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Abstract

We study the stability of two related classical solutions in an effective gauge theory, which correctly describes the properties of π and ρ mesons at low energies. The first solution (sphaleron), which excites only the ρ field, with baryon number $B = 0$ and energy $E \approx 1.5$ GeV, is unstable. The second (Skyrmion), which excites both the π and ρ fields, with $B = 1$ and $E \approx 1.0$ GeV, is stable locally. We show how to make this Skyrmion absolutely stable, which is desirable for identification with the nucleon. This Skyrmion solution may also have some relevance for the electroweak interactions (now $E \approx 10$ TeV).

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1. Introduction

We think we know what fundamental theory underlies the strong interactions, namely Quantum Chromodynamics (QCD), but we cannot solve it analytically. Even numerical simulations of lattice QCD [1] are not expected to give in the near future (a few years, say) reasonably accurate values for all quantities of interest. Awaiting these results it may be worthwhile to agree on an effective Lagrangian, which describes the low energy physics. Ultimately the values of the free parameters in this Lagrangian should be determined from the lattice calculations, or from other non-perturbative methods should they be found. Of course, a lot is known already about the effective low energy theory, especially the body of results from current algebra [2]. In particular, the pion dynamics is given by the Lagrangian of the non-linear sigma model [3] supplemented by the Wess-Zumino term [4], which incorporates the effects of anomalies.

Although we cannot solve QCD, we are able to obtain some information about what its low energy behaviour may be expected to look like. The idea [5] is to consider not exactly QCD with its N = 3 colors, but the version with infinitely many colors ($N = \infty$). One takes the point of view that real QCD is obtained from an expansion in 1/N evaluated at N = 3, i.e. symbolically

$$\text{"QCD"} \equiv [1 + 1/N + 1/N^2 + \ldots]_{N=3} \quad , \qquad (1.1)$$

and that the leading term, where only planar Feynman graphs contribute, is already a reasonable approximation. The reason for (1.1) is perhaps the fact that the starting point planar QCD is a better defined theory, i.e. the Feynman expansion may be summable [6]. For matters pertaining to confinement it is thought that the pure gauge theory (no dynamical quark fields) contains the essential ingredients, in which case the expansion parameter in (1.1) becomes $1/N^2$ instead of $1/N$. If this holds planar QCD may be within a few tenths of a percent ($1/N^2 \sim 10\%$) from the full theory. Indeed large-N QCD displays qualitative features as observed in the real world, see Ref. 7 for a review. For our present purpose the most important are the existence of infinitely many narrow meson resonances and the spontaneous breakdown [8] of the chiral symmetry $SU(N_F) \times SU(N_F)$ to $SU(N_F)$ for N_F flavors of massless quarks. Furthermore, baryons are thought to arise [9] as solitons of the effective meson theory, which confirms the prophetic ideas of Skyrme [10]. In obtaining these impressive results from large-N QCD one crucial assumption was made, namely that confinement survives the limit $N \to \infty$. Recent numerical results appear to support this assumption, see Ref. 11 for further discussion. We see that the large-N point of view gives a picture of the low energy world in terms of mesons only, but alas it has been impossible, even in this simplified planar theory, to calculate the specific effective Lagrangian of the mesons.

As mentioned above we know what Lagrangian [3,4] describes the low energy pions, but it is a non-trivial matter how to include the vector and axial vector mesons. Already in 1960 Sakurai [12] argued rather persuasively that the ρ vector meson might be a "massive Yang-Mills" field, i.e. that the ρ's might be related to a local gauge symmetry. His problem was how to give the ρ a mass without violating this gauge symmetry. As is well known, the very same problem was solved in the context of the weak interaction through the Higgs mechanism and the resulting Weinberg-Salam theory [13] has been verified experimentally over the last two decennia, culminating in the discovery of the massive gauge bosons $W^{\pm}$ and Z^0 [14]. The Weinberg-Salam theory for Higgs coupling $\lambda \to \infty$ take the form of a non-linear sigma model [15]. The historic circle has been closed recently by Bando et al. [16] (independently by Hung [17] also), who argue that the strong interactions, in guise of the non-linear sigma model, may have a hidden SU(2) gauge symmetry whose massive gauge bosons are to be identified with the ρ mesons. The effective Lagrangian correctly describes many properties of pions and ρ's interacting with each other and also with the photon field of electromagnetism. In fact, this coupling is precisely of the same form [15] as in the Weinberg-Salam theory and there is the following correspondence between the gauge bosons:

gauge group:	SU(2)	$\otimes$	U(1)	
electroweak:	$W^{\pm}, Z^0$	+	γ	(1.2)
electrostrong:	$\rho^{\pm}, \rho^0$	+	γ .	

In (1.2) we glossed over the mixing of the neutral gauge boson of the SU(2) factor and the one of the U(1) factor, which gives the physical states Z^0,γ and ρ^0,γ. From the observed [14,18] mass ratios ($m_W/m_Z = \cos\theta_W$ and similarly for the ρ's) one gets the following mixing angles

$$\begin{aligned} \sin^2\theta_W &\approx 0.20 \\ \sin^2\theta_S &\approx 0.0092 \quad . \end{aligned} \tag{1.3}$$

Apparently there is less mixing with the strong sector than with the weak sector, which is a fact that must be explained by some Grand Unified Theory or

Technicolor/Compositeness theory (for three reviews see Ref. 19). The present author would like to see (1.3) as a hint that the Z^0 is a composite just as the ρ^0 is and that the different binding forces (?? and QCD) account for the different mixing with the fundamental massless photon γ (for some further discussion see at the end of this article).

After these speculative flights let us return down to earth, viz. low energy π's and ρ's as described by the effective SU(2) Lagrangian. In fact, it is possible to enlarge [20,21,16] the gauge group to U(2) x U(2), where the massive gauge bosons are identified with the ρ and ω vector mesons and the A and D axial vector mesons. But the enlarged recipe leaves something undetermined, so that in this paper we will be mainly concerned with the SU(2) sector for the ρ's, which as said above are the cousins of the W and Z. In this there are two types of classical solutions, which we will study in detail in the present article. The first is a static, but unstable, solution, for which we coined [22] in general the word "sphaleron" (the Greek sphaleros means unstable) in order to distinguish it from the stable "soliton." As discussed by Manton and the present author [22] the Weinberg-Salam theory has a sphaleron, whose ansatz was first proposed by Dashen, Hasslacher and Neveu [22]. For the Higgs coupling constant $\lambda = \infty$ its energy is [22]

$$E_{Sph}^{(WS,\lambda=\infty)} = 2.70\,\frac{4\pi v}{g} = 5.40\,\frac{4\pi}{g^2}\,m_W \approx 13.5\ \text{TeV} \quad , \tag{1.4}$$

where $v \approx 250$ GeV is the Higgs vacuum expectation value, $g \approx 0.632$ the SU(2) coupling constant and $m_W = \frac{1}{2}\,gv$. Turning to the effective $\rho\pi$ gauge theory of Bando et al. [16] we remarked [20] that the field equations for the ρ, while setting the π field zero, allow for the same solution under a simple rescaling, so that the energy (1.4) becomes

$$E_{Sph}^{(\rho\pi)} = 2.70\,\frac{4\pi}{g}\,2\sqrt{a}\;f_\pi = 5.40\,\frac{4\pi}{g^2}\,m_\rho \approx 1.5\ \text{GeV} \quad , \tag{1.5}$$

where $f_\pi \approx 93$ MeV is the pion decay constant, $g \approx 5.87$ the SU(2) gauge coupling of the ρ Yang-Mills field, and $a \approx 2$ a free parameter of the Lagrangian, which appears in the expression $m_\rho^2 = a\,g^2 f_\pi^2$ for the ρ mass. In fact, Boguta [24] was to our knowledge the first to entertain the possibility of a classical solution in an effective gauge theory for the ρ meson, for which he used a rescaled Weinberg-Salam theory without the U(1) factor, but this theory differs from the one [16] considered here in having a scalar degree of freedom.* Boguta rediscovered the Dashen, Hasslacher, Neveu [23] solution and named it "hadroid," but he erroneously claimed it to be (meta)stable and we prefer to call this solution a ρ-sphaleron.

The second solution we will study here is closely related to the one Skyrme [10] found 25 years ago, so let us review his results briefly. The theory under consideration is the non-linear sigma model with the following Lagrangian [3,10]

$$L^{(\pi)} = L_{NL\sigma}^{(\pi)} + L_{Sk}^{(\pi)} \tag{1.6}$$

$$L_{NL\sigma}^{(\pi)} \equiv \frac{1}{4}\,f_\pi^2\ \mathrm{Tr}\ \partial_\mu U\ \partial^\mu U^\dagger \tag{1.7}$$

$$L_{Sk}^{(\pi)} \equiv \frac{1}{32\ e^2}\ \mathrm{Tr}\left[U\ \partial_\mu U^\dagger,\ U\ \partial_\nu U^\dagger\right]^2 , \tag{1.8}$$

where for $N_F = 2$ flavors $U \equiv \exp(i\ \sigma^a \pi^a / f_\pi) \in SU(2)$ contains the three

*Later [25] he realised that the Sakurai theory [12], with an explicit mass term violating gauge invariance, allows for the same $\lambda = \infty$ solution (1.5).

massless pion fields π^a. The chiral symmetry $SU(N_F)_L \otimes SU(N_F)_R$ is realised as $U \to g_L U g_R^\dagger$. The Skyrme ansatz [10]

$$U = U_{Sk} \equiv \exp(i\ F(r)\ \vec{x} \cdot \vec{\sigma} / r^2) \tag{1.9}$$

$$F(0) = \pi \quad , \quad F(\infty) = 0$$

gives a solution of the field equations, which has an energy [26]

$$E_{Sk}^{(\pi)} = 79.3\ f_\pi/e \quad . \tag{1.10}$$

It was necessary to include the higher derivative term (1.8) in the Lagrangian in order to prevent the solution from shrinking to a "spike" ($F(0) = \pi$, $F(r > 0) = 0$) with zero energy; indeed (1.10) $\to 0$ for $e \to \infty$. But the fourth order term (1.8) is not unique and there are two other terms consistent with the chiral symmetry, c.f. [27]. Skyrme made the important observation that his solution (1.9) has a topological charge $T = 1$ and he argued to identify this with the baryon number B

$$B = T \equiv \frac{1}{24\ \pi^2} \int_{S^3} d^3x \quad \varepsilon_{ijk} \mathrm{Tr}(U^{-1}\partial_i U \ \ U^{-1}\partial_j U \ \ U^{-1}\partial_k U), \tag{1.11}$$

which was later confirmed [28]. The expression on the right hand side of (1.11) measures the winding number of the map $U(x) : S^3 \to SU(2) \simeq S^3$. This identification and static properties [26] leads one to conclude that the Skyrmion (1.9) describes the nucleon as a soliton in the effective theory $L^{(\pi)}$ of (1.6), which, as discussed above, seems reasonable from a large-N point of view.

After this summary of Skyrme's work, let us return to the effective gauge theory for the π and ρ, which basically is the covariant generalization of (1.7) with an additional kinetic term for the ρ, but without higher derivative terms as (1.8). Igarashi et al. [29] recently showed that there is again a Skyrme-like solution given by the ansatz (1.9) supplemented by a hedgehog for the ρ field $V_\mu(x) \in SU(2)$

$$gV_0^a = 0$$

$$gV_i^a = G(r)\,\epsilon_{ija}\;x_j/r^2 \tag{1.12}$$

$$G(0) = 2 \quad , \quad G(\infty) = 0$$

and the energy of the solution is [29]

$$E_{Sk}^{(\rho\pi)} = 3.696\,\frac{4\pi}{g^2}\,m_\rho \approx 1.04 \text{ GeV} \quad . \tag{1.13}$$

The repulsion that prevents this Skyrmion from shrinking comes from the ρ self interaction.

In summary, the effective gauge theory [16] for the π and ρ fields has spherically symmetric sphaleron and skyrmion solution with energies (1.5) and (1.13), respectively. In addition, there may be other (non-spherically symmetric) sphalerons [30], which excite only the ρ field, and Skyrme-like solutions, which excite both π and ρ fields, perhaps one related to the B = 2 solution [28] in the pure π theory (1.6). But before we get excited, we have to conjure a potential disaster for the identification of the Skyrmion

(1.9) + (1.12) with the nucleon: the ρ field (1.12) of the Skyrmion is precisely the same as that of the unstable sphaleron and perhaps the Skyrmion is unstable too? In this article we attack this problem and obtain a (partial) victory. Incidentally, Igarashi et al. [29] have disproven the claim by Adkins and Nappi [31] that they could stabilise the Skyrmion without the term (1.8) by introducing the ω vector meson (coupled to the baryon number current as $\omega_\mu B^\mu$).

The outline of this paper is as follows. In Section 2 we present the effective gauge theory for the low energy mesons. In Sections 3 and 4 we discuss the sphaleron and skyrmion solutions and discuss their stability properties. We will find that the $\rho\pi$-Skyrmion is locally stable, and in the final Section 5 we suggest some fortifying measures to make it absolutely stable, which is a desirable property for the identification of the Skyrmion with the nucleon. Also we discuss the elevation of the $\rho\pi$-Skyrmion to the energy scale of the electroweak interactions.

2. Effective gauge theory

In this Section we discuss the derivation of the effective gauge theory for low energy ρ vector mesons (V^a, a = 1,2,3) and pions (π^a). In the rest of this article we consider 2 flavors, but extension to $N_F > 2$ massless flavors is straightfoward. Actually "derivation" may be too big a word; rather we have a "recipe" that gives us a successful effective Lagrangian. In fact, this Lagrangian contains a single free parameter a and remarkably many experimental results are obtained by simply setting a = 2. The crucial step towards an effective Lagrangian is the identification of the relevant collective variables. The physical starting point for us is the non-linear sigma model (1.7), but instead of a single variable $U \in SU(2)$ the recipe [16,20] introduces two

$$U = L^{\dagger}R \quad , \tag{2.1}$$

where $L,R \in U(2)$ and $\det L = \det R$. In general, more variables means more symmetries and with (2.1) there arises a hidden gauge symmetry with group $H = U(2)$ in addition to the global $SU(2)_L \otimes SU(2)_R$

$$\begin{pmatrix} L(x) \\ R(x) \end{pmatrix} \rightarrow \begin{pmatrix} h(x)\ L(x)\ g_L^{\dagger} \\ h(x)\ R(x)\ g_R^{\dagger} \end{pmatrix} \quad . \tag{2.2}$$

Next we define the covariant derivative $D_{\mu}\xi \equiv (\partial_{\mu} - ig\, W_{\mu})\xi$, where W_{μ} is the gauge field corresponding to the new local symmetry (2.2). Instead of (1.7) we can write a more general Lagrangian [16]

$$L_{-} + aL_{+} \quad , \tag{2.3}$$

where a is a free parameter and

$$L_{\pm} \equiv f_{\pi}^{2}\ \mathrm{Tr}\left(J_{\mu}^{\pm}\right)^{2} \tag{2.4}$$

with

$$J_{\mu}^{+} \equiv (1/2i)(D_{\mu}L \cdot L^{\dagger} + D_{\mu}R \cdot R^{\dagger}) = (1/2i)(\partial_{\mu}L \cdot L^{\dagger} + \partial_{\mu}R \cdot R^{\dagger}) - gW_{\mu} \tag{2.5a}$$

$$J^-_\mu \equiv (1/2i)(D_\mu L \cdot L^\dagger - D_\mu R \cdot R^\dagger) = (1/2i)(\partial_\mu L \cdot L^\dagger - \partial_\mu R \cdot R^\dagger) \quad . \tag{2.5b}$$

Note that the square in (2.4) involves Hermitian conjugation. Up till now we have only changed variables and (2.3) can be seen to contain the same physics as (1.7). The crucial step now is to assume that QCD generates in addition to (2.3) a kinetic term for the W_μ field, so that

$$L^{(\rho\pi)} = L_- + aL_+ - \frac{1}{2} \mathrm{Tr}\, W_{\mu\nu}W^{\mu\nu} \quad , \tag{2.6}$$

where $W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu - i[W_\mu, W_\nu]$ is the Yang-Mills field strength. We write $W_\mu = W^\alpha_\mu T^\alpha$, where T^α are the generators of the algebra of U(2) normalised as $T^\alpha = (1,\vec{\sigma})/2$ with $\vec{\sigma}$ the Pauli matrices. If we want to single out the SU(2) part we will write V^a instead of W^a, a = 1,2,3. In the symmetric gauge*

$$L^\dagger = R = \xi \equiv \exp(i\, T^a\, \pi^a/f_\pi) \quad , \tag{2.7}$$

one easily checks that $L^{(\rho\pi)}$ contains a mass term

$$\frac{1}{2}\, a(g\, f_\pi)^2 \left(W^\alpha_\mu\right)^2 \quad . \tag{2.8}$$

If we identify the W^α gauge bosons with the ω and ρ vector mesons (2.8) gives the mass relation [16,20]

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*Another useful gauge is R = 1, $L = U^\dagger$, which we used [20] to show the relation with the Weinberg-Salam theory.

$$m_\omega = m_{\rho^\pm} = \sqrt{a}\, g\, f_\pi \qquad . \tag{2.9}$$

As we remarked in the Introduction the ρ^0 mass gets a small contribution from mixing with the U(1) field B (see (2.12) below). But the first equality in (2.9) is not rock solid. As we noted in [20] the field strength term in (2.6) could in principle have had different weights for the two factors of the Lie algebra su(2) x R, so that the respective coupling constants g_2 and g_1 would differ. In that case (2.8) would become

$$\frac{1}{2}\, a(g_2 f_\pi)^2 \left(V_\mu^a\right)^2 + \frac{1}{2}\, a(g_1 f_\pi)^2 \left(W_\mu^0\right)^2 \qquad . \tag{2.10}$$

An heuristic argument [20] for having $g_2 = g_1 = g$ is that the generation of the kinetic term is a non-perturbative effect, which probably depends on the gauge <u>group</u> H rather than its algebra h, c.f. the discussion in [32] for a lattice theory, where it is indeed the group that matters. If we take H the largest and most symmetric group allowed by (2.1), namely H = U(2) and not H = SU(2) ⊗ U(1), say, then we have a <u>single</u> coupling constant for this <u>group</u> and the mass equality (2.9) follows.

In the Lagrangian (2.6) we can include the coupling to an external U(1) gauge field $B_\mu(x)$ just as in the Weinberg-Salam theory [15] by use of the following covariant derivative

$$D_\mu \xi \equiv \partial_\mu \xi - ig\, W_\mu \xi + i\tilde{e}\, B_\mu\, \xi\, T^3 \qquad . \tag{2.11}$$

Diagonalizing for the physical ρ^0 and γ states one gets the ρ^0 mass^2 and the electric charge [16]

$$m^2_{\rho^0} = a(g^2 + \tilde{e}^2)f^2_\pi \tag{2.12a}$$

$$e = \tilde{e}\left(1 + \tilde{e}^2/g^2\right)^{-1/2} \equiv \tilde{e}\left(1 + \tan^2\theta_s\right)^{-1/2} \quad , \tag{2.12b}$$

and the θ_s value of (1.3) then gives $e = 0.9954\ \tilde{e}$. Furthermore (2.6) with (2.11) give the relations [16]

$$g_{B\pi\pi} = ga/2 \tag{2.13a}$$

$$g_{\beta\pi\pi} = \tilde{e}(a/2 - 1) \tag{2.13b}$$

$$g_{\rho 0} = 2g_{\rho\pi\pi}f^2_\pi \quad , \tag{2.13c}$$

which for the parameter value a = 2 agree quite well with the experimental facts [12] of (a) universality, (b) vector dominance of the pion electromagnetic form factor, and (c) the KSRF relation, in which $g_{\rho 0}$ is the coupling in the $\rho^0 - \gamma$ mixing term of the Lagrangian. In addition to (2.13) we have also the relation (2.9) for the ρ mass. With no disturbance of these remarkable relations it is possible [21] to extend the gauge group to $U(2)_V \otimes U(2)_A$, where the gauge bosons are now identified with the ω and ρ vector mesons and A and D axial vector mesons with masses

$$m_D = m_A = \sqrt{a}\ g_A f_\pi \quad . \tag{2.14}$$

The ratio g_A/g has to be fixed by hand (in fact, the hand of QCD) so as to give the experimental results [18] $m_A/m_\rho \approx 1.7$. A disappointment in this approach is that no $A\rho\pi$ coupling occurs, which could have been anticipated since A and ρ belong to different factors of the gauge group. Adding by hand a term to the Lagrangian

$$-\frac{1}{2} f_\pi g_A \vec{A}_\mu \cdot (\vec{V}^\mu \times \vec{\pi}) \qquad , \tag{2.15}$$

where the vectors are for isospin, breaks the gauge symmetry softly, and we conclude that knowledge of the compositeness mechanism that gives the A and ρ is required to explain residual terms such as (2.15). For this reason we will omit the axial vector in the considerations of this article.

In addition to (2.6) there are anomalous terms. For $N_F = 2$ there is, of course, no Wess-Zumino term $\Gamma_{WZ}(U^{-1}d\,U)$ in the action, but if we couple to external fields A_L and A_R one must make Γ_{WZ} covariant. This entails, even for $N_F = 2$, new contributions to the action, which in contradistinction to Γ_{WZ} proper <u>can</u> be written as a 4d integral. In [33] the anomalous Lagrangian is discussed in detail

$$L_{an}^{(\rho\pi)} = L_{WZ,cov.}(U = L^\dagger R, V, A_L, R_R) + \sum_{i=1}^{6} c_i L_i(L,R,V,A_L,A_R) \qquad , \tag{2.16}$$

where the first term and the last six are given in their equations (2.8) and (4.5), respectively. Even if the external fields A_L, A_R vanish (2.16) still has non-zero terms L_i, $i = 1,2,3,4$; for example

$$L_4 = i\,\mathrm{Tr}(F_V[DL \cdot L^\dagger, DR \cdot R^\dagger]) \qquad , \tag{2.17}$$

where we use differential forms (F_V is the 2-form field strength of gauge field V). For the sake of simplicity we neglect ($c_i = 0$) such higher derivative terms as (2.17), but in a final analysis geared to precise values, e.g. for (1.13), their contribution should be included.

In the next Sections we consider some classical solutions that occur in the theory (2.6) detailed here, and for simplicity we only consider the SU(2) part ($\omega_\mu = 0$).

3. ρ-Sphaleron

In the Introduction we briefly described the two types of solution in the effective theory (2.6) which we will study in this article. We start with the sphaleron, first in the context of the Weinberg-Salam theory. The ansätze for the SU(2) gauge field $V_\mu(x)$ and the Higgs doublet $\phi(x)$ are [34,22]

$$
\begin{aligned}
gV_0 &= 0 \\
gV_m &= -g(r)i\,\partial_m U^\infty \cdot U^{\infty-1} \qquad\qquad (3.1) \\
\phi &= (v/\sqrt{2})\, h(r) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad ,
\end{aligned}
$$

where g and h are radial functions vanishing at the origin and approaching unity at infinity and where $U^\infty = U^\infty(\mu = \pi/2;\ \hat{x},\hat{y},\hat{z}) \in SU(2)$ describes the behaviour of the fields at infinity (pure gauge). After a gauge transformation with $\Omega \equiv U^{\infty-1}$ (3.1) becomes

$$gV_0 = 0 \qquad (3.2a)$$

$$gV_m = -\frac{G(r)}{2}\, i\, \partial_m\Omega \cdot \Omega^{-1} \tag{3.2b}$$

$$\phi = (v/\sqrt{2})h(r)\begin{pmatrix}0\\1\end{pmatrix} \quad , \tag{3.2c}$$

where $G(r) \equiv 2(1 - g(r))$. The specific form of U^∞ implies that $\Omega \equiv U^{\infty-1} = -U^\infty\ (\mu = \pi/2;\ \hat{x},\hat{y},-\hat{z})$. As mentioned in [22] (3.2b) can be transformed to the hedgehog form of [23] by a global gauge transformation with $i\sigma_x$. In the Weinberg-Salam theory there are two sets of field equations resulting from variations δV or $\delta\phi$. If we were to <u>freeze</u> the theory to one where ϕ has a fixed amplitude $v^2/2$ and work in the unitary gauge ϕ = constant, then the ansatz (3.2ab) still gives a solution, of course. In fact, this is the sphaleron ansatz for the effective $\rho\pi$ gauge theory (2.6) with vanishing π field:

$$gV_0^a = 0$$

$$gV_m^a = G(r)\, \epsilon_{mja}\, x_j/r^2 \tag{3.3}$$

$$\pi^a = 0\ .$$

The ansatz reduces the field equations to a single differential equation for $G(r)$ and its solution $\bar{G}(r)$ gives the energy (1.5), see (3.4) below. The form (3.2b) of the ansatz is useful to demonstrate the instability of the sphaleron. Manton [34] constructed a non-contractible loop (NCL; parameter $\mu = 0 \to \pi$) in the space of 3d configuration, which starts from and ends at the vacuum ($\mu = 0,\pi$) and passes through the sphaleron at $\mu = \pi/2$. The configurations of this loop are of the form (3.2b) but with $\Omega = \Omega(\mu)$, see (4.5) below. The energy functional for these NCL configurations is

$$E(\mu) = \frac{1}{g^2} m_\rho \int d^3\xi \left[\frac{1}{\xi^2} \left\{ \left(\frac{\partial G}{\partial \xi}\right)^2 + G^2 \right\} \sin^2\mu + \right.$$
$$\left. + \frac{1}{2\xi^4} G^2 (G - 2)^2 \sin^4\mu \right] \quad , \qquad (3.4)$$

where we introduced the dimensionless radial distance $\xi \equiv m_\rho r$ and $G = G(\xi)$. The energy density (3.4) is spherically symmetric for all configurations of the loop, in particular for the sphaleron at $\mu = \pi/2$. One sees that (3.4) decreases monotonically away from $\mu = \pi/2$ and that the sphaleron merits its name. Inserting $\hat{G}$, which is the numerical solution of G for the sphaleron at $\mu = \pi/2$, the energy functional gives the dashed curve in Fig. 1. Note that one could make the drop away from the sphaleron sharper by varying G also: $G = G(\mu)$ with $G(\pi/2) = \hat{G}$. Apart from this, we conjecture that the non-contractible loop [34] lies along the unstable eigenvector (in configuration space, of course) of the sphaleron at $\mu = \pi/2$, or at least close to it. We base this conjecture on the remarkably simple form of (3.4).

4. $\rho\pi$-Skyrmion

The ansatz of Igarashi et al. [29] was given in (1.9) for the pion field and in (1.12) to the ρ field. In the previous Section we showed that this hedgehog ρ field is unstable by itself and this Section is devoted to a study of the stability* of the $\rho\pi$-Skyrmion.

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*Despite the title of their paper Igarashi et al. do not really discuss stability.

With this ansatz the energy functional becomes

$$E_{Sk}^{(\rho\pi)} = \frac{1}{g^2} m_\rho \int d^3\xi \left[\frac{1}{2a}\left(F'^2 + 2\,\frac{\sin^2 F}{\xi^2}\right) + \frac{1}{\xi^2}(1 - \cos F - G)^2 + \right.$$

$$\left. + \frac{1}{\xi^2} G'^2 + \frac{1}{2\xi^4} G^2(G - 2)^2\right] \quad , \tag{4.1}$$

where $\xi \equiv m_\rho r$ and a prime denotes differentiation with respect to ξ. The field equations reduce under the ansatz to the following two differential equations, which also follow from variation of (4.1),

$$\xi^2 F'' + 2\xi F' + (a - 1)\sin 2F + 2a(G - 1)\sin F = 0 \tag{4.2a}$$

$$G'' + (1 - \cos F - G) - \frac{1}{\xi^2} G(G - 1)(G - 2) = 0 \quad . \tag{4.2b}$$

Remark that the energy (4.1) for F = 0 equals the sphaleron energy (3.4) at $\mu = \pi/2$. Remarkably, the introduction of the pions keeps the energy density (4.1) spherically symmetric. The reason is that in the current (2.5a) the π and ρ parts have the same structure and combine as

$$J_m^{a+} = (1 - \cos F - G)\, \epsilon_{mja}\, x_j \,/\, r^2 \quad . \tag{4.3}$$

The numerical solutions $\bar{F},\bar{G}$ of (4.2), with the correct boundary conditions, give for the energy (4.1) the value (1.13), which is only 10% higher than the nucleon mass. The solution looks a trifle small (75% of its energy within a radius of 0.43 fm), but before we start worrying about the details we must be sure if the overall picture is correct; in particular is the solution stable?

Let us return for a moment to the Skyrmion (1.9) in the pure pion theory (1.6) with an explicit higher derivative term (1.8). At the risk of belabouring the obvious we give a heuristic argument for the stability of this Skyrmion (the reader may picture configuration space as a plane above which the energy functional traces out a surface):

1. The ansatz (1.9) has a topological charge (1.11) $T = 1$, so that under smooth deformations of the configuration T will remain equal to 1 always.
2. Suppose there is an unstable direction at $U = U_{Sk}$ and let us follow the steepest descent path. Where do we end up? Not at the vacuum $U_V = \text{const.}$, because $T(U_V) = 0 \neq 1$. There must be a non-trivial configuration $\tilde{U}$ with $T(\tilde{U}) = 1$ and $E(\tilde{U}) < E(U_{Sk})$.
3. The ansatz (1.9) arranges its energy so nicely (spherical symmetry) that it is hard to believe another configuration $\tilde{U}$ exists with even lower energy. Hence the assumption in 2. must be false and the Skyrmion is stable.

In short, the Skyrmion (1.9) is absolutely stable, because it is able to distribute its energy density so smoothly that it has the lowest total energy of all configurations in the $T = 1$ sector. Recently we became aware of Ref. 35, where point 3. is shown mathematically. The rôle of the higher derivative term (1.8) is to prevent the size (R) of the Skyrmion from shrinking, and it contributes the second term to the energy $E_{Sk} = af_\pi^2 R + b/e^2 R$, where a and b are some numbers, c.f. (1.10). The stability of the Skyrmion is both dynamic and topological in origin.

Returning to the Skyrmion in the effective $\rho\pi$ theory, Igarashi et al. [29] argued that the Yang-Mills field strength of the ρ could produce a term $b/g^2 R$ in the energy, which prevents shrinking. But what if the ρ-configuration can be "unwrapped" to zero, so that b = 0? As we have seen the ρ configuration of the ρ-sphaleron and $\rho\pi$-Skyrmion is the same. The danger for the $\rho\pi$-Skyrmion lies in the ρ sector and for this reason we consider the following variation

$$\begin{aligned} &U \to U \\ &V \to V + \delta V]_{\text{along NCL}} \quad . \end{aligned} \tag{4.4}$$

Specifically, V + δV is given by (3.2b) with

$$\Omega = \begin{pmatrix} -i \sin\mu \sin\theta\, e^{-i\phi} & ie^{i\mu}(\cos\mu - i\sin\mu\cos\theta) \\ ie^{-i\mu}(\cos\mu + i\sin\mu\cos\theta) & i\sin\mu\sin\theta\, e^{i\phi} \end{pmatrix} , \tag{4.5}$$

where θ and ϕ are the standard spherical coördinates. With (4.5) we get $V_m = 0$ at $\mu = 0,\pi$ and the hedgehog at $\mu = \pi/2$. Apart from some strategic signs and a multiplication on the left by $i\sigma_x$, (4.5) is the same as the U^∞ constructed by Manton [34]. For this loop of configurations the energy functional is given by

$$E(\mu) = E_{Sk}^{(\rho\pi)} + \frac{4\pi}{g^2} m_\rho \int_0^\infty d\xi \left[(G^2 + G'^2)(\sin^2\mu - 1) + \frac{1}{2\xi^2} G^2(G-2)^2(\sin^4\mu - 1) + 2\,(1 - \cos F)G\left(1 - \frac{1}{3}\sin^2\mu(1 + 2\sin\mu)\right)\right] , \tag{4.6}$$

where the expression for $E_{Sk}^{(\rho\pi)}$ has already been given in (4.1). All terms in (4.6) come from a spherically symmetric energy density, except for the one proportional to $\sin^2\mu\ (1 + 2\sin\mu)$, whose energy density was axisymmetric

$$(1 - \cos F)G\sin^2\mu\ [-\sin^2\theta - (1 + \cos^2\theta)\sin\mu] \quad . \tag{4.7}$$

It is precisely this term, which originated from $Tr(J_m^+)^2$, that turns the instability of the sphaleron around to stability for the Skyrmion, as we will see in a moment. The reason for this is that the delicate balance in (4.3) is upset for $\mu = \pi/2 + \varepsilon$ $(0<|\varepsilon|<<1)$, so that the energy increases as ε^2. In fact, close to the Skyrmion we have

$$E(\frac{\pi}{2} + \varepsilon) = E_{Sk}^{(\rho\pi)} + \varepsilon^2 \int d\xi \left[\frac{8}{3}(1 - \cos F)G - (G^2 + G'^2) - \right.$$
$$\left. - \frac{1}{\xi^2} G^2(G - 2)^2\right] \quad , \tag{4.8}$$

and for the solution $\bar{F},\bar{G}$ of (4.2) the coefficient of ε^2 becomes

$$[8.45 - 5.37 - 1.05] = 2.03 > 0 \quad . \tag{4.9}$$

As we see it, the local stability (4.9) is just a numerical "coincidence." In Fig. 1 we plot $E(\mu,\bar{F},\bar{G})$ as the full curve and it looks as if the Skyrmion is not only locally stable, but actually the lowest energy configuration in the B = 1 sector. Alas, our happiness does not last very long, when we realise what we have at $\mu = 0$: a configuration $U = U_{Sk}(\bar{F})$, $gV_m = 0$, which by a scale transformation $r \to \sigma r$, $\sigma < 1$, can be reduced to a spike with

$B = 1$ and $E = 0$. Furthermore, we can perform this scale transformation while we travel on the loop ($\mu = \pi/2 \to 0$), so that the energy functional $E(\mu,\bar{F},\bar{G})$ becomes

$$E(\mu,\sigma) = \frac{4\pi}{g^2} m_\rho \int_0^\infty d\xi \left[\sigma\left\{\frac{1}{2a}\left(\xi^2\bar{F}'^2 + 2\sin^2\bar{F}\right)\right.\right.$$
$$+ (1 - \cos\bar{F} - \bar{G})^2 + \bar{G}^2(\sin^2\mu - 1)$$
$$\left. + 2(1 - \cos\bar{F})\bar{G}\left(1 - \frac{1}{3}\sin^2\mu(1 + 2\sin\mu)\right)\right\}$$
$$\left. + \frac{1}{\sigma}\left\{\bar{G}'^2\sin^2\mu + \frac{1}{2\xi^2}\bar{G}^2(\bar{G} - 2)^2\sin^4\mu\right\}\right] \quad . \tag{4.10}$$

In Fig. 2a we show this energy behaviour for the optimal $\bar{\sigma}(\mu)$ (Fig. 2b), which follows from a simple variational calculation. We see that the Skyrmion remains locally stable even for the "double barrelled" attack, but the height of the barrier becomes rather small

$$\Delta E^{(\rho\pi)}_{Sk,barrier} \approx 0.05\ E^{(\rho\pi)}_{Sk} \tag{4.11}$$

for $\mu \approx 1.12$ and $\bar{\sigma} \approx 0.73$.

At this moment the obvious question is if there are other variations for which the Skyrmion is really unstable (no barrier). We think the answer is negative. The potential danger is in the first place the unwrapping of the ρ configuration, which has a single negative eigenmode equal to the one for the ρ-sphaleron. At the end of Section 3 we conjectured that the path of the non-contractible loop at $\mu = \pi/2 + \varepsilon$ lies along this eigenvector (in configuration space), or at least has a significant projection on it. For $|\varepsilon| < 0.3$, say, there remains enough of the ρ field to give repulsion against collapse, but when too much of it is gone ($\mu < 1.12$ in Fig. 2a) the configuration collapses to the spike configuration. Provided our conjecture holds we have shown in (4.8-9) that the Skyrmion remains locally stable against the compounded dangers of unwrapping and collapse. By the way, if we only shrink the $\bar{F}$ configuration and keep the hedgehog for V while allowing G(r) to adjust itself, the energy increases from $E = E_{Sk}^{(\rho\pi)}$ at $\sigma = 1$ to $E = E_{Sph}^{(\rho\pi)}$ at $\sigma = 0$, where the final configuration is a ρ-sphaleron with at its center a π-spike, which has $B = 1$.

To summarise, we have shown (modulo a conjecture) that the Skyrmion solution, which Igarashi et al. [29] considered to be a candidate for the nucleon, is locally stable. Whether or not this is sufficient will be the subject of the next Section.

5. Stability and speculation

An effective low energy Lagrangian should contain at least the pions and the ρ vector mesons. Sakurai [12] has drawn attention to many facts (universality, vector dominance, current field identity) that point towards some kind of gauge invariance. Only recently was an effective Lagrangian proposed [16] that realizes Sakurai's idea. This Lagrangian was, of course, not really derived from QCD, rather it emerged from postulating appropriate collective variables (2.1). With these variables and the basic dynamics of the non-linear sigma model there is little freedom left, a single parameter a in fact. For the value a = 2 several interesting relations (2.13) follow. For this reason we think that the Lagrangian (2.6) with a = 2 is an important part of the full effective theory of low energy mesons. It is then appropriate to look for classical solutions. Normally we would not be interested in classical solutions of a theory with $g^2/4\pi \approx 2.8$, which certainly does not look very small. But the underlying theory QCD does have a "small" parameter 1/N, which translates [7] to f_π and g being of order $N^{1/2}$ and $N^{-1/2}$, respectively. Rescaling $gW_\mu \rightarrow W_\mu$, this shows that the Lagrangian (2.6) is proportional to N, which makes a saddle-point approximation quite reasonable [7,9].

In this article we studied two simple types of classical solutions, but there may be others [28,30] probably with higher energy. The first was a static, but unstable, solution which excites only the ρ field and which we called the ρ-sphaleron (another name proposed in [24] was "hadroid"). This solution lies on top of an energy barrier between the vacuum in different gauges, but the same physical vacuum all the same [22]. In the Weinberg-Salam theory the passage over this barrier induces a change in baryon number ΔB = 1, and in fact the WS-sphaleron has B = 1/2 [22]. In the $\rho\pi$ theory (2.6), where there is no anomaly for the baryon number current (left-right symmetry), there is no such signal, and the ρ-sphaleron has baryon number 0, c.f. (1.11). Still, if one would try to excite a field configuration close to that of a sphaleron one would stand a better chance for the $\rho\pi$ theory. Not only are energies of 1 GeV more manageable than those of 10 TeV, but precisely the fact that $g^2/4\pi$ is not very small in the $\rho\pi$ context, whereas it is in the electroweak theory, is of practical importance. The size of a classical solution is in general proportional to $(4\pi/g^2)E^{-1}$, so that the ρ-sphaleron is reasonably small. In fact, the sphaleron radii are approximately [22]

$$R_{Sph}^{(\rho\pi)} \approx 2\ m_{\rho}^{-1} \approx 0.5\ \text{fm} \tag{5.1a}$$

$$R_{Sph}^{(WS)} \approx 2\ m_{W}^{-1} >> 1/E_{Sph}^{(WS)} \tag{5.1b}$$

Twice these distances give the size of the region over which the field should be coherent. For this reason the experimental study of configuration space seems more feasible in relativistic heavy ion collisions. For the moment we have not thought much about possible signals. In [25] it was shown that fermions coupled covariantly to the ρ, i.e. in the Lagrangian

$$\bar{\psi}(\gamma^{\mu}D_{\mu} + m)\psi \quad , \tag{5.2}$$

have a large cross section with the ρ-sphaleron. But what are these fermions in (5.2) if we take the view that the nucleons are not elementary, but solitons rather of the π and ρ fields? At the very least, (5.2) for nucleons must be changed by the effect of a form factor.

The ρ-sphaleron excites the ρ field but not the π field. In general the opposite is not possible. In the ρ field equation

$$D_{\mu}V_{\mu\nu} = S_{\nu}(V,L,R) \tag{5.3}$$

the source term S_{ν}, c.f. (2.5a), does not vanish in general for $V = 0$ and $L,R \neq 0$. Our equations (4.2) for the ansätze (1.9) and (1.12) illustrate this: we have a solution with $F = 0$ and $G \neq 0$, i.e. the ρ-sphaleron, but not a naked Skyrmion with $F \neq 0$ and $G = 0$. The ansatz of Igarashi et al. [29] gives the ρ-field response to a non-trivial π field, which provides the non-zero baryon number (1.11). We have shown in Section 4 that the $\rho\pi$ interaction is of such a form (2.5a) as to compensate for the instability of the ρ field itself, c.f. (4.8). We have not proven this rigorously, which would be a difficult matter, but argued that it is very likely. But is local stability enough? On face value Fig. 2a would give a tunneling rate of order

$$\Gamma \sim m_\rho e^{-\Delta E/\hbar m_\rho} \sim m_\rho \quad , \tag{5.4}$$

which gives an uncomfortably short lifetime for a proton. There are two attitudes one can take: 1. not to worry, or 2. to return as the prodigal son to Skyrme. The first alternative then is to say that we are doing only classical field theory and quantum mechanical tunneling effects should not worry us (in (5.4) $\Gamma \to \infty$ for $\hbar \to 0$). In fact, quantum corrections to the sigma model are pretty sick [36]. The strategy would be to hope that the classical Lagrangian (2.6) is the relevant partfor the low energy phenomenology. There are, of course, an infinite number of higher derivative terms, which all together should be a renormalizable theory, since QCD is. The statement in the previous sentence is somewhat cavalier, since QCD itself may be sick towards the infrared and our effective Lagrangian is precisely at this IR end. We do not know if QCD is a "theory," c.f. our remarks below (1.1).

For the moment we favor the second attitude. Probably higher derivative terms arise anyway in addition to (2.6). Their contributions should prevent in Fig. 2a the downward slide for small values of $\bar{\sigma}$. Of course, it is disappointing to depend on these terms, since we lose in predictive power when there are a priori many higher derivative terms, some of which should somehow be more important than the others. We have a fancy for two particular terms, the first of which is

$$L^{(\rho\pi)}_{Sk,++} \equiv \frac{1}{2e^2_{++}} \mathrm{Tr}\left[J^+_\mu, J^+_\nu\right]^2 \tag{5.5}$$

and the second the original term (1.8) of Skyrme, which basically is $L^{(\rho\pi)}_{Sk,--}$ in the notation of (5.5).

Consider first the pure Skyrme term (1.8) added to $L^{(\rho\pi)}$ of (2.6).* One could think of the Lagrangian (1.6) as describing the "fundamental" pion field, for which the hidden gauge mechanism turns (1.7) into (2.6) by making the ρ a propagating composite of π's. The ρ-sphaleron is unaffected by the $L^{(\pi)}_{Sk}$ term, since its field equation (4.2b) with $F = 0$ is unchanged. For the $\rho\pi$-Skyrmion an additional contribution to its energy density (4.1) arises [26]

$$(g^2/e^2)\ \frac{1}{2}\ \frac{\sin^2 F}{\xi^2}\left(\frac{\sin^2 F}{\xi^2} + 2F'^2\right) \quad , \tag{5.6}$$

which gives some extra terms in the field equation (4.2a). In order to get a qualitative picture and to estimate what value of e is needed to make the Skyrmion absolutely stable, we given in Fig. 3a the energy (4.5) + (5.7) evaluated with our old functions $\bar{F},\bar{G}$. We estimate from this that for

$$e < e_{crit} \approx \sqrt{2}\ g \tag{5.7}$$

*At this point the reader may very well object: "Good Lord, after all the fuss about the ρ you are back at the Skyrme term, you better stick with the pure pion theory (1.6), where there is no problem with the stability of the Skyrmion!" To answer him we submit that 1. the ρ is important for phenomenology [12] and should be included; and 2. just setting the ρ field zero probably does not lead to a solution, see our discussion below (5.3).

the $\rho\pi$-Skyrmion becomes absolutely stable. The resulting configurations at $\mu = 0$ are smoothed out versions of the spike Skyrmion, see a few lines below (4.9). They are not solutions of the field equations, c.f. (4.2b), and lie in a cusp of the energy surface over configuration space. Note that for $\mu = 0$ we return to the ansatz (1.9) + (1.12), but in a different topological sector $G(r = 0) = 0$, and that the energy density (4.1) is infinite at the origin. There is a (small) chance that for $e^2 \sim 3\, g^2$ this $\mu = 0$ configuration is the relevant one for the nucleon, whereas the $\rho\pi$-Skyrmion would be an excitation at $\approx$20% higher energy. In the following, we will try to see the $\rho\pi$-Skyrmion as the nucleon, but this other possiblity should be kept in mind. For comparison the arrows show in Fig. 3a the Skyrmion energy (1.10) for the pure pion theory normalised to $E_{Sk}^{(\rho\pi)}$ of (1.13). There is no need for concern that these values are lower, since the theories are different; in particular (4.1) has a term $(1 - \cos F)^2/\xi^2$, which does not arise in the pure pion theory (1.6).

For e just below e_{crit} (5.7) the true Skyrmion energy will be a little less than the value of Fig. 3a ($5.03\ (4\pi/g^2)m_\rho \approx 1.4$ GeV), since the $\bar{F}$ and $\bar{G}$ have not precisely the optimal shapes to minimize the full energy including (5.6). In fact, because $\bar{F},\bar{G}$ are not optimal it is advantageous to do a scale transformation even at $\mu = \pi/2$, c.f. Fig. 3b.

We see that the pure Skyrme term (1.8) can stabilize the $\rho\pi$-Skyrmion, but at the expense of increasing its energy by a few 100 MeV; which may be too much perhaps. Therefore we turn to the higher derivative term (5.5), which gives an additional contribution to the energy density (4.1) of

$$8(g^2/e_{++}^2)\ \frac{1}{\xi^4}\ (1 - \cos F - G)^4 \tag{5.8}$$

which is spherically symmetric also (probably this is not the case for the contribution of an eventual $L^{(\rho\pi)}_{Sk,+-}$ term). With (5.8) we calculate for $\bar{F},\bar{G}$ the increase of the total energy

$$\Delta E/E^{(\rho\pi)}_{Sk} \approx 0.084\ g^2/e^2_{++} \quad . \tag{5.9}$$

For the same type of variation as used above, the qualitative picture will be the same as in Fig. 3a (reading g^2/e^2_{++} for g^2/e^2), with similar energy values at $\mu = 0$, where the gauge field vanishes, and at $\mu = \pi/2$ the energy values from (5.9), namely $E(\pi/2,\sigma) = 1.084$, 1.061, 1.044 and 1.027 for $g^2/e^2_{++} = 1$, 0.72, 0.52 and 0.32, respectively. This indicates that the higher derivative term (5.5) stabilizes the $\rho\pi$-Skyrmion absolutely for

$$e_{++} < e_{++,crit} \approx \sqrt{3}\ g \quad , \tag{5.10}$$

while its energy becomes (1.13) + (5.9). It is remarkable that the energy value of the Skyrmion changes little, whereas the global stability properties are completely "turned around" (c.f. Fig. 3a). On the other hand for the ρ-sphaleron, which has $F = 0$ and $G(0) = 2$, (5.8) would give an infinite contribution to the energy. One could consider a sphaleron configuration (mark we do not say "solution") with a hole around the origin, but clearly this would be rather inelegant.

In a future communication we hope to report on an extensive numerical analysis of the interplay of higher order terms, which should also include the ones of (2.16) and other terms not mentioned here. Another outstanding problem is the response of the ω and the axial vector meson fields, see our discussion below (2.15).

We conclude that in the effective gauge theory (2.6) for the π and ρ the generalized Skyrme terms can make the Skyrmion absolutely stable, which is not so surprising after all. If the parameters of these 3 terms, one of which is given explicitly in (5.5), are related as

$$e_{+-} \gg \max(e_{++}, e_{--} = e) \tag{5.11}$$

there are at least two classical solutions:

1. a ρ-sphaleron, with an energy value close to (1.5) (for finite e_{++} this will be an "approximate" solution only);
2. an absolutely stable $\rho\pi$-Skyrmion, with energy $\approx$1.1 GeV if $e_{++} \approx e_{++crit} - \delta \ll e_{--}$ or energy $\approx$1.4 GeV if $e_{--} \approx e_{--crit} - \delta \ll e_{++}$, where δ is a small positive number.

Although we have lost in predictive power (why should (5.11) hold?), we are gratified that in what is probably the relevant effective theory for low energy mesons there is indeed a candidate $\rho\pi$ solution for the nucleon. Furthermore, a choice of parameters is possible so that $E \approx 1.1$ GeV! In addition there may be a ρ-sphaleron (approximate) solution, and other possible solutions, of course.

Finally, we want to follow up our speculative remarks below (1.2) and (1.3) of the Introduction. The discussion of a Skyrmion solution in (2.6) may be of some relevance for the physics of the electroweak interactions. Gipson [37] discussed a naked Skyrmion in a non-linear sigma model for the Higgs [15], but as we mentioned above this probably is not a true solution of the full theory, which contains massive gauge fields also.* In fact, the SU(2) theory (2.6)

*In Ref. 38 the Skyrme model was gauged $SU(2)_L$ and the authors discuss the stability of the "Skyrmion." This theory is different from ours; still we invite the reader to look at their results and to compare with our picture.

could be seen as an effective theory of composite W and Z* and "technipions" (Π) provided these technipions can be given a large enough mass ($m_\Pi < m_W$?). Our model (2.6) for SU(2), with the rescaling [20] indicated in (1.4-5), is related to the model of Abbott and Farhi [39], who consider also the coupling to composite quarks and leptons (vector meson dominance may be important). Perhaps our $W^\pm$ and Z^0 are joined by another boson Ω^0 in analogy with the ω^0, but neutral current phenomenology [39] requires $m_\Omega^2 > 20\ m_W^2$, which in principle is possible, c.f. (2.10). With the caveat on the technipion mass we can scale the previous solution down in size to get the WΠ-Skyrmion, for which the energy (1.13) becomes

$$E_{Sk}^{(W\Pi)} \approx 3.969\ \frac{4\pi}{g^2}\ m_W + \Delta E \approx 9.2\ \text{TeV} + \Delta E \qquad , \qquad (5.9)$$

where ΔE contains a contribution due to the technipion mass and perhaps a further contribution from higher derivative terms. In addition, there is the W-sphaleron with energy (1.4). Depending on what higher derivative terms occur this WΠ-Skyrmion is or is not absolutely stable (in our discussions replace ρ by W). Ultimately it will decay, of course, since its building blocks (W,Π) are unstable, see the discussions in [37], but perhaps vacuum polarization effects of "heavy" fermions suppress the Skyrmion decay rate [40]. These are rather wild speculations and problems abound; happily there can be no doubt that at least the world of π's and ρ's is real.

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*The U(1) factor can be coupled in easily, and the parameter $\rho \equiv \cos\theta_W\ m_Z/m_W$ is unity, c.f. (1.2), (1.3) and (2.12). This effective theory has no scalar particle, in contrast to the Weinberg-Salam theory [13]. Of course, for λ large its mass becomes large also and the particle does not propagate far.

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Figure captions:

Fig. 1 <u>Dashed curve</u>:

Energy along a non-contractible loop ($\mu = 0 \rightarrow \pi$) in configuration space. There is symmetry around $\mu = \pi/2$. At $\mu = \pi/2$ the loop passes through the ρ-sphaleron solution, whose energy is given in (1.5). This shows that the solution is unstable.

<u>Full curve</u>:

Energy along a loop, $\mu \in [0,\pi]$, of configuration (4.4). There is again symmetry around $\mu = \pi/2$. At $\mu = \pi/2$ the loop passes through the $\rho\pi$-Skyrmion solution, whose energy is given in (1.13). The Skyrmion appears to be stable, at least for this variation, see text.

Fig. 2 a) Change in energy (4.10) of the $\rho\pi$-Skyrmion ($\mu = \pi/2$) under variation of the ρ-field (parameter $\mu = \pi/2 \rightarrow 0$) and a simultaneous scale transformation $r \rightarrow \sigma r$. For comparison the curve for $\sigma = 1$ is shown also (= full curve of Fig. 1). The energy is normalised to that of the $\rho\pi$-Skyrmion solution (1.13).

b) The optimal values $\bar{\sigma}$ for the scale transformation parameter σ as used in a).

Fig. 3 a) Same as in Fig. 2a, but now the energy contains a contribution from a Skyrme term (1.8), (5.7), which has a prefactor g^2/e^2. The functions $\bar{F},\bar{G}$ were used. For $g^2/e^2 = 0$ the curve is the same as in Fig. 2a. The energy is normalised to that (1.13) of the $\rho\pi$-Skyrmion solution for $g^2/e^2 = 0$. For comparison, the arrows give the pure Skyrmion energy (1.10) for the non-zero values of g^2/e^2.

b) Optimal scale parameters $\tilde{\sigma}$ used in a); they are in the same order.

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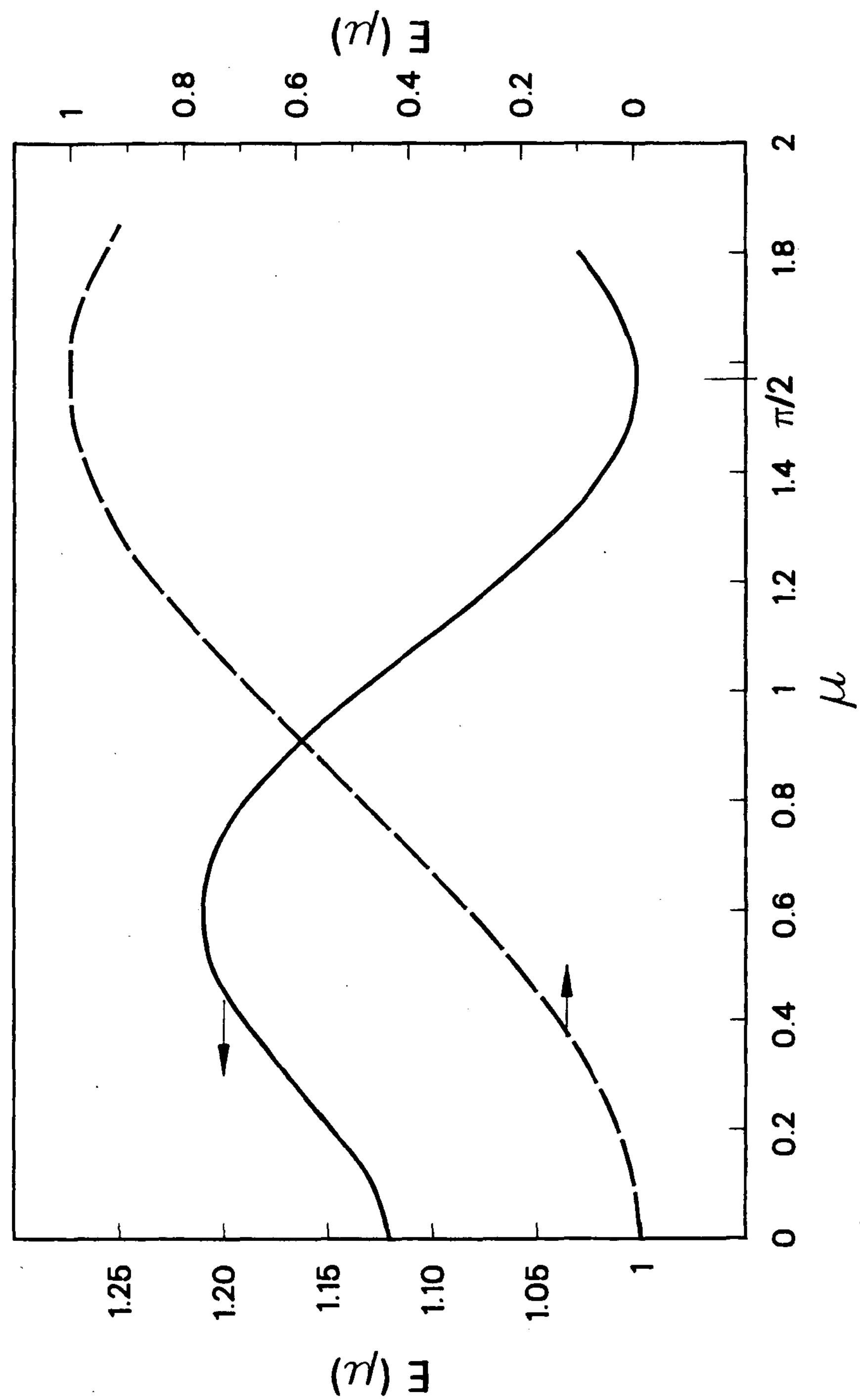

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Fig.1

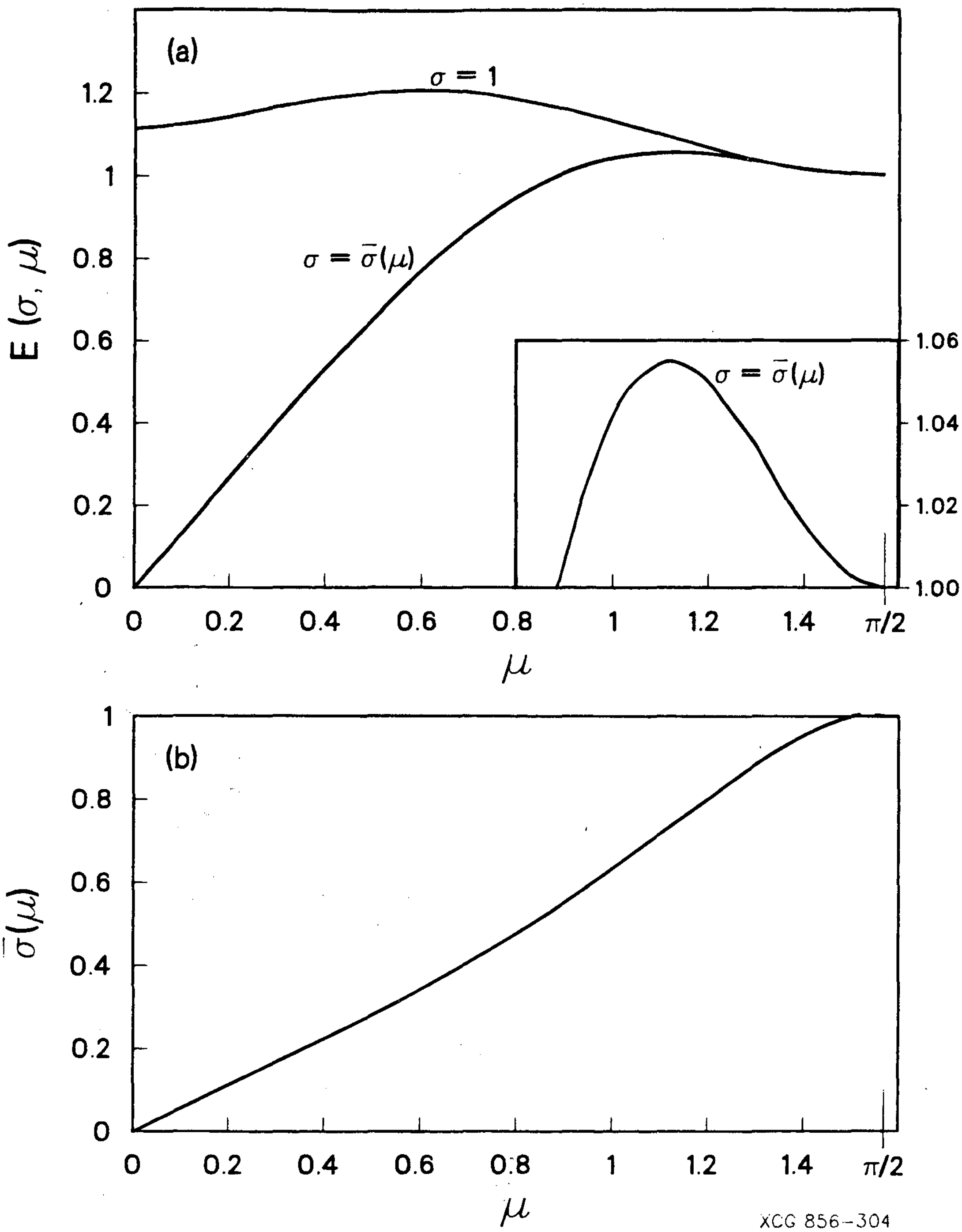
(a)
σ = 1
σ = σ̄(μ)
σ = σ̄(μ)
E (σ, μ)
1.2
1
0.8
0.6
0.4
0.2
0
1.06
1.04
1.02
1.00
0.2
0.4
0.6
0.8
1
1.2
1.4
π/2
μ
(b)
σ̄(μ)
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
π/2
μ
XCG 856-304

Fig.2

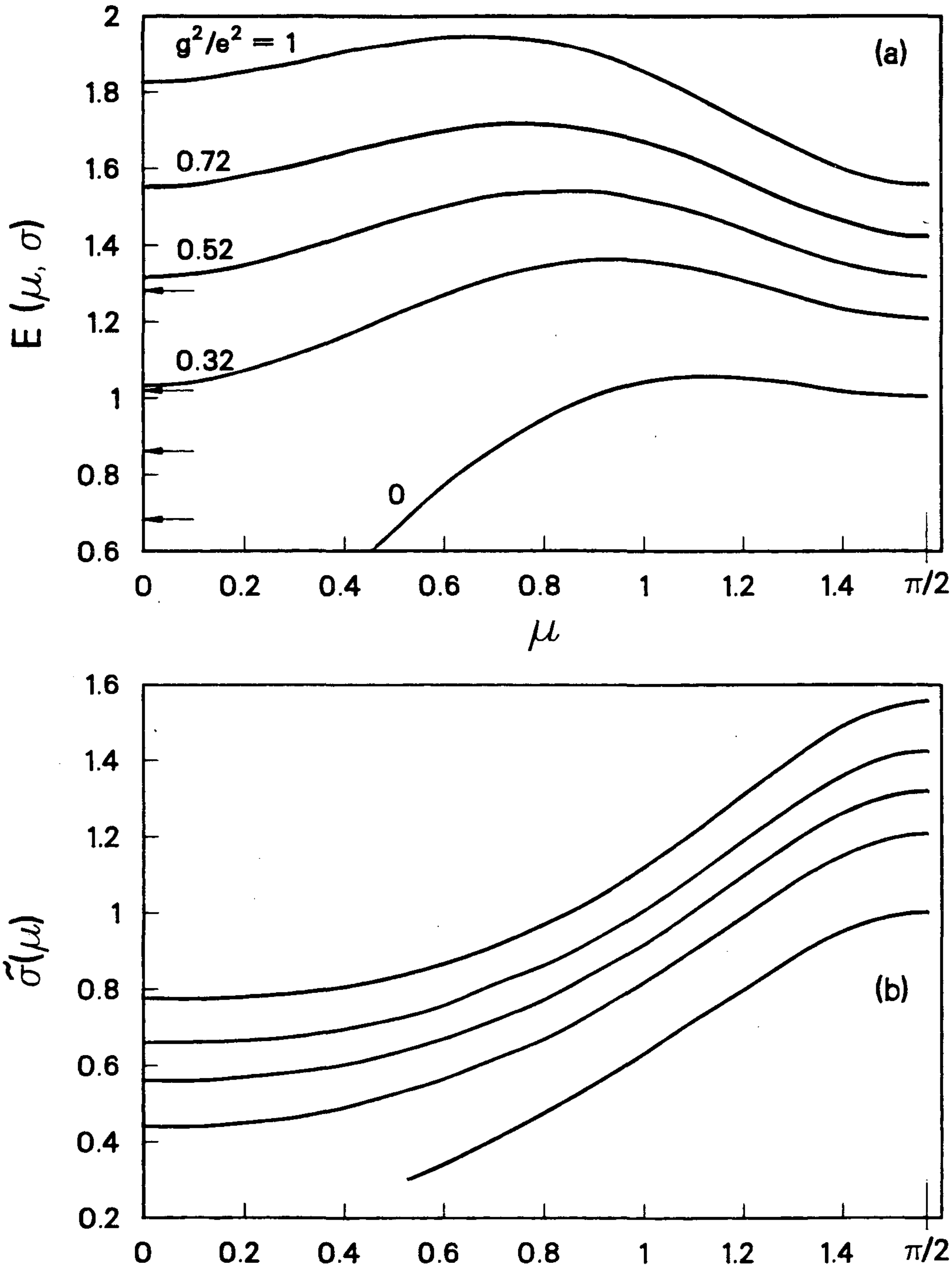

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Fig.3

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