Title
Microscopic Car Following Models Simulation Study

Permalink
https://escholarship.org/uc/item/8tx731bq

Author
Luong, Elaine

Publication Date
2018

License
CC BY 4.0

Peer reviewed|Thesis/dissertation
UNIVERSITY OF CALIFORNIA, IRVINE

Microscopic Car Following Models Simulation Study

THESIS

submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in Civil Engineering

by

Elaine Luong

Thesis Committee:
Professor Wenlong Jin, Chair
Professor R. Jayakrishnan
Professor Stephen Ritchie

2018
DEDICATION

To

my parents

for their guidance, patience, and support
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>ABSTRACT OF THE THESIS</td>
<td>vii</td>
</tr>
<tr>
<td>CHAPTER 1: Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Motivation for Study</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Thesis Outline</td>
<td>5</td>
</tr>
<tr>
<td>CHAPTER 2: Literature Study</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Notation</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Safe Distance Models</td>
<td>8</td>
</tr>
<tr>
<td>2.3 Optimal Velocity Models</td>
<td>10</td>
</tr>
<tr>
<td>2.4 Linear Models</td>
<td>12</td>
</tr>
<tr>
<td>2.5 Non-linear Models</td>
<td>15</td>
</tr>
<tr>
<td>2.6 Fritzsche Model</td>
<td>18</td>
</tr>
<tr>
<td>CHAPTER 3: Model Development</td>
<td>20</td>
</tr>
<tr>
<td>3.1 Model parameters and conditions</td>
<td>20</td>
</tr>
<tr>
<td>3.2 Procedure</td>
<td>21</td>
</tr>
<tr>
<td>CHAPTER 4: Simulation Results</td>
<td>24</td>
</tr>
<tr>
<td>4.1 Safe Distance Models</td>
<td>25</td>
</tr>
<tr>
<td>4.2 Optimal Velocity Models</td>
<td>27</td>
</tr>
<tr>
<td>4.3 Linear Models</td>
<td>29</td>
</tr>
<tr>
<td>4.4 Non-linear Models</td>
<td>32</td>
</tr>
<tr>
<td>4.5 Fritzsche Model</td>
<td>36</td>
</tr>
<tr>
<td>4.6 Overall Comparisons</td>
<td>36</td>
</tr>
<tr>
<td>CHAPTER 5: Summary and Conclusions</td>
<td>38</td>
</tr>
<tr>
<td>5.1 Limitations</td>
<td>39</td>
</tr>
<tr>
<td>5.2 Suggestions for Future Research</td>
<td>40</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>42</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Diagram of a car following model</td>
<td>2</td>
</tr>
<tr>
<td>3.1</td>
<td>Basic block diagram</td>
<td>22</td>
</tr>
<tr>
<td>3.2</td>
<td>Controller and plant for the intelligent driver model</td>
<td>23</td>
</tr>
<tr>
<td>4.1</td>
<td>The spacing (left) and velocity (right) data for the intelligent driver model</td>
<td>24</td>
</tr>
<tr>
<td>4.2</td>
<td>Velocity data for Pipes model</td>
<td>26</td>
</tr>
<tr>
<td>4.3</td>
<td>Velocity data for Gipps model</td>
<td>26</td>
</tr>
<tr>
<td>4.4</td>
<td>Velocity data for optimal velocity model</td>
<td>28</td>
</tr>
<tr>
<td>4.5</td>
<td>Velocity data for full velocity difference model</td>
<td>28</td>
</tr>
<tr>
<td>4.6</td>
<td>Velocity data for generalized force model</td>
<td>29</td>
</tr>
<tr>
<td>4.7</td>
<td>Velocity data for Newell model</td>
<td>31</td>
</tr>
<tr>
<td>4.8</td>
<td>Velocity data for follow the leader model</td>
<td>31</td>
</tr>
<tr>
<td>4.9</td>
<td>Velocity data for Helly model</td>
<td>32</td>
</tr>
<tr>
<td>4.10</td>
<td>Velocity data for Gazis, Herman, and Potts model</td>
<td>34</td>
</tr>
<tr>
<td>4.11</td>
<td>Velocity data for Edie model</td>
<td>34</td>
</tr>
<tr>
<td>4.12</td>
<td>Velocity data for May and Keller model</td>
<td>35</td>
</tr>
<tr>
<td>4.13</td>
<td>Velocity data for intelligent driver model</td>
<td>35</td>
</tr>
<tr>
<td>4.14</td>
<td>Velocity data for Fritzsche model</td>
<td>36</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Summary of General Motors model parameters</td>
<td>16</td>
</tr>
<tr>
<td>4.1</td>
<td>Summary of simulations</td>
<td>37</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

I would like to express my gratitude to everyone who has helped me on my academic journey. I would never have been able to complete my academic goals or dreams without the advice and help that I have received.

I would like to thank my committee chair, Professor Wenlong Jin for always being available to provide guidance. I hold a great appreciation for his generosity in offering research materials and possible solutions to any complications. He possessed an endless supply of advice and I greatly appreciate his willingness to meet in person whenever I encountered a setback.

I would like to thank my thesis committee members, Professor Stephen Ritchie and Professor R. Jayakrishnan, whose passion for research and recommendations for graduate school provided one of the guiding reasons for my decision to continue my studies.

Additionally, I thank the Pacific Southwest Region University Transportation Center (PSRUTC) for providing the financial support for my research and University of California, Irvine Institute of Transportation Studies and the Department of Civil and Environmental Engineering for offering me the Graduate Research Fellowship.
ABSTRACT OF THE THESIS

Microscopic Car Following Models Simulation Study

By

Elaine Luong

Master of Science in Civil Engineering

University of California, Irvine, 2018

Professor Wenlong Jin, Chair

This study aims to provide a comparison of different car following models in terms of safe distance, stability, and velocity. Although initially developed to model traffic, these algorithms can be applied to autonomous vehicles as a way to control following speed and acceleration. Each model was judged based on their performance in a theoretical ring road simulation. For the simulation, 22 vehicles were initially evenly spaced on a 230 meter length ring road. Every vehicle was assumed to follow the same car following algorithm and be connected to the leading vehicle in terms of information sharing. The following vehicle adjusted its acceleration and velocity based on stimuli such as relative velocity and spacing to the leading vehicle according to a car following algorithm. Most of the algorithms were able to produce adequate spacing between vehicles. The major differences between models were the average velocities and stability.
CHAPTER 1

INTRODUCTION

1.1. Background

Models are an abstraction of reality and can be used to study complicated real-life scenarios in a simplified manner. Using models to simulate and study different traffic scenarios is a cost effective and efficient way to determine the solutions to many traffic problems. Although driving is a common activity, human behavior on the roadway is complicated and difficult to understand. As a result, simulations and models have been used to study and predict driving behaviors in certain conditions. These include mathematical and theoretical models. Models can be classified as microscopic or macroscopic. Macroscopic models simulate characteristics of traffic flow including the overall average density, flow and speed. These models are flow-based. On the contrary, microscopic models can be described as vehicle-based (Bourrel and Lesort, 2003). Microscopic models simulate single units such as driver-vehicle units. Car following models are an example of microscopic simulation models. Each vehicle unit is modeled with
individual properties including the position, headway, and velocity and used to observe the following behavior individually in various scenarios.

Car following and cruise control algorithms are an important area of study as it enables a better understanding of traffic flow and driving behaviors in instances where vehicles are constricted into a single lane. The basic components of car following models include the characteristics, such as location, velocity, acceleration, and car length of the leading and following vehicle. This is illustrated in the diagram below where \( X(n) \) is the position of vehicle \( n \), the lead vehicle, and \( X(n+1) \) is the position of vehicle \( n+1 \), the following vehicle.

![Diagram of a car following model.](image)

**Figure 1.1: Diagram of a car following model.**

Car following simulations can further be classified as a stimulus-response-type model (Gerlough and Huber, 1976). These models offer a way to observe driver behaviors and reactions, which are limited to braking and accelerating. They offer a cost-effective way to relate theoretical experiments to real-world data to test a hypothesis. However, models can always be improved upon and thus, there have been many variations of car following models. Though originally developed to model driving behaviors, car-following models can be applied to connected and autonomous vehicles as an algorithm to safely and efficiently control a following vehicle.
1.2 Motivation for Study

Technological advances in vehicles are constantly improving and gaining recognition. Many companies in the automobile industry have been shifting their attention towards connected and autonomous vehicles. In recent years, interest and investments in connected and autonomous vehicles has grown. As research into connected and autonomous vehicles is gaining traction, so has public knowledge about its benefits and unfortunately, its drawbacks. There is a certain degree of mistrust in the public due mainly to viral reports about the ethics and dangers of autonomous vehicles. As a result, it is imperative that the algorithms in these self-driving vehicles be optimized. With the right algorithms and optimizations, autonomous vehicles can actually improve the safety of the transportation system as well as the efficiency.

According to the National Highway Traffic Safety Administration in a report from 2016, rear-end collisions are the most common type of accident, accounting for almost 30 percent of accidents throughout the nation (National Highway Traffic Safety Administration, 2016). In a study conducted by Virginia Tech Transportation Institute, it was determined that most of these accidents resulted from the leading vehicle moving at a very low velocity or being stopped. In most of these circumstances, the following vehicle was too close to the lead vehicle and did not leave adequate stopping distance between them. (Neale et al., 2005). Ironically, the roads on which these accidents occurred were rarely considered dangerous. Most of these accidents occurred on straight and level roads during the day. The accidents resulted from distracted driving. As long as some drivers continue to stay connected on their phones or wireless devices while driving, distracted driving behaviors will continue to cause accidents and decrease the safety of the streets. In
order to improve the safety of the roads, there is a need to decrease the number of accidents and distracted driving behaviors that occur.

Autonomous and connected vehicles will take the dangers of distraction completely out of the picture. These vehicles can obtain information about the leading vehicle such as speed or distance and adjust their own speed accordingly. Although they were initially developed to study traffic, car following models can be utilized as algorithms in autonomous vehicles to determine following accelerations and velocities. Many car-following models have been proposed with different safety and efficiency objectives in mind to model human behavior in traffic systems. For example, Pipe’s car following model requires that the vehicle maintain a safe braking distance between the lead and following vehicle. For every 10 miles per hour increase in velocity, an extra car space distance must be added between the lead and following vehicle (Pipes, 1953). Unfortunately, these algorithms cannot accurately depict human behavior as driving behaviors vary between each driver. The algorithms assume that all drivers behave in the same way with respect to the same stimulus. As a result, there is some error when applied to traffic studies. However, such algorithms, if applied to autonomous vehicles, can be used to control vehicle behavior and remove the error associated with random human behaviors. The different safety and efficiency objectives can be utilized to ensure that following vehicles provide safe and comfortable trips for all passengers. With a safe distance objective, vehicles will maintain a safe distance from leading vehicles and thus, there will be no need to use an extreme deceleration rate to prevent collisions. Accelerations and decelerations will be at a comfortable rate, thus increasing the safety and satisfaction of passengers.
In order to model real traffic as accurately as possible, several car following models have been tested and calibrated. These models have been calibrated using field and experimental data. This thesis serves to provide a comprehensive comparison of different car following models that have been proposed and improved upon through research and calibrations. Each model was developed with slightly different objectives, which will be examined in further detail in chapter 2. As most rear-end accidents occur on straight and level roads, the car following models will be simulated in a theoretical ring-road that will serve to illustrate the effects of vehicle behavior. For simplicity, each vehicle is assumed to follow the same car-following algorithm and is connected with the vehicle in front of it in terms of information sharing. The following vehicles will have access to information about the lead vehicle including the speed, acceleration, and location. For further simplicity, reaction times, minimum gap, and vehicle lengths are assumed to be the same for all vehicles.

1.3 Thesis Outline

A general outline of this thesis will be as follows. Chapter 1 will serve as an introduction into the motivation and relevance of the car following research. Chapter 2 will provide a detailed description of each of the car following models. The differing objectives and variables used in each model will be explained in detail. The models will be compared and grouped into similar objectives or equations. The third chapter will provide a description of the test and the variables that will be observed. The fourth chapter will provide the results from the experiments and what effects they have on the car following
model. Finally, the fifth chapter will present the conclusion and suggestions for future research.
CHAPTER 2

LITERATURE STUDY

Car following models have been developed and researched as a method to model driver behaviors. Different algorithms and objectives have been considered in order to maximize the safety and welfare of consumers. The models depicted in this thesis determine the acceleration of the following vehicle as a response to stimuli from the leading vehicle. The stimuli can be relative position, velocity, or other parameters. These models have been researched and improved upon since the 1950s (Siuhi and Kaseko, 2016). The sensitivity variables for these models were calibrated through data and research collected from traffic streams through the years.

2.1 Notation

Various papers on traffic theory utilize different sets of notations. As a result, it is imperative to describe the notation that will be used in this thesis. The leading vehicle will be denoted as \( n \) and the following vehicle as \((n+1)\). The location, speed, and acceleration of each vehicle is dependent upon the location and time and are written as \( x(n,t) \), \( x_t(n,t) \), and \( x_{tt}(n,t) \) respectively where \( t \) is the time. Across all models, the free-flow or desired velocity
is denoted as \( v_f \), the time gap is \( \tau \), the minimum gap is \( d \), the reaction time is \( T \), and the car length is \( L_n \).

### 2.2 Safe Distance Models

Some of the earlier car following models relied on leaving a large enough gap between vehicles to ensure safety by allowing an acceptable braking distance. The objective of these models are to provide a safe braking distance and as a result, these models are known as the safe distance models. AIMSUN, a traffic simulation software, uses the safe following distance model in their algorithms.

#### 2.2.1. Pipes Model

Some of the first car following models were developed from the California DMV safe distance rule which states that a general rule for allowing enough distance between vehicles is to leave the length of a car for every ten miles per hour velocity. One such model is the Pipes car following model (Pipes, 1953). Forbes improved upon this model by including a driver reaction time (Siuhi and Kaseko, 2016). This model, if based on the triangular fundamental diagram has the following model equation:

\[
x_t(n, t) = w(z)
\]

Where the general form \( W(z) \) is dependent upon the fundamental diagram. This equation can be rewritten in terms of vehicular location and reaction time.

\[
x_{tt}(n + 1, t) = \min \left[ v_f, \frac{\Delta x(n, t) - L_n}{\tau} \right]
\]

This model is similar to the optimal velocity model which will be described in the later sections of this chapter.
2.2.2. Gipps Model

The Gipps car following model is a successful collision avoidance car following model that is commonly referenced (Gipps, 1981). A modified version is used in Aimsun traffic modelling software. This model includes the reaction time for drivers and maximum acceleration and deceleration rates. It is considered to be one of the more complicated models as it takes into account several parameters that correspond to different driver behaviors. For simplicity, the reaction time for each vehicle is assumed to be the same. The velocity equation for the model is as follows:

\[
x_t(n+1,t+\tau) = \min \left\{ x_t(n+1,t) + 2.5 * a\tau \left( 1 - \frac{x_t(n+1,t)}{v_f} \right) \left( 0.025 + \frac{x_t(n+1,t)}{v_f} \right)^{\frac{1}{2}}, \right. \\
\left. b\tau + \sqrt{b^2 \tau^2 - b \left( 2(\Delta x(n,t) - L_{n} - d) - x_t(n+1,t)\tau - \frac{x_t(n,t)^2}{b} \right)} \right\}
\]

Where \( b \) is the maximum deceleration rate and \( a \) is the maximum acceleration rate. If the vehicle has a large gap, the first expression is the minimum and thus, the vehicle accelerates to the desired speed. This model also ensures that the maximum deceleration can be used without compromising the minimum headway between the two vehicles.

The simplified acceleration model based on the triangular fundamental diagram and in steady states is as follows:

\[
x_{tt}(n+1,t) = \left( \frac{1}{T} \right) \left[ \min \left( v_f, \frac{1}{T} \left( \Delta x(n,t) - L_n - d + \frac{x_t^2(n,t)}{2b} - \frac{x_t^2(n+1,t)}{2b} \right) \right) \right] - x_t(n+1,t)
\]

Gipps model assumes that the following vehicle does not begin the braking process until the reaction time has passed. As a result, this results in a safety margin assumed to be
half the reaction time that accounts for when the following vehicle is unable to brake or accelerate at the maximum rate.

### 2.3 Optimal Velocity Models

The optimal velocity model (Bando et al., 1995) has been helpful in explaining the traffic flow behaviors. This model utilizes an optimal velocity, $V(s)$ that is determined by the driver based on the gap between the following and lead vehicle. Generally, there is a difference between this optimal velocity and the velocity at which the vehicle is traveling. The driver notes the difference and attempts to minimize it by accelerating at a rate proportional to this difference and a sensitivity parameter. This model can be written as:

$$x_{tt}(n, t) = \lambda (V(s) - x_t(n, t))$$

where $\lambda$ is the sensitivity parameter and $V(s)$ is the desired velocity based on the gap.

Helbing and Tilch calibrated this model and developed the optimal velocity through empirical data as the following:

$$V(s) = V_1 + V_2 \tanh(C_1 (\Delta x(n, t) - L_n) - C_2)$$

In their calibrations, they determined that the optimal parameter values are $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13$ m$^{-1}$, $C_2 = 1.57$, and $\lambda = 0.85$ s$^{-1}$. A noted downfall of this model when compared to field data is that the optimal velocity model can cause unrealistic acceleration and deceleration rates (Jiang et al., 2001). However, it has been used to depict several characteristics of traffic flow including the instability of traffic flow and traffic congestion.

The Optimal velocity model is similar to Pipes model when modified after the triangular fundamental diagram.
\[ x_{tt}(n, t) = \frac{w(z) - x_t(n, t)}{T} \]

The optimal velocity acceleration model can be simplified in terms of vehicular location and speed into the following equation:

\[ x_{tt}(n + 1, t) = \left( \frac{1}{T} \right) \left( \min \left[ v_f, \frac{\Delta x(n, t) - L_n}{\tau} \right] - x_t(n + 1, t) \right) \]

Many car following models have been developed with the optimal velocity model as a sub-model, for example, the generalized force model and the full velocity difference model. Both these models consist of the optimal velocity model with the incorporation of additional terms.

### 2.3.1. Full Velocity Difference Model

The full velocity difference model improves upon the optimal velocity model by including an additional term. The model equation is as follows:

\[ x_{tt}(n + 1, t) = \left( \frac{1}{T} \right) \left( \min \left[ v_f, \frac{\Delta x(n, t) - L_n}{\tau} \right] - x_t(n + 1, t) \right) + \lambda(\Delta x_t(n, t)) \]

The additional term accounts for both the positive and negative velocity differences. The sensitivity term \( \lambda \) has been experimented with different forms including a constant or step function. The step function can be written as

\[ \lambda = \begin{cases} x, & s \leq s_c \\ y, & s > s_c \end{cases} \]

where \( x, y, \) and \( s_c \) are constraints to be determined. The parameters used to simulate the full velocity difference model are \( x = 0.5 \text{ sec}^{-1}, y = 0, \) and \( s_c = 100 \text{ meters}. \) Since the headway for the simulation will not exceed \( s_c = 100 \text{ meters}, \) the sensitivity term can be set to a constant \( 0.5 \text{ sec}^{-1} \) instead of relying on a step function (Jiang et al., 2001).
2.3.2. Generalized Force Model

The optimal velocity model was a bit flawed in that it led to unrealistic acceleration and deceleration rates. In order to combat this problem, Helbing and Tilch proposed a new car following model: the generalized force model. This model specifies effective acceleration and deceleration variables. This model utilizes the optimal velocity model as its base and adds on additional parameters. The parameters are easily calibrated based on vehicular speeds, weather, and speed limit. Using the optimal velocity model based on the triangular fundamental diagram, the generalized force model is as follows:

\[
x_{tt}(n + 1, t) = \left(\frac{1}{T}\right) \left(\min\left[v_f, \frac{\Delta x(n, t) - L}{\tau}\right] - x_t(n + 1, t)\right) + \frac{\Delta x_t(n, t)\Theta(-\Delta x_t(n, t))}{T^{*'}} e^{-\frac{(\Delta x(n, t) - (d + \tau x_t(n + 1, t)))}{R'}}
\]

Where \(T^{*'}\) is the braking time, which is less than the acceleration time, \(R'\) is the range of braking interaction, and \(\Theta(\cdot)\) is the Heaviside function. A calibration based on follow-the-leader data determined that the optimal parameters are \(T^{*'} = 0.77\) seconds and \(R' = 98.78\) meters (Helbing and Tilch, 1998).

2.4. Linear Models

Linear car following models typically define the acceleration as a linear function that comprises of the speed and/or location differences between leading and following vehicles. The most commonly referenced linear models includes the Newell model, follow-the-leader model, and the Helly model.
2.4.1 Newell Model

The Newell model analyzes car-following behavior through the use of the time-space trajectory (Newell, 2002). It is assumed that the trajectory for both the following and leading vehicle is the same, merely translated in time and space. It is also assumed spacing and velocity vary linearly.

\[ T_n x_{tt}(n + 1, t) = \frac{1}{\tau} (\Delta x_n(n, t) - d) - x_t(n + 1, t) \]

Where \( T_n \) is the relaxation time that is equivalent to \( \tau/2 \). This model is similar to the optimal velocity model with the inclusion of the gap rather than only the car length. This model also closely resembles the linear General Motors model, which will be covered in a later section of this chapter.

This thesis will focus on the linear Newell model, however, a stochastic version of this model has been proposed in which speed dependent stochastic parameters are added (Tian et al., 2016). The model is as follows.

\[ \bar{v}(n, t + \tau) = \min\left( v(n, t) + a_n \tau, v_f, \frac{\Delta x(n, t) - d - L_n}{\tau} \right) \]

\[ x_t(n, t + \tau) = \max(\bar{v} - a_n H(r, p_n) \tau, 0) \]

\[ a_n = \alpha \ast \text{rand}(0,1) \]

\[ H(r, p_n) = 1 \text{ if } r < p_o \]

\[ r = \text{rand}(0,1) \]

\[ p_o = f(x) = \left\{ \begin{array}{ll}
 p_{\text{STR}}, & \text{if } v(n, t) < \alpha \tau \text{ and } d(n, t) < d_{\text{stop}} \\
 \frac{p_{\text{max}}}{1 + e^{-k\left(\frac{v(n,t)}{v_f} - \gamma}\right)}, & \text{otherwise}
 \end{array} \right. \]

This stochastic model produces results consistent to NGSIM data.
2.4.2. Linear Helly Model

The Helly model (Helly, 1959) is a car following model that is comprised of linear equations. Helly proposed this model by claiming that the following vehicle seeks to minimize headway and the velocity difference. The simplified Helly model equation is as follows.

\[ x_{tt}(n + 1, t) = \lambda_k(\Delta x_n - d - \tau x_t(n + 1, t)) + \lambda_y(\Delta x_t(n, t)) \]

Where \( \lambda_k \) is the sensitivity parameter with respect to distance and \( \lambda_y \) is the sensitivity parameter with respect to velocity. For simplicity in this model, delay and brake factors, which are included in the original model is ignored. One disadvantage of this model is that if the gap is infinitely large, the vehicle accelerates indefinitely, allowing its velocity to reach infinity (Taniguchi et al., 2015). The sensitivity parameters have been calibrated as \( \lambda_k = 0.2 \text{ sec}^{-2} \) and \( \lambda_y = 0.6 \text{ sec}^{-1} \). These constants were calibrated through data observed from the performance of 14 vehicles. This model is similar to the follow-the-leader model, which is covered in the next section, with the addition of a location difference or safety distance parameter.

2.4.3. Follow-the-Leader Model

The follow-the-leader model, also known as the linear General Motors model or Chandler et al. is another simple car following model (Chandler et al., 1958). The model equation is as follows:

\[ x_{tt}(n + 1, t) = \lambda(\Delta x_t(n, t)) \]

Where the sensitivity parameter \( \lambda \) is suggested to be \( 0.37 \text{ sec}^{-1} \). This model which was proposed by Chandler, Herman, and Potts at the General Motors research laboratories, was proposed as a result of two core beliefs. These beliefs are that the higher velocities result in
higher spacing and that a safe distance must be maintained between vehicles at all times (Matthew, 2014). They found that velocities impacted accelerations and that spacing was not significant in determining acceleration. However, there have been some weakness in this model that have been identified. This model does not consider different following behaviors that occur due to different traffic situations. The tests for this model utilized low speed or stop and start traffic conditions, and as a result, may not be able to accurately model other traffic conditions.

2.5 Non-Linear Models

Non-Linear models, with the exception of the intelligent driver model, tend to follow the General Motors model. This model depicts driving behavior in the following equation:

\[ x_{tt}(n + 1, t) = cx_t^m(n + 1, t) \left( \frac{\Delta x_t(n, t)}{\Delta x_t'(n, t)} \right) \]

where \( c, m, \text{ and } l \) are perimeters that are to be determined through calibrations. This model was first developed by Chandler et al. in what is known as the follow the leader model. This model was discussed in the above section as the linear form of the General Motors model. The sensitivity variable was calibrated using wire-linked vehicles to determine the reactions of the following vehicles (Ossen, 2005).

This model is a popular car-following model as it closely resembles field data and speed-density relationships can be derived from this model. The basics behind this model is that acceleration is viewed as a response to a stimulus that the driver responds to. This belief is similar to Newton’s laws of motion. The mechanics behind Newton’s law describes the acceleration of a vehicle being a response to stimuli that it receives from external forces.
as well as interaction with other vehicles in the observed system. This stimulus can be related to the actual velocities, relative velocities, headway, or and sensitivity parameters. The linear model takes only the relative velocities into account, however, several models have been proposed that takes a combination of the stimuli listed above into consideration. MITSIM, a traffic simulator, uses this type of algorithm in their car following model. A table summary of the coefficients used for each General Motors model is shown below.

<table>
<thead>
<tr>
<th>Model</th>
<th>$m$</th>
<th>$l$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear General Motors</td>
<td>0</td>
<td>0</td>
<td>0.37</td>
</tr>
<tr>
<td>Gazis, Herman, Potts</td>
<td>0</td>
<td>1</td>
<td>14.62</td>
</tr>
<tr>
<td>Edie</td>
<td>1</td>
<td>1</td>
<td>0.99</td>
</tr>
<tr>
<td>May and Keller</td>
<td>0.8</td>
<td>2.8</td>
<td>$1.33 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of General Motors model parameters.

### 2.5.1. Gazis, Herman, Potts Model

Gazis, Herman, and Potts improved upon this model in 1959 when they found that the linear model did not accurately depict situations for which higher densities were encountered in traffic (Gazis et al., 1959). In higher densities, it was not accurate to assume that drivers behave as a response only to relative velocities. As a result, a new model was proposed, which included an additional parameter.

$$x_{tt}(n + 1, t) = c \left( \frac{\Delta x_t(n, t)}{\Delta x(n, t)} \right)$$

The relative spacing was added to the linear model in order to account for driver behaviors in dense traffic, which rely more on relative spacing rather than relative velocity. Through studies
involving vehicle trajectory data obtained from images taken by a helicopter, the optimal c value
for peak flow traffic is calibrated to be 14.62 (Ossen, 2005).

2.5.2. Edie Model

Edie further improved upon the model by including the velocity of the following
vehicle, resulting a model that includes all the parameters in the general form of the
General Motors algorithm (Edie, 1961).

\[ x_{tt}(n + 1, t) = cx_t(n + 1, t) \left( \frac{Ax_t(n, t)}{Ax(n, t)} \right) \]

Edie calibrated his model through comparisons to macroscopic data. The sensitivity parameter,
when calibrated through macroscopic data obtained from high-frequency images, is determined
to be 0.99 (Ossen, 2005).

2.5.3. May and Keller Model

Gazis, Herman, and Rothary studied the general form with Edie’s added velocity
parameter and introduced the exponential variables l and m, resulting in the general non-linear
form of the model. Thus, the General Motors car following model is often referred to as the
Gazis-Herman-Rothary (GHR) model. While the previous two models either chose 0 or 1 as the
exponential values for m and l, May and Keller took the model a bit further by calibrating the
values to be 0.8 and 2.8 respectively (May and Keller, 1967).

\[ x_{tt}(n + 1, t) = cx_t^m(n + 1, t) \left( \frac{Ax_t(n, t)}{Ax^l(n, t)} \right) \]

Data obtained from the Eisenhower Expressway, including speed and density, was used to
calibrate the optimal l, m, and c parameters. Regression analysis has determined that the optimal
value for c can be taken as $1.33 \times 10^{-4}$. 
2.5.4 Intelligent Driver Model

The intelligent driver model is a nonlinear car following model that has been described as the simplest accident-free model which produces acceleration behaviors that closely mimics realistic scenarios for single-lane traffic. This model does not follow the general form of General Motors as in the other nonlinear models depicted prior. The intelligent driver model formulation is as follows.

\[ x_{tt}(n + 1, t) = a \left( 1 - \left( \frac{x_t(n + 1, t)}{v_f} \right)^\delta \right) - \left( \frac{d + \tau x_t(n + 1, t) + \frac{x_t(n + 1, t) \Delta x_t(n, t)}{2(\sqrt{ab})}}{\Delta x(n, t) - L_n} \right)^2 \]

where \( \delta \) is typically taken to be 4. This is a time-continuous model that can be considered a part of the optimal velocity-type models (Malinauskas, 2014). This model does not typically exhibit abnormal data such as negative velocities or spacing. As a result, it can be considered a relatively safe and accurate model.

2.6 Fritzsche Model

A Psycho-physical model, also known as an action point model, uses thresholds for changes in driving behavior. It is assumed that drivers only react when the thresholds are perceived. These thresholds can be relative velocities or spacing. VISSIM, a traffic simulation software, utilizes a psycho-physical model in their algorithms. The Fritzsche model utilizes four spacing thresholds and two relative velocity thresholds to determine driver behavior.

The two relative velocity thresholds are for driver perception of negative and positive relative speeds. These differences must be a certain level for the driver to respond. The distance thresholds are for desired distance, risky distance, safe distance, and braking distance. The
desired distance is the distance that the driver wishes to keep. The risky distance requires the
driver to decelerate to avoid a collision. The safe distance is the smallest distance in which the
driver accelerates. The braking distance is used to prevent collisions in the instance where
maximum deceleration may be limited.

There are five actions that the driver can take, which are split into different regimes. The
danger zone is when distance is smaller than the risky distance. In this scenario, the driver
utilizes the maximum deceleration rate. The closing in regime are for distances between the
braking distance or desired distance and the risky distance. In this regime, the vehicle decelerates
to match the velocity of the leading vehicle. The acceleration equation for the vehicle is as
follows.

\[
x_{tt}(n + 1, t) = \frac{x_t^2(n, t) - x_t^2(n + 1, t)}{2(\Delta x(n, t) - d + x_t(n + 1, t))}
\]

The following 1 regime is entered when speed differences are between the two thresholds
and distance is between the desired and risky distance. In this instance, the driver does not make
any conscious action. The following 2 regime is for headways larger than the desired distance
with a speed larger than the thresholds. The driver does not take any action in this regime as
well. The final regime is the free driving regime. In this instance, the speed difference is smaller
than the thresholds and the distance is larger than the desired distance. The vehicle accelerates to
the desired speed.
CHAPTER 3

MODEL DEVELOPMENT

The car following models as described in chapter 2 were each modeled in Simulink in order to compare the advantages and disadvantages of each model. Each car following model was simulated using the same scenario and observed for stability and anomalies.

The scenario chosen was modeled after the shockwave traffic jam experiment first introduced by researchers from Japanese universities (Glaskin, 2008). This ring road is comparable to real data as it closely imitates the shockwaves that are typically present on highways. Although the vehicles are evenly spaced, random driving behaviors can lead to congestion in certain areas of the road or possibly back-traveling in some models. For simplicity, the simulation will take place on a single-lane road. As a result, lane-changing behaviors will not be taken into account in the simulation.

3.1 Model Parameters and Conditions

As in the traffic jam experiment in Japan, the simulation for this thesis features a ring road with a circumference of 230 meters. 22 vehicles were evenly placed throughout the ring road. For each simulation, all the vehicles were assumed to be connected and
utilize the same car-following behavior in determining their accelerations. The following vehicles received feedback information about the leading vehicle’s velocity, position, and acceleration. The follower vehicles used the algorithms for that simulation to adjust their velocity and acceleration as needed based on the stimulus.

With an initial velocity set between 5 to 10 meters per second, the vehicles were allowed to run their course. In order to accurately compare the models, the same parameters were used through all the models. These parameters include the free-flow speed, maximum deceleration rate, maximum acceleration rate, car length, and minimum gap. The free-flow or desired velocity was set to 26 m/s. For the models that require maximum deceleration or acceleration rates, the maximum deceleration rate was set at a comfortable 1.5 m/s$^2$ and the maximum acceleration rate was set to 1 m/s$^2$. The car length is assumed to be 4.8 meters, which is the average length of a mid-sized car, and the minimum gap is assumed to be 2.2 meters, resulting in a minimum spacing of 7 meters. Vehicles will not move unless the minimum gap is greater than 2.2 due to safety reasons. If the spacing between vehicles is less than 7 meters, there is a chance that an accident may occur. Each model will be simulated for 1000 seconds. The spacing between each vehicle as well as the velocities was plotted for the entire 1000 seconds in order to fully observe the stability of the model.

### 3.2. Procedure

The Simulink models require the use of a block diagram. These block diagrams require the input values or reference state, control, plant, and feedback if necessary. A
general block diagram is shown in the figure below. This block diagram is the basic building block of the model. Each block diagram represents one vehicle. As all the vehicles are connected, the output of a single vehicle's block diagram feeds into the input of the following vehicle's block diagram.

![Figure 3.1: Basic block diagram.](image)

The input for the diagrams consists of the lead vehicle's speed, position, or minimum headway. The controller would designate the following vehicle's response. In the case of the car following models, it would be the acceleration of the vehicle as a response to a stimulus presented by the input. The figure above utilizes a PID controller, which stands for proportional-integral-derivative controller. This controller, when presented with an input value of headway, results in a linear combination of headway, relative velocity, and absement. This is a common controller in many car following models such as the optimal velocity model and the Helly model. However, not all models follow the simple PID controller. The figure below depicts the controller and plant for the intelligent driver model.
Another component of the car following block diagram is the plant. The plant dynamics depict the input and output relationship between the control signal and the state variable. In the case of a car following model, the control signal would be the acceleration rate and the state variable would be the velocity. As a result, the plant would be an integration of the acceleration rate that results from the stimulus. An additional integration of the velocity will result in an output of the vehicle’s trajectory, which can lend itself as an input value for the following vehicle.

Once the models were completed and simulated, any anomalies were noted and the models were compared. In order to counter unrealistically high velocities or negative velocities, the velocity ranges were restricted to be between 0 m/s and the free-flow speed, 26 m/s. The results were used to determine the advantages or disadvantages in terms of safety and efficiency of each model. This will be further discussed in the later chapters.
CHAPTER 4

SIMULATION RESULTS

Each car following model was simulated in the ring road scenario for 1000 seconds. The trajectory, spacing, and velocity with respect to time was plotted for each model. The models were compared by “family,” for example, the safe distance models, and with respect to the other models. The spacing data was closely correlated with the velocity data. Similar patterns were observed on both plots. As a result, only the velocity plots will be shown. The figure below illustrates the similarities between the spacing and velocity plots.

![Figure 4.1: The spacing (left) and velocity (right) data for the intelligent driver model.](image)

Figure 4.1: The spacing (left) and velocity (right) data for the intelligent driver model.
4.1. Safe Distance Models

In terms of the safe distance models, Gipps model was both safer and more efficient than the Pipes model. This is to be expected as the Pipes model is regarded as one of the earlier car following models. Initially, the Pipes model showed negative velocities and trajectories, which is both unrealistic and unsafe. With the addition of velocity ranges, the data improved, but did not stabilize.

Gipps model showed more stability when compared to Pipes model. The velocity stabilized at around 50 seconds at around 2.5 m/s. The spacing between each vehicle was maintained at around 10.5 meters, which is well above the 7 meters minimum. As a result, the Gipps model, is a safe model that can be improved in terms of efficiency. Between the safe distance models tested, Gipps model is superior to Pipes model. This is to be expected as Pipes model was one of the earlier models to be developed. Gipps model was developed after Pipes model and includes several additional parameters such as maximum deceleration rate and velocity, which in turn, resulted in data with more stability and safety. The velocity plots for Gipps and Pipes model is shown below.
Figure 4.2: Velocity data for Pipes model.

Figure 4.3: Velocity data for Gipps model.
4.2. Optimal Velocity Models

The optimal velocity models all exhibited an average velocity of 2.5 m/s. The original optimal velocity model experienced the most severe velocity fluctuations of the three optimal velocity models between 2 m/s and 3 m/s. The full velocity difference model experienced smaller fluctuations between 2.5 m/s and 2.6 m/s once the simulation reached the 50 second mark. The generalized force model achieved a stabilized velocity of 2.5 m/s at around 150 seconds into the simulation.

The spacing results from these models exhibited similar patterns to their respective velocity results. The original optimal velocity model experienced severe fluctuations between 9.8 meters to 11.5 meters. This fluctuation appears throughout the entire simulation and does not seem to stabilize. The full velocity difference model experienced smaller spacing fluctuations between 10.3 meters and 10.6 meters once the simulation reached 50 seconds. The spacing results from the generalized force model stabilized at 150 seconds with a constant spacing of 10.5 meters.

These results are expected as the full velocity difference model is essentially the original optimal velocity model with the added parameter of relative velocity. This added parameter adds some stability into the results. However, the best optimal velocity type model would be the generalized force model. The added parameters of this model achieves stabilization in both the velocity and spacing results, something the other two models failed to achieve.
Figure 4.4: Velocity data for optimal velocity model.

Figure 4.5: Velocity data for full velocity difference model.
4.3. Linear Models

Of the Linear models, Newell’s linear model showed the most velocity fluctuations throughout the simulation. The velocity fluctuated between 2 and 3 m/s and did not seem to stabilize. The linear General Motors model showed velocity fluctuations in the first 250 seconds of simulation, but the fluctuations lessened as time progressed to 8.8 m/s. The Helly model also showed some fluctuations in velocity in the first 250 seconds of the simulation, but eventually stabilized to 6.1 m/s. The linear General Motors model was able to achieve the highest average velocity, followed by Helly, and finally, Newell.

The spacing results also showed similar patterns to the velocity results. Newell’s model had fluctuations between 9.8 meters to 11 meters throughout the entire simulation.
The linear General Motors model showed some fluctuations between 8 meters and 11 meters of spacing in the first 250 seconds of simulation, but the fluctuations smoothed to around 9.8 meters. The Helly model had larger fluctuations in the first 250 seconds, but eventually stabilized to slight fluctuations around 10.4 meters. The linear General Motors model showed the highest spacing values, followed closely by Newell, and finally, the Helly model.

Of the linear models, the linear General Motors model allowed the vehicles to reach the highest average velocity of 8.8 m/s but produced the lowest following distance. The Helly model showed the second highest average velocity of 6.1 m/s and the highest average spacing of 10.4 meters. The Newell model was the least stable of the linear models, it achieved a high following distance but unfortunately resulted in the lowest average velocity of 2.5 m/s. Additionally, the fluctuations and instability for this model is an issue. As a result, of the linear car following models, the Helly model shows the most promise. It allows the following vehicles to maintain a large safe distance from the leading vehicle while also allowing the vehicle to travel at a relatively high velocity.
Figure 4.7: Velocity data for Newell model.

Figure 4.8: Velocity data for follow the leader model.
4.4. Non-linear Models

May and Keller’s model showed a relatively steady velocity around 6 m/s. The Gazis, Herman, and Potts model experienced some fluctuations in velocity in the first 100 seconds between 6.6 m/s and 7.5 m/s but stabilized to an average of around 7.5 m/s. Edie’s model experienced fluctuations between 7 m/s and 8 m/s in the first 100 seconds of simulation. However, the fluctuations lessened to an average of 7.6 m/s. The intelligent driver model showed great fluctuations between 2 m/s and 13 m/s throughout the entire simulation with an average of around 7.5 m/s.

May and Keller’s model resulted in spacing less than the minimum 7 meters in some vehicles and as a result, it an unsafe model to use. The Gazis, Herman, and Potts model had
some spacing fluctuations between 8.5 meters and 9.5 meters in the first 100 seconds of simulation but stabilized at an average of 9 meters. Edie’s model also had some spacing fluctuations between 9.8 meters and 11.5 meters but becomes relatively stable with an average spacing of 10.8 meters. The intelligent driver model experienced great fluctuations in spacing between 10 meters and 30 meters.

In terms of safety distance, the intelligent driver model exhibits the greatest spacing and is, thus, the safest. However, this model is very unstable as it closely follows stop-and-go traffic, and the velocity and spacing fluctuates greatly throughout the simulation. As May and Keller’s model shows spacing less than the minimum of 7 meters, this model is not as safe as the other non-linear models. Gazis, Herman, and Potts shows a stable velocity at 7.5 m/s and an average spacing of 9 meters, which is above the minimum spacing requirement. This model produces results that are more stable than the intelligent driver model and Edie’s model. However, Edie’s model produced an average spacing of 10.8 meters, which in turn, makes it a safer model in terms of following distance. As a result, of the non-linear models, the Gazis, Herman, and Potts model is the optimal model in terms of stability while the intelligent driver model shows promise in safe following distance.
Figure 4.10: Velocity data for Gazis, Herman, and Potts model.

Figure 4.11: Velocity data for Edie Model.
Figure 4.12: Velocity data for May and Keller model.

Figure 4.13: Velocity data for intelligent driver model.
4.5. Fritzsche Model

The Fritzsche Model experienced some velocity fluctuations between 6.5 m/s and 7.5 m/s in the first 100 seconds of simulation. However, these fluctuations lessened and averaged around 7 m/s. A similar pattern is observed in the spacing results. There are larger fluctuations between 9 meters and 10 meters in the initial 100 seconds of simulation. However, the fluctuations decrease and average around 9.5 meters of spacing between vehicles. The velocity data for this model is shown below.

![Fritzsche Model Velocity Data](image)

**Figure 4.14: Velocity data for Fritzsche model.**

4.6. Overall Comparisons

Most of the models were able to produce spacing greater than the minimum allowed 7 meters. As a result, they are adequate in providing a safe distance. As the ring road does
not allow for a large spacing between vehicles, most of the velocities were relatively low in this simulation. The highest velocity of 13 m/s as well as the highest following distance of 30 meters was produced in the intelligent driver model simulation. However, this model also produced the lowest velocity of 2 m/s due to the stop-and-go nature of the algorithm, which is not very efficient. The second highest velocity occurred in the Newell model. Unfortunately, this model does not stabilize during the 1000 second simulation. A summary of the velocity, spacing, and stability in each model is summarized in the table below.

<table>
<thead>
<tr>
<th>Model</th>
<th>Velocity (m/s)</th>
<th>Spacing (m)</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipes</td>
<td>2.5</td>
<td>10</td>
<td>none</td>
</tr>
<tr>
<td>Gipps</td>
<td>2.5</td>
<td>10.5</td>
<td>70 seconds, relatively stable</td>
</tr>
<tr>
<td>Optimal velocity</td>
<td>2-3</td>
<td>9.8-11.5</td>
<td>none</td>
</tr>
<tr>
<td>Velocity difference</td>
<td>2.5</td>
<td>10.3-10.6</td>
<td>50 seconds, but some instability still present</td>
</tr>
<tr>
<td>Generalized force</td>
<td>2.5</td>
<td>10.5</td>
<td>150 seconds, relatively stable</td>
</tr>
<tr>
<td>Newell</td>
<td>2-3</td>
<td>9.8-11</td>
<td>none</td>
</tr>
<tr>
<td>Helly</td>
<td>6.1</td>
<td>10.4</td>
<td>250 seconds, relatively stable</td>
</tr>
<tr>
<td>Follow the leader</td>
<td>8.8</td>
<td>9.8</td>
<td>250 seconds, relatively stable</td>
</tr>
<tr>
<td>Gazis, Herman, Potts</td>
<td>7.5</td>
<td>9</td>
<td>100 seconds, relatively stable</td>
</tr>
<tr>
<td>Edie</td>
<td>7.6</td>
<td>10.8</td>
<td>100 seconds, but some instability still present</td>
</tr>
<tr>
<td>May and Keller</td>
<td>6</td>
<td>Has negative spacing</td>
<td>Gradual increase</td>
</tr>
<tr>
<td>Intelligent driver</td>
<td>2-13</td>
<td>10-30</td>
<td>None</td>
</tr>
<tr>
<td>Fitzsche</td>
<td>7</td>
<td>9.5</td>
<td>100 seconds, but some instability still present</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of simulations.
CHAPTER 5

SUMMARY AND CONCLUSIONS

There are many advantages for studying car following behavior. With the technological advances in autonomous and connected vehicles, it is imperative to ensure that the algorithms used in determining the vehicle’s speed and acceleration result in safe and efficient traffic flows.

The many car following algorithms developed in the past to study traffic behaviors is a unique and useful tool in determining the optimal methods for controlling the vehicle’s behavior. The car following algorithms take into account information from the leading vehicle such as velocity and spacing. As connected vehicles are readily able to share this information through vehicle-to-vehicle communication, these algorithms can prove useful in creating a safe and comfortable trip for passengers utilizing these connected and autonomous vehicles.

From the results, it can be determined that the best model in terms of speed and following distance is the intelligent driver model. This model resulted in the greatest spacing between vehicles of 30 meters as well as the largest velocity of 13 m/s at times. Unfortunately, the stop-and-go nature of this algorithm results in an inefficient traffic flow.
system. Most of the models with the exception of May and Keller, were able to obtain following distances around 10 meters, which is approximately the initial evenly spaced value. Vehicles were able to travel at a greater distance than the minimum spacing, and as a result, it is unlikely for an accident to happen. A proper balance between efficiency and safety is important. Passengers value time and it is necessary to reach their destination in a timely and comfortable fashion, otherwise they may choose other means of transportation or utilize unsafe driving behaviors to meet their schedules. However, leaving an adequate gap between vehicles can help reduce accidents and provide comfortable deceleration rates for passengers.

5.1 Limitations

Despite these results, there are some limitations in this research that need to be addressed. The simulations performed for each model was a theoretical ring road. The ring road is a good indicator of how connected vehicles react to information such as the velocity or spacing from leading vehicles. Although this ring road has been shown to closely simulate real scenarios for a single lane road without any outside disturbances, it fails to take into account outside disturbances or randomness. Each vehicle was assumed to be equal in length and following the same algorithms. In the real world, this is rarely the case.

Another limitation is that the sensitivity parameters were taken from calibrations done by other researchers using different simulations or traffic data. As a result, the parameters were chosen based on different optimization methods, such as speed or safety. Further calibrations may result in better data for some of the models’ results in this particular scenario.
Another limitation is the assumption that all the cars in the scenario utilize the same car following algorithm. In earlier car following research, the randomness of human behavior made it difficult to pinpoint the perfect algorithm to study traffic. Different people react to different stimuli, whether it be a certain following distance, relative velocity, or other information.

### 5.2 Suggestions for Future Research

As stated above, the models fail to take into account different vehicle characteristics. Every vehicle is assumed to be the same, in both physical and driving characteristics. Future research can delve into incorporating other elements such as differing vehicle lengths and mixed traffic. Another improvement can be including lane changing elements into the model. This may be difficult, and further research may be needed to accurately portray human following behaviors.

Additionally, different autonomous vehicle companies as well as different consumers may value different characteristics for their algorithms, whether it be efficiency, safety, or a combination of other qualities. As a result, it may not be accurate to assume that all vehicles on the road will utilize the same driving behavior. Further research into how the model reacts in a mixed car following behavior environment may be necessary to determine the optimal model. With safety as a priority, the following vehicle must leave a large gap in order to allow for a comfortable braking distance and collision avoidance. However, leaving a large gap may result in a less than optimal velocity profile and thus, hinder efficiency. Each algorithm results in different acceleration and velocity profiles for the vehicles in the traffic stream. Consequently, these profiles influence the emission rates.
and energy consumption of the platoon of vehicles. As there are policies and standards in effect to curb emissions, energy efficiency may be a priority to companies, especially when marketing to larger cities where pollution poses a problem. As a result, the energy and emissions rates should be investigated for each model to determine the most energy efficient algorithm.

There is also a stability issue in many of the models, such as Pipes model. Although the same base model was used for the General Motors models, the May and Keller model produced anomalies in the spacing results while Edie and Gazis, Herman, and Potts model did not. These instabilities and anomalies can be due to the accumulation of numerical errors or bugs. Further research is needed to determine the cause of the instability and possible solutions to combat this issue.
REFERENCES


