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ABSTRACT

We have studied the effect of mass differences within multiplets on relations between scattering amplitudes predicted by SU(3) in a model in which the matrix ND^{-1} method and determinantal approximation are used to calculate the one-baryon exchange contribution to the $P_{3/2}$ partial wave in octet-octet meson-baryon scattering; the model is the same as that used by Wali and Warnock to calculate decuplet mass splittings. The unphysical cut is taken to be completely SU(3)-symmetric, so that the only symmetry breaking arises from the mass differences in the scattering particles. We have examined the predicted relation among the amplitudes for the reactions $\pi^+p \to \pi^+p$, $\pi^+p \to K^+\Sigma^+$, and $K^+p \to K^+p$, and the predicted equality of the amplitudes for $K^-p \to K^o\Xi^o$ and $K^-p \to \Sigma^-\pi^+$. The results confirm an earlier calculation based on a simple

potential model, in that one finds the mass splittings may produce very large violations (as much as a factor of 10 in the amplitudes in one case) of the predicted SU(3) relations at low energies; one cannot be sure of the precise validity of these relations until the center-of-mass energy becomes greater than the masses. Thus, even the large discrepancy between the $\Xi^0 K^0$ and $\Sigma^- \pi^+$ production cross sections near threshold, as well as the perhaps 20% disagreement in these cross sections reported at higher energy, could be due to mass differences alone; similar conclusions hold for all other apparent violations of SU(3) in scattering processes so far reported. It seems to make little difference whether one compares different reactions at, e.g., the same final or same initial Q values. We note that the production cross sections near threshold for $K^+ \Sigma^+$ and $K^0 \Xi^0$, as predicted by this rather crude dynamical model, are much too large when SU(3) and the usual value of the D - F mixing parameter, $f = 0.35$, are used; the $P_{3/2}$ contribution to the former, alone, is about ten times the experimental value. One can obtain qualitative agreement with these small production cross sections in the context of the model by taking f to be about 0.7; this, however, results in a value of the $N \Lambda K$ coupling constant which may be unacceptably large.

A number of authors have studied the problem of obtaining, from the assumption of SU(3) symmetry, relations between scattering amplitudes for different reactions, and hence equalities or inequalities which should hold among experimental cross sections, and have compared some of the resulting predictions with experiment; further predictions have also been obtained from SU(6). Examples of such calculations are given in Ref. 1. Similar results could, of course, be obtained from the assumption of other symmetry groups for the Hamiltonian. The difficulty arises, however, that one knows the higher symmetries are at best approximate. Even if no other violations of the symmetry were present, it would be broken by the appreciable mass differences which exist between members of the same multiplet. These mass differences not only break the symmetry, but they introduce ambiguities as to how the predicted relations between cross sections for different reactions should be compared with experiment. Most comparisons that have been made have followed a recipe suggested by Meshkov, Snow, and Yodh (MSY).[2] MSY suggest that one should compare cross sections at energies such that the final-state Q value, Q_f (i. e., the total kinetic energy of all final-state particles in the center-of-mass system), is the same for each reaction. They also point out that the appropriate quantities to compare are probably the cross sections multiplied by the kinematic factor F, where

$$F = q_i E^2 / q_f . \qquad (1)$$

In Eq. (1), q_i and q_f are the magnitudes of the initial and final three-momenta in the center-of-mass system, and E is the total center-of-mass energy. The cross section multiplied by F is proportional to the square of the absolute value of the invariant amplitude for the reaction.

In a previous article[3] one of the present authors explored, within the context of a simple relativistic potential scattering model, the question of the effect of mass splittings on symmetry relations, as well as the problem of whether any of the possible procedures, e. g., that of MSY, for comparing data from different reactions seemed clearly preferable to the others. The conclusions of the potential model calculation can be summarized as follows. First, one finds that near threshold, mass differences of the order of those within hadronic multiplets cause violations of the symmetry relations which can be extremely severe, to the extent of an order of magnitude or more in the amplitude, and remain appreciable until the total center-of-mass energy becomes large compared with the masses (not just the mass differences) involved; simple qualitative arguments show that this is not surprising. Secondly, it was found that there was no reason to prefer the MSY procedure to, e. g., comparing different reactions at the same initial-state Q value, Q_i; indeed, if the two procedures differed markedly, it was an indication that the effect of the mass differences was so large that neither procedure could be expected to work very well. It was also concluded that existing discrepancies between SU(3) predictions and scattering experiments could easily be attributed to the effect of

mass differences, without invoking other symmetry-breaking mechanisms, since they occurred in relatively low-energy data.

The purpose of this work is to study the same questions in a more realistic model. We calculate the $P_{3/2}$ partial wave for some pseudoscalar meson-baryon scattering processes, under the assumption that the entire force, or unphysical cut, is due to baryon exchange. Our procedure follows exactly that of Wali and Warnock (WW),[4] who in turn use a somewhat simplified version of an earlier calculation by Martin and Wali.[5] WW were interested in studying the mass splitting of the decuplet of meson-baryon resonances produced by the mass differences among the members of the meson and baryon octets. They employed the matrix ND^{-1} method,[6] and the determinantal approximation. The unphysical cut functions were calculated by using degenerate masses and coupling constants which obey exact $SU(3)$ symmetry. Hence the only symmetry breaking in the model arises from the fact that the masses of the scattering particles, which enter in the phase-space factors appearing in the integrals for the matrix elements of the D matrix, are nondegenerate. We apply WW's procedure to the calculation of amplitudes for several processes, rather than simply to the search for the zeros of Re det D which indicate the positions of resonances or bound states. The only differences between our calculation and that of WW is that we made some small alterations in the parameters which enter the calculation. We found we were unable to reproduce the values for the decuplet resonance positions quoted by WW without discrepancies of the order of

a pion mass, and hence made some changes in our parameter values so that our calculations yielded the decuplet resonances at more nearly their experimental positions. In the calculations reported below, we took the subtraction point $\hat{w} = 0.66$ BeV, the exchanged mass $M_i = 1.385$BeV, and the pion-nucleon coupling constant $g^2/4\pi = 15$. The notation follows WW, to which the reader is referred for a detailed definition of these parameters. The corresponding values in WW were $\hat{w} = 0.42$ BeV, $M_i = 1.47$ BeV, and $g^2/4\pi = 19$. We note that our value of the exchanged mass is still large enough so that the unphysical cuts never overlap the physical cuts, even when the degenerate mass values--i. e., the masses of the Λ and the η--are used for the external masses in computing the Born approximation amplitudes. We define a parameter f such that the coefficients of the F- and D-type couplings are proportional to f and $1 - f$ respectively. With $f = 0.35$, one finds that only the 10 representation of SU(3) has a low-energy $P_{3/2}$ resonance, in agreement with experiment. Taking $f = 0.35$, and the above values of the other parameters, we obtained the 33 πN resonance about 16 MeV higher than the experimental value, with comparable discrepancies for the positions of the other members of the decuplet. No attempt was made to find a set of parameters which would give a best fit to the experimental decuplet values.

We directed our investigation to the five reactions

$$K^+ + p \rightarrow K^+ + p \,, \qquad (2)$$

$$\pi^+ + p \to \pi^+ + p \,, \tag{3}$$

$$\pi^+ + p \to K^+ + \Sigma^+ \,, \tag{4}$$

$$K^- + p \to \pi^+ + \Sigma^- \,, \tag{5}$$

$$K^- + p \to K^o + \Xi^o \,. \tag{6}$$

Let T_n be the amplitude for the reaction numbered n above in the $P_{3/2}$ partial wave, normalized so that in the single-channel case $T = \sin\delta \; e^{i\delta}/q$, where q is the center-of-mass momentum. We define $A_n = E_n T_n$, with E_n being the total energy at which T_n is evaluated; thus A_n is the invariant amplitude for the reaction given by Eq. (n). The assumption of SU(3) invariance, together with the kinematic correction procedure described by Eq. (1), then leads to the relations[1]

$$A_2 - A_3 = A_4 \,, \tag{7}$$

$$A_5 = A_6 \,. \tag{8}$$

Equation (7) implies the inequality

$$|A_2| - |A_3| \leq |A_4| \leq |A_2| + |A_3| \,. \tag{9}$$

The inequality (9) is, of course, much weaker than (7), but the quantities entering it can be directly related to the $P_{3/2}$ contributions to the partial cross section for reaction n, σ_{Pn}, by the relation

$$|A_n| = \sqrt{F}\ \sigma_{Pn}/8\pi , \tag{10}$$

where F is defined in Eq. (1).

We note here that there is a question of phase convention in computing the amplitudes for Reactions (5) and (6). If one uses the isotopic spin factors given in Appendix II of Ref. 5 to compute the amplitudes for the reactions $\bar{K}N \to \pi\,\Sigma$ and $\bar{K}N \to K\,\Xi$ in states of pure isotopic spin, then

$$T_5 = (\bar{K}N \to \pi\,\Sigma)_0 / (6)^{\frac{1}{2}} + (\bar{K}N \to \pi\,\Sigma)_1/2 , \tag{11}$$

$$T_6 = (\bar{K}N \to K\,\Xi)_0/2 + (\bar{K}N \to K\,\Xi)_1/2 , \tag{12}$$

where by, for example, $(\bar{K}N \to K\,\Xi)_0$ we mean the amplitude for $\bar{K} + N \to K + \Xi$ in a state of total isotopic spin 0 computed from Martin and Wali's[5] Appendix II. The ambiguity arises from the freedom one has to choose, e. g., the $\bar{K}N$ and $K\,\Xi$ states with isospin = 0 to be, respectively, either $\pm\,(|K^- p> - |\bar{K}^0 n>)/\sqrt{2}$ or $\pm\,(|K^+ \Xi^-> - |K^0 \Xi^0>)/\sqrt{2}$. A particular relative choice of these signs might change both the sign of the first term in (12) and the sign of the expression for $(\bar{K}N \to K\,\Xi)_0$ given by Martin and Wali, leaving T_6,

of course, unchanged. That is to say, the coefficients of Martin and Wali's Appendix II imply a convention on the choice of some relative signs such as the one mentioned. In order to be sure whether one wishes, e.g., a plus or minus sign on the right side of Eq. (12), one must go back to the Lagrangian given in Appendix I of Ref. 5 and find out how to choose the signs so that the first Born approximation to, e. g., $(\bar{K}N \to K \Xi)_0$ is given correctly by the Martin and Wali isospin factors. Equations (11) and (12) then result.

The actual numerical calculations, involving numerical integrations and the inversion of 4×4 and 5×5 denominator matrices (in computing $\bar{K}N$ scattering with isospin 0 and 1) were, of course, done on a computer. Most of the work was done on the IBM 1620 at the Tufts University Computation Center, while some final production runs were made on a CDC 6400 at the Smithsonian Astrophysical Observatory.

We have investigated the validity of Eqs. (7) - (9) at various energies. We have compared amplitudes for different reactions both at the same value of Q_f, as suggested by MSY, and also at the same value of Q_i. We have also varied the D - F mixing parameter f over a fairly wide range of values. Our reason for varying f is twofold. Firstly, our basic philosophy is that we are studying the effect of mass splittings in a typical model relativistic scattering problem. We do not believe that our model, with its drastic simplifications (neglect of all unphysical cut contributions other than one-baryon exchange, determinantal approximation, neglect of other than two-particle final states) can be expected to provide a good description of the actual

physical processes at the energies involved, which are typically of the order of 2 BeV in our calculations, since we are dealing with the production of heavy particles; we do hope that the mass difference effects encountered will be similar to those in a more realistic theory. By varying f we can, in effect, study several different models with the same computer program, and perhaps get a better idea of the range of phenomena one may expect to encounter in nature. Secondly, as we shall see, when an orthodox value of f is used together with SU(3) symmetry, we find that the $P_{3/2}$ contribution alone to the cross sections for Reactions (4) and (6), at least near threshold, is much larger than the experimental values. Since it is not clear that one expects the model to be quite as bad as that, it seems worthwhile to see if one can obtain qualitative agreement with the small experimental cross sections for Reactions (4) and (6) within the framework of an SU(3) symmetric theory by varying f , whose value is at best rather uncertain.

Our results are shown in Tables I, II, and III. Table I gives the values of the amplitudes for the various reactions under consideration, and at various total energies, along with the initial and final state Q values. We have presented results for f = 0.35, and also for f = 0.7. Calculations were also performed for two other values of f, 0.1 and 0.55, within the range for which Martin and Wali[5] find a resonance only in the 10 representation. The results were qualitatively very similar to those for f = 0.35, and it does not seem worthwhile presenting them.

In Table II, we give a comparison of the results of the symmetry predictions (7) and (9) with the results of our calculations using the exact masses. We have confined ourselves to energies above that at which the 33 resonance occurs, as it is obvious that, since the mass splitting causes the resonance energy to be kinematically allowed in only one of the three channels, it will have a drastic effect for Q values near that of the resonance. It will be seen, however, that even above the 33 resonance region, Eq. (7) is badly violated, especially for f = 0.7. The violations remain large for Q's of the order of twice the mass differences, and even at the very high energy (a bombarding energy of about 14 BeV) corresponding to Q values of 4 BeV, the two sides of Eq. (7), although qualitatively similar, are not in quantitative agreement. The triangle inequality (9), is, of course, in better shape, since it does not necessarily reflect even large violations of (7); nevertheless, it is very badly violated for f = 0.7 at Q_f = 0.37, and violated to a lesser extent in all four cases at Q = 0.75.

As for the question of how one compares different reactions, as in Ref. 3 there does not seem to be a great deal to choose. At Q = 0.37, of course, choosing to put the Q_i's equal means putting the amplitude for Reaction (4) equal to zero, since it is not kinematically allowed. In this case, one does somewhat better to make the comparison at the same Q_f. Where both values of Q are allowed, there is little choice and, in fact, putting the Q_i's equal gives somewhat better results in this case, though this is probably not significant. (At Q = 4.0 BeV, the two methods yield essentially equivalent answers, and

we have given only one of the results.) As concluded in Ref. 3, it seems likely that if the results are sensitive to how one compares different reactions, e. g., in the case of Eq. (7) with $f = 0.7$ at $Q = 0.75$ BeV, this is in itself an indication that the effects of mass differences are large enough that any agreement, by any procedure, with a symmetry prediction must be considered essentially accidental.

Meshkov and Yodh[7] have recently surveyed the experimental situation with respect to the inequality (9). The only violation they find is at $Q_f \approx 0.8$ BeV, where the left-hand member of the first inequality in (9) is about a factor of three larger than $|A_4|$. They comment that this is not surprising, because this is approximately the energy of the $N^*(1920)$. This seems to us to be begging the question somewhat, since, if SU(3) is really a valid symmetry, the resonance should manifest itself in the various channels in such a way as to preserve Eq. (7), and therefore (9); unlike the 33 case, the resonance is kinematically allowed in all three of the reactions here. However, in view of our results, it seems perfectly possible that the mass differences alone could account for a violation of this magnitude at this energy, even if SU(3) is otherwise a valid symmetry. [Our assumption, of course, is that the violation of relations like (7) for the sum of partial waves comprising the total amplitude will, typically, be of the same order of magnitude as those we find for a single partial wave.]

The results in Table II do, however, indicate one difficulty with the SU(3) symmetric theory, at least with the value of f in the

usually accepted range. Experimentally, the cross section for Reaction (4) is quite small; near threshold, values of 0.21 ± 0.03 and 0.37 ± 0.07 mb have been obtained at values of Q_f of 0.1 and 0.37 BeV.[8] Taking $f = 0.35$, our model yields a $P_{3/2}$ cross section of 4.5 mb at $Q_f = 0.13$ BeV (or 2.9 mb at $Q_f = 0.1$ BeV, if one extrapolates assuming the cross section is proportional to q_f^3, as one would expect for a P-wave cross section near threshold), and 3.5 mb at $Q_f = 0.37$ BeV. That is, even neglecting other partial waves, our model with the usual value of f yields a cross section for Reaction (4) that is too large by a factor of 10. As indicated above, the same comments hold true for $f = 0.1$ and 0.55. The cross section decreases rather rapidly as a function of f for $f > 0.55$. For $f = 0.7$, the $P_{3/2}$ contribution is compatible with experiment, yielding cross sections of 0.22 and 0.022 mb at $Q_f = 0.13$ and 0.37 BeV. We shall see that a roughly similar situation occurs with respect to Reaction (5), and we shall postpone consideration of its possible significance until after reviewing the situation as regards the validity of Eq. (8) in our model.

Table III summarizes the situation on Reactions (5) and (6). We again give the kinematically corrected amplitudes A_5 and A_6, which are predicted to be equal by the symmetry assumption, for several different energies, and with the two reactions compared both at the same Q_f and at the same Q_i. The situation is very much like that we have already encountered. Near threshold we find one catastrophic

violation of the symmetry relation; at $Q_f = 0.18$, the two amplitudes which are predicted to be equal disagree by a factor of 10. The disagreement is perhaps not as bad at intermediate energies as in Eq. (7). Again, even at $Q = 4.0$ BeV, there are still differences of the order of 10% in the amplitudes. We also examined $f = 0.1$ and 0.55 for these reactions; they were once more qualitatively similar to $f = 0.35$, and we have not included the results. Once again, there seems no terribly clear-cut choice between the two prescriptions for comparing the reactions. Using Q_i does avoid the catastrophe at $Q_f = 0.18$ which we have just mentioned. Again we feel that if the two methods disagree significantly in analyzing experimental data, the main conclusion to be drawn is that one is not at sufficiently high energy to have eliminated the mass difference effects.

Experimentally, there are data on Reaction (6) at very low Q values. Berge et al.[9] have measured the cross section for Reaction (6) for values of Q_f ranging from 0 to 295 MeV, and obtain cross sections ranging from 46 ± 35 to 113 ± 27 μ b. These results differ from the corresponding cross sections for Reaction (5), after the kinematic corrections, by factors of the order of 50. As we have seen, however, even discrepancies of this magnitude could result from the mass differences alone. If one compares the two reactions at the same Q_i, one finds the cross sections, after correction, disagree by a factor of 4 rather than 50,[10] thus indicating the mass differences are likely to be extremely important.

There is one measurement on Reaction (6) at a moderately high energy.[11] It yields a cross section of 32 ± 8 μb at an incident K^- momentum of 3 BeV/c, corresponding to $Q_i = 1180$ BeV and $Q_f = 806$ BeV. Measurements of the cross section for Reaction (5) yield 45 ± 9 μb at $Q_i = 1180$ BeV, and 60 μb at $Q_f = 1030$ BeV.[12] These numbers yield $(F\sigma)_6/(F\sigma)_5 = 0.82$ evaluated at equal values of Q_i, and 0.75 evaluated at (approximately) equal values of Q_f. From Table II one observes that there is no reason to expect better agreement with Eq. (8) at these energies.

Once again, however, our calculations are not in agreement with the very small value of the cross section for Reaction (6) if the usual value of f is used, although the disagreement is not so severe as for Reaction (4). With $f = 0.35$, we obtain a $P_{3/2}$ contribution to the cross section for $K^o \Xi^o$ production at $Q_f = 0.18$ BeV of 0.21 mb, or about three times the experimental total cross section for the reaction. This seems a large discrepancy, especially since this close to threshold one might expect important S-wave contributions. We have again considered $f = 0.1$ and $f = 0.55$, and find that the real and imaginary parts of the amplitude vary with f in such a way that the predicted cross section for these f's is actually larger than for $f = 0.35$. For $f > 0.55$, the cross section decreases rapidly; for $f = 0.7$, the predicted $P_{3/2}$ cross section is 0.12 mb at $Q_f = 0.18$, or about one sixth of the experimental cross section for the reaction.

We turn now to the conclusions which seem to follow from these calculations. First of all, the general results of Ref. 3 are sustained

by the more realistic model considered here. For mass differences of the order of those within the hadronic SU(3) multiplets, one finds that, for Q values of the order of 100 or 200 MeV, the SU(3) predictions of relations among the cross sections for various reactions may be violated beyond all recognition. Moreover, the mass difference effects remain appreciable in many cases until the center-of-mass energy becomes comparable to, or better, larger than, the baryon masses. As far as the method of comparing different reactions is concerned, the procedure suggested by MSY, of looking at each reaction at the same Q_f, is probably as good as, but no better than, other plausible methods such as taking the Q_i's to be equal as done here, or taking the initial or final center-of-mass three-momenta to be the same.

It is, perhaps, worth commenting briefly on the choice of amplitude to which to apply the symmetry relation. We have been accepting, more or less tacitly, that one should use the invariant amplitude A rather than the scattering amplitude T, i. e., that the factor F proposed by MSY and defined in Eq. (1) is appropriate. In the symmetry limit, one could use either A or T, as the energy E would be the same for each reaction. The use of A is reasonable if one makes the usual assumptions about the analytic properties of the amplitudes, which are, of course, built into the present calculation, since then T has a $1/E$ singularity which is removed by considering A, which is free of singularities at $E = 0$. At low energies, where the effects of mass differences are very large, it is probably a somewhat academic question, as it must be considered accidental if the

symmetry relations hold for either A or T. [The reader can, however, easily find that even the catastrophic failures of Eqs. (7) and (8) in our model calculation would have been worse had we applied these relations to T rather than A.] At high energies, the difference between A and T becomes small, since the ratios of the total energies for different reactions tend to 1. It is, however, worth noting that at $Q = 4$ BeV the percentage error in both (7) and (8) is cut roughly in half by using A rather than T. For $Q = 4$ BeV, one has $T_2 - T_3 = 0.029 - 0.042\ i$ and $T_4 = 0.056 - 0.027\ i$, while $T_5 = -0.015 - 0.005\ i$ and $T_6 = -0.013 - 0.004\ i$. Comparing these with the corresponding values in Tables II and III for the A's one finds, as expected, that our model confirms the propriety of comparing the invariant amplitudes.

As far as the experimental conclusions to be drawn on SU(3) from symmetries in scattering experiments are concerned, it seems fair to say that such discrepancies as exist could easily be accounted for in terms of the mass differences alone. On the other hand, because of the large statistical errors in much of the data, the confirmatory evidence is also not convincing. More accurate data, and at much higher Q values, is required. It may be worthwhile adding that the possibility of studying nonrelativistic SU(6) by means of the additional symmetries in scattering which it predicts is very discouraging. Because it is a nonrelativistic theory, its presumed region of validity is just that in which, as we have seen, the mass differences may destroy all similarities between the experimental data and the predictions of the symmetry.

As we have seen, however, our calculations have developed an additional difficulty for an SU(3) symmetric theory, arising from the fact that the predicted cross sections for Reactions (4) and, to a lesser extent, (6) are too large. There are three possible explanations of this difficulty. The first is that our model is wrong by the amount of the discrepancy. The likelihood of this is very difficult to estimate. The model is crude in several respects; whether it is likely to be off by a factor of 10 in the K^+ Σ^+ production cross section near threshold is not clear.

A second possibility is that the value of the parameter f is larger than currently believed. The evidence on f is rather tenuous. As we have noted, Martin and Wali found they obtained a decuplet resonance, and no other, for $0.1 < f < 0.55$; Capps[13] obtains a similar range by similar arguments. In particular, for $f = 0.7$, Martin and Wali predict a resonance also in the 27 representation. In addition, Wali and Warnock, as well as the present work, find that one can achieve rather good agreement with the actual parameters of the decuplet states for $f = 0.35$. For $f = 0.7$, we obtain the 33 resonance, e. g., at an energy which is too large by about 200 MeV, though this could perhaps be improved upon somewhat by varying the subtraction point, which we have not attempted doing. In any event, it is not clear that better agreement than this should be expected from the model; it seems possible that, in an exact calculation, one might find a value of f, probably in the range of 0.5 to 0.7, that would yield the decuplet resonances at the right positions, produce either no resonances or resonances only at

quite high energy in the 27-dimensional representation, and yield suitably small values of the cross sections for Reactions (4) and (6). The biggest difficulty with a value of f in this region is in the predicted value of the $N\Lambda K$ coupling constant. For $f = 0.5$ one finds[5] $g_{N\Lambda K}^2/g_{NN\pi}^2 = 4/3$, while for $f = 0.7$, $g_{N\Lambda K}^2/g_{NN\pi}^2 = 1.9$. Lusignoli et al.[14] obtain $g_{N\Lambda K}^2/g_{NN\pi}^2 = 0.33 \pm 0.7$ from the dispersion relations for forward Kp scattering; this is consistent with values obtained from Λ production by photons and π^- mesons,[15] though the latter calculations are highly model-dependent, and probably not very reliable. Even allowing, however, for the difficulties in analyzing the Kp dispersion relations, because of the presence of unphysical regions, it is rather hard to accept values of the $N\Lambda K$ coupling constant consistent with the value of f suggested by our calculations.

The third solution to the problem is, of course, to abandon, or at least to allow severe breaking of, $SU(3)$ symmetry for the coupling constants. If this is done, then one has enough parameters so that one can probably achieve consistency with the resonance structure, the small production cross sections in Reactions (4) and (6), and the Kp forward dispersion relations. Indeed, Ref. (15) already has suggested that there are substantial violations of $SU(3)$, since it is impossible to fit simultaneously for any value of f the value obtained for $g_{N\Lambda K}$, and to satisfy an upper bound which is obtained for $g_{N\Sigma K}$. If this is the case, one would hope to find some clear disagreements with some of the $SU(3)$ cross sections predictions persisting even at center-of-mass energies large compared with hadronic masses.

It is a pleasure for one of us (A. E. E.) to express his thanks for the hospitality of Prefessor Geoffrey Chew in the Theoretical Group at the Lawrence Radiation Laboratory of the University of California in Berkeley during the completion of this work.

TABLE I. Values of the real and imaginary parts of the amplitudes, T, as defined in the text, for the various reactions at the indicated initial and final Q values and values of the D-F mixing parameter, f. The numbers in the first column refer to the reactions given by the correspondingly numbered equations in the text. The last column gives the total center-of-mass energy. We use units with h = c = 1.

Reaction	f	Q_i(BeV)	Q_f(BeV)	Re T(BeV^{-1})	Im T(BeV^{-1})	E
2	0.35	0.37	0.37	0.939	0.852	1.81
2	0.7	0.37	0.37	-0.881	1.280	1.81
2	0.35	0.75	0.75	0.372	1.117	2.19
2	0.7	0.75	0.75	-0.483	1.010	2.19
2	0.35	4.0	4.0	0.141	0.321	5.48
3	0.35	0.37	0.37	-0.947	2.041	1.45
3	0.7	0.37	0.37	0.053	2.479	1.45
3	0.35	0.75	0.75	-0.387	0.535	1.83
3	0.7	0.75	0.75	-0.338	1.413	1.83
3	0.35	4.0	4.0	0.112	0.363	5.08
4	0.35	0.75	0.13	0.797	-0.331	1.83
4	0.7	0.75	0.13	-0.123	-0.146	1.83
4	0.35	0.98	0.37	0.715	0.092	2.06
4	0.7	0.98	0.37	-0.035	-0.046	2.06
4	0.35	1.36	0.75	0.515	0.089	2.44
4	0.7	1.36	0.75	0.100	0.104	2.44
4	0.35	4.0	3.38	0.056	-0.027	5.08

Table I continued

Table I continued

Reaction	f	Q_i(BeV)	Q_f(BeV)	Re T(BeV^{-1})	Im T(BeV^{-1})	E
5	0.35	0.08	0.185	-0.105	0.151	1.52
5	0.7	0.08	0.185	0.440	-0.493	1.52
5	0.35	0.57	0.68	0.011	0.242	2.00
5	0.7	0.57	0.68	0.050	-0.061	2.00
5	0.7	0.64	0.75	0.040	-0.060	2.07
5	0.7	1.14	1.25	0.000	-0.047	2.58
5	0.7	3.89	4.0	-0.015	-0.005	5.34
6	0.35	0.57	0.185	0.001	0.178	2.00
6	0.7	0.57	0.185	0.048	-0.018	2.00
6	0.7	1.14	0.75	0.011	-0.042	2.58
6	0.7	4.39	4.0	-0.013	-0.004	5.81

TABLE II. The amplitude A_i represents the amplitude for reaction i, as given in Table I, multiplied by the center-of-mass energy, i. e., the invariant amplitude for reaction i . Column 1 gives the Q value, and whether initial or final, at which the amplitudes are evaluated, and column 2 gives the coupling parameter f. SU(3) predicts that corresponding entries in columns 3 and 5, and 4 and 6, should be equal, as well as the less stringent inequality that the entries in column 7 should be less than the corresponding ones in column 8. In the first two rows of the table, the amplitudes are compared following the prescription of MSY, Ref. 2. The units are the same as in Table I.

Q	f	$Re(A_2 - A_3)$	$Im(A_2 - A_3)$	$Re\ A_4$	$Im\ A_4$	$\lvert A_2\rvert - \lvert A_3\rvert$	$\lvert A_4\rvert$
$Q_f = 0.37$	0.35	3.07	-1.42	1.45	-0.61	1.02	1.57
$Q_f = 0.37$	0.70	-1.52	-1.28	-0.07	-0.09	0.83	0.11
$Q_f = 0.75$	0.35	1.52	1.58	1.44	0.19	1.59	1.45
$Q_f = 0.75$	0.70	-0.44	-0.37	0.24	0.29	0.19	0.11
$Q_i = 0.75$	0.35	1.52	1.58	1.47	0.19	1.59	1.48
$Q_i = 0.75$	0.70	-0.44	-0.37	-0.23	-0.27	0.19	0.11
$Q_i = 4.0$	0.35	0.20	-0.08	0.28	-0.14	0.01	0.32

TABLE III. Comparison of the invariant amplitudes A, defined as in the caption to Table II, for Reactions 5 and 6, at the indicated values of Q, in the initial or final state, and f. SU(3) predicts the equality of corresponding entries in columns 3 and 5, 4 and 6 and 7 and 8, and the prescription of MSY corresponds to carrying out the comparison as in rows 1, 3, and 5.

Q	f	Re A_5	Im A_5	Re A_6	Im A_6	$\|A_5\|$	$\|A_6\|$
$Q_f = 0.18$	0.35	-0.160	0.230	0.002	0.356	0.276	0.356
$Q_i = 0.57$	0.35	0.022	0.484	0.002	0.356	0.484	0.356
$Q_f = 0.18$	0.7	0.667	-0.748	0.096	-0.036	1.003	0.105
$Q_i = 0.57$	0.7	0.100	-0.122	0.096	-0.036	0.158	0.105
$Q_f = 0.75$	0.7	0.083	-0.127	0.028	-0.108	0.152	0.139
$Q_i = 1.14$	0.7	0.000	-0.121	0.028	-0.108	0.121	0.139
$Q_f = 4.0$	0.7	-0.080	-0.029	-0.075	-0.027	0.091	0.079

FOOTNOTES AND REFERENCES

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