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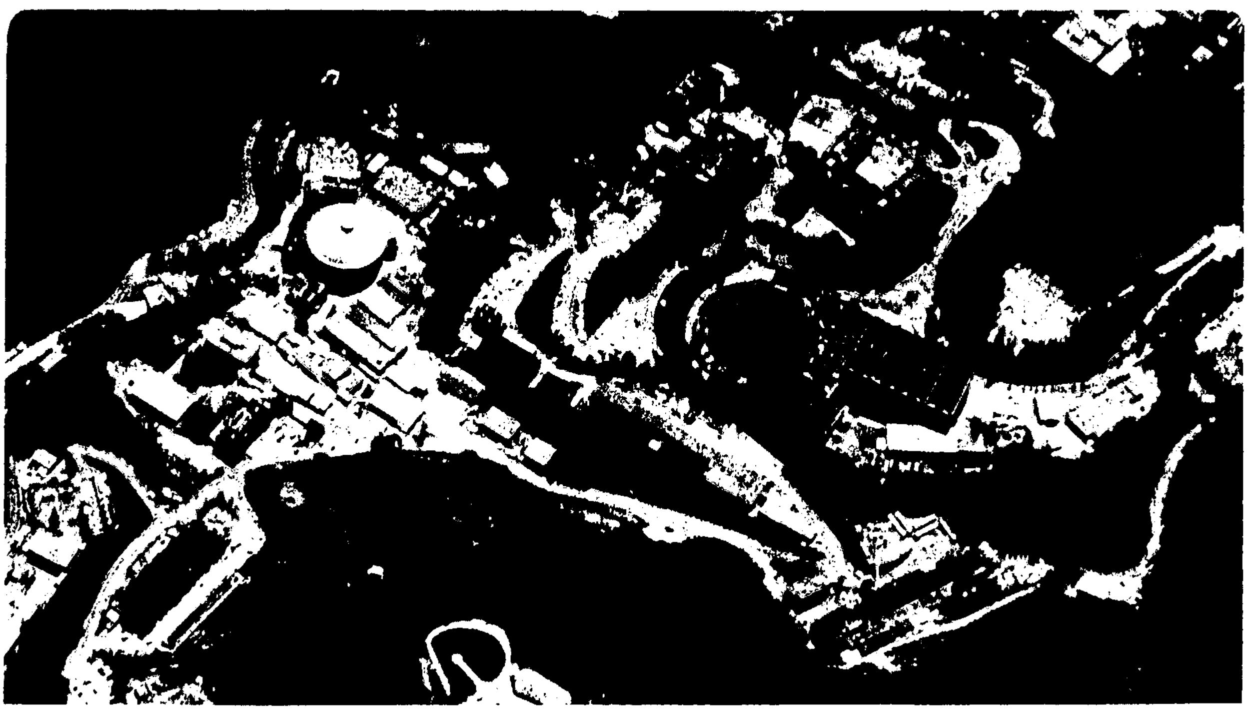

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TRIVIALITY OF THE TWO-DIMENSIONAL DIRECTED SELF-AVOIDING WALK*

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ABSTRACT

The two-dimensional directed self-avoiding walk (SAW) can be solved exactly by elementary means.

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In a recent letter to this Journal (Chakrabarti, Manna 1983) the directed SAW in two-dimensions was proposed as a significant and interesting variation on the usual problem of SAWs. In this comment I show that having the direction of walk specified in one of the two-dimensions trivializes the problem and renders it exactly soluable.

The directed SAW in two-dimensions is equivalent to the directed, non-backtracking walk. Unlike ordinary SAWs where the allowed possibilities of each step can depend on the entire past history of the walk, only the direction of the previous step must be specified for the directed SAW.

For a walk directed in the y direction, let the state of a step be labeled as,

1: + x direction

2: + y direction

3: − x direction.

The i, j element of the matrix,

$$(M^N)_{ij} = \begin{pmatrix} e^{i\alpha} & e^{i\alpha} & 0 \\ e^{i\beta} & e^{i\beta} & e^{i\beta} \\ 0 & e^{-i\alpha} & e^{-i\alpha} \end{pmatrix}^N_{ij}$$

consists of a sum of terms each corresponding to a particular directed SAW of N steps beginning from the state j and ending in the state i. If the walk begins at the origin (0, 0) and arrives at the lattice point (x,y), the term contributed by the walk will be,

$$e^{i\alpha x}\, e^{i\beta y}$$

The total number of walks of N steps, G_N, can be obtained by setting $\alpha = \beta = 0$, j = 2, and summing over final states,

$$G_N = \sum_{i=1}^{3} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}^N_{i2}$$

$$= \tfrac{1}{2}(1+\sqrt{2})^{N+1} + \tfrac{1}{2}(1-\sqrt{2})^{N+1}$$

The probability of arriving at (x,y) after N steps is now simply,

$$P_N(x,y) = \frac{1}{G_N} \sum_{i=1}^{3} \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} \int_{-\pi}^{\pi} \frac{d\beta}{2\pi} (M^N)_{i2}\, e^{-i\alpha x - i\beta y}$$

The $N \to \infty$ limit is easily obtained by expanding M in the neighborhood of $\alpha = \beta = 0$ where it has its largest eigenvalue. The result is the gaussian distribution:

$$P_N(x,y) \sim \frac{1}{4\pi N} \frac{1}{\sqrt{AB}}\, e^{-\frac{1}{4N}\left[\frac{(y-\frac{N}{2})^2}{A} + \frac{x^2}{B}\right]}$$

$$A = \frac{2+\sqrt{2}}{16} \qquad B = \frac{1+\sqrt{2}}{4}$$

All average values can now be evaluated in terms of P_N. For example,

$$\langle R_N \rangle = \sum_{x,y} P_N(x,y)\sqrt{x^2+y^2}$$

$$\sim \frac{N^2}{4\pi} \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv\, e^{-\frac{N}{4}(u^2+v^2)} \sqrt{(\tfrac{1}{2} + \sqrt{A}\, v)^2 + Bu^2}$$

$$\sim \frac{N}{2} \quad (N \to \infty)$$

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1. B. K. Chakrabarti, and S. S. Manna, *J. Phys. A: Math Gen.* ***16, L113 (1983).***

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