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Synthetic river valleys: creating prescribed topography for form-process inquiry and river rehabilitation design

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Abstract

The synthesis of artificial landforms is complementary to geomorphic analysis because it affords a reflection on both the characteristics and intrinsic formative processes of real world conditions. Moreover, the applied terminus of geomorphic theory is commonly manifested in the engineering and rehabilitation of riverine landforms where the goal is to create specific processes associated with specific morphology. To date, the synthesis of river topography has been explored outside of geomorphology through artistic renderings, computer science applications, and river rehabilitation design; while within geomorphology it has been explored using morphodynamic modeling, such as one-dimensional simulation of river reach profiles, two-dimensional simulation of river networks, and three-dimensional simulation of subreach scale river morphology. To date, no approach allows geomorphologists, engineers, or river rehabilitation practitioners to create landforms of prescribed conditions. In this paper a method for creating topography of synthetic river valleys is introduced that utilizes a theoretical framework that draws from fluvial geomorphology, computer science, and geometric modeling. Such a method would be valuable to geomorphologists in understanding form-process linkages as well as to engineers and river rehabilitation practitioners in developing design surfaces that can be rapidly iterated. The method introduced herein relies on the discretization of river valley topography into geometric elements associated with overlapping and orthogonal two-dimensional planes such as the planform, profile, and cross section that are represented by mathematical functions, termed geometric element equations. Topographic surfaces can be parameterized independently or dependently using a geomorphic covariance structure between the spatial series of geometric element equations. To illustrate the approach and overall model flexibility examples are provided that are associated with mountain, lowland, and hybrid synthetic river valleys. To conclude, recommended advances such as multithread channels are discussed along with potential applications.

Keywords: river topography; digital elevation modeling; channel form; river restoration; synthetic river topography

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1. Introduction

Geomorphologists fundamentally explore land-surface patterns on Earth and beyond through the analysis of natural landscape formation and evolution as well as anthropogenic activities. As important as it is to study what exists in the world, it is equally significant to create and study synthetic data unbounded by the existing set of known natural landforms and anthropogenic alterations. The synthesis (i.e., artificial physical or virtual reproduction) of Earth properties and landforms is a valuable component of scientific research because it enables inquiry into why landforms have specific shapes and characteristics (Rodriquez-Iturbe and Rinaldo, 1997; Perron et al., 2009), how they evolve over time , what consequences arise from anthropogenic intervention, and how life and human populations relate to natural and human-altered landforms. Furthermore, it can also be used to contextualize known relationships of synthetic landforms to real world cases for analysis (Perron et al., 2008). The creation of synthetic data is valuable to Earth sciences specifically because it allows testing of conditions that may not be accessible in nature such that underlying causalities can be explored (Richards, 1978). Synthetic analysis is not a replacement for engaging the real world but a natural outgrowth of play-driven scientific curiosity (Wolman, 1995) in the digital age that can catalyze bigger conceptual leaps. The purpose of this paper is to introduce a new method for creating prescribed topography of river valleys using a geometric modeling framework that can be easily adjusted to create landforms of varying complexity.

2. Background

Landscape evolution modeling— from real world or artificial initial conditions— is a common method for the synthesis of landforms. While promising in replicating general landform characteristics at basin and larger scales (Perron et al., 2009), landscape evolution models (LEMs) are not appropriate for river reaches (e.g., 10^1–10^3 channel widths) and fluvial morphological units (e.g., 10^0–10^1 channel widths). In the case of deterministic river-corridor LEMs, the common approach is to use *reduced complexity* (e.g., cellular automata) models that either exclude or greatly simplify momentum conservation to obtain a basic representation of channel nonuniformity at morphological-unit to reach scales (Murray and Paola, 1994; Coulthard and Van De Weil, 2006; Coulthard et al., 2007). Unfortunately, none of these LEM schemes account for first-order controlling processes founded on landform nonuniformity (Hancock and Anderson, 1998; MacWilliams et al., 2006; Thompson, 2006), without which no accurate topographic representation can be reproduced in many fluvial settings. Therefore, as important as they are as a general learning tool at the landscape scale, these models are not well suited for exploring the full range of likely (and conversely impossible yet interesting to consider) morphologies of typical nonuniform river valleys at the river channel scale.

The synthesis of river topography has been approached implicitly and explicitly from a variety of methods focusing on specific fluvial attributes at a variety of spatial scales ranging from particle clusters to river reaches. At the reach scale, one-dimensional (1D) aspects of river geometry, such as the longitudinal profile, have been modeled using fractals (Robert, 1991), stochastic time series methods (Richards, 1976;

Knighton, 1983), diffusion modeling (Begin, 1988), and hybrid combinations of analytical and empirical relationships (Naden, 1987; Cao et al., 2003). In addition, two-dimensional (2D) simulations of bed particle arrangements based on probabilistic and rule-based techniques are only appropriate in one specific landscape setting to date (Malmaeus and Hassan, 2002). The explicit treatment of morphodynamically derived models of river topography have shown considerable promise (Seminara, 2006; Luchi et al., 2010), but are still in their infancy when it comes to simulating large river reaches with multiple scales of material heterogeneity and the ability to resolve dynamic features at the channel width to subwidth scale. Computationally, these types of models are very consumptive and the resulting topographies are only as dynamic as provided in known equations. The inherent crux in the above approaches is that a 1D description is incomplete for many rivers, 2D treatments are limited in geographic scope, and three-dimensional (3D) treatments are limited in their present simplicity, computational efficiency, and spatial domain.

In the absence of model-derived synthetic landscapes for the sake of scientific inquiry, river engineers create design topographic surfaces from a blend of empiricism and heuristics. For example, in the Spawning Habitat Integrated Rehabilitation Approach, landforms are determined by establishing the key physical processes for the site and then using a blend of empirical functions and heuristics to draw the contours of complex fluvial features in CAD (Wheaton et al., 2004a; Elkins et al., 2007; Pasternack and Brown, 2013). Regardless of the underlying theoretical foundations, concepts are developed in 2D in the form of planform alignments, cross sections, and vertical thalweg profiles; then a computer-assisted drafting (CAD) program is used to link the three planes into a topographic design surface. This has been problematic because the complexity of fluvial form is often reduced during communication from geomorphologist to engineer via typical sections and standard details. Moreover, CAD programs do not have a spatially explicit method for the iterative manipulation of topographic surfaces, which is often needed to optimize river rehabilitation designs for competing uses (Wheaton et al., 2004b).

Contrasting the above approaches, landscape and river synthesis has also been approached from a computer science perspective with an entirely different goal. In this framework the goal is to generate realistic looking landforms with little or no processes considered and a modicum of computer-aided artistry (Fournier et al, 1982; Musgravet et al., 1989; Prusinkiewicz and Hammel, 1993; Zhou et al., 2007). These techniques were initially born out of the idea that fractals could be used as representative models of landscapes (Fournier et al., 1982). They have the advantage that all scales of variance are generated. Since then, numerous algorithms have been developed to create entire landscapes and more recently river systems (Prusinkiewicz and Hammel, 1993). However, as these approaches seek visual realism as the goal of synthesis, they have no utility in yielding landforms that would naturally result from processes associated with landform characteristics. They also are not helpful in adjusting a landform after it has been created or creating a prescribed topographic configuration.

There is tremendous scientific and applied benefit to having methods capable of creating river valleys of prescribed (e.g., explicitly defined) conditions without arriving at them through dynamic process-based modeling in that exploratory testing can allow testing into why specific landform configurations do and do not exist. Specifically, all of the approaches described above that could generate a channel or river corridor would rely on the outcome of the underlying rules or governing equations, but cannot create a river of prescribed conditions. It may seem counterintuitive at first to want to study arbitrary landforms that naturally never form, but in a world heavily altered by humans this is exactly what society is confronted with. Thus, it is equally important to be able to understand why any arbitrary fluvial landscape is nonfunctional as a process-based landscape is functional. Lastly, river restoration practice would benefit from the ability to rapidly create design topographic surfaces that embrace fluvial geomorphic theory and can be easily adjusted for multiple design iterations.

3. Objectives

The specific objectives of this study were to (i) develop a geometric modeling framework for creating prescribed synthetic river valley topography that can be readily adjusted, (ii) illustrate the synthesis method with examples, (iii) offer suggestions on extending this approach, and (iv) discuss potential applications for form–process inquiry and river engineering. The significance of this study is that river scientists and professional practitioners may now use this approach as an alternative means of synthesizing numerous, diverse project design scenarios for numerical experimentation to advance theory (e.g., Cao et al., 2003; Pasternack et al., 2008) and river rehabilitation (e.g., Pasternack and Brown, 2013) as an alternative to problematic empirical methods of channel design (e.g., Simon et al., 2007; Pasternack, 2013). The point of this study was to introduce the method with the minimum required elements and illustrate what can be achieved with additional complexity, not enumerate all possible capabilities. While only single thread channels were explored in this paper, the approach is general enough to accommodate multiple threads (see section 6.3).

This approach draws on aspects of the streamline-based modeling of river channels presented by Merwade et al. (2005), Prycz et al. (2009), and Legleiter (2014) as well as geostatistical modeling of river channels by Legleiter and Kyriakidis (2008), but is thought to be more general and flexible by providing a theoretical framework for general artificial synthesis of entire river valleys. An additional novel contribution is the use of geomorphic covariance structures (explained in section 4.5.1) in dependent parameterization of specific geometric elements. The approach herein also incorporates floodplain topography. Once the model is established attributes of the synthetic river valley can be adjusted for design or scientific exploration depending on the exact fluvial geomorphic elements of interest and the mathematical functions used to represent them. The novelty in this approach is that it draws from computer science, geometric modeling, and fluvial geomorphology to create topographic surfaces that— with parametric equations— can be used to efficiently evaluate relationships between river form and subsequent processes

4. Synthesis framework

Geometric modeling is implemented in a variety of ways, but a common approach is to mathematically represent an object in overlapping and orthogonal 2D planes to determine the 3D geometry of the modeled form— either through explicit or implicit mathematical equations (Mortenson, 1997; Tao Ju et al., 2005). With that in mind, consider that a river valley in Cartesian space can be decomposed into— at a minimum— a primary channel and a floodplain that could be enclosed within a valley (Fig. 1; Knighton, 1998); and each of these elements can be viewed in 2D space in the planform, profile, or cross section views. Through this decomposition, river valley topography can be created by modeling specific fluvial geometric elements associated with each 2D plane via the development of mathematical functions that control the shape of each element and link elements to adjacent planes. Geometric element equations describe the shape of each fluvial element, treated herein as finite approximations. Each Cartesian plane has a minimum number of geometric elements needed to create a simple synthetic river valley. For example, the thalweg elevation, top of bank, and valley height are all fluvial geometric elements in the profile (xz) plane. Similarly, the planform alignment, channel banks, and valley width limits are all fluvial geometric elements in the planform (xy) plane. The channel cross section lies in the cross section (yz) plane. River channel geometry is expressed in a channel-referenced coordinate system (sn), and this is discussed in section 4.2 below. Within each fluvial geometric element equation, control functions can also be used to describe subreach scale variance of a particular fluvial element. Control functions can be deterministic equations, stochastically generated spatial series, or a blend. Herein the focus is on deterministic functions that allow parametric manipulation because when parametric mathematical equations are used the coefficients can adjust each geometric channel attribute— via 2D attribute linkage— to manipulate the topographic surface. To provide real-world context for prototype simulation, some of these control functions associated with fluvial geometric elements are scaled by the reach-average properties of the river valley, such as the bankfull width and depth, which could be selected independently or calculated using existing analytical or empirical relationships. Once programmed, an xyz output of a synthetic river valley can be derived from a small set of parameters, such as values for the reach-average bankfull width, depth, slope, median sediment size, critical Shields stress, valley heights, and valley widths—although even simpler models can be developed. The final xyz output can then be used to construct topographic surfaces through interpolation that also can be adjusted via independent and dependent parameter manipulation.

The basic steps in developing a geometric model of a synthetic river valley are (i) conceptualize, (ii) specify model domain, (iii) determine 2D fluvial geometric elements in the model, (iv) determine reach-average values of geometric elements, (v) develop geometric element equations, (vi) construct model, and (vii) parameterize. After summarizing the steps in this paragraph, details for each are explained in sections 4.1–4.5. The first step is a conceptualization of the desired fluvial elements to be created, (including a clear description of the scale, type, and resolution of morphologic features), how the landform is represented in 2D planes, and the extent and resolution of modeling. Next, the model domain, resolution, horizontal coordinate system, horizontal datum, and vertical datum need to be selected. Following this, the average properties of the river reach need to be selected or determined (e.g., width, median particle size, and depth), since they are used to scale specific fluvial geometric elements to prototypical conditions. Scaling is defined in this context as the multiplicative adjustment of a control function by reach-average properties such as bankfull width and depth, where control functions are a subset of specific geometric element equations. Once the basic attributes of the reach are defined, the fluvial geometric elements and their equations for the synthetic river valley need to be determined for each of the three planes. After this, control functions are selected to represent the subreach scale variance of each fluvial geometric element of the river valley, some of which are scaled by specific reach-average river properties. Then, the model needs to be programmed to produce an xyz output, but this aspect is not explored in this article because model programming is outside the present scope. Finally, after model construction the synthetic river valley can be adjusted, through dependent and independent parameterization, to create varying landform conditions.

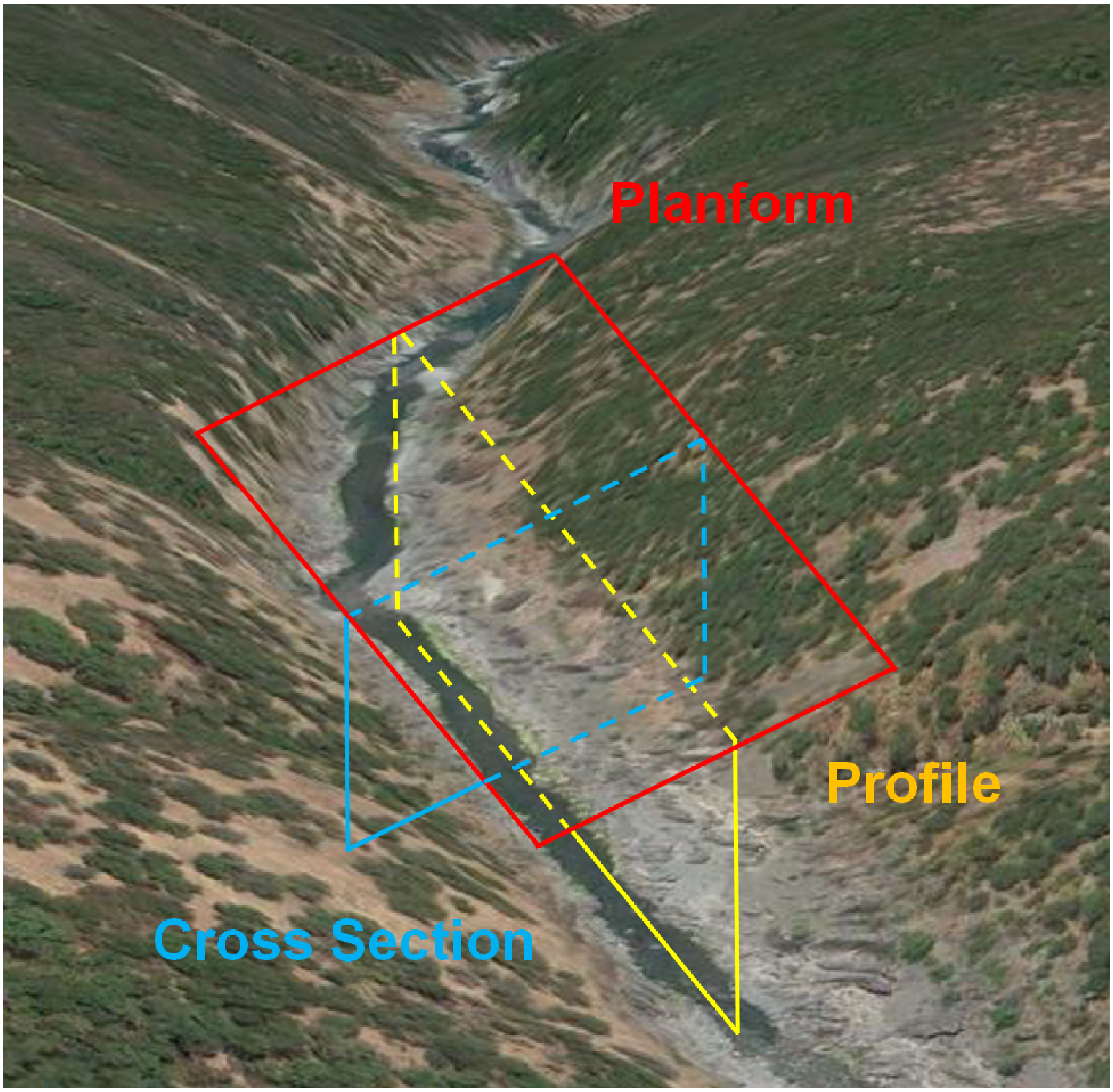

Fig. 1. The major components of channel form represented as different planes after Knighton (1998) overlaid on an oblique section of the Eel River in California. Image from Google Earth.

4.1 Conceptualize

As a first step in creating a synthetic river valley the landform of interest should be conceptualized by identifying (i) the purpose of modeling, (ii) the primary morphological features of the river valley, and (iii) the scale and resolution of these attributes in each 2D plane. Conceptualization provides the broad template upon which model components and their characteristics are based. Purposes of modeling could be to understand how specific channel and floodplain configurations affect ecological and geomorphic processes (Pasternack et al., 2008), to create prototypes of channel configurations for historical analysis (Jacobson and Galat, 2006), to develop river and stream rehabilitation scenarios (Elkins et al., 2007), or to evaluate land management impacts and engineering scenarios (Pasternack and Brown, 2013). Each of these applications could have varying model resolutions, spatial domains, number and type of geometric elements, and subsequent control functions. As part of conceptualization, several modeling aspects should be explicitly considered, such as river valley length, number of channels, scale-dependent variability of geometric elements, river morphology, and the model's geometric elements— including their resolution. Scale is important because geomorphologists are now learning more than ever that processes and landform variability are scale dependent. For example, river profiles show varying characteristics depending on whether a single bedform, morphological unit, or entire river system is being considered. The number of channels also has a strong bearing on model construction because it drives the underlying placement of control functions for river planform and cross section models. River morphology is another important consideration that will affect the number and type of geometric elements and control functions. For example, a reach-scale model of a gravel-bed river could have frequent riffle–pool units necessitating a control function that allows vertical bed undulations. In addition, depending on valley type there could be a floodplain with gradually sloped sides, confining terraces, or a narrow V-shaped bedrock wall (for more examples, see Grant and Swanson, 1995). All of these valley states can be incorporated into the geometric model. Lastly, model resolution refers to the planform, profile, and cross sectional point density that the control functions populate.

4.2 Specify model domain

Specifying the model domain includes defining— in no order— a vertical datum, units, coordinate systems for fluvial geometric elements, model bounds, and longitudinal and cross section node intervals. The model operates in space so all units are for distance. A vertical datum, Z_D, can be set arbitrarily or selected if the model is for a real river with a prespecified datum. For the horizontal plane, the model uses Cartesian as well as curvilinear, orthogonal coordinate systems. The curvilinear, orthogonal coordinate system is used initially for channel geometric elements such as the thalweg and channel banks (Smith and McLean, 1984) and is then transformed to Cartesian coordinates (Leigleiter and Kyriakidis, 2006). Toward describing the channel centerline in curvilinear coordinates, discrete nodes i with Cartesian coordinates (x_i, y_i) are found for each centerline node i ranging from 1 to m (Fig. 2A). Spacing between nodes in the X direction is $\Delta x_i = x_{i+1} - x_i$, which is equal to the total length of the model in the x direction, L_X, divided by $m - 1$, the total number of increments. Notice that $\Delta y_i = y_{i+1} - y_i$ varies from segment to segment as the path changes direction along the channel centerline (Fig. 2A).

With the Cartesian coordinates available for each centerline node, curvilinear, orthogonal coordinates, in the direction along the channel thalweg, s_i, and perpendicular (orthogonal) to it, n_j, are calculated (see Fig. 2B). For each forward increment of Δx_i and Δy_i, the change in the channel referenced coordinate, Δs_i, in the curvilinear, orthogonal system is first determined from the Pythagorean Theorem as $\Delta s_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$ (Fig. 3). It follows that because $\Delta s_i = s_{i+1} - s_i$, the cumulative distance in s (coordinate) is found from $s_{i+1} = s_i + \Delta s_i$. For most rivers the minimum longitudinal channel node spacing, Δs_i, should be on the order of approximately $\frac{1}{4}\overline{W_{BF}}$, because at least four points are needed to capture topographic highs and lows of undular bed relief (Rayburg and Neave, 2008). Orthogonal to the s direction is the n direction, which is also discretized (Figs. 2B and 3). Spatial increments $\Delta n_j = n_{j+1} - n_j$ are found by dividing half the total width of the cross section at each channel referenced node, $\frac{w_{BF}(s_i)}{2}$, by k for river left $(n > 0)$ and river right $(n < 0)$ sides of the channel centered node, where $w_{BF}(s_i)$ is the local bankfull channel width and k is the number of nodes (Fig. 2D). There are $k - 1$ increments for each side of the channel. Nodes n_j are indexed by j, where j ranges from 1 at the centerline to k at the river left side and at the river right side (Fig. 2D), and the cumulative distance is hence $n_{j+1} = n_j + \Delta n_j$. For the cross-sectional node spacing there should be increments, Δn_j, no greater than ~ $\frac{1}{5}\overline{W_{BF}}$, with the simplest, though unsatisfactory, representation being a three-point distribution that has two banks and a thalweg. Regardless of the coordinate system used, all geometric elements are described through finite difference approximations.

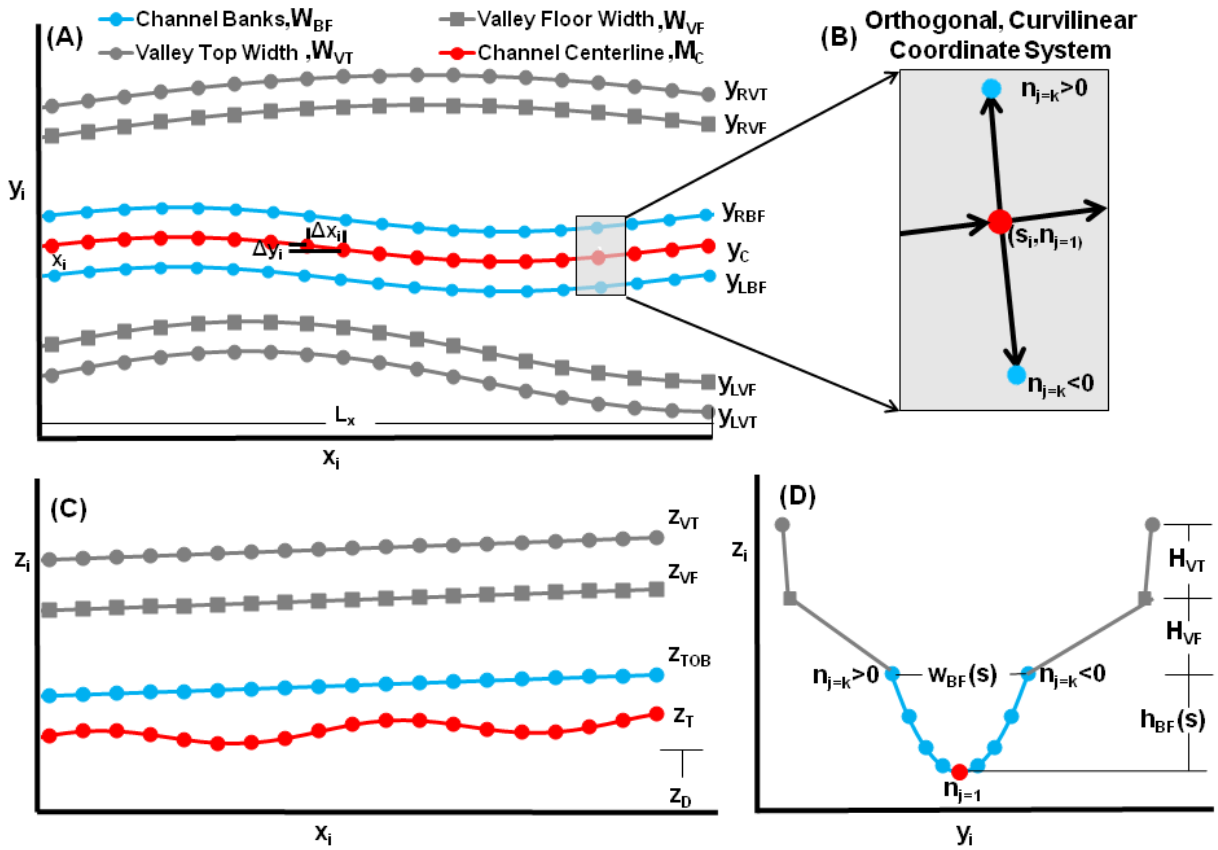

Fig. 2. Schematic of simple synthetic river valley geometric elements in (A) planform along with description of (s, n)coordinate system (inset), (B) profile, and (C) cross section.

Because there are two coordinate systems, a coordinate transformation from curvilinear, orthogonal coordinates to Cartesian coordinates is needed for nonchannel geometric elements such as the valley floodplain. In Fig. 3 curvilinear, orthogonal and Cartesian coordinates are given subscript i for the centerline direction and j for the orthogonal direction according to the node numbering scheme. Using similar triangles $\frac{\Delta y_i}{\Delta s_i} = -\frac{\Delta x_j}{n_{j+1}}$ and $\frac{\Delta x_i}{\Delta s_i} = \frac{\Delta y_j}{n_{j+1}}$ as shown in Fig. 3, the Cartesian coordinates of the nodes perpendicular to the center line may be expressed as a function of curvilinear coordinates as:

$$x_{i+1,j+1} = x_{i+1,j} - n_{j+1}\frac{\Delta y_i}{\Delta s_i} \quad (1)$$

$$y_{i+1,j+1} = y_{i+1,j} + n_{j+1}\frac{\Delta x_i}{\Delta s_i} \quad (2)$$

where $(x_{i+1,j=1}, y_{i+1,j=1})$ are the known Cartesian coordinates along the channel center having curvilinear, orthogonal coordinates $(s_{i+1}, n_{j=1} = 0)$ and $(x_{i+1,j+1}, y_{i+1,j+1})$ are the calculated Cartesian coordinates of the nodes perpendicular to the centerline having curvilinear coordinates (s_{i+1}, n_{j+1}) where Δy_i, Δs_i, and Δx_i are also known along the centerline (Fig. 3).

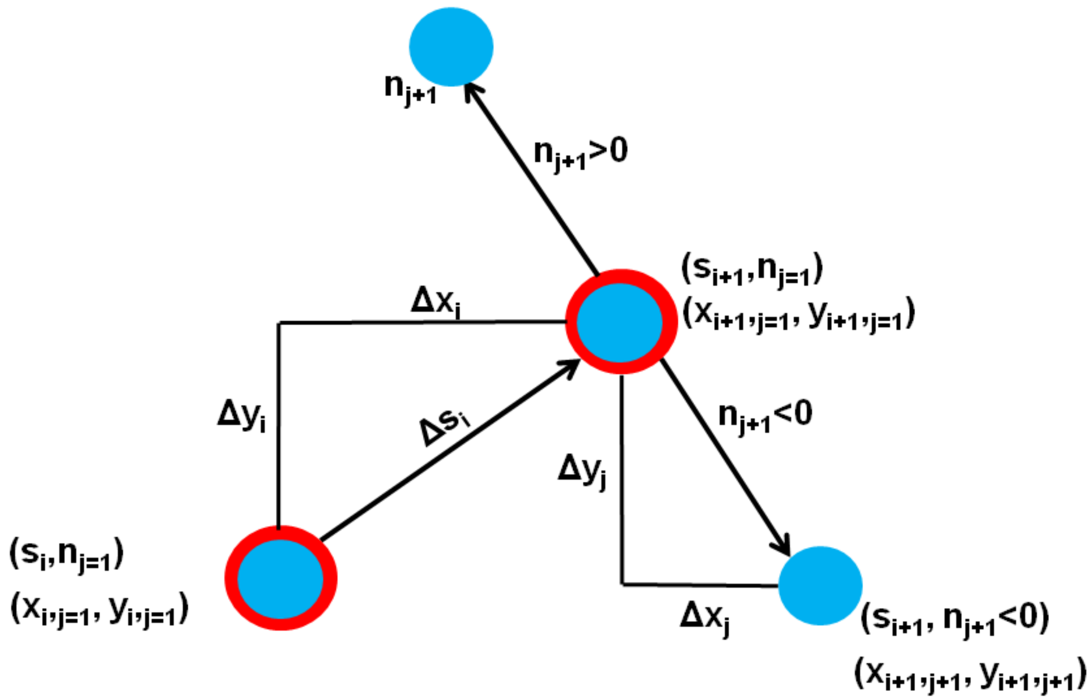

Fig. 3. Schematic and nomenclature for inverse coordinate transformation from (s,n) to (x,y).

4.3 Determine fluvial geometric elements

For this set of steps the fluvial geometric elements of the synthetic river valley need to be determined in each 2D plane so that in subsequent steps mathematical functions can be used for their spatial description in the model domain. As defined earlier, a geometric element is a 2D aspect of the synthetic river valley that will be locally modeled using a mathematical control function and is denoted here with a capital letter (e.g., bankfull width geometric element is denoted as W_{BF}). For the case of a simple synthetic river valley, a minimum set of fluvial geometric elements can be defined (Fig. 2) in planform (xy/sn), profile (xz/sz), and cross section planes (yz/nz). In planform the minimum recommended elements are W_{BF}, which defines the local bankfull channel extent, the channel centerline, M_C, left and right valley floor widths, W_{VF}, and valley top widths, W_{VT}. For the profile, the thalweg, Z_T, top of bank, Z_{TOB} floodplain floor elevation, Z_{VF}, and floodplain top elevation, Z_{VT}, are needed. Finally, the channel cross section is a key geometric element, linking planform and profile elements, and is bounded vertically by the thalweg trough and bank crest as well as horizontally by the bankfull width and the position of the onset of the floodplain.

4.4 Develop equations for fluvial geometric elements

In this step one must create geometric element equations that are representative of the scales of variability desired in each 2D plane for each geometric element. First, the type and scale(s) of variability in prototypical spatial series for each geometric element should be identified as an extension of the conceptualization step. For single-thread gravel-bed channels in a simple valley, Table 1 shows each of the primary fluvial geometric elements considered so far by 2D plane with scale-dependent attributes of natural spatial series (e.g., variables that change with distance) reported in the literature. These attributes are simplified from spatial series descriptions— specific landscapes will have exogenic controls that can override any of these descriptions at any point in space. In addition, each element has some level of inherent randomness found in nature in each of the spatial series and this aspect is omitted in Table 1. What is most important is that the creation of each geometric element equation is dependent on the type of feature and its relevant spatial scales to be modeled. For example, to simulate a longitudinal river profile of a single-thread riffle – pool channel at the reach scale would require functions that are at least quasi-oscillatory and linearly sloping. However, if a basin-scale synthetic river valley is simulated, then the longitudinal profile may also have an exponentially decreasing function to represent the larger scale shift in landscape position from mountain to lowland.

Once geometric elements are defined and the type(s) of spatial series attribute(s) are selected, the user can then begin to create geometric element equations that model the basic attributes but also allow manipulation through parameters. Each geometric element equation may have several basic components, including a fluvial scaling component, consisting of the reach-average value and a scaled control function for morphological-unit-scale variance, a reach and/or basin scale trend, a random component, coordinates of connected geometric elements, and spatial offsets. The control functions used can be scaled or unscaled, but the channel geometric elements such as W_{BF}, Z_T, and the cross section are recommended to be scaled. Each of these components can be linearly related to form a final geometric element equation as will be shown in section 4.4 below. A general expression for an arbitrary geometric element equation is not presented here because not all geometric elements necessarily require each of the above components, depending on the scale of the model and application. Next, the basis for the fluvial scaling component and the selection of control functions are discussed.

Fluvial geometric elements by 2D plane	Spatial series attributes		
	Morphologic scale (10^0-10^1 channel widths)	**Reach scale (10^1-10^2 channel widths)**	**Basin scale (>10^2 channel widths)**
Planform			
Planform alignment	Oscillatory (Langbein and Leopold , 1966)	Oscillatory (Ferguson, 1973, 1976)	Fractal (Nikora, 1991; Rodriquez-Iturbe and Rinado, 1997)
Bankfull width	Phased to bed elevation (Richards, 1976b;Wilkinson et al., 2004) and Centerline (Luchi et al., 2012)	Oscillatory (Richards, 1976b)	Increases downstsream as a power function (Leopold and Maddock, 1953)
Valley width	Constant	Quasi-oscillatory or constant (Grant and Swanson, 1995; Wohl et al., 1993; McDowell, 2001)	Fractal or multi-fractal (Beauvais and Montgomery, 1996; Gangodagamage et. al., 2007)
Profile elevation			
Thalweg	Quasi-periodic (Yang, 1971; Ferguson, 1973; Richards, 1976a; Knighton, 1983)	Linear sloping (Richards, 1976a; Ferguson, 1973; Knighton, 1983)	Concave (Langbein and Leopold, 1964; Langbein and Leopold, 1966; Tanner 1971; Sinha and Parker, 1996)
Top of bank	Follows Valley Floor		
Valley floor	Constant	Linear sloping or oscillatory (White et. al., 2010; Gangodagamage et al., 2007)	Concave (Gangodagamage et al., 2007)
Valley top	Constant	Constant	Concave; oscillatory (Montgomery, 1994)
Channel cross section	Asymetrical to symmetric; concave down (Knighton, 1982; Milne, 1983; Rayburg and Neave, 2008)	Phased to curvature of channel meander alignment (Knighton, 1982; Milne, 1983)	Decreasing confinement and width-to-depth ratio in downstream (Leopold and Maddock, 1953)

Table 1. Scale-dependent attributes of spatial series associated with common geometric elements

4.4.1 Fluvial scaling

While the fluvial geometric elements are governed by mathematical equations that describe their shape, some form of scaling using real world dimensions has to occur to relate the function to prototypical dimensions of the desired river valley, especially for channel elements. In this article, channel elements are all scaled by reach-average properties of the river valley, where the notation for all reach-average properties will be shown as capital letters with an over bar (e.g., the reach-average bankfull width is denoted as $\overline{W_{BF}}$). Valley elements are not scaled herein for simplicity, although this does not always have to be the case. The theoretical foundation for the fluvial scaling of a single-thread river valley is that the local bankfull value of a geometric element, $G_{BF}(s_i)$, may be expressed as the combination of the reach-average bankfull value, $\overline{G_{BF}}$, plus this value multiplied by a control function, $f(s_i)$. The control function models the morphological-unit variance at the scale of the channel through linear scaling by $\overline{G_{BF}}$. Thus, the fluvial scaling component of the general geometric element equation is:

$$G_{BF}(s_i) = \overline{G_{BF}} f(s_i) + \overline{G_{BF}} \quad (3)$$

Note that the geometric elements and control functions above are for orthogonal, curvilinear coordinates, but some geometric elements can be constructed in Cartesian coordinates as a function of x_i. The theoretical basis for Eq. (3) is rooted in the concept of quasi-equilibrium (Langbein and Leopold, 1964), whereby rivers are dynamic systems with a multitude of time and space varying conditions, but still maintain some modal condition (Huang et al., 2004; Huang and Nanson, 2007; Nanson and Huang, 2008).

Typical scaling variables for alluvial river morphology are reach-average slope ($\bar{S}$), bankfull width ($\overline{W_{BF}}$), median sediment size ($\overline{D_{50}}$), and either hydraulic radius ($\overline{R_{BF}}$) or bankfull depth ($\overline{H_{BF}}$) (Gioia and Bombardelli, 2001; Church, 2006; Parker et al., 2007; Eaton et al., 2010). For the channel geometric elements $\overline{W_{BF}}$, $\bar{S}$, and $\overline{D_{50}}$ are used as initial inputs governed by the landscape setting of interest for synthesis. Ultimately, these may be thought of as deriving from the balance of genetic controls, such as lithology, tectonics, anthropogenic impacts, landscape position, flow regime, and sediment supply. From these initial scaling values, $\overline{H_{BF}}$ is determined in this model based on determining the critical depth for incipient motion so that the geometry reflects a quasi-equilibrium state. This approach is used because many gravel- and sand-bed rivers adjust their bankfull depth such that the channel is in pseudo-equilibrium with the reach-average critical Shields stress, $\overline{\tau_c^*}$ for bed material entrainment (Lisle et al., 2000; Parker et al., 2007; Wilkerson and Parker, 2011). Thus, for a given alluvial river in quasi-equilibrium, the critical depth for incipient motion should be comparable to $\overline{H_{BF}}$. This allows a unique $\overline{H_{BF}}$ to be determined for each synthetic river valley using the Shields equation (or any user-preferred equation that related $\overline{H_{BF}}$ to input values) along with an estimate of $\bar{S}$, $\overline{D_{50}}$, and $\overline{\tau_c^*}$. Assuming $\overline{H_{BF}}$ is approximated by the hydraulic radius, the critical depth at incipient motion can be approximated by,

$$\overline{H_{BF}} \sim \overline{H_{critical}} = \frac{(\gamma_S - \gamma_W)\overline{D_{50}}\,\overline{\tau_c^*}}{\gamma_W \bar{S}} \quad (4)$$

where γ_s and γ_w are the specific weight of sediment and water, respectively. As this shows, geometric modeling can be mindful of geomorphology; it can strictly adhere to known empiricism or allow deviations from convention to explore wide-ranging possibilities.

For a single-thread river valley, the control functions governing the variable shapes of the channel geometric elements (e.g., local bankfull width, thalweg elevation, and cross-sectional form) are scaled using $\overline{W_{BF}}$ and $\overline{H_{BF}}$. For the bankfull width control function the obvious scaling variable is $\overline{W_{BF}}$. The thalweg elevation control function could be scaled by $\overline{D_{50}}$ or $\overline{H_{BF}}$, but the latter is derived from $\overline{W_{BF}}$ so it does not make sense to use it. Local cross-sectional form, however, is commonly scaled by local values of the bankfull width and depth, $w_{BF}(s_i)$ and $h_{BF}(s_i)$, as lateral and vertical scale factors, respectively (Deutch and Wang, 1996; James, 1996; Merwade, 2004; Jacobson and Galat, 2006).

4.4.2 Control functions

Recall that a scaled control function, which expresses the subreach scale variance of the river valley, may be nested within each geometric element equation. The type of control function(s) used for each geometric element will ultimately dictate the level of parametric manipulation possible in the final xyz representation of the synthetic river valley. As a guide, Table 2 shows example mathematical functions commonly used to model the Z_T, W_{BF}, and M_C geometric elements, which are the most commonly modeled attributes of river channels. The selection of control functions is driven initially by the conceptualization of the synthetic river valley where the type and scale(s) of variability in prototypical spatial series for each geometric element should be considered. While each geometric element has been modeled with a variety of approaches, the types of control functions can be broadly classified as deterministic, stochastic, or hybrid combinations. Deterministic functions are beneficial for creating river valleys of prescribed conditions because parameter manipulation of each control function can allow parsimonious manipulation of the 3D river valley topography. Stochastic functions differ in that they have coefficients that are empirically derived but are driven by pseudo-random algorithms built on predefined random seeds. For these types of functions control over the shape of a landform is limited, but multiple realizations can be generated and if the same seed is used, then different users can repeat the same outcome.

Of all the potential mathematical and functions available for this article the examples shown in the following section used sinusoids as control functions. First, sinusoids succinctly capture the essence of control function selection, scaling, and manipulation because the oscillations are controlled by parameters that adjust the frequency, amplitude, and phase of the waveform. Second, rivers are a continuum of topographic variations shaped by a quasi-harmonic interaction between water flow and material transport (Nelson, 1990; Furbish, 1998). Lastly, it is also possible to spectrally decompose specific river properties and inversely use specific frequencies in synthesis, similar to Clarke (1988).

The general sinusoidal model used in the examples below is:

$$y(x_i) = \sum_{p=1}^{q} (a_c \, cos(b_c x_r + \theta_c) + a_s \, sin(b_s x_r + \theta_s))_p \qquad (5)$$

where y_i is the dependent control function value, a_c, b_c, and θ_c are the amplitude, angular frequency, and phase for the cosine component and a_s, b_s, and θ_s are the amplitude, angular frequency, and phase for the sinusoidal component, p denotes the order and ranges from 1 to q, and x_r is the Cartesian stationing in radians. The Cartesian stationing in radians, x_r, is scaled to L_x by the relationship $x_r = 2\pi \frac{x_i}{L_x}$ and similarly for s_i. Thus, amplitude, phase, and frequency are contextualized by reach length.

Geometric element	Mathematical function/ model type	Scale	Sources
Thalweg	Exponential	B	Tanner, 1971; Yang, 1971; Snow and Slingerland, 1987
	Power	B	Yang, 1971; Snow and Slingerland, 1987
	Logarithmic	B	Yang, 1971; Snow and Slingerland, 1987
	Hybrid	B	Schumm, 1960; Langbein and Leopold, 1964; Ohmori, 1991
	2nd order, autoregressive	R, M	Knighton, 1983; Richards, 1976a
	Variogram	R, M	Robert and Richards, 1988
	Regression	R, M	Anderson et al., 2005
	Linear trend	R, M	Leopold et al., 1964; Knighton, 1998
	Variogram	R	Legleiter and Kyriakidis, 2006; Legleiter, 2014
Cross section	Polynomial	NA	James, 1996
	Statistical distribution	NA	Merwade, 2004; Jacobson and Galat, 2006
	Curvature based asymetry	NA	Deutch and Wang, 1996
	Analytical	NA	Bridge, 1977; Beck, 1988
	Rectangular	NA	Chow, 1959
	Semi-circle	NA	Chow, 1959
	Triangular	NA	Chow, 1959
	Trapezoid	NA	Chow, 1959
Channel alignment	2nd order, autoregressive	M,R,B	Ferguson, 1976
	Analytical	M,R,B	Kinoshita, 1961
	Sinusoid	M,R,B	Langbein and Leopold, 1966

Table 2. Mathematical functions used to model the meander planform alignment, thalweg profile, and cross section geometric elements drawn from the literature. M refers to morphologic unit scale (100-101 channel widths), R refers to reach scale (101-102 channel widths), and B refers to basin scale (>101 channel widths)

4.4.3 Planform geometric elements

In the planform dimension, this study evaluated only the basic geometric elements of a single thread synthetic river valley: channel planform alignment, M_C, bankfull width, W_{BF}, valley floor width, W_{VF}, and valley top width, W_{VT}. In the following subsections, the modeling approach and potential control functions for the local values of each geometric element are explained in detail. The notation for local values of geometric elements is lowercase symbols (e.g., the local bankfull width is denoted as $w_{BF}(s_i)$).

4.4.3.1 Channel centerline

Many rivers are sinuous at some length scale, so having the ability to simulate a meandering channel centerline, M_C, is essential for the creation of most synthetic river valleys. The geometric element equation for M_C could be constructed from existing deterministic equations that are unscaled or alternatively from empirical relationships that scale wavelength based on discharge or channel dimensions (Schumm, 1960; Ferguson, 1975). For meandering streams M_C can be defined most simply using a sinusoidal model (Langbein and Leopold, 1966; Darby and Delbono, 2002), although other more complex models exist (Kinoshita, 1961; Ferguson, 1973, 1976; Nikora and Sapozhnikov, 1993). Regardless of the geometric element equation constructed, in this paper M_C is first developed in Cartesian coordinates by modeling the position of the alignment in which y is a function of distance along the x coordinate. The channel centerline coordinate in the y plane at node i is $y_C(x_i)$, which is described by Eq. (5) in the examples.

4.4.3.2 Local bankfull width

The geometric element for the bankfull width, W_{BF}, describes planform alignment of the channel extents at each bank and marks the beginning of the valley floodplain, following the channel planform alignment as a function of s_i. However, before coordinates can be developed for the extent of the channel banks a spatial width series needs to be developed that allows for variations in the local bankfull channel width. Based on the scaling approach described above, $w_{BF}\,(s_i)$ is determined according to Eq. (3) such that the geometric element equation for the local bankfull width at each location s_i is given by

$$w_{BF}\,(s_i) = \overline{W_{BF}}\,f(s_i) + \overline{W_{BF}} \tag{6}$$

where $w_{BF}\,(s_i)$ is the local bankfull width at location s_i and $\overline{W_{BF}}$ is the reach-average bankfull width. This is a simple relationship that states that the bankfull width at any location is the average value for the reach plus a scaled component that may be more or less than the average value at any location in the channel depending on the control function, $f(s_i)$. Note that $w_{BF}\,(s_i)$ is first developed in the curvilinear, orthogonal coordinate system following s_i, and the Cartesian transformed coordinates for both banks are:

$$y_{RB}(x_i) = y_C(x_i) \; + \left(n_{j=k} < 0\,\frac{w_{BF}(s_i)}{2}\right)\left(\frac{\Delta x_i}{\Delta s_i}\right) \tag{7}$$

$$y_{LB}(x_i) = y_C(x_i) \; + \left(n_{j=k} > 0\,\frac{w_{BF}(s_i)}{2}\right)\left(\frac{\Delta x_i}{\Delta s_i}\right) \tag{8}$$

where $y_{RB}(x_i)$ and $y_{LB}(x_i)$ are the right and left bank coordinates at each location x_i and all other variables are defined as above.

4.4.3.3 Local valley width

In planform the valley consists of two geometric elements, the width of the valley floor, W_{VF}, and the valley top width, W_{VT}. There exists a paucity of information on the spatial series attributes for these geometric elements other than they both oscillate at reach and basin scales and exhibit multifractal scaling (Table 1). Rather than use control functions for both of these elements, only W_{VF} varies as a function of distance downstream, while W_{VT} is treated as a constant spatial offset defined by $\overline{W_{VT}}$ alone, but this could easily be changed. The basis for this approach is that in conjunction with a vertical offset for $\overline{H_{VT}}$, sloping valley walls can be created. Therefore, the focus in this section is on determining a geometric element equation only for W_{VF}.

The local valley floor width could be scaled according to Eq. (3) and follow s_i similar to $w_{BF}(s_i)$, but this is not done here for two reasons. First, Cartesian coordinates are used for W_{VF}, because floodplain and valley topography can be driven by nonchannel processes (e.g., mass movements of hillslopes and anthropogenic activities) as well as channel processes (Grant and Swanson, 1995). Second, using a scaled control function for $w_{VF}(s_i)$ as in Eq. (6) limits the potential for nonuniformity of the valley floor. A different approach, used herein, is to consider each side (e.g., right and left) of W_{VF} and W_{VT} as components of the geometric elements and use control functions with separate parameters that are functions of x_i.

With that in mind, at a minimum the geometric element equation for each side of W_{VF} should have components that allow for (i) an offset of an average valley toe width $\overline{W_{VF}}$, (ii) a control function to create subreach variability, and (iii) terms that prohibit overlapping of valley and channel elements. First, constant offsets for $\overline{W_{VF}}$ are applied equally by dividing this value in half for both sides –for the left side this value is subtracted and for the right side it is added (Fig. 2B). Second, by definition the spatial relationship of $w_{VF}(s_i)$ to $w_{BF}(s_i)$ is such that the right and left valley floor coordinates $y_{VFR}(x_i)$ and $y_{VFL}(x_i)$ should lay outside of the channel bank coordinates, $y_{RB}(x_i)$ and $y_{LB}(x_i)$. However, as each valley floor coordinate is defined by an unscaled control function, $f(x_i)$, it is necessary to eliminate potential overlaps of $y_{VFR}(x_i)$ and $y_{VFR}(x_i)$ with $y_{RB}(x_i)$ and $y_{LB}(x_i)$. This is done for $y_{RVF}(x_i)$ by determining the maximum value of $y_{RB}(x_i)$ and the minimum value of $f(x_i)$ and adding those values as additional spatial offsets. Similarly, for $y_{LVF}(x_i)$, the minimum value of $y_{LB}(x_i)$ is determined and added as a spatial offset while the minimum value of $f(x_i)$ is determined and subtracted from that offset. Therefore, the geometric element equations for the valley floor toe for the right and left sides are expressed as, respectively:

$$y_{RVF}(x_i) = f(x_i) + \frac{\overline{W_{VF}}}{2} + max(y_{RB}(x_i)\,) - min(f(x_i)) \tag{9}$$

$$y_{LVF}(x_i) = -f(x_i) - \frac{\overline{W_{VF}}}{2} + min(y_{LB}(x_i)\,) + min(f(x_i) \tag{10}$$

where $y_{RVF}(x_i)$ and $y_{LVF}(x_i)$ are the right and left local y_i coordinates of the floodplain toe at location x_i, $\overline{W_{VF}}$ is the user specified average width of the valley floor, and all other variables are defined as above. This equation states that the valley floor coordinates are dependent on a control function, $f(x_i)$, plus half of the reach-average valley floor width, $\overline{W_{VF}}$, and two additional terms that account for restricting the valley floor geometric element from overlapping with the channel elements. A detriment to this approach is that the local values of W_{VF} and W_{VT} are determined by the largest oscillations in the bank coordinates, $y_{RB}(x_i)$ and $y_{RB}(x_i)$, rather than solely $\overline{W_{VF}}$ and $\overline{W_{VT}}$. While for Eqs. (9-10) the minimum and maximum bounds of the bank coordinates are taken for the entire reach, although a moving average could also be used. Further, while in Eqs. (9) and (10) the valley widths are unscaled by any of the reach-average properties, relationships based on watershed hydrology for valleys formed by fluvial or glacial processes (Montgomery, 2002) could be used to incorporate this aspect.

Because only an offset for $\overline{W_{VT}}$ is used, the geometric element equations for the left and right valley top coordinates, $y_{LVT}(x_i)$ and $y_{RVT}(x_i)$, are:

$$y_{RVT}(x_i) = y_{RVF}(x_i) + \frac{\overline{W_{VT}}}{2} \quad (11)$$

$$y_{LVT}(x_i) = y_{LVF}(x_i) - \frac{\overline{W_{VT}}}{2} \quad (12)$$

4.4.4 Local profile attributes

In profile there is a minimum of four geometric elements needed to create a synthetic river valley: (i) thalweg elevation, Z_T; (ii) bank top, Z_{TOB}; (iii) height of the valley floor, Z_{VF}; and (iv) height of the valley top, Z_{VT}. For the model considered herein, Z_T initially has a control function dependent on s_i, but is later transformed to $x_{i,}y_i$. The basic rational and equations for each of these elements are presented next.

4.4.4.1 Local thalweg elevation

In profile Z_T is specified as a function of s_i, located in planform tangent to M_C, describing the characteristics of the bed slope and vertical undulations in the channel. Longitudinal profiles are the most heavily studied geometric element and this is reflected in the amount of information available for their spatial descriptions (Tables 1–2). According to theory, scale-dependent properties of longitudinal profiles of rivers suggest that such profiles have specific mathematical components depending on their length and upstream contributing area (Tanner, 1971; Ohmori, 1991). Many mathematical functions have been reported for river profiles (Table 2) at different scales. At the reach scale there is a linear trend commonly referred to as the average slope (Einstein, 1950; Leopold et al., 1964; Knighton, 1998). In addition, theory and observation also show that the bed elevation in alluvial rivers is quasi-oscillatory at morphological-unit to reach scales owing to periodic bedforms, whose frequency varies with its landscape position and/or gradient (Keller and Melhorn, 1978; Wohl et al., 1993; Montgomery and Buffington, 1997). Thus, the creation of a geometric element equation for Z_T for the reach-scale channel morphology will at a minimum need to provide a linear trend and a quasi-oscillatory component that is scaled according to Eq. (3).

While there are a variety of expressions and combinations thereof (Table 2) that could be used to construct a geometric element equation for the local vertical bed elevation $z_T(s_i)$, this study constructed a geometric element equation based on (i) a scaled component that has subreach scale oscillations determined by a control function, $f(s_i)$; (ii) a linear sloping component; and (iii) a vertical datum. Thus, the geometric element equation is:

$$z_T(s_i) = (\overline{H_{BF}} f(s_i) + \overline{H_{BF}}) + \bar{S}(\Delta s_i) + Z_D \quad (13)$$

where $Z_D$is a user-defined datum.

4.4.4.2 Local top of bank elevation

The top of bank represents the vertical extent of the channel at the intersection with the valley floodplain. Information on the scale-dependent spatial series attributes of Z_{TOB} is lacking in the peer-reviewed literature (Table 1). However, a simple geometric element equation can be constructed using a sloping line that is offset some fixed height by $\overline{H_{BF}}$. This is performed by first determining the height of the maximum bed undulation for the detrended thalweg series, $max(z_{TR}(s_i))$ and then adding $\overline{H_{BF}}$ to the downstream most model node $s_{i=1}$; while for subsequent nodes at $z_{TOB}(s_{i+1})$, it is modeled as a simple sloping line. The geometric element equations are thus:

$$z_{TOB}(s_{i=1}) = max(z_{TR}(s_i)) + \overline{H_{BF}} \tag{14}$$

$$z_{TOB}(s_{i+1}) = z_{TOB}(s_i) + \bar{S}(\Delta s_i) \tag{15}$$

where $z_{TOB}(s_i)$ is the top of bank elevation and $z_{TR}(s_i)$ is the detrended residual series for the thalweg along the channel at location s_i. This relationship assumes that maximums in thalweg elevation are associated with riffle crests that are in quasi-equilibrium with the reach-average bed slope, bankfull depth, and median sediment size such that the riffle crest is at equilibrium with its sediment supply via grain size and incipient motion characteristics for the bankfull geometry. Note that the first term in Eq. (14) assumes all bed undulations have uniform height. For bed undulations with variable frequencies and amplitudes this could be modified using a window size commensurate with the spacing of bedforms so that the top of bank is defined relative to local riffle crests. Moreover, the top of bank in natural channels varies depending on channel type, which could be expressed through an additional control function nested within the geometric element equation.

4.4.4.3 Local valley floor elevation

In this model there are two valley elevations, the valley floor, Z_{VF} and valley top, Z_{VT}. There is relatively very little information in the peer-reviewed literature on the spatial series of valley floor and top elevations at scales other than the basin (Table 1). In general, the valley slope, $\overline{S_V}$, is thought to equal $\bar{S}$ for equilibrium conditions, but can deviate for nonequilibrium conditions (Nanson and Huang, 2008). Moreover, undulations in this profile can occur from tributary fans, mass movement and wasting, geologic controls, and land use practices for human-modified river valleys. For Z_{VT}, the elevation profile can be highly variable and range from completely flat over plateaus, oscillatory owing to uplift and incision (Montgomery, 1994), and/or downstream sloping for river valleys that transition to lowlands.

In this article a geometric element equation for Z_{VF} is constructed using two basic components consisting of (i) a vertical offset for the average valley floor height above the channel, $\overline{H_{VF}}$ (Fig. 2), and (ii) a sloping term that allows for a valley slope, $\overline{S_V}$. Note that only a simple offset is used herein, while a scaled control function could be used in applications to enable vertical variability of the two valley heights, for example to add terraces. Then, for Z_{VT}, an additional vertical offset can be added for the average valley height, $\overline{H_{VT}}$. While a control function could be used to simulate oscillations in Z_{VT}, we omit this term in this model. The geometric element equations for this configuration are:

$$z_{VF}(x_i) = z_{TOB}(x_i) + \overline{H_{VB}} + \overline{S_V}(\Delta x_i) \tag{16}$$

$$z_{VT}(x_i) = \overline{H_{VT}} + z_{VF}(x_i) \tag{17}$$

where $\overline{H_{VT}}$ is a user-defined vertical height of the valley from bottom to top, $z_{TOB}(x_i)$ is the Cartesian transformed top of bank coordinate, $z_{VF}(x_i)$ is local valley floor height, and $z_{VT}(x_i)$ is local valley top height as well as vertical extent of the model (Fig. 2).

4.4.5 Local channel cross-sectional geometry

The channel cross section topography is defined by a geometric element equation that populates the cross section points, n_j, whose generation is described above (section 4.2). Cross section elevations are defined by mathematical functions that describe the elevation of each node $z_{n_j}(s_i)$ at locations orthogonal to the channel centerline. In this article, cross sections are developed in curvilinear, orthogonal coordinates and then transformed to Cartesian coordinates using Eqs. (1–2). Each cross section is bounded vertically by $z_{TOB}(s_i)$ and $z_T(s_i)$ defining the bankfull depth as $h_{BF}(s_i) = z_{TOB}(s_i) - z_T(s_i)$. In addition, the local bankfull width, $w_{BF}(s_i)$, bounds the cross section laterally as $n_j \in \left(\frac{w_{BF}(s_i)}{2}, \frac{-w_{BF}(s_i)}{2}\right)$. As discussed above, cross section functions are commonly scaled locally by $w_{BF}(s_i)$ and $h_{BF}(s_i)$. Therefore, a generic expression for the cross section geometric element is:

$$z_{n_j}(s_i) = f(h_{BF}(s_i), w_{BF}(s_i)) + z_T(s_i) \tag{18}$$

Several potential control functions exist for channel cross sections (Table 2). These include power functions (James, 1996), polynomials (James, 1996), probability distributions (Merwade, 2004; Jacobson and Galat, 2006), simple geometric shapes (Chow, 1959), and analytical functions (Bridge, 1977; Beck, 1988; Deutch and Wang, 1996). For cross sections to have asymmetric properties they are commonly related to local parameters of the channel centerline such as curvature (Deutch and Wang, 1996). To model multiple channels in a cross section, the four-parameter Weibull distribution used by Jacobson and Galat (2006) can be used.

For this paper the examples described below utilized the channel cross section control function of Deutsch and Wang (1996). The model is based in the curvilinear, orthogonal coordinate system and is presented here based on modifications by Leigleiter and Kyriakidas (2008). The parameter, $B(s_i)$, determines the position of lateral depth given by:

$$B(s_i) = \frac{1}{2}(1 - |C_S(s_i)|/C_{MS}) \text{ when } C_{MS}(s_i) < 0 \tag{19}$$

$$B(s_i) = \frac{1}{2}(1 + |C_S(s_i)|/C_{MS}) \text{ when } C_{MS}(s_i) > 0 \tag{20}$$

$$B(s_i) = \frac{1}{2} \text{ when } C_S(s_i) = 0 \tag{21}$$

where $C_S(s_i)$ is centerline curvature and C_{MS} is the maximum curvature of the centerline. When the channel curves to the right the channel is deepest toward the left and the elevation at each location, n_j, is given by:

$$z_{n_j}(s_i) = 4h_{BF}(s_i)\left(\frac{w_{BF}(s_i)-n}{w_{BF}(s_i)}\right)^{l_1(s_i)}\left[1-\left(\frac{w_{BF}(s_i)-n}{w_{BF}(s_i)}\right)^{l_1(s_i)}\right] \tag{22}$$

where $l_1(s_i) = -\ln(2)/\ln B(s_i)$. Then, when the channel curves to the left it is deepest toward the right and the elevation at each location, n_j, is given by:

$$z_{n_j}(s_i) = 4h_{BF}(s_i)\left(1-\frac{w_{BF}(s_i)-n}{w_{BF}(s_i)}\right)^{l_2(s_i)}\left[1-\left(1-\frac{Ww_{BF}(s_i)-n}{w_{BF}(s_i)}\right)^{l_2(s_i)}\right] \tag{23}$$

where $l_2(s_i) = -\ln(2)/(1-\ln B(s_i))$.

The channel centerline and its curvature can be linked or separate control functions can be used for both. A benefit of having a separate expression for C_S is that channels can be created with asymmetrical cross sections at locations other than pools in meander bends and this approach is utilized herein. To model the centerline curvature in the examples in this paper, Eq. (5) is used with $p = 1$ and no cosine term, because computing the derivatives of a single sinusoid is trivial and the function still allows for control of the phase and frequency of curvature, thus cross-sectional asymmetry. Based on the standard definition for the curvature of a plane curve described by a sinusoid, the curvature of the channel alignment is given by:

$$C_S(s_i) = \frac{-a_p \sin(b_p x_r + \theta_p)}{(1 + (a_p \cos(b_p x_r + \theta_p))^2)^{3/2}} \quad (24)$$

where the numerator in Eq. (24) is the second derivative and the denominator is the first derivative of Eq. (5) with $p = 1$.

4.5 Parameterization

Parameterization is the final step of creating a synthetic river valley where the parameters of the geometric element equations are adjusted to meet user-specified attributes defined through the conceptualization process. This includes specification of reach-average properties of the river corridor and also each control function parameter independently (e.g., the frequency of bed oscillations) and in some cases dependently (e.g., the relationship between thalweg elevation and bankfull width). The extent of independent and dependent parameterization will depend on the purpose of modeling, which fluvial elements are being included, the mathematical function used, and expert judgment. For independent parameterization, reach-average properties are specified to match the attributes set during conceptualization. Moreover, the river valley can further be put into this context by casting the reach-average values in terms of other geomorphic indices such as the width-to-depth ratio, channel sinuosity, and confinement ratio. Channel sinuosity, S_L, is determined as $\frac{L_s}{L_x}$ and also can be independently parameterized by the sinuosoidal parameters in Eq. (5). Moreover, the degree of channel confinement within the valley floor can be contextualized by a confinement ratio, $\overline{W_C} = \frac{\overline{W_{BF}}}{\overline{W_{BF} + W_{VF} + W_{VT}}}$. In addition, the control functions provide another avenue for parameterizing the model. For example, the control function for Z_T could be used to manipulate the number and spacing of riffle–pools or even residual pool depth. For dependent parameterization, the spatial covariance between geometric elements may be used to contextualize parameterization of the synthetic river valley as described next.

4.5.1 Geomorphic covariance structure

In this study, dependent parameterization of the geometric model is contextualized by the spatial covariance of specific geometric elements, which is termed the geomorphic covariance structure (GCS). A GCS is a spatial covariance relationship between two or more geometric attributes in a river valley. For example, the serial covariance, $C_i(xy)$, between pairs $x_i \text{ and } y_i$ can be calculated from detrended and standardized series residuals, x_i and y_i by the product $x_i * y_i$. The basis for the use of a GCS in dependent parameterization is that alluvial river systems in dynamic equilibrium can have ideal relationships between the covariance of specific geometric variables, such as Z_T and W_{BF} or Z_T and C_S that can aid in dependent parameterization (Brown and Pasternack, 2012). Interpretation of a GCS is based on the sign of the covariances. For example consider three basic cases: (i) if $x_i > 0$ and $y_i > 0$ then $C_i(xy) > 0$, (ii) if $x_i < 0$ and $y_i < 0$ then $C_i(xy) > 0$, and (iii) if x_i or y_i are < 0 then $C_i(xy) < 0$. Thus, when $C_i(xy) > 0$ the local geometric elements covary and when otherwise do not. By thinking through how and where variables that control geomorphic process need to covary, it is possible to design a GCS to operationalize a geomorphic process concept via tailored nonuniform, stage-varying channel forms to yield functional form–process dynamics.

5. Geometric model construction

To illustrate river valley synthesis via geometric modeling, the above framework and equations were used to build a geometric model capable of creating a wide array of topographic configurations. This approach was chosen instead of merely adjusting each parameter and coefficient in the geometric model and illustrating arbitrary topographic configurations. Specifically, a single geometric model was built for a single-thread, gravel-bed river valley to illustrate how independent and dependent parameterization can be employed to make a prototypical straight lowland river (S1), straight mountain river (S2), meandering lowland river (S3), meandering mountain river (S4), and two hybrid configurations (S5–S6). Generating these examples involved drawing on fundamental descriptions of reach-scale and morphological-unit-scale variability in two distinct physiographic settings, mountains and lowlands, and then extrapolating 2D geometric functions to yield 3D surfaces. Independent parameterization was needed to provide reach-average values for geometric elements for mountain and lowland valleys. Dependent parameterization was performed using ideal GCS relationships that contrast fluvial landforms in these settings. The simulations presented herein were programmed in Microsoft Excel® and exported into an xyz format so that topographic surfaces with contour overlays could be rendered with a program (herein, SURFER v. 8) to visualize results. Upon creating surfaces, a low-pass Gaussian filter was used with 30 passes to smooth out visual irregularities produced in the interpolation process, since no breaklines or other visualization aids were used with the native point cloud. The steps from section 4 were used to create the examples; below only the specific conceptualization and parameterization steps unique to each example are discussed. After that, the utility of GCS in directly creating topographic features is presented in section 6.1. The use of GCS functions to create indirect features— specifically between channel and floodplain elements— is presented in Brown (2014).

5.1 Example conceptualizations

Lowland and mountain gravel-bed rivers are typically differentiated by valley and channel slope, where the former typically has $\bar{S}$ < 0.01 (Leopold et al., 1964; Wohl et al., 1993; Montgomery and Buffington, 1997). Moreover, lowland rivers are ostensibly thought to be less confined, having lower values of the confinement ratio ($\overline{W_C}$) and median sediment size ($\overline{D_{50}}$) as well as higher values of sinuosity ($\overline{S_L}$) and width-to-depth ratio$\left(\frac{\overline{W_{BF}}}{\overline{H_{BF}}}\right)$than mountain rivers (Schumm, 1960; Leopold et al., 1964; Knighton, 1998; Church, 2006; Wohl, 2010). Despite these variables being used heavily in geomorphology to provide a broader understanding of landform changes that occur along physiographic gradients, mountain and lowland rivers can also be differentiated by their GCS functions.

Alluvial lowland rivers that lack significant topographic forcing elements and are unconfined ($\overline{W_C} << 1$) exhibit positively correlated peaks in spatial series of bankfull width and thalweg. This means that topographic highs along the longitudinal bed profile are correlated with higher than average values of bankfull width (Richards, 1976b; Hey and Thorne, 1983; Hudson, 2002; White et al., 2010), which happens because mass continuity for the channel-forming flow dictates that wider than average cross-sectional areas have a lower τ_{BF}^{*} than areas that have narrower cross-sectional area (Carling and Wood, 1994; Repetto et al., 2002; Caamaño et al., 2009). Mechanistically, this is related to the presence of velocity reversals or flow convergence routing, whereby the channel form is indicative of a joint geomorphic and hydrodynamic interaction (Wilkinson et al., 2004; MacWilliams et al., 2006; Sawyer et al., 2010). Moreover, it is common in freely meandering alluvial rivers that pools are located at outside bends owing to secondary flows driven by centripetal forces (Rhoads and Welford, 1991), which implies a negative serial covariance between the spatial series of channel curvature and thalweg elevation. Thus, alluvial lowland pool–riffle channels in quasi-equilibrium may be characterized as having quasi-oscillatory GCS patterns where $C_i(z_T, w_{BF}) \geq 0$ and $C_i(z_T, c_s) \leq 0$.

Mountain rivers, especially in confined canyons where $\overline{W_C} \sim 1$, predominantly have forcing bedrock elements, large wood, and coarse sediments that promote structurally locked steps and cascades (Montgomery and Buffington, 1997; Church and Zimmerman, 2007). Mechanistically these can occur from a variety of interactions between flow, sediment, wood, and structural forcing elements in a manner analogous to the *jammed-state* hypothesis for step–pool formation (Church and Zimmerman, 2007). This can result in bankfull width and thalweg elevation having a negative spatial covariance as well as channel curvature and thalweg elevation having a positive one. In the Grand Canyon for example, rapids form at constrictions that accumulate coarse material supplied from tributaries and lateral erosion, creating locally narrow topographic highs (Dolan et al., 1978). Thus, GCS functions for confined mountain rivers can be the opposite of those for unconfined alluvial rivers, characterized as also having quasi-oscillatory GCS patterns with $C_i(z_T, w_{BF}) \leq 0$ and $0 \leq C_i(z_T, c_s) \geq 0$. The benefit of this type of conceptualization whereby GCS properties of prototypical landforms are utilized is that the architecture of the geometric model can remain static across diverse settings, while dependent control function manipulation of geometric elements can allow the creation of different topographic surfaces.

5.2 Parameterization

Independent parameterization is needed for reach-average variables and for each control function. Reach-average values for lowland (S1 and S3) and mountain rivers (S2 and S4) were selected from the literature (Table 3; e.g., Leopold et al., 1964; Knighton, 1998; Lisle et al., 2000). For the hybrid scenarios (S5 and S6) values were chosen for large confined alluvial rivers (e.g., Grant and Swanson, 1995; White et al., 2010). For S1 through S6 the values for the amplitude, frequency, and phase shift in Eq. (5) were specified for control functions for Z_T, W_{BF}, W_V, M_C, and C_S (see Table 4 for additional parameterization values). Similarly, sinuosity, S_L, was determined by $\frac{L_S}{L_x}$ as described above in section 4.5 with S3 having a higher sinuosity than S4.

Dependent parameterization for these landforms relied on relating each GCS to conceptual or even quantitative aspects of a river valley landscape and are shown as 2D plots for each set of variables in section 5.3. For a positive GCS, the sinusoid control function between two elements should have no equal frequencies and no phase shift. For a negative GCS, the sinusoid control functions should have a phase shift between the two elements and their frequencies should also be equal. For these examples, S1 and S3 were unconfined and had positive GSCs for thalweg elevation and bankfull width. In addition, for meandering S3 to have a pool at each outside bend and a riffle at each bend apex, thalweg elevation and channel curvature were set out of phase by π/2 and centerline frequency was set to 1/2 of the thalweg frequency. Scenario 2 and S4 required a negative GCS with W_{BF} out of phase with Z_T, so a phase shift of π was applied. For S5 and S6, each GCS was arbitrarily manipulated to create complex topography to illustrate the overall approach.

Reach Average Parameters		**Example**					
		S1	***S2***	***S3***	***S4***	***S5***	***S6***
Channel slope	S	0.002	0.02	0	0.02	0.001	0.001
Valley slope	S_V	0.002	0.02	0	0.02	0.001	0.001
Bankfull width (m)	W_{BF}	30.00	10.00	30.00	10.00	50.00	50.00
Median sediment size (m)	D_{50}	0.064	0.26	0.03	0.26	0.032	0.032
Bankfull depth (m)	H_{BF}	2.11	1.13	1.06	1.13	2.11	2.11
Valley top height (m)	Z_{VT}	0.00	5.00	1.00	5.00	10.00	5.00
Valley bottom height (m)	Z_{VB}	0.00	5.00	0.00	5.00	4.00	2.00
Valley bottom width (m)	W_{VB}	25.00	1.00	50.00	1.00	25.00	25.00
Valley top width (m)	W_{VT}	25.00	1.00	50.00	1.00	100.00	200.00
Confinement ratio	W_C	0.38	0.83	0.23	0.83	0.29	0.18
Width to depth ratio	W_{BF}/H_{BF}	14.20	8.88	28.41	8.88	23.67	23.67
Sinuosity	S_L	1.00	1.00	1.13	1.01	1.17	1.07
Longitudinal node spacing	Δx	0.53	0.18	0.53	0.18	1.27	2.54
Cross section node spacing	Δn	0.10	0.10	0.10	0.10	0.10	0.10
Model length	Lx	420.00	140	420	140	1000	2000
Vertical datum	Z_D	1000.00	1000	1000	1000	1000	1000
Number of increments	m	784	784	784	784	784	784

Table 3. Reach average synthetic river simulation parameters for examples

5.3 Evaluation of GCS parameterization on topographic surfaces

This section visually evaluates the topographic surfaces for S1–S6 and discusses how complex topography can be created and subsequently adjusted by dependent GCS parameterization. The intent is not to individually manipulate each geometric element while holding all others constant, but rather to show how GCS manipulation provides a compact method for holistically creating and adjusting topographic surfaces with a single geometric model. This is the most useful aspect of geometric modeling for synthesizing river valleys with mutually or independently adjustable geometric elements. This section also highlights the role of varying control functions in creating topographic complexity.

For S1 and S2, a single waveform (Eq. 5; $p = 1$; $a_C, b_C, \theta_C = 0$) was used for Z_T and W_{BF} to illustrate the utility of GCS manipulation in creating straight channels with varying topographic surfaces (Table 4; Fig. 4). For S1, $C_i(z_T, w_{BF}) \geq 0$, which has the effect of creating topographic high points in areas that are locally wide and topographic low points in areas that are locally wide, consistent with geomorphic theory (Fig. 4a). Using the sign of $z_{TR}(s_i)$ as an index for riffle and pool locations, where $z_{TR}(s_i) > 0$ are riffles and $z_{TR}(s_i) < 0$ are pools, station 100 would be classified as the center of a riffle where stations 0 and 325 would be pool centers. In contrast, S2 has $C_i(z_T, w_{BF}) \leq 0$, which has the effect of creating topographic high points in areas that are locally narrow, which is also consistent with the conceptualizations envisioned in the first step of the modeling process (Fig. 4B). Using relative bed elevation as to discriminate between riffles and pools, stations 50 and 125 would be the center of a riffle or step, while stations 18 and 88 would be pool centers. Together with independent parameterization for the reach-average variables (Table 3), the GCS of geometric elements that have simple control functions can create the topography of idealized end-member landforms associated with mountain and lowland settings.

		S1	**S2**	**S3**	**S4**	**S5**		**S6**			
Control Curve Parameters	**Symbol**	**Sine**	**Sine**	**Sine**	**Sine**	**Sine**	**Cosine**	**Sine 1**	**Cosine 1**	**Sine 2**	**Cosine 2**
Thalweg amplitude	a_{zt}	0.25	2.00	1.00	2.00	0.50	2.00	1.00	0.50	0.25	0.50
Thalweg frequency	b_{zt}	1.00	2.00	2.00	2.00	1.25	2.00	4.00	4.00	5.00	2.00
Thalweg phase shift	θ_{zt}	0.00	3.14	1.57	0.00	3.14	3.14	0.00	0.00	0.00	0.00
Channel width amplitude	a_{wbf}	0.25	0.25	0.25	0.25	0.20	0.30	0.30	0.20	0.10	0.10
Channel width frequency	b_{wbf}	1.00	2.00	2.00	2.00	2.00	0.50	4.00	4.00	2.00	2.00
Channel width phase shift	θ_{wbf}	0.00	0.00	1.57	3.14	1.57	0.00	0.00	0.00	0.00	0.00
Centerline curvature amplitude	a_{cc}	0.00	0.00	0.10	0.10	0.25	NA	0.25	NA	NA	NA
Centerline curvature frequency	b_{cc}	0.00	0.00	1.00	2.00	1.00	NA	2.00	NA	NA	NA
Centerline curvature phase shift	θ_{cc}	0.00	0.00	0.00	3.14	1.00	NA	3.14	NA	NA	NA
Channel meandering amplitude	a_{cm}	0.00	0.00	50.0	5.00	100.00	50.00	175.0	75.00	50.0	10.00
Channel meandering frequency	b_{cm}	0.00	0.00	1.00	1.00	1.00	2.00	0.20	2.00	1.00	7.00
Channel meandering phase shift	θ_{cm}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Left valley floor amplitude	a_{rvf}	0.00	2.00	50.0	5.00	75.00	25.00	100.	75.00	50.0	10.00
Left valley floor frequency	b_{rvf}	0.00	2.00	1.00	1.00	1.50	4.00	0.20	2.00	1.00	7.00
Left valley floor phase shift	θ_{rvf}	0.00	0.00	3.14	3.14	0.00	3.14	0.00	3.14	0.00	0.00
Right valley floor amplitude	a_{lvf}	0.00	2.00	50.00	5.00	25.00	55.00	100.	75.00	50.0	10.00
Right valley floor frequency	b_{lvf}	0.00	2.00	1.00	1.00	4.50	1.50	0.20	2.00	1.00	7.00
Right valley Floor phase shift	θ_{lvf}	0.00	0.00	0.00	3.14	3.14	0.00	3.14	0.00	3.14	3.14

Table 4. Control function parameters for examples S1 through S6.

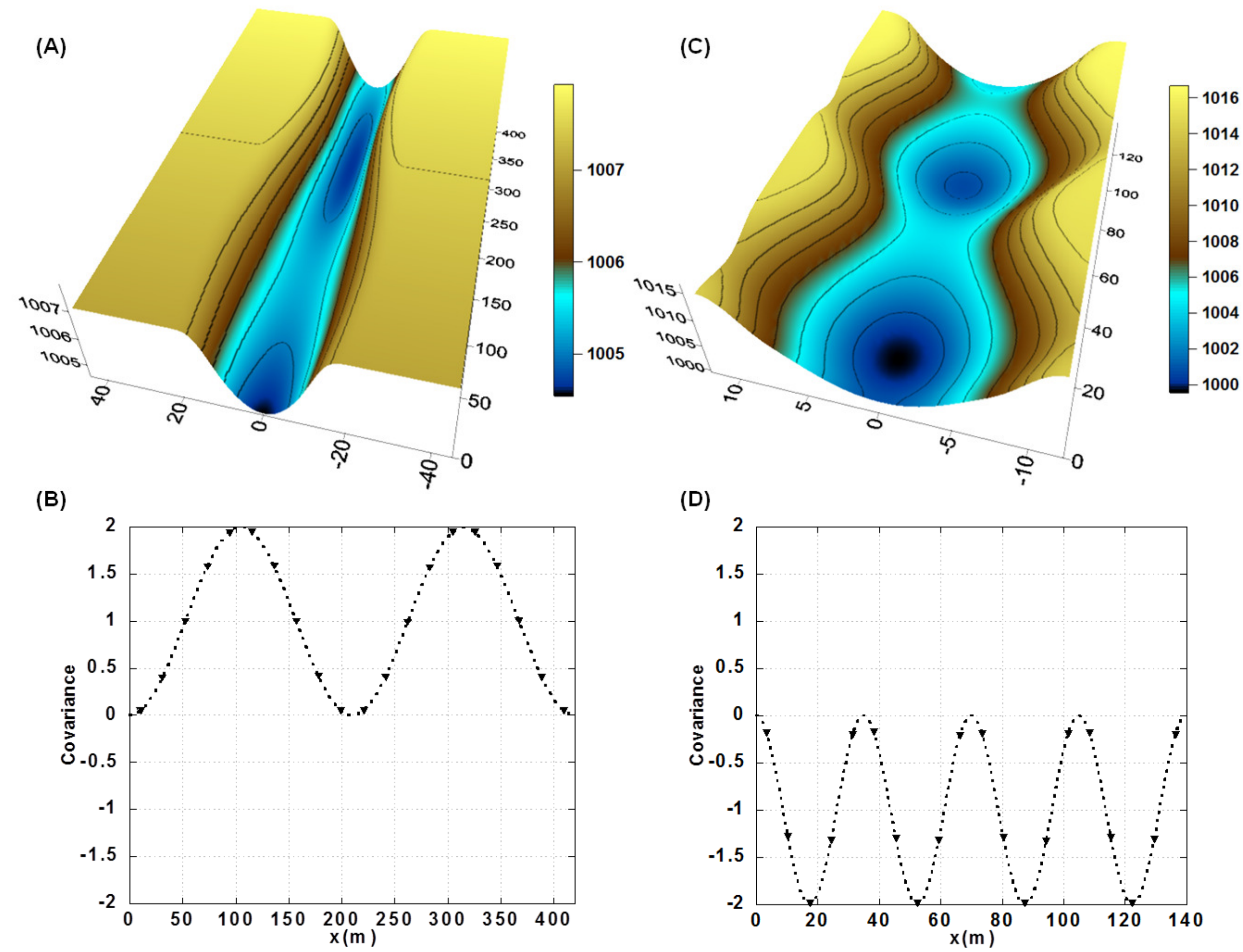

Fig. 4. Topographic surface and GCSs for S1 (A,B) and S2 (C,D) illustrating how the varying GCS's can create different channel configurations. Contour lines are at intervals of 2 m and labels are omitted for clarity.

For S3 and S4, the channels meander with a similar $C_i(z_T, w_{BF})$ as S1 and S2, respectively, to explore the role of the GCS, $C_i(z_T, c_s)$, in creating topography that matches the conceptualizations of bed elevation and channel curvature in section 5.1 (Fig. 5). In S3 $C_i(z_T, c_s) \leq 0$ and the GCS oscillates from zero to negative along x_i, with a positive peak at approximately station 100 and a negative peak at station 275, both of which would be commonly classified as pools because of the presence of lower bed elevation values (Fig. 5B). This is consistent with commonly reported relationships of pool locations and channel curvature for meandering alluvial rivers in quasi-equilibrium. There are also two positive smaller peaks at stations 225 and 350 that coincide with riffles. For S4, $C_i(z_T, c_s) \geq 0$ and mirrors $C_i(z_T, w_{BF})$, the result is that topographic high points are associated with channel bends as on stations 20 and 85 (Fig. 5C).

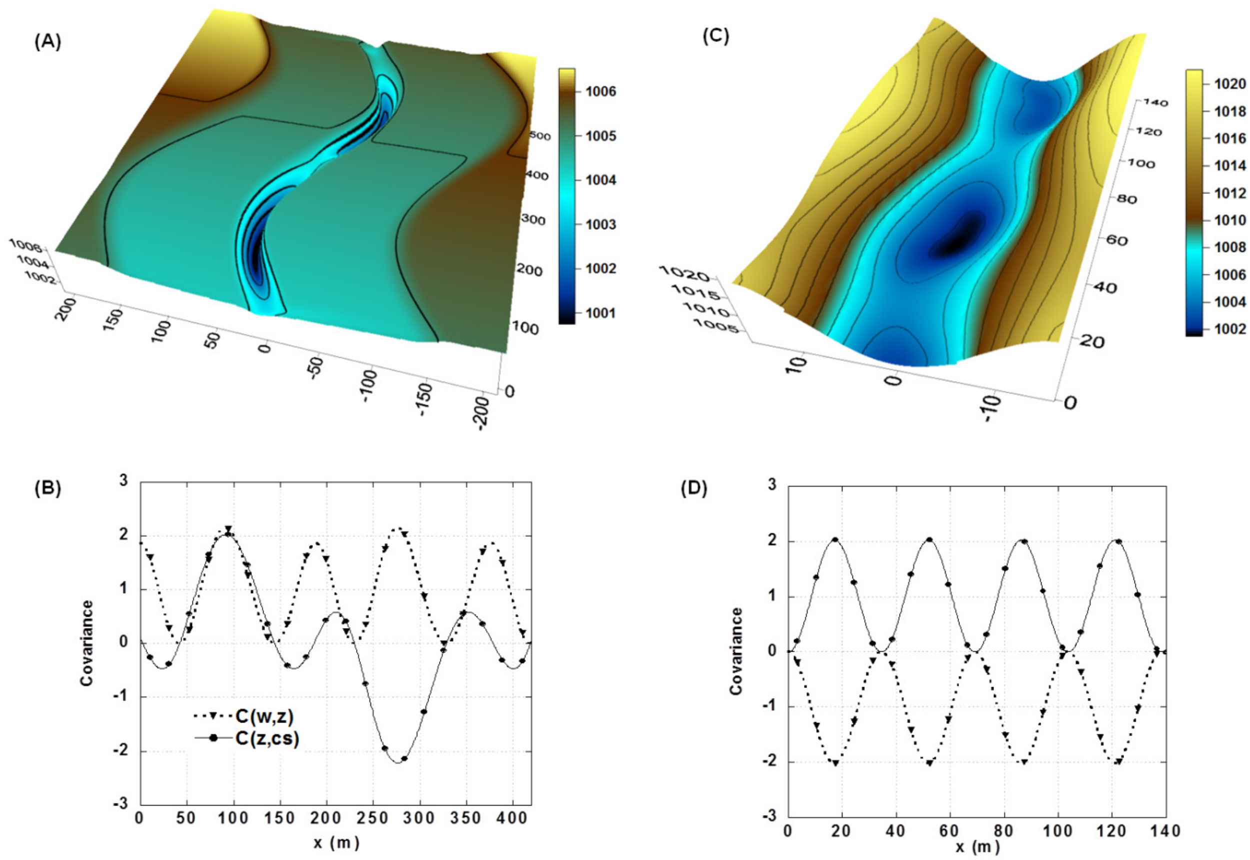

Fig. 5. Topographic surface and GCSs for S3 (A,B) and S4 (C,D) illustrating the topographic response of channel meandering. Contour lines are at intervals of 2 m and labels are omitted for clarity.

River valleys S1–S4 illustrate two important components of geometric modeling. First, even with only a single waveform (Eq. 5; of $p = 1;\ a_C, b_C, \theta_C = 0$) and simple input parameters, GCS manipulation of otherwise identical models can create topographic surfaces with drastically different characteristics (Figs. 4–5). The addition of higher harmonics in Eq. (5) can yield topographic surfaces that have the potential for vastly increased complexity (Fig. 6), but GCS relations for $C_i(z_T, c_s)$ and $C_i(z_T, w_{BF})$ also become more complex. Simply by adding the cosine component of Eq. (5) for $p = 1$ for the Z_T, W_{BF}, W_V, M_C, and C_S geometric elements as in S5, a much more complex topographic surface and associated GCS relations are created (Figs. 6A,B). By increasing the order of Eq. (5) to $p = 2$, the topographic surface and GCS relations become even more complex (Figs. 6C,D).

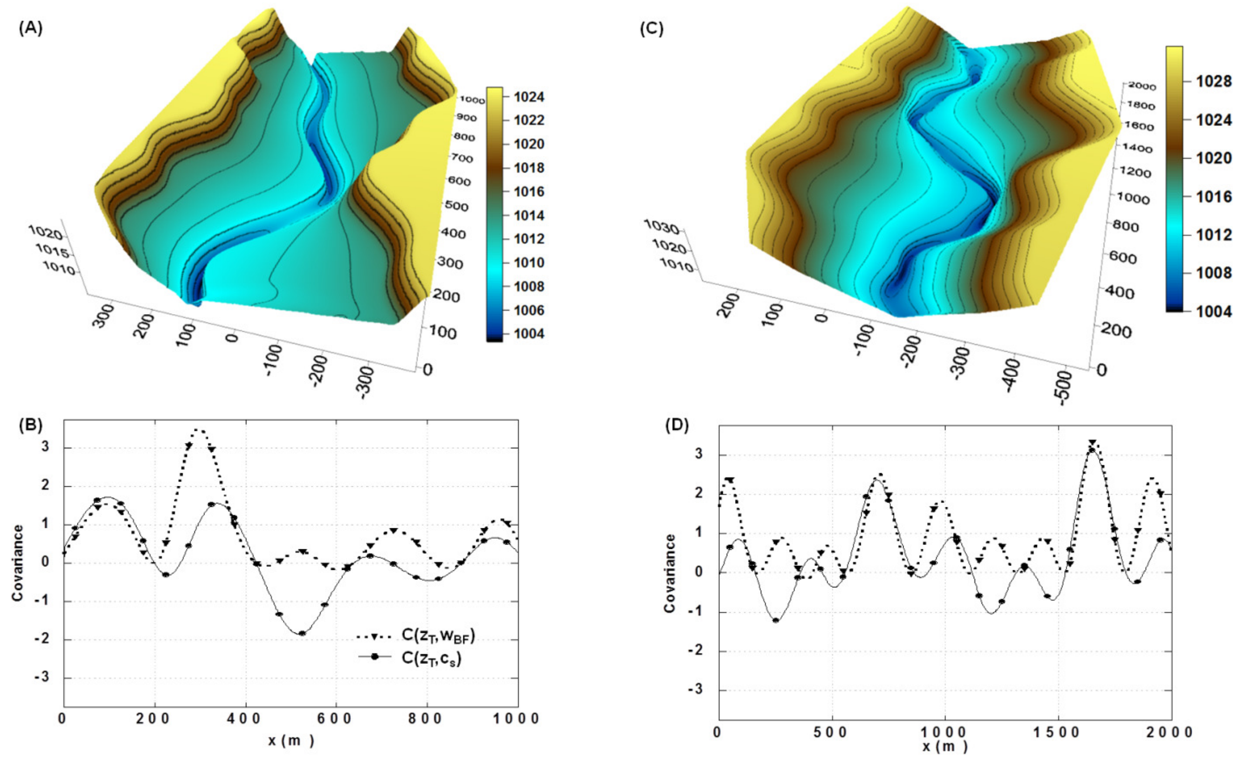

Fig. 6. Topographic surface and GCSs for S5 (A,B) and S6 (C,D) illustrating the GCS structure and topographic response when more complex waveforms are used as control functions for Z_T, W_{BF}, W_V, M_C, and C_S geometric elements. Contour lines are at intervals of 2 m and labels are omitted for clarity.

6. Discussion

6.1 Utility of geometric modeling for creating fluvial topography

Geometric modeling was able to create river valley topography of prescribed conditions to have fundamental attributes of lowland and mountain rivers at the reach scale. The technique was able to simulate reach-scale river attributes in idealized mountain and lowland settings that included well known GCS relations between geometric attributes such as bankfull width and thalweg elevation. Alteration of the sinusoidal amplitude of Eq. (5) resulted in increased longitudinal variance of bed elevation. The frequency component— when used with reach-average bankfull width scaling— allows simple modeling of riffle and pool spacing in these simulations to capture morphological-unit-scale patterns. While independent parameterization may be user-defined to produce simpler landscapes, dependent parameterization of all model variables can yield extremely complex yet organized river valleys. The GCS relationships of the local z_T to w_{BF} and c_S provide a compact and unique method of creating different topographic surfaces based on individual 2D relationships. Specifically, the conceptual attributes discussed that differentiate lowland and mountain rivers were captured through these GCS relations and become more valuable as more complex control functions are used, such as in S5 and S6. For example, if a specific geomorphic process or ecological function is associated with a GCS, then the identification of the GCS can reveal the presence or absence of the process on the basis of pure topographic analysis without hydrodynamic or morphodynamic simulation, which is a grand challenge in geomorphology today. While they may appear simplistic at this scale, future geometric and/or stochastic models can incorporate subreach-scale topographic variability— the possibilities are limitless. Notably, subwidth-scale features can be an important attribute of these types of rivers, especially in mountain settings, but this phenomenon is not addressed here. Section 6.5 discusses how this can be tackled in the existing framework.

6.2 Appropriate control functions

The approach to geometric modeling of river valleys presented in this paper relies on using control functions for parametric descriptions of 2D topographic variability. Understandably, the question as to what is the most representative control function for specific geometric elements is presently open-ended. Multiple applications exist, each perhaps requiring a different function to represent the signature of dominant processes as well as controls such as lithology and vegetation. Therefore, exploring the utility of diverse mathematical functions as control curves would be a valuable addition. Some examples that show immediate promise are kriging (Leigleiter and Kyriakidis, 2008), using specific variogram models for different scales (Robert and Richards, 1988), auto-regressive moving average stochastic models (Richards, 1976a; Knighton, 1983), specified power spectral density structures for spatial variables in the frequency domain with randomly re-assigned phases (Newland, 1984), nonlinear wave functions (e.g., cnoidal waves), and harmonic synthesis as shown herein. In addition, splines and Bezier curves could prove beneficial (Mortenson, 1997) using control points in lieu of control functions.

There are tradeoffs in using specific control functions in generating complexity, especially in terms of control over landform synthesis. For example, compared to a simple sinusoid the addition of harmonics (e.g., Fig. 6B) makes it more difficult to analytically derive expressions for morphometric parameters, such as number of riffles and pools, their spacing, residual pool depth, and riffle-riffle height. While an AR2 model, such as in Ferguson (1976), is capable of producing a noisier spatial series, it is still driven by randomness that limits full parametric adjustment. Further, Fourier series are recommended over AR2 models because the same level of complexity can be achieved with much more parametric control. At this stage of river valley synthesis the priority is on mindful control and unrestrained synthetic exploration.

6.3 Modeling multithread channels

To lay out the geometric-modeling foundation for river valley synthesis this article only addressed single-thread river channels. The extension of this approach to anabranching and braided channels could be made using several approaches. First, in the conceptualization several geometric element sets could be modeled for each channel so that each thread is explicitly defined. A drawback to such an approach is that the intersection of channel threads would cause an overlap unless junctions are modeled as specific geometric elements. In the case where geometric elements for channel junctions are not used, simple rule-based filtering of overlaps could also give this approach promise. An entirely different approach is to rely on using cross section functions that explicitly have multithread characteristics. For example, the four-parameter Weibull distribution used by Jacobson and Galat (2006) models multiple channels in a cross section, though with less options possible when treating each thread with independent functions. Moreover, cross section control functions based on sinusoids could be harmonically created by analogy with bar modes (Furbish, 1998). In this approach the bar mode could vary with the valley width, channel curvature, or bed elevation defining an entirely new GCS altogether. Further research is needed to evaluate the relative merits of these approaches, but extension of the modeling approach herein to more complex channel patterns is foreseeable.

6.4 Potential applications

Four potential applications that this technique could have an immediate impact in are illustrated in this section, including form–process inquiry, river restoration design, historical river reconstruction, and geomorphology education.

6.4.1 Form–process inquiry

Form–process inquiry could benefit from geometric modeling through direct testing of GCS structures and topographic response as well as creating digital elevation models (DEMs) for hydrodynamic and/or morphodynamic modeling. The use of GCS functions coupled with subsequent hydrodynamic, sediment transport, and ecological modeling of synthetic DEMs could help geomorphologists understand form–process mechanisms. Starting with a GCS configuration, one can iteratively adjust river valley DEMs allowing a stepwise analysis into the role of specific landform characteristics. For example, several studies have relied on creating prototypical riffle–pool units to understand geometric controls on riffle–pool maintenance (Richards, 1978; Carling and Wood, 1994; Pasternack et al., 2008), and this tool would easily accommodate such inquiry. However, such studies typically look at one to a few units, whereas geometric modeling would enable inquiry of longer corridors and include floodplain elements. By creating topographic surfaces such as S1 through S4 subsequent flow modeling would elucidate which GCS relations create conditions for riffle–pool maintenance. Insight into chaotic dynamics and nonlinear complexity could also be gained by subsequent morphodynamic modeling of simulated terrains to understand the role of initial conditions in landscape evolution (Perron and Fagherazzi, 2012).

6.4.2 River engineering and restoration design

River engineering and restoration could benefit from this approach because it allows the design of synthetic (i.e., alternative new and restored) river topography that can be rapidly adjusted and iterated upon. Currently river restoration designs typically create *restored* river reaches through graphical manipulation of surface contours via CAD programs (Wheaton et al., 2004a). The approach presented herein facilitates similar user flexibility, but allows the generation, and any subsequent iterations, of channel form to be linked to parametric equations that can be further related to hydrodynamic and ecohydraulic design criteria, removing much subjectivity from synthesis of restored river reaches. This extension is easily met, as real world coordinates can be used in lieu of specific planform geometric elements that are often fixed in river rehabilitation design, such as the centerline alignment or valley width. In this context the user can manipulate the GCS of $C_i(z_T, c_s)$ and $C_i(z_T, w_{BF})$ to initially represent ideal channel topography. For example, single-thread meandering rivers are a common planform and channel typology used in river restoration and S3 could be used as a guide for their creation. Based on subsequent flow, sediment transport, and habitat modeling, GCS relations can then be manipulated through simple parameter adjustments to meet specific stage-dependent ecohydraulic design criteria.

6.4.3 Historical reconstruction

This approach could also facilitate mechanistic modeling of historical reach-scale and morphological-unit scale landforms. For example, historical data often lack 3D detail making them difficult to visualize and analyze. While river synthesis cannot create completely unknown conditions, it could approximate landforms using the best available information, analogous to forensic facial reconstruction. For example, used in conjunction with historical maps and geologic data the synthesis of ancient channel morphology would allow an evaluation of aquatic habitats potentially present in these environments (e.g., Jacobson and Galat, 2006). Alternate approaches could use paleochannel information to determine the number, type, and subsequent characteristics of channels with geometric modeling used to create synthetic DEMs (Pyrcz et al., 2009).

6.4.4 Education

Synthetic river valleys could be a major tool in educating students on what makes rivers unique as topographic surfaces via active learning pedagogy. Renwick (1985) and Fonstad (2006) both suggested that LEMs and synthetic data have value in Earth sciences education. In particular, Fonstad (2006) argued that a stepwise progression in model complexity can be helpful for students to understand the roots and consequences of complexity. For the model presented herein, undergraduate students could build synthetic DEMs with varying GCS functions and then evaluate surfaces heuristically from hydrodynamic, geomorphic, ecologic, and civil engineering perspectives. In building a synthetic river, students also come to understand firsthand the basic mathematical properties of rivers relative to other landforms and civil structures, an insight that should always be appreciated. Meanwhile, graduate students can study similar river valley processes by combining geometric models with numerical flow and sediment transport modeling to understand relationships between channel geometry, sediment transport, and aquatic ecology (Pasternack, 2011).

6.5 Recommended advances

Recommended advances to further develop and improve geometric modeling of synthetic river valleys include the use of more sophisticated control curves (section 6.2), hierarchical modeling of scale-dependent variability, the inclusion of stochastic modeling, object-based modeling or import of nonfluvial elements (e.g., boulders, bridges, levees, and wood jams) as highly detailed, independent meshes or voxels, and the modeling of multithread channels (section 6.3). Hierarchical modeling is a preferred approach in simulating landscape complexity because scale-dependent topographic features can be explicitly modeled (Clarke, 1988; Werner, 1999). A hierarchical modeling approach that represents the total variance of each control function as the linear sum of scale-specific variance may afford insights into scale-dependent effects of river topography on habitat, sediment transport, and landscape change. Using control functions, such as Eq. (5), it could also be possible to simulate real or artificial river topography by identifying desirable scale-dependent harmonics of fluvial geometric elements (Clarke, 1988). In this case, one could determine the harmonics of a geometric element via spectral analysis and use that information to populate the sinusoidal parameters of the geometric model. The inclusion of object-based geometric models, such as individual boulders that can be scaled by flow, drainage area, or sediment type would allow for added layers of physical complexity required for microhabitat. Moreover, the use of spatially explicit patches of known bounds could be used to allow rapid synthesis and modification of surface roughness for modeling bedrock and alluvial surfaces.

7. Conclusions

This article introduced a geometric modeling framework for synthesizing prescribed river valley topography. A theoretical framework and workflow demonstrated how to create geometric models capable of generating adjustable topographic surfaces. The flexibility of the model was illustrated with six lowland and mountain topographic surfaces adjusted through independent and dependent GCS parameterization to create landforms of varying complexity. A key aspect of the model was the dependent parameterization of GCS relations between geometric elements that can be used to set the location of riffles and pools relative to the bankfull width and channel centerline, among other geometric elements. The approach presented herein can be used by geomorphologists for form–process inquiry and by engineers in designing river restoration projects. Thus, synthesis of prescribed river topography is a significant advance in understanding linkages between form, flow, ecology, and society.

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