Title
Essays on the Macroeconomics of the Labor Market, Human Capital, and Markets with Frictions

Author
Jackson, Paul

Publication Date
2020

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA,
IRVINE

Essays on the Macroeconomics of the Labor Market, Human Capital, and Markets with Frictions

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

Paul Gabriel Jackson

Dissertation Committee:
Professor Guillaume Rocheteau, Chair
Associate Professor Damon Clark, Co-chair
Distinguished Professor David Neumark
Associate Professor Victor Ortego-Marti

2020

DEDICATION

For my parents, in recognition of their love, support, and teaching the value of education.

TABLE OF CONTENTS

LIST OF FIGURES

Page

LIST OF TABLES

ACKNOWLEDGMENTS

First and foremost, I thank my wife, Maggie, for her love and support over the last five years. There were some difficult times throughout writing this dissertation where it was hard to push forward and make progress. She always showed patience and helped me carry on. I'm lucky and grateful to have that support.

My family, Mom, Dad, and David, have always supported me finding my own path and pursuing my passions. I would not be at this point without that support and I can't say thank you enough to them for all they have done to help me reach the end of the Ph.D.

I'm deeply indebted to Guillaume Rocheteau, who has provided a tremendous amount of support and guidance throughout the process of writing this dissertation and becoming a research economist. It hasn't always been easy, but I thought working with Guillaume would bring the best out of me and I'm grateful that turned out to be true.

I also appreciate the support from my committee, Damon Clark, David Neumark, and Victor Ortego-Marti. Each of them were heavily involved in my research progress at various points throughout writing the dissertation and I'm grateful for their perspectives and support.

I thank my friend and co-author, Florian Madison. We have had countless conversations about economics over the years that have changed the way I view questions and made me a much better economist. I'm looking forward to working together and continuing those conversations for years to come.

To all participants and organizers of the Macro Ph.D. workshop at UCI over the last few years, I thank you for your attendance, participation, questions, and feedback over the years. You played a large role in shaping the research in this dissertation.

I would not be here today without Wayne Nirode, who set me on the path towards studying economics by bringing AP Economics to Troy High School.

Financial support from Department of Economics during the 2015-2018 summers, the Sheen T. Kassouf Fellowship, the UCI School of Social Sciences Fellowship in Honor of Christian Werner, and for the Associated Graduate Students is gratefully acknowledged.

VITA

Paul Gabriel Jackson

EDUCATION

Ph.D. Economics **2020**
University of California, Irvine

M.A. Economics **2015**
University of California, Irvine

B.S. Economics **2014**
The Ohio State University

RESEARCH FIELDS

Macroeconomics, Labor Economics, Monetary Economics

ACADEMIC PUBLICATIONS

Revisiting Unemployment in Intermediate Macroeconomics: A New Approach for Teaching Diamond-Mortensen-Pissarides **2017**
Journal of Economic Education

MANUSCRIPTS IN PREPARATION

"Equilibrium Underemployment"

"The Impact of the Great Recession on Educational Attainment: Evidence from Florida"

"Entrepreneurial Finance, Home Equity, and Monetary Policy" (with Florian Madison)

"Search Intensity of Employed Workers"

VISITING POSITIONS

Visiting Doctoral Scholar, University of Basel **2017**

CONFERENCES AND SEMINARS

Carleton College; Grinnell College; University of San Diego; National University of Singapore; Federal Reserve Bank of Boston; UC Riverside **2020**

3rd Dale T. Mortensen Centre Conference; West Coast Search and Matching Workshop Washington University in St. Louis Grad Student Conference; 9th European Search & Matching Workshop; Paris II; Midwest Macro; UC Irvine; Econometric Society **2019**

Econometric Society; UC Irvine **2018**

4th Macro Marrakesh Workshop **2017**

Undergraduate Economics Society, Ohio State **2014**

TEACHING EXPERIENCE

Teaching Assistant **2014–2019**
University of California, Irvine

Instructor for UCI Macro Summer School **2018**

PEDAGOGICAL DEVELOPMENT

Teaching Assistant Professional Development Program **2018**
University of California, Irvine

Pedagogical Fellows Program **2018**
University of California, Irvine

AWARDS

Sheen T. Kassouf Fellowship	**2019**
Charles A. Lave Prize for Creative Modeling in Social Sciences	**2018**
UCI School of Social Sciences Fellowship in Honor of Christian Werner	**2018**
UCI Pedagogical Fellowship	**2018**
Best Teaching Assistant, UCI Department of Economics	**2018**
Summer Research Fellowship, UCI Department of Economics	**2016-2018**
Distinction on Microeconomics & Macroeconomics Qualifying Exams	**2015**
Merit Fellowship, UC Irvine	**2014-2020**
Undergraduate Research Award, Ohio State	**2014**

ABSTRACT OF THE DISSERTATION

Essays on the Macroeconomics of the Labor Market, Human Capital, and Markets with Frictions

By

Paul Gabriel Jackson

Doctor of Philosophy in Economics

University of California, Irvine, 2020

Professor Guillaume Rocheteau, Chair

This dissertation studies several topics: the macroeconomics of the labor market, human capital accumulation, and entrepreneurial finance using both theoretical and empirical methods. The common theme across the theoretical chapters is the use of search theory, i.e. the view that there are various frictions in markets that prevent instantaneous trade. The first chapter develops a search model of the labor market to rationalize why employed workers would exert more effort into looking for a job in a downturn when there are less jobs available. The mechanism that can explain this observation is that workers near the bottom of the job ladder will exert more effort in a job search to avoid becoming unemployed, even if there are relatively few jobs available. In the second chapter, I draw on empirical methods popular within labor economics to estimate the cyclicality of human capital investment. A key innovation in my empirical approach is to focus on one state, Florida, which helps to simplify the interpretation of any estimated relationship between the state of the economy and human capital investment. I find little evidence to suggest that measures of human capital investment respond to changes in macroeconomic conditions. In the third chapter, I study a phenomena called underemployment, which occurs when a worker is employed in a job that requires less education than they have. A calibrated version of the model implies that the U.S. exhibits too little human capital investment and too much underemployment,

an issue that is only exacerbated by subsidizing human capital investment. In the last chapter, I develop a monetary search model of entrepreneurial finance where entrepreneurs can use home equity to secure capital. A crash in the pledgeability of housing can have large impacts on entrepreneurial investment and output, especially if entrepreneurs face a high interest rate environment and are more reliant on external finance.

Chapter 1

Search Intensity of Employed Workers

This chapter studies search intensity among workers searching on the job. The Mortensen and Pissarides (1994) model of the labor market with ex-post random matching is extended to allow employed workers to choose their search intensity and to include bargaining over a pairwise Pareto-efficient contract. The model is used to study, both analytically and quantitatively, the effect of changes in labor market fundamentals on search intensity. The model can help explain recent empirical evidence that documents a countercyclical pattern in search intensity among employed workers and offers new insights on the relationship between search effort among employed workers and entry of firms.

1.1 Introduction

In the U.S., job-to-job transitions are abundant and account for nearly one-half of all new hires.[1] While the prevalence of these transitions has been known for some time, more de-

[1] See Parsons (1973, 1991), Pissarides and Wadsworth (1994), Fallick and Fleischman (2004), Nagypál (2008), and Hall and Schulhofer-Wohl (2018).

tailed evidence on the behavior among workers searching on the job has recently emerged.[2] In particular, Elsby et al. (2015) and Ahn and Shao (2017) find that search intensity among employed workers is countercyclical.[3] Additional search effort from employed workers could cause congestion for unemployed workers and further increase the average duration of unemployment during recessions.

While this new evidence is informative, it has exposed a gap between the theory and evidence on the cyclicality of search effort, as baseline models predict that search effort is procyclical.[4] The intuition is straightforward: in good times, there are many job opportunities, so the marginal benefit of exerting more effort into a job search is higher. In recessions, where opportunities are scarce, workers should exert less effort in looking for a job.[5]

The purpose of this paper is simple: I develop a model that can help explain the recent evidence on search intensity among employed workers. A model that is successful at this will offer an economic intuition as to why employed workers search more intensely during recessions and could lead to quantitative predictions as to how much the search intensity decision of employed workers contributes to increased unemployment durations in recessions.

I study this question within the Diamond-Mortensen-Pissarides (DMP) framework.[6] In particular, I consider an economy similar to Pissarides (1985) with the following ingredients: ex-post match heterogeneity, on-the-job search, and two types of matches: low and high productivity. Some workers will find themselves in a low productivity job and, by exerting more effort in their job search, they increase their chance of finding a new, high productivity

[2] Faberman et al. (2017) find that on-the-job search is prevalent, with 23% of employed workers actively looking for another job. They also find evidence that employed workers are more efficient in their job search, as they tend to generate more offers per unit of search intensity.

[3] Shimer (2004), Krueger and Mueller (2010), Aguiar et al. (2013), and Mukoyama et al. (2018) have also found a countercyclical pattern in search intensity among unemployed workers.

[4] See, for example, ch. 4 of Pissarides (2000).

[5] This is if searching for a job is costly.

[6] An alternative framework used for studying on-the-job search is Burdett and Mortensen (1998). This framework usually assumes an exogenous arrival rate of outside offers to those searching on the job. I use the DMP framework because it generates an endogenous arrival rate of job offers for both employed and unemployed workers through free entry of firms.

match.

I show that the search intensity among employed workers can increase when the output produced in a low productivity match decreases. This result has two interpretations. The first is that when workers in the low productivity match produce less output, the surplus from moving from the low to high productivity job increases, causing workers to increase their search intensity. The second is that when the output in a low productivity match decreases, the surplus in the match decreases, causing workers to exert more effort to leave a job that is close to not generating a positive surplus.[7]

Search effort may not always increase following a decline in the output produced in the low productivity jobs. This is because a decrease in the output in a low productivity match may cause less firms to post vacancies, which reduces the returns to searching more intensely. Whether the search intensity among employed workers increases following such a change in the economy depends on the relative sizes of the effect on a workers' optimal search intensity decision and the effect of a decline in the output in a low productivity match on firm entry. To address this ambiguity, I calibrate the model to U.S. labor market data.

A calibrated version of the model shows a few examples of search intensity increasing following a decline in output produced in low productivity matches. The mechanism which emerges is that the change in worker's optimal search decision can outweigh a decrease in firm entry, as workers search more to leave their current position. I interpret this as workers searching more intensely when their job becomes less productive relative to others in the economy, and to avoid unemployment as their job becomes closer to not generating a positive amount of surplus.[8]

[7]In a model with endogenous job destruction, these workers would face a higher job destruction rate following a decline in the output unique to their match.

[8]This is consistent with empirical evidence reported in Fujita (2010), in that a non-trivial fraction of workers search on the job because they fear losing their current job or are not satisfied with the quality of their current match.

A distinctive feature of this paper is how bargaining over employment contracts is modeled. In the DMP framework, wages are typically part of an employment contract that specifies a flat wage. In a model of on-the-job search, this contract is not pairwise Pareto-efficient (Stevens, 2004a) and the set of feasible payoffs is not guaranteed to be convex (Shimer, 2006a; Bonilla and Burdett, 2010). I allow the worker and firm to bargain over a contract which specifies a flat wage paid to the worker and a hiring fee paid by the worker to the firm. As shown in Stevens (2004a) and Shimer (2006a), the contract is pairwise Pareto-efficient and the bargaining set is convex.[9] A second feature of bargaining in this paper is that employed workers use their current value of employment as their disagreement payoff.[10] This follows from Hall and Krueger (2012), who show that many employed workers who bargain with a prospective employer could return to their current job if negotiations broke down. I find that this formulation leads to new insights, particularly that there can be a non-monotonic relationship between search effort among employed workers and firm entry.

Related Literature This paper is most closely related to Nagypál (2005) and Ahn and Shao (2017). Both of these papers developed models of search intensity among employed workers and showed that search intensity among employed workers may increase as a means to avoid unemployment. Nagypál (2005) develops a model where the value of an employment relationship is subject to negative shocks. When a job is hit by a negative shock, workers increase their search effort to find a new job before they lose their own. While I find a similar mechanism, I find that this can occur when the output produced in all low productivity jobs declines, not just when a single job is hit by an idiosyncratic shock. I argue that this is important because it shows that search effort can increase when a significant fraction of jobs in the economy are hit by a shock and that search effort can increase despite a decline in market tightness. The model presented in Ahn and Shao (2017) is partial-equilibrium and

[9] This optimal contract was also applied in Engelhardt et al. (2008).

[10] Other models of on-the-job search such as Pissarides (1994a) and Nagypál (2007) model the disagreement point of employed workers as the value of unemployment, implying that the worker receives the value of unemployment during negotiations and would return to unemployment if negotiations broke down.

does not explain the joint determination of search intensity, the arrival rate of job offers (through firm entry), and wages. In this setup, one cannot explain why search intensity may increase despite a decline in market tightness.

1.2 Environment

Time is continuous and goes on forever. There are two types of agents: a large measure of firms and a unit measure of workers. All agents are risk neutral and discount the future at rate $r > 0$.

Workers can be unemployed or employed. Unemployed workers receive a flow payoff b,[11] and are endowed with $\rho_u \in (0, 1]$ units of search intensity. Firms pay a flow cost γ to fill their vacancy. Vacancies produce no output.

When a worker and firm meet, they draw productivity p_i with probability α_i for $i \in \{l, h\}$ and $p_l < p_h$. If the worker and firm form a match, they produce a flow output $p_i z$ where z is a measure of aggregate productivity and $z(\alpha_l p_l + \alpha_h p_h) > b$. Workers employed at the low productivity job are endowed with $\rho_e \in [0, 1]$ units of search intensity. Filled jobs receive destruction shocks according to a Poisson process with arrival rate δ.

Matching function The labor market is subject to search and matching frictions. To account for these frictions, the flow of contacts between workers and vacancies is governed

[11]The flow utility b can be interpreted as unemployment benefits, the value of home production, and leisure. Common across the components of b are that they are given up when the worker becomes employed.

by the matching function:

$$m = m\bigg(\int_0^1 s(i)di, v\bigg), \tag{1.1}$$

$$s(i) = \begin{cases} \rho_u & \text{if unemployed,} \\ \rho_e & \text{if employed at } p_l, \\ 0 & \text{if employed at } p_h, \end{cases} \tag{1.2}$$

and where v is the measure of vacancies. The matching function is assumed to be continuous, strictly increasing in each input, strictly concave with respect to each input, and exhibits constant returns to scale. Also, $m(0,\cdot) = m(\cdot,0) = 0$ and $m(\infty,\cdot) = m(\cdot,\infty) = \infty$.

Define $\theta \equiv {}^{v}/_{\rho_u u + \rho_e e_l}$ as market tightness where u is the measure of unemployed workers and e_l is the measure of employed workers searching on the job. Let $q(\theta) \equiv \frac{m(\rho_u u + \rho_e e_l, v)}{v}$. Firms contact unemployed workers according to a Poisson process with arrival rate $q_u(\theta) \equiv \frac{\rho_u u}{\rho_u u + \rho_e e_l} q(\theta)$ and workers searching on the job according to a Poisson process with arrival rate $q_e(\theta) \equiv \frac{\rho_e e_l}{\rho_u u + \rho_e e_l} q(\theta)$.

Let $f(\theta) \equiv \frac{m(\rho_u u + \rho_e e_l, v)}{\rho_u u + \rho_e e_l}$. Unemployed workers meet firms according to a Poisson process with arrival rate $\rho_u f(\theta)$ and workers searching on the job meet firms according to a Poisson process with arrival rate $\rho_e f(\theta)$.

1.2.1 Employment Contracts

This paper departs from the common approach to wage determination in search models of the labor market, where wages are usually determined through a contract that only specifies a flat wage. Typically, a flat wage contract will achieve the same division of the surplus as a contract with tenure dependent wages and a hiring fee because of agents' risk neutrality. In these cases, there is no need to go beyond the contract with a flat wage.

As shown in Stevens (2004a), in a model with on-the-job search, the bargaining set over a contract with a flat wage may not achieve a pairwise Pareto-efficient outcome. This is because a worker does not account for the turnover costs that the firm will have to pay following a quit. Also, the feasible set payoffs when bargaining over a flat wage need not be convex (Shimer, 2006a; Bonilla and Burdett, 2010). This is because an incumbent firm could offer a raise to a worker who has an outside offer to keep the worker from quitting. The possibility of giving a small raise to avoid large turnover costs creates a discontinuity in the firm's set of feasible payoffs.[12]

Stevens (2004a) shows that when an employment contract specifies a one-time hiring fee and a flat wage, the worker and firm can reach a pairwise Pareto-efficient outcome. Given that this is the appropriate employment contract for an environment with on-the-job search, it is the one I implement.[13]

The one time hiring fee compensates a firm when they hire a worker who searches on the

[12]Nagypál (2007) argues that if there is a lack of commitment to carry out the contract then there is no reason to depart from the flat wage contract. I argue that a contract is a commitment and could be enforced by relevant institutions.

[13]There are a few alternatives to determine wages that do not follow the axiomatic approach of Nash bargaining. Shimer (2006a) determines wages through an infinitely repeated bargaining game of alternating offers. Others, such as Postel-Vinay and Robin (2002, 2004) and Cahuc et al. (2006), model the wage as the outcome of Bertrand price competition when a worker is contacted by an outside firm. I do not allow workers to exploit outside offers for two reasons. The first is that Fujita (2010) found little evidence of firms making counteroffers. The second is that allowing for counteroffers gives workers an incentive to inefficiently search on the job leading to inefficient rent sharing between the worker and the firm (Mortensen, 1978). The third is that the model would not be as analytically tractable.

job and eventually quits and causes the firm to pay turnover costs. While this may seem empirically extreme, I claim that it is a simplification of tenure contracts which specify an upward sloping wage in tenure as in Burdett and Coles (2003a). A worker who pays an upfront hiring fee today and is paid a higher wage tomorrow approximates an upward sloping wage which increases between today and tomorrow.

Also, many models of on-the-job search allow a worker who is currently employed to use the value of unemployment as their disagreement payoff.[14] The interpretation of this is that in a Rubinstein (1982) infinitely repeated game of alternating wage offers with a risk of breakdown, the worker would return to unemployment if negotiations were to break down.[15]

This paper assumes the following underlying bargaining structure between a firm and employed worker. When a worker and firm meet, they play an infinitely repeated bargaining game of alternating wage offers with an exogenous risk of breakdown. If negotiations break down, the worker would return to their current job and the firm returns to the labor market with a vacancy. Another interpretation of this is that the worker continues to earn the payoff of employment while bargaining.[16] As shown in Hall and Krueger (2012), this is empirically supported by the fact that a non-trivial fraction of workers who search on the job say that they could have returned to their current employer if negotiations broke down.

1.3 Equilibrium

I focus on stationary steady-state equilibria, where the distribution of workers across their respective states and market tightness are constant over time.

[14] See, among others, Pissarides (1994a), Barlevy (2002), and Nagypál (2005, 2007).

[15] As discussed in Binmore et al. (1986), this also means that the unemployed worker enjoys the flow value of unemployment while bargaining.

[16] It is also true that the firm who currently employs the worker would be indifferent between retaining and firing the worker if they are found to have bargained with another firm and negotiations broke down. This is because the firm has already earned their share of the match surplus from the hiring fee.

1.3.1 Bellman Equations

Firms A firm can be in one of the following states: (i) vacancy or (ii) filled job with a worker at productivity i. The value of being in each state is $\mathcal{V}$ and $\mathcal{J}_i$, respectively. The flow Bellman equations for the firms are

$$r\mathcal{V} = -\gamma + q_u(\theta) \sum_{i:p_i \geq p^*} \alpha_i[\mathcal{J}_i - \mathcal{V} + \phi_u(p_i)] + q_e(\theta)\alpha_h[\mathcal{J}_h - \mathcal{V} + \phi_e(p_h)], \tag{1.3}$$

$$r\mathcal{J}_l = p_l z - w_l - [\delta + \rho_e f(\theta)\alpha_h][\mathcal{J}_l - \mathcal{V}], \tag{1.4}$$

$$r\mathcal{J}_h = p_h z - w_h - \delta[\mathcal{J}_h - \mathcal{V}]. \tag{1.5}$$

From (1.3) the firm pays a vacancy filling cost γ while waiting to meet a worker. At rate $q_u(\theta)$ the firm meets a worker who is currently unemployed. Upon meeting an unemployed worker, the firm and worker draw productivity p_i with probability α_i. If $p_i \geq p^*$, the match generates a positive surplus and is created. The firm enjoys a capital gain of the profits a filled job and hiring fee from the unemployed worker net the value of an open vacancy, $\mathcal{J}_i - \mathcal{V} + \phi_u(p_i)$. A firm with an open vacancy meets a worker who is currently employed at rate $q_e(\theta)$. The firm will be able to poach the worker if they draw the high productivity, which occurs with probability α_h. In this case, the firm enjoys a capital gain of the profits of a filled job at the high productivity and the hiring fee from the employed worker net the value of an open vacancy, $\mathcal{J}_h - \mathcal{V} + \phi_e(p_h)$.

Equation (1.4) shows that a firm in the low productivity match earns flow profits of $p_l z - w_l$ and the firm experiences a capital loss of the profits of a filled job net the value of vacancy when the job is destroyed, which occurs at rate δ, or when their employee meets another firm and draws the high productivity. This occurs at rate $\rho_e f(\theta)\alpha_h$.

According to (1.5), a firm with a high productivity job enjoys flow profits of $p_h z - w_h$ until the match is destroyed, at rate δ, and experiences a capital loss of the value of the filled job net the value of a vacancy, $\mathcal{J}_h - \mathcal{V}$.

Workers Workers can be in one of following states: (i) unemployed or (ii) employed at productivity i. The value of being in each state is $\mathcal{U}$ and $\mathcal{W}_i$, respectively. The flow Bellman equations for workers are

$$r\mathcal{U} = b + \rho_u f(\theta) \sum_{i:p_i \geq p^*} \alpha_i[\mathcal{W}_i - \mathcal{U} - \phi_u(p_i)], \tag{1.6}$$

$$r\mathcal{W}_l = w_l - \delta[\mathcal{W}_l - \mathcal{U}] + \rho_e f(\theta)\alpha_h[\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h)], \tag{1.7}$$

$$r\mathcal{W}_h = w_h - \delta[\mathcal{W}_h - \mathcal{U}]. \tag{1.8}$$

Equation (1.6) shows that an unemployed worker receives a flow utility b and meets a firm at rate $\rho_u f(\theta)$. The worker and firm then draw a productivity p_i. If $p_i \geq p^*$, the match is created. The worker enjoys a capital gain of the value of working net the value of being unemployed and the hiring fee paid to the firm, $\mathcal{W}_i - \mathcal{U} - \phi_u(p_i)$.

According to (1.7), a worker employed at the low productivity job earns a wage w_l. At rate δ, the job is destroyed and the worker experiences a capital loss of the value of working net the value of unemployment, $\mathcal{W}_l - \mathcal{U}$. At rate $\rho_e f(\theta)$, the worker finds an outside offer. With probability α_h, the worker and new firm will draw the high productivity. In this case, the worker leaves their current job for the new one. Upon moving to the new job, the worker experiences a capital gain of the value of working at the high productivity job net the value at the low productivity job and the hiring fee paid to the new firm, $\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h)$. Notice that the worker will quit if and only if

$$\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h) \geq 0. \tag{1.9}$$

From (1.8), a worker employed at the high productivity job earns a wage w_h until the job is destroyed, which occurs at rate δ. Upon losing their job, the worker receives a capital loss of the value of working at the high productivity job net the value of unemployment, $\mathcal{W}_h - \mathcal{U}$.

1.3.2 Employment Contracts

To determine the optimal contact, define $\mathcal{S}_u(p_i) = \mathcal{W}_i - \mathcal{U} + \mathcal{J}_i$ as the total surplus of a match between a firm and worker hired from unemployment. From (1.4) and (1.7),

$$r\mathcal{S}_u(p_l) = p_l z - r\mathcal{U} - \delta\mathcal{S}_u(p_l) + \rho_e f(\theta)\alpha_h[\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h) - (\mathcal{J}_l - \mathcal{V})]. \tag{1.10}$$

Equation (3.11) has the following interpretation. A match with an unemployed worker at productivity p_l generates a flow surplus $p_l z - r\mathcal{U}$. The match is destroyed and the surplus is lost at rate δ. The worker receives an outside offer, draws the high productivity, and quits at rate $\rho_e f(\theta)\alpha_h$. Following a quit, the worker gains the surplus $\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h)$ and the firm has a capital loss of $\mathcal{J}_l - \mathcal{V}$. From (1.5) and (1.8),

$$r\mathcal{S}_u(p_h) = p_h z - r\mathcal{U} - \delta\mathcal{S}_u(p_h). \tag{1.11}$$

Equation (1.11) has the following interpretation: a high productivity match with an unemployed worker creates a flow surplus of $p_h z - r\mathcal{U}$, and the match is destroyed at rate δ.

If the worker and firm could jointly decide when the worker quits, they would choose the opportunities for which (3.11) is maximized. From (3.11), the surplus of the match is maximized if

$$\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h) \geq \mathcal{J}_l - \mathcal{V}. \tag{1.12}$$

Comparing equations (3.13) and (3.12) show that if $\mathcal{J}_l - \mathcal{V} > 0$, the worker's decision to quit is not aligned with the optimal decision for the match. This occurs when the firm makes higher profits with a filled job than a vacancy and the worker does not internalize the effect of their quit on the firm, namely the reduction in profits the firm faces following a quit.

The worker and firm can reach a pairwise agreement over an employment contract that achieves efficient separations. This is done by bargaining over an employment contract which specifies a hiring fee and a constant wage paid to the worker. The contract satisfies the following generalized Nash solution where $\beta \in [0, 1]$ is the worker's bargaining power,

$$(w_i, \phi_u(p_i)) \in \arg\max(\mathcal{W}_i - \mathcal{U} - \phi_u(p_i))^{\beta}(\mathcal{J}_i + \phi_u(p_i))^{1-\beta}. \tag{1.13}$$

Lemma 1.1. *The employment contract as the solution to (3.14) is*

$$w_i = p_i z, \tag{1.14}$$

$$\phi_u(p_i) = (1-\beta)[\mathcal{W}_i - \mathcal{U}]. \tag{1.15}$$

Proof. See Appendix A.1.1. □

Proofs of all lemmas and propositions can be found in the appendix. We see that the worker earns the entire output of the match so that $\mathcal{J}_i = 0$ and internalizes the effect of their decision to quit on the total surplus of the match. The hiring fee will split the surplus of the match according the worker and firm's bargaining power.[17]

Defining $\mathcal{S}_e(p_h) = \mathcal{W}_h - \mathcal{W}_l + \mathcal{J}_h$ as the surplus generated in a match between a worker hired from employment to the high productivity job, where from (1.5) and (1.8),

$$r\mathcal{S}_e(p_h) = p_h z - r\mathcal{W}_l - \delta\mathcal{S}_e(p_h). \tag{1.16}$$

Equation (1.16) shows that a match generates a flow surplus of $p_h z - r\mathcal{W}_l$ and the surplus is destroyed at rate δ.[18] A worker employed at p_l and firm can reach a pairwise Pareto-efficient agreement by bargaining over a two-part employment contract. The intuition for the contract is the same as with the unemployed worker and satisfies the following generalized

[17]Note that the employment contract also applies to workers who are hired from unemployment to the high productivity job. This is because from (1.11), the surplus of a match is the same regardless of the value of $\mathcal{J}_h$.

[18]Notice that the flow surplus takes into account that an employed worker's disagreement payoff is the value of working at the low productivity job.

Nash solution,

$$(w_h, \phi_e(p_h)) \in \arg\max(\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h))^{\beta}(\mathcal{J}_h + \phi_e(p_h))^{1-\beta}. \tag{1.17}$$

Lemma 1.2. *The employment contract as the solution to (1.17) is*

$$w_h = p_h z, \tag{1.18}$$

$$\phi_e(p_h) = (1-\beta)[\mathcal{W}_h - \mathcal{W}_l]. \tag{1.19}$$

Proof. See Appendix A.1.2. □

The contract still ensures that the worker earns the entire output and $\mathcal{J}_h = 0$. Comparing (1.19) with (3.16), the hiring fee depends on whether the worker is hired from unemployment or employment. Since the worker who is currently employed uses their current value of working as a disagreement payoff, the match surplus and hiring fee paid to the firm will be lower in a match where firms hire workers from employment. The implications of this result will be discussed in Section 1.3.5.

Discussion When a worker is hired and draws the high productivity, they do not search on the job and thus there are no voluntary separations. In this case it may seem unnecessary to specify a contract in which the worker pays a hiring fee to the firm. It can be shown that in these cases, the employment contracts are payoff-equivalent to the Nash bargaining solution over a contract which only specifies a flat wage and no hiring fee. For the sake of consistency and allowing the worker and firm to bargain over an optimal contact in all situations, I implement the generalized Nash solution described above for all employment contracts.

1.3.3 Worker Flows

Unemployment Workers flow in to unemployment at a rate δ. Workers find a job when they get an offer, at rate $\rho_u f(\theta)$ and when the productivity they draw is large enough to start a match. The law of motion of unemployment is

$$\dot{u} = \delta(1-u) - \rho_u f(\theta)(\alpha_l \mathbb{1} + \alpha_h)u, \tag{1.20}$$

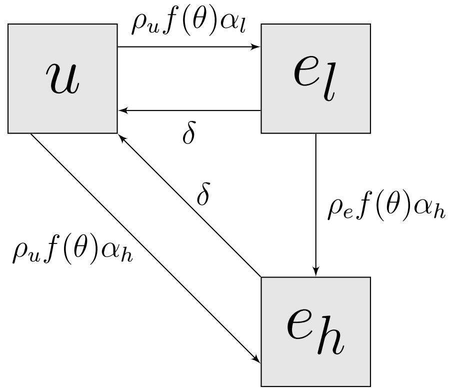
where u is the measure of unemployed workers and $\mathbb{1}$ is an indicator function that equals 1 when the low productivity match generates a positive surplus and 0 when it does not. This gives a steady unemployment rate of

$$u = \frac{\delta}{\delta + \rho_u(\alpha_l \mathbb{1} + \alpha_h)f(\theta)}. \tag{1.21}$$

Employment Workers flow in to employment at the low productivity job when they meet an employer and draw the low productivity. This occurs at rate $\rho_u f(\theta)\alpha_l$. They flow out of this state when they meet another employer and draw the high productivity or when the job is destroyed. This occurs at rate $\rho_e f(\theta)\alpha_h + \delta$. Let e_l be the measure of workers employed at the low productivity match. The law of motion of workers employed in the low productivity match is given by

$$\dot{e}_l = \rho_u f(\theta)\alpha_l u \mathbb{1} - (\rho_e f(\theta)\alpha_h + \delta)e_l. \tag{1.22}$$

Figure 1.1: Worker Flows



Solving for e_l such that $\dot{e}_l = 0$ gives the steady state measure of workers employed in the low productivity job

$$e_l = \mathbb{1}\frac{\alpha_l \rho_u f(\theta) u}{(\rho_e f(\theta)\alpha_h + \delta)}. \tag{1.23}$$

Finally, let e_h be the measure of workers employed at the high productivity job. It follows that

$$e_h = 1 - u - \mathbb{1}e_l. \tag{1.24}$$

A summary of worker flows is shown in Figure 1.1.[19]

Composition of job seekers With the distribution of workers defined, it is possible to solve for the ratio of each type of job seeker to all job seekers in the economy. Here, I assume

[19]This implicitly assumes that low productivity matches generate a positive surplus.

that the low productivity job generates a positive surplus. From (1.21) and (1.23),

$$\frac{\rho_u u}{\rho_u u + \rho_e e_l} = \frac{\rho_e f(\theta)\alpha_h + \delta}{\rho_e f(\theta) + \delta}, \tag{1.25}$$

$$\frac{\rho_e e_l}{\rho_u u + \rho_e e_l} = \frac{\rho_e f(\theta)\alpha_l}{\rho_e f(\theta) + \delta}. \tag{1.26}$$

Equation (1.25) shows the ratio of the measure of unemployed workers to effective measure of all job seekers, and the ratio of workers searching on the job to all job seekers is given by (1.26). From the firm's perspective, these ratios represent the probability of contacting each type of job seeker, conditional on meeting a worker.

1.3.4 Free Entry

Free entry of firms implies that the expected profits of posting a vacancy will be equal to zero, i.e. $\mathcal{V} = 0$. From equation (1.3), and substituting the solution to the optimal contracts, free entry of firms gives

$$\frac{\gamma}{1-\beta} = q_u(\theta) \sum_{i:p_i \geq p^*} \alpha_i [\mathcal{W}_i - \mathcal{U}] + q_e(\theta)\alpha_h[\mathcal{W}_h - \mathcal{W}_l]. \tag{1.27}$$

Further simplification of equation (1.27) gives,[20]

$$\frac{\gamma}{(1-\beta)q(\theta)} = \frac{\rho_e f(\theta)\alpha_h + \delta}{\rho_e f(\theta) + \delta}\left[\frac{z(\alpha_l p_l + \alpha_h p_h) - b}{r + \delta + \beta\rho_u f(\theta)}\right]$$
$$+ \frac{z(p_h - p_l)f(\theta)\rho_e\alpha_l\alpha_h}{(\rho_e f(\theta) + \delta)(r + \delta + \beta\rho_e f(\theta)\alpha_h)}\left[\frac{(r + \delta + \beta\rho_u f(\theta)) + \beta(\rho_e f(\theta)\alpha_h + \delta)}{r + \delta + \beta\rho_u f(\theta)}\right]. \quad (1.28)$$

Equation (1.28) determines the equilibrium value of market tightness, θ. The left-hand side of (1.28) is the cost that a firm expects to pay while waiting to fill a vacancy.[21] The right-hand side of (1.28) is the expected surplus of a filled job. The first term of the right-hand side is the surplus from hiring an unemployed worker and the second term is the surplus created when the firm poaches an employed worker. Each of the surpluses are weighted for the respective probability that this particular type of worker is hired. Under free entry, enough firms will enter the market so that the expected profits of a filled job are equal to the costs incurred to fill a vacancy.

Definition 1.1. *A steady state equilibrium is a list* $\{\theta, u, e_l, e_h\}$ *such that:* θ *satisfies* (1.28)*,* u *satisfies* (1.21)*,* e_l *satisfies* (1.23)*, and* e_h *satisfies* (1.24)*.*

1.3.5 Results

The model can be solved recursively by first determining θ from (1.28). Given θ, one can obtain the distribution (u, e_l, e_h) from (1.21), (1.23), and (1.24). The determination of θ is shown in Figure 1.2. The upward sloping cost curve represents the left-hand side of (1.28). Along the cost curve, as market tightness increases, firms face more competition, lower vacancy filling rates, and pay more to fill a vacancy. The downward sloping surplus curve represents the right-hand side of (1.28). Along the surplus curve, as market tightness

[20] See sections A.1.3-A.1.4 for more details on the derivation.

[21] The expected costs are adjusted by a factor of $\frac{1}{1-\beta}$. As $\beta \to 0$, the firm has more bargaining power and they face effectively lower costs to fill their vacancy.

increases, workers face higher job finding rates. This increases the workers' disagreement point and they can be choosier in their job search. The intersection of the cost and surplus curve gives the equilibrium value of market tightness, which can then be mapped in to the unemployment rate via the Beveridge curve.[22]

Proposition 1.1. *There exists an equilibrium such that* $\theta > 0$*. Suppose that the low productivity match is always formed. Comparative statics are summarized in Table 1.1:*

Table 1.1: Comparative statics: Exogenous Search Intensity

	γ	b	z	ρ_u	β	p_h
θ	$-$	$-$	$+$	$-$	$-$	$+$
u	$+$	$+$	$-$	$+$	$+$	$-$
e_l	$-$	$-$	$+$	$-$	$-$	$+$

Proof. See Appendix A.1.5. □

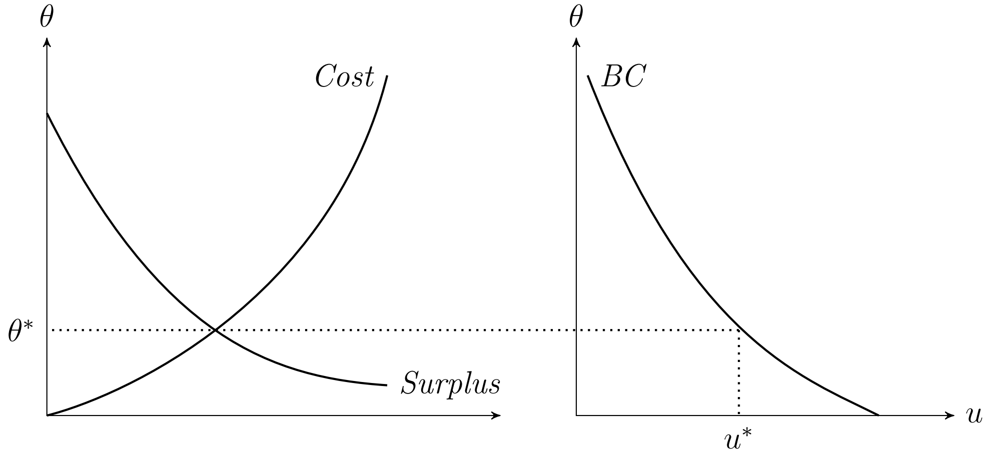
An increase in the search intensity of unemployed workers, ρ_u, decreases market tightness because higher search intensity of unemployed workers allows them to be choosier in their job search, which in turn decreases the surplus created in a match with an unemployed worker. An increase in b has a similar effect.

An increase in both z and p_h increases the surplus generated in all matches and thus, market tightness increases. As γ increases, firms will face higher costs to fill their vacancies, which causes market tightness to decrease. An increase in β means that firms receive a lower fraction of the surplus of a match, which results in a lower market tightness.

Search intensity of employed workers Suppose that the search intensity of employed workers, ρ_e, increases. One may think that this increases market tightness because it in-

[22]Strictly speaking, the Beveridge curve gives the equilibrium relationship between the vacancy rate and the unemployment rate. A modified Beveridge curve, as shown in Figure 1.2, shows the equilibrium relationship between market tightness and the unemployment rate.

Figure 1.2: Equilibrium



creases the pool of job searchers and thus leads to higher vacancy filling rates. On the other hand, market tightness may decrease because a match with a worker who is currently employed generates less surplus than a match with an unemployed worker. This occurs because employed workers' disagreement payoff is their current value of employment. It turns out that equilibrium market tightness is non-monotonic in the search intensity of employed workers, a key result summarized in the following Lemma.

Lemma 1.3. *Suppose that the search intensity of employed workers increases. If,*

1. $\alpha_l = 0$:

$$\frac{\partial \theta}{\partial \rho_e} = 0,$$

2. $\alpha_l = 1$:

$$\frac{\partial \theta}{\partial \rho_e} < 0.$$

3. Finally, there exists an $\alpha_l^* \in (0,1)$ and $\rho_e^* \in [0,1]$ such that

$$\left.\frac{\partial \theta}{\partial \rho_e}\right|_{\alpha_l^*, \rho_e \leq \rho_e^*} \geq 0, \ \left.\frac{\partial \theta}{\partial \rho_e}\right|_{\alpha_l^*, \rho_e > \rho_e^*} < 0.$$

Proof. See Appendix A.1.6. □

Lemma 1.3 is represented graphically in Figure 1.3. Over $\rho_e \geq 0$, market tightness is constant for $\alpha_l = 0$, decreasing for $\alpha_l = 1$, and non-monotonic for $\alpha_l = \alpha_l^*$. In this case, θ increases for $\rho_e \in [0, \rho_e^*)$ and then decreases for $\rho_e \geq \rho_e^*$. The intuition behind this result is as follows.

If firms never expect to meet a worker searching on the job, $\alpha_l = 0$, their search intensity will have no effect on the expected surplus in a match and hence no effect on firm entry and market tightness. If firms only meet workers searching on the job with no chance to hire them by drawing the high productivity, $\alpha_l = 1$, then only matches with unemployed workers generate a positive match surplus. Increasing the search intensity of employed workers creates congestion and makes it more difficult for firms to contact unemployed workers. It follows that firms post fewer vacancies and market tightness decreases.

The last case in Lemma 1.3 states that for some probability of drawing a low productivity, α_l^*, there is a range of search intensity where market tightness is increasing in ρ_e for all $\rho_e \in [0, \rho_e^*)$ and when ρ_e becomes sufficiently large, market tightness is decreasing in ρ_e for all $\rho_e \geq \rho_e^*$.

There are two channels which drive this result. The first is the component of the match surplus which is generated by hiring an unemployed worker to the low productivity job, $\mathcal{W}_l - \mathcal{U}$, that comes from the worker eventually transitioning from the low to high productivity job, $\mathcal{W}_h - \mathcal{W}_l$. As the firm captures a fraction $(1-\beta)$ of the total surplus, it captures a fraction of the expected surplus a worker can gain by moving from the low to high productivity job. As the expected surplus of moving from the low to high productivity job increases, the total surplus in a match with an unemployed worker increases and may lead to higher firm entry and market tightness.

A key parameter that determines whether θ is increasing in ρ_e is α_l. As α_l approaches α_l^*, it becomes more likely that a firm meets a worker who is currently employed and that they will be able to draw the high productivity match. This means that the expected surplus $\mathcal{W}_l - \mathcal{U}$ will be increasing in α_l for $\alpha_l \in [0, \alpha_l^*)$. By increasing ρ_e, this allows workers the possibility to transition from one job to another and increases the expected surplus at α_l^*. Firms will benefit from these changes by capturing more surplus in a match with an unemployed worker, namely the surplus that is generated by allowing the worker to transition from one job to another.

The second channel driving the result is the level of ρ_e. If ρ_e increases from $0 \to \epsilon$, this generates the effect described in the preceding paragraphs and market tightness will increase. However, there will be a point where there is too much search on the job from the perspective of the firm. If ρ_e increases too much, then it will be difficult for firms to meet unemployed workers.[23] The point at which employed workers create too much congestion for firms in finding unemployed workers is ρ_e^*.

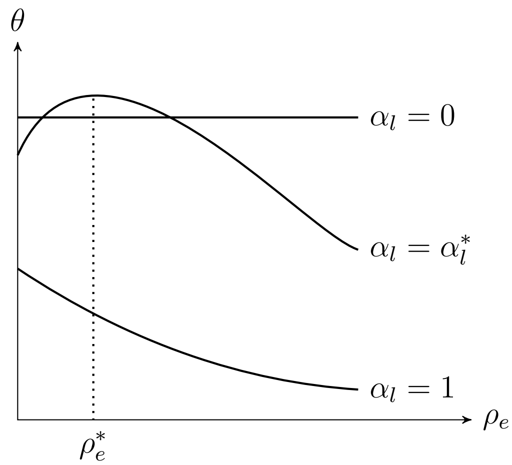
Reservation productivity All jobs may not be created in an environment with stochastic job matchings.[24] With only two productivities, it can be shown when the low productivity match will be created.

Lemma 1.4. *The surplus in a low productivity match is greater than or equal to zero,*

[23]Recall that firms prefer to meet unemployed workers because they generate a higher match surplus.

[24]To avoid the trivial case of no firm entry, $\theta = 0$, by assumption the high productivity match will always be formed.

Figure 1.3: Search Intensity of Employed Workers and Market Tightness



$\mathcal{W}_l \geq \mathcal{U}$, and hence the low productivity match is formed if and only if:

$$\frac{p_l z(r+\delta) + \rho_e f(\theta)\alpha_h \beta p_h z}{r+\delta+\rho_e f(\theta)\alpha_h\beta} \geq \frac{b(r+\delta) + \rho_u f(\theta)\alpha_h \beta p_h z}{r+\delta+\rho_u f(\theta)\alpha_h\beta}. \tag{1.29}$$

Proof. See Appendix A.1.7. □

Equation (1.29) states that if the discounted utility of working in the low productivity job is greater than or equal to the discounted utility of remaining unemployed, then the low productivity match will be formed. It is natural to ask whether increasing ρ_e makes it easier or more difficult for the low productivity match to be formed.

As the left-hand side of (1.29) is increasing in ρ_e, it becomes more likely that the low productivity match will generate a positive surplus and be formed. This means that workers will accept a low productivity match if they are endowed with more ability to transition to the high productivity job. Also notice that as workers are endowed with the ability to transition between the low and high productivity job, they will accept the low productivity job even if $p_l z < b$. This is because the low productivity job offers them the opportunity to move to the high productivity job.

1.4 Endogenous Search Intensity

In the following section, I extend the model to allow employed workers to choose their search effort. After defining a steady state equilibrium with endogenous search intensity, I analyze how search intensity is affected by fundamentals.

1.4.1 Environment with Endogenous Search Intensity

All features of the environment from Section 1.2 are maintained except that now workers employed at the low productivity job choose their units of search intensity, s_e. The cost of intensity is given by $\sigma(s_e)$, where $\sigma'(s_e) > 0$, $\sigma'(0) = 0$, $\sigma''(s_e) \geq 0$, and $\sigma(0) = 0$.

Also, let $q(\theta) = \frac{m(\rho_u u + s_e e_l, v)}{v}$. The rate at which firms meet unemployed workers is now given by $q_u(\theta) \equiv \frac{\rho_u u}{\rho_u u + s_e e_l} q(\theta)$ and the rate at which they meet employed workers is given by $q_e(\theta) \equiv \frac{s_e e_l}{\rho_e u + s_e e_l} q(\theta)$.

Let $f(\theta) = \frac{m(\rho_u u + s_e e_l, v)}{\rho_u u + s_e e_l}$. The rate at which unemployed workers contact firms is given by $\rho_u f(\theta)$. Employed workers contact firms at rate $s_e f(\theta)$.

1.4.2 Bellman Equations

The value of a vacancy and filled high productivity job are still given by (1.3) and (1.5). The value of a filled low productivity job is now given by

$$r\mathcal{J}_l = p_l z - w_l - [\delta + s_e f(\theta)\alpha_h][\mathcal{J}_l - \mathcal{V}]. \tag{1.30}$$

A comparison of equations (1.4) and (1.30) shows that the rate at which workers quit is given by $s_e f(\theta)\alpha_h$ instead of $\rho_e f(\theta)\alpha_h$.

The value of unemployment and value of working at the high productivity are still given by equations (1.6) and (1.8). The value of working at the low productivity job is now given by

$$r\mathcal{W}_l = \max_{s_e} \left\{ w_l - \sigma(s_e) - \delta[\mathcal{W}_l - \mathcal{U}] + s_e f(\theta)\alpha_h[\mathcal{W}_h - \mathcal{W}_l - \phi_e(p_h)] \right\}. \tag{1.31}$$

Comparing equations (1.7) and (1.31) shows that the worker chooses s_e to maximize the expected lifetime utility of working in the low productivity job and pays a corresponding search cost $\sigma(s_e)$.

Employment contracts Notice that by having employed workers choose their search intensity, there is no fundamental change the match surpluses. It follows that the solutions to the optimal employment contracts as defined in Section 1.3.2 are the same as when the search intensity is endogenous.[25]

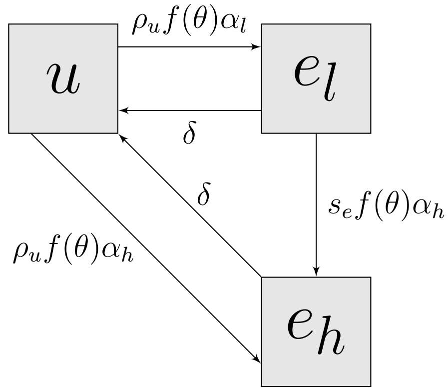
1.4.3 Worker Flows

The flows in to and out of unemployment are unchanged, hence the law of motion is given by (1.20) and the steady state measure of unemployed workers is given by (1.21).

For flows in to and out of employment at the low productivity job, the only difference with the model of exogenous search intensity is the rate at which workers flow out of employment at the low productivity job. This occurs at rate $s_e f(\theta)\alpha_h + \delta$. The law of motion of workers

[25]The hiring fee will be different because the optimal search intensity s_e need not be equal to the exogenous search intensity ρ_e. That being said, the wage and hiring fee which splits the surplus according to the agents' bargaining power does not change.

Figure 1.4: Worker Flows with Endogenous Search Intensity.



employed in the low productivity match is given by

$$\dot{e}_l = \rho_u f(\theta)\alpha_l u \mathbb{1} - (s_e f(\theta)\alpha_h + \delta)e_l, \tag{1.32}$$

where u is the measure of unemployed workers at the level of aggregate productivity where the low productivity match is formed. Solving for e_l such that $\dot{e}_l = 0$ gives

$$e_l = \mathbb{1}\frac{\alpha_l \rho_u f(\theta) u}{(s_e f(\theta)\alpha_h + \delta)}. \tag{1.33}$$

From (1.33), the fraction of employed workers searching on the job is now given by

$$\bar{e}_l = \mathbb{1}\frac{\alpha_l \rho_u f(\theta) u}{(1-u)(s_e f(\theta)\alpha_h + \delta)}. \tag{1.34}$$

Let e_h be the measure of workers employed at the high productivity job. It follows that

$$e_h = 1 - u - e_l \mathbb{1}. \tag{1.35}$$

A summary of worker flows with endogenous search intensity is shown in Figure 1.4.

In deriving the composition of job seekers, I again suppose that the low productivity job generates a positive surplus. From (1.21) and (1.33),

$$\frac{\rho_u u}{\rho_u u + s_e e_l} = \frac{s_e f(\theta)\alpha_h + \delta}{s_e f(\theta) + \delta}, \tag{1.36}$$

$$\frac{s_e e_l}{\rho_u u + s_e e_l} = \frac{s_e f(\theta)\alpha_l}{s_e f(\theta) + \delta}. \tag{1.37}$$

Notice that as $s_e \to 0$ the ratio of unemployed workers to all job seekers approaches 1 and the ratio of workers employed at p_l to all job seekers tends approaches 0. As in Section 1.3.3, these ratios are the probability that a firm contacts each type of job seeker conditional on meeting a worker.

1.4.4 Equilibrium

This section closes the model by deriving two equilibrium conditions that jointly determine optimal search intensity and market tightness. A worker employed at the low productivity will choose their units of search intensity to maximize the lifetime expected utility of working at the low productivity job.

Lemma 1.5. *The optimal search intensity, s_e, is determined by*

$$\sigma'(s_e) = f(\theta)\alpha_h\beta\left[\frac{z(p_h - p_l) + \sigma(s_e)}{r + \delta + s_e f(\theta)\alpha_h\beta}\right]. \tag{1.38}$$

Proof. See Appendix A.1.8. □

Equation (1.38) shows that the optimal search intensity of employed workers, s_e, is chosen so that the marginal cost of effort is equated to the marginal benefit of an additional unit of effort. The marginal benefit of an additional unit of effort is comprised of the additional arrival of outside offers that extra effort brings and the lifetime discounted surplus of transitioning from the low to high productivity job. The surplus of moving from the low to high productivity job is a discounted sum of the additional wages a worker earns and the search costs saved by transitioning.

Equation (1.38) also establishes the first equilibrium relationship between optimal intensity and market tightness. It shows that if $\theta = 0$, a worker would choose $s_e = 0$ as there are no returns to their search intensity. As $\theta \to \infty$, the returns to search intensity also tend to infinity. Workers respond by choosing more search intensity. It follows that (1.38) can be represented by an upward sloping relationship, labelled the SI curve, in (s_e, θ) space.

Free entry again implies that $\mathcal{V} = 0$. Following the derivation of Section 1.3.4, the condition which determines equilibrium market tightness is given by

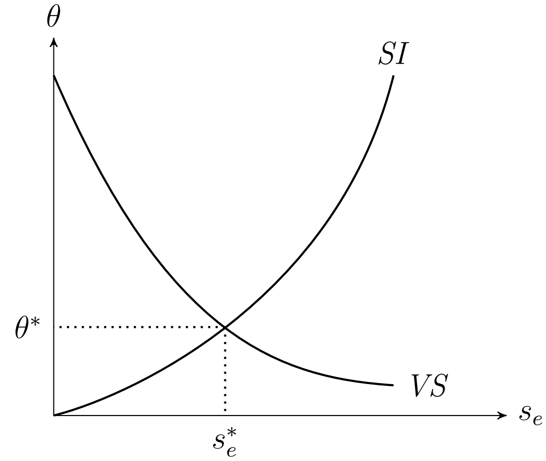
$$\begin{aligned} \frac{\gamma}{(1-\beta)q(\theta)} &= \frac{s_e f(\theta)\alpha_h + \delta}{s_e f(\theta) + \delta}\left[\frac{z(\alpha_l p_l + \alpha_h p_h) - b}{r + \delta + \beta\rho_u f(\theta)}\right] \\ &\quad + \frac{[z(p_h - p_l) + \sigma(s_e)]f(\theta)s_e\alpha_l\alpha_h}{(s_e f(\theta) + \delta)(r + \delta + \beta s_e f(\theta)\alpha_h)}\left[\frac{(r + \delta + \beta\rho_u f(\theta)) + \beta(s_e f(\theta)\alpha_h + \delta)}{r + \delta + \beta\rho_u f(\theta)}\right]. \end{aligned} \tag{1.39}$$

The only difference between equations (1.28) and (1.39) is the exogenous search intensity of employed workers, ρ_e, is replaced with the amount of search intensity that is optimally chosen by the worker, s_e. For each level of search intensity among employed workers s_e, equation (1.39) gives the equilibrium level of market tightness and the second equilibrium relationship between θ and s_e.

Definition 1.2. *A steady state equilibrium with endogenous search intensity of employed workers is a list* $\{s_e, \theta, , u, e_l, e_h\}$ *such that:* s_e *satisfies* (1.38)*,* θ *satisfies* (1.39)*,* u *satisfies* (1.21)*,* e_l *satisfies* (1.33)*, and* e_h *satisfies* (1.35)*.*

The model is now solved recursively by jointly determining s_e and θ by (1.38) and (1.39). The

Figure 1.5: Equilibrium with Endogenous Search Intensity



steady state distribution of workers is then given by (1.21), (1.33), and (1.35). As alluded to earlier, equation (1.38) describes an upward sloping relationship in the (s_e, θ) space, as market tightness increases and the returns from search intensity increases, workers would choose higher search intensity. The relationship is shown in Figure 1.5 as the *SI* curve.

Equation (1.39) gives the second equilibrium relationship between s_e and θ. In the (s_e, θ) space this relationship is given by the *VS* (vacancy supply) curve. From the logic of Lemma 1.3, the *VS* curve need not be always downward sloping in (s_e, θ) space. Depending on the value of α_l, the *VS* curve may be flat, monotonically decreasing, or hump shaped as it is varied over a range of values for s_e. For simplicity, it is shown to be downward sloping in Figure 1.5.[26] The intersection of the *VS* and *SI* curves determine the equilibrium values of market tightness and the search intensity of employed workers.

Proposition 1.2. *There exists an equilibrium such that $\theta > 0$. Again, suppose that the low productivity match is always formed. Comparative statics are summarized in Table 1.2:*

[26]Proposition 1.2 shows that the *VS* curve approaches 0 as s_e approaches ∞.

Table 1.2: Comparative Statics: Endogenous Search Intensity

	γ	b	z	ρ_u	β	p_h
θ	$-$	$-$	$+$	$-$	$-$	$+$
s_e	$-$	$-$	$+$	$-$	$+/-$	$+$
u	$+$	$+$	$-$	$+$	$+$	$-$
e_l	$-$	$-$	$+$	$-$	$+/-$	$+$

Proof. See Appendix A.1.9. □

Suppose that the flow cost to fill a vacancy, γ, increases. From Section 1.3, this will shift the *VS* curve to the left. The *SI* curve is unaffected by the change in γ, causing the *VS* and *SI* curves to intersect at a lower value of market tightness and search intensity. An increase in the flow utility when unemployed, b, will have a similar effect.

An increase in aggregate productivity, z, will increase the surplus in all types of matches. Firms will post more vacancies and the *VS* curve shifts to the right. From (1.38), employed workers will choose more search intensity at every given level of market tightness as there are higher returns to effort, causing the *SI* curve to shift to the right. The new *VS* and *SI* curves will intersect at a higher value of market tightness and search intensity of employed workers. Notice that an increase to the productivity generated in the high productivity match, p_h, has the same effect as it causes both the *VS* and *SI* curves to shift to the right.

Now suppose that the search intensity of unemployed workers, ρ_u, increases. From (1.39) and similar logic to Section 1.3, this will shift the *VS* curve to the left. From (1.38), an increase to ρ_u does not affect the *SI* curve. Thus, the new *VS* and *SI* curves will intersect at a lower value of market tightness and search intensity of employed workers.

An increase in β has an ambiguous effect on the search intensity of employed workers. First, an increase in β will cause the *VS* curve to shift to the left as firms capture less surplus in all matches. From (1.38), an increase in β will cause the *SI* curve to shift to the right as workers have higher returns to search effort. Whether search intensity ultimately increases or decreases depends on whether the shift in the VS curve dominates the shift in the SI curve (s_e decreases) or whether the shift in the SI curve dominates the shift in the VS curve (s_e increases).

Search intensity in a downturn Here, I revisit the central motivation of the paper: why would the search intensity of employed workers increase following a deterioration of economic conditions? From the comparative statics, if aggregate productivity decreases then both market tightness and search intensity of employed workers decreases. It can also be seen from the comparative statics that an increase in workers' bargaining power may cause the search intensity of employed workers to increase. Seldom do we associate an increase in workers' bargaining power with the onset of an economic downturn, so it is unlikely that this is the mechanism behind the change in workers' search intensity following the onset of the Great Recession.

Consider the productivity unique to the low quality job, p_l. Suppose that there is a permanent decrease in p_l. From (1.38), this will cause workers to choose more search intensity for any given level of market tightness shifting the SI curve to the right.

This happens because workers have a larger surplus to gain by transitioning from the low to high productivity job. One interpretation of this is that the high productivity job becomes more productive relative to the low productivity job. The other interpretation alludes to the instability of the low productivity job. Remember that the low productivity job need not always be formed. If p_l decreases, it is as if the workers currently employed at p_l sense that their job is closer to not producing a positive surplus. Workers would like to move away from the low productivity job, causing them to search with more effort.

From (1.39), a decrease in p_l has an ambiguous effect on the VS curve. This is because a decrease in p_l decreases the output in a low productivity match. But a decrease in p_l

also increases the surplus a worker gains by changing from the low to high productivity job. These competing forces result in an ambiguous effect of the VS curve.

Combining the effects of a decline in p_l on both the SI and VS curves shows that when both the VS and SI curves shift outwards, s_e will increase while θ may increase or decrease. The top left of Figure 1.6 shows a case where both s_e and θ increase while the top right shows the case where s_e increases and θ decreases. We see from the top right panel that if the change in the SI curve is very large while the VS is not responsive to the decline in p_l, s_e will increase while θ decreases.

If the VS curve shifts inwards, it is still possible that s_e increases. This occurs when the outward shift in the SI curve outweighs the inward shift of the VS curve. This causes s_e to increase, θ to decrease, and is shown in the bottom left of Figure 1.6. If the decrease in the VS curve outweighs the outward shift of the SI curve, both s_e and θ will decrease (shown in the bottom right panel).

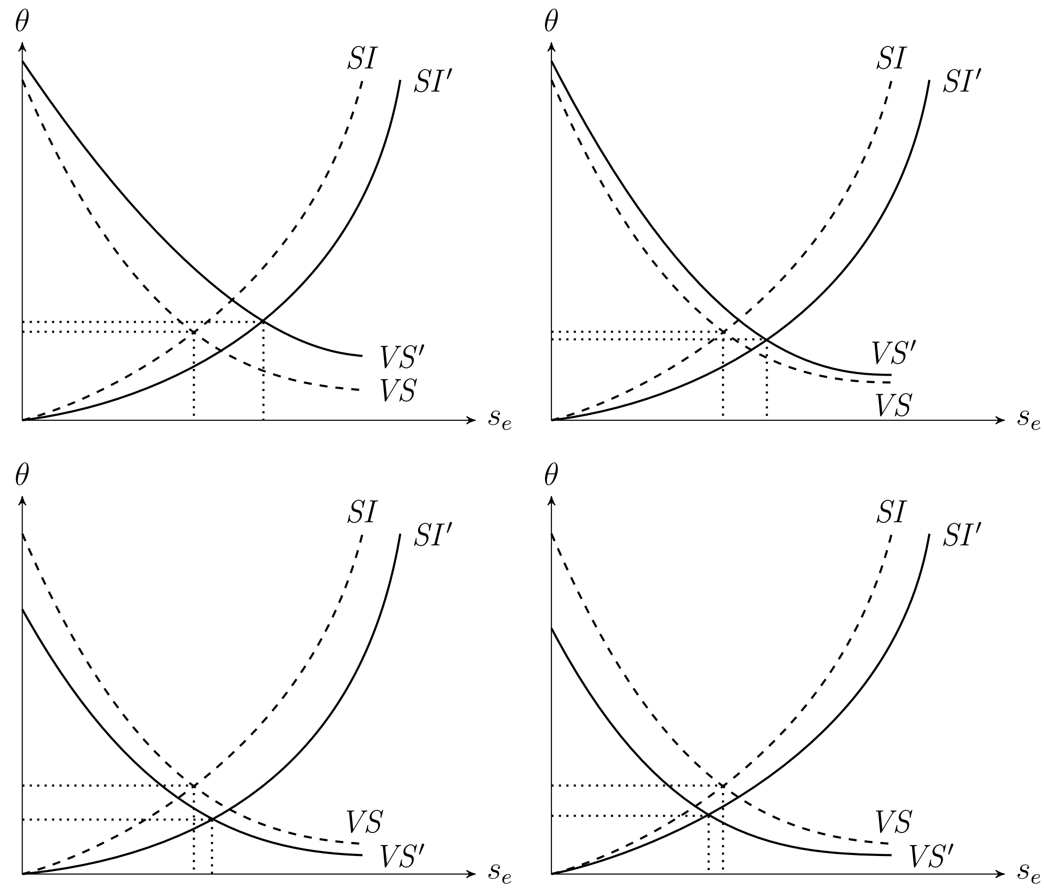
Notice that the top right and bottom left panels have the same outcome following a decline in p_l: search effort among employed workers increases despite a decline in market tightness. To analyze which case emerges, I turn to quantitative examples when the model is calibrated to U.S. labor market data.

1.5 Calibrated Example

A unit of time corresponds to one month. Following Shimer (2005b), the rate of time preference is set to $r = .004$ to target an annual discount factor of 0.953. The flow value of unemployment is set to $b = 0.71$, following Hall and Milgrom (2008).[27]

[27] The choice of b can be controversial in the literature. See Shimer (2005b) and Hagedorn and Manovskii (2008) for an alternative calibration.

Figure 1.6: The Effect of a Decline in p_l



The matching function is Cobb-Douglas, $m(\bar{s}, v) = A\bar{s}^{\eta}v^{1-\eta}$, where $\bar{s} = \rho_u u + s_e e_l$. I set $\eta = 0.5$, as this is within a range which is empirically supported.[28] I then set $\beta = 0.5$ so that the division of match surplus between the worker and firm take into account the worker and firm's contributions to the match surplus (Hosios, 1990a).

I normalize θ to 1 in the steady state and choose A to match the unemployed worker's job finding rate as 0.45 per month, following Shimer (2005b). This gives $A = 0.45$. The monthly job destruction rate also follows Shimer (2005b) and gives $\delta = 0.034$. The flow cost to fill a vacancy follows from Hall and Milgrom (2008) and is set to $\gamma = 0.43$. The cost of search intensity is given by $\sigma(s_e) = \frac{s_e^2}{2}$.

[28]See Petrongolo and Pissarides (2001a) for a more detailed discussion.

The probability of drawing the low productivity match, α_l, is chosen to target 5.5% of employed workers searching on the job and an average monthly job-to-job transition rate that is twice as high as the job destruction rate, i.e., $2 \times 0.034 = 0.068$.[29] From the model, the job-to-job transition rate is $s_e f(\theta)(1-\alpha_l)$. Setting this equal to 0.068 and solving for s_e gives

$$s_e = \frac{0.068}{0.45(1-\alpha_l)}. \tag{1.40}$$

By setting $\bar{e}_l = 0.055$ and combining (1.34) with (1.40), one can solve for α_l. This gives $\alpha_l = 0.165$.[30]

After solving for α_l, s_e can be obtained from (1.40). Then (1.38) can be rewritten as

$$p_h = p_l + \Delta, \tag{1.41}$$

where Δ is obtained by substituting all parameter values and s_e into (1.38). By substituting (1.41) into (1.39), one can obtain p_l. Then p_h is given by (1.41). This gives $p_l = 1.201$ and $p_h = 1.254$. Aggregate productivity, z, is normalized to 1. Table 1.3 summarizes the parameters.

1.5.1 Results

This section performs steady state exercises to analyze the effect of a change in the output produced in a low productivity match on the search intensity of employed workers, the unemployment rate, the job-to-job transition rate, and the fraction of employed workers searching on the job.

[29] See Fujita (2010) for evidence on the fraction of employed workers who search on the job and both Fallick and Fleischman (2004) and Nagypál (2005) for the relationship between the job-to-job transition rate and the destruction rate.

[30] Note that α_h follows from α_l as $\alpha_h = 1 - \alpha_l$.

Table 1.3: Calibrated Parameters

Parameter	Definition	Value	Source/Target
r	Rate of time preference	0.004	Annual discount factor
b	Unemployed utility flow	0.71	Hall and Milgrom (2008)
γ	Vacancy filling cost	0.43	Hall and Milgrom (2008)
δ	Job destruction rate	0.034	Shimer (2005b)
A	Efficiency of matching technology	0.45	Job finding rate
η	Elasticity of matching function	0.5	Petrongolo and Pissarides (2001a)
β	Worker bargaining power	0.5	Hosios (1990a)
z	Aggregate productivity	1	Normalization
p_l	Low match productivity	1.201	Optimal SI condition
p_h	High match productivity	1.254	Optimal SI condition
α_l	Pr of drawing low productivity	0.165	Job-to-job transition rate
ρ_u	Search intensity of unemployed	1	Normalization

Table 1.4: Effect of Changes in p_l

	p_l			
	1.16	1.175	1.201	1.24
SI of employed workers	0.28	0.26	0.18	0.06
Unemployment rate	6.95	6.98	7.02	6.99
Job-to-job transition rate	0.1057	0.0928	0.068	0.023
Percentage of employed SOJ	5.45	5.47	5.5	5.48

From Table 1.4, search intensity of employed workers is decreasing and the unemployment rate is non-monotonic in p_l. Both the job-to-job transition rate and percentage of employed workers who are searching on the job is decreasing in p_l.

One may think that if economic conditions improve, when p_l increases, that more firms would post vacancies and the unemployment rate would decrease. To see if the model can generate such a scenario where both the search intensity of employed workers and unemployment rate is decreasing in p_l, I perform the same steady state experiments with $\alpha_l = 0.75$.[31]

Table 1.5 shows that, when $\alpha_l = 0.75$, search intensity of employed workers and the unemployment rate are decreasing in p_l. This quantitatively illustrates the intuition of Lemma 1.3. When α_l is high, firms do not want to post vacancies following an increase in search intensity

[31] Recall that α_l plays a key role in the effect of changes in search intensity on firm entry.

Table 1.5: Effect of Changes in p_l when $\alpha_l = 0.75$

	p_l			
	1.16	1.175	1.201	1.24
SI of employed workers	0.11	0.09	0.07	0.02
Unemployment rate	8.54	8.44	8.18	7.37
Job-to-job transition rate	0.009	0.008	0.006	0.004
Percentage of employed SOJ	58.3	60.1	63.4	70.6

because employed workers cause congestion in the market. More employed job seekers make it more difficult for employers to find unemployed workers. A high value of α_l also implies that it is difficult to hire employed workers, due to the low probability of drawing the high productivity match. Combining these effects gives a result of employed workers choosing more search intensity while less firms enter the market following a decrease in p_l.

Also notice that despite workers choosing more search intensity and less firms entering the market, the job-to-job transition rate is monotonically decreasing in p_l; implying that an increase in the search intensity of employed workers outweighs the decrease in firm entry.

1.6 Conclusion

This paper has attempted to fill the gap between the theory and evidence on how search intensity among employed workers responds to a deterioration in economic conditions. To do this, I extended the Diamond-Mortensen-Pissarides framework to allow employed workers to choose their search intensity while workers and firms bargain over pairwise Pareto-efficient contracts. Analytically, a decrease in the output produced in a low productivity match has ambiguous effects on the search intensity of employed workers. A calibrated version of the model shows that the search intensity of employed workers increases following a decline in the output produced in a low productivity match.

There are two interpretations of this result. The first is that workers in a low productivity match have more to gain by moving from the low to high productivity job, thus increasing the returns from search. The second interpretation is workers in a low productivity match sense that their job is closer to not generating a positive surplus. In this interpretation, it is as if workers choose more search intensity as a means to avoid unemployment. This is consistent with Fujita (2010) who found that a non-trivial fraction of workers search on the job out of fear of losing their job. This paper has suggested that this may be one of the reasons why search intensity among employed workers increases in recessions.

Chapter 2

The Effect of the Great Recession on Educational Attainment: Evidence from Florida

This chapter estimates the effect of labor demand shocks caused by the Great Recession on high school dropout, high school graduation, postsecondary enrollment, and postsecondary completion rates. The analysis leverages a rich administrative data set from Florida that contains thorough information on student's background and human capital investments. I find little evidence to suggest that the Great Recession affected educational attainment or that these effects vary across characteristics of students and colleges. I interpret these findings within the context of Florida's educational institutions that provide students opportunities to continue progressing towards a college degree while adjusting to economic shocks.

2.1 Introduction

There have been many anecdotal reports within the last decade that the labor demand shocks associated with the Great Recession caused a large increase in college enrollment.[1] If true, this result has both short- and long-run implications. First, in the short-run, students entering college in a recession are somewhat shielded from the scarring effects of entering a depressed labor market (Kahn, 2010; Oreopoulos et al., 2012). This has important implications for the costs of recessions, which is an important input into macroeconomic policy discussions (Barlevy, 2005). Second, since these reports suggest that much of the increased enrollment occurred in community colleges and reflects students switching from four- to two-year colleges, it implies that recessions are associated with changes in the composition of college students and that students may have difficulties financing college investments (Lovenheim, 2011; Lovenheim and Reynolds, 2013). Third, an increased demand for college coupled with declining revenues can exacerbate budget crises and cause higher payments for college (Barr and Turner, 2013). In the long-run, college enrollment would decrease as the economy recovers, which has ramifications for outcomes ranging from the aggregate human capital and economic growth at the macroeconomic level to individual labor market outcomes such as lifetime earnings and unemployment risk.

It is difficult to know whether, as the anecdotal reports suggest, the labor demand shocks associated with the Great Recession increased college enrollment. Answering this question requires variation in local labor conditions along with thorough measures of educational attainment and student type. A precise answer would also rule out two channels that could complicate any estimated relationship between local labor conditions and college enrollment:

[1]See, for example: "Data Reveal a Rise in College Degrees Among Americans", "College Enrollment Falls as Economy Recovers", and "Half of People Who Went to College In the Recession Haven't Graduated". The anecdotal evidence in these articles is supported by simple models that predict educational attainment is counter-cyclical, as recessions decrease the foregone earnings associated with education investments. In more complex environments where students have difficulty financing education investments, educational attainment could be pro-cyclical (Dellas and Sakellaris, 2003).

the school resource and college supply channels.[2] While the prior literature suggest that the data might support these anecdotal reports, this is largely based on cross-state approaches that do not meet those requirements. In particular, states operate their own education systems and budgets that exhibit heterogeneity in their response economic shocks,[3] implying that magnitude of the school resource and college supply channels differs across states. Moreover, studies using cross-state variation often use national surveys that contain relatively limited information on educational attainment and student type.[4]

In this paper, I address these challenges by leveraging (i) variation in economic conditions within one state, Florida, and (ii) a rich administrative data set to estimate the effect of labor demand shocks during the Great Recession on educational attainment. There are several advantages to this approach. First, the primary source of funding for Florida's public K-12 schools is invariant to local economic conditions. This establishes that changes in economic conditions will not affect student achievement through a school resources channel (Jackson et al., 2016). Second, Florida is also one of a few states where the state legislature sets tuition at public colleges which limits the scope for colleges to respond to economic conditions. Third, the Florida Department of Education maintains administrative records that follow students from grade 8 through college. I have obtained access to these records and hence can examine investment decisions including high school graduation, enrollment in college, type of college enrolled in, and college degree obtained. Fourth, as well as being a large and diverse state, Florida has several educational programs that could dampen the channels that drive the cyclicality of educational attainment.[5] Finally, as seen in Figure 2.1 by the large increase in the unemployment rate relative to the U.S., Florida was hit particularly hard by

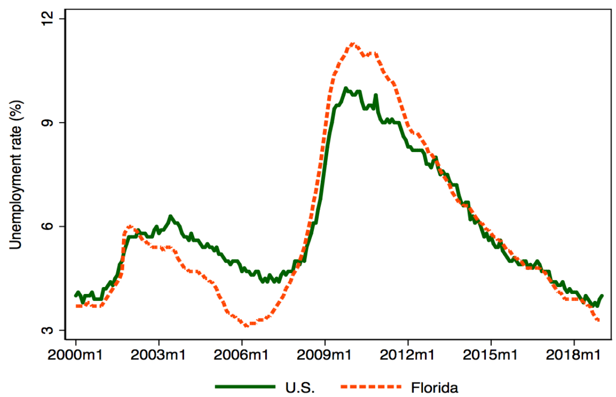
[2]The school resource channel is the effect of school resources on student achievement (Jackson et al., 2016) while the college supply channel is the response of colleges to economic and budget shocks (e.g., adjusting tuition) (Barr and Turner, 2013).

[3]For example, there was variation across states in the extension of unemployment benefits. Barr and Turner (2015) find that these extensions increased the propensity to enroll in college.

[4]Most of the previous literature which estimates the effect of labor demand shocks on educational attainment uses data from all 50 states with 50 different education systems. See, for example, Card and Lemieux (2001) and Charles et al. (2018).

[5]Section 2.2 provides an overview of these programs.

Figure 2.1: Unemployment Before, During, and After the Great Recession



Notes: This figure shows the the monthly, seasonally adjusted, unemployment rate for U.S. and Florida. Source: Bureau of Labor Statistics.

the Great Recession.

To conduct the analysis, I use an administrative dataset on the universe of students enrolled in eighth grade in Florida every year from 1996 through 2006. I follow these students over time, observing valuable information on socio-economic background, academic characteristics, and human capital investments. Because the students are followed from middle school through college, the data allow for an analysis on a large set of education investments including high school completion, enrollment in college, type of college enrolled in (two-year versus four-year), and type of degree attained (associates versus bachelors).

I find little evidence to suggest that the decline in local labor conditions during the Great Recession affected neither enrollment demand nor educational attainment in Florida. This contrasts with the balance of prior evidence which, as I discuss in Section 2.6, suggests that a decline in economic conditions increases educational attainment. Florida has several institutions that might explain this finding. The first is a program which provides scholarships to Florida residents who meet certain eligibility criteria and attend college in Florida. This program could ensure that the benefits of attending college outweigh the opportunity costs and allow students to rely less on the family resource channel (Lovenheim, 2011; Lovenheim and Reynolds, 2013). The second program lets students who have earned an associates degree at one of the state's community colleges to transfer to a state university and work towards a four-year degree. Thus, this program provides students a flexible path to continue working towards a four-year degree at a community college irrespective of local labor market conditions.

I also find no evidence to imply that the relationship between local labor conditions and educational attainment varies across different types of students. In particular, when I characterize students by academic ability (as proxied by standardized test scores) or by socio-economic status (as proxied by eligibility for a free or reduced price lunch), I find no evidence to suggest that the relationship between local labor conditions and educational attainment varies with these characteristics. Moreover, I do not find evidence to suggest that the effect of labor demand on college enrollment varies across two- and four-year colleges or that the effect on college degree attainment differs across associates and bachelors degrees. These results suggest that future levels of educational attainment in Florida will be determined largely by what happens in families, schools and colleges, and much less by what happens to local economic conditions. It is an open question as to whether these results apply to other states. For example, Lovenheim and Reynolds (2013) find, using the NLSY97, that changes in household wealth has significant effects on the quality of colleges attended.

2.2 Education Institutions and Related Literature

2.2.1 Key Institutions

This section introduces four institutions that have shaped public K-12 schools though the state university system in Florida and are particularly relevant for interpreting this paper's estimates of the Great Recession's impact on educational attainment.[6]

The first is the Florida Education Finance Program (FEFP). The FEFP was established in 1973 and is the main source of funding for Florida's public K-12 schools. The program's primary goal is to equalize funding for education programs across public schools and for funding to be independent of local economic conditions. Among other components, the FEFP considers differences in local property tax bases and costs of living. This program helps to ensure that variation across counties in economic conditions will not affect student achievement through a school resources channel (Jackson et al., 2016).

The second institutional feature is the fact that Florida is one of only a few states in which the state legislature sets tuition at four-year colleges. Moreover, tuition is the same in each college.[7] The 28 two-year colleges in the state have only slightly more flexibility over tuition. Given these tuition constraints and since state funding to colleges is largely based on the number of full-time equivalent students enrolled, colleges have limited scope for responding to changes in local economic conditions. As such, it seems reasonable to view the supply side of the human capital market as fixed when comparing counties within Florida and to interpret the findings of this paper as identifying the effects of economic conditions on the demand side of the human capital market.

The third institution is the Statewide 2+2 Articulation Agreement. This agreement was established in 1971 and provides a framework for students who have earned credit hours at one of Florida's 28 two-year colleges, referred to as the Florida College System (FCS), to transfer to one of Florida's 12 state universities to continue working towards a bachelors degree. All students enrolled in associates and bachelors programs at public universities in Florida must complete 36 general education credit hours. The 2+2 system ensures that these credit hours can be transferred to any public college or university within the state.

[6]These descriptions are based on the following reports from the Florida Department of Education (FDOE): FDOE (2017, 2018) and OSFA (2018a,b).

[7]See OPPAGA (2004).

This program is widely used: nearly half of the juniors and seniors at Florida's four-year state universities transferred from the FCS. In addition to being widely used, this program is important because it provides students an option to work part-time and earn credit hours at a two-year college that can also be used towards a four-year degree. Thus, enrollment may be less sensitive to economic conditions under the 2+2 system as students have more options to work and make progress towards a four-year degree.

The fourth program is the Bright Futures Scholarship. This program was established in 1997 to provide scholarships to students attending college in Florida. There are two award amounts. The first, Florida Medallion Scholars (FMS), awards 75% of tuition and fees at public universities.[8] The largest award, Florida Academic Scholars (FAS), pays for 100% of tuition and fees at public universities. To receive an award, FMS recipients must achieve at least a 3.00 high school GPA and a 26 (1170) on the ACT (SAT) exams.[9] FAS are awarded to students with at least a 3.50 GPA and a 29 (1290) on the ACT (SAT). The Bright Futures program could contribute to enrollment decisions being independent of economic conditions as it makes award recipients less reliant on the family resource channel, which has been shown to be an important force in shaping enrollment decisions (Lovenheim, 2011; Lovenheim and Reynolds, 2013).

2.2.2 Related Literature

This paper is most closely related to the literature which estimates the cyclicality of human capital investment. The previous literature has used one of three approaches: a cross-sectional approach, a state panel approach, and a college panel approach.[10] Gustman and Steinmeier (1981) and Bozick (2009) implement the cross-sectional approach. Studies that

[8]Students can also apply their scholarship to nonpublic institutions within Florida.

[9]Students must also graduate high school from a Florida public school, fulfill high school credit hour criteria, and record a minimum number of service hours.

[10]Studies focused on other countries, such as Clark (2011) for the UK, Petrongolo and Segundo (2002) for Spain, and Atkin (2016) for Mexico typically use variations on the state panel approach.

have used the state panel approach in the U.S. include Card and Lemieux (2001), Barr and Turner (2013, 2015), Lovenheim (2011), Long (2015), and Charles et al. (2018). Examples of papers using the third approach, college panels with two-year colleges, include Betts and McFarland (1995), Hillman and Orians (2013), and Nutting (2008).

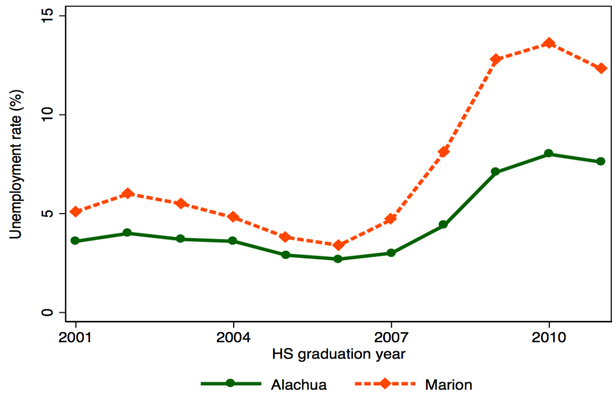
Relative to cross-sectional variation, variation in economic conditions within areas and over time is less likely to be correlated with unobserved determinants of human capital investment. Previous studies using the state-panel approach use survey data from all 50 states which has the following limitation: states operate their own education systems and budgets, which may change in response to changing economic conditions. Previous studies try to hold the supply side fixed by controlling for features of the state education system (e.g., Card and Lemieux (2001) control for college tuition), but supply side features are hard to measure making this strategy problematic.[11]

The college panel approach with two-year colleges can also present at least two difficulties. First, this approach cannot determine whether enrollment increases in two-year colleges come at the expense of enrollment decreases in four-year colleges. Second, since college enrollment data contain little information on the types of students enrolled, it is difficult to determine which groups of students are driving the estimated effects.

This paper advances the literature on the cyclicality of human capital investment by analyzing an administrative data set that combines appealing features of the three existing approaches. These data contain information on the student's background and a variety of measures of human capital investment from high school through college. The data is a panel and thus exploit variation across local labor markets and over time. By analyzing a single state, the analysis does not need to control for differences in state education systems that

[11] One could argue that these supply-side responses are a part of the effect of interest and hence should not be controlled for. But in that case estimates would be difficult to interpret. For example, an alternative interpretation of the Barr and Turner (2013) finding that investment was more counter-cyclical during the 2004-2011 period is that it reflects a tighter connection between economic conditions and state education budgets over this period.

Figure 2.2: Overview of the Research Design



Notes: This figure shows the annual average of the monthly unemployment rate in Alachua and Marion Counties between 2001 and 2011.

can be difficult to measure, leading to estimates that are interpreted as the effects of the Great Recession on the demand for education within Florida.

2.3 Research Design and Methods

2.3.1 Research Design

Figure 2.2 provides some intuition for the research design. This plots the unemployment rate facing tenth grade students in Alachua County and Marion County, two counties in central Florida that share a border. As discussed in more detail below, this unemployment rate is defined as the county unemployment rate in the calendar year in which these students are scheduled to graduate high school. Over the first half of the observation period, the unemployment rate was slightly lower in Alachua than in Marion. But over the Great Recession it increased much less in Alachua than it did in Marion. A plausible explanation is that Alachua was sheltered from the Great Recession by the large fraction (more than 50%) of its residents employed in Government, Education and Health industries (Alachua County contains the University of Florida).

If these divergent unemployment trends are unrelated to other county-level determinants of educational attainment (e.g., socio-economic composition), a comparison of educational attainment across the two counties can shed light on how educational attainment responds to local labor conditions conditions. For example, if a measure of educational attainment (e.g., college enrollment) increases in response to local labor conditions, we would expect to see a relative increase in the college enrollment rate among Alachua students towards the end of the observation window; if it decreases in response to local labor conditions we would expect to see a relative decrease.

2.3.2 Methods

The two-county example provides some intuition for the research design. I estimate versions of the following model using data from all 67 Florida counties:

$$E_{ct} = \beta_0 + \beta_1 LD_{ct} + \beta_2 X_{ct} + \mu_c + \gamma_t + \varepsilon_{ct}, \tag{2.1}$$

where the subscript ct refers to county c in grade ten in year t, E is a measure of educational attainment (e.g., high school graduation rates), LD measures local labor conditions, X is a vector of student characteristics (e.g., demographics), μ_c and γ_t are vectors of county and year fixed effects and ε is an error term. I estimate this model using ordinary least squares. Since the error term may contain a county-level component that is correlated over time, I cluster standard errors at the county level.[12]

The county fixed effect captures permanent differences across counties in the outcome of

[12] An alternative specification to (2.1) would be to conduct the analysis at the individual level, where individual student characteristics are controlled for. I do not present these analyses as the treatment (local labor conditions) varies at the county level and the individual characteristics do not explain much of the variation in educational outcomes. The main conclusions I find when estimating variations of (2.1) do not change when estimated at the individual level. All estimates of models estimated at the individual level are available upon request.

interest. Since Florida counties coincide with school districts, these include permanent differences in the quality of education provided by the district, as well as permanent factors that might influence postsecondary outcomes (e.g., distance to the nearest college). The year fixed effects capture statewide trends in educational attainment. Among other things, these could be driven by changes to state education policy (e.g., the stringency of high school exit exams and the generosity of financial aid).

2.4 Data and Descriptive Statistics

This paper uses data supplied by the Florida Department of Education (FDOE). The "base file" contains enrollment information for the universe of students enrolled in eighth grade in every year from 1996 through 2006. There is at least one such record for every individual in the dataset (more if they attended different schools in eighth grade). Samples sizes range between 167,698 in 1996 and 224,430 in 2006. Several files (also supplied by FDOE) were then matched to these base files. These files include information on public school enrollment in subsequent academic years (including the identity of the school and grade level), standardized test scores in grades eight and ten (including the scores on all test attempts), eligibility for a free or reduced price lunch in ever year of enrollment, all diplomas received, enrollment in two- and four-year colleges, and the details of any postsecondary degrees received. The college enrollment data contain information on the semesters enrolled, the relevant institution, and whether the enrollment status was full- or part-time. The postsecondary degree file specifies the type of degree, the relevant institution, and date of the award.

My analysis sample includes students that were enrolled in the fall of grade ten. The implicit assumption is that dropout before grade ten is not driven by economic conditions. I define student type by ability (proxied by tenth grade test scores) and income (proxied by eligibility for a free or reduced-price lunch).[13]

Table 2.1 provides some summary statistics for the samples used. Nearly 60% of the students graduated from high school.[14] Among the high school graduates, slightly more than half enrolled in college. Over this period, it was more common to enroll in the community college system than the State University System (SUS). The second through fourth columns show the expected relationships between achievement, free-lunch status, and educational attainment.

2.4.1 Educational Attainment

The resulting dataset yields numerous indicators of educational attainment. I focus on six indicators that fall into three broad categories ("High School", "College Enrollment" and "College Completion"). The indicators with the "High School" category are (a) high school completion (i.e., whether a student completes high school, defined as enrollment in the spring of twelfth grade) and (b) high school diploma receipt (i.e., whether a student receives a high school diploma). The "College Enrollment" category contains indicators for (a) enrollment among high school graduates (i.e., whether a high school graduate enrolled in any type of college in the fall of the year they graduated from high school) and (b) SUS enrollment (i.e., the fraction of enrollees that enroll in the SUS).[15] The last category, "College Completion", consists of (a) whether a student obtains an AA degree within four years of graduating high school and (b) whether a student earns a BA degree within six years of graduating high school.

One concern specific to the college outcomes is that I do not capture enrollment in private or

[13]I record a student as eligible for free or reduced-price lunch if they were eligible at any point during grades 8-12.

[14]The high school graduation rates in Table 2.1 and Figure 2.3 may be slightly biased downwards due to students in the base sample moving to a different state during high school.

[15]These will be measured at the point at which individuals enroll in college for the first time.

Table 2.1: Summary Statistics

	Overall	Low achievement	Medium achievement	High achievement	Eligible for free lunch	Ineligible for free lunch
Sample size (N)	1,976,342	446,077	444,370	440,613	771,038	1,205,304
FCAT	320.52	275.13	324.16	362.82	304.04	329.31
High school completion	64.2	63.1	84.4	91.6	58.4	67.9
High school degree	58.2	53.5	82.7	91.0	49.1	63.9
College enrollment	53.8	40.3	54.2	66.9	43.8	58.6
SUS enrollment	39.0	9.50	27.2	62.7	27.3	43.2

Notes: The low achievement group are students whose average FCAT score fell in the first tercile of the distribution of scores in the year that they took the exam, whereas medium (high) achievement students are those who fell in the second (third) tercile of the distribution. The number of students with an FCAT score does not add up to the overall sample size as not all students had an FCAT score (e.g., exemptions and absences). Free lunch applies to students who applied for free lunch and were not eligible, qualified for free lunch, or qualified for reduced-price lunch. The first row shows the average FCAT score among the students in each respective group. The lowest (highest) possible score was 100 (500). The second row shows the percentage of students who completed high school. The third row is the percent of students who obtained a high school diploma. The fourth row is the percentage of students, among high school graduates, who enrolled in college in the fall term following their high school graduation. The last row is the percentage, among those who enrolled in college, who enrolled in the SUS.

out-of-state colleges. To the extent that one would like to measure the impact of local labor conditions on enrollment at both public and private colleges, this generates measurement error in the college-related outcomes and could bias some estimates. For two reasons however, I suspect that any such biases will be small. First, for-profit colleges are oriented towards older individuals returning to education after a spell of unemployment or not being in the labor force. Since I focused on college enrollment patterns among students completing high school, this is not a major concern. Second, with expensive private colleges and out-of-state colleges in mind, I can assess how the estimates change when focusing on lower-ability and lower-income students.

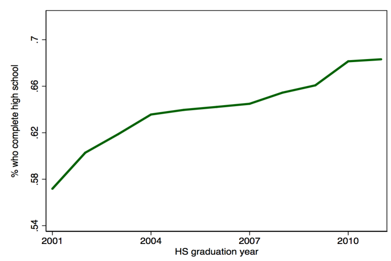
To set the scene for the empirical analysis, Figure 2.3 shows how these outcomes have evolved over the sample period. Educational attainment increased along all dimensions over the period, with only two exceptions. First, Panel (c) shows that the fraction of high school graduates who enrolled in college decreased between 2007 and 2011. Second, Panel (d) shows that the fraction of college enrollees who enrolled in the SUS steadily declined over this time period.[16]

2.4.2 Local Labor Conditions

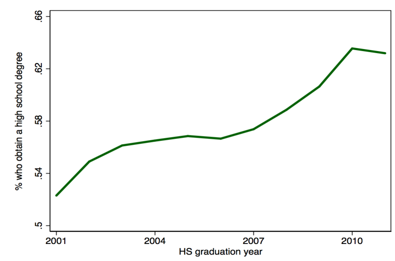
My analysis uses several measures of local labor conditions. The primary measure is the county unemployment rate in the calendar year in which a student is scheduled to graduate high school. The county unemployment rate is a natural measure of economic conditions and a student's opportunity cost of enrolling in college, and is available on a monthly basis from the Bureau of Labor Statistics (BLS).

[16]Note that the series in panels (e) and (f) end before 2011. As discussed above, I focus on the attainment of AA degrees within four years of graduating high school. Since the data ends in 2012, the last cohort of students we could observe for four years after high school graduation were students who graduated in 2008. Similarly, for attainment of BAs, the last cohort of students we could observe for six years after high school graduation were students who graduated in 2006.

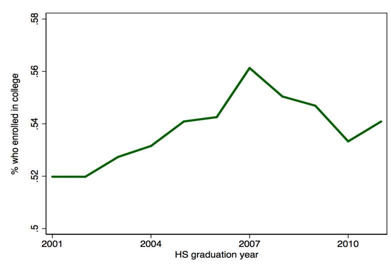
Figure 2.3: The Evolution of Educational Attainment



(a) High School Completion

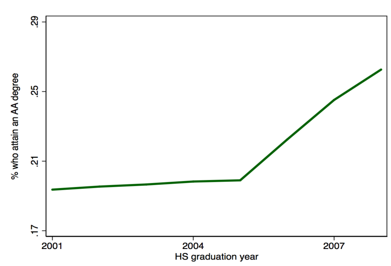


(b) High School Diploma

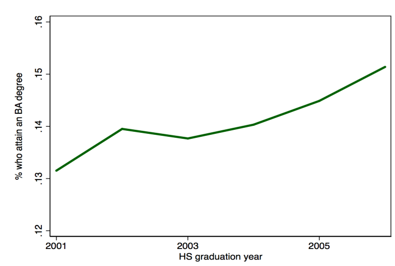


(c) Enrollment in College

(d) Enrollment in the SUS



(e) Attainment of AA Degrees



(f) Attainment of BA Degrees

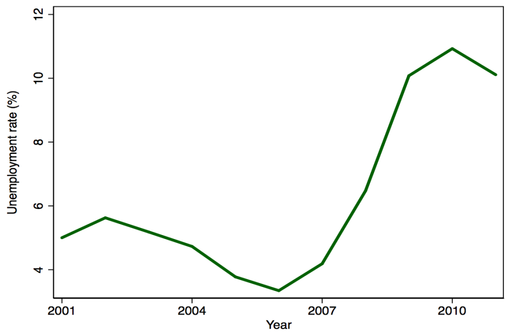
Notes: High school completion, Panel (a), shows the fraction of students who were enrolled in spring of twelfth grade. High school diploma, Panel (b), is the fraction of students who received a high school diploma. Enrollment in college, Panel (c), is the fraction of high school graduates who enrolled in the community college (CC) or state university system (SUS) in the fall of the year they graduated from high school. Enrollment in SUS, Panel (d), shows the fraction of college enrollees who enrolled in the SUS. Attainment of AA degrees, Panel (e), is the fraction of students who obtained an AA degree within four years of graduating high school. Attainment of BA degrees, Panel (f), shows the fraction of students who obtained a BA degree within six years of graduating high school.

The year in which students are scheduled to graduate is a natural point at which to assess the effects of economic conditions, since it covers the period over which many students drop out of high school (i.e., the twelfth grade) and the period in which postsecondary plans are finalized. A concern specific to the college completion outcomes is that these are measured several years after the base year. I will interpret these estimates as the effects of economic conditions in the years in which I measure them given the expected evolution of subsequent economic conditions. In other words, the estimates will reflect the direct effects of economic conditions in the years in which I measure them plus the indirect effects of economic conditions in subsequent years (operating via the correlation between these conditions and the measured conditions).

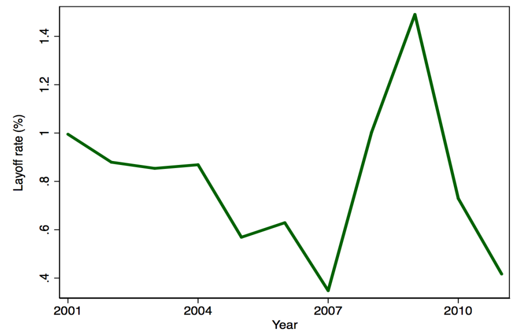
Although county-level unemployment rates are a natural measure of local labor conditions, I assess the robustness of the findings to two alternative measures. First, I will measure local labor conditions by the number of mass layoffs (involving more than 50 workers) in a particular county in a particular year (also obtained from the BLS). One could argue that this measure captures local labor conditions conditions better than the unemployment rate, which is also influenced by the number of people that choose to search for a job. Second, I will estimate the effects of the various local labor conditions measures at the Metropolitan Statistical Area (MSA) rather than the county level. Florida counties are a natural geographical unit, since they are reasonably large (around 1000 square miles on average) and they coincide with school districts. Nevertheless, one could argue that many counties can be considered part of a larger labor market, and that it is local labor conditions conditions in this larger labor market that will impact educational attainment. The Florida MSAs capture these broader labor markets.

To provide an overview of these measures, Figure 2.4 shows how the state-wide unemployment and layoff rates evolved over the sample period. The unemployment and layoff rates followed similar patterns: both decreased between 2001 and 2007 before increasing dramati-

Figure 2.4: Measures of Labor Demand



(a) Unemployment Rate



(b) Layoff Rate

Notes: The unemployment rate, Panel (a), shows the annual average of the monthly unemployment rate in Florida. The layoff rate, Panel (b), shows the annual layoff rate in Florida. The layoff rate is calculated by dividing the total number of workers in mass layoffs by the size of the labor force. Mass layoffs are defined by the Bureau of Labor Statistics (BLS) as layoffs involving at least 50 workers.

cally in the Great Recession. One difference is that the layoff rate returned to its pre-recession level by 2011, whereas the unemployment rate remained at an elevated level.

2.5 Results

This section presents the results. I take a chronological perspective, focusing first on high school outcomes, then on college enrollment outcomes and finally on college completion outcomes. In the next subsection I present the main results. In the following subsection, I probe the robustness of these results to different measures of local labor conditions.

2.5.1 Main Results

Table 2.2 presents the main results. The table includes six columns, one for each outcome. Each column reports the estimate of the effect of local unemployment on this outcome, along with the standard error on this estimate (in parentheses). For example, column (1) reports the estimate of the effect of local labor conditions (measured by county unemployment) on high school completion. This estimate is based on a county-level regression of high school completion rates on local labor conditions, county fixed effects and year fixed effects (see equation (2.1)).[17] The column also includes the number of observations underlying the regression (labeled "*N*"). Since there are 67 counties observed for 11 years (2001-2011), this model uses 737 observations. The column also reports statewide average high school completion rates in the first and last year of the observation window. The other columns are organized in a similar way.[18]

Recall that local labor conditions is measured as the average unemployment rate in the year in which a cohort is scheduled to complete grade 12. The estimate reported in column (1) implies that a 1 percentage point increase in the unemployment rate is associated with a 0.33 percentage point increase in high school completion.[19] It is not surprising that we estimate a positive effect - we would expect high school completion to increase in a recession - but the estimate is small.[20] For example, this effect implies that a six percentage point increase in unemployment (the increase seen during the Great Recession) increases the high school completion rate by only two percentage points (6×0.33). This is small relative to the ten-point increase seen over the time period studied. Moreover, the standard error on this estimate implies that a zero effect cannot be ruled out.

For completeness, I also study high school diploma receipt ("HS Degree"). One would expect

[17] Estimates of regressions that include demographic controls (fraction of students in county c who qualify for free- and reduced-price lunch and average grade ten FCAT scores among students in county c) do not generate different results for any of the subsequent analysis. These estimates are available upon request.

[18] Recall that since I focus on attainment of AA (BA) degrees within 4 (6) years of high school graduation, the reported average from the last year of the sample window is 2008 for column (5) and 2006 for column (6).

[19] I abstain from elasticity models so that the results can be interpreted in terms of percentage point increases in the unemployment rate and directly related to the increase in the unemployment rate seen during the Great Recession.

[20] Black et al. (2005) and Cascio and Narayan (2015) find that high school dropout rates can increase in response to a positive shock to a particular industry (coal and fracking, respectively).

Table 2.2: Baseline Results of the Impact of Changes in the Unemployment Rate on Educational Attainment

	(1)	(2)	(3)	(4)	(5)	(6)
	HS Completion	HS Degree	Enroll in College	Enroll in SUS	AA receipt	BA receipt
Unemployment rate	0.333	0.455	-0.340	0.342	0.151	0.242
	(0.234)	(0.274)	(0.250)	(0.409)	(0.325)	(0.226)
N	737	737	737	737	737	737
Average, first year	0.571	0.523	0.519	0.420	0.193	0.131
Average, last year	0.683	0.631	0.540	0.359	0.262	0.151

Notes: All regressions include county and year fixed effects. Robust standard errors are in parentheses and are clustered at the county level. The dependent variable in column (1) is the fraction of students to complete high school. The dependent variable in column (2) is the fraction of students to obtain a high school degree. Column (3)'s dependent variable is the fraction of students, among the students to obtain a high school diploma, who enrolled in college in the Fall term in the year in which they graduated high school. Column (4)'s dependent variable is the fraction of students, among those who enrolled in college, to enroll in the SUS. Column (5)'s dependent variable is the fraction of students, among those who enrolled in college, to obtain an associates degree within four years of graduating from high school. The dependent variable in column (6) is the fraction of students, among those who enrolled in college, to obtain a bachelors degree within six years of graduating from high school. Each column indicates the mean of the dependent variable in the first and last year of the sample. The first year is 2001. For columns (1)-(4), the last year of the sample is 2011. For column (5), the last year refers to 2008. The last year is 2006 for column (6). Levels of statistical significance are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

estimates for this outcome to be similar to those for high school completion: the difference between the two outcomes is that students can complete high school without obtaining a diploma (e.g., if they fail to meet the exit exam requirement). As expected, I estimate similar effects on this outcome. Note that while these estimates are on the margins of statistical significance, they are not robust to the sensitivity analyses reported below. I conclude that local labor conditions has little impact on high school completion and diploma receipt, at least in Florida over the time period studied.

With this conclusion in mind, I condition college outcomes on receiving a high school diploma. For example, column (3) reports the impact of local labor conditions on college enrollment among students with a high school diploma. More specifically, among the students that complete grade 12 and receive a high school diploma, this is the fraction that enroll in college (Community College or State University) in the Fall of that year. I restrict to diploma recipients because a much smaller fraction of the students without a diploma enroll in college. The resulting data should be less noisy and should generate more precise estimates.

As seen in column (3), I estimate that a one percentage point increase in the unemployment rate reduces college enrollment by 0.34 percentage points. Although one might have expected the effect to be positive, three points are worth noting. First, one can rationalize negative effects, particularly if college enrollment requires family financial commitments and if these are more difficult to make in a weaker economy. Second, the standard error on this estimate implies that the 95% confidence interval spans zero. In other words, zero effects and small positive effects cannot be ruled out. Third, the negative effect is not large. Again, taking the six-point unemployment increase associated with the Great Recession as a benchmark, it implies that an unemployment increase of this magnitude will decrease the college enrollment rate by two percentage points.

As noted, the college enrollment measure used in column (3) encompasses enrollment in both Community Colleges (CC) and the State University System (SUS). For students and families, these might represent very different types of investment. In particular, one might expect SUS attendance to represent a greater financial commitment for families. For example, for most students, it will be easier to live at home while attending CC than SUS. This choice of CC and SUS is especially relevant in Florida, since students graduating from CC with an AA degree are guaranteed admission to the SUS system.

The fourth outcome is focused on this choice of enrolling in the CC or SUS system and will indicate if, conditional on enrolling, students change their choice of college type in response to changes in local labor conditions. This is the fraction of college enrollees in SUS, where the sample of college enrollees is again restricted to high school diploma recipients. I estimate that a one percentage point increase in the unemployment rate increases this by 0.34 percentage points. Again, this is a "wrong-signed" effect, albeit one that is small in magnitude and not statistically significant.

Finally, I consider the receipt of AA and BA degrees. Since these are defined conditional on enrolling in college (which is again conditional on obtaining a high school diploma), I think of these as a summary measure of college dropout. Note that I define BA receipt conditional on enrolling in any type of college, not conditional on enrolling in an SUS. This will capture any effects of local labor conditions on the CC-SUS transfer process.

Since I found small effects of unemployment on college enrollment and on the type of college enrollment, it is not surprising that I find small effects of unemployment on AA and BA receipt. Indeed, both estimates are very small. For example, the AA estimate implies that a six percentage point increase in the unemployment rate facing a cohort when they graduate high school would increase the fraction that receive an AA degree by less than 1 percentage point.

To summarize, I find little evidence that local labor conditions impacts high school completion, high school diploma receipt, the probability of attending college conditional on receiving a high school diploma, the probability of attending SUS conditional on attending college or the probability of receiving an AA or BA degree conditional on attending college. I now proceed to explore whether these findings hold across different types of students and whether these findings are robust to the various specification and data choices that were made.

2.5.2 Subgroups

One might expect the education decisions of some types of students to be especially sensitive to local labor conditions. For example, irrespective of local labor conditions conditions, high-achieving students (as measured by scores on standardized tests) may be very likely to complete high school and attend college. The opposite may be true for low-achieving students. Students with mid-level achievement may be those most sensitive to local labor conditions conditions. I check for differences across these three groups by re-estimating the models after splitting our sample into low-, middle- and high-achieving students.

A second dimension of interest is family wealth. There are at least two reasons why students from economically advantaged families may be less sensitive to local labor conditions than students from less advantaged families. Starting with the opportunity cost channel, if more advantaged families subsidize their children's education, those children may be less sensitive to opportunity costs (e.g., they will attend college irrespective of their alternative labor market options). Turning to the family resources channel, if more advantaged families are less exposed to economic downturns, then their children's educational decisions will likely be less sensitive to local economic conditions. I have only a crude proxy of family wealth: a student's eligibility for a free or reduced-price lunch. Nevertheless, I re-estimate the models after splitting our sample into those two groups.

The estimates are reported in Tables 2.3 and 2.4. In both tables, Panel A reproduces the main estimates (for ease of comparison). The remaining panels then report the estimates split by achievement level and lunch status. Focusing first on differences by achievement, and looking at high school outcomes, there little variation in demand impacts across achievement level: the estimates are close to zero and there is no obvious pattern across the achievement levels. For example, for the diploma outcome, the effects are negative for the "low achievement" group and positive for the medium and high achievement groups, but larger for the medium achievement group than the high achievement group (0.123 versus 0.116).

The picture is similar when disaggregating by achievement level and looking at effects on other outcomes. For example, local labor conditions effects on college enrollment are increasing in ability level. While this might suggest a pattern, these estimates are not statistically different from zero. Moreover, against the hypothesis of generally larger effects among higher-achieving students, I do not find this pattern when we looking at the probability of receiving an AA degree. When looking at college outcomes, note that the concept of "low achievement" is a tricky one. For example, a student that earns a diploma despite low levels of achievement (as measured by grade 10 scores) may have especially strong non-cognitive skills (e.g., determination and perseverance). This must be kept in mind when looking at college enrollment outcomes and especially when looking at college diploma outcomes (which condition on high school diploma receipt and college enrollment).

Turning to subgroups defined by lunch status, the question is how local labor conditions impacts depend on measures of socio-economic status (as measured by lunch status). In nearly every case, I find that the impacts are larger among the group ineligible for free or reduced price lunch. However, these differences are much too small to read anything into: the only reasonable conclusion is that across both groups, there is no evidence to suggest that local labor conditions had any impact on educational attainment.

Table 2.3: The Impact of Changes in the Unemployment Rate on Educational Attainment by Academic Achievement

	(1)	(2)	(3)	(4)	(5)	(6)
	HS Completion	HS Degree	Enroll in College	Enroll in SUS	AA receipt	BA receipt
Panel A: Baseline results						
Unemployment rate	0.333	0.455	-0.340	0.342	0.151	0.242
	(0.234)	(0.274)	(0.250)	(0.409)	(0.325)	(0.226)
N	737	737	737	737	737	737
Panel B: Results for students with low academic achievement						
Unemployment rate	-0.196	-0.060	-0.560	-0.176	0.341	-0.155
	(0.389)	(0.440)	(0.426)	(0.279)	(0.425)	(0.194)
N	737	737	737	735	735	735
Panel C: Results for students with medium academic achievement						
Unemployment rate	0.127	0.123	-0.054	0.477	0.359	0.042
	(0.288)	(0.322)	(0.290)	(0.381)	(0.450)	(0.205)
N	737	737	737	737	737	737
Panel D: Results for students with high academic achievement						
Unemployment rate	0.166	0.116	0.232	1.104	0.235	0.690*
	(0.216)	(0.225)	(0.332)	(0.977)	(0.553)	(0.398)
N	737	737	737	737	737	737

Notes: All regressions include county and year fixed effects. Robust standard errors are in parentheses and are clustered at the county level. The dependent variables are the same as in Table 2.2. Panel A reproduces the baseline results that are reported in Table 2.2. Panel B contains results among only low-achievement students. Panel C contains results among medium-achievement students and Panel D reports estimates among only high-achievement students. The number of observations in Panel B decreases from 737 to 735 in columns (4)-(6) as not all counties had at least one low-achievement student enroll in college directly after graduating high school. Levels of statistical significance are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Table 2.4: The Impact of Changes in the Unemployment Rate on Educational Attainment by Lunch Status

	(1)	(2)	(3)	(4)	(5)	(6)
	HS Completion	HS Degree	Enroll in College	Enroll in SUS	AA receipt	BA receipt
Panel A: Baseline results						
Unemployment rate	0.333	0.455	-0.340	0.342	0.151	0.242
	(0.234)	(0.274)	(0.250)	(0.409)	(0.325)	(0.226)
N	737	737	737	737	737	737
Panel B: Results for students who are eligible for free and reduced price lunch						
Unemployment rate	-0.245	-0.059	-0.654*	0.217	0.199	-0.119
	(0.536)	(0.382)	(0.381)	(0.588)	(0.535)	(0.253)
N	733	733	728	723	724	724
Panel C: Results for students who are ineligible for free and reduced price lunch						
Unemployment rate	0.201	0.454	0.084	0.330	0.244	0.365
	(0.513)	(0.525)	(0.481)	(0.725)	(0.496)	(0.290)
N	736	736	734	729	729	729

Notes: All regressions include county and year fixed effects. Robust standard errors are in parentheses and are clustered at the county level. The dependent variables are the same as in Table 2.2. Panel A reproduces the baseline results that are reported in Table 2.2. Panel B contains results among students who were eligible for free and reduced price lunch. Panel C contains results among students who were ineligible for free and reduced price lunch. The number of observations in Panel B, columns (1) and (2), are smaller than 737 as there were four county, year observations where no students were eligible for free or reduced price lunch. In columns (3)-(6), the number of observations further decreases as not all counties had a student who was eligible for free and reduced price lunch graduate from high school. The number of observations in Panel C and columns (1) and (2) are less than 737 as there was one county, year observation where all students were eligible for free and reduced price lunch. Similar to Panel B, the number of observations decline in columns (3)-(6) as not all counties had a student who was ineligible for free and reduced price lunch graduate from high school. Levels of statistical significance are denoted by ***$p < 0.01$, **$p < 0.05$, *$p < 0.10$.

2.5.3 Robustness

I now probe the robustness of these results by analyzing alternative measures of local labor conditions.[21]

local labor conditions measure

My main demand measure is the county unemployment rate. Although this is a standard measure of labor market conditions, a common concern with this measure is that it does not capture non-participation in the labor market, a phenomenon much discussed during the Great Recession. With this in mind, I re-estimate the models using a more demand-driven measure of labor market conditions: the occurrence of mass layoffs. This measure is especially attractive if young people form expectations of labor market conditions based on salient events such as mass layoffs. One drawback of this measure is that it is much noisier than the county unemployment rate.

Table 2.5 presents the layoff rate estimates. Layoffs are measured as the number of layoffs per worker in the labor force. The average layoff rate is 0.798; while over the period of the Great Recession (between 2007 and 2009) the layoff rate increased from 0.348 to 1.490. Looking first at high school completion, the estimated effect of mass layoff occurrences is -0.314. This implies that an increase in layoffs commensurate with the Great Recession reduces the high school completion rate by 0.358 percentage points.[22] This is a relatively small effect and it is statistically indistinguishable from zero. Moreover, the direction of this effect differs from that estimated using the county unemployment rate (which implied that completion rates

[21]Additional robustness checks that probe the sensitivity of estimates to the econometric specification include models with county-specific linear trends and weighted estimates by county labor force size. All of these models show statistically insignificant effects of labor demand and are available by request.

[22]I calculate this estimate by first noting that the layoff rate increased by 1.142 percentage points between 2007 and 2009. Column (1) and Panel B in Table 6 show then that an increase of the layoff rate by 1.142 percentage points would reduce the high school completion rate by $1.142 \times 0.314 = 0.358$ percentage points.

Table 2.5: The Impact of Changes in the Layoff Rate on Educational Attainment

	(1)	(2)	(3)	(4)	(5)	(6)
	HS Completion	HS Degree	Enroll in College	Enroll in SUS	AA receipt	BA receipt
Panel A: Baseline results						
Unemployment rate	0.333	0.455	-0.340	0.342	0.151	0.242
	(0.234)	(0.274)	(0.250)	(0.409)	(0.325)	(0.226)
N	737	737	737	737	737	737
Panel B: Results using county layoffs						
Layoff rate	-0.314	-0.289	-0.081	0.980	-1.598***	-0.289
	(0.838)	(0.724)	(0.581)	(0.854)	(0.568)	(0.416)
N	711	711	711	711	711	711

Notes: All regressions include county and year fixed effects. Robust standard errors are in parentheses and are clustered at the county level. The dependent variables are the same as in Table 2.2. Panel A reproduces the baseline results that are reported in Table 2.2. Regressions in Panel B show the effect of changes in the county layoff rate. The number of observations in Panel B (711) is less than 737 as there were 26 county, year combinations for which layoff data was unavailable. Levels of statistical significance are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

rise in a recession). We see similarly small estimates when looking at the other outcomes, the one exception being AA completion. Since the estimated effect of local labor conditions on the AA outcome is much smaller (and takes the opposite sign), I suspect this could be a chance finding and hence do not put much weight on this outcome.

Local versus non-local labor conditions measures

One issue facing any measure of local conditions is that young people may base expectations of labor market conditions on state-level or even national-level factors. In that case, we would not expect county-level measures to have any impact. In practice, behavior is likely influenced by a combination of local and more aggregated factors.

I address this by estimating models at the MSA level. The estimates are reported in Table 2.6. Again, I report baseline estimates to aid comparison. These MSA-level models are

Table 2.6: The Impact of Changes in the Unemployment Rate on Educational Attainment at the MSA Level

	(1)	(2)	(3)	(4)	(5)	(6)
	HS Completion	HS Degree	Enroll in College	Enroll in SUS	AA receipt	BA receipt
Panel A: Baseline results						
Unemployment rate	0.333	0.455	-0.340	0.342	0.151	0.242
	(0.234)	(0.274)	(0.250)	(0.409)	(0.325)	(0.226)
N	737	737	737	737	737	737
Panel B: Results at the MSA level						
Unemployment rate	0.148	0.021	0.215	0.497	0.167	0.770***
	(0.355)	(0.376)	(0.250)	(0.550)	(0.441)	(0.227)
N	242	242	242	242	242	242

Notes: All regressions in Panel A (B) include county (MSA) and year fixed effects. In Panel A (B), robust standard errors are in parentheses and are clustered at the county (MSA) level. The dependent variables are the same as in Table 2.2. Panel A reproduces the baseline results that are reported in Table 2.2. Regressions in Panel B show the effect of changes in the unemployment rate on measures of educational attainment at the MSA level. The number of observations in Panel B (242) is less than 737 as there are 22 MSAs observed over 11 years. Levels of statistical significance are denoted by ***$p < 0.01$, **$p < 0.05$, *$p < 0.10$.

based on far fewer observations (242 - 22 MSAs observed in 11 years) but generate similar estimates to those generated by the baseline analysis (i.e., point estimates are small and statistically indistinguishable from zero). Where there are differences between the baseline and the MSA-level estimates, these reveal no obvious patterns: in some cases, the MSA-level estimates take a different sign; where the estimates take the same sign the MSA-level estimates are sometimes larger and sometimes smaller. In one case ("BA receipt") the MSA-level estimates are much larger (and statistically significant). Although this might add weight to the impression that BA receipt increases in recessions, I note that this estimate takes the opposite sign when demand is measured by mass layoffs (not reported but available upon request). I conclude that the baseline estimates are not especially sensitive to the level of aggregation.

2.6 Discussion and Relation to Prior Literature

I estimate that local labor conditions has small effects on several indicators of educational attainment, but how do these estimates compare with those reported in the prior literature? The most natural comparison is between my estimates and those reported by Barr and Turner (2013) for the period 2004-2011. Recall that these estimates are based on an analysis that exploits differences across states and over time. They estimate effects on the order of "one for one". That is, a one percentage point increase in local unemployment generates a one percentage point increase in enrollment among 18-19 year olds. I do not analyze this outcome, but approximate it as:

$$\text{Enrollment among 18-19 year olds} = \text{P(High School Diploma)*P(Enroll in College—High School Diploma)}. \quad (2.2)$$

This approximation implies that impacts on enrollment among 18-19 year olds can be viewed as the sum of impacts on diploma receipt (multiplied by the probability that a student enrolls in college) and impacts on college enrollment (multiplied by the probability that a student earns a diploma). If this approximation is accurate, then I can reject effects on the order of "one for one". First, my baseline estimates imply that a one percentage point increase in county unemployment increases high school diploma receipt by 0.45 percentage points, with the upper bounds on the respective confidence intervals of around one percentage point. Second, my baseline estimates imply that a one percentage point increase in county unemployment decreases the college enrollment rate by 0.34 percentage points. I find a positive effect (around 0.2) in the MSA-level analyses although the associated confidence interval excludes effects larger than about 0.7. When these two upper bounds (one and 0.7) are multiplied by the relevant fractions (around 0.50 and 0.57 respectively), they sum to less than one. It is more difficult to compare the completion estimates to the prior literature, since they do not consider these outcomes. Nevertheless, I conclude that with respect to high school completion and college enrollment, my estimates are smaller than those obtained in the most recent prior literature.

Since the Barr and Turner (2013) estimates refer to the US as a whole, they can be viewed as identifying local labor conditions effects on the typical state. This begs the question of why Florida estimates might be smaller. One explanation centers on the 2+2 system. Recall that this provides Florida students with an unusually flexible path to both an AA and BA degree. To be more concrete, suppose that a student in another state must either stay at home and take a job or go to college and work towards a BA. If the economy is doing well, they might be tempted to stay home, take the job and forego the BA. A Florida student can stay home, take the part-time job, enroll in a community college, earn the credits needed for an AA degree, and then transfer to a BA degree. Ordinarily, this would imply that strong local labor conditions will increase the fraction of students in the SUS. The absence of such effects might be explained by the Bright Futures program that equalized tuition across two- and four-year colleges for scholarship recipients. Of course this explanation is speculative, but it seems reasonable that these features of Florida's higher education landscape play an important role in mediating the effects of local labor conditions conditions.

2.7 Conclusion

The Great Recession was an important event in Florida, as it was in the rest of the country, and there was much speculation about its impacts on educational attainment. With this in mind, some may find this paper's results rather striking: I find that changes in local economic conditions have little impact on educational attainment when measured in terms of high school completion, high school diploma receipt, college enrollment or college completion. An important caveat to these results is that I can not rule out economically significant large positive or negative effects of economic conditions on educational attainment.

Another important caveat is that this paper focuses on young people. It may be that the Great Recession exerted a much stronger influence on educational attainment among older workers, specifically those who enrolled in community college after losing their job. Despite this caveat, one may ask why this analysis in Florida's setting generates different results than the previous literature. I have argued that two features of the Florida higher education system could mitigate the impact of economic shocks: the "2+2" system and the Bright Futures scholarship. These policies offer students an extremely flexible path to both AA and BA degrees by allowing them to adjust course in response to shocks but without changing the final destination. I leave it to future research to provide a more rigorous assessment of this explanation.

Chapter 3

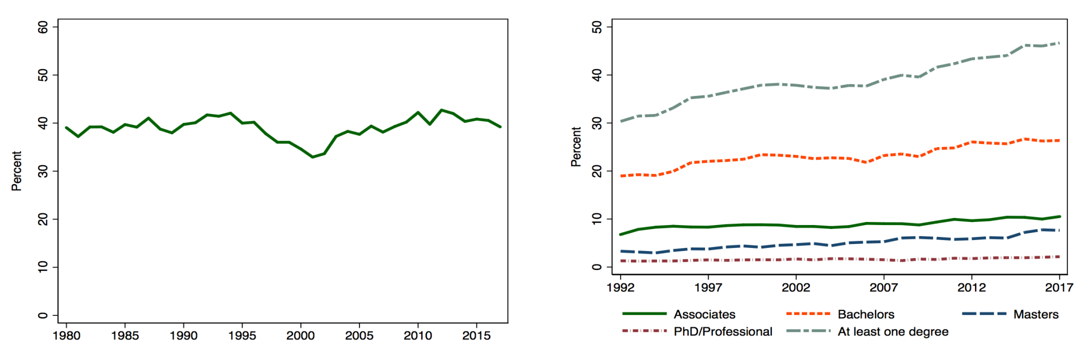
Equilibrium Underemployment

This chapter develops and calibrates a model of human capital investment in a frictional labor market with two-sided heterogeneity. The model generates underemployment in equilibrium: highly-educated workers are employed in jobs that do not require human capital to be productive. The decentralized equilibrium is never constrained efficient and exhibits an inefficient amount of human capital investment and underemployment. The model is calibrated to the U.S. economy to compare the decentralized and constrained-efficient allocations and to perform counterfactual policy experiments. The baseline calibration implies that the U.S. economy exhibits under-investment in human capital and an inefficiently high underemployment rate. Fully subsidizing education increases both human capital investment and the underemployment rate. The benefit of increasing investment in human capital outweighs the inefficiencies associated with a higher underemployment rate, leading to a net increase in welfare.

3.1 Introduction

College graduates in the U.S. are frequently underemployed, i.e. working in occupations that do not require a college degree.[1] As seen in Figure 3.1, nearly 40% of recent graduates are underemployed.[2] Moreover, nearly 60% of underemployment durations last at least 1 year (Barnichon and Zylberberg, 2019) and 48% of those who begin their career underemployed remain so 10 years later (BGT and SI, 2018). Meanwhile, as seen in panel (b) of Figure 3.1, attainment of college degrees continues to expand and there has been an enduring priority to further increase their attainment via policy levers such as offering free tuition and relaxing student loan borrowing limits.[3]

Figure 3.1: Underemployment and College Degree Attainment



(a) Underemployment Rate (b) College Degree Attainment

Notes: Data come from the Current Population Survey and O*NET survey. Panel (a) shows the fraction of recent graduates who are employed in non-college occupations. Each line in Panel (b) represents the percentage of 25-30 year olds whose highest degree completed is the corresponding degree. Section 3.2 provides details on definitions and calculations.

Underemployment is typically viewed as an inefficient outcome. The prominent example that exemplifies this sentiment is the story of ordering from a barista who has an advanced

[1]This has been extensively discussed in the media. See, for example, recent articles in *The Washington Post*: "First jobs matter: Avoiding the underemlpoyment trap" by Michelle Weise and "College students say they want a degree for a job. Are they getting what they want?" by Jeffrey J. Selingo.

[2]Abel et al. (2014); BGT and SI (2018); Barnichon and Zylberberg (2019) also find that nearly 40% of recent graduates are underemployed.

[3]Section 3.6 presents more details on trends in Federal student loans and grants.

degree.[4] Building on that logic, one may oppose the idea of subsidizing education as many graduates will ultimately spend portions of their career underemployed. This rationale, however, neglects that underemployment is an outcome that arises from (i) education choices based on factors such as the college earnings premium and the composition of job complexity and (ii) firms choosing their job's complexity based on the supply of college educated workers. With this perspective, the positive impact of education policies on underemployment is not trivial, as the ultimate effect on the *underemployment rate* depends on the policy's impact on educational attainment, the returns from education, and job creation decisions of firms. Additionally, the normative implications of such policies likely depends on whether the economy exhibits under-investment, over-investment, or the socially efficient level of human capital investment.

In this paper, I develop a model of equilibrium underemployment and study the model's implications for aggregate underemployment, job creation, the supply of human capital, and efficiency of equilibrium allocations. I then calibrate the model to quantify the effects of increasing education subsidies and student loan borrowing limits on underemployment, human capital investment, and welfare. My theory builds on previous literature that has emphasized two channels in studying the effects of education policy on aggregate outcomes. The first, emphasized by Heckman et al. (1998), Lee (2005), Krueger and Ludwig (2016), Abbott et al. (2018), and others, study competitive environments with an aggregate production technology that exhibits diminishing returns to labor. These frameworks highlight the effect increasing the supply of highly-educated workers on the returns to labor and human capital investment. The second, emphasized in Shephard and Sidibé (2019), considers an environment with search and matching frictions where firms choose their job's skill requirements based on the supply of education. In this setting, an increase in the supply of highly-educated workers causes shifts in the composition of job complexity to be directed

[4]See, for example "Welcome to the Well-Educated-Barista Economy" by William A. Galston in the *Wall Street Journal*.

to more skill intensive occupations. What has not been studied, to date, is a theory that accounts for both channels.

The model features a frictional labor market with two-sided heterogeneity. There are two types of jobs (simple and complex) and two education groups among workers (less- and highly-educated) which determines their capacity to be productive in complex jobs (Albrecht and Vroman, 2002). The labor market is unsegmented and, due to random matching, highly-educated workers meet simple jobs according to a Poisson process. If this meeting turns into a match, the worker forms a *cross-skill match* (Albrecht and Vroman, 2002), becomes underemployed, and searches on the job to eventually meet a complex job (Dolado et al., 2009). The decision to become underemployed is endogenous and is a function of two quantities. The first is the relative productivity of simple to complex jobs, which is made endogenous through a final goods technology as in Acemoglu (2001). The second is the worker's opportunity cost of giving up their job search that is determined by how much faster they expect to meet a complex job searching from unemployment than employment.

There are overlapping generations of workers who face a constant risk of death (Blanchard, 1985). Newborn workers are ex-ante heterogenous along two dimensions and make an extensive-margin human capital investment decision before entering the labor market. Workers differ in their innate ability, which affects their productivity and returns to human capital. They are also endowed with a technology to produce the final good that can be used to finance human capital. The notion of differences in familial wealth and transfers is introduced by assuming workers differ in their cost to produce the final good.

The set of equilibria contains pure- and mixed-strategy equilibria in the formation of cross-skill matches. Additionally, the model exhibits multiplicity of steady-state equilibria in some regions of the parameter space that is driven by two coordination problems. The first is a two-sided entry problem: workers choose how much human capital to accumulate whereas firms choose their vacancy's skill level. There is a complementarity between the supply of highly-educated workers and complex jobs: firms create more complex jobs if more workers invest in human capital, whereas more workers will invest in human capital if firms create more complex jobs. The second coordination problem is in the formation of cross-skill matches. Cross-skill matches are more likely to be formed if there are more simple jobs, as this reduces the worker's opportunity cost of giving up their job search, while firms will create more simple jobs if highly-educated workers are more likely match with them.[5]

The model can be used to study a wide variety of comparative statics on the *underemployment rate*, i.e. the fraction of employed, highly-educated workers who are employed in simple jobs. To build intuition, I study a simplified version of the model where output from the two sectors are perfect substitutes and there are no differences across workers in their innate ability. Increasing the productivity of complex jobs increases the expected profits of posting a complex vacancy and benefit of investing in human capital. As more workers invest in human capital, the vacancy filling rate of firms with complex vacancies increases which further incentivizes the creation of complex jobs. As more complex jobs are created, the underemployment rate decreases as highly-educated workers are less likely to match with simple jobs. I then relax the simplifying assumptions and numerically compute comparative statics. By a similar intuition, increasing the productivity of complex jobs causes more complex jobs to be created and increases the benefits of investing in human capital. However, with imperfect substitutability between sectors, the relative price of output produced in simple jobs to complex jobs increases as the supply of highly-educated workers increases. This increases the probability that highly-educated workers form cross-skill matches and ultimately increases the underemployment rate.

The decentralized equilibrium is never constrained efficient.[6] There are several inefficiencies in human capital investment and the formation of cross-skill matches that give rise to this.

[5]This coordination problem is discussed in Albrecht and Vroman (2002) and also generates multiplicity of steady-state equilibria in their environment.

[6]Brunnermeier and Julliard (2008) find the same result in a similar environment but one where the supply of highly-educated workers is exogenous.

The first is a hold-up problem where workers incur the full cost of human capital and earn a share of the returns (Acemoglu, 1996; Moen, 1998). The second is that workers do not internalize that the magnitude of thick market and congestion externalities they generate as a job seeker differs across education groups. For example, if the planner does not form cross-skill matches and creates a majority of jobs that are simple, highly-educated workers create more congestion in the labor market relative to less-educated workers. Finally, the planner is typically choosier in the formation of cross-skill matches, as the planner considers the total expected forgone surplus of a match between a highly-educated worker and complex job when deciding to form a cross-skill match or not. Workers in the decentralized equilibrium, however, only consider their private share of the forgone surplus.

With these inefficiencies in hand, the focus of the paper narrows to compare the constrained efficient and decentralized allocations and to study the effects of education policies, changes to education subsidies and student loan borrowing limits, on underemployment and welfare. I begin by identifying three channels through which these policies affect the underemployment rate. The first is a supply channel where increasing the supply of highly-educated workers shifts the composition of unemployed workers towards highly-educated workers, causing firms to create more complex jobs. The second, the composition channel, occurs when there are shifts in the average innate ability within the pools of less- and highly-educated workers. To illustrate, suppose that only high-ability workers invest in human capital. Decreasing the price of human capital will increase the supply of complex jobs through the supply channel to a point where low-ability workers begin to invest in human capital. This decreases the average ability among the pool of highly-educated workers and reduces the creation of complex jobs. I show, however, that the supply channel always outweighs the composition channel. The final channel, the relative price channel, is active if the final goods technology is not linear: policies that increase the supply of highly-educated workers decrease the price of output produced in complex jobs, leading to a decline in the creation of complex jobs and higher underemployment. I show existence of cases where the relative price channel outweighs the supply channel and therefore where underemployment increases following an increase in the supply of highly-educated workers.

Having identified these mechanisms, I calibrate the model to the U.S. economy over the period 1992-2017 and compute the constrained efficient and decentralized allocations. The baseline calibration implies that workers under-invest in human capital and form cross-skill matches at an inefficiently high rate, leading to an inefficiently high underemployment rate. As workers under-invest in human capital, I use the model to study the effects of education policies which aim to increase the supply of highly-educated workers.

In the first experiment, I study the effects of fully subsidizing human capital through lump-sum taxes. I find that this policy increases the supply of highly-educated workers by 4.828%, which in turn increases the price of output produced in simple jobs relative to complex jobs. Through this change in the relative prices, the probability of forming a cross-skill match increases from 83.5% to 100% and thereby increases the underemployment rate from 26.5% to 30.0%. While the policy increases underemployment, it also increases welfare by 1.171%.[7] This result illustrates that while education subsidies can further increase the underemployment rate, which was already at an inefficiently high level, subsidies can improve welfare if the initial level of human capital investment is inefficiently low.

In a second experiment, I increase the maximum amount that workers can produce to be equal to the price of human capital so that there is no borrowing limit. This policy reduces the underemployment rate by 8.67%, reduces human capital investment by 2.78%, and reduces welfare by 0.749%. The reason for these effects is that the workers who are initially constrained are those with a relatively high innate ability. When they become unconstrained and invest in human capital, they crowd out the returns from human capital investment for all other workers, particularly those with a low innate ability, leading to a net decrease in

[7] Welfare is measured as the economy's net output, i.e. production of the final good and home production from unemployed workers net of vacancy creation costs and costs incurred investing in human capital.

human capital investment, the underemployment rate, and welfare. The quantitative effects of relaxing borrowing limits are relatively small due to the fact that less than 1% of workers are constrained by the initial borrowing limit.[8]

3.1.1 Related Literature

This paper contributes to several literatures. The first is search models with heterogeneity among workers and firms. Albrecht and Vroman (2002) developed a model of heterogenous jobs and workers where highly-educated workers can end up working in low-skill jobs and characterize two equilibrium regimes: cross-skill matching and ex-post segmentation.[9] Dolado et al. (2009) extend Albrecht and Vroman (2002)'s model by allowing underemployed workers to search on the job. Barnichon and Zylberberg (2019) develop a model with segmented labor markets and non-random matching in which high-skill workers are preferred to lower-skill competitors. Underemployment is generated in this model as high-skill workers can escape competition from their highly-skilled peers and can more easily find jobs for which they are over-qualified.[10] This paper builds on these studies by endogenizing the supply of highly-educated workers through a human capital decision and the relative productivity of jobs through a final goods technology as Acemoglu (2001). This enriches a theory of underemployment in several ways. The first is that I am able to characterize when workers (i) invest in human capital and (ii) form cross-skill matches. The second is my model highlights additional inefficiencies that can lead to an inefficiently high or low amount of underemployment that is tied to workers' human capital decision. Finally, my theory illustrates the importance of allowing the productivity of jobs to respond to changes in the supply of highly-educated workers.

[8] This is consistent with previous work that has tested for, and found little evidence on the importance of borrowing constraints. See Lochner and Monge-Naranjo (2012) for a survey.

[9] See Gautier (2002) for a similar framework as Albrecht and Vroman (2002).

[10] Examples related models with segmented labor markets, directed search with heterogenous workers and firms include Shi (2001, 2002), and Shimer (2005a). The models of Shi (2001, 2002) do not generate mismatch between highly-skilled workers and unskilled jobs in equilibrium, while Shimer (2005a) does.

This paper is also related to models of human capital investment in frictional labor markets. Acemoglu (1996) and Moen (1998) study the hold-up problem that arises in these environments. Moen (1999) studied an environment where, due to a particular form for the matching technology, investing in human capital increased worker's job-finding rates. The models of Charlot and Decreuse (2005), Flinn and Mullins (2015), and Macera and Tsujiyama (2017) also have workers who are heterogenous in ability and invest in human capital before entering the labor market. Relative to these studies, this paper is the first to characterize the efficient allocation of human capital investment and composition of jobs in an unsegmented labor market where the productivity of jobs is endogenous.

The normative analysis in this paper is related to the literature which studies efficiency in frictional labor markets. The most relevant paper is Brunnermeier and Julliard (2008) who show that the decentralized equilibrium in the same environment as Albrecht and Vroman (2002) can never be efficient.[11] Acemoglu (2001) shows that in a model with heterogenous jobs and homogenous workers that there can be a composition in the bias of jobs towards low-wage jobs. Charlot and Decreuse (2005) show that due to the composition effects associated with low-ability workers investing in human capital that there can be over-education relative to what a social planner would choose.[12] This paper advances this literature by emphasizing a wedge between the social and private returns to human capital that results from workers not internalizing the net search externalities they generate by investing in human capital.

Finally, this paper contributes to the growing literature which studies the effects education policies on aggregate labor market outcomes. Ji (2018) studies student loan repayment plans and emphasizes how the structure of student loan repayment plans impacts the decision to accept low-wage jobs. Shephard and Sidibé (2019) study the effect of education subsidies

[11] They show that at the Hosios (1990b) condition, that the total number of vacancies is efficient, but that there is a bias in the composition of jobs.

[12] More generally, the normative analysis is related to recent work by Mangin and Julien (2018) who study efficiency in economies where the productivity of matches depend on market tightness. This situation is relevant in my model as the output produced in jobs is a function of market tightness through the linkage generated by the final goods technology.

and compulsory schooling on wage inequality and mismatch within a framework where the distribution of job complexity responds endogenously to the supply of education. This paper advances this literature by developing a framework that emphasizes both the job creation channel studied in Shephard and Sidibé (2019) and a relative price channel. Additionally, this paper connects the effects of education policies on welfare to inefficiencies identified in comparing the centralized and decentralized allocations.[13]

3.2 Underemployment in the Data

This section presents the empirical definition of the aggregate underemployment rate. Section 3.2.2 illustrates differences in the underemployment rate across education and demographic groups while Section 3.2.3 summarizes evidence on the duration of underemployment.

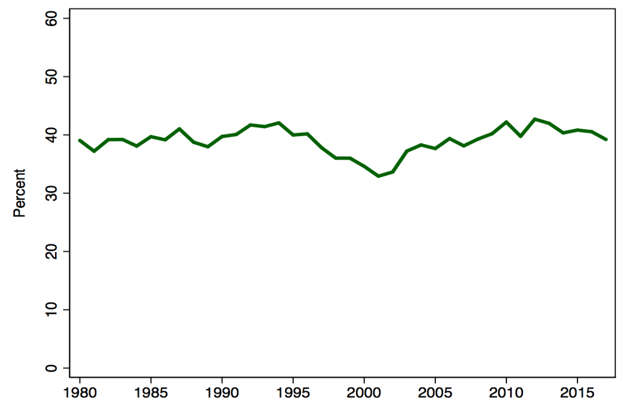
3.2.1 Measuring Underemployment

I define a recent graduate (ages 22-27) with at least a Bachelors degree to be underemployed if they work in an occupation that requires less than a Bachelors degree.[14] An occupation is defined to require at least a Bachelors degree if at least 50% of respondents in the O*NET survey respond that a Bachelors degree or above is required to perform that occupation. Figure 3.2 presents the aggregate underemployment rate.

[13]Ionescu (2009), Ionescu and Simpson (2016) and Abbott et al. (2018) develop heterogenous agent life-cycle models to study the effects of various student loan policies on, among other outcomes, college enrollment, borrowing, and defaults on student loans. While these studies address many interesting questions, they do not analyze underemployment.

[14]This definition follows from Abel et al. (2014).

Figure 3.2: Underemployment Among Recent College Graduates



Notes: Data come from the March Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS), the U.S. Department of Labor's Occupational Information Network (O*NET), and the Bureau of Labor Statistics (BLS). A college graduate is defined to be underemployed if they work in an occupation where less than 50% of respondents in the O*NET survey respond that a college degree is necessary to perform that occupation. The graph shows the fraction of recent graduates (ages 22-27) with a bachelors degree and above who are underemployed, where educational attainment comes from the ASEC. All calculations use the ASEC person weight.

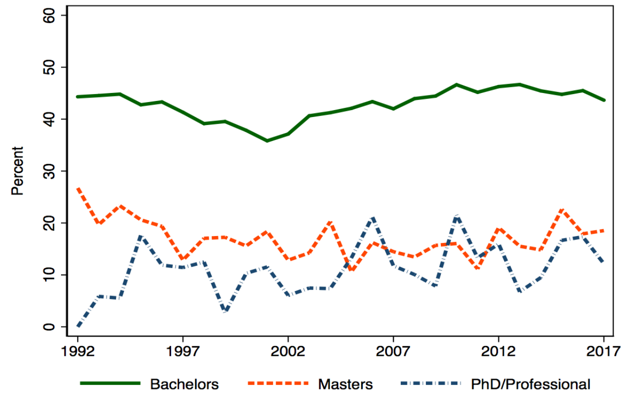
3.2.2 Heterogeneity in Underemployment

As with the unemployment, the aggregate underemployment rate in Figure 3.2 masks differences across education and demographic groups. Starting with highest degree obtained, Figure 3.3 illustrates that the underemployment rate among those whose highest degree is a Bachelors is typically between 40-50%. As one may expect, the underemployment rate is substantially lower among those with a Masters, PhD, or Professional degree.

While Figure 3.3 shows that the underemployment rate among those with a Bachelors degree is much larger than those with advanced degrees, there can also be substantial heterogeneity among those with a Bachelors degree. Figure 3.4 illustrates this by showing the underemployment rate for several undergraduate majors. For these majors, the underemployment rate varies from 17.9% (Engineering) to 52.46% (Communications).[15]

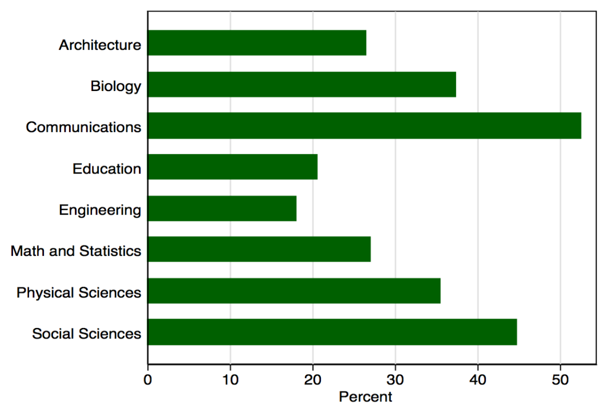
[15] The full list of underemployment rates by major for those available in the ACS is available by request.

Figure 3.3: Underemployment by College Degree



Notes: Data come from the March Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS), the U.S. Department of Labor's Occupational Information Network (O*NET), and the Bureau of Labor Statistics (BLS). All calculations use the ASEC person weight.

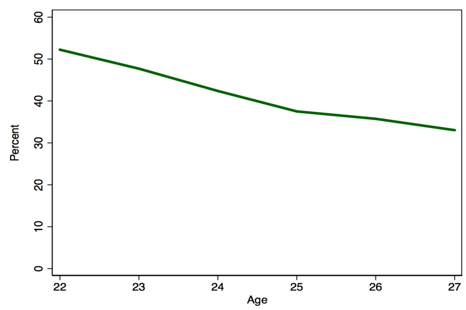
Figure 3.4: Underemployment by Undergraduate Major



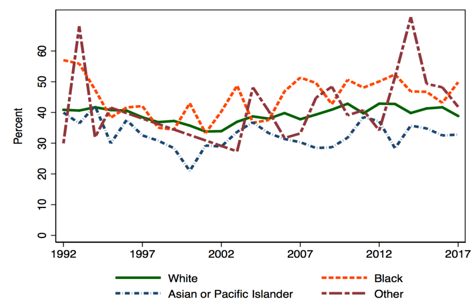
Notes: Data come from the American Community Survey (ACS). The graph shows the fraction of recent graduates (ages 22-27) with a bachelors degree and above who are underemployed between 2009-2016. Calculations use the ACS person weight.

With an overview of underemployment across several measures of education attainment, I proceed to present the underemployment rate across several demographic variables. Starting with panel (a) of Figure 3.5, one can see that the underemployment rate is decreasing in age. This is consistent with evidence that it takes time for young workers to find suitable matches early in their career and they may need to change employers several times to do so (Topel and Ward, 1992). Panel (b) presents the underemployment rate for different racial groups and shows that whites and asians are typically less likely to be underemployed. Finally, panel (c) shows that there has been relatively little differences in underemployment rates across females and males.

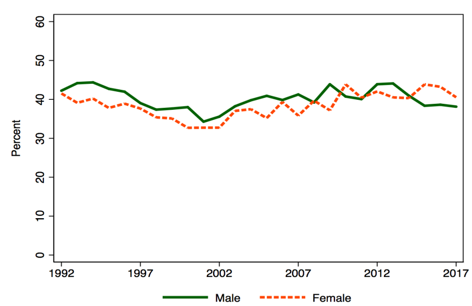
Figure 3.5: Underemployment Across Demographic Groups



(a) Age



(b) Race



(c) Sex

Notes: Notes: Data come from the March Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS), the U.S. Department of Labor's Occupational Information Network (O*NET), and the Bureau of Labor Statistics (BLS). All calculations use the ASEC person weight.

3.2.3 The Duration of Underemployment

The degree to which underemployment is viewed as an inefficient outcome may rest on how transitory underemployment is as many college graduates may take a temporary job that they are overqualified for as they transition from college to a career in their field. A recent report by Burning Glass Technologies and the Strada Institute, BGT and SI (2018), sheds light on this. They find that (i) nearly 43% of college graduates begin their career underemployed, (ii) out of those initially underemployed, 67% are underemployed five years later, (iii) ten years after entering the labor market, 72% of the group that is underemployed after five years remain underemployed, and (iv) overall, 48% of those initially underemployed are still underemployed ten years after entering the labor market. Additionally, Barnichon and Zylberberg (2019) find that nearly 60% of workers who become underemployed are underemployed one year later.

3.3 Environment

Time is continuous and indexed by $t \in \mathbb{R}_+$. There are two types of agents: workers and intermediate-good firms. There are three goods: two intermediate goods and a final good. All agents are risk neutral, discount the future at rate $\rho > 0$, and only value consumption of the final good. The final good is taken as the numeraire. The lifetime discounted utility of a worker born at time t is given by

$$\mathbb{E} \int_t^{t+T} e^{-\rho(\tau - t)} c_\tau d\tau, \tag{3.1}$$

where c_τ is consumption of the numeraire and T is the worker's lifespan that is exponentially distributed with mean $1/\sigma$, i.e. workers die at Poisson rate σ.[16] I define $r \equiv \rho + \sigma$ as the effective discount rate.

Over an infinitesimal time interval dt, a measure σdt of workers are born. Each flow of newborn workers at time t are comprised of different combinations of three characteristics: innate ability, human capital, and a technology to produce the numeraire. Workers draw

[16]This gives the feature of "perpetual youth" as in Blanchard (1985).

their innate ability $a \in \{a_L, a_H\}$, where $a_H > a_L$ and a_L is drawn with probability π.[17] There are two values of human capital denoted $h \in \{0, 1\}$.[18] When workers are born, they are endowed with $h = 0$ and must make an irreversible decision of how much to invest in human capital. Following the human capital decision, workers enter the labor market as unemployed where they receive a flow utility ba while unemployed.

There are two costs to human capital: a pecuniary cost in terms of the numeraire, p_h, and psychic cost, ς. When workers are young, i.e. have not entered the labor market, they can produce ℓ units of the numeraire at cost

$$\varphi(\ell) = \begin{cases} \ell & \text{if } \ell \leq \underline{\ell}, \\ c(\ell) & \text{if } \underline{\ell} < \ell < \overline{\ell}, \end{cases} \tag{3.2}$$

where $c(\underline{\ell}) = \underline{\ell}$, $c'(\underline{\ell}) = 1$, $c'(\ell) > 0$, $c''(\ell) > 0$, and $c'(\overline{\ell}) = \infty$. The production cost is linear for $\ell \in [0, \underline{\ell}]$, where $\underline{\ell}$ is drawn from the cumulative distribution $F(\underline{\ell})$ over $[0, \infty)$ when the worker is born and represents familial transfers/wealth that reduce their need to finance human capital through borrowing. The second component of $\varphi(\ell)$ that is strictly convex and approaches ∞ as $\ell \to \overline{\ell}$ is interpreted as borrowing costs incurred once the worker exceeds a production of $\underline{\ell}$. The value of $\overline{\ell}$ is common to all workers and is interpreted as a policy parameter that represents either relaxing ($\overline{\ell}$ increasing) or tightening ($\overline{\ell}$ decreasing) of borrowing limits. After workers enter the labor market, they can produce the numeraire at unit cost.

Intermediate-good firms are infinitely lived, comprised of one job that can be one of two types indexed by $\chi \in \{s, c\}$: simple (s) and complex (c), and incur a flow cost γ while

[17] Innate ability is innate characteristics, parental investments, and any other factors which affect the returns to human capital.

[18] Human capital represents a worker's stock of skills which determine their capacity to be productive in the labor market.

searching for a worker.[19] A worker and firm produce flow output $y_\chi(a,h)$ of intermediate goods as shown in Table 3.1:

Table 3.1: Production of Intermediate Goods

Human capital	Simple	Complex
0	$x_s a$	0
1	$x_s a$	$x_c a$

with $x_c > x_s$. Intermediate goods are sold to competitive firms who produce the final good according to

$$Y = \left[\mu\big(Y_s\big)^\epsilon + (1-\mu)\big(Y_c\big)^\epsilon\right]^{\frac{1}{\epsilon}}, \tag{3.3}$$

where Y_χ is the aggregate output from type χ jobs, $\mu \in [0,1]$ measures the relative importance of Y_s, and $1/[1-\epsilon]$ is the elasticity of substitution between Y_s and Y_c.

The labor market is unsegmented. The flow of contacts between workers and vacancies is given by the aggregate meeting technology

$$\mathcal{M} = \mathcal{M}\bigg(\int_{i\in\mathbb{W}} e(i)di, v\bigg), \tag{3.4}$$

where $\mathbb{W}$ is the set of all workers workers alive at a point in time, $e(i)$ is worker i's search effort, and v is the measure of vacancies. Unemployed workers are endowed with 1 unit of search intensity whereas employed workers are endowed with $\lambda \in [0,1]$ units of search intensity.[20] The meeting technology is continuous, strictly increasing and concave in each argument, and exhibits constant returns to scale. Defining $\Omega \equiv \int_{i\in\mathbb{W}} e(i)di$ as the aggregate search effort and $\theta \equiv v/\Omega$ as labor market tightness, firms meet workers at rate $q(\theta) = \mathcal{M}/v = \mathcal{M}(\theta^{-1},1)$. Unemployed workers meet firms at rate $f(\theta) = \mathcal{M}/\Omega = \mathcal{M}(1,\theta)$ and

[19] Hereafter, I refer to intermediate-good firms as "firms".

[20] If there is an infinitely small fixed cost to searching for a job, only unemployed and underemployed workers (those with $h = 1$ and employed at $\chi = s$ jobs) will search for jobs.

employed workers meet firms at rate $\lambda f(\theta)$. Upon meeting, the worker's ability and human capital are observable to the firm. Filled jobs are destroyed at rate δ.

3.4 Equilibrium

The description of the equilibrium is presented as follows. I start by defining the flow Bellman equations in Section 3.4.1. Section 3.4.2 then presents the optimal employment contracts that are bargained over in a meeting between a worker and firm. The two subsection sections, 3.4.3 and 3.4.4, describe the entry of firms and human capital investment among workers. Section 3.4.5 then characterizes the formation of cross-skill matches. The final equilibrium conditions that determine the distribution of workers across their states are presented in Section 3.4.6. Section 3.4.7 then defines a steady-state equilibrium and characterizes the set of equilibria. Finally, Section 3.4.8 presents comparative statics.

3.4.1 Bellman Equations

The lifetime discounted utility of a worker when they are born, $W(a, \underline{\ell})$, solves:

$$W(a, \underline{\ell}) = \max_{c,\ell,h \in \{0,1\}} \left\{ c - \varphi(\ell; \underline{\ell}) + h\big[U(a,1) - \varsigma\big] + (1-h)U(a,0) \right\}, \tag{3.5}$$

$$\text{s.t.} \quad c + p_h h = \ell. \tag{3.6}$$

Workers choose their consumption, c, production of the numeraire, ℓ, and human capital, h, to maximize their lifetime discounted utility. The budget constraint shows that workers allocate their production of the numeraire between consumption and the pecuniary cost of human capital.

Let $\zeta = v_s/[v_s+v_c]$ denote the share of vacancies that are simple. The flow Bellman equations for workers in the labor market are given by

$$rU(a,h) = ba + f(\theta)\big\{\zeta(\mathbb{I}_{h=0} + \mathbb{I}_{h=1}\max_{\kappa\in[0,1]}\kappa)\big[E_s(a,h) - U(a,h) - \phi_s^u(a,h)\big] \tag{3.7}$$

$$+ (1-\zeta)\mathbb{I}_{h=1}\big[E_c(a,h) - U(a,h) - \phi_c^u(a,h)\big]\big\},$$

$$rE_\chi(a,h) = w_\chi(a,h) + \lambda f(\theta)(1-\zeta)\mathbb{I}_{h=1,\chi=s}\big[E_c(a,h) - E_s(a,h) - \phi_c^e(a,h)\big] \tag{3.8}$$

$$+ \delta\big[U(a,h) - E_\chi(a,h)\big],$$

where $\phi_\chi^{lf}(a,h)$ is a hiring fee paid by a worker who is hired from labor force status lf (unemployed or employed).[21] Equation (3.7) shows that unemployed workers earn a flow utility ba and meet firms at rate $f(\theta)$. With probability ζ, they meet a simple vacancy. If they are highly-educated, they form a cross-skill match with probability κ.[22] With probability $1-\zeta$ they meet a complex vacancy and become employed if they are highly-educated. From (3.8), workers earn a wage $w_\chi(a,h)$, lose their job at rate δ, and meet a complex job at rate $\lambda f(\theta)(1-\zeta)$.

Denoting $\psi = u/\Omega$ as the share of job seekers who are unemployed and η as the fraction of

[21] The determination of the hiring fee is presented in Section 3.4.2.

[22] Equation (3.7) assumes that the probability of forming a cross-skill match is independent of the worker's innate ability. Lemma 3.3 proves this. One can interpret κ as being chosen by the worker subject to a participation constraint for the firm, where the firm forms a match if and only if it generates a positive surplus.

unemployed workers who are less-educated, the flow Bellman equations for firms are

$$\rho V_\chi = -\gamma + q(\theta)\Big\{\psi\big\{\eta\mathbb{I}_{\chi=s}\mathbb{E}_{a|h=0}\big[J_\chi(a,0) - V_\chi + \phi^u_\chi(a,0)\big] \tag{3.9}$$

$$+ (1-\eta)\big(\mathbb{I}_{\chi=s}\max_{\kappa\in[0,1]}\kappa + \mathbb{I}_{\chi=c}\big)\mathbb{E}_{a|h=1}\big[J_\chi(a,1) - V_\chi + \phi^u_\chi(a,1)\big]\big\}$$

$$+ (1-\psi)\mathbb{I}_{\chi=c}\mathbb{E}_{a|h=1}\big[J_\chi(a,1) - V_\chi + \phi^e_\chi(a,1)\big]\Big\},$$

$$rJ_\chi(a,h) = p_\chi y_\chi(a,h) - w_\chi(a,h) + \big[\lambda f(\theta)(1-\zeta)\mathbb{I}_{h=1,\chi=s} + \delta\big]\big[V_\chi - J_\chi(a,h)\big], \tag{3.10}$$

where p_χ is the price of output produced in type χ jobs. According to (3.9), firms pay a flow cost γ until they meet a worker at rate $q(\theta)$. With probability ψ, they meet an unemployed worker. Conditional on meeting an unemployed worker, they meet a less-educated worker with probability η, where $\mathbb{E}_{a|h}$ is the expected value with respect to innate ability within education group h. Firms initially meet an employed worker with probability $1-\psi$ and form the match if they have a complex vacancy. Equation (3.10) shows that firms earn flow profits of the output net of the wage until either the job is destroyed or a highly-educated worker quits.

3.4.2 Optimal Employment Contracts

In this section I show that, with no loss in generality, an employment contract can be reduced to a pair (w, ϕ) that specifies a wage paid to the worker by the firm and a one-time hiring fee paid by the worker to the firm.[23] To determine the employment optimal contract, let

[23]Stevens (2004b) shows that, in a model with on-the-job search, the bargaining set over a contract with only a flat wage may not achieve a pairwise Pareto-efficient outcome. This is because a worker does not account for the turnover costs paid by the firm following a quit. Also, the feasible set payoffs when bargaining over a flat wage may not be convex (Shimer, 2006b). An employment contract that specifies a one-time hiring fee paid by the worker to the firm and a flat wage is Pareto-efficient as the hiring fee compensates a firm when they hire a worker who searches on the job and eventually quits. While this may seem empirically unrealistic, it is a simplified version of contracts where wages increase with tenure as in Pissarides (1994b) and Burdett and Coles (2003b).

$S^u_\chi(a,h) = E_\chi(a,h) - U(a,h) + J_\chi(a,h) - V_\chi$ be the total surplus of a match between a firm and worker hired from unemployment. It follows that $S^u_\chi(a,h)$ solves

$$
\begin{aligned}
rS^u_s(a,h) = {} & p_s y_s(a,h) - rU(a,h) - \delta S^u_s(a,h) \\
& + \lambda f(\theta)(1-\zeta)\mathbb{I}_{h=1}\big[E_c(a,h) - E_s(a,h) - \phi^e_c(a,h) - (J_s(a,h) - V_s)\big], \quad (3.11)
\end{aligned}
$$

Equation (3.11) has the following interpretation: a match at a simple job generates a flow surplus $p_s y_s(a,h) - rU(a,h)$ and the match is destroyed at rate δ. A highly-educated worker quits at rate $\lambda f(\theta)(1-\zeta)$, gains the surplus $E_c(a,h) - E_s(a,h) - \phi^e_c(a,h)$, and the firm incurs the capital loss $J_s(a,h) - V_s$.

If the worker and firm could jointly decide when the worker quits, they would choose the opportunities for which (3.11) is maximized, which occurs if

$$
E_c(a,h) - E_s(a,h) - \phi^e_c(a,h) \geq J_s(a,h) - V_s. \quad (3.12)
$$

However, I assume that the decision to separate is non-contractable. It follows that when the worker makes the quit decision on their own, they will quit if the private net benefit from doing so is positive, i.e. if

$$
E_c(a,h) - E_s(a,h) - \phi^e_c(a,h) \geq 0. \quad (3.13)
$$

Comparing (3.12) and (3.13) shows that if $J_s(a,h) - V_s > 0$, the worker's private decision decision rule and the choice that maximizes the match surplus differs. That is, the match surplus is not maximized because workers do not internalize the negative externality that they impose on the incumbent firm when they quit. Only when $J_s(a,h) = V_s$ will the worker's decision to quit be aligned with the choice that maximizes the match surplus.

The worker and firm can reach a pairwise agreement over an employment contract that achieves efficient separations. The contract satisfies the following generalized Nash solution where $\beta \in [0,1]$ is the worker's bargaining power and $w_\chi(a,h)$ is the wage:

$$w_s(a,h), \phi^u_s(a,h) \in \arg\max \big[E_s(a,h) - U(a,h) - \phi^u_s(a,h)\big]^\beta \big[J_s(a,h) - V_s + \phi^u_s(a,h)\big]^{1-\beta}. \quad (3.14)$$

Lemma 3.1. *The employment contract as the solution to (3.14) is*

$$w_s(a,h) = p_s y_s(a,h), \quad (3.15)$$

$$\phi^u_s(a,h) = (1-\beta)[E_s(a,h) - U(a,h)]. \quad (3.16)$$

Proof. See Appendix B.2.1. □

The worker earns a wage that is equated with their marginal product so that they earn the entire flow surplus and fully internalize their decision to quit on the match surplus. The hiring fee is then used to split the total match surplus according to the agent's bargaining power.[24]

3.4.3 Entry of Firms

Firms post vacancies until the expected profits of doing so are equal to zero, i.e. $V_\chi = 0$ for $\chi \in \{s, c\}$. This gives the free-entry condition for type χ jobs:

$$\frac{\gamma}{q(\theta)} = (1-\beta)\mathbb{E}\big[\psi S^u_\chi(a,h) + (1-\psi) S^e_\chi(a,h)\big]. \quad (3.17)$$

[24] There are a variety of matches for which the worker does not search on the job. It may seem unnecessary to specify the two-part contracts in these matches. However, in these matches, the employment contracts are payoff-equivalent to the Nash bargaining solution over a contract which only specifies a flat wage. For consistency, I allow the generalized Nash solution described above for all meetings between workers and firms. The solution to the optimal employment contracts in these other types of meetings is delegated to Appendix B.2.2.

The left side of (3.17) is the expected costs to meet a worker whereas the right side is the expected surplus from meeting a worker. The expected value is taken with respect to the heterogeneity within the pool of unemployed workers (less- and highly-educated) and differences in innate ability within education groups.[25]

3.4.4 Human Capital Investment

A worker will invest in human capital if the benefits outweigh the opportunity costs, i.e. if

$$U(a,1) - U(a,0) \geq \varsigma + \varphi(p_h; \underline{\ell}). \tag{3.18}$$

Lemma 3.2 characterizes the worker's optimal choice of human capital investment.

Lemma 3.2. *Define* $\Gamma(a,\underline{\ell}) \equiv U(a,1) - U(a,0) - (\varsigma + \varphi(p_h;\underline{\ell}))$ *as the net gain of investing in human capital and* $\underline{\ell}^*(a)$ *as the solution to* $\Gamma(a,\underline{\ell}^*(a)) = 0$.

1. *If* $U(a,1) - U(a,0) > 0$, *then* $\partial\Gamma(a,\underline{\ell})/\partial a > 0$.
2. $\partial\underline{\ell}^*(a)/\partial a \leq 0$.
3. $\partial\Gamma(a,\underline{\ell})/\partial\underline{\ell} > 0$ *if* $\underline{\ell} < p_h$.

Proof. See Appendix B.2.4. □

Equation (3.18) shows that workers choose $h = 1$ if the capital gain of doing outweighs the sum of the psychic and production costs. The reservation property in innate ability follows from the complementarity between the worker's ability and productivity of job, x_χ.[26] This reservation property leads to the next result which states that higher ability workers are

[25] A closed-form derivation of (3.17) is delegated to Appendix B.2.3.
[26] See Table 3.1.

willing to incur higher costs to invest in human capital. The third result shows that the capital gain is increasing in the worker's endowment if $\underline{\ell} < p_h$ as this reduces the costs the costs incurred in the strictly convex region of $\varphi(\ell)$.

The aggregate supply of highly-educated workers, H, is given by

$$H = \pi h(a_L) + (1-\pi)h(a_H), \tag{3.19}$$

where $h(a)$ is the fraction within an ability group who invest in human capital and is given by

$$h(a) = \begin{cases} 0 & \text{if } \Gamma(a, p_h) < 0, \\ 1 - F(\underline{\ell}^*(a)) & \text{if } \Gamma(a, p_h) \geq 0. \end{cases} \tag{3.20}$$

Equation (3.20) illustrates that any worker, of ability a, who draws an endowment below the critical value, $\underline{\ell}^*(a)$, will not invest in human capital.

3.4.5 Cross-skill Matches

I have assumed that the probability of forming a cross-skill match is independent of the worker's innate ability. Lemma 3.3 proves this and characterizes the optimal choice of κ.

Lemma 3.3. *The formation of cross-skill matches is independent of a. Moreover, $\kappa \in [0,1]$ if*

$$\frac{p_s x_s - b}{p_c x_c - b} = \frac{\beta f(\theta)(1-\zeta)(1-\lambda)}{r + \delta + \beta f(\theta)(1-\zeta)}. \tag{3.21}$$

Proof. See Appendix B.2.5. □

When deciding to form a cross-skill match, a worker and firm compare the relative productivities of simple and complex jobs to the opportunity cost of the worker giving up their job search. The relative productivities of the job, the right side of (3.3), is independent of the worker's ability as both their productivity and flow value of unemployment are scaled by their innate ability. This implies that what is important for determining the relative productivities of the jobs is the differences between the unique components of the output in a match, $p_\chi x_\chi$, and the flow value of unemployment, b. Both the frequency at which the worker meets vacancies, $f(\theta)$, and composition of vacancies determine the worker's opportunity cost.

3.4.6 Distribution of Workers

The remaining equilibrium conditions are steady-state flow conditions that determine the distribution of workers across states:

$$\delta[1 - H - \eta u] = f(\theta)\zeta\eta u, \tag{3.22}$$

$$\delta[H - (1-\eta)u] = f(\theta)(\zeta\kappa + 1 - \zeta)(1-\eta)u, \tag{3.23}$$

$$f(\theta)(1-\eta)u = [\delta + \lambda f(\theta)(1-\zeta)][H - (1-\eta)u]. \tag{3.24}$$

Equation (3.22) states the flow of less-educated workers from employment to unemployment is equal to the flow from unemployment to employment, where ηu is the measure of less-educated unemployed workers. Equation (3.23) is the same condition for highly-educated workers. Equation (3.24) states that the flow of workers into underemployment is equal to the separations and quits among underemployed workers.

With the steady-state conditions above, one can define the steady-state underemployment rate, i.e. the fraction of employed highly-educated workers in simple jobs. I denote the underemployment rate by $\underline{u}$ and it is given by

$$\underline{u} = \frac{(\delta+\sigma)\zeta\kappa}{(\delta+\sigma+\lambda f(\theta)(1-\zeta))(\zeta\kappa+1-\zeta)}, \tag{3.25}$$

which is decreasing in θ, as an increase in market tightness increases the flow out of underemployment through job-to-job transitions. The underemployment rate is increase in both ζ and κ, as both increase the flow into employment at simple jobs.

The aggregate unemployment rate, u, is given by

$$u = \frac{(\delta+\sigma)(1-H)}{\delta+\sigma+f(\theta)\zeta} + \frac{(\delta+\sigma)H}{\delta+\sigma+f(\theta)(\zeta\kappa+1-\zeta)}, \tag{3.26}$$

where the first term on the right side of (3.26) is the measure of less-educated workers who are unemployed and the second term is the measure of unemployed, highly-educated workers. Comparing equations (3.25) and (3.26) shows that both the underemployment and unemployment rates are decreasing in market tightness, θ, while an increase in $\zeta\kappa$ increases the underemployment rate while decreasing the unemployment rate.

3.4.7 Definition and Characterization of Equilibria

Definition 3.1. *A steady-state equilibrium is a list of value functions* $\{W(\cdot), U(\cdot), E_\chi(\cdot), V_\chi, J_\chi(\cdot)\}$ *and prices* p_χ *for* $\chi \in \{s, c\}$, *aggregate supply of highly-educated workers* H, *the probability to form a cross-skill match* κ, *a vector* $\{\theta, \zeta, \eta, \psi\}$, *and distribution of workers across their states such that: The value functions satisfy* (3.5)-(3.10), *intermediate good prices are equated with marginal product,*[27] *the supply of highly-educated workers is given by* (3.19), *the probability to form a cross-skill match satisfies* (3.21), *and the vector* $\{\theta, \zeta, \eta, \psi\}$ *and distribution of workers satisfies the free-entry condition,* (3.17) *for* $\chi \in \{s, c\}$, *and steady-state conditions*

[27] It is straitforward to show that $p_s = \mu(Y_s)^{\epsilon-1}Y^{1-\epsilon}$ and $p_c = (1-\mu)(Y_c)^{\epsilon-1}Y^{1-\epsilon}$.

(3.22)-(3.24).

Proposition 3.1. *The following results describe the existence of steady-state equilibria.*

(i) Assume $b < \min\{\mu x_s, (1-\mu)x_c\}$. An active steady-state equilibrium with $\theta > 0$ exists.

(ii) If $\epsilon < 1$, then $\zeta \in (0,1)$ and $H > 0$.

(iii) If $\epsilon = 1$ and $p_h + \varsigma > \underline{\iota}$, where $\underline{\iota}$ is defined in Appendix B.2.6, then $\zeta = 1$ and $H = 0$.

Proof. See Appendix B.2.6. □

The first result in Proposition 3.1 shows that if the flow utility while unemployed is sufficiently small, then a positive measure of firms will create vacancies. The set of equilibria contains various combinations of human capital investment, job creation, and matching patterns within the labor market. Proposition 3.1 shows that if the final goods technology is not linear, then both types of jobs are created and a positive amount of workers invest in human capital. There can be equilibria where no workers invest in human capital, $H = 0$, and only simple jobs are created, $\zeta = 1$. A necessary condition for this to occur is that the final goods technology is linear. Workers may still find it optimal to invest in human capital if the cost to acquire human capital is relatively small. Proposition 3.1 shows that if the final goods technology is linear and the cost of human capital is sufficiently large, then no workers will invest in human capital and only simple jobs will be created.

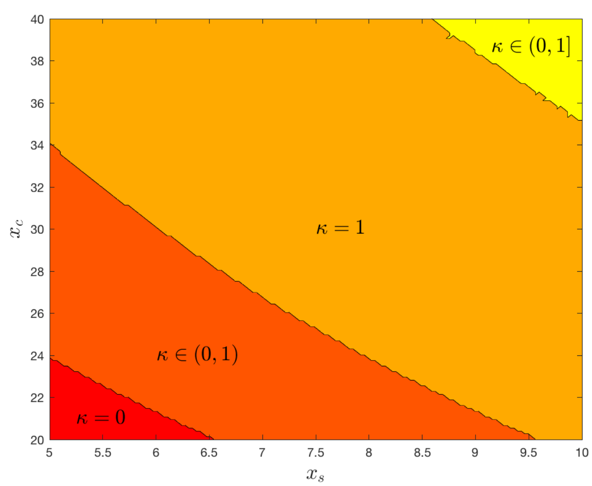
Within equilibria with $H > 0$, there can be cross-skill matching equilibria with $\kappa \in (0,1]$ and ex-post segmentation equilibria with $\kappa = 0$. Proposition 3.2 establishes that if employed workers are endowed with enough search intensity, then cross-skill matches will always be formed and the underemployment rate will be positive. This is because having a higher search intensity while in a cross-skill match increases the rate at which underemployed workers can meet complex jobs relative to unemployed workers, thus reducing the opportunity cost of forming a cross-skill match.[28] If however, the search intensity of underemployed works is low enough, and the productivity of complex jobs is large, then there is a large opportunity cost of forming a cross-skill matches, resulting in an ex-post segmentation equilibrium with no underemployment.

Proposition 3.2. *If $\lambda \geq \underline{\lambda}$, then $\underline{u} > 0$. If $\lambda < \underline{\lambda}$ and $x_c \geq \underline{x}_c$, then $\underline{u} = 0$.*

Proof. See Appendix B.2.7. □

While an equilibrium with $\theta > 0$ typically exists, it is not always unique. This is illustrated in Figure 3.6 which shows the equilibrium regime in the (x_s, x_c) parameter space.[29] Starting on the left side of the figure, the equilibrium is a unique mixed-strategy equilibrium in the formation of cross-skill matches, i.e. $\kappa \in (0, 1)$. As x_s increases, the economy switches to a region with a unique pure-strategy equilibria in the formation of cross-skill matches, $\kappa = 1$. As x_s continues to increase, the economy enters a region of the parameter space that exhibits both a pure- and mixed-strategy equilibria in the formation of cross-skill matches, $\kappa \in (0, 1]$.

Figure 3.6: Topology of Equilibria



[28] See Dolado et al. (2009) for a complete analysis of how search intensity in cross-skill matches affects the formation of cross-skill matches.

[29] The parameter values used to construct Figure 3.6 are the same as in the calibration presented in Section 3.7, except I set $p_h = 0$ in the construction of Figure 3.6.

Multiplicity arises from two coordination problems. The first is the complementarity between the firm's entry decision and the worker's human capital decision. If firms create more complex vacancies, then the value of investing in human capital is larger. Additionally, the expected profits of posting a complex vacancy are increasing in the supply of highly-educated workers. The second coordination problem is in the formation of cross-skill matches. If highly-educated workers match with any job, then the composition of vacancies will shift towards simple jobs. If firms create more simple jobs, then cross-skill matches will be formed with a higher probability.

3.4.8 Comparative Statics

The effects of a change in the model's parameters can (i) move the economy from a pure-strategy equilibrium, $\kappa \in \{0, 1\}$, to another pure-strategy equilibrium, (ii) cause the economy to shift from a pure-strategy equilibrium to a mixed-strategy equilibrium, or (iii) switch the economy from a mixed- to a pure-strategy equilibrium. To illustrate the model's key mechanisms, I study comparative statics within a pure-strategy cross-skill matching equilibrium. After studying a few cases analytically, I present numerical examples that allow for changes to the equilibrium regime.

The outcome of interest is the underemployment rate, $\underline{u}$. I first study comparative statics within a simplified version of the model. Specifically, I assume that the supply of highly educated workers is fixed at $H \in (0, 1)$, shut down search on the job, $\lambda = 0$, consider a final goods technology that is linear, $\epsilon = 1$, eliminate heterogeneity in the workers' innate ability, $a_L = a_H = 1$, and assume $\beta \approx 0$. I also assume parameter values are such that $\zeta \in (0, 1)$. From (3.25), in the case of a $\kappa = 1$ and $\lambda = 0$, the underemployment rate is simply given by ζ. Proposition 3.3 summarizes comparative statics on market tightness and the underemployment rate.

Proposition 3.3. *Assume that $H \in (0, 1)$ and is exogenous, $\lambda = 0$, $\epsilon = 1$, $a_L = a_H = 1$, $\beta \approx 0$, and the remaining parameters are such that $\kappa = 1$. Comparative statics are summarized in the table below.*

	μ	x_s	x_c	γ	H
θ	+	+	0	−	0
$\underline{u}$	+	+	−	+	−

Proof. See Appendix B.2.8. □

An increase in the relative importance of simple jobs, μ, or the productivity of simple jobs, x_s, increases the expected profits of posting a simple job, causing the composition of vacancies to shift towards simple jobs and for the underemployment rate to increase. This also increases the outside option of highly-educated workers in meetings with complex vacancies, thereby reducing the expected profits of posting a complex vacancy. However, the increased supply of simple jobs outweighs the decrease in complex jobs to result in a larger value of market tightness. Increasing the productivity of complex jobs has the opposite effect: the expected profits of posting a complex (simple) vacancy increase (decrease), as highly-educated workers have a larger outside option when bargaining with simple jobs. This causes the composition of vacancies to shift towards complex jobs and for the underemployment rate to decrease. The increase in complex vacancies and decrease in simple vacancies cancel each other out to leave market tightness unchanged.[30] If the vacancy flow cost increases, it becomes more costly for firms to fill a vacancy, reducing market tightness. The composition of vacancies shifts towards simple jobs and the underemployment rate increases because as firms with complex vacancies expect to incur the vacancy costs over a longer duration. An increase in

[30] In order for both jobs to be created, the effective productivity of the two jobs has to be equalized. As $x_c > x_s$, the effective productivities are equalized when accounting for the fact that it is more difficult to fill complex vacancies. Changes to x_c or H affect the effective productivity of complex jobs. However, since the effective productivity of complex jobs must be equal to that of simple jobs, a change in x_c or H is accounted for by a shift in the composition of jobs. Market tightness is independent of changes to x_c or H because there is no change to the effective productivities of jobs after accounting for a shift in the composition of vacancies to equalize the productivities of the jobs.

the supply of highly-educated workers shifts the composition of unemployed workers towards highly-educated workers which increases the vacancy filling rate of complex vacancies. In this simplified case, market tightness is independent of the composition of unemployed workers but due to the increased vacancy filling rate, the composition of vacancies shifts towards complex jobs and the underemployment rate decreases.

In the next set of comparative statics, I allow for H to be endogenous and consider comparative statics with respect to the same parameters in Proposition 3.3 in addition to the effects of changes to the cost of human capital. Proposition 3.4 summarizes the results.

Proposition 3.4. *Assume that* $\lambda = 0$, $\epsilon = 1$, $a_L = a_H = 1$, $\beta \approx 0$, *and the remaining parameters are such that* $\kappa = 1$. *Comparative statics are summarized in the table below.*

	μ	x_s	x_c	γ	p_h	ς
θ	$+$	$+$	0	$-$	0	0
$\underline{u}$	$+/-$	$+/-$	$-$	$+$	$+$	$+$
H	$+/-$	$+/-$	$+$	$-$	$-$	$-$

Proof. See Appendix B.2.9. □

With the supply of human capital endogenous, an increase in either μ or x_s causes market tightness to increase and has ambiguous effects on the the underemployment rate and supply of highly-educated workers. This is because, as discussed in Proposition 3.3, an increase in μ or x_s causes ζ to increase. However, an increase in market tightness (simple jobs) increases (decreases) the benefits of investing in human capital. If the increased supply of simple jobs outweighs the effect of a higher market tightness on the benefits of human capital, then the supply of highly-educated workers will decrease. Alternatively, if the market tightness effect dominates, then the supply of highly-educated workers will increase which causes the underemployment rate, ζ, to decrease.

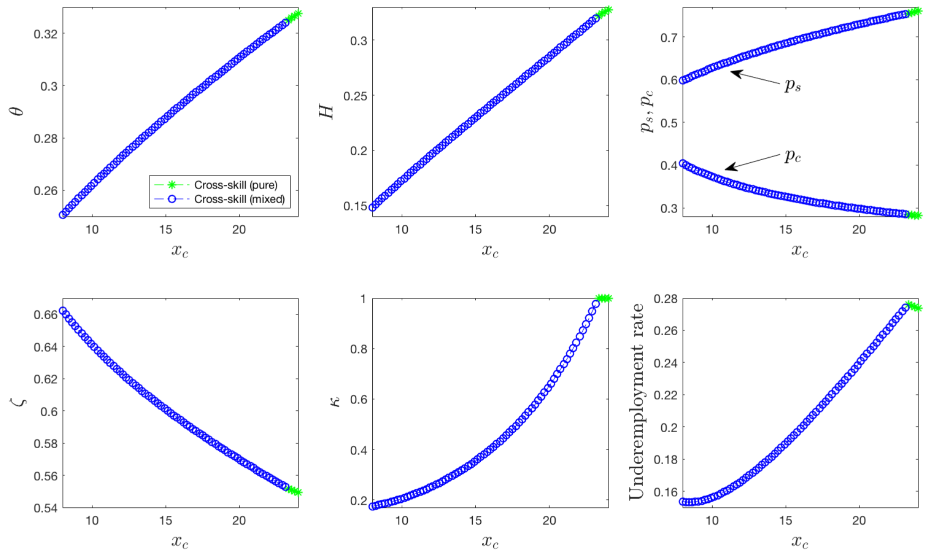
An increase in x_c increases both the benefits of investing in human capital and posting a complex vacancy, causing the underemployment rate to decrease and for the supply of highly-educated workers to increase. If the vacancy flow cost increases, market tightness will decrease and cause more simple jobs to be created and the underemployment rate to increase, as in Proposition 3.3, which decreases the benefits of human capital. Finally, as market tightness is still independent of the composition of jobs seekers, changes to p_h or ς have no effect on θ. An increase to either p_h or ς reduces the net benefit of human capital. As H decreases, the composition of unemployed workers shifts towards less-educated workers and increases the vacancy filling probability of firms with a simple job, causing the underemployment rate to increase.

With the mechanisms in hand from these simplified cases, I proceed to demonstrate a few numerical examples that relax the simplifying assumptions made in the previous examples.[31] As mentioned above, I also allow for all types of equilibria, $\kappa \in [0, 1]$. To understand the effects of changes to parameters on the underemployment rate, it is helpful to present the effects on market tightness, θ, and the prices of intermediate goods, p_χ.

Figure 3.7 shows the effects of changes to the productivity in complex jobs, x_c. As x_c increases, more firms post vacancies and more workers invest in human capital. As the supply of highly-educated workers increases, the composition of vacancies shifts towards complex jobs and the ratio p_s/p_c increases. Through the changes to the intermediate-good prices, an increase in x_c causes the probability of forming a cross-skill match and the underemployment rate to increase. This result differs from the previous analytical results, where an increase in x_c caused a reduction in the underemployment rate, and is driven by the endogenous response of the relative prices, p_χ. This channel was shut down in the analytical cases by assuming a linear final goods technology.

[31] Numerical examples not presented in this section are available upon request. The parameter values used to construct these examples are the same as those in Table 3.4 with the exception of $x_s = 5$, $x_c = 20$, and $p_h = 0$ in the numerical examples.

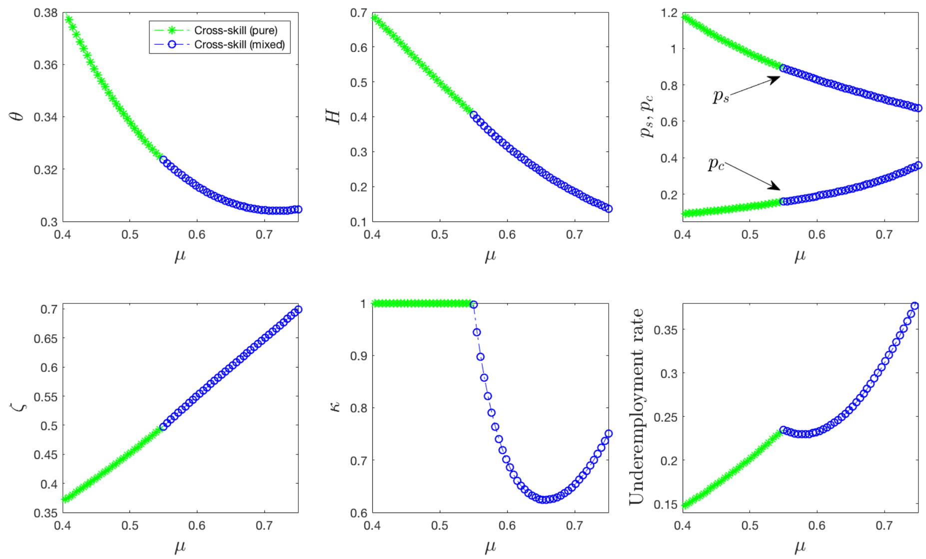
Figure 3.7: Comparative Statics With Respect to x_c



Consider the effects of changes to the relative importance of simple jobs, μ. Figure 3.8 shows increasing μ can cause a decrease in market tightness. This differs from previous results because market tightness is no longer independent of the composition of unemployed workers when workers are heterogenous in their innate ability and the final goods technology is not linear. Increasing μ decreases the benefit of investing in human capital, causing H to decrease, and for the composition of vacancies to shift towards simple jobs. The effects on the probability of forming a cross-skill match, κ are non-monotonic as well. This is because, as μ increases, the increased price of output produced in complex jobs outweighs the effects of an increase in ζ on the worker's opportunity cost of giving up their job search, causing κ to decrease. Eventually, as μ increases, the increase in ζ outweighs the effects of changes to p_χ and causes κ to increase. For most of the parameter space, an increase in ζ and decrease in θ cause the underemployment rate to increase.

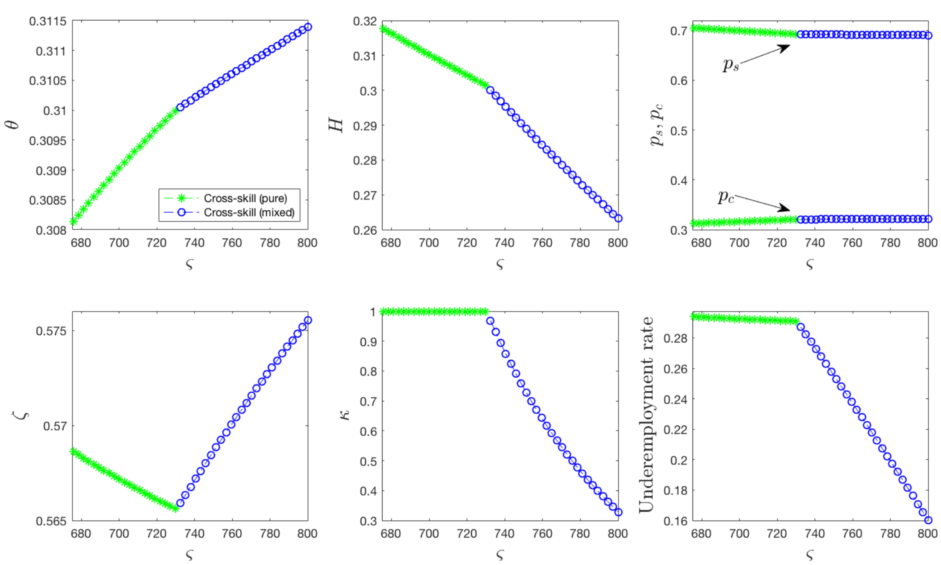
The last example that I present is the effects of changes to the psychic cost of education, ς. As seen in Figure 3.9, increasing ς causes the supply of highly-educated workers to decrease. As H decreases, the composition of vacancies shifts towards simple jobs. The increase in

Figure 3.8: Comparative Statics With Respect to μ



θ outweighs the effect of an increase in ζ on the opportunity cost of forming a cross-skill match and eventually causes κ to decrease. The bottom right panel shows that the decline in κ outweighs the increase in ζ, ultimately causing the underemployment rate to decrease.

Figure 3.9: Comparative Statics With Respect to ς



3.5 Planner's Problem

Consider a social planner whose objective is to maximize society's net output subject to the search frictions that agents face in a decentralized equilibrium. The planner chooses the amount of simple and complex vacancies to open, v_s and v_c, whether a worker endowed with the pair $(a_i, \underline{\ell})$ should invest in human capital, $h(a_i, \underline{\ell}) \in [0, 1]$, and the fraction of meetings between highly-educated workers of ability a_i and simple jobs that should become matches, $\kappa_i \in [0, 1]$. The planner's objective function is given by

$$\max_{\{v_\chi, h(a_i,\underline{\ell}), \kappa_i\}} \int_0^\infty e^{-\rho t} \Bigg[Y + \sum_i \sum_h u_i^h b a_i - \\ \gamma(v_s + v_c) - \sigma \sum_i \pi_i \int_0^\infty h(a_i, \underline{\ell})[\varsigma + \varphi(p_h; \underline{\ell})] dF(\underline{\ell}) \Bigg] dt, \quad (3.27)$$

for $\chi \in \{s, c\}$, $i \in \{L, H\}$, $h \in \{0, 1\}$, and where u_i^h is the measure of unemployed workers with human capital h and ability a_i. From (3.27), the planner maximizes production of the final good, Y, and home production from unemployment net of vacancy costs and costs incurred to produce highly-educated workers. The planner maximizes (3.27) subject to the laws of motion of workers across the states of employment and unemployment. Proposition 3.5 compares the decentralized and efficient steady-states under the simplifying assumptions that there is no search on the job and the final goods technology is linear.

Proposition 3.5. *Suppose that $\lambda = 0$ and $\epsilon = 1$. A decentralized steady-state equilibrium never coincides with the efficient steady-state.*

Proof. See Appendix B.2.10. □

There are several inefficiencies in each of the agents's key decisions (human capital investment, vacancy creation, and formation of cross-skill matches) which lead to the result shown in Proposition 3.5. The first is a hold-up problem in the worker's human capital investment decision.[32] Workers only obtain a share β of the total returns associated with investing in human capital due to ex-post surplus sharing with firms. Thus, there is a share, $1-\beta$, of the total gains from accumulating human capital that workers do not internalize when they make their investment decision. This can be seen by comparing equation (B.2.12), a private agent's benefit of human capital, to equation (B.2.50), the social benefit of investing in human capital, as the private agent's benefits to human capital are scaled by their share of the match surplus, β.

A second inefficiency relates to the thick market and congestion externalities generated by job seekers in frictional labor markets. Job seekers produce congestion externalities as an additional job seeker reduces the job-finding rate of all other job seekers while the thick market externality arises as job seekers increase firms' vacancy filling rate. In a model of homogenous workers, these externalities cancel each other out when the Hosios (1990b) condition holds. As shown in Brunnermeier and Julliard (2008), this is not true in an unsegmented labor market with heterogenous workers as the search externalities generated by a job seeker differ across education groups. When cross-skill matches are formed, highly-educated workers improve the vacancy filling rates of both simple and complex vacancies, whereas less-educated workers only improve the vacancy filling rate of simple vacancies. This can be seen in the social benefits of human capital investment, (B.2.50), by a term that I define as the *net thick market externality*, Θ, where

$$\Theta \equiv \frac{f(\theta)(1-\nu)\Psi[1+\zeta(\kappa_i-2)]}{(r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i))(r+\delta+f(\theta)\zeta)}, \tag{3.28}$$

where ν is the elasticity of the meeting technology with respect to job seekers and Ψ is the average value of a match.[33] It is straitforward to see that $\Theta > 0$ if $\kappa_i = 1$ or $\zeta < 1/2$, i.e. if it is relatively easy for the planner to form matches with highly-educated workers.

[32] See Acemoglu (1996) and Moen (1998) for earlier discussions of hold-up problems in human capital investment.

[33] See Appendix B.2.10 for a formal definition of Ψ.

There are additional differences between the decentralized equilibrium and efficient steady-state in the conditions that govern the formation of cross-skill matches. The first difference is similar to a hold-up problem: workers in the decentralized equilibrium weigh the benefits of accepting a simple job offer against the opportunity cost of giving up their job search. The opportunity cost is given by the right hand side of equation (3.21) and is scaled by the worker's bargaining power, β. The opportunity cost to the planner, however, is not scaled by β as the planner considers the total expected surplus that is forgone by forming a cross-skill match.

A second difference between the decentralized and centralized solutions in the formation of cross-skill matches is that the planner accounts for the fact that forming a cross-skill match reduces the congestion faced by other unemployed workers. This is seen by the term $(1-\nu)f(\theta)\Psi$ in (B.2.42). It is due to this that the rate at which the planner forms cross-skill matches is a function of the worker's ability, whereas the formation of cross-skill matches was independent of the worker's ability in the decentralized equilibrium. From (B.2.42), the benefit from reducing congestion by forming cross-skill matches is larger for low-ability workers. The intuition for this is simple: the planner forms cross-skill matches among low-ability workers at a higher rate because this reduces congestion and allows for highly-educated, high-ability workers.

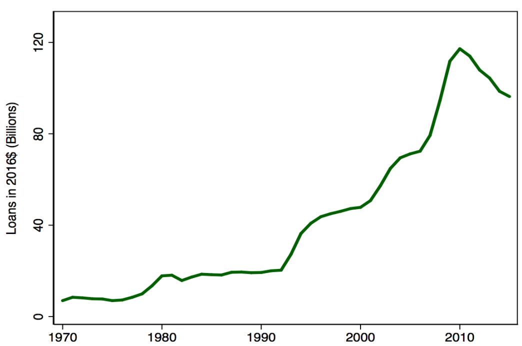
3.6 Education Policy

3.6.1 Background and Empirical Evidence

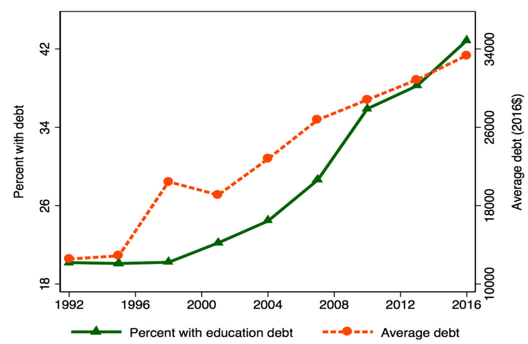
One of the most striking developments in the attainment college degrees is the use of student loans. In fact, borrowing to finance college has increased to the point where student debt is the second largest type of consumer debt behind only mortgage debt (FRBNY, 2018).

To illustrate, panel (a) in Figure 3.10 shows that federal student loan disbursements have been increasing since the 1970s and that the pace of disbursements increased in the early 1990s and continued until 2010. Panel (b) shows how the extensive and intensive margins of borrowing have evolved since 1992. The solid line (left axis) shows that the percentage of U.S. households with education debt increased from 20% to 43% while the dashed line (right axis) shows that, among those with a positive amount of education debt, the average amount borrowed increased by nearly $20,000.

Figure 3.10: Trends in Student Loans



(a) Federal Student Loan Disbursements



(b) Extensive and Intensive Margins

Notes: The data in panel (a) come from College Board (2017) and shows the total amount of Federal student loans disbursed in an academic year. Data in panel (b) comes from the Survey of Consumer Finances (SCF). The solid line (left axis) shows the percentage of respondents who report having a positive amount of education debt. The dashed line (right axis) shows, among those with a positive amount of education debt, the average amount of education debt. Calculations only include households where the head of the household is between 20-40 years old. All calculations use SCF weights where, per-recommendation of the Federal Reserve Board, the weights are divided by 5 before performing calculations.

I focus on Federal student loans because they made up 87% of all student loan disbursements between 1995 and 2015 (College Board, 2017). The Federal student loan program offers Stafford, Perkins, and PLUS/GradPLUS loans. I focus on Stafford Loans as they made up an average of 87% of Federal loan disbursements between the 1992-93 and 2015-16 academic years (College Board, 2017).[34] There are two types of Stafford loans: subsidized

[34]For more details on the other types of loans available through the Federal student loan program, see

and unsubsidized. Interest accrues on unsubsidized loans while enrolled in school, whereas it does not on subsidized loans. Both undergraduates and graduate students can obtain unsubsidized loans without demonstrating financial need, while eligibility for subsidized loans is restricted to undergraduates who demonstrate financial need.[35]

Stafford loans have both annual and cumulative borrowing limits that are determined by the student's dependency status and year in school. The borrowing limits are set by congress and are fixed in-between policy changes. Table 3.2 shows the cumulative limit based on a student's dependency status and their loan type and also illustrates that the only change to the cumulative limits in 2008 increased the borrowing limit for dependent (independent) students by 34.7% (25%).[36]

Table 3.2: Stafford Loan Cumulative Borrowing Limits

		Dependent			Independent	
	Subsidized	Unsubsidized	Combined	Subsidized	Unsubsidized	Combined
1993-2008	23,000	23,000	23,000	23,000	46,000	46,000
2008-09 and after	23,000	31,000	31,000	23,000	57,500	57,500

Notes: Each column shows the maximum amount that a student can borrow based on their year dependency status and loan type. Students whose parents do not qualify for PLUS loans are eligible to borrow up the limit for independent students. Prior to 1993, independent students and some dependent students could borrow from the Supplemental Student Loan for Students (SLS) program.

To provide some context for the relevance of these limits, panel (a) in Figure 3.11 shows that nearly 50% of undergraduates use Stafford loans.[37] Panel (b) shows that, among those

Lochner and Monge-Naranjo (2016).

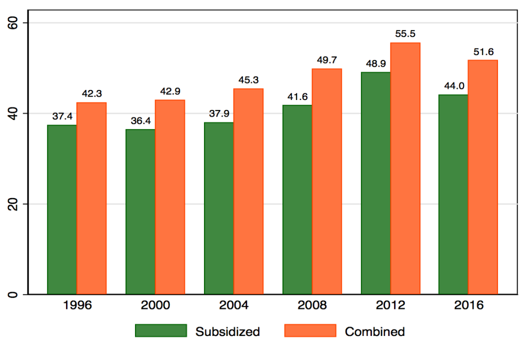
[35]Factors determining eligibility for subsidized loans include dependency status, family income, and cost of the institution attended. Students under age 24 are considered to be dependent.

[36]The borrowing limits in Table 3.2 are often referred to as "program limits" as opposed to "individual limits". An individual limit specifies that a student may not borrow more than their student budget (total price of attendance) or financial need (student budget net expected family contribution). A student is therefore constrained by the individual limit if it is less than the program limit. See Table 2 of Wei and Skomsvold (2011) for borrowing limits by year in school.

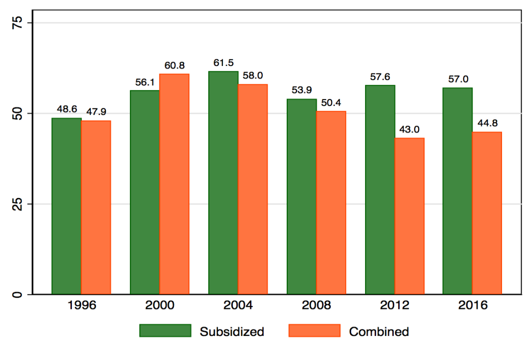
[37]See Figure B.1 in Appendix B.1 for the fraction of students who borrow Stafford loans and borrow the maximum by dependency status.

who use Stafford loans, approximately 50% borrow the maximum that they are eligible for.[38] These borrowing limits are also relevant because students who hit the maximum are more likely to have to take out private student loans which often have higher interest rates and less flexible repayment options (Wei and Skomsvold, 2011).

Figure 3.11: The Use of Stafford loans



(a) Students with Stafford Loans



(b) Students who Borrow the Maximum Amount

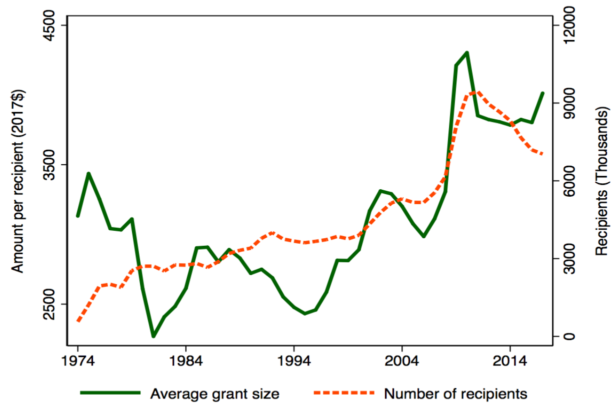
Notes: Data come from the National Postsecondary Student Aid Survey (NPSAS). Panel (a) shows the fraction of undergraduates who borrow a positive amount of Stafford loans. Panel (b) shows the percentage of undergraduates who borrowed the "usual maximum" amount of Stafford loans. Sample include students who were enrolled full-time, full-year. Percentages are calculated using the NPSAS weights.

Another education policy that has been extensively discussed are subsidies/grants. While most of the discussion and debate in recent years has centered around whether college should be fully subsidized, Federal pell grants have increasingly been used a policy tool since the mid 1990s. The solid line (left axis) in Figure 3.12 shows that, between 1994 and 2017, the average grant amount per recipient increased from nearly \$2500 to \$4000, a 60% increase. The dashed line (right axis) shows that the number of grant recipients steadily has steadily increased.

To more formally test for whether higher education policy is useful for predicting the underemployment rate, I perform a VAR analysis and subsequently perform tests for Granger

[38]As for before 1996, Berkner (2000) found that 17.8% of full-time, full-year undergraduates borrowed the maximum combined amount of Stafford loans in the 1989-90 academic year.

Figure 3.12: Trends in Federal Pell Grants



Notes: Data come from College Board (2017). The left axis (solid line) shows the average grant amount per recipient in 2017$. The right axis (dashed line) shows the amount of grant recipients per academic year (measured in thousands).

causality. Consider the five variable VAR:

$$\begin{bmatrix} \Delta \text{BA}_t \\ \Delta \text{Underemp}_t \\ \Delta \text{Disburse}_t \\ \Delta \text{Recipients}_t \\ \Delta \text{Grant}_t \end{bmatrix} = \beta_0 + \boldsymbol{\beta}_1 \begin{bmatrix} \Delta \text{BA}_{t-1} \\ \Delta \text{Underemp}_{t-1} \\ \Delta \text{Disburse}_{t-1} \\ \Delta \text{Recipients}_{t-1} \\ \Delta \text{Grant}_{t-1} \end{bmatrix} + \ldots + \boldsymbol{\beta}_k \begin{bmatrix} \Delta \text{BA}_{t-k} \\ \Delta \text{Underemp}_{t-k} \\ \Delta \text{Disburse}_{t-k} \\ \Delta \text{Recipients}_{t-k} \\ \Delta \text{Grant}_{t-k} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{bmatrix}, \quad (3.29)$$

where BA_t is the fraction of 25-30 year olds in year t who have at least a Bachelors degree, Underemp_t is the fraction of underemployed college graduates in year t who work in occupations with an average annual salary below \$25,000, Disburse_t is the total amount of Federal student loans disbursed in the academic year $t-t+1$, Recipients_t is the number of recipients of a Federal Pell Grant in the academic year $t-t+1$, and Grant_t is the average per-capita Federal Pell grant award in the academic year $t-t+1$, β_0 is a matrix of intercept terms, and $\boldsymbol{\beta}_k$ is a matrix of coefficients for $t \in \{1, \ldots, k\}$. I estimate (3.29) with $k = 3$ (per the

Akaike Information Criterion) and using data between 1974 and 2015.[39]

Using the estimates of (3.29), I test the null hypothesis that Federal loan disbursements, number of grant recipients, and per-capita grant awarded do not Granger-cause the underemployment rate. I find that the amount of Federal loan disbursements and per-capita grant amount Granger cause the underemployment rate at the 1% significance level and fail to reject the null hypothesis that the number of grant recipients Granger causes the underemployment rate.[40] These results indicate that changes in the amount of Federal loans disbursed and changes to average grant sizes are useful for predicting future changes to the underemployment rate.

3.6.2 Analytical Channels

With an overview of developments in higher-education policy in recent decades and evidence of a connection between education policy and underemployment, I return to the model to isolate the channels through which changes to education policy affect the equilibrium underemployment rate. These channels can be illustrated through studying comparative statics with respect to the pecuniary cost of human capital, p_h. I proceed by outlining the intuition behind these channels and summarize the formal results in Proposition 3.6.

Suppose that $\epsilon = 1$ and p_h decreases. This causes the net benefit of investing in human capital to increase and for more workers to invest in human capital. As the supply of highly-educated workers increases, the composition of unemployed workers shifts from less- to highly-educated workers. From equation (3.9), as η decreases, the expected profits of posting a complex vacancy increase. When more complex vacancies are created, highly-educated workers have a higher opportunity cost of forming a cross-skill match. Thus, the *supply channel* induces more complex jobs to be created and for highly-educated workers to

[39]These results are available upon request.

[40]See Table B.1 in Appendix B.1.3 for test statistics generated by the Wald tests of joint significance.

become less-likely to form a cross-skill match.

Recall, from Lemma 3.2, that the net benefit of investing in human capital is increasing in the worker's innate ability. Consider an equilibrium in which only high ability workers invest in human capital. As p_h declines and the composition of vacancies shifts towards complex jobs through the supply channel, the net benefit of investing in human capital will increase. Eventually, low-ability workers will find it beneficial to invest in human capital. When low-ability workers invest, the average innate ability within highly-educated workers decreases which, from (3.9), decreases the expected profits of posting a complex vacancy. It follows that the *composition channel* induces less complex jobs to be created, the opposite effect of the supply channel.

Despite their competing effects on the expected profits of posting a complex vacancy, it can be shown that the supply channel outweighs the composition channel. The intuition for this is the fact that when low-ability workers invest in human capital, they enter a group of highly-educated workers which already contains high-ability workers, which diminishes the impact of low-ability workers on the average ability within highly-educated workers.

Now suppose that $a_L = a_H = 1$, which shuts down the composition channel, and there is curvature in the final goods technology, i.e. $\epsilon < 1$. As p_h decreases and more workers to invest in human capital, the price of output produced in complex jobs decreases, as there are diminishing returns to production of the final good, which reduces the expected profits of posting a complex vacancy. Thus, the *relative price channel* causes the composition of jobs to shift towards simple vacancies, the opposite effect of the supply channel. Proposition 3.6 shows that as there is a stronger complementarity between output from simple and complex jobs, that the effect of the relative price channel can outweigh the effect of the supply channel on the expected profits of a complex job.

Proposition 3.6. *The following cases summarize the results mentioned above:*

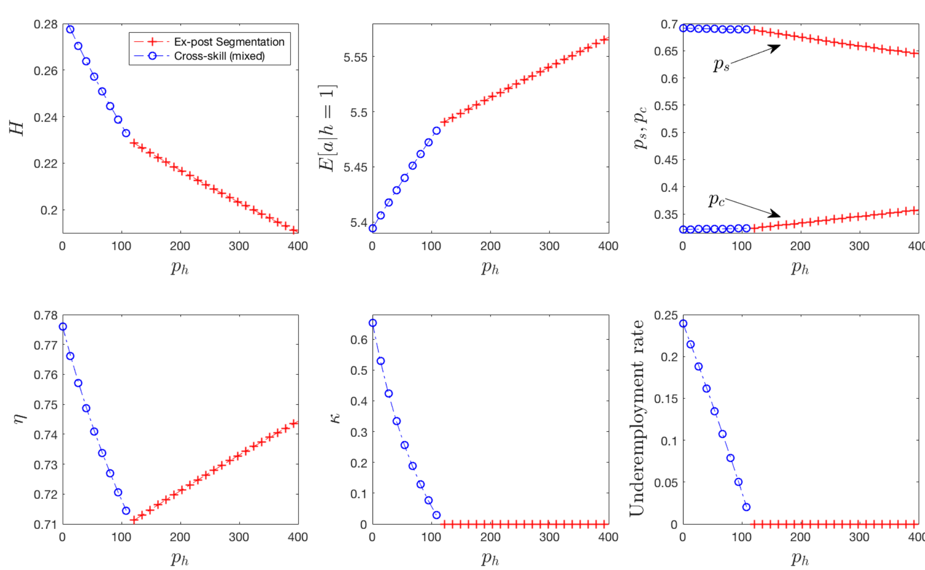
1. *Suppose that $\epsilon = 1$. The effect of an increase in the supply of highly-educated workers on the expected profits of positing a complex vacancy outweighs the effects of changes to the average innate ability within highly-educated workers.*

2. *Suppose that $a_L = a_H = 1$ and $\epsilon < 1$. As $\epsilon \to -\infty$, the effect of an increase in the supply of highly-educated workers on the expected profits of posting a complex vacancy through the relative price channel outweighs the effect of the supply channel.*

Proof. See Appendix B.2.11. □

Figure 3.13 illustrates comparative statics with respect to p_h and the aforementioned channels. The top row shows that as p_h increases and less workers invest in human capital that the average innate ability within highly-educated workers increases and the relative prices adjust. The bottom row shows that as less workers invest in human capital, the composition of unemployed workers shifts towards less-educated workers. The effects of the relative price and composition channels outweigh the effect of the supply channel workers become less likely to form cross-skill matches and the underemployment rate decreases.

Figure 3.13: Comparative Statics With Respect to p_h



3.7 Calibration and Policy Experiments

This section presents the calibrated version of the model and performs counterfactual policy experiments. In Section 3.7.1, I introduce a few modifications to the model that are unique to the quantitative version of the model. Section 3.7.2 details the calibration strategy, Section 3.7.3 compares the decentralized and constrained efficient allocations, and Section 3.7.4 performs education policy experiments.

3.7.1 Quantitative Version of the Model

I introduce two differences in the quantitative model relative to the baseline environment in Section 3.3. The first is that a worker's innate ability is drawn from a continuous distribution $G(a)$. The second modification is that I assume the following structure for the production costs, $\varphi(\ell)$:

$$\varphi(\ell) = \begin{cases} 0 & \text{if } \ell \leq \underline{\ell}, \\ \alpha\ell & \text{if } \underline{\ell} < \ell < \overline{\ell}, \end{cases} \tag{3.30}$$

with $\alpha > 1$. The interpretation of (3.30) is that a worker's endowment, $\underline{\ell}$, is a familial transfer that only can be used for educational expenses. The linear portion of $\varphi(\ell)$ is now interpreted as the borrowing costs incurred to finance human capital if their endowment is less than the pecuniary cost of human capital, i.e. if $\underline{\ell} < p_h$.

3.7.2 Calibration Strategy

The model is calibrated to the U.S. economy between 1992-2017. A unit of time is interpreted as one month. I assume that the aggregate meeting technology is Cobb-Douglas: $\mathcal{M}(\Omega, v) =$

$A(\Omega)^{\nu}v^{1-\nu}$. The elasticity of the meeting technology is set to $\nu = 0.5$, as this is within an empirically supported range (Petrongolo and Pissarides, 2001b) and I subsequently assume $\beta = 0.5$. The elasticity of substitution between simple and complex jobs, $1/_{1-\epsilon}$, is set equal to 1.41 following Katz and Murphy (1992), which implies $\epsilon = 0.29$.[41] The death and birth rate, σ, is calculated as the average mortality rate among 15-34 year olds in 2007 from the Centers for Disease Control (CDC) National Vital Statistics System and gives $\sigma = 0.000924$. The separation rate is set equal to the monthly separation rate among 22-27 year olds in the Current Population Survey (CPS). Following Shimer (2012)'s method for constructing transition rates, I find $s = 0.021$.

The parameter which determines borrowing costs, α in (3.30), is calculated by equating the production costs $\alpha\ell$ to the total cost incurred by a borrower who borrowed the amount $p_h - \underline{\ell}$ and made monthly repayments over 10 years at an annual interest rate of 5%.[42] This gives the following form for $\varphi(\underline{\ell})$:

$$\varphi(\ell) = \frac{120 \max\{\hat{p}_h - \underline{\ell}, 0\} \left[\frac{.05}{12}\left(1 + \frac{.05}{12}\right)^{120}\right]}{\left[1 + \frac{.05}{12}\right]^{120} - 1}, \tag{3.31}$$

where $\hat{p}_h$ is the net price of human capital.

The strategy for choosing the distribution of innate ability follows Braun (2019) who matches the distribution of ASVAB scores in the NLSY and estimates that $a - 1$ is distributed log-normal with a mean of 4.62 and a standard deviation of 0.62.

The rest of the model's parameters are chosen to target empirical moments. The first five targets from the data are the following: (i) The average value of market tightness from the Job Openings and Labor Turnover Survey (JOLTS) between December 2000 and December 2017

[41] This value for the elasticity of substitution is also used in Krueger and Ludwig (2016). Borjas (2003) finds a similar estimate for the elasticity of substitution.

[42] Ten-year repayment plans are typical for Federal student loans and a 5% interest rate is within the range of interest rates seen over the last decade. See https://studentaid.ed.gov/sa/types/loans/interest-rates for an overview of Federal student loan interest rates.

of 0.3857 (ii) an underemployment rate of 24.6%,[43] and three estimates of the college earnings premium.[44] I estimate these premia by estimating variations of the following regression:

$$y_{ist} = \alpha + \beta_1 \text{college}_i + \boldsymbol{\beta} X_i + \lambda_s + \delta_t + \varepsilon_{ist}, \tag{3.32}$$

where the subscript ist refers to individual i in state s and year t, y is an outcome of interest (log earnings), college is an indicator for whether the individual has at least a bachelors degree, X is a vector of individual characteristics (e.g., demographics and industry), λ_s is a year fixed effect, δ_t is a year fixed effect, and ε_{ist} is an error term that captures shocks and omitted variables. I estimate variations of (3.32) by ordinary least squares.

Table 3.3 reports the estimates of the college earnings premium. Column (1) includes all individuals in the sample and shows that on average a college degree is associated with an increase in earnings of 43.8%. Column (2) restricts the sample by excluding workers with at least a bachelors degree who work in college occupations. It shows that within non-college occupations that a college degree is associated with an earnings premium of 19.4%. Column (3) excludes those with at least a bachelors degree who work in non-college occupations and shows that a college degree is associated with a 60.7% increase in earnings in occupations that typically require a college degree. Five parameters, $(x_s, x_c, \mu, \gamma, b)$, are chosen to match these five targets. I find $x_s = 7.97$, $x_c = 22.06$, $\mu = 0.619$, $\gamma = 86.12$, and $b = -2.73$.

The value of the matching efficiency, A, is chosen to match the monthly job-finding rate of 0.504 among college educated workers ages 22-27 in the CPS. Combining with the target

[43] The target I use for the underemployment rate is the average of the average fraction of workers who work in occupations where less than 50% of respondents say that a college degree is required to perform that occupation (39.6%) and the fraction who work in occupations where less than 5% of respondents say that a college degree is required to perform that occupation (9.6%).

[44] I focus on annual earnings rather than hourly wages because it is more transparent to interpret a worker's expected earnings in a job due to the two-part employment contract rather than the flow wage earned by a worker. Appendix B.1.4 contains estimates of the same estimation strategy with hourly wages and shows that the gaps between the estimated premia are relatively unchanged when considering hourly wages.

Table 3.3: Regression Estimates: College Earnings Premia

	(1)	(2)	(3)
	Overall	All non-college occupations	Appropriately matched
College	0.438***	0.194***	0.607***
	(0.039)	(0.036)	(0.058)
N	213,778	182,125	193,917
R^2	0.118	0.085	0.134

Notes: All regressions include state fixed effects, year fixed effects, control for demographics (age, sex, race, marital status), whether the individual works in a city, and the individual's industry of employment. The sample covers 1992-2017 and is composed of individuals between the ages of 22-27 who are not currently enrolled in school. Column (1) includes all individuals in the constructed sample. Column (2) excludes workers with at least a bachelors degree who work in a college occupation. Column (3) excludes workers with a college degree who work in non-college occupations. Standard errors are clustered at the occupation level and are in parentheses. Levels of statistical significance are denoted by $^{***}p < 0.01$.

of $\theta = 0.3857$, I find $A = 0.943$. The search intensity of employed workers, λ, is chosen to match the ratio of the monthly job-to-job transition rate among mismatched college educated workers (0.0379) to the monthly job-finding rate of college educated workers (0.504) in the CPS. This gives $\lambda = 0.125$. The rate of time preference is chosen to target an annual effective discount factor of 0.953 (Shimer, 2005c). Combining with σ gives $\rho = 0.003076$.

The pecuniary cost is chosen to match the estimated rate of return of college of 15% in Abel and Deitz (2014), which gives $p_h = 799.80$. The debt limit, $\bar{\ell}$, is chosen to match the ratio of the cumulative Stafford loan borrowing limits to the average four-year sticker price of public universities in the U.S. of 43%. This gives $\bar{\ell} = 343.91$. The psychic cost is chosen to match the fraction of 25-30 year olds with at least a bachelors degree between 1992-2017 in the CPS of 30.5%. This corresponds to targeting $H = 0.305$ and gives $\varsigma = 755.60$.

The distribution $F(\underline{\ell})$ is a Generalized Pareto distribution with location parameter 0. The shape and scale parameters are chosen to match the mean and median of expected family contributions (EFC) for education purposes relative to the average sticker price of public universities.[45] This gives a shape parameter of 0.2136 and a scale parameter of 4278.4. Table 3.4 summarizes the parameter values and Table 3.5 shows that the model is able to closely match the empirical targets.

Table 3.4: Parameter Values

Parameter	Definition	Value
A. Externally calibrated		
σ	Death & birth rate	0.000924
δ	Separation rate	0.021
ν	Elasticity of meeting function w.r.t. job seekers	0.50
ϵ	Elasticity of substitution in final goods technology	0.29
β	Worker's bargaining power	0.50
B. Internally calibrated		
x_s	Productivity of simple jobs	7.97
x_c	Productivity of complex jobs	22.06
μ	Relative importance of simple jobs	0.619
γ	Vacancy flow cost	86.12
b	Unemployed flow utility	−2.73
A	Efficiency of meeting function	0.943
λ	Search intensity of mismatched workers	0.125
ρ	Discount rate	0.003076
p_h	Price of education	799.80
$\bar{\ell}$	Borrowing limit	343.91
ς	Psychic cost of education	755.60

3.7.3 Centralized vs. Decentralized Allocations

In this section, I compare the constrained efficient and decentralized allocations under the calibrated parameters presented in Table 3.4. For the constrained efficient allocation, I compute the planner's choice of market tightness, θ, the composition of vacancies, ζ, hu-

[45]Data on EFC comes from the National Postsecondary Student Aid Survey (NPSAS) for the survey years 1996, 2000, 2004, 2008, 2012, and 2016. The estimates of the mean and median only includes dependent students and includes those students who had an EFC of 0. The mean (median) in 2017$ was $14,684 ($8,627). Combined with data on average sticker prices of four-year public universities from the College Board implies that the ratio of the mean (median) ω to p_h is 0.9423 (0.5536).

Table 3.5: Targeted Moments

	Data	Model
Market tightness	0.385	0.388
Underemployment rate	0.246	0.265
Earnings premia: non-college	0.194	0.192
Earnings premia: college only	0.607	0.585
Earnings premia: overall	0.438	0.414
Job-finding rate: highly-educated	0.504	0.504
JtJ rate among underemp.	0.037	0.030
Fraction with a BA	0.305	0.308
College RoR	1.150	1.150
Debt limit/sticker price	0.430	0.430
Mean EFC/sticker price	0.942	0.940
Median EFC/sticker price	0.553	0.552

man capital choice $h(a_i, \underline{\ell})$, and cross-skill matching rules κ_i to maximize steady-state net output.[46] Table 3.6 summarizes the results.

Table 3.6: Comparison of Allocations

	Centralized	Decentralized
Market tightness	0.320	0.388
Composition of vacancies	0.570	0.580
Supply of highly-educated workers	0.325	0.308
Pr(form cross-skill match)	0.000	0.835
Underemployment rate	0.000	0.265
Aggregate unemployment rate	0.098	0.072
Net output	20.56	18.13

As seen in Table 3.6, the constrained efficientallocation exhibits a lower amount of job creation and a composition of vacancies that consists of less simple jobs. Relative to the constrained efficientallocation, the decentralized equilibrium exhibits under-investment in human capital. Moreover, the underemployment rate in the constrained efficientallocation is 0%, due to the fact that the social planner chooses not to form cross-skill matches. Finally, the aggregate unemployment rate is higher and net output is 13.4% larger under the

[46] Shimer and Smith (2001) show that the efficient allocation with ex-ante heterogenous agents may be a limit cycle if both the production function is supermodular and the planner has the option to break up existing matches. I abstract from allowing the planner to break up existing matches. Moreover, the steady-state output may not be the optimal solution but I show that the steady-state output produced under the planner's choice is larger than that of the decentralized equilibrium.

constrained efficient allocation relative to decentralized outcome.

The discrepancies between the constrained efficientand decentralized outcomes can be tied to the inefficiencies discussed in Section 3.5. We see that workers under-invest in human capital, which results from the hold-up problem in human capital investment. Moreover, workers in the decentralized equilibrium are not choosy enough in the formation of cross-skill matches. This is due to the fact that their opportunity cost of forming a cross-skill match is scaled by their share of the match surplus, β, whereas the social planner considers the total forgone surplus of forming a cross-skill match. As workers in the decentralized outcome form cross-skill matches at an inefficiently high rate, the composition of jobs creates too many simple jobs and leads to an inefficiently high amount of job creation.

In the subsequent policy experiments, I study the effect of policies that have a goal of increasing investment in human capital to address the under-investment seen in Table 3.6. In particular, I study the effect of subsidizing education through lump-sum taxes and relaxing borrowing constraints by increasing student loan borrowing limits.

3.7.4 Policy Experiments

The first set of policy experiments that I consider are varying the size of education subsidies. I assume that the subsidies are financed by lump-sum taxes on workers.[47] The tax revenue is equally distributed among the workers who invest in human capital and the subsidy is not correlated with a worker's endowments. The first experiment is to implement subsidies that finance 27% of the pecuniary cost of human capital, which is the average amount observed in the U.S. between 1992-2016 (College Board, 2017). The second experiment is one where the tax is increased to the point where education is fully subsidized. Table 3.7 contains the results of both experiments.

[47] The amount of the tax is independent of a worker's employment status.

Table 3.7: Policy Experiment: Education Subsidies

	Benchmark	1992-2017 increase		Fully subsidized education	
		Level	% change	Level	% change
Underemployment					
P(form cross-skill match)	0.835	1.000	19.76	1.000	19.76
Share of simple vacancies	0.580	0.584	0.585	0.593	2.220
Underemployment rate	0.265	0.291	9.811	0.300	13.20
Unemployment rates					
Less-educated	0.081	0.081	−0.122	0.080	−0.859
Highly-educated	0.053	0.049	−8.905	0.049	−7.977
Aggregate	0.072	0.071	−2.606	0.070	−3.017
Earnings premia					
Non-college	0.192	0.190	−1.400	0.190	−1.244
College only	0.585	0.574	−1.878	0.578	−1.297
Overall	0.414	0.368	−11.08	0.366	−11.56
Education sector					
Supply of human capital	0.308	0.321	4.082	0.323	4.828
Net price/sticker price	1.000	0.730	−27.00	0.000	−100.0
Welfare	18.13	18.26	0.693	18.34	1.171

As seen in columns (2) and (3), when 27% of p_h is subsidized, the probability to form a cross skill match increases from 85.3% to 100%. Combined with little change to the composition of vacancies, this increases the underemployment rate by 9.81%. Rows (3) through (6) show that the subsidy decreases the unemployment rate for highly-educated workers as they are much less likely to become employed at simple jobs. The bottom half of the table shows that the subsidy increases the supply of human capital by 4.08%. As the supply of highly-educated workers increases, the relative prices p_χ adjust and decreases the overall college earnings premium by 11.08%. The last row shows that this policy increases welfare by 0.693% due to its effect on human capital accumulation and reduction in unemployment.

Columns (4) and (5) of Table 3.7 show the effects of fully subsidizing human capital. This policy also causes κ to increase from 0.835 to 1 and for the composition of vacancies to slightly shift towards simple jobs. These changes cause the underemployment rate to increase by 13.20% and for the unemployment rate among highly-educated workers to decrease by 7.977% relative to the baseline allocation. The supply of highly-educated workers increases by 4.828% and due to the changes in the ratio p_s/p_c, the earnings premium in non-college jobs decreases by 11.56%. Overall, fully subsidizing human capital increases welfare by 1.17% and, taken together, Table 3.7 illustrates that while subsidizing education can cause the underemployment rate to increase, these policies also increase welfare.

The second set of policies studies changes to the borrowing limit $\overline{\ell}$. The first change that I consider is eliminating the option to borrow to finance human capital by setting $\overline{\ell} = 0$. Secondly, I consider the opposite extreme of fully relaxing borrowing limits and setting $\overline{\ell} = p_h$. Table 3.8 contains the results.

Columns (2) and (3), which correspond to $\underline{\ell} = 0$, show that tightening borrowing constraints causes the underemployment rate to decrease from 26.5% to 13.1%. This is driven by the decrease in the supply of highly-educated workers, H, due to the tightened borrowing constraints, as this policy causes H to decrease by 18.98%. This policy causes a decline in welfare of 8.96%. Columns (4) and (5) illustrate that the the quantitative effects of relaxing borrowing constraints to the point where $\overline{\ell} = p_h$ are relatively small. This is because, in the baseline calibration, less than 1% of workers are constrained by the original borrowing limit. The workers who are constrained have a relatively high innate ability (see Lemma 3.2). When these workers begin to invest in human capital, it reduces the benefits of human capital investment for other workers, particularly those with a relatively low innate ability. In net, the supply of highly-educated workers decreases, which leads to a decline in the underemployment rate of 8.67% and a 0.749% decrease in welfare.

Table 3.8: Policy Experiment: Borrowing Limits

	Benchmark	No borrowing		No borrowing limit	
		Level	% change	Level	% change
Underemployment					
P(form cross-skill match)	0.835	0.244	−70.77	0.684	−18.08
Share of simple vacancies	0.580	0.592	1.910	0.582	0.189
Underemployment rate	0.265	0.131	−50.566	0.242	−8.679
Unemployment rates					
Less-educated	0.081	0.080	−1.71	0.081	−0.491
Highly-educated	0.053	0.085	57.69	0.059	9.461
Aggregate	0.072	0.081	11.11	0.074	1.508
Earnings premia					
Non-college	0.192	0.117	−39.31	0.190	−1.452
College only	0.585	0.582	−0.631	0.586	0.051
Overall	0.414	0.526	26.99	0.446	7.677
Education sector					
Supply of human capital	0.308	0.250	−18.98	0.300	−2.786
Net price/sticker price	1.000	1.000	0.000	1.000	0.000
Welfare	18.13	16.51	−8.965	18.00	−0.749

3.8 Conclusion

This paper has developed a theory of equilibrium underemployment. The model generates a rich set of equilibria, including the multiplicity of equilibria with different combinations of matching and human capital investment patterns. The introduction of a human capital investment decision among workers allows for a positive and normative analysis of the effects of increasing student loan borrowing limits and education subsidies. The analytical results show that the effects educational policies on the labor market are driven by a supply, composition, and relative price channels. A normative analysis shows that the decentralized equilibrium is never efficient and can exhibit an inefficiently high or low amount of underemployment. A calibrated version of the model shows that there is under-investment in human capital and an inefficiently high amount of underemployment in the U.S. Increasing education subsidies can increase welfare by inducing more workers to investment in human capital, despite the fact that this policy also increases the underemployment rate.

Chapter 4

Entrepreneurial Investment, Home Equity, and Monetary Policy

This chapter models entrepreneurial finance using a combination of fiat money, credit cards, traditional bank loans, and home equity loans.[1] The banking sector is over-the-counter, where bargaining determines the pass-through from the nominal interest rate to the bank lending rate, characterizing the transmission channel of monetary policy. The strength of this channel depends on the combination of nominal and real assets used to finance investments, and declines in the extent to which housing is accepted as collateral. A calibration to the U.S. economy complements the theoretical results and provides novel insights on entrepreneurial finance between 2000 and 2016.

[1]This chapter is written jointly with Florian Madison, Assistant Professor of Economics at Claremont McKenna College.

4.1 Introduction

This paper provides a monetary search model subject to commitment and bargaining frictions to study entrepreneurial finance in the United States. Capital accumulation can be financed internally using savings and credit card debt, and externally through traditional bank loans and home equity loans, allowing for coexistence of money and credit. The banking sector is over-the-counter, where bilateral bargaining determines the terms of trade, and thus the pass-through from the nominal interest rate to the real lending rate, characterizing the transmission channel of monetary policy.

It is well documented that apart from housing services, home equity is commonly used as collateral to secure loans and lines of credit (HELs and HELOCs) when markets are imperfect.[2] While a fair share of these loans are used to finance consumption (Greenspan and Kennedy, 2007), they further find high demand among entrepreneurs facing capital expenditures. According to the Annual Survey of Entrepreneurs, in 2016, roughly 6.3% of all entrepreneurs in the U.S. reported to have used home equity loans to start or acquire their business.[3] The most prominent competing means of financing were savings, traditional banks loans, and credit cards, used by 73.1%, 16.5%, and 14% respectively, as summarized in Table 4.1, characterizing the four main funding channels of entrepreneurs in the United States.

Table 4.1: Entrepreneurs' Primary Funding Sources - Annual Survey of Entrepreneurs (2016)

Funding Source	%-used
Savings	73.1%
Credit cards	14%
Traditional bank loans	16.5%
Home equity loans	6.3%

[2]This does not come as a surprise since home equity accounts for nearly 50% of a household's net wealth and is amongst the most valuable asset an average household holds – Iacoviello (2011).

[3]Furthermore, according to he Survey of Consumer Finances, between 2001 and 2016, 12% of entrepreneurs financed expenditures through HELs, accounting for a 52% (74%) higher average use (amount borrowed) among self-employed relative to households working for someone else.

The limited sources of external funding, as opposed to larger corporations who can issue equity and debt on international financial markets, make entrepreneurs highly susceptible to fluctuations in the housing market and frictions in the local banking sector. Recent data from the Small Business Credit Survey, as summarized in Table 4.2, shows that in 2017, 22% of all applicants did not receive a loan at all (extensive margin), while 78% received some or all the funding applied for (intensive margin). The provided framework incorporates these margins through search frictions and liquidity constraints, whereas the latter highlights the importance of housing as collateral in the entrepreneurial sector, backing the empirical results by Schmalz et al. (2017) and Corradin and Popov (2015) among others discussed in the related literature.

Table 4.2: Total Funding Received - Small Business Credit Survey (2017)

Credit application outcomes	All firms
All (100%)	46%
Most (> 50%)	12%
Some (< 50%)	20%
None (0%)	22%

Next to the frictions in the banking sector, monetary policy affects the financing landscape of firms, whereas the transmission channel is twofold. Directly, through the inflation tax on savings, or indirectly, via the pass-through from nominal to real interest rates, and thus the lending environment of banks. Using a monetary search framework, our model allows us to encompass this dimension and provide normative statements with regards to optimal monetary policy.

The environment is infinite horizon and draws from Rocheteau et al. (2018), extended by a frictionless housing market. Under uncertainty regarding future investment and financing opportunities, entrepreneurs choose to fund capital expenditures using a combination of internal (fiat money and unsecured credit card debt) and external financing (bank loans). Contrary to the convention in the literature, the banking sector is over-the-counter and terms of the loan contract are determined by bilateral bargaining between banks and entrepreneurs.[4] Frictions including a lack of commitment and record keeping make collateral essential for bank loans to be incentive compatible, where distinction is made between traditional bank loans secured by claims on future profits and home equity loans secured by real estate. The latter endows housing with a liquidity premium, adding an additional role next to consumption of housing serves, underpinning its ability to facilitate trade when credit markets are imperfect. Furthermore, the introduction of banks extends the supply of (outside) fiat money by tradeable bank liabilities (inside money), governed by collateral constraints.

Outside money has two roles in our environment. First, it serves as insurance against tightened borrowing constraints in the banking sector, and second, it is used as a strategic device to demonstrate 'skin in the game', allowing entrepreneurs to obtain more favorable terms of trade when bargaining with a bank.[5] This multiplier makes outside money and real estate imperfect substitutes, directly affecting the pass-through of the nominal interest rate to the bank lending rate.

Relying on said pass-through and the two roles of outside money, the established model provides novel insights on the transmission of monetary policy. The results show that the magnitude of the real effects following a change in the nominal interest rate depends on the composition of internal and external financing used, and hence the size of the haircuts applied on the provided collateral. If housing is sufficiently pledgeable, the demand for fiat money is low, weakening the transmission channel of monetary policy. If, however, housing is barely accepted as collateral and capital is primarily financed through internal financing and traditional bank loans, the transmission channel is strong. The sensitivity of the pass-through underpins the importance of the individual financing channels and highlights the

[4]In doing so, we rely on the observations of Mora (2014), relating the observed dispersion in loan rates to differences in banks' bargaining power.

[5]The former is in line with Bates et al. (2009), Sánchez and Yurdagul (2013), and Campello (2015) on corporate liquidity management, where fiat money serves as an insurance against the risk of idiosyncratic financing opportunities.

close relationship between the entrepreneurial sector, the housing market, and monetary policy.

Given the environment, however, tractability of the analytical results is limited. Thus, to complement the theoretical results, we calibrate the model to the U.S. economy for 2000-2016. In doing so, we focus on the semi-elasticity of aggregate entrepreneurial investment in response to a change in the nominal interest rate, capturing the strength of the transmission of monetary policy. Varying the pledgeability of housing allows to account for fluctuations in the housing market and consequential changes in the composition of internal and external finance. The calibrated results confirm the aforementioned theoretical outcomes. For example, at a nominal interest rate of 0.05 (as seen in 2007), if housing is denied as collateral, a one percent increase in the nominal interest rate decreases entrepreneurial investment by 7.1%. The larger the share of investments financed by fiat money, i.e., the lower the nominal interest rate, the stronger the real effects of a change in monetary policy.

4.1.1 Related Literature

This paper is deeply founded in the literature on monetary search-theory and markets with frictions, as surveyed in Rocheteau and Nosal (2017) and Lagos et al. (2017). The first to apply the theoretical toolkit of this literature to study corporate finance and monetary policy, and the paper most closely related to ours, were Rocheteau et al. (2018). We build on this framework by tailoring the environment to the entrepreneurial sector in the United States, i.e, introducing credit card debt and a housing market, where private real estate has two roles: consumption and saving. The latter allows entrepreneurs the use of home-equity to secure bank loans, capturing the relationship between the housing market, the the pass through from nominal to real rates, and the transmission channel to entrepreneurial investment.[6]

Since the Great Recession, a vast literature on the use of home equity to secure bilateral credit transactions emerged, whereas distinction is made between consumption and investment loans. The former are studied by He et al. (2015), incorporating HELs into a Lagos and Wright (2005) environment. The endogenously arising liquidity premium on housing generates dynamics in the value of real estate, explaining parts of the housing boom experienced in the early 2000s.[7] Using similar toolkits, Branch et al. (2016) provide an application to unemployment by endogenizing the construction sector. Their results show that financial innovations that raise the acceptability of homes as collateral increase house prices and reduce unemployment. Among the first to abstain from consumption loans were Liu et al. (2013), introducing land as collateral in firms' credit constraints using a DSGE model, showing how co-movements of land prices and business investments propagate macroeconomic fluctuations. While the provided results apply for firms of all sizes, a more detailed application to entrepreneurs is provided by Decker (2015). Within a heterogeneous agent DSGE model with housing and entrepreneurship, his results show that while recessions accompanied by a housing crash can explain the decline in entrepreneurial activity experienced in the early 2000s, a broader financial crisis would have no such effects. To further explain the recent synchronization of house prices and entrepreneurial activity, Lim (2018) develops an occupational choice model incorporating home equity loans. His results are in line with Decker (2015) and the paper at hand, showing that a rise in house prices increases entrepreneurial investment. However, a role for monetary policy remains absent. Our paper combines these components, allowing for an analysis of entrepreneurial finance, home equity, and monetary policy.

We also draw on empirical work studying the importance of housing as collateral in en-

[6]Another extension to study how heterogeneity of financial frictions and monopolistic competition influences this the transmission channel was provided by Silva (2017).

[7]Complementary results are provided by Justiniano et al. (2015) and Garriga et al. (2018), studying the relationship between the liquidity of real estate and house prices in the United States.

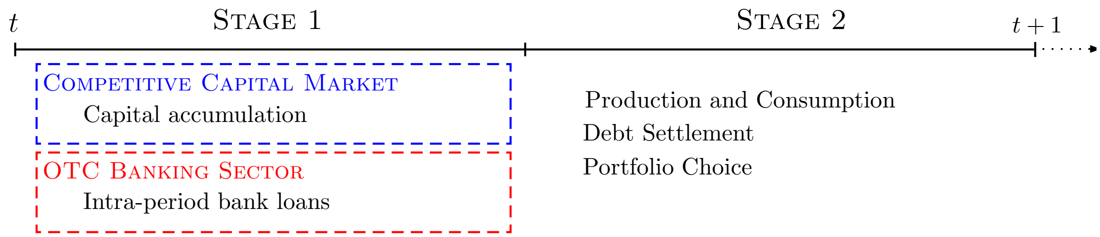
trepreneurial finance. Schmalz et al. (2017) found that higher values of collateral increased the likelihood of becoming an entrepreneur and that those with higher values of collateral took on more debt, started larger firms, and were more likely to remain in the long run. Corradin and Popov (2015) in turn focus on real estate prices in the U.S. and found that a 10% increase in home equity increased the probability of becoming an entrepreneur by 7%. Adelino et al. (2015) estimated that the collateral lending channel could explain 15-25% of employment variation in the U.S. between 2002-2012, showing the value of housing as collateral is directly tied to the formation of new businesses. Black et al. (1996) used data from the UK and found that a 10% increase in the value of home equity increased VAT registrations by nearly 5%, suggesting that an increase in the value of housing led to more small business formation. Harding and Rosenthal (2017) estimated that a 20% increase in real home value over two years increased entry into self-employment by 15 percentage points and that self-employed homeowners are more likely to use home equity lines of credit. Abstaining from the value of housing but focusing on its pledgeability, Jensen et al. (2014) found that a reform in Denmark which increased the availability of home equity loans increased entry into entrepreneurship.

4.2 Environment

Time is discrete, continues forever, and each period is divided into two stages, as displayed in Figure 4.1. In stage 1, two markets open simultaneously. An over-the-counter (OTC) banking sector, allowing agents to obtain intra-period bank loans, and a competitive capital market. In stage 2, agents produce and consume a numéraire good, settle outstanding debt obligations, and reallocate their portfolios in a frictionless market.

There are three types of agents, $j \in \{e, s, b\}$ – entrepreneurs, capital suppliers, and banks –

Figure 4.1: Timing of Events



each in a continuum $[0, 1]$, and three types of goods, $\{k, c, a\}$ – capital, a consumption good, and housing. Capital, k, produced by capital suppliers at unit cost in stage 1, is used by entrepreneurs as the sole input to produce output, $f(k)$, in units of a numéraire good c, in stage 2, where $f'(k) > 0 > f''(k)$, with $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 0 \; \forall \; k$. At the end of stage 2 both the numéraire good and capital fully depreciate, eliminating gains from storage across periods. Contrary to capital and the numéraire good, housing, a, is durable, in fixed supply A, and one unit buys q_a units of the numéraire good at stage 2. Furthermore, each unit of housing generates one unit of housing services each period, analogue to a Lucas tree. All agents have linear utility over the consumption good, c, while only entrepreneurs value housing services, $\vartheta(a)$, where $\vartheta'(a) > 0 > \vartheta''(a)$, $\vartheta(0) = 0$, $\vartheta'(0) = \infty$, and $\vartheta'(\infty) = 0 \; \forall \; a$. The discount factor across periods is $\beta = (1 + r)^{-1}$ and $r > 0$ the rate of time preference, characterizing the entrepreneur's lifetime utility:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t [c_t + \vartheta(a_t)]. \tag{4.1}$$

There is no record keeping of transactions in the competitive capital market and entrepreneurs have limited ability to commit to future actions. Hence, given the timing of events (supplier providing k in stage 1 and entrepreneur producing c in stage 2) and the non-durability of capital and the numéraire good, media of exchange – money and/or credit – are essential for trade to occur.

We allow for internal and external financing. Internal financing consists of fiat money (savings), m, and intra-period credit card debt, $b \in [0, \bar{b}]$, up to an exogenous debt limit $\bar{b}$. There is a central bank managing the supply of fiat money in the economy according to $M' = (1+\tau)M$, where M denotes the stock of fiat money in the current period, M' the stock in the next period, and expansion/contraction is conducted through lump-sum transfers, $T = \tau M_t$. One unit of money can buy q_m units of the numéraire good in stage 2. Since we focus on symmetric and stationary monetary equilibria, it holds that $M'/M = q_m/q_m' = \gamma$ with γ being the exogenous gross growth rate of the fiat money supply. External financing, on the other hand, can be obtained through banks via intra-period loans, consisting of perfectly divisible and recognizable one-period liabilities (inside money), i.e., banks can commit.[8] Given the lack of record-keeping and an entrepreneur's limited ability to commit to future actions, bilateral loans need to be collateralized to be incentive compatible. We consider (partial)-pledgeability of housing (HELs), $\rho q_a a$, and future output (traditional bank loans), $\chi f(k)$, with $\rho \leq 1$ and $\chi \leq 1$.[9] Hence, expansion and contraction of inside money is bounded by collateral constraints. Settlement of loan obligations, l, takes place in stage 2, where given the banks' ability to commit to future actions, redemption of collateral is guaranteed. In case of default on the part of the entrepreneur, the bank keeps the collateral.

Two idiosyncratic uncertainties determine the composition of internal and external financing: the availability of production/investment opportunities (as in Kiyotaki and Moore (1997)) and the availability of financing opportunities (as in Wasmer and Weil (2004)). With probability $\lambda \in [0, 1]$, an entrepreneur encounters an investment opportunity at the beginning of stage 1, guaranteeing access to the production technology $f(k)$ in stage 2. Once encountered, with probability $\alpha \in [0, 1]$, he meets a banker in the OTC banking sector who is willing to provide a loan. Assuming independence, with probability $\lambda\alpha$ an investment opportunity is financed internally and externally, while with probability $\lambda(1-\alpha)$, an investment opportunity

[8]Said liabilities are perfectly recognizable within a period, but can be counterfeited thereafter, which precludes them from circulating across periods.

[9]We assume that housing and future output are pledgeable only to banks, as banks are the only agents who have the technology to verify home-ownership and recover investments. Thus, there is no trade credit between capital suppliers and entrepreneurs.

is solely financed internally.

At the end of stage 2, entrepreneurs consume the produced numéraire good, c, housing services, $\vartheta(a)$, settle outstanding credit obligations, $l \in \mathbb{R}_+$ and $b \in \mathbb{R}_+$, and adjust their portfolio consisting of fiat money, and housing, m and a.

4.3 Model

An entrepreneur enters stage 2 with k units of capital and financial wealth, ω, denoted in the numéraire consisting of real money balances $q_m m$ and housing $q_a a$, minus bank and credit card liabilities, l and b, to solve:

$$W^e(k,\omega) = \max_{c,m',a'} c + \vartheta(a) + \beta V^e(m',a') \tag{4.2}$$

$$\text{s.t.} \quad c = f(k) + T + \omega - q_m m' - q_a a', \tag{4.3}$$

where $\beta V^e(m',a')$ is the continuation value in stage 1 of the next period. Plugging (4.3) into (4.2), yields:

$$W^e(k,\omega) = f(k) + \vartheta(a) + \omega + T + \max_{m',a'}\{-q_m m' - q_a a' + V^e(m',a')\}. \tag{4.4}$$

Hence, since $W^e(k,\omega)$ is linear in current wealth, an entrepreneur's choice of fiat money and housing for the subsequent stage 1, (m',a'), is independent of current balances (k,ω). The entrepreneur's value function in stage 1 is:

$$V^e(m,a) = (1-\lambda)W^e(0,\omega) + \lambda[(1-\alpha)W^e(k_I,\omega) + \alpha W^e(k_E,\omega)], \tag{4.5}$$

where capital accumulation is conditional on α and λ with k_I representing only internally financed capital and k_E internally and externally financed capital. Plugging (4.5) into (4.4) and updating reduces the portfolio choice in stage 2 to:

$$\max_{m',a'} -[q_m/\beta - q'_m]m' - [q_a/\beta - q'_a]a' + \vartheta(a') + \lambda[(1-\alpha)\Delta_I^e(m',b') + \alpha\Delta_E^e(m',a',b')], \quad (4.6)$$

with $\Delta_I^e(m,b)$ and $\Delta_E^e(m,a,b)$ denoting the entrepreneur's surpluses using internal and external financing, respectively. Following perfect competition in the capital market, $\Delta_I^e(m,b)$ is defined as $\Delta_I^e(m,b) = f(k_I) - k_I$ with $k_I = \min\{q_m m + \bar{b}, k^*\}$, where k^* solves $k^* \in \arg\max_k[f(k) - k] > 0$, analogue to the planner's problem. It represents the entrepreneur's outside option financing investments with savings and credit card debt only, playing a crucial role when determining $\Delta_E^e(m,a,b)$ via bilateral bargaining between a bank and an entrepreneur in Section 4.4.[10] Once determined, Section 4.5 characterizes the entrepreneur's optimal portfolio choice in stage 2, paving the way to study optimal policy in Section 4.6.

Analogue to the entrepreneur, the value functions of a bank and a capital supplier in stage 2 and 1 are:

$$W^{b,s}(k,\omega) = \max_{m',a'} \omega - q_m m' - q_a a' + \beta V^{b,s}(m',a') \quad (4.7)$$

and

$$V^b(m,a) = (1-\lambda)W^b(0,\omega) + \lambda[(1-\alpha)W^b(0,\omega) + \alpha W^b(0,\omega+\phi)] \quad (4.8)$$

$$V^s(m,a) = \max_k\{-k + W^s(0,\omega + q_k k)\}, \quad (4.9)$$

where ϕ denotes the bank's intra-period loan profits determined via bilateral bargaining in Section 4.4 and $q_k k$ the capital supplier's proceeds from capital sales in stage 1. The linearity of W^s implies $q_k = 1$, and hence $W^s = V^s$. Plugging either (4.8) into (4.7) or (4.9) into (4.7)

[10] Savings and credit card debt are used as monetary downpayment to represent 'skin in the game' when negotiating the terms of the loan contract with a bank.

yields:

$$\max_{m',a'} -[q_m/\beta - q'_m]m' - [q_a/\beta - q'_a]a', \tag{4.10}$$

for both banks and capital suppliers. Thus, if the rate of return of money and housing is non-positive, i.e., if $q_m/\beta \geq q'_m$ and $q_a/\beta \geq q'_a$, neither capital suppliers nor banks have an incentive to hold positive positions, and hence $(m^s, a^s) = (m^b, a^b) = (0, 0)$.

4.4 Bargaining

The terms of the loan contract, (k_E, ϕ, d, y, b), are determined via proportional bargaining, where k_E denotes the total amount of capital financed, ϕ the bank's service fee, $y \in [0, a]$ the amount of housing used as collateral, $d \in [0, m]$ the monetary downpayment, and $b \in [0, \bar{b}]$ credit card debt. Let $S = S^e + S^b$ be the total surplus to be bargained over:

$$S^e \equiv W^e(k_E, \omega) - W^e(k_I, \omega) = f(k_E) - k_E - \phi - \Delta^e_I(m, b), \tag{4.11}$$

$$S^b \equiv W^b(0, \omega + \phi) - W^b(0, \omega) = \phi, \tag{4.12}$$

characterizing the following bargaining problem:

$$(k_E, \phi, d, y, b) \in \arg\max f(k_E) - k_E - \phi - \Delta^e_I(m, b), \tag{4.13}$$

$$\text{s.t. } \theta[f(k_E) - k_E - \phi - \Delta^e_I(m, b)] \geq (1 - \theta)\phi, \tag{4.14}$$

$$\text{s.t. } l \equiv k_E - q_m d - b + \phi \leq \chi f(k_E) + \rho q_a y, \tag{4.15}$$

where $\theta \in [0, 1]$ represents the bank's bargaining power, and (4.14) governs how the total surplus is split proportionally between the bank and the entrepreneur. Equation (4.15) is the entrepreneur's liquidity constraint and determines the size of the future loan obligation, l, where $\rho q_a y$ is the collateral value of housing and $\chi f(k_E)$ the collateral value of future output. Debt limits are imposed exogenously following Kiyotaki and Moore (1997) with χ (ρ) being the fraction of future output (housing) pledgeable to the bank.[11] It follows immediately that the size of the loan obligation, l, decreases with the monetary downpayment, i.e., the amount of capital financed internally using savings and credit card debt, $q_m d$ and b.

Definition 4.1. *An equilibrium of the bargaining game between an entrepreneur and a bank is a pair of strategies,* (k_E, ϕ, d, y, b)*, such that the terms of trade,* (k_E, ϕ, d, y, b)*, are a solution to the bargaining problem* (4.13)-(4.15)*.*

If the entrepreneur's liquidity constraint, (4.15), does not bind, the entrepreneur may achieve the socially-efficient level of investment, $k_E = k^*$. Using (4.14) and solving for ϕ gives:

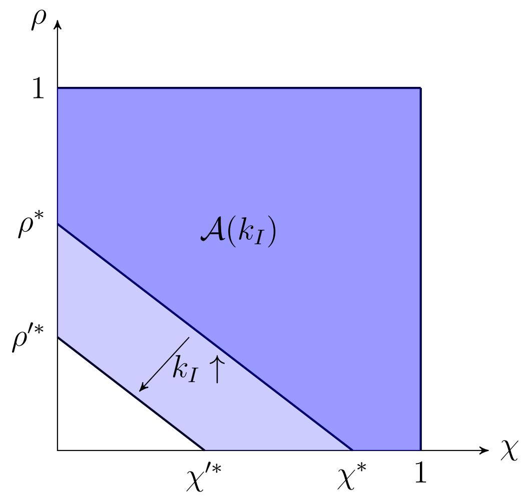
$$\phi = \theta[f(k^*) - k^* - \Delta_I^e(m, b)], \tag{4.16}$$

denoting the bank's fraction of the total match surplus. Comparative statics show that $\partial\phi/\partial\Delta_I^e(m, b) < 0$, so the fee collected by the bank is decreasing in the value of the entrepreneur's outside option. Thus, apart from being an insurance against not receiving a bank loan, fiat money incorporates a strategic role in the bargaining game, reducing the bank's surplus. In the limiting case with $\chi = 0$ and $\rho = 0$ (no bank loan), $k_E = k_I = k^*$ if $q_m m + \bar{b} \geq k^*$. If, however, $q_m m + \bar{b} < k^*$, then $k_I < k_E \leq k^*$, where $k_E \leq k^*$ holds with equality if either output or housing is sufficiently pledgeable. Plugging (4.16) into (4.15), we characterize a set of pairs, $(\hat{\chi}, \hat{\rho})$, such that $k_E = k^*$:

$$\mathcal{A}(k_I) = \Big\{(\hat{\chi}, \hat{\rho}) \in \mathbb{R}_+^2 : \hat{\chi} f(k^*) + \hat{\rho} q_a y \geq \theta[f(k^*) - f(k_I)] + (1-\theta)(k^* - k_I)\Big\}, \tag{4.17}$$

[11] One can interpret ρ as a loan-to-value ratio representing various transaction costs and information asymmetries regarding the resale value of a house, and χ as the parts of the capital input a bank can recover in case the entrepreneur defaults on his credit obligation (scrap value).

Figure 4.2: Set $\mathcal{A}(k_I)$ in an Unconstrained Equilibrium



with the threshold values $\hat{\chi} \leq 1$ and $\hat{\rho} \leq 1$. From (4.17), $\hat{\chi}$ is decreasing in ρ and $\hat{\chi} \to 0$ as $\rho \to \rho^*$, where ρ^* allows entrepreneurs to accumulate k^* when $\chi = 0$ (analogue for χ^*). The same, but vice versa, holds for $\hat{\rho}$. Furthermore, with an increase in k_I, $\mathcal{A}(k_I)$ converges towards the origin, since there are more combinations of χ and ρ that allow for $k_E = k^*$. Figure 4.2 illustrates.

Consider now the case where the entrepreneur's liquidity constraint, (4.15), is binding. Solving (4.14) for ϕ and substituting into (4.15) with $d = m$, $y = a$, and $b = \bar{b}$ gives:

$$(1-\theta)k_E + \theta[f(k_E) - \Delta_m^e(m)] = \chi f(k_E) + \rho q_a a + q_m m + \bar{b}, \tag{4.18}$$

which determines k_E. The corresponding comparative statics show that $\partial k_E/\partial\theta < 0$, $\partial k_E/\partial k_I > 0$, $\partial k_E/\partial q_a a > 0$, $\partial k_E/\partial\rho > 0$, and $\partial k_E/\partial\chi > 0$. Lemma 4.1 summarizes.

Lemma 4.1. *There exists a unique solution to* (4.13) *with* $k_I \in \min\{q_m m + \bar{b}, k^*\}$. *If* $\chi < \chi^*$ *and* $\rho < \rho^*$, *there exists an* m^* *such that* $k^* > q_m m^* + \bar{b}$ *and the following is true: If* $m \geq m^*$, *the solution to* (4.13) *is:*

$$k_E = k^*, \tag{4.19}$$

$$\phi = \theta[f(k^*) - k^* - \Delta_I^e(m,b)]. \tag{4.20}$$

If $m < m^*$*, however, then* (ϕ, k_E) *solves:*

$$\phi = \theta[f(k_E) - k_E - \Delta_I^e(m,b)], \tag{4.21}$$

$$\theta[f(k_E) - k_E - \Delta_I^e(m,b)] = \chi f(k_E) + \rho q_a a - k_E + k_I, \tag{4.22}$$

and $k_E \geq \underline{k}_E$*, where* $\chi f'(\underline{k}_E) = 1$*.*

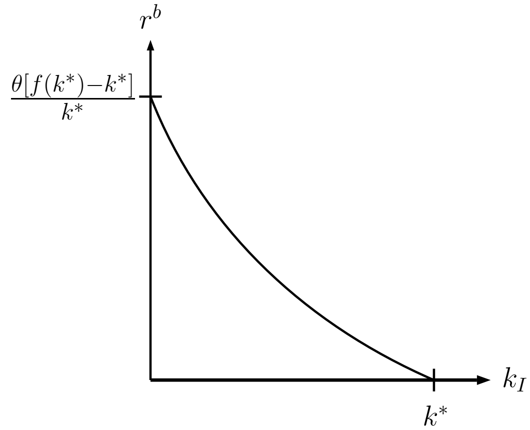
Proof. See Appendix C.1.1. □

It is important to note that in equilibrium $\partial[k_I + \chi f(k_E) + \rho q_a a]/\partial k_I > 1$, and thus by carrying an additional unit of fiat money along the period, entrepreneurs can increase their accumulated capital by more than one unit. The intuition behind this result is straightforward. An additional unit of fiat money does not only buy the entrepreneur more capital from the supplier, but also signalizes a higher investment to the bank, enabling the entrepreneur to credibly pledge more future output. Lemma 4.2 revisits this result and determines its implications for the bank lending rate.

Lemma 4.2. *The bank lending rate,* r^b*, defined as the ratio of the fee,* ϕ*, to the loan size,* $k_E - k_I$*, is given by:*

$$r^b = \frac{\phi}{k_E - k_I} = \begin{cases} \dfrac{\theta[f(k^*) - k^* - \Delta_I^e(m,b)]}{k^* - k_I} & \text{if } m \geq m^*, \\ \dfrac{\theta[f(k_E) - k_E - \Delta_I^e(m,b)]}{k_E - k_I} & \text{if } m < m^*, \end{cases} \tag{4.23}$$

Figure 4.3: Bank Lending Rate



where, given q_m, χ and ρ, m^* is the minimal amount of fiat money the entrepreneur needs to attain k^* through bank credit.

Proof. See Appendix C.1.2. □

From Lemma 4.2 and Figure 4.3, the bank lending rate is decreasing in k_I, as the entrepreneur faces a more valuable outside option, $\Delta_I^e(m, b)$. Hence, the more real money balances an entrepreneur is able to bring into the stage 1, i.e., the more capital is financed internally, the lower the real lending rate, $\partial r^b/\partial k_I < 0$. This pass-through is revisited in more detail in Sections 4.6 and 4.7.

4.5 Portfolio Choice

To determine the entrepreneur's optimal portfolio choice in stage 2, we revisit (4.6):

$$\max_{m',a'} -[q_m/\beta - q_m']m' - [q_a/\beta - q_a']a' + \vartheta(a') + \lambda[(1-\alpha)\Delta_I^e(m', b') + \alpha\Delta_E^e(m', a', b')], \quad (4.24)$$

with

$$\Delta_I^e(m,b) = \begin{cases} f(k^*) - k^* & \text{if} \quad q_m m + \bar{b} \geq k^* \\ f(k_I) - k_I & \text{if} \quad q_m m + \bar{b} < k^*, \end{cases} \tag{4.25}$$

$$\Delta_E^e(m,a,b) = \begin{cases} (1-\theta)\big[f(k^*) - k^*\big] + \theta\Delta_I^e(m,b) & \text{if} \quad m \geq m^* \\ (1-\chi)f(k_E) - \rho q_a a - k_I & \text{if} \quad m < m^*, \end{cases} \tag{4.26}$$

where the terms of trade $[k_I(m,\bar{b}), k_E(m,a,\bar{b}), \phi(m,a,\bar{b}), d(m,a,\bar{b}), y(m,a,\bar{b}), b(m,a,\bar{b})]$ are a function of the entrepreneur's credit card limit and aggregate money and housing. The first term, $-\left[q_m/\beta - q_m'\right] m'$, represents the opportunity costs of carrying fiat money across the period, while $-[q_a/\beta - q_a']a'$ is the cost of holding real estate.

Definition 4.2. *An equilibrium in the stage 2 is a list of portfolios, terms of trade in stage 1, and aggregate balances,* $\{[m(\cdot), a(\cdot)], [k_I(\cdot), k_E(\cdot), \phi(\cdot), d(\cdot), y(\cdot), b(\cdot)], M, A\}$*, such that:*

(i) $[m(\cdot), a(\cdot)]$ *is a solution to* (4.24)*;*

(ii) $k_I = \min\{q_m m + \bar{b}, k^*\}$*;*

(iii) $[k_E(\cdot), \phi(\cdot), d(\cdot), y(\cdot), b(\cdot)]$ *is a solution to* (4.13)*;*

(iv) $M' = (1+\tau)M$ *is the law of motion of the fiat money stock;*

(v) A *is the total supply of housing in the economy; and*

(vi) Market clearing conditions, $\int_0^1 a(j)dj = A$ *and* $\int_0^1 m(j)dj = M$*, hold.*

Lemma 4.3. *There exists a unique solution to* (4.24)*:*

$$q_m = \beta q_m'[1 + \mathcal{L}_m], \tag{4.27}$$

$$q_a = \beta[q_a'(1 + \mathcal{L}_a) + \vartheta'(a)], \tag{4.28}$$

with $(\mathcal{L}_m, \mathcal{L}_a) = (0,0)$ *for* $k_I = k^*$*, and:*

$$\mathcal{L}_m = \begin{cases} \lambda[1-\alpha(1-\theta)][f'(k_I)-1] & \textit{for } k_E \geq k^*, \\ \lambda\alpha\left[\dfrac{(1-\chi)f'(k_E)[1+\theta(f'(k_I)-1)]}{(1-\theta)-(\chi-\theta)f'(k_E)} - 1\right] + \lambda(1-\alpha)\left[f'(k_I)-1\right] & \textit{for } k_E < k^*, \end{cases}$$

$$\mathcal{L}_a = \begin{cases} 0 & \textit{for } k_E \geq k^*, \\ \lambda\alpha\rho\left[\dfrac{(1-\chi)f'(k_E)}{1-\theta-(\chi-\theta)f'(k_E)} - 1\right] & \textit{for } k_E < k^*, \end{cases} \tag{4.29}$$

where $\mathcal{L}_m$ *and* $\mathcal{L}_a$ *correspond to the liquidity premia of money and housing, respectively.*

Proof. See Appendix C.1.3. □

Consider the three cases from the bargaining game in Section 4.4: $k_I \geq k^*$, $k_E \geq k^*$, and $k_E < k^*$. If money is costless to hold, i.e. $\mathcal{L}_m = 0$, an entrepreneur obtains enough fiat money to purchase $k_I = k^*$. As a consequence, housing is priced at its fundamental value with $\mathcal{L}_a = 0$. If $\mathcal{L}_m > 0$, however, money is costly to hold and $k_I < k^*$. As a consequence, there is demand for external finance. Two cases need to be considered: If the entrepreneur's liquidity constraint, (4.15), is non-binding, then $k_E \geq k^*$. Nonetheless, money incorporates a positive liquidity premium, $\mathcal{L}_m > 0$, since an additional unit would increase the entrepreneur's outside option, $\Delta_I^e(m,b)$, and hence decrease the cost of borrowing form a bank. Housing, however, still trades at the fundamental value, i.e., $\mathcal{L}_a = 0$, since the entrepreneur is not constrained in the amount of collateral carried into stage 1. If (4.15) is binding, however, then $k_E < k^*$. As a consequence, $\mathcal{L}_a > 0$ since an additional unit of housing would relax (4.15). The costlier fiat money, the larger the premium.

4.6 Monetary Policy and the Transmission Mechanism

Having determined the bargaining game and the general equilibrium results of the model, this section characterizes optimal monetary policy. We start by characterizing the nominal interest rate, followed by an analysis of the pass-through to understand how changes in monetary policy affect the terms of trade in the banking sector. Last but not least, we then analyze the transmission of monetary policy to aggregate investment and lending.

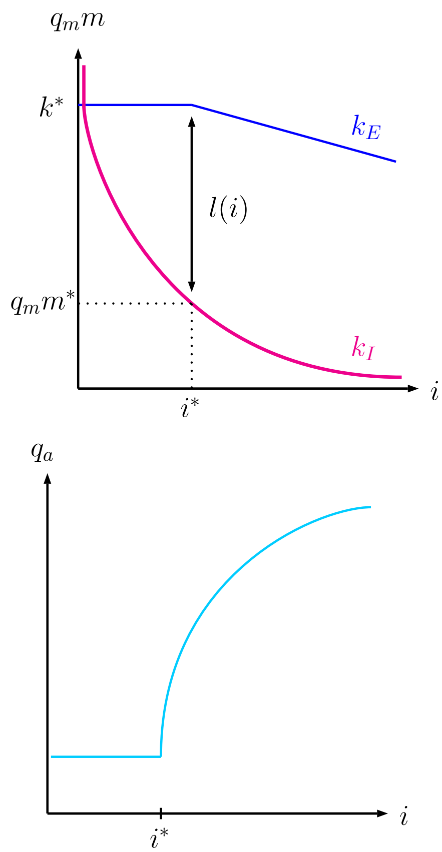
Proposition 4.1. ***(Nominal Interest Rate)*** *Define the nominal interest rate as* $i = \gamma/\beta - 1$ *and* i^**, where* i^* *corresponds to* $m = m^*$*. If* $i = 0$ *(the Friedman rule), then* $k_I = k_E = k^*$*. If* $0 < i \leq i^*$*, then* $k_I < k_E = k^*$*. If* $i > i^*$*, then* $k_E < k^*$*. Comparative statics involve* $\partial i^*/\partial \rho > 0$ *and* $\partial i^*/\partial \chi > 0$*.*

Proof. See Appendix C.1.4. □

Proposition 4.1 characterizes optimal monetary policy using the general equilibrium results in Section 4.5, as visually represented in Figure 4.4. Comparative statics show that an increase in the pledgeability of housing or future output, ρ and χ, increases i^*.

In order to study the pass-through of the nominal interest rate, i, to the bank lending rate, r^b, we rely on first-order Taylor approximations. Distinction is made between an unconstrained and a constrained equilibrium. In an *unconstrained equilibrium*, we use a first-order approximation of the equilibrium for i close to 0, and hence k_I close to k^*. We take this approach for two reasons. First, if $i \approx 0$, then $k_I < k^*$, and thus bank credit is essential. Second, it allows for closed form solutions. To analyze a *constrained equilibrium* and maintain analytical tractability, we set $\theta = 0$ and take a first-order approximation of an equilibrium where $i \approx i^*$ and thus $k_E \approx k^*$. While setting the bank's bargaining power to zero implies $r^b = 0$, we are still able to derive closed form approximations for k_I and k_E. A more general analysis

Figure 4.4: Money and Housing Demand



with $\theta > 0$ is provided in the calibration in Section 4.7. With this in mind, Proposition 4.2 summarizes.

Proposition 4.2. ***(Pass-Through)*** *For $i \approx 0$, the pass-through of the nominal interest rate to the bank lending rate is approximated by:*

$$r^b \approx \frac{\theta i}{2\lambda[1-\alpha(1-\theta)]}. \tag{4.30}$$

For $i \approx i^*$, however, $r^b = 0$ since $\theta = 0$. Comparative statics involve $\partial r^b/\partial\lambda < 0$, $\partial r^b/\partial\theta > 0$, and $\partial r^b/\partial\alpha > 0$.

Proof. See Appendix C.1.5. □

For $i \approx 0$, equation (4.30) identifies a positive pass-through from the nominal interest rate to the bank lending rate, $\partial r^b/\partial i > 0$, since entrepreneurs rely more on external finance. For $i \approx i^*$, however, the pass-through cannot be characterized, since we set $\theta = 0$ for analytical tractability. Further comparative statics show that an increase in λ weakens the pass-through from the nominal interest to the bank lending rate, while an increase in θ or α strengthens the pass-through. Moreover, a change in ρ or χ has no effect on the pass through.

With this understanding, we proceed to analyze the transmission of monetary policy to aggregate investment and lending, $K \equiv \lambda[(1-\alpha)k_I + \alpha k_E]$ and $L \equiv \lambda\alpha(k_E - k_I)$. Let $\bar{k} = k^* - \chi f(k^*) - \frac{\rho a \beta \vartheta'(a)}{1-\beta}$ and $i^* = \lambda(1-\alpha)[f'(\bar{k}) - 1]$, where $\bar{k}$ is defined as the minimum k_I an entrepreneur needs to obtain $k_E = k^*$ after having pledged all of his private real estate and claims on future output, and i^* is the corresponding nominal interest rate. For $i \approx 0$ and thus $k_E \approx k^*$, the transmission of the nominal interest rate to aggregate investment and lending is approximated by:[12]

$$K \approx \lambda k^* + \frac{(1-\alpha)i}{f''(k^*)[1-\alpha(1-\theta)]}, \tag{4.31}$$

$$L \approx \frac{-\alpha i}{f''(k^*)[1-\alpha(1-\theta)]}. \tag{4.32}$$

For $i - i^* \approx 0$ and $\theta = 0$, however,

$$k_E - k^* \approx \frac{k_I - \bar{k}}{1-\mathcal{O}} \approx -\frac{i-i^*}{D}, \tag{4.33}$$

$$L \approx \lambda\alpha\left[k^* - \bar{k} - \frac{\mathcal{O}(i-i^*)}{D}\right], \tag{4.34}$$

where $D > 0$, and:

$$\mathcal{O} = \chi + \left(\frac{\beta\rho}{1-\beta}\right)^2 \left[\frac{\alpha\lambda a\vartheta'(a) f''(k^*)}{1-\chi}\right]. \tag{4.35}$$

Proposition 4.3 summarizes the transmission mechanism.

Proposition 4.3. ***(Transmission Mechanism)*** *For $\theta = 0$, $\chi < \chi^*$, and $\rho < \rho^*$, transmission of monetary policy to aggregate investment and lending is characterized by the following three regions:*

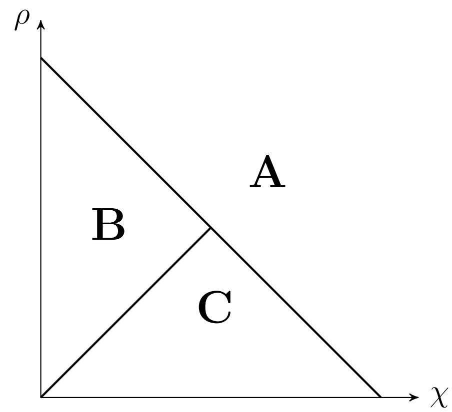
$$\begin{aligned} A &: \; i \leq i^* \;\; \textit{with} \;\; \partial k_I/\partial i < \partial k_E/\partial i = 0, \;\; \textit{and} \;\; \partial L/\partial i > 0, \\ B &: \; i > i^* \;\; \textit{with} \;\; \partial k_I/\partial i < \partial k_E/\partial i < 0, \;\; \textit{and} \;\; \partial L/\partial i > 0, \\ C &: \; i > i^* \;\; \textit{with} \;\; \partial k_E/\partial i < \partial k_I/\partial i < 0, \;\; \textit{and} \;\; \partial L/\partial i < 0, \end{aligned}$$

as represented in Figure 4.5 and equations (4.31)-(4.35)*. Comparative statics show:*

$$\begin{aligned} \partial|\partial k_E/\partial i|/\partial\rho = 0 \quad &\textit{for} \quad i \leq i^*, \\ \partial|\partial k_E/\partial i|/\partial\rho < 0 \quad &\textit{for} \quad i > i^*. \end{aligned}$$

[12] Details to the derivation can be found in Appendix C.2.2.

Figure 4.5: Transmission to Aggregate Investment and Lending



Proof. See Appendix C.1.6. □

Proposition 4.3 analyzes the effect of a change in the nominal interest rate on aggregate investment and lending, characterizing three distinct regions. In region A, $i \leq i^*$, and thus the entrepreneur's liquidity constraint, (4.15), does not bind. As a consequence, k_I decreases with an increase in i, while k_E remains unaffected. From (4.32), aggregate lending is increasing in i, as entrepreneurs rely more on external finance. For $i > i^*$, however, an increase in i decreases both internally and externally financed capital. Given (4.33) and (4.35), the strength of this effect depends on the composition, i.e., the combination of savings, credit card debt, traditional bank loans, and HELs used to purchase k_E. The effect on aggregate lending, i.e., $\partial L/\partial i$, follows this outcome. Consider first region B, in which an entrepreneur relies extensively on HELs. In this scenario, k_I will decrease more than k_E in response to an increase in i, as entrepreneurs successfully hedge against inflation by using their real asset (housing) when bargaining with the bank. The increased demand for housing and the consequential price appreciation increases L and weakens the transmission channel of monetary policy. In contrast, in region C, an increase in the nominal interest rate, i, decreases k_E more than k_I, implying a decrease in aggregate lending, L. Thus, whenever the pledgeability of housing is limited, entrepreneurs are unable to alleviate the inflation tax and as a result, the appreciation in the house prices does not compensate for the reduction in real money balances and the consequentially worse terms of trade when bargaining with a bank. Further, comparative statics show that for region B and C, an increase in ρ weakens the transmission mechanism, while in region A, a change in ρ has no effect on $|\partial k_E/\partial i|$.

4.7 Calibrated Results

Having characterized the transmission channel analytically, we now complement the previous results with a calibrated version of the model. The key difference to the previous Taylor approximations is that we allow banks to have a positive bargaining power ($\theta > 0$), enabling an analysis of the pass-through in a constrained equilibrium.

The model is calibrated to U.S. data covering 2000-2016.[13] We start by setting the discount factor to $\beta = 0.97$. Our measure for the nominal interest rate is the 3-month T-bill secondary market rate with an average of $i = 0.0163$. The probability to have a bank loan approved, α, is 0.80 following the 2003 Survey of Small Business Finances. The pledgeability of future output is calculated as the average ratio of liabilities to assets among small businesses. From the Federal Reserve Flow of Funds Accounts, we calculate $\chi = 0.24$. The pledgeability of housing, ρ, is set equal to the average ratio of home equity loan limits to the average sale price of U.S. homes between 2006 and 2016. Using data from the Federal Reserve Bank of New York's Quarterly Report on Household Debt and Credit and the FRED database, we find $\rho = 0.18$. We then choose $\bar{b}$ to match the average ratio of credit card limits to home equity loan limits and find $\bar{b} = 0.3783$.[14] We estimate the probability of receiving an investment opportunity, λ, by calculating the average percentage of entrepreneurs who started their business within the last year. Using data from the Survey of Consumer Finances

[13] See Appendix **??** for details on data sources and calculations.

[14] Using data from the Federal Reserve Bank of New York's Quarterly Report on Household Debt and Credit we find the average ratio of credit card limits to home equity loan limits, between 2003-2016, to be 0.13.

(SCF) between 2001 and 2016, we estimate $\lambda = 0.0628$. To pin down θ, we follow Rocheteau et al. (2018) in targeting the spread between the prime bank rate and the 3-month T-bill rate of 3.25%, i.e. $r^b - i = 0.0325$.[15] Last but not least, we define the functional forms for the entrepreneur's production function, $f(k) = \nu k^\eta$, and the utility of housing services, $\vartheta(a) = a^{1/2}$, where $\nu = \beta/(1-\beta)$ is a scaling parameter.[16]

Table 4.3: Parameter Values

Parameter	Definition	Value	Target
β	Discount factor	0.97	Annual frequency
i	Nominal interest rate	0.0163	3-month T-bill rate
α	Probability of receiving bank credit	0.80	Loan acceptance rate
λ	Probability of receiving an investment	0.0628	Formation of businesses
θ	Bank's bargaining power	0.162	Spread
χ	Pledgeability of output	0.24	Asset-to-liability ratio
ρ	Pledgeability of housing	0.18	HE limit-to-home price ratio
$\bar{b}$	Maximum amount of unsecured credit	0.3783	Credit card limits
η	Capital share	1/3	Fixed

To study the transmission mechanism of monetary policy, we focus on the pass-through rate, $\partial r_b/\partial i$, and the semi-elasticity of aggregate investment and aggregate lending, i.e., the percentage change in response to a one percentage point increase in the nominal interest rate, $\partial \log(K)/\partial i$, and $\partial \log(L)/\partial i$. By varying the pledgeability of housing, ρ, between 0 and 1, we account for changes in the composition of financing. We focus on these effects for $i \in [0, 0.12]$, whereas the calibration determines $i^* = 0.0308$.

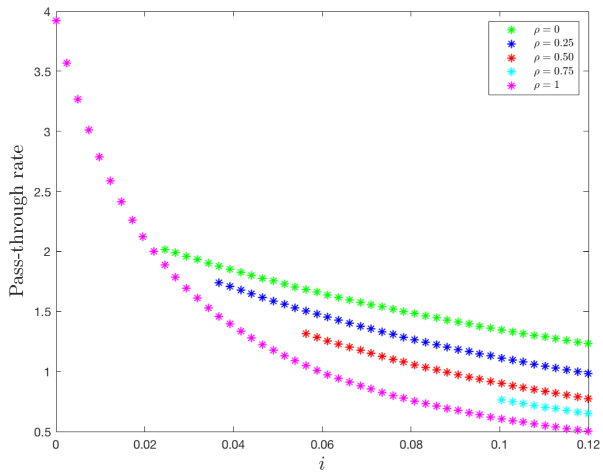
Starting with the pass-through from the nominal interest rate to the bank lending rate, Figure 4.6 displays a positive pass-through rate, $\partial r^b/\partial i$, for all values of ρ, where $|\partial r^b/\partial i|/\partial\rho < 0$. Hence, an increase in the pledgeability of housing dampens the pass-through rate, confirming the theoretical results in Proposition 4.3. For example, at $i \approx 0.05$, a decrease in ρ from 0.2 to 0 increases the pass-through rate from 1.588 to 1.723, an increase of 9%, illustrating the

[15] For $0 < i < i^*$, r^b is given by equation (4.23) for the case of $m > m^*$. We check that under the parameters in Table 4.3, an entrepreneur's liquidity constraint does not bind for $i < 0.0233$.

[16] Recall that the fundamental value of housing is $q_a = {}^{\beta\vartheta'(A)}/{}_{1-\beta}$. Without scaling the production function, say if $f(k) = k^\eta$, entrepreneurs would mechanically be able to obtain k^* through bank loans (as $f'(k^*) = 1$) for $\rho > 0$.

sensitivity of the pass-through to the pledgeability of housing.

Figure 4.6: The Pass-through of Monetary Policy

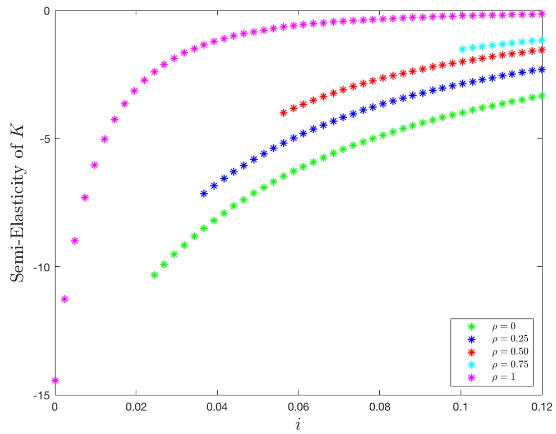


Focusing now on the transmission to aggregate investment and lending, Figure 4.7 presents our results. Panel (a) shows the semi-elasticity of aggregate investment as the pledgeability of housing varies from 0 to 1. From Proposition 4.3 we know that $\partial \log(K)/\partial i < 0$, whereas the magnitude depends on whether the entrepreneur faces a binding liquidity constraint or not. The calibrated results confirm and show that for $i < 0.0220$, the equilibrium is unconstrained and hence the semi-elasticity independent of ρ, since $\partial k_I/\partial i < \partial k_E/\partial i = 0$ for $i \leq i^*$. For $i > i^*$, however, the liquidity constraint binds, $\partial k_E/\partial i < 0$, and thus $\partial \log(K)/\partial i$ depends on ρ. When ρ is lower, the semi-elasticity is larger, and thus the stronger the transmission channel. For example, at $i \approx .05$, the semi-elasticity varies from -7.1 to -5.8 as ρ varies from 0 to 0.2. In the limiting case with $\rho = 1$, $\partial \log(K)/\partial i$ is close to zero and monetary policy has relatively little impact on entrepreneurial investment.

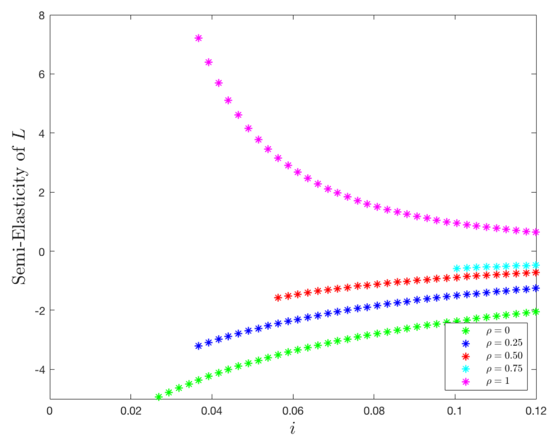
Panel (b) in turn focuses on the semi-elasticity of aggregate lending, $\partial \log(L)/\partial i$. The results show that the semi-elasticity is positive in case of a non-binding liquidity constraint and negative if the liquidity constraint binds, whereas $|\partial L/\partial i|/\partial \rho < 0$.[17] Thus, for most values

[17]The values of $\partial \log L/\partial i$ for low values of i do not appear on the graph because they approach ∞ as $i \to 0$ (as $L = 0$ when $i = 0$ and $k_I = k^*$).

Figure 4.7: The Transmission of Monetary Policy



(a) Aggregate Investment



(b) Aggregate Lending

of the nominal interest rate over the sample period, the response of aggregate lending is strictly negative, even for large values of ρ (region C in Proposition 4.3).

4.8 Conclusion

Motivated by the recent financial crisis and the subsequent crash in home prices, we introduced housing into a monetary corporate finance model to study the relationship between entrepreneurial finance, home equity, and monetary policy. Entrepreneurs have access to internal and external financing, where a decentralized banking sector allows for pledgeability of real estate and determines the pass-through from the nominal interest rate to real bank lending rates. In doing so, the model explicitly characterizes the transmission channel of monetary policy as a function of external and internal finance. A calibration complements the analytical results and provides novel insights on entrepreneurial finance in the United States.

Bibliography

Abbott, B., G. Gallipoli, C. Meghir, and G. L. Violante (2018): "Education Policy and Intergenerational Transfers in Equilibrium," NBER Working Paper No. 18782.

Abel, J. R. and R. Deitz (2014): "Do the Benefits of College Still Outweigh the Costs?" *Current Issues in Economics and Finance*, 20, 1–11.

Abel, J. R., R. Deitz, and Y. Su (2014): "Are Recent College Graduates Finding Good Jobs?" *Current Issues in Economics and Finance*, 20, 1–8.

Acemoglu, D. (1996): "A Microfoundation for Social Increasing Returns in Human Capital Accumulation," *Quarterly Journal of Economics*, 111, 779–804.

——— (2001): "Good Jobs versus Bad Jobs," *Journal of Labor Economics*, 19, 1–21.

Adelino, M., A. Schoar, and F. Severino (2015): "House Prices, Collateral, and Self-Employment," *Journal of Financial Economics*, 117, 288–306.

Aguiar, M., E. Hurst, and L. Karabarbounis (2013): "Time Use during the Great Recession," *American Economic Review*, 103, 1664–1696.

Ahn, H. J. and L. Shao (2017): "Precautionary On-the-Job Search over the Business Cycle," Working paper.

Albrecht, J. and S. Vroman (2002): "A Matching Model with Endogenous Skill Requirements," *International Economic Review*, 43, 283–305.

Atkin, D. (2016): "Endogenous Skill Acquisition and Export Manufacturing in Mexico," *American Economic Review*, 106, 2046–2085.

Barlevy, G. (2002): "The Sullying Effect of Recessions," *Review of Economic Studies*, 69, 65–96.

——— (2005): "The Cost of Business Cycles and the Benefits of Stabilization," *Federal Reserve Bank of Chicago Economic Perspectives*, 29, 32–49.

Barnichon, R. and Y. Zylberberg (2019): "Under-Employment and the Trickle-Down of Unemployment," *American Economic Journal: Macroeconomics*, 11, 40–78.

Barr, A. and S. Turner (2015): "Out of work and into school: Labor market policies and college enrollment during the Great Recession," *Journal of Public Economics*, 124, 63–73.

Barr, A. and S. E. Turner (2013): "Expanding Enrollments and Contracting State Budgets: The Effect of the Great Recession on Higher Education," *The ANNALS of the American Academy of Political and Social Science*, 650, 168–193.

Bates, T. W., K. M. Kahle, and R. M. Stulz (2009): "Why Do U.S. Firms Hold So Much More Cash than They Used To?" *The Journal of Finance*, 64, 1985–2021.

Berkner, L. (2000): "Trends in Undergraduate Borrowing: Federal Student Loans in 1989-09, 1992-93, and 1995-96," Tech. Rep. NCES 2000-151, U.S. Department of Education, National Center for Education Statistics, Washington D.C.

Betts, J. and L. McFarland (1995): "Safe Port in a Storm: The Impact of Labor Market Conditions on Community College Enrollments," *Journal of Human Resources*, 30, 741–765.

BGT and SI (2018): "The Permanent Detour: Underemployment's Long-Term Effects on the Careers of College Grads," Tech. rep., Burning Glass Technologies and Strada Institute for the Future of Work, technical report.

Binmore, K., A. Rubinstein, and A. Wolinsky (1986): "The Nash Bargaining Solution in Economic Modelling," *RAND Jouranl of Economics*, 17, 176–188.

Black, D. A., T. G. McKinnish, and S. G. Sanders (2005): "Tight Labor Markets and the Demand for Education: Evidence from the Coal Boom and Bust," *ILR Review*, 59, 3–16.

Black, J., D. de Meza, and D. Jeffreys (1996): "House Prices, The Supply of Collateral and the Enterprise Economy," *Economic Journal*, 106, 60–75.

Blanchard, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223–247.

Bonilla, R. and K. Burdett (2010): "On-the-Job Search and Labor Market Equilibrium," *The B.E. Journal of Macroeconomics*, 10, 1–28.

Borjas, G. J. (2003): "The Labor Demand Curve is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market," *Quarterly Journal of Economics*, 118, 1335–1374.

Bozick, R. (2009): "Job Opportunities, Economic Resources and the Postsecondary Destinations of American Youth," *Demography*, 46, 493–512.

Branch, W. A., N. Petrosky-Nadeau, and G. Rocheteau (2016): "Financial Frictions, the Housing Market, and Unemployment," *Journal of Economic Theory*, 164, 101–135.

Braun, C. (2019): "Crime and the minimum wage," *Review of Economic Dynamics*, 32, 122–152.

Brunnermeier, M. K. and C. Julliard (2008): "Money Illusion and Housing Frenzies," *The Review of Financial Studies*, 21, 135–180.

Burdett, K. and M. Coles (2003a): "Equilibrium Wage-Tenure Contracts," *Econometrica*, 5.

——— (2003b): "Equilibrium Wage-Tenure Contracts," *Econometrica*, 5.

Burdett, K. and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39, 257–273.

Cahuc, P., F. Postel-Vinay, and J.-M. Robin (2006): "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 74, 323–364.

Campello, M. (2015): "Corporate Liquidity Management," *NBER Reporter*, 1–3.

Card, D. and T. Lemieux (2001): *Dropout and Enrollment Trends in the Postwar Period: What Went Wrong in the 1970s?*, Chicago, IL: University of Chicago Press, chap. 9, 439–482, Risky Behavior Among Youths: An Economic Analysis.

Cascio, E. U. and A. Narayan (2015): "Who Needs a Fracking Education? The Educational Response to Low-Skill Biased Technological Change," NBER Working Paper 21359.

Charles, K. K., E. Hurst, and M. J. Notowidigdo (2018): "Housing Booms and Busts, Labor Market Opportunities, and College Attendance," *American Economic Review*, 108, 2947–2994.

Charlot, O. and B. Decreuse (2005): "Self-selection in education with matching frictions," *Labour Economics*, 12, 251–267.

Clark, D. (2011): "Do recessions keep students in school? The impact of youth unemployment on enrolment in post-compulsory education in England," *Economica*, 78, 523–545.

College Board (2017): "Trends in Student Aid," Tech. rep., College Board Publications, New York, NY.

Corradin, S. and A. Popov (2015): "House Prices, Home Equity Borrowing, and Entrepreneurship," *Review of Financial Studies*, 28, 2399–2428.

Decker, R. A. (2015): "Collateral Damage: Housing, Entrepreneurship, and Job Creation," *Working Paper*, 1–57.

Dellas, H. and P. Sakellaris (2003): "On the cyclicality of schooling: theory and evidence," *Oxford Economic Papers*, 55, 148–172.

Dolado, J. J., M. Jansen, and J. F. Jimeno (2009): “On-the-Job Search in a Matching Model with Heterogenous Jobs and Workers,” *Economic Journal*, 119, 200–228.

Elsby, M. W., R. Michaels, and D. Ratner (2015): “The Beveridge Curve: A Survey,” *Journal of Economic Literature*, 53, 571–630.

Engelhardt, B., G. Rocheteau, and P. Rupert (2008): “Crime and the labor market: A Search Model with Optimal Contracts,” *Journal of Public Economics*, 92, 1876–1891.

Faberman, J. R., A. I. Mueller, A. Şahin, and G. Topa (2017): “Job Search Behavior Among the Employed and Non-Employed,” NBER Working Paper No. 23731.

Fallick, B. and C. A. Fleischman (2004): “Employer-to-Employer Flows in the U.S. Labor Market: The complete Picture of Gross Worker Flows,” Board of Governors of the Federal Reserve Finance and Economics Discussion Paper 2004-34.

FDOE (2017): “Funding For Florida School Districts,” Tech. rep., Florida Department of Education.

——— (2018): “Statewide $2 + 2$ Articulation Agreement,” Tech. rep., Florida Department of Education.

Flinn, C. and J. Mullins (2015): “Labor Market Search and Schooling Investment,” *International Economic Review*, 56, 359–398.

FRBNY (2018): “Quarterly Report on Household Debt and Credit,” Tech. rep., Federal Reserve Bank of New York Research and Statistics Group.

Fujita, S. (2010): “Reality of On-the-Job Search,” Federal Reserve Bank of Philadelphia Working Paper No. 10-34.

Garriga, C., R. Manuelli, and A. Peralta-Alva (2018): “A Model of Price Swings in the Housing Market,” *American Economic Review*, forthcoming.

Gautier, P. A. (2002): “Unemployment and Search Externalities in a Model with Heterogenous Jobs and Workers,” *Economica*, 69, 21–40.

Greenspan, A. and J. Kennedy (2007): “Sources and Uses of Equity Extracted from Homes,” *Federal Reserve Board Finance and Economics Discussion Series*, 1–49.

Gustman, A. and T. Steinmeier (1981): “The impacts of wages and unemployment on youth enrollment and labor supply,” *The Review of Economics and Statistics*, 63, 553–560.

Hagedorn, M. and I. Manovskii (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 1692–1706.

Hall, R. E. and A. B. Krueger (2012): “Evidence on the Incidence of Wage Posting, Wage Bargaining, and On-the-Job Search,” *American Economic Journal: Macroeconomics*, 4, 56–67.

Hall, R. E. and P. R. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 98, 1653–1674.

Hall, R. E. and S. Schulhofer-Wohl (2018): "Measuring Job-Finding Rates and Matching Efficiency with Heterogeneous Job-Seekers," *American Economic Journal: Macroeconomics*, 10, 1–32.

Harding, J. P. and S. S. Rosenthal (2017): "Homeownership, housing capital gains and self-employment," *Journal of Urban Economics*, 99, 120–135.

He, C., R. Wright, and Y. Zhu (2015): "Housing and liquidity," *Review of Economic Dynamics*, 18, 435–455.

Heckman, J. J., L. Lochner, and C. Taber (1998): "General-Equilibrium Treatment Effects: A Study of Tuition Policy," *American Economic Review: Papers & Proceedings*, 88, 381–386.

Hillman, N. W. and E. L. Orians (2013): "How does enrollment demand change relative to local unemployment rates?" *Research in Higher Education*, 54, 765–780.

Hosios, A. J. (1990a): "On the efficiency of matching and related models of search and unemployment," *Review of Economic Studies*, 57, 279–298.

——— (1990b): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279–298.

Iacoviello, M. (2011): "Housing Wealth and Consumption," *Board of Governors International Finance Discussion Papers*, 1–15.

Ionescu, F. (2009): "The Federal Student Loan Program: Quantitative implications for college enrollment and default rates," *Review of Economic Dynamics*, 12, 205–231.

Ionescu, F. and N. Simpson (2016): "Default risk and private student loans: Implications for higher education policies," *Journal of Economic Dynamics & Control*, 64.

Jackson, C. K., R. C. Johnson, and C. Persico (2016): "The effects of school spending on educational and economic outcomes: Evidence from school finance reforms," *Quarterly Journal of Economics*, 131, 157–218.

Jensen, T. L., S. Leth-Petersen, and R. Nanda (2014): "Housing collateral, credit constraints and entrepreneurship - evidence from a mortgage reform," *NBER Working Paper 20583*, 1–46.

Ji, Y. (2018): "Job Search under Debt: Aggregate Implications of Student Loans," Working paper.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2015): "Credit Supply and the Housing Boom," *Working Paper.*

Kahn, L. B. (2010): "The long-term labor market consequences of graduating from college in a bad economy," *Labour Economics*, 17, 303–316.

Katz, L. F. and K. M. Murphy (1992): "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *Quarterly Journal of Economics*, 107, 35–78.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Krueger, A. B. and A. Mueller (2010): "Job Search and Unemployment Insurance: New Evidence from Time Use Data," *Journal of Public Economics*, 94, 298–307.

Krueger, D. and A. Ludwig (2016): "On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium," *Journal of Monetary Economics*, 77, 72–98.

Lagos, R., G. Rocheteau, and R. Wright (2017): "Liquidity: A New Monetarist Perspective," *Journal of Economic Literature*, 55, 371–440.

Lagos, R. and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463–484.

Lee, D. (2005): "An Estimable Dynamic General Equilibrium Model of Work, Schooling, and Occupational Choice," *International Economic Review*, 46, 1–34.

Lim, T. (2018): "Housing as collateral, financial constraints, and small businesses," *Review of Economic Dynamics*, 30, 68–85.

Liu, Z., P. Wang, and T. Zha (2013): "Land-price dynamics and macroeconomic fluctuations," *Econometrica*, 81, 1147–1184.

Lochner, L. and A. Monge-Naranjo (2012): "Credit Constraints in Education," *Annual Review of Economics*, 4, 225–256.

——— (2016): "Student Loans and Repayment: Theory, Evidence, and Policy," in *Handbook of the Economics of Education*, ed. by E. A. Hanushek, S. Machin, and L. Woessmann, Elsevier, vol. 5, chap. 8, 397 – 478.

Long, B. T. (2015): *The Financial Crisis and college enrollment: how have students and their families responded?*, Chicago, IL: University of Chicago Press, chap. 7, 209–233, How the Financial Crisis and the Great Recession affected higher education.

Lovenheim, M. F. (2011): "The effect of liquid housing wealth on college enrollment," *Journal of Labor Economics*, 29, 741–7.

Lovenheim, M. F. and C. L. Reynolds (2013): "The Effect of Housing Wealth on College Choice," *Journal of Human Resources*, 14, 3–37.

Macera, M. and H. Tsujiyama (2017): "Frictional Labor Markets, Education Choices, and Wage Inequality," Working paper.

Mangin, S. and B. Julien (2018): "Efficiency in Search and Matching Models: A Generalized Hosios Condition," Working paper.

Moen, E. R. (1998): "Efficient Ways to Finance Human Capital Investments," *Economica*, 65, 491–505.

——— (1999): "Education, Ranking, and Competition for Jobs," *Journal of Labor Economics*, 17, 694–723.

Mora, N. (2014): "The Weakened Transmission of Monetary Policy to Consumer Loan Rates," *Federal Reserve Bank of Kansas City Economic Review*, 93–117.

Mortensen, D. T. (1978): "Specific Capital and Labor Turnover," *The Bell Journal of Economics*, 9, 572–586.

Mortensen, D. T. and C. A. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 120, 397–415.

Mukoyama, T., C. Patterson, and A. Şahin (2018): "Job Search Behavior over the Business Cycle," *American Economic Journal: Macroeconomics*, 10, 190–215.

Nagypál, E. (2005): "On the extent of job-to-job transitions," Mimeo.

——— (2007): "Labor-Market Fluctuations and On-The-Job Search," Mimeo.

——— (2008): "Worker Reallocation Over the Business Cycle: The Importance of Employer-to-Employer Transitions," Mimeo.

Nutting, A. W. (2008): "Costs of attendance and the educational programs of first-time community college students," *Economics of Education Review*, 27, 450–459.

OPPAGA (2004): "The State Has Several Options Available When Considering the Funding of Higher Education," Tech. Rep. 04-54, Office of Program Policy Analysis and Government Accountability.

Oreopoulos, P., T. von Wachter, and A. Heisz (2012): "The Short- and Long-Term Career Effects of Graduating in a Recession," *American Economic Journal: Applied Economics*, 4, 1–29.

OSFA (2018a): "Bright Futures Student Handbook Chapter 1: Initiation Eligibility Requirements," Tech. rep., Office of Student Financial Assistance Florida Department of Education.

——— (2018b): "Bright Futures Student Handbook Chapter 2: What You Need to Know Now that You are Eligible," Tech. rep., Office of Student Financial Assistance Florida Department of Education.

Parsons, D. O. (1973): "Quit Rates Over Time: A Search and Information Approach," *American Economic Review*, 63, 390–401.

——— (1991): "The Job Search Behavior of Employed Youth," *The Review of Economics and Statistics*, 73, 597–604.

Petrongolo, B. and C. A. Pissarides (2001a): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39, 390–431.

——— (2001b): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39, 390–431.

Petrongolo, B. and M. J. S. Segundo (2002): "Staying-on at school at 16: the impact of labor market conditions in Spain," *Economics of Education Review*, 21, 353–365.

Pissarides, C. A. (1985): "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review*, 75, 676–690.

——— (1994a): "Search Unemployment with On-the-job Search," *Review of Economic Studies*, 61, 457–475.

——— (1994b): "Search Unemployment with On-the-job Search," *Review of Economic Studies*, 61, 457–475.

——— (2000): *Equilibrium Unemployment Theory*, MIT Press, second ed.

Pissarides, C. A. and J. Wadsworth (1994): "On-the-job search: Some empirical evidence from Britain," *European Economic Review*, 38, 385–401.

Postel-Vinay, F. and J.-M. Robin (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–2350.

——— (2004): "To Match or Not to Match? Optimal Wage Policy with Endogenous Worker Search Intensity," *Review of Economic Dynamics*, 7, 297–330.

Rocheteau, G. and E. Nosal (2017): *Money, Payments, and Liquidity*, MIT Press, 2nd ed.

Rocheteau, G., R. Wright, and C. Zhang (2018): "Corporate Finance and Monetary Policy," *American Economic Review*, 108, 1147–86.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, 97–109.

Sánchez, J. M. and E. Yurdagul (2013): "Why Are U.S. Firms Holding So Much Cash? An Exploration of Cross-Sectional Variation," *Federal Reserve Bank of St. Louis Review*, 95, 293–325.

Schmalz, M., D. A. Sraer, and D. Thesmar (2017): "Housing Collateral and Entrepreneurship," *Journal of Finance*, 72, 99–132.

Shephard, A. and M. Sidibé (2019): "Schooling Investment, Mismatch, and Wage Inequality," Working paper.

SHI, S. (2001): "Frictional Assigment. I. Efficiency," *Journal of Economic Theory*, 98, 232–260.

——— (2002): "A Directed Search Model of Inequality with Heterogenous Skills and Skill-Biased Technology," *Review of Economic Studies*, 69, 467–491.

SHIMER, R. (2004): "Search Intensity," Mimeo.

——— (2005a): "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 113, 996–1025.

——— (2005b): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25–49.

——— (2005c): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25–49.

——— (2006a): "On-the-job Search and Strategic Bargaining," *European Economic Review*, 50, 811–830.

——— (2006b): "On-the-job Search and Strategic Bargaining," *European Economic Review*, 50, 811–830.

——— (2012): "Reassessing the ins and outs of unemployment," *Review of Economic Dynamics*, 15, 127–148.

SHIMER, R. AND L. SMITH (2001): "Nonstationary Search," Mimeo.

SILVA, M. (2017): "Corporate Finance, Monetary Policy, and Aggregate Demand," *Working Paper*, 1–50.

STEVENS, M. (2004a): "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention," *Review of Economic Studies*, 71, 535–551.

——— (2004b): "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention," *Review of Economic Studies*, 71, 535–551.

TOPEL, R. H. AND M. P. WARD (1992): "Job Mobility and the Careers of Young Men," *Quarterly Journal of Economics*, 107, 439–479.

WASMER, E. AND P. WEIL (2004): "The Macroeconomics of Labor and Credit Market Imperfections," *American Economic Review*, 94, 944–963.

WEI, C. C. AND P. SKOMSVOLD (2011): "Borrowing at the Maximum Undergraduate Stafford Loan Borrowers in 2007-08," Tech. Rep. NCES 2012-161, U.S. Department of Education National Center for Education Statistics.

Appendix A

Supplementary Material for Search Intensity of Employed Workers

A.1 Proofs and Derivations

A.1.1 Proof of Lemma 3.1

Proof. The contract $(w_i, \phi_u(p_i))$ must be pairwise Pareto efficient. After the hiring fee, $\phi_u(p_i)$, is transferred from the worker to the firm the wage, w_i, is chosen to maximize the surplus of the match, $\mathcal{S}_u(p_i)$. Comparing (1.7) and (3.11) show that the surplus is maximized if and only if $\mathcal{V} = \mathcal{J}_i$. From (1.3)-(1.4), this is true if and only if $w_i = p_l z$. The hiring fee in (3.16) is derived by taking the first order condition of (3.14) with respect to $\phi_u(p_i)$. □

A.1.2 Proof of Lemma 1.2

Proof. The solution to (1.17) follows from the proof of Lemma 3.1. □

A.1.3 Derivation of $r\mathcal{U}$

From equations (1.7)-(1.8),

$$\mathcal{W}_l - \mathcal{U} = \frac{p_l z + \beta\rho_e f(\theta)\alpha_h \left[\frac{z(p_h - p_l)}{r+\delta+\rho_e f(\theta)\beta\alpha_h}\right] - r\mathcal{U}}{r+\delta}, \mathcal{W}_h - \mathcal{U} = \frac{p_h z - r\mathcal{U}}{r+\delta}.$$

Combining with (1.6) and solving for $\frac{r\mathcal{U}-b}{\beta f(\theta)}$ and gives

$$\frac{r\mathcal{U}-b}{\beta f(\theta)} = \frac{[\alpha_l\{p_l z + \rho_e f(\theta)\beta\alpha_h \left[\frac{z(p_h - p_l)}{r+\delta+\rho_e f(\theta)\beta\alpha_h}\right]\} + \alpha_h p_h z - b]}{r+\delta+\beta f(\theta)}.$$

A.1.4 Derivation of the free entry condition

From (1.7)-(1.8), $\mathcal{W}_h - \mathcal{W}_l = \frac{z(p_h - p_l)}{r+\delta+\rho_e f(\theta)\beta\alpha_h}$ and rewriting $q_e(\theta) = \frac{\rho_e e}{\rho_u u + \rho_e e} q(\theta)$ and $q_u(\theta) = \frac{\rho_u u}{\rho_u u + \rho_e e} q(\theta)$,

$$\frac{\gamma}{1-\beta} = \frac{\rho_u u}{\rho_u u + \rho_e e} q(\theta) \sum_{i:p_i \geq p^*} \alpha_i [\mathcal{W}_i - \mathcal{U}] + \frac{\rho_e e}{\rho_u u + \rho_e e} q(\theta)\alpha_h \left[\frac{z(p_h - p_l)}{r+\delta+\rho_e f(\theta)\beta\alpha_h}\right]. \quad \text{(A.1)}$$

Substituting equations (1.25)-(1.26) and dividing by $q(\theta)$ gives

$$\frac{\gamma}{(1-\beta)q(\theta)} = \frac{\rho_e f(\theta)\alpha_h + \delta}{\rho_e f(\theta) + \delta}\left[\frac{r\mathcal{U}-b}{\beta f(\theta)}\right] + \frac{\rho_e f(\theta)\alpha_l}{\rho_e f(\theta)+\delta}\left[\frac{z(p_h - p_l)}{r+\delta+\rho_e f(\theta)\beta\alpha_h}\right], \quad \text{(A.2)}$$

where, from (1.6), $\frac{r\mathcal{U}-b}{\beta\rho_u f(\theta)} = \sum_{i:p_i \geq p^*} \alpha_i [\mathcal{W}_i - \mathcal{U}]$. Substituting for $\frac{r\mathcal{U}-b}{\beta\rho_u f(\theta)}$ and simplifying gives equation (1.28).

A.1.5 Proof of Proposition 1.1

Proof. Define $\Gamma(\theta)$ as

$$\Gamma(\theta) \equiv \frac{\gamma}{(1-\beta)q(\theta)} - \frac{\rho_e f(\theta)\alpha_h + \delta}{\rho_e f(\theta) + \delta}\left[\frac{z(\alpha_l p_l + \alpha_h p_h) - b}{r + \delta + \beta\rho_u f(\theta)}\right] - \frac{z(p_h - p_l) f(\theta)\rho_e \alpha_l \alpha_h}{(\rho_e f(\theta) + \delta)(r + \delta + \beta\rho_e f(\theta)\alpha_h)}\left[\frac{(r + \delta + \beta\rho_u f(\theta)) + \beta(\rho_e f(\theta)\alpha_h + \delta)}{r + \delta + \beta\rho_u f(\theta)}\right].$$

This is the expected costs of posting a vacancy net the expected surplus of posting a vacancy. First note that $\Gamma(\theta)$ is a continuous function as $q(\theta)$ and $f(\theta)$ are themselves continuous functions. To show existence of an equilibrium such that $\theta > 0$, it must be shown that there exists at least one θ such that $\Gamma(\theta) = 0$, as $\Gamma(\theta) = 0$ implies that (1.28) is satisfied.

Step 1. Show that $\lim_{\theta\to 0}\Gamma(\theta) < 0$. As $\lim_{\theta\to 0} q(\theta) = \infty$ and $\lim_{\theta\to 0} f(\theta) = 0$, it follows that

$$\lim_{\theta\to 0}\Gamma(\theta) = 0 - \frac{z(\alpha_l p_l + \alpha_h p_h) - b}{r + \delta} < 0.$$

Step 2. Show that $\lim_{\theta\to\infty}\Gamma(\theta) = \infty$. Note that $\lim_{\theta\to\infty} q(\theta) = 0$ and $\lim_{\theta\to\infty} f(\theta) = \infty$. It follows that

$$\lim_{\theta\to\infty}\frac{\gamma}{(1-\beta)q(\theta)} = \infty.$$

Consider

$$\lim_{\theta\to\infty} \frac{\rho_e f(\theta)\alpha_l}{\rho_e f(\theta)+\delta}\alpha_h \left[\frac{z(p_h-p_l)}{r+\delta+\rho_e f(\theta)\beta\alpha_h}\right]$$

Rewriting, and evaluating by $\lim_{\theta\to\infty} \frac{\rho_e f(\theta)\alpha_l}{\rho_e f(\theta)+\delta}\alpha_h$ by L'Hôpital's rule,

$$\lim_{\theta\to\infty} \frac{\frac{\rho_e f(\theta)\alpha_l\alpha_h}{\rho_e f(\theta)+\delta}}{\frac{r+\delta+\rho_e f(\theta)\beta\alpha_h}{z(p_h-p_l)}} = 0.$$

Similarly for the remaining term, rewriting and evaluating by L'Hôpital's rule,

$$\lim_{\theta\to\infty} \frac{\frac{\rho_e f(\theta)\alpha_h+\delta}{\rho_e f(\theta)+\delta}}{\frac{(r+\delta+\rho_e f(\theta)\beta\alpha_h)(r+\delta+\beta\rho_u f(\theta))}{(r+\delta+\rho_e f(\theta)\alpha_l\beta\alpha_h)[z(\alpha_l p_l+\alpha_h p_h)-b]+\rho_e f(\theta)\beta\alpha_h z(p_h-p_l)}} = 0.$$

Putting all terms together gives $\lim_{\theta\to\infty}\Gamma(\theta)=\infty$. Thus, as $\lim_{\theta\to 0}\Gamma(\theta)<0$ and $\lim_{\theta\to\infty}\Gamma(\theta)=\infty$, there exists at least one value of θ such that $\Gamma(\theta)=0$ and (1.28) holds.

For comparative statics, if a change in the parameter causes $\Gamma(\theta)$ to increase (decrease), then the sign of the comparative static is positive (negative). This follows from the properties of $\Gamma(\theta)$: $\lim_{\theta\to 0}\Gamma(\theta)<0$ and $\lim_{\theta\to\infty}\Gamma(\theta)=\infty$. Comparative statics on u and e_l follow from the fact that u is decreasing in θ and e_l is increasing in θ.

□

A.1.6 Proof of Lemma 1.3

Proof. Recall $\Gamma(\theta)$ as defined in the proof of Lemma 1.1. Suppose that $\alpha_l=0$. Clearly, in this case, $\Gamma(\theta)$ is not a function of ρ_e and $\frac{\partial\theta}{\partial\rho_e}=0$. Now suppose that $\alpha_l=1$,

$$\Gamma(\theta)_{|\alpha_l=1} = \frac{\gamma}{(1-\beta)q(\theta)} - \frac{\delta}{\rho_e f(\theta)+\delta}\left[\frac{zp_l - b}{r+\delta+\beta\rho_u f(\theta)}\right].$$

Clearly $\frac{\partial\Gamma(\theta)}{\partial\rho_e}|_{\alpha_l=1} > 0$. It follows that $\frac{\partial\theta}{\partial\rho_e}|_{\alpha_l=1} < 0$. Now suppose that $\alpha_l \in (0,1)$ and ρ_e is close to zero. It follows that $\frac{\partial\Gamma(\theta)}{\partial\rho_e}$ is negative if and only if

$$\left[\frac{z(\alpha_l p_l + \alpha_h p_h) - b}{r+\delta+\beta\rho_u f(\theta)}\right] > \left[\frac{-\delta}{f(\theta)\alpha_l}\right]\left[\frac{(r+\delta+\beta\rho_u f(\theta)) + \beta\delta}{r+\delta+\beta\rho_u f(\theta)}\right]\bigg\{ \frac{z(p_h-p_l)\alpha_l\alpha_h}{\delta(r+\delta)} - \frac{z(p_h-p_l)f(\theta)\alpha_l\alpha_h[f(\theta)(r+\delta)+\delta\beta f(\theta)\alpha_h]}{[\delta(r+\delta)]^2}\bigg\},$$

which is satisfied for some $\alpha_l^* \in (0,1)$ and for a sufficiently small δ and if $\theta < \infty$. This shows that if ρ_e is sufficiently close to zero, then market tightness is increasing in ρ_e at α_l^*.

The next step is to show $\frac{\partial^2\Gamma(\theta)}{\partial\rho_e^2} > 0$, as this would imply the hump shaped curve in Figure 3. The second derivative is given by

$$\frac{\partial^2\Gamma(\theta)}{\partial\rho_e^2} = -\left[\frac{2[\rho_e f(\theta)\alpha_h + f(\theta)]f(\theta)^2}{[\rho_e f(\theta)+\delta]^3} - \frac{2f(\theta)^2\alpha_h}{[\rho_e f(\theta)+\delta]^2}\right]\left[\frac{z(\alpha_l p_l+\alpha_h p_h)-b}{r+\delta+\beta\rho_u f(\theta)}\right] - \left[\frac{(r+\delta+\beta\rho_u f(\theta)) + \beta(\rho_e f(\theta)\alpha_h+\delta)}{r+\delta+\beta\rho_u f(\theta)}\right]\bigg\{$$
$$\frac{-z(p_h-p_l)f(\theta)^2\alpha_l\alpha_h[f(\theta)(r+\delta+\beta\rho_e f(\theta)\alpha_h + (\rho_e f(\theta)+\delta)\beta f(\theta)\alpha_h]}{[(\rho_e f(\theta)+\delta)(r+\delta+\beta\rho_e f(\theta)\alpha_h)]^2} -$$
$$\frac{2z(p_h - p_l f(\theta)^2\rho_e\alpha_l\alpha_h[f(\theta)(r+\delta+\beta\rho_e f(\theta)\alpha_h + (\rho_e f(\theta)+\delta)\beta f(\theta)\alpha_h]^2}{[(\rho_e f(\theta)+\delta)(r+\delta+\beta\rho_e f(\theta)\alpha_h)]^4} -$$
$$\frac{z(p_h-p_l)f(\theta)^2\alpha_l\alpha_h\{[f(\theta)(r+\delta+\beta\rho_e f(\theta)\alpha_h + (\rho_e f(\theta)+\delta)\beta f(\theta)\alpha_h] + 2\beta\alpha_h f(\theta)^2\}}{[(\rho_e f(\theta)+\delta)(r+\delta+\beta\rho_e f(\theta)\alpha_h)]^2}\bigg\} -$$
$$2\left\{\left[\frac{\beta f(\theta)\alpha_h}{r+\delta+\beta\rho_u f(\theta)}\right]\left[\frac{-z(p_h-p_l)f(\theta)^2\rho_e\alpha_l\alpha_h[f(\theta)(r+\delta+\beta\rho_e f(\theta)\alpha_h) + (\rho_e f(\theta)+\delta)\beta f(\theta)\alpha_h]}{[(\rho_e f(\theta)+\delta)(r+\delta+\beta\rho_e f(\theta)\alpha_h)]^2} + \frac{z(p_h-p_l)f(\theta)^2\alpha_l\alpha_h}{(\rho_e f(\theta)+\delta)(r+\delta+\beta\rho_e f(\theta)\alpha_h)}\right]\right\}.$$

There is one term in the second derivative, namely the last term, which decreases the derivative. Without this term the second derivative is unambiguously positive, which would imply a negative second derivative between θ and ρ_e. It can be shown that the last term in the derivative is negative if and only if

$$(\rho_e f(\theta))^2\beta\alpha_h > \delta(r+\delta).$$

Which is clearly satisfied for a sufficiently small δ and for some $\rho_e > 0$. This means that at some level of ρ_e, $\frac{\partial \theta}{\partial \rho_e} < 0$. Combining this with the fact that the first derivative is positive at $\rho_e > 0$, there exists a hump shape relationship between θ and ρ_e for some $\alpha_l^* \in (0, 1)$. $\square$

A.1.7 Proof of Lemma 1.4

Proof. The worker and firm are indifferent between forming the low productivity match when $\mathcal{W}_l = \mathcal{U}$. Assume that $\mathcal{W}_l = \mathcal{U}$. From (1.6),

$$r\mathcal{U} = \frac{b(r+\delta) + \rho_u f(\theta)\alpha_h \beta p_h z}{r + \delta + \rho_u f(\theta)\alpha_h \beta},$$

where $\mathcal{W}_h - \mathcal{U} = \frac{p_h z - r\mathcal{U}}{r+\delta}$ has been substituted into equation (1.6). From (1.7), $r\mathcal{W}_l = p_l z - \delta[\mathcal{W}_l - \mathcal{U}] + \rho_e f(\theta)\alpha_h \beta[\mathcal{W}(p_{h)} - \mathcal{W}_l]$. Substituting $\mathcal{W}_l = \mathcal{U}$ and solving for $r\mathcal{U}$ gives,

$$r\mathcal{U} = \frac{p_l z(r+\delta) + \rho_e f(\theta)\alpha_h \beta p_h z}{r + \delta + \rho_e f(\theta)\alpha_h \beta}.$$

The second expression for $r\mathcal{U}$ shows the discounted utility of working under the assumption that $\mathcal{W}_l = \mathcal{U}$. It follows that $\mathcal{W}_l \geq \mathcal{U}$ if

$$\frac{p_l z(r+\delta) + \rho_e f(\theta)\alpha_h \beta p_h z}{r + \delta + \rho_e f(\theta)\alpha_h \beta} \geq \frac{b(r+\delta) + \rho_u f(\theta)\alpha_h \beta p_h z}{r + \delta + \rho_u f(\theta)\alpha_h \beta}.$$

$\square$

A.1.8 Proof of Lemma 1.5

Proof. Taking the first order condition of (1.31) and equating to 0 gives

$$\sigma'(s_e) = f(\theta)\alpha_h\beta[\mathcal{W}_h - \mathcal{W}_l],$$

where the $\phi_e(p_h)$ has been substituted by its solution, given by (1.19). From equations (1.8) and (1.31),

$$\mathcal{W}_h - \mathcal{W}_l = \frac{z(p_h - p_l) + \sigma(s_e)}{r + \delta + s_e f(\theta)\alpha_h\beta}.$$

Substituting into the first order condition gives (1.38). □

A.1.9 Proof of Proposition 1.2

Proof. First, show that the *VS* curve lies above the *SI* curve at $s_e = 0$. From (1.38), evaluated at $s_e = 0$:

$$0 = f(\theta)\alpha_h\beta\left[\frac{z(p_h - p_l)}{r + \delta}\right],$$

which implies $\theta = 0$ so that (1.38) is satisfied. Equation (1.39), evaluated at $s_e = 0$:

$$\frac{\gamma}{(1-\beta)q(\theta)} = \frac{z(\alpha_l p_l + \alpha_h p_h) - b}{r + \delta + \rho_u\beta f(\theta)},$$

which implies that $\theta > 0$. Thus, the *VS* curve lies above the *SI* curve at $s_e = 0$. The next step is to show that the *SI* curve lies above the VS curve as $s_e \to \infty$. From (1.38), as $s_e \to \infty$, $\theta \to \infty$ so that (1.38) is satisfied. From (1.39), as $s_e \to \infty$, the right hand side of (1.39) approaches ∞. To satisfy the *VS* curve, the left hand side must also approach ∞, which is achieved by $\theta \to 0$. Thus, the SI curve lies above the VS curve as $s_e \to \infty$ and there exists at least one equilibrium (s_e^*, θ^*) where the SI curve intersects the VS curve from below.

Comparative statics follow from the effect of a change in the exogenous parameters on the VS and SI curves. Comparative statics on u and e_l follow from the fact that u is decreasing in θ and e_l is increasing in θ.

$\square$

Appendix B

Supplementary Material for Equilibrium Underemployment

B.1 Empirical appendix

B.1.1 Data Sources and Construction

Data on underemployment and college degree attainment comes from four sources: (i) the ASEC, (ii) the U.S. Department of Labor's Occupational Information Network (O*NET), (iii) The Bureau of Labor Statistics (BLS), and (iv) The American Community Survey (ACS).[1] Occupations are defined to require a college degree if at least 50% of respondents in the August 2017 O*NET survey (release 222) respond that a college degree is necessary to perform that occupation. The sample of recent college graduates comes from the ASEC and only includes workers who are ages 22-27 (inclusive) and have at least a Bachelors degree, where educational attainment is based on the EDUC variable. For the years 1992-2017,

[1] The ASEC and ACS data was download from the IPUMS at `https://ipums.org`.

Bachelors recipients are those with EDUC $\geq$ 111. Prior to 1992, Bachelors recipients are those with at least four years of college (EDUC $\geq$ 110). A recent graduate is defined to be underemployed if the respondent's primary occupation is one in which less than 50% of respondents in the O*NET survey say that a college degree is required to perform that job. Occupations are matched between the ASEC and O*NET survey using 2010 Census Bureau occupation codes. In Figure 3.2 panel (b), high-wage (low-wage) jobs are those where the average annual salary of that occupation is more than $\$35,000$ (less than $\$25,000$), where data on average salary by occupation is based on 2012 data published by the BLS.[2]

The sample in panel (b) of 3.1 comes from the ASEC and is restricted to ages 25 to 30 (inclusive). Educational attainment is derived from the variable EDUC. Associates are those where the highest degree attained is an Associates degree occupational/vocational program or Associates degree academic program (EDUC = 91 or EDUC = 92). Bachelors corresponds to EDUC = 111 and Masters degree is EDUC = 123. Ph.D. & Professional is those with a Professional School Degree or Doctorate Degree (EDUC = 124 or EDUC = 125). Percentages are calculated using the ASEC person weight (ASECWT).

Data on Federal student loan disbursements in Figure 3.10 panel (a) comes from College Board (2017). Data for Figure 3.10 panel (b) comes from the Survey of Consumer Finances. The average amount of debt is calculated by summing the amount owed on education loans among those who report having a positive amount of education debt. The sample contains households where the head of the household is between 20-40 years old (inclusive), where age is calculated by subtracting birth year from year of the survey. Calculations use SCF weights where the weights are divided by 5.[3]

The regressions in Table 3.3 use data on annual earnings from ASEC data. College graduates are those with at least a Bachelors degree and follows the same definition as described above.

[2]This data is downloaded from `https://www.bls.gov/oes/tables.htm`.
[3]This follows the recommendation made by the Federal Reserve Board.

The definition of underemployment follows the same definition described above. The sample is restricted to ages 22-27 (inclusive) and those who report having positive earnings in the previous year (INCWAGE > 0). Earnings observations in the top and bottom 1% of the earnings distribution in each year are dropped. Real earnings are in 2016$ and are deflated using the CPI-U, annual average.

B.1.2 The use of Stafford Loans by Dependency Status

Panels (a) and (b) in Figure B.1 illustrates the percentage of full-time, full-year undergraduate students who borrow a positive amount using Stafford loans by dependency status. Panels (c) and (d) show the percentage, among full-time, full-year students who borrow using Stafford loans, who borrow the maximum amount that they are eligible for by dependency status.

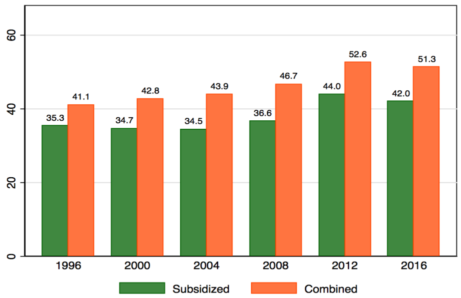
B.1.3 Granger Causality Tests

Table B.1 presents results from testing for Granger causality. Specifically, the second column presents the χ^2 statistic that is generated by performing a Wald test with the null hypothesis that the coefficients of the variable in the first column are jointly zero in the equation for the underemployment rate that is estimated in (3.29).

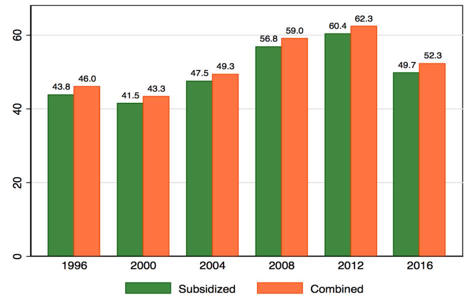
Table B.1: Granger Causality Wald Tests

Excluded variable	χ^2	P-value
ΔFederal	11.498	0.009
ΔRecipients	11.74	0.008
ΔGrants Size	4.9837	0.173

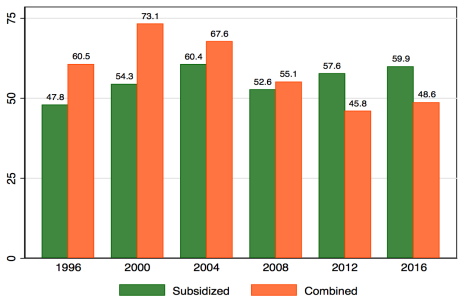
Figure B.1: The Use of Stafford Loans by Dependency Status



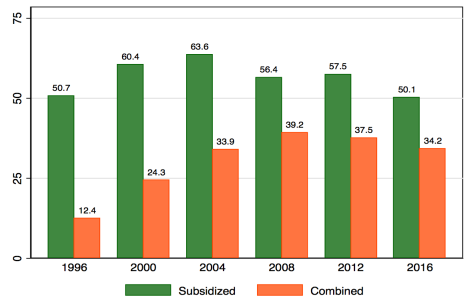
(a) Use of Stafford loans, Dependent



(b) Use of Stafford loans, independent



(c) Borrow Maximum Amount, Dependent



(d) Borrow Maximum Amount, Independent

Notes: Data come from the National Postsecondary Student Aid Survey (NPSAS). These calculations only include students who were enrolled full-time, full-year in that particular academic year. Panels (c) and (d) show the percentage of undergraduates, by dependency status, who borrowed the "usual maximum" amount of Stafford loans.Percentages are calculated using the NPSAS weights.

B.1.4 College Wage Premiums

Table B.2 presents estimates of the college hourly wage premium and is also constructed using ASEC data. The sample is the same as the annual earnings regressions. Hourly wages are calculated by dividing total income (INCWAGE) by total hours worked in the last year (WKSWORK1 × UHRSWORKLY). Wage observations in the top and bottom 1% of the wage distribution in each year are dropped. Real wages are in 2016$ and are deflated using the CPI-U, annual average.

Table B.2: Regression Estimates (Wages)

	(1)	(2)	(3)
	Overall	All non-college occupations	Appropriately matched
College	0.342***	0.179***	0.454***
	(0.022)	(0.018)	(0.030)
N	213,778	182,125	193,917
R^2	0.136	0.076	0.156

Notes: All regressions include state fixed effects, year fixed effects, control for demographics (age, sex, race, marital status), whether the individual works in a city, and the individual's industry of employment. The sample covers 1992-2017 and is composed of individuals between the ages of 22-27 who are not currently enrolled in school. Column (1) includes all individuals in the constructed sample. Column (2) excludes workers with at least a bachelors degree who work in a college occupation. Column (3) excludes workers with a college degree who work in non-college occupations. Standard errors are clustered at the occupation level and are in parentheses. Levels of statistical significance are denoted by $^{***}p < 0.01$.

B.2 Proofs and Derivations

B.2.1 Proof of Lemma 3.1

The contract $w_s(a,h), \phi_s^u(a,h)$ must be pairwise Pareto efficient. After the hiring fee is transferred from the worker to the firm the wage is chosen to maximize the surplus of the match. The surplus is maximized if and only if $J_s(a,h) = V_s$. This is true if and only if $w_s(a,h) = p_s y_s(a,h)$. The hiring fee in (3.16) is derived by taking the first order condition of (3.14) with respect to $\phi_s^u(a,h)$. □

B.2.2 Additional Employment Contracts

There are two other types of meetings to consider.

1. Highly-educated workers and complex jobs (via employment).
2. Highly-educated workers and complex jobs (via unemployment).

The flow surpluses of each match type are given by:

$$rS_c^e(a,h) = p_c y_c(a,h) - rE_s(a,h) - sS_c^e(a,h), \tag{B.2.1}$$

$$rS_c^u(a,h) = p_c y_c(a,h) - rU(a,h) - sS_c^u(a,h). \tag{B.2.2}$$

Equations (B.2.1)-(B.2.2) have a similar interpretation as equation (3.11), except that there are no voluntary separations in the three types of meetings described above. One feature that is unique to equation (B.2.1) is that the worker's outside option in a meeting between a complex job and an employed, highly-educated worker is the value of employment at the simple job. The contracts solve:

$$w_c(a,h), \phi_c^e(a,h) \in \arg\max \left[E_c(a,h) - E_s(a,h) - \phi_c^e(a,h)\right]^{\beta} \left[J_c(a,h) - V_c + \phi_c^e(a,h)\right]^{1-\beta}, \tag{B.2.3}$$

$$w_c(a,h), \phi_c^u(a,h) \in \arg\max \left[E_c(a,h) - U(a,h) - \phi_c^u(a,h)\right]^{\beta} \left[J_c(a,h) - V_c + \phi_c^u(a,h)\right]^{1-\beta}. \tag{B.2.4}$$

The solution to the wage contracts follows the same logic as in the Proof to Lemma 3.1. The wage is chosen to maximize the join surplus after the hiring fee is transferred from the worker to the firm. From equations (B.2.1)-(B.2.2), the joint surplus will be maximized independent of the wage (because there are no voluntary separations in these meetings). It follows that, without loss of generality, the wage can be set equal to the marginal product and the hiring fee splits the match surplus according to the respective bargaining weights:

$$w_c(a,h) = p_c y_c(a,h); \phi_c^u(a,h) = (1-\beta)[E_c(a,h) - U(a,h)];$$
$$\phi_c^e(a,h) = (1-\beta)[E_c(a,h) - E_s(a,h)]. \tag{B.2.5}$$

It is straitforward to show that the solutions in (B.2.5) are payoff equivalent to Nash bargaining over an employment contract which only specifies a flat wage as the wage does not affect the joint surplus in a match without voluntary separations.

B.2.3 Derivation of Closed-form Free-entry Conditions

Combining equations (3.7)-(3.8) with the optimal contracts in Section 3.4.2 gives the values of unemployment for less- and highly-educated workers:

$$rU(a,0) = \frac{a[b(r+\delta) + \beta f(\theta)\zeta p_s x_s]}{\rho_0}, \tag{B.2.6}$$

$$rU(a,1) = \frac{a\big[b(r+\delta)\rho_2 + \beta f(\theta)\{\zeta\kappa p_s x_s(r+\delta) + (1-\zeta)p_c x_c \rho_3\}\big]}{\rho_1\rho_2}, \tag{B.2.7}$$

where $\rho_0 \equiv r + \delta + \beta f(\theta)\zeta$, $\rho_1 \equiv r + \delta + \beta f(\theta)(\zeta\kappa + 1 - \zeta)$, $\rho_2 \equiv r + \delta + \beta\lambda f(\theta)(1-\zeta)$, and $\rho_3 = r + \delta + \beta\lambda f(\theta)(\zeta\kappa + 1 - \zeta)$ are discount factors.

Next, combining equations (3.9)-(3.10), the optimal employment contracts, and equations (B.2.6)-(B.2.7), the free-entry conditions are given by

$$\begin{aligned} \frac{\gamma}{q(\theta)(1-\beta)} &= \psi\Bigg\{\eta\frac{p_s x_s - b}{\rho_0}\mathbb{E}[a|h=0] \\ &+ (1-\eta)\kappa\frac{p_s x_s(r+\delta) - \beta f(\theta)(1-\zeta)[(1-\lambda)p_c x_c - p_s x_s] - b\rho_2}{\rho_1\rho_2}\mathbb{E}[a|h=1]\Bigg\}, \end{aligned} \tag{B.2.8}$$

$$\begin{aligned} \frac{\gamma}{q(\theta)(1-\beta)} = \mathbb{E}[a|h=1]\Bigg\{&\psi(1-\eta)\frac{p_c x_c \rho_4 - \beta f(\theta)\zeta[\kappa p_s x_s - (\kappa-\lambda)p_c x_c] - b\rho_2}{\rho_1\rho_2} \\ &+ (1-\psi)\frac{p_c x_c - p_s x_s}{\rho_2}\Bigg\}, \end{aligned} \tag{B.2.9}$$

where $\rho_4 \equiv r + \delta + \beta\lambda f(\theta)$.

B.2.4 Proof of Lemma 3.2

Substituting the worker's budget constraint, (3.6), into (3.5) reduces their maximization problem to consumption and human capital investment:

$$W(a,\underline{\ell}) = \max_{c,h} \left\{ c - \varphi(c + p_h h; \underline{\ell}) + h\big[U(a,1) - \varsigma\big] + (1-h)U(a,0) \right\}. \tag{B.2.10}$$

The first order condition with respect to consumption given by

$$1 \leq \varphi'(c + p_h h; \underline{\ell}). \tag{B.2.11}$$

From (B.2.11), a worker will only produce the numeraire good for consumption if they are at the linear portion of $\varphi(\ell)$. Even then, a worker is indifferent between producing the numeraire for consumption, as the marginal benefit is equal to the marginal cost. I consider an equilibrium where workers choose $c = 0$. It follows that a worker will choose $h = 1$ if (3.18) is satisfied.

From equations (B.2.6)-(B.2.7), the net gain of investing in human capital is given by

$$\begin{aligned} \Gamma(a,\underline{\ell}) = {} & -[\varphi(p_h;\underline{\ell}) + \varsigma] + \\ & \frac{a}{r}\left\{ \beta f(\theta)\left[\frac{\zeta\kappa p_s x_s (r+\delta) + (1-\zeta) p_c x_c \rho_3}{\rho_1\rho_2} - \frac{\zeta p_s x_s}{\rho_0}\right] + b(r+\delta)\frac{\rho_0 - \rho_1}{\rho_0\rho_1}\right\}. \end{aligned} \tag{B.2.12}$$

From equation (B.2.12), the capital gain of investing in education can only be positive when

$$\beta f(\theta)\left[\frac{\zeta\kappa p_s x_s (r+\delta) + (1-\zeta) p_c x_c \rho_3}{\rho_1\rho_2} - \frac{\zeta p_s x_s}{\rho_0}\right] + b(r+\delta)\frac{\rho_0 - \rho_1}{\rho_0\rho_1} > 0. \tag{B.2.13}$$

It follows that (B.2.13) is a necessary condition to have a positive measure of workers invest in human capital. It also follows that $\partial\Gamma(a,\underline{\ell})/\partial a > 0$ if (B.2.13) is satisfied.

From equation (B.2.12), $\underline{\ell}^*(a)$ satisfies:

$$\varphi(p_h,\underline{\ell}^*(a))+\varsigma = \frac{a}{r}\left\{\beta f(\theta)\left[\frac{\zeta\kappa p_s x_s(r+\delta)+(1-\zeta)p_c x_c\rho_3}{\rho_1\rho_2} - \frac{\zeta p_s x_s}{\rho_0}\right] + b(r+\delta)\frac{\rho_0-\rho_1}{\rho_0\rho_1}\right\}. \tag{B.2.14}$$

Assume that (B.2.13) is satisfied which implies that the right hand side of (B.2.14) is increasing in a. Thus, if a increases, $\varphi(p_h,\underline{\ell}^*(a))$ must increase so that (B.2.14) is satisfied. As $\varphi(p_h;\underline{\ell})$ is weakly decreasing in $\underline{\ell}$, $\partial\underline{\ell}^*(a)/\partial a \leq 0$. □

It is also clear from (B.2.12) that $\partial\Gamma(a,\underline{\ell})/\partial\underline{\ell} > 0$ if $\underline{\ell} < p_h$ as $\partial\varphi(\ell)/\partial\underline{\ell} < 0$ if $\underline{\ell} < p_h$.

B.2.5 Proof of Lemma 3.3

Cross-skill matches are formed if they generate a positive surplus. From equation (3.11), the surplus in a cross skill match is positive if

$$p_s x_s a + \beta\lambda f(\theta)(1-\zeta)\frac{p_c x_c a - p_s x_s a}{\rho_2} > rU(a,1). \tag{B.2.15}$$

Combining with (B.2.7) and simplifying shows that the surplus for forming a cross-skill match is positive if

$$\frac{p_s x_s - b}{p_c x_c - b} > \frac{\beta f(\theta)(1-\zeta)(1-\lambda)}{r+\delta+\beta f(\theta)(1-\zeta)}. \tag{B.2.16}$$

Thus, $\kappa = 1$ if (B.2.16) is satisfied, $\kappa = 0$ if (B.2.16) holds with opposite inequality, and $\kappa \in [0,1]$ if (B.2.16) holds at equality. It is straitforward to see from (B.2.16) that the decision to form a cross-skill match is independent of the worker's ability and financial wealth. □

B.2.6 Proof of Proposition 3.1

Part (i): The proof of existence of an active steady-state equilibrium with $\theta > 0$ is done by contradiction.

Case 1: $\epsilon < 1$. Suppose that $\theta = 0$. If $\theta \to 0$, then the left hand side of the free-entry conditions, (B.2.8)-(B.2.9), approach 0 as $q(\theta) \to \infty$ as $\theta \to 0$. However, if $\theta \to 0$, the right-hand side of at least one of the free-entry conditions is strictly positive for all $H \in [0, 1]$ as $p_\chi \to \infty$ as $\theta \to 0$. Thus, firms have an incentive to post at least one type of a vacancy giving $\theta > 0$.

Case 2: $\epsilon = 1$. As $\theta \to 0$, the right-hand side of at least one of the free-entry conditions is strictly positive for all $H \in [0, 1]$ if $b < \min\{\mu x_s, (1-\mu)x_c\}$. Thus, firms have an incentive to post at least one type of a vacancy giving $\theta > 0$.

Part (ii): I now proceed to prove that if $\epsilon < 1$, then $\zeta \in (0, 1)$ and $H > 0$. Suppose that firms only post simple vacancies ($\zeta = 1$). It follows that the value of a simple vacancy is given by

$$V_s = -\gamma + q(\theta)(1-\beta)\mathbb{E}_{a,h}\left[\frac{p_s y_s(a,h) - rU(a,h)}{r+\delta}\right] = 0, \tag{B.2.17}$$

where

$$rU(a,h) = ba + \beta f(\theta)\left[\frac{p_s x_s a - rU(a,h)}{r+\delta}\right], \tag{B.2.18}$$

defines the worker's reservation wage which is independent of the worker's human capital if $\zeta = 1$. Define θ^* as the unique value of market tightness that jointly solves equations (B.2.17) and (B.2.18).

Now suppose that a small measure of firms deviate to post a complex vacancy so that the fraction of complex vacancies is given by $1-\zeta = \varepsilon$ where ε is positive and small. Additionally, let $\Gamma^*(a, \underline{\ell})$ denote the capital gain of investing in human capital for $\theta = \theta^*$ and $1 - \zeta = \varepsilon$. If $\Gamma^*(a_H, p_h) \geq 0$, then the minimum supply of highly-educated workers will be given by $(1 - \pi)[1 - F(p_h)]$.[4] Firms have an incentive to deviate and post a complex vacancy if the expected profits of doing so are positive, i.e. if

$$\rho V_c = -\gamma + q(\theta)(1-\beta)(1-\pi)[1 - F(p_h)]\mathbb{E}_{a,h}\left[\frac{p_c x_c - rU(a,h)}{r+\delta}\right] > 0. \tag{B.2.19}$$

Combining equation (B.2.18) with equation (B.2.19), it follows that (B.2.19) is satisfied if

$$(1-\pi)[1 - F(p_h)]\mathbb{E}_{a,h}\left[\frac{p_c x_c - rU(a,h)}{r+\delta}\right] > \mathbb{E}_{a,h}\left[\frac{p_s y_s(a,h) - rU(a,h)}{r+\delta}\right]. \tag{B.2.20}$$

Solving for $rU(a,h)$ from equation (B.2.18), substituting into (B.2.20) and simplifying shows that (B.2.20) is satisfied if

$$\frac{(1-\pi)[1 - F(p_h)]\big[(r+\delta)(p_c x_c - b) + \beta f(\theta^*)(p_c x_c - p_s x_s)\big]\mathbb{E}[a|h=1]}{(r+\delta)(p_s x_s - b)\mathbb{E}[a]} > 1. \tag{B.2.21}$$

The prices of intermediate goods are given by

$$p_s = \mu\left[\mu + (1-\mu)\left(\frac{Y_c}{Y_s}\right)^{\epsilon}\right]^{\frac{1-\epsilon}{\epsilon}}, \tag{B.2.22}$$

$$p_c = (1-\mu)\left[\mu\left(\frac{Y_s}{Y_c}\right)^{\epsilon} + (1-\mu)\right]^{\frac{1-\epsilon}{\epsilon}}, \tag{B.2.23}$$

where, from the steady-state conditions (3.22)-(3.24), the aggregate output from type χ

[4] See equation (3.20)

matches is given by

$$Y_s = x_s\left[\frac{(1-H)f(\theta)\zeta\mathbb{E}[a|h=0]}{\delta+f(\theta)\zeta} + \frac{Hsf(\theta)\zeta\kappa\mathbb{E}[a|h=1]}{(s+f(\theta)[\zeta\kappa+1-\zeta])[\delta+\lambda f(\theta)(1-\zeta)]}\right], \tag{B.2.24}$$

$$Y_c = x_c\frac{Hf(\theta)(1-\zeta)\mathbb{E}[a|h=1]}{\delta+f(\theta)(\zeta\kappa+1-\zeta)}\left[\frac{\delta+\lambda f(\theta)(\zeta\kappa+1-\zeta)}{\delta+\lambda f(\theta)(1-\zeta)}\right]. \tag{B.2.25}$$

Suppose that $\epsilon < 1$. From equation (B.2.23),

$$\lim_{Y_c/Y_s\to 0} p_c = \infty. \tag{B.2.26}$$

Combining (B.2.26) with the fact that $Y_c \to 0$ as $\zeta \to 1$,[5] it follows that

$$\lim_{\zeta\to 1} p_c = \infty. \tag{B.2.27}$$

It is straitforward to see that (B.2.21) is always satisfied if $p_c \to \infty$ as $\zeta \to 1$.

Part (iii): From the Proof of Lemma 3.2, the first workers to invest in human capital are those with (i) the high ability and (ii) lowest cost to finance the pecuniary cost of human capital. It follows that the minimum supply of highly-educated workers is given by $H^* = (1-\pi)(1-F(\bar{\ell}))$. Suppose that the supply of highly-educated workers is given by H^*. Define θ^*, ζ^*, and κ^* as the corresponding market tightness, composition of vacancies, and cross-skill match formation rule that solve the entry conditions, (B.2.8)-(B.2.9), and (3.21) when the supply of highly-educated workers is given by H^*. Workers with the high innate ability and low cost to produce the numeraire will not invest in human capital if

$$p_h + \varsigma > \frac{a_H}{r}\left\{\beta f(\theta^*)\left[\frac{\zeta^*\kappa^*\mu x_s(r+\delta)+(1-\zeta^*)(1-\mu)x_c\rho_3}{\rho_1\rho_2} - \frac{\zeta^*\mu x_s}{\rho_0}\right] + b(r+\delta)\frac{\rho_0-\rho_1}{\rho_0\rho_1}\right\}. \tag{B.2.28}$$

[5] This follows from equation (B.2.25).

It follows that if $\iota \equiv p_h + \varsigma$ is greater than $\underline{\iota}$, which is defined as the right side of (B.2.28), then no workers will invest in human capital. If $H = 0$, then the right hand side of (B.2.9) is equal to zero and no complex jobs will be created leaving $\zeta = 1$. □

B.2.7 Proof of Proposition 3.2

Assume that parameter values are such that $H > 0$ and $\zeta \in (0, 1)$. Recall that cross-skill matches are formed, i.e. $\kappa \in (0, 1]$ if

$$\frac{p_s x_s - b}{p_c x_c - b} > \frac{\beta f(\theta)(1-\zeta)(1-\lambda)}{r + \delta + \beta f(\theta)(1-\zeta)}. \tag{B.2.29}$$

It is clear that as $\lambda \to 1$, the right hand side of (B.2.29) approaches 0. It follows that as $\lambda \to 1$, then $\kappa \in (0, 1]$. From (3.25), $\underline{u} > 0$ if $\zeta\kappa > 0$.

Cross-skill matches are not formed, i.e. $\kappa = 0$, if

$$\frac{p_s x_s - b}{p_c x_c - b} < \frac{\beta f(\theta)(1-\zeta)(1-\lambda)}{r + \delta + \beta f(\theta)(1-\zeta)}. \tag{B.2.30}$$

Suppose that $\lambda < \underline{\lambda}$. As x_c increases, the left-hand side of (B.2.30) decreases, making it more likely that (B.2.30) is satisfied. In the extreme case of $x_c \to \infty$, it is clear that (B.2.30) will be satisfied and cross-skill matches will not be formed and $\underline{u} = 0$. □

B.2.8 Proof of Proposition 3.3

With a fixed supply of highly-educated workers and $\kappa = 1$, the two-equilibrium conditions that simultaneously determine θ and ζ are given by the following entry conditions.

$$\frac{\gamma}{q(\theta)(1-\beta)} = \eta\frac{\mu x_s - b}{\rho_0} + (1-\eta)\frac{(\mu x_s - b)(r+\delta) - \beta f(\theta)(1-\zeta)[(1-\mu)x_c - \mu x_s]}{\rho_1\rho_2}, \tag{B.2.31}$$

$$\frac{\gamma}{q(\theta)(1-\beta)} = (1-\eta)\frac{[(1-\mu)x_c - b](r+\delta) - \beta f(\theta)\zeta[\mu x_s - (1-\mu)x_c]}{\rho_1\rho_2}. \tag{B.2.32}$$

It is strait-forward to make use of $V_s = V_c$ to show

$$(r+\delta)(\mu x_s - b) = (1-\eta)\big[[(1-\mu)x_c - b](r+\delta) + \beta f(\theta)\zeta[(1-\mu)x_c - \mu x_s]\big], \tag{B.2.33}$$

where

$$1-\eta = \frac{H[\delta + f(\theta)\zeta]}{\delta + Hf(\theta)\zeta + (1-H)f(\theta)}. \tag{B.2.34}$$

Substituting (B.2.33) into (B.2.32) and simplifying gives

$$\frac{\gamma}{q(\theta)(1-\beta)} = \frac{(r+\delta)(\mu x_s - b)}{r+\delta+\beta f(\theta)}, \tag{B.2.35}$$

which defines a unique value of θ that is independent of ζ. Given θ that is determined by (B.2.35), (B.2.33) determines the equilibrium value of ζ. It is strait-forward to show that the right hand side of (B.2.33) is decreasing in θ (recall that I have assumed $\beta \approx 0$). It is also straitforward to see that the right hand side of (B.2.33) is increasing in ζ, as $1-\eta$ is decreasing in ζ.

If μ or x_s increases, then, from (B.2.33), θ will increase. As μ or x_s and θ increase, the left side of (B.2.33) increases while the right side decreases. It follows that ζ must increase to satisfy (B.2.33).

From (B.2.35), θ is independent of x_c and H. From (B.2.33), if x_c or H increases, then the right side will increase. It follows that ζ must decrease to satisfy (B.2.33).

Finally, if γ increases, then θ will decrease to satisfy (B.2.35). If θ decreases, the left side of (B.2.33) decreases which implies that ζ must increase so that (B.2.33) is satisfied. □

B.2.9 Proof of Proposition 3.4

The net benefits of human capital, under the simplifying assumptions, is given by

$$\Gamma(\underline{\ell}) = -[\varphi(p_h;\underline{\ell}) + \varsigma] + \frac{\beta f(\theta)}{r}\left\{\frac{\zeta\mu x_s + (1-\zeta)(1-\mu)x_c}{r+\delta+\beta f(\theta)} - \frac{\zeta\mu x_s}{r+\delta+\beta f(\theta)\zeta} - \frac{b(r+\delta)(1-\zeta)}{(r+\delta+\beta f(\theta))(r+\delta+\beta f(\theta)\zeta)}\right\}, \quad \text{(B.2.36)}$$

with $\partial\Gamma(\underline{\ell})/\partial\theta > 0$ and $\partial\Gamma(\underline{\ell})/\partial\zeta < 0$. The remaining equilibrium conditions that determine ζ and θ are given by equations (B.2.33)-(B.2.35).

Suppose that μ or x_s increases. From (B.2.33), it is clear that θ will increase. From the Proof of Proposition 3.3, this causes ζ to increase. An increase in θ and ζ have opposite effects on the net benefits of human capital, (B.2.36). If the effect of θ outweighs the effect of ζ on $\Gamma(\underline{\ell})$, then the supply of highly-educated workers will increase, which puts downward pressure on ζ. On the other hand, if the effect of ζ increasing dominates the effect of θ, then H will decrease which further causes ζ to increase. Thus, the end result on both H and ζ is ambiguous.

If γ increases, then θ will decrease (from (B.2.35)). From the Proof of Propsition 3.3, this increase ζ. A decrease in θ and increase in ζ both reduce the benefits from investing in human capital, causing H to decrease. A decrease in H subsequently causes ζ to decline (see the Proof of Proposition 3.3). Thus, an increase in γ will cause both θ and H to decrease and for ζ to increase.

Market tightness is independent of the remaining parameters, x_c, p_h, and ς, as neither enter equation (B.2.35). From the Proof of Proposition 3.3, if x_c increases, then ζ decreases. As ζ decreases, the benefits of investing in human capital increase and thus H increases, which further causes ζ to decrease. It follows that if x_c increases, then θ is unaffected, ζ decreases, and H increases. Now suppose that p_h or ς increase. From (B.2.36), this decreases the benefits of investing in human capital. As H decreases, ζ will increase (see the Proof of Proposition 3.3), which further decreases the benefits of investing in human capital. It follows that ζ increases while H decreases following an increase in p_h or ς. $\square$

B.2.10 Proof of Proposition 3.5

I proceed to derive the efficient steady-state allocations by writing down the current-value Hamiltonian, where $\Upsilon^h_{\chi,i}$ is the multiplier on the $\dot{e}^h_{\chi,i}$ laws of motion and Λ^h_i is the multiplier on the $\dot{u}^h_i$ laws of motion:

$$\begin{aligned}\mathcal{H} = {} & Y(e^h_{s,i}, e^1_{c,i}) + \sum_i \sum_h u^h_i b a_i - \gamma v - \sigma \sum_i \pi_i \int_0^\infty h(a_i, \underline{\ell})[\varsigma + \varphi(p_h; \underline{\ell})] dF(\underline{\ell}) \\ & + \sum_i \Bigg\{ \sum_h \Upsilon^h_{s,i} \Big[\mathcal{M}(\cdot,\cdot) \frac{v_s}{v} \frac{u^h_i}{u} [\kappa_i h + (1-h)] - (\delta+\sigma) e^h_{s,i}\Big] + \Upsilon^1_{c,i} \Big[\mathcal{M}(\cdot,\cdot) \frac{v_c}{v} \frac{u^1_i}{u} - (\delta+\sigma) e^1_{c,i}\Big] \\ & + \Lambda^0_i \left[\sigma \pi_i \int_0^\infty [1 - h(a_i, \underline{\ell})] dF(\underline{\ell}) + \delta e^0_{s,i} - \mathcal{M}(\cdot,\cdot) \frac{v_s}{v} \frac{u^0_i}{u} - \sigma u^0_i\right] \\ & + \Lambda^1_i \left[\sigma \pi_i \int_0^\infty h(a_i, \underline{\ell}) dF(\underline{\ell}) + \delta [e^1_{s,i} + e^1_{c,i}] - \mathcal{M}(\cdot,\cdot) \left(\frac{v_s}{v} \kappa_i + \frac{v_c}{v}\right) \frac{u^1_i}{u} - \sigma u^1_i\right] \Bigg\}. \end{aligned} \quad \text{(B.2.37)}$$

I begin by characterizing the total surplus generated in each type of match which are derived from the standard co-state equations. They are given by:

$$\Upsilon^0_{s,i} - \Lambda^0_i = \frac{a_i[\mu x_s - b] + (1-\nu)f(\theta)\Psi}{r + \delta + f(\theta)\zeta}, \tag{B.2.38}$$

$$\Upsilon^1_{s,i} - \Lambda^1_i = \frac{a_i[(\mu x_s - b)(r+\delta) - f(\theta)(1-\zeta)((1-\mu)x_c - \mu x_s)] + (r+\delta)f(\theta)(1-\nu)\Psi}{(r+\delta)(r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i))}, \tag{B.2.39}$$

$$\Upsilon^1_{c,i} - \Lambda^1_i = \frac{a_i[((1-\mu)x_c - b)(r+\delta) - f(\theta)\zeta\kappa_i(\mu x_s - (1-\mu)x_c)] + (r+\delta)f(\theta)(1-\nu)\Psi}{(r+\delta)(r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i))}, \tag{B.2.40}$$

where $\nu \equiv \frac{\theta q'(\theta)}{q(\theta)}$ is the elasticity of the matching function with respect to job seekers and Ψ is the average value of a match:

$$\Psi \equiv \eta\zeta\sum_i \hat{\pi}^0_i[\Upsilon^0_{s,i} - \Lambda^0_i] + (1-\eta)\big\{\zeta\sum_i \hat{\pi}^1_i\kappa_i[\Upsilon^1_{s,i} - \Lambda^1_i] + (1-\zeta)\sum_i \hat{\pi}^1_i[\Upsilon^1_{c,i} - \Lambda^1_i]\big\}, \tag{B.2.41}$$

and $\hat{\pi}^h_i$ is the probability of matching a worker with ability i conditional on meeting a worker with human capital h. Equation (B.2.38) is the surplus generated in a match between a less-educated worker and simple job, (B.2.39) shows the surplus in cross-skill matches between highly-educated workers and simple jobs, and finally equation (B.2.40) is the total surplus in a match between a highly-educated worker in a complex job.

The planner's choice of forming cross-skill matches is governed by the sign of $\partial\mathcal{H}/\partial\kappa_i$, which gives $\kappa_i \in [0,1]$ if

$$\frac{\mu x_s - b}{(1-\mu)x_c - b} + \frac{(1-\nu)f(\theta)\Psi}{a_i((1-\mu)x_c - b)(r+\delta+f(\theta)(1-\zeta))} = \frac{f(\theta)(1-\zeta)}{r+\delta+f(\theta)(1-\zeta)}. \tag{B.2.42}$$

The planner's optimal choice for vacancies satisfies $\partial\mathcal{H}/\partial v_\chi = 0$ for $\chi \in \{s, c\}$:

$$\frac{\gamma}{q(\theta)} = \eta \sum_i \hat{\pi}_i^0 [\Upsilon_{s,i}^0 - \Lambda_i^0] + (1-\eta) \sum_i \hat{\pi}_i^1 \kappa_i [\Upsilon_{s,i}^1 - \Lambda_i^1] - \nu\Psi, \tag{B.2.43}$$

$$\frac{\gamma}{q(\theta)} = (1-\eta) \sum_i \hat{\pi}_i^1 [\Upsilon_{c,i}^1 - \Lambda_i^1] - \nu\Psi. \tag{B.2.44}$$

The planner opens vacancies until the expected cost to fill the vacancy is equal to the expected surplus generated when the vacancy is filled. Equations (B.2.43) and (B.2.44) can be combined to show that

$$\eta \sum_i \hat{\pi}_i^0 [\Upsilon_{s,i}^0 - \Lambda_i^0] + (1-\eta) \sum_i \hat{\pi}_i^1 \kappa_i [\Upsilon_{s,i}^1 - \Lambda_i^1] = (1-\eta) \sum_i \hat{\pi}_i^1 [\Upsilon_{c,i}^1 - \Lambda_i^1] = \Psi, \tag{B.2.45}$$

which states the the expected surplus generated by each type of vacancy is equal to the average surplus generated by a match. Combining the surplus terms, (B.2.38)-(B.2.40), and (B.2.45) with (B.2.43)-(B.2.44) gives

$$\frac{\gamma}{q(\theta)(1-\nu)} = \eta \sum_i \hat{\pi}_i^0 \frac{a_i[\mu x_s - b] + (1-\nu) f(\theta)\Psi}{r + \delta + f(\theta)\zeta} +$$
$$(1-\eta) \sum_i \hat{\pi}_i^1 \kappa_i \frac{a_i[(\mu x_s - b)(r+\delta) - f(\theta)(1-\zeta)((1-\mu)x_c - \mu x_s)] + (r+\delta) f(\theta)(1-\nu)\Psi}{(r+\delta)(r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i))}, \tag{B.2.46}$$

$$\frac{\gamma}{q(\theta)(1-\nu)} = (1-\eta) \sum_i \hat{\pi}_{1,i} \times \tag{B.2.47}$$

$$\frac{a_i[((1-\mu)x_c - b)(r+\delta) - f(\theta)\zeta\kappa_i(\mu x_s - (1-\mu)x_c)] + (r+\delta) f(\theta)(1-\nu)\Psi}{(r+\delta)(r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i))}. \tag{B.2.48}$$

Equations (B.2.46)-(B.2.48) are analogous to the free-entry conditions in the decentralized equilibrium.

The final component of the solution to the planner's problem is the choice of human capital investment, which is governed by the signed of $\partial\mathcal{H}/\partial h(a_i,\omega)$:

$$h(a_i,\omega) = \begin{cases} 0 & \text{if } \Lambda_i^1 - \Lambda_i^0 < \varsigma + \varphi(\ell), \\ \in [0,1] & \text{if } \Lambda_i^1 - \Lambda_i^0 = \varsigma + \varphi(\ell), \\ 1 & \text{if } \Lambda_i^1 - \Lambda_i^0 > \varsigma + \varphi(\ell), \end{cases} \tag{B.2.49}$$

where

$$\Lambda_i^1 - \Lambda_i^0 = \frac{a_i f(\theta)}{r}\Bigg\{\frac{\zeta\kappa_i\mu x_s + (1-\zeta)(1-\mu)x_c}{r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i)} - \frac{\zeta\mu x_s}{r+\delta+f(\theta)\zeta} + \frac{b(r+\delta)[\zeta(2-\kappa_i)-1]}{(r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i))(r+\delta+f(\theta)\zeta)} + \frac{f(\theta)(1-\nu)\Psi[1+\zeta(\kappa_i-2)]}{(r+\delta+f(\theta)(1-\zeta+\zeta\kappa_i))(r+\delta+f(\theta)\zeta)}\Bigg\}. \tag{B.2.50}$$

Suppose that parameter values are such that all cross-skill matches are formed in both the decentralized equilibrium and efficient steady-state (i.e., $\kappa_i = 1$ for $i \in \{L,H\}$). A comparison of (B.2.8)-(B.2.9) and (B.2.46)-(B.2.48) show that these pairs of equations may only coincide with each other when $(\nu,\beta) = (1,1)$. But in the case where $\beta = 1$, there is no interior solution as firms have no incentive to post vacancies. $\square$

B.2.11 Proof of Proposition 3.6

To help illustrate the results, I make an additional simplifying assumption of $\lambda = 0$. Suppose that $\epsilon = 1$. I begin by showing that the supply channel outweighs the composition channel. This can be illustrating by showing

$$\frac{(1-\eta)\mathbb{E}[a|h=1]}{\partial H} > 0, \tag{B.2.51}$$

where

$$\eta(\theta, \zeta) = \frac{(1-H)(\delta + f(\theta))}{\delta + Hf(\theta)\zeta + (1-H)f(\theta)}, \tag{B.2.52}$$

and

$$\mathbb{E}[a|h=1] = \frac{\pi h(a_L)a_L + (1-\pi)h(a_H)a_H}{\pi h(a_L) + (1-\pi)h(a_H)}. \tag{B.2.53}$$

It is straitforward to show that (B.2.51) is true if and only if

$$\frac{(1-\pi)h(a_L)(a_H - a_L)}{\pi h(a_L)a_L + (1-\pi)h(a_H)a_H} < \frac{\delta + f(\theta)}{\delta + Hf(\theta)\zeta + (1-H)f(\theta)}. \tag{B.2.54}$$

The right hand side of equation (B.2.54) is always greater than 1. The left hand side is greater than 1 if

$$a_H \left[\frac{h(a_L) - h(a_H)}{h(a_L)} \right] > \frac{a_L}{1-\pi}. \tag{B.2.55}$$

From Lemma 3.2, we know that $h(a_H) \geq h(a_L)$. Therefore, (B.2.55) is never satisfied, the left hand side of (B.2.54) is never greater than 1, and (B.2.54) always holds.

Now suppose that $a_L = a_H = 1$ and $\epsilon < 1$. Consider the term $(1 - \eta(\theta, \zeta))p_c$, where the prices of intermediate goods are given by equations (B.2.22) and (B.2.23). The sign of $\partial(1 - \eta(\theta, \zeta))p_c / \partial H$ determines the net effect of the supply and relative price channels on the expected profits of posting a complex vacancy. Differentiating with respect to H, one can show that

$$\frac{\partial(1 - \eta(\theta, \zeta))p_c}{\partial H} < 0 \tag{B.2.56}$$

if and only if

$$(1-\eta(\theta,\zeta))\left(\mu\left({}^{Y_s}/_{Y_c}\right)+1-\mu\right)<\frac{x_s\zeta(1-\epsilon)\mu}{x_c(1-\zeta)}\left({}^{Y_s}/_{Y_c}\right)^{\epsilon-1}, \tag{B.2.57}$$

which is true as $\epsilon \to -\infty$. $\square$

B.3 Segmented Markets

It may be unrealistic to assume a completely unsegmented labor market. For example, if the differences between simple and complex jobs are large enough, it may be more reasonable to assume that the markets for these types of jobs are completely segmented into their own submarkets. To study the implications of this type of market structure, this appendix analyzes a version of the model where there are separate submarkets for simple and complex vacancies. Workers direct their search into a submarket to where, within each market, there is still random search and ex-post bargaining over employment contracts. That is, firms cannot commit to a posted wage or a single type of worker to hire. To simplify the exposition while maintaining the same insights, I consider the case without on-the-job search.

Suppose that there are two markets indexed by job type $\chi \in \{s, c\}$. I assume that the flow cost to post vacancies differs across markets and is given by γ_χ with $\gamma_s < \gamma_c$. Meetings within each market are generated by the same matching technology as in the baseline model.

Let ϱ denote the probability that the worker permanently enters the market for simple jobs. It follows that market tightness in the market for skilled jobs is given by $\theta_s \equiv v_s/[u_0 + \varrho u_1]$ and market tightness in the market for complex jobs is given by $\theta_c \equiv v_c/[(1 - \varrho)u_1]$. The share of less-educated workers among all unemployed workers searching in the market for simple jobs is given by $\eta \equiv u_0/[u_0 + \varrho u_1]$. All other assumptions are the same as in Section 3.3.

The main changes in this version of the model's equilibrium begin with the value of being an unemployed worker, as seen below:

$$rU(a,0) = ba + \beta f(\theta_s)\big[E_s(a,0) - U(a,0)\big], \tag{B.3.1}$$

$$rU(a,1) = ba + \max_{\varrho \in [0,1]} \beta\Big(\varrho f(\theta_s)\big[E_s(a,1) - U(a,1)\big] + (1-\varrho) f(\theta_c)\big[E_c(a,1) - U(a,1)\big]\Big), \tag{B.3.2}$$

where $f(\theta_\chi)$ is the job finding rate in market for type χ jobs.[6] According to (B.3.1), less-educated workers only search in the market for simple jobs. This is because less-educated workers produce zero output in complex jobs and thus would never search in that market with a positive probability.

Equation (B.3.2) shows that highly-educated workers choose the probability of permanently searching in the market for simple jobs, ϱ, to maximize their lifetime discounted utility. I assume that highly-educated workers always form matches with simple jobs as they would never enter the market for simple jobs (i.e. $\varrho = 0$) if cross-skill matches generated a negative surplus.

Free entry of firms into each sub-market implies:

$$\frac{\gamma_s}{q(\theta_s)(1-\beta)} = \eta \sum_i \hat{\pi}_{i,0}\big[E_s(a_i,0) - U(a_i,0)\big] + (1-\eta)\sum_i \hat{\pi}_{i,1}\big[E_s(a_i,1) - U(a_i,1)\big], \tag{B.3.3}$$

$$\frac{\gamma_c}{q(\theta_c)(1-\beta)} = \sum_i \hat{\pi}_{i,1}\big[E_c(a_i,1) - U(a_i,1)\big]. \tag{B.3.4}$$

Conditions (B.3.3) and (B.3.4) simply state that market tightness in each submarket adjusts so that the expected costs to fill a vacancy in each market are equal to the surplus of filling a vacancy.

[6]Equations (B.3.1) and (B.3.2) already account for the solution to the optimal employment contracts and show that the worker receives a share β of the total match surplus.

Taking the first order condition of (B.3.2) with respect to ϱ gives

$$\varrho = \begin{cases} 0 & \text{if } f(\theta_s)\big[E_s(a,1) - U(a,1)\big] < f(\theta_c)\big[E_c(a,1) - U(a,1)\big], \\ \in [0,1] & \text{if } f(\theta_s)\big[E_s(a,1) - U(a,1)\big] = f(\theta_c)\big[E_c(a,1) - U(a,1)\big], \\ 1 & \text{if } f(\theta_s)\big[E_s(a,1) - U(a,1)\big] > f(\theta_c)\big[E_c(a,1) - U(a,1)\big]. \end{cases} \tag{B.3.5}$$

From (B.3.5), highly-educated workers only search in the market for complex jobs ($\varrho = 0$) if the interaction between the job-finding rate and surplus in that type of match is larger than the same term in the market for simple jobs. This type of equilibrium is referred to as complete separation: highly-educated workers only search for complex jobs.[7] An equilibrium of partial mixing, $\varrho \in (0,1)$, occurs if the highly-educated worker is indifferent between the two markets. After some simple algebra, one can obtain the following surplus terms:

$$E_s(a,1) - U(a,1) = \frac{a\{p_s x_s(r + \delta + \beta(1-\varrho)f(\theta_c)) - \beta(1-\varrho)f(\theta_c)p_c x_c - (r+\delta)b\}}{(r+\delta)(r+\delta+\beta[\varrho f(\theta_s) + (1-\varrho)f(\theta_c)])}, \tag{B.3.6}$$

$$E_c(a,1) - U(a,1) = \frac{a\{p_c x_c(r + \delta + \beta\varrho f(\theta_s)) - \beta\varrho f(\theta_s)p_s x_s - (r+\delta)b\}}{(r+\delta)(r+\delta+\beta[\varrho f(\theta_s) + (1-\varrho)f(\theta_c)])}. \tag{B.3.7}$$

From equations (B.3.6) and (B.3.7), it is straitforward to show that the surplus generated in a complex match is greater than the surplus in a simple match if $p_c x_c > p_s x_s$, which I assume is always true. Combining this result with equation (B.3.5) reveals that a necessary condition for highly-educated workers to be indifferent between searching in the simple or complex markets and choose $\varrho \in (0,1)$ if $f(\theta_s) > f(\theta_c)$. The intuition for this is simple: highly-educated workers must be compensated with a higher job-finding rate in the market with simple jobs in order to search there to offset the lower surplus that they earn from a match. This is the standard tradeoff found in models of directed search whereby workers

[7] I do not consider the case of $\varrho = 1$ as workers would never invest in human capital to only search for simple jobs.

can either search in markets with a low (high) wage and a high (low) job-finding rate.

Suppose that workers are indifferent between searching for simple and complex jobs. It follows that the workers are then indifferent between the pure strategies of $\varrho = 0$ and $\varrho = 1$ and that $U(a, 0) = U(a, 1)$, as less- and highly-educated workers are equally productive in simple jobs. It is easy to show that under these assumptions that the free-entry conditions are given by

$$\frac{r + \delta + \beta f(\theta_s)}{q(\theta_s)} = \frac{(1 - \beta)(p_s x_s - b) \sum_i \pi_i a_i}{\gamma_s}, \tag{B.3.8}$$

$$\frac{r + \delta + \beta f(\theta_c)}{q(\theta_c)} = \frac{(1 - \beta)(p_c x_c - b) \sum_i \hat{\pi}_{1,i} a_i}{\gamma_c}. \tag{B.3.9}$$

The left hand side of equations (B.3.8) and (B.3.9) are increasing in θ_s and θ_c, respectively. It follows that $\theta_s > \theta_c$ if and only if

$$\frac{\gamma_c}{\gamma_s} > \frac{(p_c x_c - b) \sum_i \hat{\pi}_{1,i} a_i}{(p_s x_s - b) \sum_i \pi_i a_i}. \tag{B.3.10}$$

Equation (B.3.10) shows that if the relative flow cost to fill a complex vacancy is larger than the relative productivity of a complex vacancy, then $\theta_s > \theta_c$ and it is possible to have a partial mixing equilibrium.[8]

Consider the case where (B.3.10) holds and $\theta_s > \theta_c$. From equations (B.3.6) and (B.3.7), workers are indifferent between searching for simple and complex jobs if and only if

$$f(\theta_c)\{p_c x_c(r + \delta + \beta f(\theta_s)) - (r + \delta)b\} = f(\theta_s)\{p_s x_s(r + \delta + \beta f(\theta_c)) - (r + \delta)b\}. \tag{B.3.11}$$

Equation (B.3.11) defines an increasing relationship between θ_c and θ_s that will ensure a highly-educated worker's indifference between searching for simple and complex jobs. Thus,

[8] Brunnermeier and Julliard (2008) consider the case where the flow vacancy costs are equal across jobs and show that there can never be a partial mixing equilibrium.

in an environment with segmented markets, it is only in a knife-edge case where (B.3.11) holds that highly-educated workers are indifferent between searching for simple and complex jobs. This is in addition to the necessary condition that (B.3.10) holds and $\theta_s > \theta_c$.

Moreover, recall the worker's human capital decision upon birth, (B.2.10). It is straitforward to see that if $U(a, 0) = U(a, 1)$, then there is no incentive to invest in human capital. Taken together, with a segmented labor market, an equilibrium with partial mixing and underemployment is not robust as it is only sustained in a knife-edge case and workers would not invest in human capital if they were indifferent between searching in both submarkets.[9]

[9] This equilibrium also requires that the job-finding rate among less-educated workers is higher than the job-finding rate of highly-educated workers, which is counterfactual to U.S. data.

Appendix C

Supplementary Material for Entrepreneurial Finance, Home Equity, and Monetary Policy

C.1 Proofs

C.1.1 Proof of Lemma 4.1

We show uniqueness in a constrained equilibrium, as $k_E = k^*$ in an unconstrained equilibrium. We rearrange (4.22) as:

$$\theta f(k_E) + (1-\theta)k_E - \theta\Delta_I^e(m,b) = \chi f(k_E) + \rho q_a a + q_m m. \tag{C.1.1}$$

If $k_E = 0$, the left hand side of (C.1.1) is less than zero and the right hand side is greater than zero. As $k_E \to \infty$, the left hand side increases at rate $\theta f'(k_E) + (1-\theta)$ and the right hand side increases at rate $\chi f'(k_E)$. The left hand side of (C.1.1) will eventually surpass the right hand side if $1-\theta > (\chi-\theta)f'(k_E)$, which is true for some $k_E > 0$, as $f'(k_E) \to 0$ as $k_E \to \infty$. Thus, there exists a unique $k_E > 0$ that satisfies (4.22).

Second, we establish that $k_E \in [\underline{k}_E, k^*]$ where $\chi f'(\underline{k}_E) = 1$. Consider the entrepreneur's binding liquidity constraint, (4.15). Solving for the bank's surplus, ϕ, gives

$$\phi = \chi f(k_E) - k_E + \rho q_a a + k_I. \tag{C.1.2}$$

From (C.1.2), the bank's surplus is maximized at $\underline{k}_E$. Suppose that $k_E < \underline{k}_E$. A Pareto improvement can be made by increasing k_E to $\underline{k}_E$, as both the surplus of the bank and entrepreneur are strictly larger at $\underline{k}_E$. Thus, $\underline{k}_E$ is a lower bound on capital acquired through bank credit. □

C.1.2 Proof of Lemma 4.2

Consider the case where $m < m^*$. From (4.21), the real lending rate is given by

$$r^b = \frac{\theta[f(k_E) - k_E - \Delta_I^e(m,b)]}{k_E - k_I}. \tag{C.1.3}$$

Now consider when $m \geq m^*$. From (4.20), r^b is given by (C.1.3) with $k_E = k^*$. □

C.1.3 Proof of Lemma 4.3

Taking the first order condition of (4.24) with respect to m' gives

$$q_m = \beta\left\{q_m' + \lambda\left[\alpha\frac{\partial\Delta_E^e(m',a',b')}{\partial m'} + (1-\alpha)\frac{\partial\Delta_I^e(m',b')}{\partial m'}\right]\right\}, \tag{C.1.4}$$

where

$$\frac{\partial \Delta_I^e(m', b')}{\partial m'} = q_m'[f'(k_I) - 1], \tag{C.1.5}$$

and

$$\frac{\partial \Delta_E^e(m',a',b')}{\partial m'} = \begin{cases} \theta q_m'\big[f'(k_I) - 1\big] & \text{if} \quad k_E \geq k^*, \\ q_m'\big[\frac{(1-\chi)f'(k_E)[1+\theta(f'(k_I)-1)]}{(1-\theta)-(\chi-\theta)f'(k_E)} - 1\big] & \text{if} \quad k_E < k^*. \end{cases} \tag{C.1.6}$$

Combining (C.1.5) and (C.1.6) gives $\mathcal{L}_m$. The first order condition of (4.24) with respect to a' is

$$q_a = \beta\bigg\{q_a' + \vartheta'(a') + \lambda\alpha\frac{\partial \Delta_E^e(m', a', b')}{\partial a'}\bigg\}, \tag{C.1.7}$$

where

$$\frac{\partial \Delta_E^e(m',a',b')}{\partial a'} = \begin{cases} 0 & \text{if} \quad k_E \geq k^*, \\ \rho q_a'\big[\frac{(1-\chi)f'(k_E)}{(1-\theta)+(\theta-\chi)f'(k_E)}\big] & \text{if} \quad k_E < k^*. \end{cases} \tag{C.1.8}$$

Substituting (C.1.8) into (C.1.7) and rearranging gives $\mathcal{L}_a$. □

C.1.4 Proof of Proposition 4.1

The portfolio choice of money and housing can be written as

$$\max_{k_I,a'}\Big\{-iq_m m' - [1/\beta - 1]q_a a' + \vartheta(a') + \lambda\big[\alpha\Delta_c^e(k_I, a') + (1-\alpha)\Delta_m^e(k_I)\big]\Big\}, \tag{C.1.9}$$

where $i \equiv \gamma/_{\beta} - 1$. If $i = 0$, then $k_I = k_E = k^*$ so that $\partial \Delta_E^e(m', a', b')/\partial k_I = \partial \Delta_I^e(m', b')/\partial k_I = 0$. The first order condition of (C.1.9) with respect to $q_m m$ gives

$$i = \lambda\big[1 - \alpha(1-\theta)\big]\big[f'(k_I) - 1\big], \tag{C.1.10}$$

for $0 < i \leq i^*$. It follows that $k_I < k^*$ to satisfy (C.1.10) and that $\partial k_I/\partial i < 0$. If $i > i^*$, k_I is determined by

$$i = \lambda\alpha\left[\frac{(1-\chi)f'(k_E)[1+\theta(f'(k_I)-1)]}{(1-\theta)-(\chi-\theta)f'(k_E)} - 1\right] + \lambda(1-\alpha)[f'(k_I) - 1]. \tag{C.1.11}$$

If i increases, (C.1.11) is satisfied if and only if the right hand side increases. Since $f'(k_I)$ is decreasing in k_I, the first and the second term on the right hand side are decreasing in k_I. Thus, $\frac{\partial m}{\partial i} < 0$. We use this result to show that $\frac{\partial k_E}{\partial i} < 0$ and $k_E < k^*$ for $m < m^*$. Differentiation of the entrepreneur's liquidity constraint gives

$$\frac{\partial k_E}{\partial i} = \frac{\big[\theta f'(k_I) + (1-\theta)\big]\frac{\partial k_I}{\partial i}}{(\theta-\chi)f'(k_E) + (1-\theta)} < 0, \tag{C.1.12}$$

as $\chi f'(k_E) < 1$. It follows that $k_E < k^*$ for $i > i^*$.

Next, we show that $\frac{\partial a}{\partial i} \geq 0$. If $m \geq m^*$, then housing is priced at it's fundamental value and thus $\frac{\partial a}{\partial i} = 0$. In the case where $m < m^*$, the price of housing is given by

$$q_a = \frac{\beta\vartheta'(a)}{1 - \beta\left\{1 - \lambda\alpha\rho\left[\frac{(1-\chi)f'(k_E)}{(1-\theta)-(\chi-\theta)f'(k_E)} - 1\right]\right\}}. \tag{C.1.13}$$

If i increases, k_E will decrease. From (C.1.13), q_a will increase following a decline in k_E and therefore $\frac{\partial a}{\partial i} > 0$ when $m < m^*$.

To determine $\partial i^*/\partial \chi > 0$ and $\partial i^*/\partial \rho > 0$, we rewrite (4.22) and substitute $k_E = k^*$ (since

$i = i^*$) to get:

$$\theta f(\bar{k}) + (1-\theta)\bar{k} = (\theta - \chi) f(k^*) + (1-\theta)k^* - \rho q_a a, \tag{C.1.14}$$

where $\bar{k}$ is the amount of internally financed capital to acquire k^* through a bank loan at $i = i^*$. Suppose that ρ or χ increases. In order for (C.1.14) to hold with equality and maintain k^*, $\bar{k}$ must decrease. Now consider (C.1.10). Rearranging and setting $i = i^*$ gives

$$f'(\bar{k}) = \frac{i^*}{\lambda[1 - \alpha(1-\theta)]}. \tag{C.1.15}$$

It follows that i^* increases if $\bar{k}$ decreases, $\partial i^*/\partial \chi > 0$, and $\partial i^*/\partial \rho > 0$. □

C.1.5 Proof of Proposition 4.2

A second-order approximation of $f(k_I) - k_I$ around k^* is given by:

$$f(k_I) - k_I \approx f(k^*) - k^* + \frac{f''(k^*)}{2}(k_I - k^*)^2. \tag{C.1.16}$$

Recall that $\Delta_I^e(m, b) = f(k_I) - k_I$. Substituting (C.1.16) into (4.23) gives:

$$r^b \approx \frac{\theta[f''(k^*)(k_I - k^*)]}{2}. \tag{C.1.17}$$

Next, a first-order approximation of $f'(k_I)$ around k^* is given by:

$$f'(k_I) \approx 1 + f''(k^*)(k_I - k^*). \tag{C.1.18}$$

Substituting $f''(k^*)(k_I - k^*) \approx f'(k_I) - 1$ into (C.1.10) gives:

$$f''(k^*)(k_I - k^*) \approx \frac{i}{\lambda[1 - \alpha(1-\theta)]}. \tag{C.1.19}$$

Substituting (C.1.19) into (C.1.17) gives (4.30).

Last but not least, to determine comparative statics, the strength of the pass-through is given by

$$\frac{\partial r^b}{\partial i} = \frac{\theta}{2\lambda[1 - \alpha(1-\theta)]}, \tag{C.1.20}$$

which is decreasing (increasing) in λ (α) and independent of χ and ρ. Rearranging (C.1.20) gives

$$\frac{\partial r^b}{\partial i} = \frac{1}{2\lambda\Big(\frac{1}{\theta}(1-\alpha) + \alpha\Big)}, \tag{C.1.21}$$

which is increasing in θ. $\square$

C.1.6 Proof of Proposition 4.3

For $i \leq i^*$, the economy is in an unconstrained equilibrium (region A in Figure 4.5) and we refer to the derivations in Appendix **??**. From equations (C.2.2) and (4.32):

$$\frac{\partial k_I}{\partial i} = \frac{1}{\lambda f''(k^*)(1 - \alpha(1-\theta))} < 0, \tag{C.1.22}$$

$$\frac{\partial L}{\partial i} = -\frac{\alpha}{f''(k^*)(1 - \alpha(1-\theta))} > 0, \tag{C.1.23}$$

and $\partial k_E/\partial i = 0$ as $k_E = k^*$. For $i > i^*$, the economy is in a constrained equilibrium. From equations (4.33)-(4.34),

$$\frac{\partial k_I}{\partial i} = -\frac{1-\mathcal{O}}{D} < 0 \tag{C.1.24}$$

$$\frac{\partial k_E}{\partial i} = -\frac{1}{D} < 0 \tag{C.1.25}$$

$$\frac{\partial L}{\partial i} = -\lambda\alpha\frac{\mathcal{O}}{D} \gtrless 0, \tag{C.1.26}$$

as $D > 0$ and $1 - \mathcal{O} > 0$. We can see that $\partial k_E/\partial i < \partial k_I/\partial i < 0$ and $\partial L/\partial i < 0$ if $\mathcal{O} > 0$. From (4.35), $\mathcal{O} > 0$ if χ is large relative to ρ, corresponding to region C in Figure 4.5. If $\mathcal{O} < 0$, then $\partial k_I/\partial i < \partial k_E/\partial i < 0$ and $\partial L/\partial i > 0$. From (4.35), $\mathcal{O} < 0$ is ρ is large relative to χ (region B in Figure 4.5).

Consider $\partial|\partial k_E/\partial i|/\partial\rho$. From equation (C.2.9),

$$\frac{\partial k_E}{\partial i} = -\frac{1}{D} < 0, \tag{C.1.27}$$

as $D > 0$. In order to show that $|\partial k_E/\partial i|$ is decreasing in ρ, we first establish that $\frac{\partial D}{\partial \rho} > 0$. From equation (C.2.10),

$$\begin{aligned} \tfrac{\partial D}{\partial \rho} &= -\lambda(1-\alpha)(1-\chi)f'''(\bar{k})\tfrac{\partial \bar{k}}{\partial \rho} + \lambda(1-\alpha)f'''(\bar{k})\tfrac{\partial \bar{k}}{\partial \rho}\left(\tfrac{\beta\rho}{1-\beta}\right)^2\left[\tfrac{\alpha\lambda a\vartheta'(a)f''(k^*)}{1-\chi}\right] \\ &+ 2\lambda(1-\alpha)f''(\bar{k})\left(\tfrac{\beta\rho}{1-\beta}\right)\left(\tfrac{\beta}{1-\beta}\right)\left[\tfrac{\alpha\lambda a\vartheta'(a)f''(k^*)}{1-\chi}\right] > 0, \end{aligned} \tag{C.1.28}$$

as $f'''(\bar{k}) > 0$, $f''(k^*) < 0$ and $\partial\bar{k}/\partial\rho = -\frac{\beta a\vartheta'(a)}{1-\beta} < 0$. Thus, $\partial|\partial k_E/\partial i|/\partial\rho < 0$. $\square$

C.2 Data and Additional Derivations

C.2.1 Data Sources and Construction

In the calibration of Section 4.7, our measure for the nominal interest (bank lending) rate was the 3-month T-bill secondary market (bank prime) rate. Our measure of α, the probability to receive a bank loan, follows from Rocheteau et al. (2018) who found that between $78-90\%$ of respondents in the 2003 Survey of Small Business Finances had their most recent loan application approved. To calculate the pledgeability of output, we first calculated the average of (i) total loans to non-financial non-corporate businesses and (ii) total loans to non-financial non-corporate businesses net total home equity loans. We then divide the amount of loans to non-financial non-corporate businesses by total assets among non-financial non-corporate businesses. Both the data on loans and assets among non-financial non-corporate businesses come from the Flow of Funds Accounts, where we use quantities from Q4 in each year.

Our calculation of the pledgeability of housing, ρ, and maximum amount of unsecured credit, $\bar{b}$, draws on data from (i) the Federal Reserve Bank of New York's Quarterly Report on Household Debt and Credit and (ii) Average Sales Price of Houses Sold for the United States from the Federal Reserve Bank of St. Louis' FRED data base (series ASPUS). To calculate ρ, we first compute the average Home Equity Revolving Limit by dividing the total amount of Home Equity Revolving Limit by the number of Home Equity accounts between 2003:Q1-2016:Q4 using the FRBNY data. We then compute the ratio of the average home equity limit to the average sale price of U.S. homes. To calculate $\bar{b}$, we compute the average credit limit by dividing the aggregate credit card limit by the number of credit card accounts in the FRBNY data between 2003:Q1-2016:Q4. We then compute the average ratio of credit limits to home equity loan limits and choose $\bar{b}$ so the ratio of the maximum amount of unsecured credit relative to the maximum home equity loan in the model corresponds to the same ratio in the data.

Last but not least, we estimated the probability of receiving an investment opportunity, λ, by using data from the SCF between 2001-2016. Specifically, we calculated the fraction of respondents who started or acquired their business within the last year.[1]

[1] Our calculation uses sample weights provided by the SCF.

C.2.2 Derivation of Equations (4.31)-(4.35)

Consider an unconstrained equilibrium. Solving for $k_I - k^*$ from (C.1.17) gives:

$$k_I - k^* \approx \frac{2r^b}{\theta f''(k^*)}. \tag{C.2.1}$$

Substituting (4.30) into (C.2.1) gives

$$k_I \approx k^* + \frac{i}{\lambda f''(k^*)[1-\alpha(1-\theta)]}. \tag{C.2.2}$$

Plugging k_I from (C.2.2) and $k_E \approx k^*$ into $K \equiv \lambda[(1-\alpha)k_I + \alpha k_E]$ and $L \equiv \lambda\alpha(k_E - k_I)$ gives (4.31) and (4.32).

Now consider a constrained equilibrium with $\theta = 0$. The triple (k_E, k_I, q_a) is determined by equations (4.22), (C.1.11), and (C.1.13):

$$k_E = \chi f(k_E) + \rho q_a a + k_I, \tag{C.2.3}$$

$$i = \lambda\left[\alpha\frac{f'(k_E)-1}{1-\chi f'(k_E)} + (1-\alpha)\Big[f'(k_I) - 1\Big]\right], \tag{C.2.4}$$

$$q_a = \frac{(1-\chi f'(k_E))\beta\vartheta'(a)}{(1-\beta)(1-\chi f'(k_E)) - \beta\alpha\lambda\rho(f'(k_E)-1)}, \tag{C.2.5}$$

Substituting (C.2.5) into (C.2.3) gives:

$$k_E = \chi f(k_E) + \frac{\rho a(1-\chi f'(k_E))\beta\vartheta'(a)}{(1-\beta)(1-\chi f'(k_E)) - \beta\alpha\lambda\rho(f'(k_E)-1)} + k_I. \tag{C.2.6}$$

A first order approximation of (C.2.6) and (C.2.4) in the neighborhood $(k_I, k_E) = (\bar{k}, k^*)$ gives:

$$\left(1 - \chi - \left(\frac{\beta\rho}{1-\beta}\right)^2\left[\frac{\alpha\lambda a\vartheta'(a)f''(k^*)}{1-\chi}\right]\right)(k_E - k^*) \approx (k_I - \bar{k}), \tag{C.2.7}$$

$$\lambda(1-\alpha)f''(\bar{k})(k_I-\bar{k})+\lambda\alpha\frac{f''(k^*)}{1-\chi}(k_E-k^*)\approx i-i^*. \tag{C.2.8}$$

Solving (C.2.7) and (C.2.8) for $(k_I-\bar{k})$ and (k_E-k^*) gives:

$$\begin{pmatrix} k_I-\bar{k} \\ \\ k_E-k^* \end{pmatrix} = \frac{1}{D}\begin{pmatrix} \frac{\lambda\alpha f''(k^*)}{1-\chi} & -(1-\chi)+\left(\frac{\beta\rho}{1-\beta}\right)^2\left[\frac{\alpha\lambda a\vartheta'(a)f''(k^*)}{1-\chi}\right] \\ \\ -\lambda(1-\alpha)f''(\bar{k}) & -1 \end{pmatrix}\begin{pmatrix} 0 \\ \\ i-i^* \end{pmatrix}, \tag{C.2.9}$$

where

$$D=-\frac{\lambda\alpha f''(k^*)}{1-\chi}-\lambda(1-\alpha)f''(\bar{k})\left((1-\chi)-\left(\frac{\beta\rho}{1-\beta}\right)^2\left[\frac{\alpha\lambda a\vartheta'(a)f''(k^*)}{1-\chi}\right]\right)>0, \tag{C.2.10}$$

gives k_I and k_E. Finally, using $L\equiv\lambda\alpha(k_E-k_I)$ and substituting k_I and k_E gives (4.34).