 - S_a^2) - S_a}{(S_e + S_f)^2} t_e + \frac{x_e}{(S_e + S_f)^2 v_0}. \quad (94)$$

3. The back of the queue starts to travel forwards before the effect of the intervention reaches it due to the effects of the end of the incident. This case occurs if

$$t_e > \left[1 + \frac{(S_f^2 - S_a^2)}{4 S_a^2} \left(1 + \frac{2 S_a}{S_a + S_e} \right) \right] t_0 + \frac{2 x_e}{(S_e + S_a) v_0} \quad (95)$$

In this sub-case the maximum upstream position of the queue is unaffected by the intervention so its location and timing is the same as in the absence of the upstream intervention. Thus the location is given in (69) as

$$x_m = \frac{(S_a^2 - S_f^2)}{4 S_a^2} v_0 t_0 \quad (69)$$

The time at which this occurs is given in (70) as

$$t_m = \left(1 + \frac{S_f^2 - S_a^2}{4 S_a^2} \right) t_0 \quad (70)$$

4.4 Removal of the intervention

When the intervention is removed and diversion of **traffic** ceases at time $t = t_1$, the density downstream of the point of intervention $x = x_e$ will increase continuously from $k = k(q_1)$ to $k = k_1$. The first effects of this will reach the position $x = 0$ after the clearing wave has passed there provided that the condition (42) is satisfied. In the linear case, this can be stated as

$$t_z \geq \left(\frac{S_f}{S_e}\right)^2 t_0 + \frac{(S_e^2 - S_a^2)}{S_e^2} t - \frac{(S_e - 2S_a)x_e}{S_e^2 v_0} . \quad (96)$$

The effects of the intervention at the position of the incident will Cease completely at time $t = t_z$ given in the linear case by

$$t_z = t_1 - \frac{x_e}{v_0 S_a} . \quad (97)$$

CHAPTER 5

EXAMPLE CALCULATIONS

In order to illustrate the use of these formulae, they are applied to two simple example incidents. The flows and durations of the two incidents are identical: they differ in that in the first case one lane of a three-lane freeway is blocked whilst in the second case two lanes are blocked. In each case, we consider the effect of intervening by diverting some traffic from a point upstream of the incident. Different interventions are considered in each of the two examples.

Consider a freeway on which Greenshields's speed-density model is appropriate and for which

$$v_0 = 65 \text{ miles/h,}$$

$$q_x = 5400 \text{ vehicles/h, so that}$$

$$k_j \approx 332 \text{ vehicles/mile.}$$

Example 1

The arriving traffic is supposed to have a uniform flow of $q_a = 4500$ vehicles/hour . An incident occurs that reduces the capacity at position $x = 0$ to $q_f = 3600$ vehicles/hour for 15 minutes .

First of all, we find from (49) that $k_a \approx 98.3$ vehicles/mile . Proceeding according to (53) - (55) of Section 4.2, we find the downstream conditions to be

$$k_f = 70.2 \text{ vehicles/mile,}$$

$$v_f = 51.3 \text{ miles/hour, and}$$

$$u_f = 32.0 \text{ miles/hour.}$$

From these values, we have

$$S_a = 0.41, \text{ and}$$

$$S_f = 0.58 .$$

Similarly, according to (58) - (60) for the upstream conditions,

$$k_b = 262.1 \text{ vehicles/mile,}$$

$$v_b = 13.7 \text{ miles/hour, and}$$

$$u_b = -5.5 \text{ miles/hour.}$$

According to (69) and (70), the furthest upstream that the back of the queue reaches is

$$x_m = -1.66 \text{ miles}$$

at time

$$t_m = 18 \text{ minutes } 45 \text{ s after the start of the incident.}$$

Note that the time of maximum queue occurs after the incident has been cleared. This is because the clearing process does not affect behaviour at the tail of the queue until the wave of density $k = k^+(q_m)$ reaches it. Finally, from (71), the congested traffic clears the position of the incident at time

$$t_c = 30 \text{ minutes}$$

so that the effects of the incident last twice as long as the incident itself.

Suppose now that a diversion is implemented at position $x_e = -2$ miles, and that the diverted flow is $q_e = 1000$ vehicles/hour. This gives

$$S_e = 0.59 .$$

From (72), we find that the speed of the shock-wave caused by the intervention is given by

$$u_e = 33.38 \text{ miles/hour .}$$

We can determine the bounding conditions between the several cases of interaction between the shock-wave caused by the intervention with that at the back of the queue by calculating the critical values of the time of intervention from (76) and (92). In this example, $q_a - q_e < q_f$ so the back of the congested region will move downstream after the effect of the intervention reaches it, whenever that may be: accordingly, sub-case 1 of case 2 cannot occur. From (76), the latest time at which the intervention can be implemented in order to have an effect on the back of the congested region before the end of the incident does is $t = 16 \text{ minutes } 51 \text{ s}$, which is about 2 minutes after the incident itself has ended. According to (92), if the intervention is implemented after time $t = 18 \text{ minutes } 7 \text{ s}$, then the maximum upstream position of the back of the congested region is unaffected by the intervention. In order for the intervention to have any effect before the queue clears the location of the incident, we require from (88) that $t_e \leq 26 \text{ minutes } 19 \text{ s}$.

Suppose that the intervention occurs at a time $t_e = 20 \text{ minutes}$, which is before that determined by (88). The interaction between the shock wave caused by the intervention and the queue caused by the incident occurs after the queue starts to clear, but before it has cleared the location of the incident. According to (85), (86) and (87), the interaction occurs at time and position

$$t_r = 20 \text{ minutes } 49 \text{ s, and}$$

$$x_r = -1.56 \text{ miles .}$$

Because in this case (65) gives $t_m = 18 \text{ minutes } 45 \text{ s}$, $t_f \geq t_m$ so the back of the congested area is already moving forward at the time of the interaction, so the maximum upstream position of the back of the queue and its time of occurrence are unaffected by the intervention,

According to (82), the effects of the incident clear the location at which it occurred at time

$$t_c = 26 \text{ minutes } 41 \text{ s} ,$$

which is about 3 minutes 19 s earlier than would occur in the absence of an intervention.

According to (96), the earliest time that the intervention can be removed without affecting the clearing time t_c is

$$t_s = 23 \text{ minutes } 34 \text{ s} ,$$

and in that case, according to (97), the last effects of the intervention will clear the location of the incident at time

$$t_z = 28 \text{ minutes } 5 \text{ s} .$$

Example 2

The circumstances of this example are identical to those of example 1 except that $q_f = 1800$ vehicles/hour , corresponding to the closure of 2 lanes. Proceeding as before but using this value gives the downstream conditions

$$k_f = 30.5 \text{ vehicles/mile},$$

$$v_f = 59.0 \text{ miles/hour, and}$$

$$u_f = 39.8 \text{ miles/hour.}$$

The upstream conditions are

$$k_b = 301.8 \text{ vehicles/mile},$$

$$v_b = 5.96 \text{ miles/hour,}$$

$$u_b = -13.3 \text{ miles/hour, and}$$

$$S_f = 0.82 .$$

The furthest upstream that the back of the queue reaches is

$$x_m = -2.49 \text{ miles}$$

at time

$$t_m = 26 \text{ minutes } 15 \text{ s after the start of the incident.}$$

In this case the congested traffic clears the position of the incident at time

$$t_c = 1 \text{ hour}$$

so that in this case the effects of the incident last four times as long as the incident itself.

Suppose now that an intervention occurs at position $x_e = -5$ miles, and that as before, the diverted flow is $q_e = 1000$ vehicles/hour. In this case,

$$S_f = 0.82, \text{ and} \\ u_b = -0.22 \text{ miles/minute,}$$

so that the back of the queue would reach the point of intervention at time $t = 13$ minutes 34 s .

In this example, suppose that the intervention occurs at time $t_e = 5$ minutes which is before the end of the incident. Then the shock wave caused by the intervention will interact with that at the back of the queue caused by the incident before clearing starts. From (74) and (75), the time and position of this interaction will be

$$t_g = 10 \text{ minutes } 6 \text{ s , at} \\ x_g = -2.23 \text{ miles.}$$

The effects of clearing will reach the back of the queue at time and position given by (78) and (79) as

$$t_r = 18 \text{ minutes } 42 \text{ s , at} \\ x_r = -3.27 \text{ miles.}$$

In this case, the queue continues to grow after the intervention starts to effect it, so the position of the maximum queue $x = x_m$ and time of occurrence $t = t_m$ are given by (84) and (85):

$$t_m = 20 \text{ minutes } 14 \text{ s , at} \\ x_m = -3.36 \text{ miles.}$$

According to (82), the effects of the incident clear the location at which it occurred at time

$$t_c = 35 \text{ minutes } 53 \text{ s} ,$$

which is about 24 minutes 6 s earlier than would occur in the absence of an intervention.

According to (96), the earliest time that the intervention can be removed without affecting the clearing time t_c is $t_s = 28 \text{ minutes } 5 \text{ s}$ after the start of the incident and in that case the last effects of the intervention will clear the location of the incident at time

$$t_z = 39 \text{ minutes } 26 \text{ s} .$$

CHAPTER 6

CONCLUSIONS

The analysis presented in this report has shown how the wave model of traffic flow *can* be used to represent the formation, propagation and decay of congestion caused by an incident that reduces the capacity of a road system. Because it is a kinematic model, the description is spatio-temporal, so that the effect of the incident on traffic flow at different locations can be estimated. The effect of introducing a diversion of traffic upstream of the incident can also be represented in this model. In this report, consideration has been given to estimating critical times before which the intervention should be made in order to reduce the maximum extent of the congestion or to reduce the duration of congestion at the site of the incident. Similarly, attention is paid to the earliest time at which the intervention can be cancelled without extending the duration of the congestion at the location of the incident: this will always be before the congestion clears. Because the intervention is normally made some distance upstream of the incident, a spatio-temporal model is required for analysis of this kind.

Explicit equations can be derived for several of the quantities of interest without reference to any specific speed-density relationship. If Greenshields's linear speed-density model is adopted, then closed-form relationships can be derived for each of the quantities of interest in the analysis of incidents and interventions: this approach has been pursued in the present report.

Some example calculations are presented in this report. They show that interventions of a size that could reasonably be made can have a substantial effect on the congestion caused by an incident. Even if the intervention is implemented after the cause of the incident has been removed, it can hasten the clearance of congestion at the site of the incident.

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