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Author

Zhu, Qiang

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Interaction Leads to Symmetry Breaking in an Array of Cantilever Plates in Oscillatory Cross Flow

Qiang Zhu^{a)}

Department of Structural Engineering, University of California San Diego, La Jolla, California 92093-0085, USA.

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By using a computational model based on the immersed-boundary framework, a new mode of symmetry breaking is discovered in a fluid-structure interaction problem featuring an array of cantilever plates in a cross flow whose strength and direction varies sinusoidally with time. Specifically, within the physical parameters considered in this study, the motion of a single plate remains symmetric, whereas a system containing multiple plates can move asymmetrically so that the symmetry-breaking instability comes from fluid-dynamic interactions among individual plates. Further examination suggests that vortices shed from the free ends of the plates play an important role. Indeed, symmetry breaking occurs only when these vortices are sufficiently strong and when the distance between plates lies within certain range. If the distance is too small, a vortex shed from one plate can only stay in the gap between this plate and its neighboring plate for a short time so that it does not have the chance to interact extensively with the neighbor. On the other hand, if the distance is too large it is also difficult for this vortex to interact with the neighbor since it has to travel a long distance to reach there while the background flow keeps changing its direction. In either case the system does not display asymmetric behavior.

^{a)}Electronic mail: qizhu@ucsd.edu

I. INTRODUCTION

Symmetry-breaking phenomena are widely observed in fluid-structure interaction problems. For example, it was discovered that the wake created by a circular cylinder performing sinusoidal oscillations in quiescent fluid became asymmetric at sufficiently large Keulegan-Carpenter numbers¹. With the cylinder undergoing both translational motion and imposed oscillation, various symmetric and asymmetric patterns were observed in the wake^{2–4}.

Another experiment demonstrated that a symmetric wing undergoing periodic heaving motions may display forward-backward symmetry breaking so that non-zero thrust force is created⁵. Numerical simulations have shown that the symmetric and asymmetric behaviors were found to be determined by a critical Reynolds number based on flapping frequency and amplitude⁶. The cross-sectional shape of the foil may also play an important role⁷.

Likewise, there exists symmetry breaking in the transverse direction in the wakes created by a single flapping foil^{8–11} or a pair of foils undergoing anti-phase oscillations¹². In extreme cases, the wakes can go from asymmetric to chaotic^{13,14}. The effects of structural flexibility on the wake stability of flapping foils have also been examined. Experiments have demonstrated that for a foil undergoing cyclic pitching motion, its chordwise flexibility may inhibit the occurrence of symmetry-breaking instability in the wake¹⁵. However, numerical simulations of self-propelled plunging foils suggest that the influence of structural flexibility may be case dependant, i.e., it may either inhibit or trigger symmetry breaking¹⁶. The exact effect is related to its impact on the strengths of the leading edge and the trailing edge vortices.

Motivated by these previous studies as well as the recently proposed usage of flexible plates for energy harvesting from oscillatory flows^{17–22}, we hereby examine the symmetric versus asymmetric responses of cantilever plates in a cross flow whose strength and direction vary sinusoidally with time. In a recent study about the dynamics of cantilever plates in oscillatory cross flows, symmetry breaking was observed in a few cases²³. However, this phenomenon has not been systematically studied so that there is little knowledge about the conditions for it to occur or its correlation with characteristics of the structure and the flow field. These will be the focus of the current investigation. In particular, we concentrate on the dynamics of an array of such plates since it has been proven that the constructive fluid-dynamic interactions within the group can significantly boost plate deformations and thus their energy harvesting capacity^{22,23}. Compared with the existing studies about the symmetry-breaking phenomenon in fluid-structure systems, the sys-

tem considered here is more complicated as it involves multiple deformable objects as well as time-varying background flow.

Towards this end, we will utilize an immersed-boundary model to numerically simulate the two-dimensional fluid-structure interaction problem involved in the dynamics of an array of identical cantilever plates in oscillatory cross flows. Through these simulations, the occurrence of symmetry-breaking instability at different combinations of physical parameters, including the number of plates in an array, the distance between neighboring plates, the plate stiffness, as well as dimensionless parameters such as the Keulegan-Carpenter number, will be explored.

The rest of the paper is organized as following. The physical problem, including the setup of the array of plates and the incoming flow conditions, is presented in the next section. It is followed by the mathematical formulations, i.e., the governing equations of the fluid flow and the structural dynamics, and the interactions between these two. A brief description of the numerical algorithm is also included in this section. Details of the numerical method can be found in previous publications. In the results section, the symmetric and asymmetric responses at different conditions are presented and discussed, with the concentration on the role of fluid-dynamic interactions through vortices generated at the free ends of the plates. Finally, conclusions are drawn.

II. PHYSICAL PROBLEM

The physical problem to be examined is illustrated in Fig. 1. The system includes N identical flexible plates with length L in a side-by-side formation. The plates are uniformly distributed so that the distance between neighboring plates remains constant (D). The bottom end of a plate is fixed, while the top end is free.

The problem is described within a Cartesian coordinate system (x, y) , in which x is in the direction perpendicular to the undisturbed configuration of a plate. The flow speed in the x direction is u , and in the y direction is v .

An oscillatory flow is applied in the far field with $u = U(t) = U_0 \sin(\omega t)$ and $v = 0$, where U_0 is the peak speed of the oscillatory flow, $\omega = 2\pi/T$ is the frequency, and T is the period. Physically, this velocity field roughly corresponds to the flow associated with a shallow water wave in the vicinity of the bottom. The wave length is much larger than the length scale of the region we consider so that without the presence of the solid objects variation of u in the x direction is negligibly small. Under this simplification, the flow condition can be represented by only two

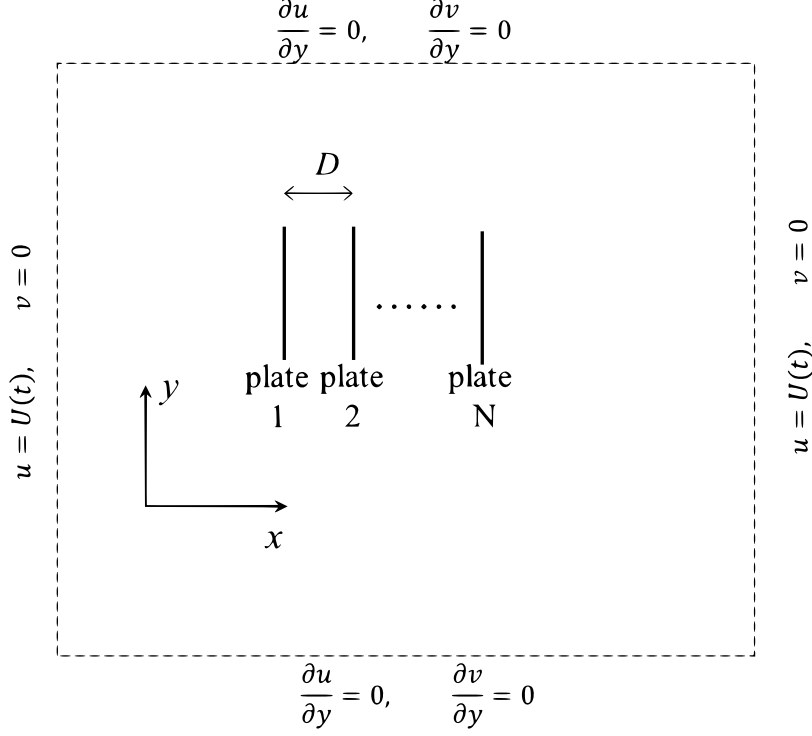


FIG. 1. The computational domain and the boundary conditions.

physical parameters, U_0 and ω (or T).

Hydrodynamically, the important dimensionless parameters that determine the physics of the system are the Reynolds number R_e and the Keulegan-Carpenter number K_C . The Reynolds number is defined as $R_e \equiv \rho U_0 L / \nu$, and the Keulegan-Carpenter is defined as $K_C \equiv U_0 L / T$.

III. MATHEMATICAL FORMULATIONS AND NUMERICAL APPROACH

A. Governing equations

By using the two-dimensional incompressible flow rendition, under the immersed-boundary framework the fluid-dynamics problem is mathematically formulated via the Navier-Stokes equations

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}(\mathbf{x}, t), \quad (1)$$

and the continuity equation

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

where the vector $\mathbf{x} \equiv (x, y)$ represents the location of a spatial point measured in the coordinate system shown in Fig. 1. $\mathbf{u} \equiv (u, v)$ is the flow velocity vector. ρ and μ are the density and dynamic viscosity of the fluid, respectively. p is the pressure. $\mathbf{f}$ is the force density exerted by the immersed structure on the fluid, a typical treatment in the immersed-boundary method.

The structural response of one plate is governed by

$$m \frac{\partial^2 \mathbf{X}}{\partial t^2} = \frac{\partial}{\partial s} (\boldsymbol{\sigma} \boldsymbol{\tau}) - \frac{\partial^2}{\partial s^2} \left(k_b \frac{\partial^2 \mathbf{X}}{\partial s^2} \right) - \mathbf{F}, \quad (3)$$

where $\mathbf{X}$ is the instantaneous location of this plate, along which a Lagrangian coordinate s is defined. The plate is inextensible so that $|\partial \mathbf{X} / \partial s| = 1$. m is the mass per unit length of the plate. k_b is the bending stiffness. $\boldsymbol{\tau} = \partial \mathbf{X} / \partial s$ is the unit tangential vector of the body. $\mathbf{n}$ is the unit normal vector. $\mathbf{F}$ is the force density applied by the surrounding fluid to the structure. $\boldsymbol{\sigma}$ is the tension inside the plate, which is determined through the inextensibility constraint^{24–26}, i.e.,

$$\frac{\partial^2}{\partial s^2} (\boldsymbol{\sigma} \boldsymbol{\tau}) \cdot \boldsymbol{\tau} = \frac{m}{s} \frac{\partial^2}{\partial t^2} (\boldsymbol{\tau} \cdot \boldsymbol{\tau}) - m \frac{\partial \boldsymbol{\tau}}{\partial t} \cdot \frac{\partial \boldsymbol{\tau}}{\partial t} + \frac{\partial}{\partial s} \left(\frac{\partial^2}{\partial s^2} \left[k_b \frac{\partial^2 \mathbf{X}}{\partial s^2} \right] + \mathbf{F} \right) \cdot \boldsymbol{\tau} \quad (4)$$

The plate is fixed at its bottom end ($s = 0$), where $\mathbf{X} = \mathbf{X}_0$ ($\mathbf{X}_0$ is a prescribed location) and $\partial \mathbf{X} / \partial s = (0, 1)$. At its top end ($s = L$), the plate is free with no bending moment and normal/shear stresses^{22,27}.

For normalization, the plate length L , the flow period T , and the fluid density ρ are chosen as the repeating variables. As to the other physical parameters, mass will be normalized by ρL^2 , mass per unit length by ρL , velocity by L/T , and vorticity by $1/T$. For convenience, hereafter we use the same symbols for dimensional variables (e.g., s , t , m , $\mathbf{X}$, $\boldsymbol{\sigma}$, and $\mathbf{F}$) to represent their dimensionless counterparts. The normalized bending stiffness κ is defined as $k_b / (\rho L^5 / T^2)$.

B. Numerical method

The fluid-structure coupling is simulated with an immersed-boundary algorithm. This approach utilizes a Dirac Delta function δ to relate the force density exerted by the immersed structure on the fluid (i.e., $\mathbf{f}$ in Eq. (1)) and the force density applied by the surrounding fluid to the structure ($\mathbf{F}$ in Eq. (3)), i.e.,

$$\mathbf{f}(\mathbf{x}, t) = \int_{\Gamma} \mathbf{F}(s, t) \delta(\mathbf{X}(s, t) - \mathbf{x}) ds, \quad (5)$$

where Γ represents the structure.

The force density $\mathbf{F}$ enforces the no-flux and no-slip conditions at the fluid-structure interface. This is achieved using the following equation

$$\mathbf{F}(s, t) = \alpha_n \int_0^t [\mathbf{U}(s, \tau) - \mathbf{V}(s, \tau)] d\tau + \beta_n [\mathbf{U}(s, t) - \mathbf{V}(s, t)], \quad (6)$$

where $\mathbf{V}$ is the structural velocity and $\mathbf{U}$ is the fluid velocity at the fluid-structure interface. α_n and β_n are numerical parameters that are sufficiently large. Physically, this corresponds to the scenario in which the neighboring fluid and solid elements are connected with a spring (whose stiffness and damping property are represented by α_n and β_n) so that they do not drift away from each other^{26,28}. Under the immersed-boundary framework, $\mathbf{U}$ is obtained from the flow velocity $\mathbf{u}$ by using the Delta function as

$$\mathbf{U}(s, t) = \int_{\Omega} \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}(s, t)) d\mathbf{x}, \quad (7)$$

where Ω is the fluid domain.

A finite difference algorithm is then adopted to solve the partial differential equations (Eqns. (1) to (3) in §III A) to obtain the fluid motion and the structural response. Details of this method can be found in previous publications^{26,29,30} so that for brevity they are not included here.

IV. RESULTS

Systematic studies have been conducted to investigate the dynamics of the system described in §II. The cases are characterized with the following dimensionless parameters:

1. The Reynolds number R_e , whose value is fixed as 100. In the simulations, this is achieved by varying the values of the velocity amplitude U_0 and the kinematic viscosity ν simultaneously so that they remain proportional to each other, thus the value of R_e remains unchanged.
2. The Keulegan-Carpenter number K_C , which is a measure of the ratio between the amplitude of the fluid particles in the oscillatory flow and the length of the body. The range of K_C considered in this study is from 1 to 3. This range covers both symmetric and asymmetric responses.
3. The dimensionless plate stiffness κ . In most cases we consider $\kappa \in [1, 5]$. According to previous studies²² as well as tests conducted in this study (see §IV A), if κ is below this range the plate motion is often irregular. Above it, on the other hand, the energy harvesting performance decays fast so that the parametric region is less interesting.

4. The distance between neighboring plates D . The range of this parameter is chosen to be $[0.1, 1.0]$. This choice is mostly due to computational concerns. If D is too small, the computational mesh near the body needs to be further refined for better resolution. If D is too large, the size of the inner computational zone (i.e., the zone with dense computational mesh) needs to be increased to accommodate the array of plates.
5. The number of plates in an array N . The range is between 1 and 4. If N is beyond this range, the inner computational zone may need to be enlarged.
6. The normalized mass per unit length of a plate m , which is chosen to be 0.5.

The numerical method has been extensively validated through comparisons with benchmark results from theoretical analysis and numerical studies as well as convergence tests. For example, the structural solver in it has been used to simulate the oscillation of a hanging filament²⁹. The fluid solver has been used to simulate the accelerating motion of a plate and the oscillatory motion of a wing have²⁹, the dynamics of flow around a fixed circular cylinder²⁶, and a cylinder undergoing prescribed oscillations²⁶. The coupled fluid-structure interaction model has been applied to simulate an elastically mounted cylinder²⁶. In all of these cases, perfect agreement with benchmark results has been obtained. Moreover, numerical sensitivity tests, including those with configurations identical to the one used in the current study, were carried out with satisfactory results^{22,23,30}.

For further validations, additional sensitivity tests have been conducted by using the case $K_C = 1$, $\kappa = 1$, $N = 3$ and $D = 0.4$. We use a square-shaped computational domain and place the plates at the center. Around the plates there is a region (i.e., the inner zone) within which denser computational mesh is applied for enhanced resolution. The size of the computational mesh within this region is $\Delta x \times \Delta y$. Out of this region (the outer zone) a coarser mesh is used for computational efficiency. The size of the computational domain is chosen as 20 on each side. Three different numerical meshes, a coarse one ($\Delta x = \Delta y = 0.034$, $\Delta s = 0.026$), a medium one ($\Delta x = \Delta y = 0.025$, $\Delta s = 0.02$), and a refined one ($\Delta x = \Delta y = 0.017$, $\Delta s = 0.013$) are chosen. Furthermore, three time steps, $\Delta t = 5 \times 10^{-5}$, $\Delta t = 2.5 \times 10^{-5}$, and $\Delta t = 1.25 \times 10^{-5}$ are used to demonstrate the accuracy of the time-evolution algorithm.

Figure 2 displays time histories of the horizontal motions of the top ends of the plates (x_t) after the steady state is established by using various combinations of numerical parameters. It is clear

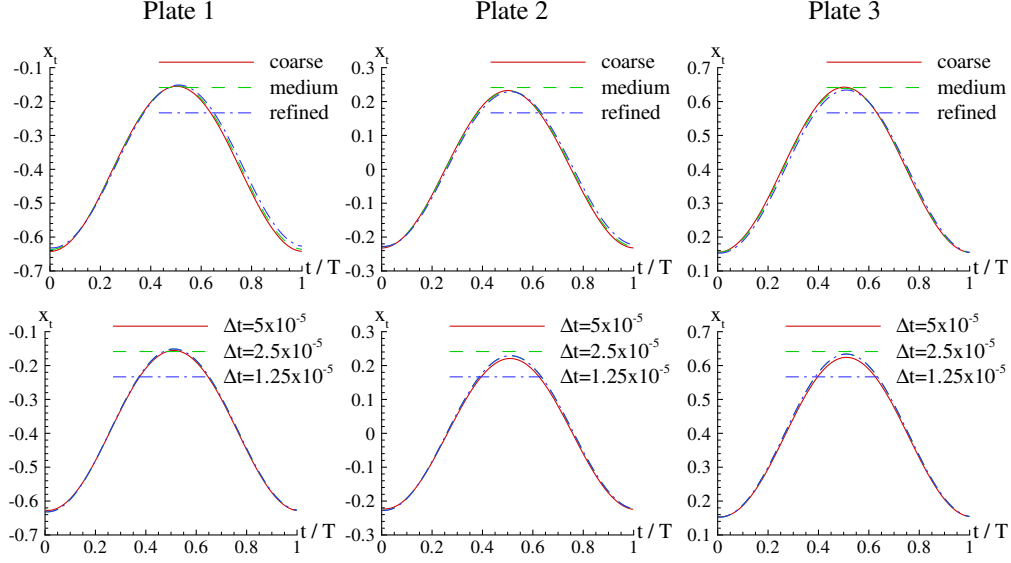


FIG. 2. Sensitivity of the horizontal motion at the top ends x_t with respect to the mesh density (upper row) and time step Δt (lower row). $N = 3$, $K_C = 1$, $\kappa = 1$. In the mesh density tests $\Delta t = 1.25 \times 10^{-5}$. In the time step tests the refined mesh is used.

that with sufficiently dense mesh and sufficiently small time step the results are not sensitive to the numerical parameters.

Sensitivity tests with regard to the size of the computational domain have also been carried out by using computational domains ranging from 20×20 to 40×40 . There is almost no change in the results. Therefore, from now on we keep the original choice of the domain with 20 in each side. We also choose the refined mesh and $\Delta t = 1.25 \times 10^{-5}$.

A. Baseline case ($N = 1$)

We start by considering the case where $N = 1$, i.e., a single stand-alone plate with no neighbors. The purpose is to provide a benchmark case for comparison, allowing the assessment of the effect of hydrodynamic interactions among neighboring plates, especially the effect upon the symmetry-breaking phenomenon, can be assessed. Moreover, this study allows us to isolate the vorticity field generated by a single plate without interference from the wakes of other plates, facilitating the analysis of underlying physics.

The simulations show that periodic responses are achieved when the stiffness κ is above a critical value. In the three values of K_C we consider (i.e., 1, 2, and 3), these critical values of κ are

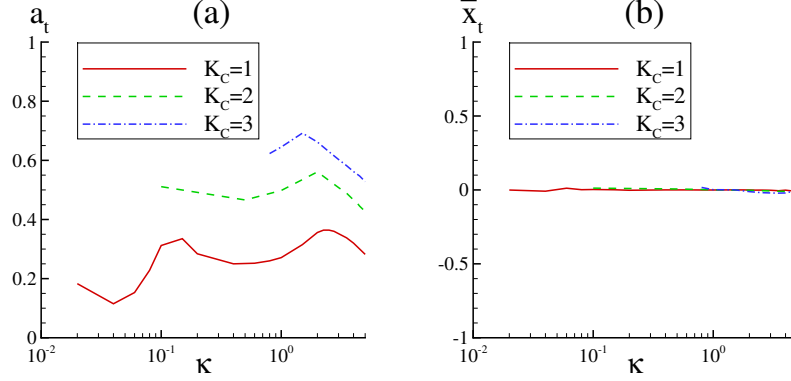


FIG. 3. (a) Amplitude and (b) mean position of the top end at various values of κ and K_C . $N = 1$.

0.02, 0.08, and 0.6, respectively. In cases with periodic response, we record the horizontal motion of the top end $x_t(t)$ and define the amplitude of this motion as $a_t = (x_{tmax} - x_{tmin})/2$, where x_{tmax} and x_{tmin} are the maximum and minimum x locations of the top end within one period, respectively.

Figure 3a demonstrates the steady-state values of a_t as functions of κ when $K_C = 1, 2$ and 3 . In the case when $K_C = 1$, two peaks are observed, corresponding to the excitation of the second (at $\kappa \sim 0.15$) and the first (at $\kappa \sim 2.5$) natural modes of the plate²². At the other two values of K_C , on the other hand, only the first natural mode is seen. Corresponding to these cases, in Fig. 3b the mean position $\bar{x}_t$ of the top end is shown. Hereby $\bar{x}_t$ is calculated as the time-averaged value of x_t over one period after the periodic response state has been established (in Fig. 3b these values are calculated within the ninth period, i.e., $9 \leq t \leq 10$). It is seen that over the whole range of parameters we consider $\bar{x}_t$ is extremely small in comparison with the amplitude of motion, suggesting that the system remains symmetric in the x direction.

The vortex generation from the top end depends on the relative motion between the background flow and the plate at that location. Specifically, the phase angle between the incoming flow speed $U(t)$ and the inline speed of the plate at the top end $\dot{x}_t$ plays a critical role²³. In Fig. 4 the vorticity fields around the body when $K_C = 1$ and $\kappa = 1, 2$, and 3 are plotted. In these figures, concentrations of positive vorticity correspond to clockwise vortices, whereas those of negative vorticity correspond to counter-clockwise vortices.

When $\kappa = 1$, the plate is sufficiently soft so that it feathers under the hydrodynamic load. In this case the phase angle between $U(t)$ and $\dot{x}_t$ is close to zero. Subsequently, there is almost no vortex shedding from the top end in comparison with that from the bottom end (Fig. 4a). When $\kappa = 3$, on the other hand, the phase angle between U and $\dot{x}_t$ reaches 57° . Correspondingly, considerable

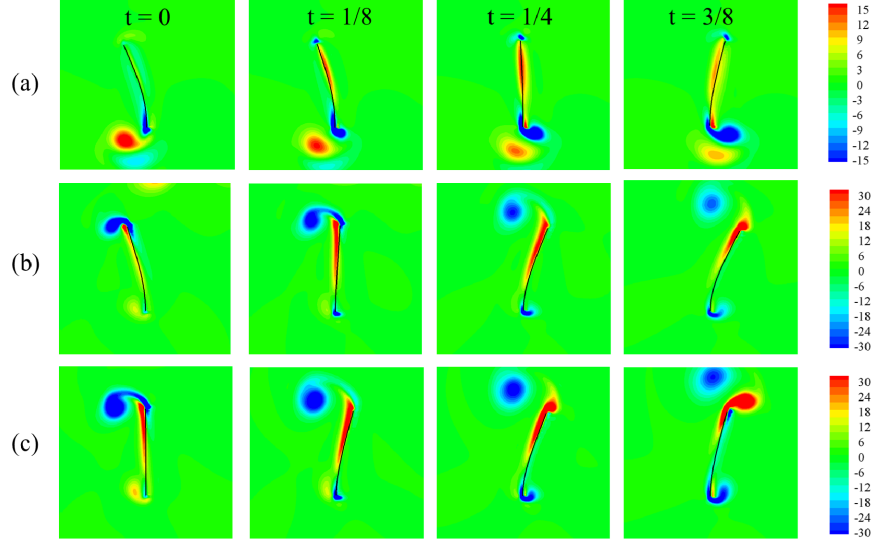


FIG. 4. Snapshots of the near-body flow field visualized through the vorticity distribution over half a period with (a) $\kappa = 1$, (b) $\kappa = 2$, and (c) $\kappa = 3$. $N = 1$, $K_C = 1$. These figures are obtained in the ninth period. For convenience in this figure (and in all the following figures showing flow fields) the starting time of the period is shifted to $t = 0$.

vortex shedding occurs at the top end (Fig. 4b). Finally, when $\kappa = 5$ the phase angle increases to 82° . The vortices generated at the top end are clearly strengthened compared with the case when $\kappa = 3$ (see Fig. 4c; note that this figure shares the same contour range with Fig. 4b).

The effect of the Keulegan-Carpenter number K_C on vortex shedding has also been examined. As shown in Fig. 5, when the stiffness κ is fixed and K_C is increased from 1 to 3, the vorticity generation from the top end of the plate also increases (all the snapshots share the same contour range).

B. The symmetry-breaking phenomenon and its correlation with near-body vorticity field

Within the range of parameters we consider, symmetry breaking only occurs in cases when N is at least 2 (the cases with irregular responses are not considered). This indicates that the phenomenon is associated with the hydrodynamic interactions among individual plates in a array.

In Fig. 6 a typical symmetry-breaking case is displayed. According to Fig. 6a, the responses of the two plates in the array are close to each other at the beginning of the simulation. They start to deviate from each other after a couple of periods and the difference between them grows with

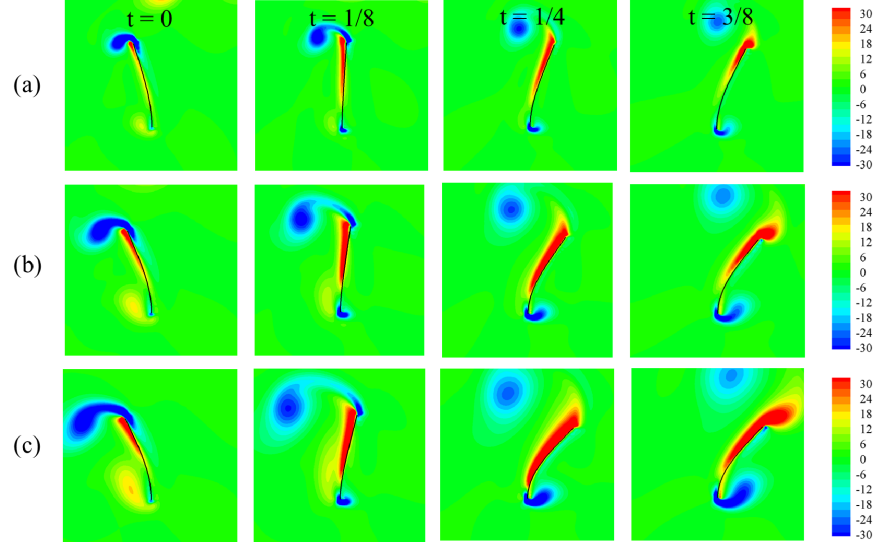


FIG. 5. Snapshots of the near-body flow field visualized through the vorticity distribution over half a period with (a) $K_C = 1$, (b) $K_C = 2$, and (c) $K_C = 3$. $N = 1$, $\kappa = 3$.

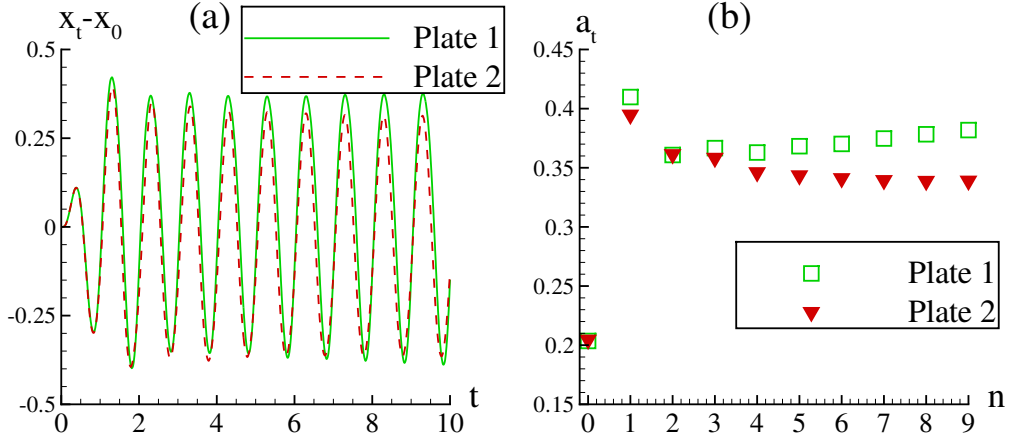


FIG. 6. (a) Time history of the motion at the top end of the plates (x_0 is the x locomotion of the bottom end of the corresponding plate). (b) Motion amplitude a_t during the n -th period (the period between $t = n$ and $t = n + 1$). $N = 2$, $K_C = 1$, $\kappa = 3$, $D = 0.4$.

time. This tendency is more clearly shown in Fig. 6b, where the amplitudes at the top ends of the plates are shown from the zeroth to the ninth period.

The corresponding flow fields for the case shown in Fig. 6 are displayed in Fig. 7. This figure implies a correlation between the vortices shed from the top ends of the plates and the development of the symmetry-breaking instability. At the beginning (the zeroth period), there is little vorticity

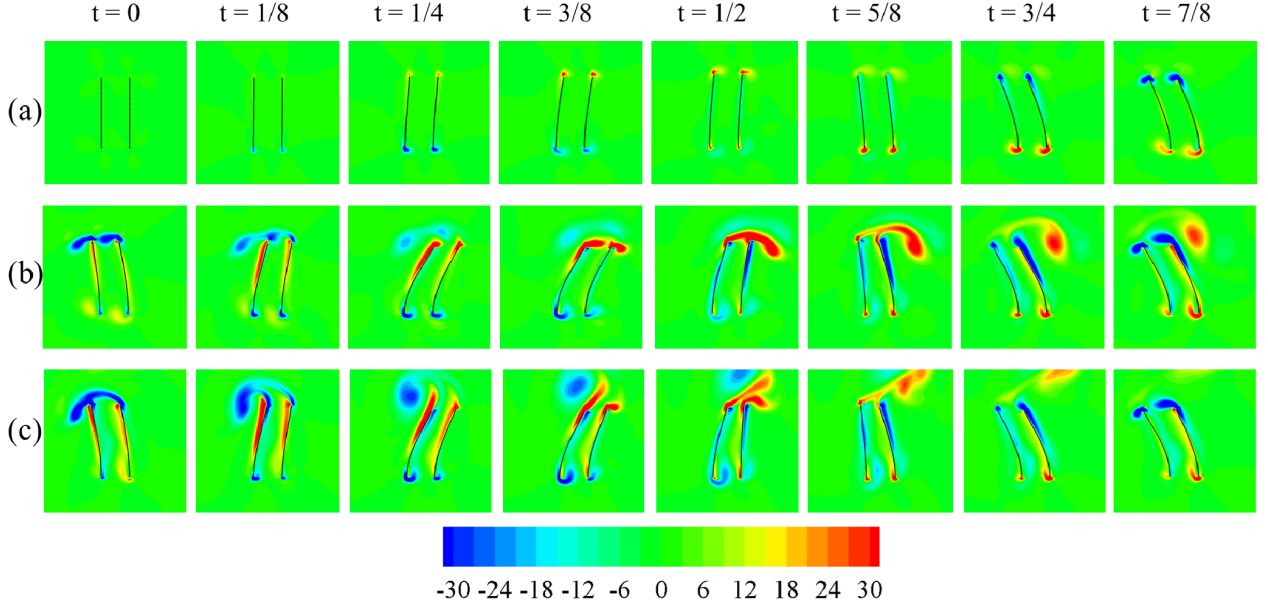


FIG. 7. Snapshots of the near-body flow field visualized through the vorticity distribution during (a) the zeroth period, (b) the first period, and (c) the ninth period. The parameters are the same as those in Fig. 6.

generation from the top ends. Meanwhile the motions of the two plates are almost identical so that the system is symmetric in the longitudinal direction (see Fig. 6). Afterwards, vortex shedding from the top ends is enhanced, accompanied by the deviation of the motions of the two plates from each other so that the system becomes more and more asymmetric.

More evidence of the role of top-end vortices on symmetry breaking comes from another case that has the same parameters to the one shown in Figs. 6 and 7 except that the size of the gap between the two plates is reduced from 0.4 to 0.1. Interestingly, despite the fact that in this case the interaction between the two plates should be stronger since they are closer to each other, the system remains symmetric as demonstrated in Fig. 8.

This paradox is explained by examining the vorticity field near the plates as shown in Fig. 9. It is seen that when the gap between the two plates is narrow, the top vortices are not able to squeeze into it. Instead, the vortices shed from the top ends of the plates merge and shed together into the wake before there is any extensive interaction between a vortex shed from one plate and the neighboring plate. Meanwhile, due to proximity the two plates become closely coupled so that their motions are identical to each other and the system response remains symmetric.

On the other hand, if the gap size D is large, it is expected that the symmetric response will be re-established as the interactions among the plates are too weak. An example is displayed in Fig.

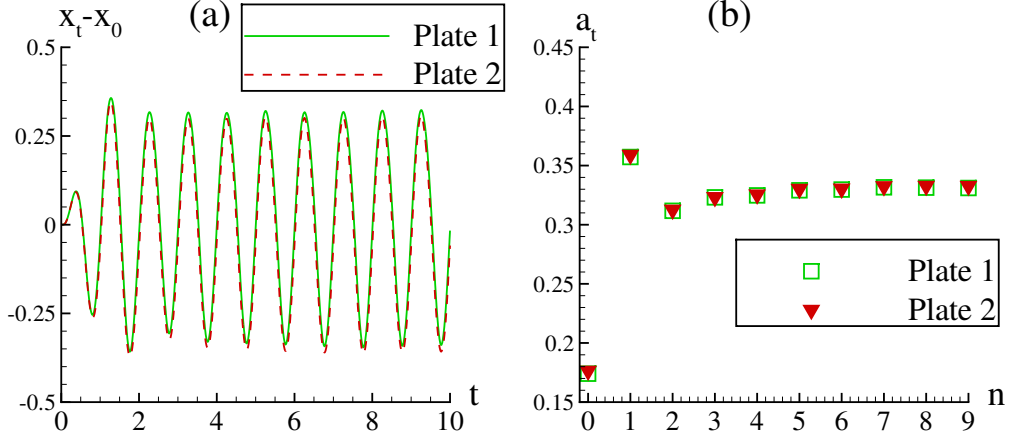


FIG. 8. (a) Time history of the motion at the top end of the plates (x_0 is the x locomotion of the bottom end of the corresponding plate). (b) Motion amplitude a_t during the $n - th$ period of the plates. The parameters are the same as those in Fig. 6 except that D is reduced from 0.4 to 0.1.

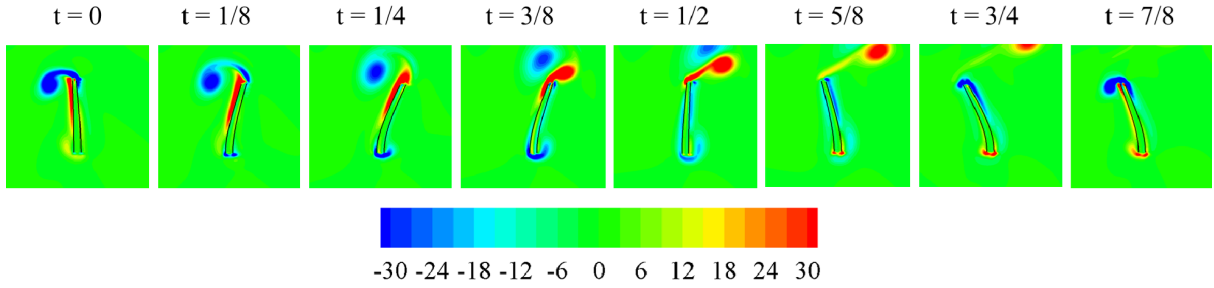


FIG. 9. Snapshots of the near-body flow field visualized through the vorticity distribution during the ninth period. The parameters are the same as those in Fig. 8.

10, in which D is increased from 0.4 to 0.8. In this case the motions of the two plates in the array become identical after around six periods when the initial disturbance is gone (see Fig. 10b).

The explanation of this symmetry-reestablishment case is again related to the evolution of the top-end vortices. As demonstrated in Fig. 11, during the zeroth period (Fig. 11a), the vortices shed from the top ends of the two plates are weak, thus the two plates undergo almost identical deformations. During the first period (Fig. 11b), the top-end vortices are much stronger in strength. Since the steady state has not been established in this period, the strengths of the vortices shed during the first half-period are weaker than those shed during the second half-period. For example, in a steady-state symmetric response, the vortex marked as ‘V1’ at $t = T/8$ and the one marked as ‘V2’ at $t = 5T/8$ in Fig. 11b should have the same strength despite their rotational directions.

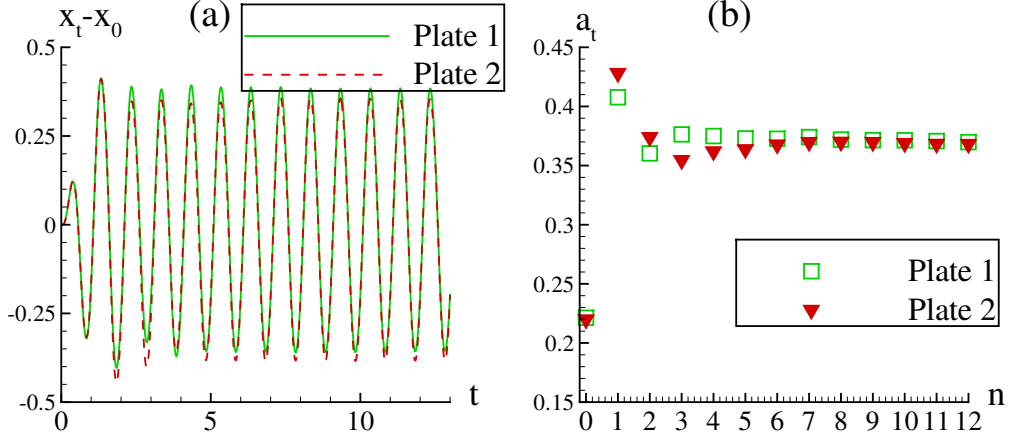


FIG. 10. (a) Time history of the motion at the top end of the plates (x_0 is the x locomotion of the bottom end of the corresponding plate). (b) Motion amplitude a_t during the $n - th$ period of the plates. The parameters are the same as those in Fig. 6 except that D is increased from 0.4 to 0.8.

Yet from the figure it is clear that ‘V2’ is much stronger than ‘V1’. The interactions between these vortices and the plates lead to the difference between the plates’ motions during this period. This initial effect is also observed in the case when $D = 0.4$ (see Fig. 6), in which it lasts for just one period. In the current case it lasts for as much as six periods, after which a steady-state response with identical motions for the two plates is reached. At this stage, unlike the case of $D = 0.4$, due to the increased distance between the plates, the top-end vortex from one plate is not able to interact thoroughly with the neighboring plate so that the vortex-plate interactions do not trigger symmetry-breaking behavior.

The difference between the wakes created by the system with different gap sizes are clearly shown in Fig. 12. When the two plates are relatively close to each other ($D = 0.4$), the vortices shed from the top ends of the two plates merge into a single wake (Fig. 12a). With larger gap ($D = 0.8$), the wakes created by the two plates remain distinguishable from each other (Fig. 12b), an indication of reduced interaction via top-end vortices between the two.

The interaction between a top-end vortex and another plate in the array, which eventually leads to the symmetry-breaking behavior, is a complicated problem. The collision between a vortex and a plate involves the following processes: 1) the interaction between the vortex with the side of the plate, which is similar to the interaction between a vortex and a solid wall³¹. 2) The collision between the vortex and the sharp top end of the plate, which is somewhat akin to the interaction between a vortex and a wall with gaps on it³². 3) the effect of structural flexibility of the plate

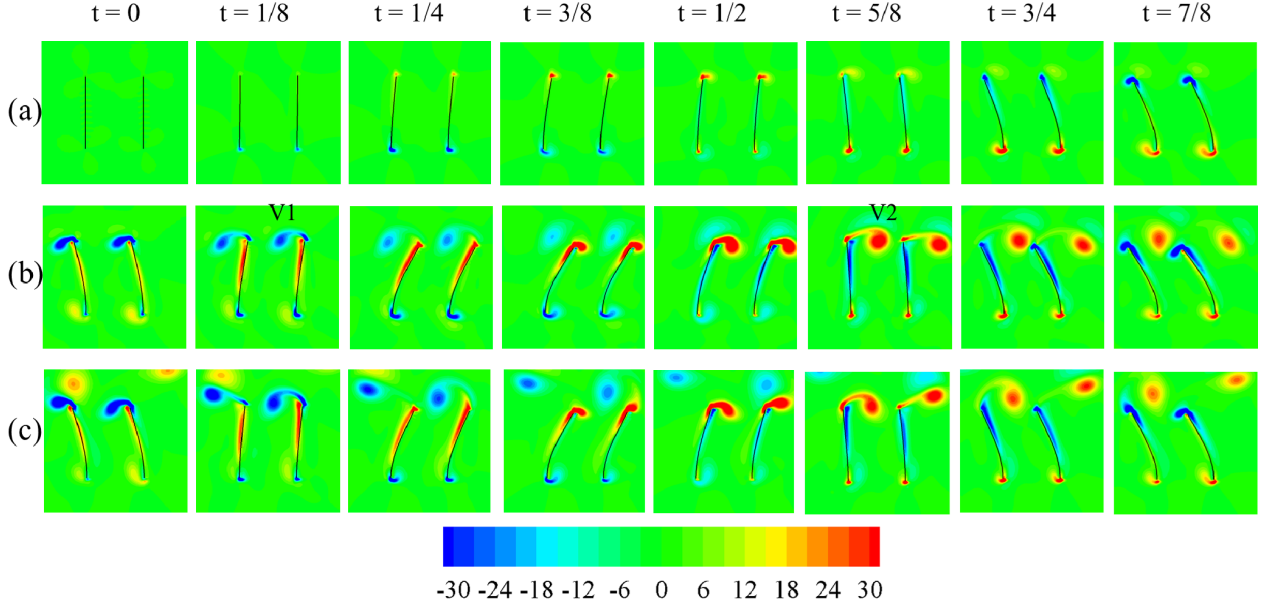


FIG. 11. Snapshots of the near-body flow field visualized through the vorticity distribution during (a) the zeroth period, (b) the first period, and (c) the ninth period. The parameters are the same as those in Fig. 10.

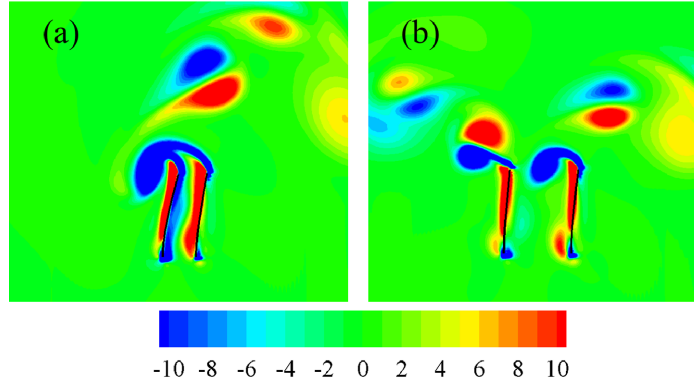


FIG. 12. Snapshots of the wake visualized through the vorticity distribution at $t = T/8$ for (a) $D = 0.4$ and (b) $D = 0.8$. $N = 2$, $K_C = 1$, $\kappa = 3$.

during this interaction³³.

To understand which process plays the dominant role in the route towards instability, in Fig. 13 the near-body flow fields in the case shown in Fig. 6 during the second oscillation period (the period when the symmetry-breaking instability starts according to Fig. 6b) are plotted. It is seen that during the vortex-plate interactions, the vortex does not penetrate deep into the gap so that it is most likely that Process 1 does not play an important role. The plate stiffness, on the other hand, affects both the vortex-plate interaction (Process 3) and the generation of the top-end vortices as

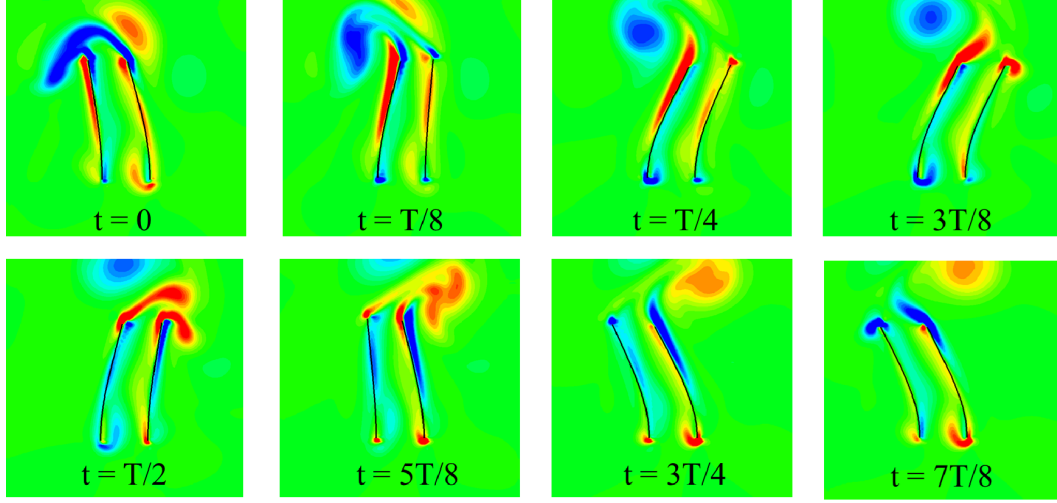


FIG. 13. Snapshots of the near-body flow field visualized through the vorticity distribution during the second period. The parameters are the same as those in Fig. 6.

illustrated in §IV A. Further studies are needed to decouple these effects.

C. Parametric studies

For further knowledge about the dynamic responses of the system at different conditions, parametric studies have been conducted within a wider range of physical parameters. Hereby symmetric responses refer to the cases in which the plates have periodic motions symmetric in the x direction. In asymmetric responses, on the other hand, the plates have (roughly) periodic motions not symmetric in the x direction. In irregular responses the plate motions are not periodic. Examples of the symmetric response can be found in Figs. 8 and 10. Figure 6 demonstrates the characteristics of an asymmetric response. The differences between these two responses have also been illustrated in a previous publication²³ (see Figs. 3, 4, and 7 in that paper). For an example of the irregular case, in Fig. 14 the structural responses of the plates in a system in which $N = 3$, $K_C = 3$, $\kappa = 1$, and $D = 0.4$ are displayed. The time histories shown in Fig. 14 suggest the competition between two modes (a high-amplitude one and a low-amplitude one), a feature usually associated with chaotic motions. Characteristics of the irregular response have also been illustrated elsewhere²² (see Fig. 4 in that paper).

In terms of vortex shedding from the top ends of the plates, within the parametric space studied here two counter rotating vortices are shed from each plate (not counting for the secondary vortices

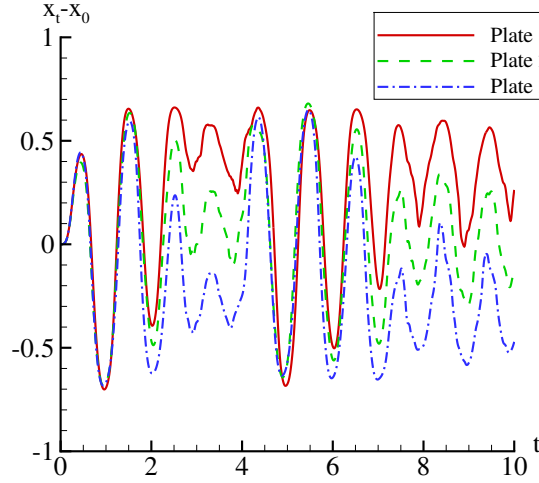


FIG. 14. Time history of the motion at the top end of the plates in an irregular response (x_0 is the x locomotion of the bottom end of the corresponding plate). $N = 3$, $K_C = 3$, $\kappa = 1$, $D = 0.4$.

that may be induced) within each incoming flow period. The asymmetric and irregular responses are caused by the interactions between these vortices that are already in the wake with other plates, rather than the generation of these vortices.

We first consider the case when $N = 2$, $\kappa \in [1, 5]$, $D \in [0.1, 1.0]$ and $K_C = 1, 2, 3$. The occurrence of symmetric and asymmetric responses at different combinations of these parameters are shown in Fig. 15a. From the figure it is seen that when κ is small (hereby when $\kappa = 1$), the response remains symmetric in most cases due to the lack of vortex shedding from the top ends. A exception happens at $K_C = 3$ and κ between 0.6 and 0.8, where irregular responses similar to those recorded at lower values of κ in the single plate case are observed. This irregularity is also triggered by the interaction between the two plates. It has been reported earlier²².

At larger values of κ or K_C , the top-end vortices are stronger (see §IV A). Moreover, with larger K_C the top-end vortices travel longer distances in the longitudinal carried by the flow so that they have more chance to interact with other plates. Subsequently, the symmetry-breaking instability is easier to happen. This is clearly shown in Fig. 15a, where the size of the asymmetric zone in terms of D (i.e., the range of D with asymmetric behavior) increases with both κ and K_C .

In comparison with the case when $N = 2$, in cases when $N > 2$ the system is more likely to turn asymmetric with smaller values of D (e.g., $D = 0.2$), as demonstrated in Figs. 15b and c. This is explained by the fact that when $N = 2$, each plate has only one neighbor. If D is small

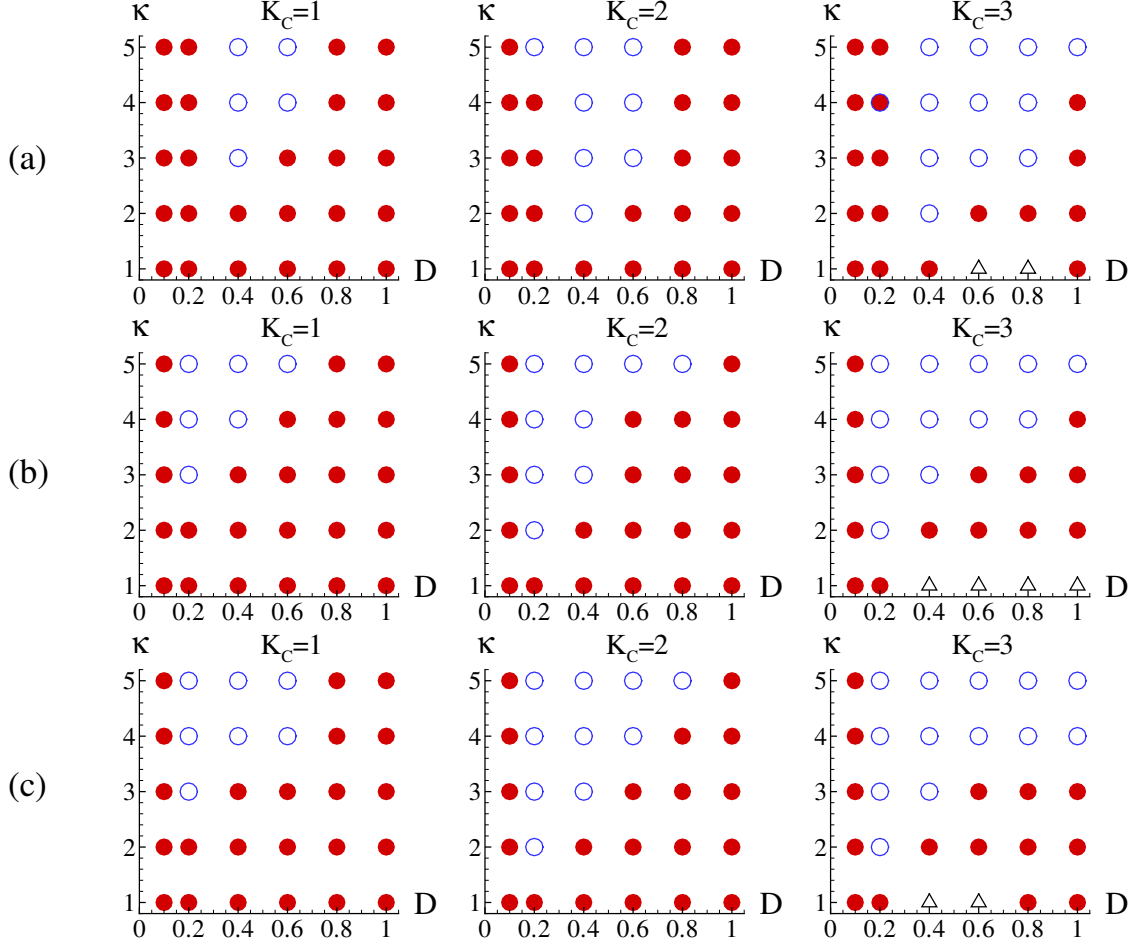


FIG. 15. Symmetric ($\bullet$), asymmetric ($\circ$), and irregular (Δ) responses at various combinations of K_C , D and κ when (a) $N = 2$, (b) $N = 3$, and (c) $N = 4$.

the vortices generated from the top end of one plate may have no chance to interact with the other plate before it is shed into the wake. When $N > 2$, on the other hand, a top-end vortex generated from one plate can not only interact with its immediate neighbors but also with other plates that are not next to it so that the chance of symmetry breaking at lower values if D is increased. Nevertheless, the difference between the case when $N = 2$ and those when $N > 2$ is relatively subtle, probably because the interaction between immediate neighbors plays a dominant role in this physical process so that adding more plates to the array does not significantly affect the instability mechanism.

V. CONCLUSIONS AND DISCUSSION

To study the dynamics of an array of cantilever plates in oscillatory cross flow, especially the symmetry-breaking instability and the subsequent transition from symmetric to asymmetric responses, a numerical model based on the immersed-boundary framework has been adopted. This model simulates the fluid-structure interaction problem under various physical parameters, including the plate stiffness, the Keulegan-Carpenter number, the number of plates in the array, and the distance between neighboring plates.

These simulations have demonstrated that symmetry breaking is closely associated with hydrodynamic interactions among individual plates enabled through vortices generated from their top ends (i.e., the free ends). The following are the key evidences for this conclusion:

1. Within the range of physical parameters (Reynolds number, plate stiffness and Keulegan-Carpenter number) considered in this study, no symmetry breaking occurs in cases with a single plate.
2. In cases with multiple plates in the array, no symmetry breaking occurs if the top-end vortices are weak.
3. In cases when the plates are too close to each other, no symmetry breaking occurs even if the top-end vortices are strong. This is explained by the fact that in these cases the top-end vortices are shed into the wake and leave the array before they have the chance to interact with other plates for any substantial period of time.
4. In cases when the plates are too far from each other, no symmetry breaking occurs since the top-end vortices do not interact extensively with other plates. Indeed, in these cases the wakes generated by different plates do not merge with each other, unlike the cases when the plates are closer to each other.

Asymmetric responses are observed when the distance between neighboring plates falls within certain range, which depends on physical parameters such as the plate stiffness and the Keulegan-Carpenter number. In general, this instability range increases when the plate stiffness and/or the Keulegan-Carpenter number increase, which is consistent with the fact that larger plate stiffness and Keulegan-Carpenter number lead to stronger vortex generation from the top ends.

Symmetry breaking often leads to uncertainty in the dynamics of the system. For example, as shown in Fig. 6b, when such instability occurs it is hard for the system to reach steady state. Indeed, in that particular case the amplitude of the motion has not been stabilized after ten periods (the same behavior is observed in other asymmetric cases), whereas in symmetric cases the initial disturbance is usually gone within a few periods. Another concern is that when the response becomes asymmetric, the individual plates in an array move differently from each other so that there is a risk of collision with each other. For these reasons, symmetry breaking is likely to be an unwelcome behavior in practice.

As the symmetry-breaking mechanism is related to the generation and evolution of top-end vortices, it is expected to be quantitatively affected by the Reynolds number. Moreover, the two-dimensional model used here is accurate only when the aspect ratio of the plates is sufficiently large. In reality three-dimensional effects may play a role so that three-dimensional simulations are needed.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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