Title

ON THE DESCRIPTION OF FERMION SYSTEMS IN BOSON REPRESENTATIONS. (III). NORMAL MODE CONSTRUCTION AND THE DERIVATION OF KINETIC AND POTENTIAL ENERGY EXPANSION

Author

Sorensen, Bent.

Publication Date

1969

Submitted to Nuclear Physics

UCRL-18691
Preprint

ey. 2

ON THE DESCRIPTION OF FERMION SYSTEMS IN BOSON REPRESENTATIONS. (III). NORMAL MODE CONSTRUCTION AND THE DERIVATION OF KINETIC AND POTENTIAL ENERGY EXPANSION

Bent Sørensen

January 1969

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

ey 2

UCRL-18691

DISCLAIMER

UCRL-18691

ON THE DESCRIPTION OF FERMION SYSTEMS IN BOSON REPRESENTATIONS. (III). NORMAL MODE CONSTRUCTION AND THE DERIVATION OF KINETIC AND POTENTIAL ENERGY EXPANSION†

Bent Sørensen††

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

January 1969

Abstract

Truncation of the boson expansions earlier suggested for the description of collective excitations in doubly-even nuclei make it important to start from the best possible boson representation. It is shown that the Tamm-Dankoff boson generally offers a better starting point than do the Random Phase boson. This is in particular true if the RPA boson is imaginary, which is shown in contrast to earlier assumptions to be the case for a majority of quadrupole-vibrating nuclei. In order to follow the quadrupole phase transition in the boson model the Hamiltonian is carried from the boson form into an expansion in terms of Hermittean variables specifying the mass quadrupole moment and its conjugate momentum.

---

†Work performed under the auspices of the U. S. Atomic Energy Commission.

††On leave from the Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark.

1. Introduction

The boson expansion method described in refs. [1,2] (to be referred to as I and II) and [3] will in the following paper [4] (IV) be applied to the branch of collective quadrupole excitations in doubly-even nuclei. The steps involved in such an application to a single branch of excitations are sketched here. The commutation relations valid among pairs of fermion operators α^+, α define the coefficients of expansions in terms of pure boson operators b^+, b

$$A^+_a = (\alpha^+_{i_a} \alpha^+_{j_a})_{J_a M_a} = \sum_{\substack{\lambda \; a_1 \ldots a_\lambda \\ \mu=(\lambda-1)/2}} x^{\lambda\mu}(a_1 \ldots a_\lambda; a)\, b^+_{a_1} \ldots b^+_{a_{\lambda-\mu}} \bar{b}_{a_{\lambda-\mu+1}} \ldots \bar{b}_{a_\lambda}, \tag{1.1}$$

$$B^+_a = (\alpha^+_{i_a} \bar{\alpha}_{j_a})_{J_a M_a} = \sum_{\substack{\lambda \; a_1 \ldots a_\lambda \\ \mu=\lambda/2}} y^{\lambda\mu}(a_1 \ldots a_\lambda; a)\, b^+_{a_1} \ldots b^+_{a_{\lambda-\mu}} \bar{b}_{a_{\lambda-\mu+1}} \ldots \bar{b}_{a_\lambda}, \tag{1.2}$$

where $\underline{a}$ is a shorthand for the quantum numbers characterizing an angular momentum coupled pair of fermions and the bars denote time-reversal. By a canonical transformation in the boson space we define normal mode operators

$$c^+_n = f_n(\{b^+_a, \bar{b}_a\}) \quad , \tag{1.3}$$

with the inverse relation

$$b_a^+ = f_a'(\{c_n^+, \bar{c}_n\}) \quad . \tag{1.4}$$

Whereas (b_a^+, b_a) correspond to a system without correlations, the idea is to construct the operators c_n in such a way, that the various branches of correlated excitations become as well separated as possible. This means that the Hamiltonian, which always can be written as a function of the operators (A_a, B_a), through the expansions (1.1) and (1.2) and the relation (1.4) is supposed to be well approximated by

$$H \approx \sum_n h_n(c_n^+, \bar{c}_n) \quad , \tag{1.5}$$

i.e. the coupling terms between different normal mode branches $\underline{n}$ are neglected. In section 2 we discuss various choices of the transformation (1.3). Each branch $\underline{n}$ can now be treated to as high an order of accuracy as desired, e.g. including anharmonic (ɔ:nonquadratic) terms only for the collective branch. The diagonalization inside each branch is carried out separately (or only that of the interesting branches, viz. only the collective), making use of the parentage properties of a chosen basis for the branch considered. Expansions similar to (1.5) can be written down for any other interesting operator, and matrix elements of such operators can be evaluated among the eigenstates (those of a single h_n if the absence of coupling terms holds also for the operator in question).

Although angular momentum is strictly conserved, the inclusion of enough anharmonic terms makes it possible to describe transitional and deformed nuclei. Apart from calculating matrix elements of the mass quadrupole operator one can gain insight into the mechanism of phase transition by expanding both the mass quadrupole operator Q_{2M} and its conjugate momentum P_{2M} in the boson basis and neglecting non-collective contributions (thus assuming that it is the collective quadrupole branch alone which is responsible for the onset of deformation)

$$Q_{2M} = q_{2M}(c^+, \bar{c}) \quad , \tag{1.6}$$

$$P_{2M} = p_{2M}(c^+, \bar{c}) \quad , \tag{1.7}$$

where we have characterized the collective boson operators by absence of subscript. Inverting the expressions (1.6) and (1.7) and inserting into (1.5) one gets

$$H = V(Q) + T(P, Q) + H_{\text{non.coll.}} \quad , \tag{1.8}$$

where the potential energy V is defined as any collective part of H which does not contain P_{2M}. As discussed in section 3, one may obtain a fair estimate of V and T just by a first order inversion of (1.6) and (1.7).

2. Normal Mode Construction

The simplest way of defining the normal mode operators of eqs. (1.3) and (1.4) is by a linear transformation

$$b_a^+ = \sum_n \left(r_n(a)\, c_n^+ + s_n(a)\, \bar{c}_n \right) \quad , \tag{2.1}$$

made unitary by the requirements

$$\sum_a \left(r_n(a)\, r_{n'}(a) - s_n(a)\, s_{n'}(a) \right) = \delta_{nn'} \quad , \tag{2.2}$$

$$\sum_a \left(r_n(a)\, s_{n'}(a) - s_n(a)\, r_{n'}(a) \right) = 0 \quad . \tag{2.3}$$

In order to establish the desired physical interpretation of the new boson operators c_n^+ it appears natural to define the transformation coefficients in such a way that (2.1) diagonalizes the Hamiltonian to second order,

$$H(\{A_a, B_a\}) = \sum_{aa'} z^{20}_{aa'} (b_a^+ b_{a'}^+ + \bar{b}_a \bar{b}_{a'}) + \sum_{aa'} z^{21}_{aa'} b_a^+ \bar{b}_{a'}$$

$$+ \sum_{\lambda \neq 2} \sum_{a_1 \cdots a_\lambda} z^{\lambda\mu}_{a_1 \cdots a_\lambda} b^+_{a_1} \cdots b^+_{a_{\lambda-\mu}} \bar{b}_{a_{\lambda-\mu+1}} \cdots \bar{b}_{a_\lambda} =$$

$$0 \leq \mu \leq \lambda$$

$$\sum_n w^{21}_{nn} c_n^+ \bar{c}_n + \sum_{\lambda \neq 2} \sum_{n_1 \cdots n_\lambda} w^{\lambda\mu}_{n_1 \cdots n_\lambda} c^+_{n_1} \cdots c^+_{n_{\lambda-\mu}} \bar{c}_{n_{\lambda-\mu+1}} \cdots \bar{c}_\lambda \, .$$

$$0 \leq \mu \leq \lambda \tag{2.4}$$

However, this requires the solution of a set of highly non-linear equation,

$$w^{20}_{nn'} = 0 \quad , \quad w^{21}_{nn'} = \delta_{nn'} w^{21}_{nn} \quad , \tag{2.5}$$

which only have been attempted in over-simplified cases (see section 2.1). We will mention a few approximate treatments of the system of equations (2.5). One is to neglect all non-quadratic (in $r_n(a)$ and $s_n(a)$) contributions to w^{20} and w^{21}, which is identical to the exact diagonalization of the second order part of the Hamiltonian (2.4)

$$H^{(2)} = \sum_{aa'} z^{20}_{aa'} (b^+_a b^+_{a'} + \bar{b}_a \bar{b}_{a'}) + \sum_{aa'} z^{21}_{aa'} b^+_a \bar{b}_{a'} \quad . \tag{2.6}$$

This approximation is usually denoted Random Phase Approximation (RPA). An improved solution can be obtained [3]) by an iterative procedure, starting with the RPA coefficients $(r^{(0)}_n(a), s^{(0)}_n(a))$ and then adding corrections

$$r^{(i)}_n(a) = r^{(i-1)}_n(a) + \delta^{(i)} r_n(a) \quad , \tag{2.7}$$

$$s^{(i)}_n(a) = s^{(i-1)}_n(a) + \delta^{(i)} s_n(a) \quad ,$$

where the corrections are determined from the insertion of (2.7) into (2.5) and the assumption that the corrections are small, so that only terms linear in $\delta^{(i)}r$ and $\delta^{(i)}s$ may be kept. Unitarity further restricts the form of (2.7) to that of infinitesimal rotations in the vector space indexed by n. If further the non-quadratic terms in $w^{2\mu}$ are considered negligible for all branches $\underline{n}$ except a single one (the collective root of RPA), we have simply

$$\delta^{(i)}{}_{r(a)} = \varepsilon^{(i-1)}\, {}_{s}{}^{(i-1)}{}_{(a)} \quad , \tag{2.8}$$

$$\delta^{(i)}{}_{s(a)} = \varepsilon^{(i-1)}\, {}_{r}{}^{(i-1)}{}_{(a)} \quad .$$

The infinitesimal rotation is purely imaginary (no minus sign in one of the eqs. (2.8)), since r and s are associated with operators which are Hermittean conjugate. The normal mode representation obtained after i iterations we will denote NMi.

The normal mode definition (2.5) and hence all of the approximations NMO = RPA, ... , NMi, ... suffers from a fundamental deficiency. Namely, although a nucleus is stable, i.e. stable when governed by the total Hamiltonian H, it may not be stable under the sole action of the second order part ($H^{(2)}$ in case of RPA). If this is not so, the equations (2.5) have at least one imaginary solution, which in the case of multipole particle-hole modes is interpreted [13]) by saying that the equilibrium shape of the system is no longer spherical, but has a multipole deformation. The quadratic part of the Hamiltonian can only describe the system near a stable equilibrium, and the failure of such a description does describe a lack of stability, although we shall see in sections 2.2 and 3 that the relation to the concept of deformation requires a further investigation.

It is thus essential to find a boson basis which is applicable for any shape of the system and which at the same time fulfills the requirement of approximately separating the collective branch of excitations from the remaining states. The only possible choices, if the relationship to the basic boson representation (2.1) shall still be linear, are bosons characterized by

$$s_n(a) = 0 \quad \text{for all} \quad a \quad . \tag{2.9}$$

If the boson conserving part of $H^{(2)}$ is simultaneously diagonalized, this defines the Tamm-Dankoff (TD) boson. We hope to prove that if reasonably many terms are kept in the Hamiltonian (2.4) then i) the results are independent of whether TD or NMi (RPA) is used, whenever NMi is not close to instability, ii) the collectivity approximation (that the collective branch may be treated independently) is fairly well justified when TD is used to define the normal mode, even in cases where the system no longer has the same equilibrium symmetry (sphericity) as the basis.

2.1. SIMPLIFIED MODEL

In this subsection we are considering a system described by a Hamiltonian

$$H = z^{20}\, b^+b^+ + z^{21}\, b^+b^+ + z^{41}\, b^+b^+b^+b + z^{42}\, b^+b^+bb + \text{h.c.} \tag{2.10}$$

in terms of a single, spinless boson b^+. This Hamiltonian contains typical anharmonic terms and the corresponding normal mode conditions (2.5) can be solved exactly[†]). The following diagonalization of the anharmonic Hamiltonian in the normal mode representation is compared to that obtained by using the NMi method and by TD. In table 1 we show the results for different magnitudes of z^{20}, the remaining parameters being fixed at $z^{21} = 1.0$, $z^{41} = 0.1$ and

[†]The code for this solution makes use of a Lawrence Radiation Laboratory library program written by G. Litton.

$z^{42} = 0.2$. These fairly typical magnitudes correspond to a situation in which the basic boson expansion of H (in terms of b^+, b) is converging. Varying z^{20} we can induce second order instability, the critical magnitude being $z^{20} = 1.0$. The reason for only carrying the iterative method NMi a few steps is that the assumption of linearity (2.8) often is rather bad and that accuracy is lost at each step because of the loss of normalization (2.2). Actually the normalization was secured to second order after each iteration, by putting

$$r^{(i)} = (r^{(i-1)} + \varepsilon^{(i-1)}\, s^{(i-1)})/(1 + (\varepsilon^{(i-1)})^2)^{1/2} \quad , \tag{2.11}$$

$$s^{(i)} = (s^{(i-1)} + \varepsilon^{(i-1)}\, r^{(i-1)})/(1 + \varepsilon^{(i-1)})^2)^{1/2} \quad ,$$

but since $\varepsilon^{(i)}$ takes values around 0.4 as soon as we approach the region of instability, the results do get unreliable after a few steps. The iteration quoted in table 1 is the one giving the smallest value of w^{20}.

The table gives the value of s (which fully specifies the transformation (2.1)) and the coefficients $w^{\lambda\mu}$ of the transformed Hamiltonian

$$H = \sum_{\substack{\lambda \\ \mu \leq \frac{1}{2}\lambda}} w^{\lambda\mu}\, (1+\delta_{\lambda\mu})^{-1}\, (c^+)^{\lambda-\mu}\, (c)^{\mu} + \text{h.c.} \tag{2.12}$$

The only way to compare the different Hamiltonians is to diagonalize them, which is done in the form

$$|E_i\rangle = \sum_{n \leq 25} \phi_n\, (E_i)(c^+)^n|0\rangle \quad . \tag{2.13}$$

Table 1 gives the six lowest roots and typical parts of the wave function of the "two-phonon" state, in order that one may estimate the number of components necessary to describe this state (which is of interest for the quadrupole calculation of paper IV, which truncates at $n = 7$). For z^{20} well below 1.0 (table 1a) all normal mode representations give equally good results, although the convergence of (2.12) is poorer for RPA than for NM1 and both poorer than TD. Yet TD requires a slightly large basis (2.13) because of the large w^{20} (only non-diagonal $w^{\lambda\mu}$ terms causes admixture). For z^{20} approaching the critical value where the second order hamiltonian $H^{(2)}$ becomes unstable (table 1b), the RPA Hamiltonian is hardly converging, yet the eigenvalues are reasonable when the diagonalization basis is large. NM2 has better convergence despite the large $\epsilon^{(i)}$ and does not require as large a basis as TD. When z^{20} is just below 1.0 (table 1c), neither RPA nor NMi work, whereas TD is exact and does not require larger diagonalization basis than before. Above $z^{20} = 1.0$ the NMi approximation starting from RPA can no longer work, since the second order Hamiltonian corresponds to instability. Yet the total H corresponds to stability, since the signs of the fourth order terms are chosen to correspond to binding. In both cases shown (table 1d) the TD provides very accurate approximations to the exact NM solution, but whereas a truncation in the diagonalization at $n = 7$ is allowed at $z^{20} = 1.2$, it will lead to considerable inaccuracy at $z^{20} = 2.0$. One might note that the exact normal mode condition $w^{20} = 0$ does not lead to a converging Hamiltonian for large z^{20} in contrast to TD. We thus

conclude that for a systematic survey of transitional nuclei TD provides the absolutely best normal mode representation among the approximate ones considered, but that the dimension $n = 7$ fixed for practical reasons in IV presumably will cause increasing inaccuracy for increasing deformation. The further expectation that in this region more terms will be important in the expansion of H has not been considered in this subsection which has assumed a Hamiltonian with no terms higher than fourth order. Still the relative magnitudes of second and fourth order terms do give some indication concerning the convergence of a more general H.

We have in table 1 shown examples where no stable RPA Hamiltonian could be formed, but where $w^{20} = 0$ still had a solution. In concluding this subsection we show in table 2 a few more unusual examples, keeping the $H^{(2)}$ stable but varying the other instability-inducing parameter, w^{41}, around its critical value $w^{42} = 0.2$. Passing this value the exact NM has become unstable, whereas TD passes the critical value smoothly. However, we also show these examples because the parameter choice $w^{41} = .199$ leads to a coincidental cancellation in RPA, so that the RPA boson Hamiltonian becomes extremely well converging. For this reason the fourth order RPA diagonalization and the NMi's based here upon yields very good results, whereas the actual largeness of anharmonicities causes TD to be rather inaccurate. Our experience with exact solutions [7]) disfavours any abrupt change in the spectrum when w^{41} is changed from 0.199 to 0.211, which means that the RPA results for $w^{41} = 0.211$ are completely wrong. We thus again conclude that although special cases do occur, where a NMi representation is the best starting point for the boson expansion, TD is much more broadly applicable and much safer, since it does not lead to sudden divergences by slight changes of parameters.

2.2. QUADRUPOLE STABILITY

In the examples studied in table 1 the exact NM was always a stable solution, even when RPA was not. We can thus hardly talk about a phase transition, defined as the point of second order instability evaluated after the transformation to the normal mode representation. The phase transition associated with onset of quadrupole deformation might be defined as corresponding to having the nucleus in a state with a non-zero expectation value of the quadrupole moment, or it might be defined as the point where the derivative of the expectation value of the Hamiltonian with respect to the quadrupole moment of the nucleus, taken at zero moment, changes sign. Since asymmetry of the energy with respect to prolate and oblate deformation makes a nucleus deformed under the first definition even if the minimum is at zero deformation, we shall mostly use the second definition, which more closely follows the intuitive concept of phase transition. However, with this definition a nucleus may have zero quadrupole moment although having experienced the phase transition, namely if the derivative of H is negative at the origin but H as function of Q is completely symmetric between oblate and prolate shape, which it might be in the middle of a shell. Now, since the quadrupole moment is a Hermittean combination of c^+ and $\bar{c}$'s, the truncation of the boson Hamiltonian to second order in any of the boson representations does in general not give a stability criterion equivalent to that based on the dependence on the quadrupole moment operator. The stability in terms of the quadrupole moment operator will be

discussed in section 3, but the stability of various normal mode representations as considered in subsection 2.1 is of course important for the course of the boson calculations.

We now turn to the problem of collective quadrupole excitations of a system of both neutrons and protons, to which a large number of single-particle levels are available. As in the calculations to be presented in IV we have here assumed a pairing plus quadrupole interaction in all cases where numerical results are quoted. The form of the boson expansion in this case is given in I, and the collective part of the fourth order Hamiltonian is

$$H = w^{00} + w^{20}(c^+c^+)_0 + w^{21}(c^+\bar{c})_0 + w^{30}(c^+c^+c^+)_0$$
$$+ w^{31}(c^+c^+\bar{c})_0 + w^{40}(c^+c^+)_0(c^+c^+)_0$$
$$+ \sum_J \{w_J^{41}(c^+c^+)_J(c^+\bar{c})_J)_0 + w_J^{42}((c^+c^+)_J(\bar{c}\,\bar{c})_J)_0\} + \text{h.c.} \quad (2.14)$$

where the boson c^+ carries angular momentum $J = 2$. If we want to study the choice of normal mode representation in analogy to what was done in subsection

2.1 for the one-level model, a reasonable way of proceeding is the following. We choose a nucleus for which the anharmonicities are small.† For such a nucleus, e.g. ^{116}Sn, we perform boson calculations in TD, RPA and in NMi. The resulting boson Hamiltonians are given in table 3, and fig. 1 we show the energy spectra for expansions truncated at second and fourth order. The results of NM0 and NM1 seems to converge toward that of TD, but following the NMi to the lowest z^{20}, which is achieved at i=3, the spectrum is now completely changed. It is tempting to ascribe this to increasing inaccuracy in the NMi, but as no exact solution of eq. (2.5) is possible, the conclusion is not sure. In connection with the one level model the question was raised, whether the normalization of the boson was lost during the iterative procedure. In the many-level model the further question arises, whether the orthogonality between the collective and non-collective branches is preserved. This question is completely neglected by the NMi, which only changes the

†This could be a doubly closed shell nucleus, but here no really collective branch exists and the lowest quadrupole branch may not be well separated from other ones. In addition, the lack of collectivity makes the spectrum more sensitive to details of the interaction, which may not be contained in our model interaction. Instead the singly closed shell nuclei are characterized by a fairly well developed collective branch of excitations, at the same time as the deviations from harmonicity are smaller than for non-closed shell nuclei.

collective boson and leaves the non-collective ones untouched. Orthogonality is only preserved during this procedure if each new boson $c^{(i)}$ is a linear combination of $c^{(i-1)}$ and $c^{+(i-1)}$. That would on the other hand make the iterative procedure superfluous, since such transformations are induced by the diagonalization, and since the successive transformations in the ^{116}Sn case far from possess this structure, one suspects that each step further destroys the separation of the Hamiltonian, so that NM1 in fact is more reliable than NM3. In the ^{116}Sn calculation the NMi do preserve normalization accurately. Another consideration also leads to the conclusion that NM3 is less reliable than NM1. Looking at the boson Hamiltonians listed in table 3, one observes that the most dangerous terms (for convergence and reliable diagonalization) in the Hamiltonian (2.14) are $w_J^{41}\{(c^+c^+)_J(c^+\bar{c})_J\}_0 + \text{h.c.}$, which change boson number by two. As shown in ref. [5]) by comparing diagonalizations of various boson Hamiltonians in spaces of up to 6 and 7 bosons, the eigenstates quickly become unreliable, when the ratio w_J^{41}/w^{21} exceeds 0.35. This ratio is around 0.5 for NM1 but w_2^{41}/w^{21} reaches 0.65 for NM3. For TD the ratio is 0.15.[†]) For this reason we believe to have proven, that, although

---

[†]It is interesting to note that this problem is hardly revealed by looking at the wave-function of the second 2+ state, given in table 3. However, in contrast to the rapidly converging TD wave function, the NM3 wave-function has one conspicuous 6 phonon component, which in conjuncture with the fact that only even boson numbers have large components might indicate that also 8 phonon and higher even-n parts could have been important.

formally attractive, RPA and the iterative normal mode method is dangerous to use in practical calculations, where the diagonalization space has to be truncated, and where the separation of the collective branch is easily lost. In contrast the TD representation appears to be very attractive, offering well converging boson Hamiltonians and being insensitive to the possible instability of the second order Hamiltonian, which bars the use of RPA as a starting representation. The negative side of the TD is the somewhat poorer separation of the collective branch, but little is lost by not keeping the so-called backward-going graphs (those associated with z^{20}-coefficients in (2.4)), since they are treated together with and on the same footing as the anharmonic terms.

We have now seen that even in the case of a single closed shell nucleus the anharmonicities play an important role. What happens when we go away from the closed shell.

By studying ^{116}Sn we have found values of the pairing and quadrupole force strengths, which roughly reproduces the lowlying part of the experimental energy spectrum. Assuming now a smooth A-dependence of the force strengths (cfr. IV), the nuclei in the neighbourhood of the closed shell can be investigated without introducing new parameters. Doing this we find that RPA breaks down already a few steps from the closed shell. Thus no RPA solution exist for ^{122}Te and ^{114}Cd is just about unstable under $H^{(2)}$. In the region of Sm, we find RPA stability for ^{148}Sm, but not for ^{150}Sm.[†]) The set-on of RPA instability occurs sooner than in the calculations of Kisslinger and Sorensen[6]) which is evident from the fact that our quadrupole strength χ

[†]Still all of the nuclei mentioned here are deformed using the criterion of evaluating the sign of the potential energy derivative at $Q=0$ (cf. IV). For the nuclei having RPA solutions, however, the magnitude of the zero point frequency exceeds that of the energy gained by deformation.

is always larger than theirs since ours has to give correct magnitudes of energies with the anharmonicities included, whereas they try to reproduce the energy of the first $2+$ state with the second order RPA itself. As fig. 1 shows, the second order RPA energy is too low, a quite general feature. Although the considerations in this section has lead as to using TD as starting representation in the calculations presented in IV, one should mention, that there is a way to circumvent the $H^{(2)}$ instability and use RPA or NMi. This consists in varying the parameters y_j^0 of the basic boson expansion (eventually dropping the state dependence of y^0 in order to get only one parameter). As shown in II the variation of y^0 corresponds to canonical transformations in the boson space and hence does not change the spectrum of an untruncated Hamiltonian, but it can very well change the relative importance of $H^{(2)}$ and the remaining part of H, so that the second order RPA is real for a given non-zero value of y^0. Fig. 2 shows some coefficients of the ^{122}Te Hamiltonian, using the iterative procedure NMi starting from RPA, as function of y^0. Not only the coefficients of the boson Hamiltonian change rapidly with y^0, but also the spectrum does. This is due to the fourth order truncation of H, which gets poorer and poorer for increasing y^0, since the $H^{(2)}$ part contains less and less correlations as y^0 increases, and the actual correlations then are pushed to still higher and higher terms in H. This is also clear from the physical interpretation of y^0 given in II. The single parameter y^0 can only compensate for the wrong counting of particle number implied by using a bad representation and truncating H at low boson number. The detailed physical effects lost by the truncation would require a large number of parameters

y_a^0, which is equivalent to retaining the correlation by shifting the truncation point upwards, but even less attractive with respect to the computational work required. With the TD representation the best choice in most cases is $y^0 = 0$. We can check this by calculating the average particle number in the final eigenstate, and for TD this came closest to the actual particle number for $y^0 = 0$ in all cases considered except ^{122}Te, where the best value was $y^0 = 0.05$. The advantage of working with small values of y^0 once more favours the TD boson representation.

3. Separation of Kinetic and Potential Energy

In order to relate our treatment of a phase transition by the boson method to the conventional ways of estimating the equilibrium shapes of nuclei by evaluating the potential energy as function of deformation we shall express our boson Hamiltonian in terms of the actual shape of the system. This offers a new way of finding the eigenstates, once the boson expansions are known (using the method of Kumar and Baranger[14]), or it allows from the solution in terms of the boson basis a construction of the kinetic and potential energy.

In exact analogy to the treatment in I of the electric quadrupole operator the mass quadrupole operator is carried through the successive set of transformations, from the real particle space[†]

$$Q_{2M} = 5^{-1/2} m \sum_{ij} \langle\, i \,\|\, (\frac{r}{b})^2 Y_2 \,\|\, j \,\rangle (a_i^{+} \tilde{a}_j)_{2M} \tag{3.1}$$

to the quasiparticle representation (corresponding to the pairing interaction alone), then to the basic boson variables $(b_a^{+}, \bar{b}_a)$ and finally to the normal mode bosons $(c_n^{+}, \bar{c}_n)$, where again a division is made between terms containing

---

[†] In papers III and IV we consistently measure all lengths in units of the oscillator parameter $b = (\frac{\hbar}{M\omega_0})^{1/2}$.

only the collective branch,†)

$$Q_{2M} = Q^{10}(c^+ + \bar{c})_{2M} + Q^{21}(c^+\bar{c})_{2M} + \sum_J Q_J^{31}((c^+c^+)_J\bar{c} + c^+(\bar{c}\bar{c})_J)_{2M} + \ldots \tag{3.2}$$

and terms involving non-collective operators (c_n^+, c_n), which are dropped. $\underline{Q}$ is a Hermittean operator, and in order to invert the relation (3.2) we need its conjugate momentum $\underline{P}$, which is easily found in the boson space

$$P_{2M} = \frac{i}{2q_{10}}(c^+ - \bar{c})_{2M} + \frac{i}{4q_{20}}(c^+c^+ - \bar{c}\bar{c})_{2M} +$$

$$\frac{i}{8}\sum_J \frac{1}{q_{31}^J}((c^+c^+)_J\bar{c} - c^+(\bar{c}\bar{c})_J)_{2M} + \ldots \ , \tag{3.3}$$

using the time reversal properties

$$\tau Q_{2M}\tau^{-1} = (-)^{2+M}Q_{2-M}$$

$$\tau P_{2M}\tau^{-1} = -(-)^{2+M}P_{2-M} \tag{3.4}$$

and the conjugateness

$$[Q_{2M}, P_{2M}] = i \ , \tag{3.5}$$

---

† For simplicity we have only written terms down which are non-zero when the TD representation is employed. With RPA or NMi also c^+c^+, $c^+c^+c^+$, ... will appear.

which relates the coefficients of (3.3) to those of (3.2), $q_{10} = Q^{10}$, etc.

However, we are going to justify the use of a first order inversion of the equations (3.2) and (3.3),

$$c^{+}_{2M} = \frac{1}{2q} Q_{2M} - iqP_{2M} \quad , \tag{3.6}$$

where q would equal Q^{10} if higher order terms are completely neglected, but where a slightly different q may represent the influence of the higher order terms in an average fashion. In this way certain higher order terms in the expansion of c^{+}_{2M} in terms of $\underline{P}$ and $\underline{Q}$ are inevitably neglected, e.g. $\underline{Q}\cdot\underline{Q}$ and $\underline{P}\cdot\underline{P}$, whereas terms like $(\underline{P}\cdot\underline{P})\underline{Q}$ and $(\underline{Q}\cdot\underline{Q})\underline{Q}$ are replaced by $\langle \underline{P}\cdot\underline{P} \rangle \underline{Q}$ and $\langle \underline{Q}\cdot\underline{Q} \rangle \underline{Q}$. We allow ourselves this simplified but very convenient form of the inverted relation (3.6) only because of the existence of the results of the boson diagonalizations which allows us to check the validity of the approximation, e.g. by calculating the ratio

$$\langle Q_{2M} \rangle / \langle (c^{+} + \bar{c})_{2M} \rangle \tag{3.7}$$

by taking expectation values in the known eigenstates after the coefficients of (3.2) have been calculated along with the boson calculation. If the ratio (3.7) equals a constant q independent of which state is used, the relation (3.6) follows immediately. Table 4 gives the value of the expression (3.7) for several eigenstates in ^{152}Sm and ^{116}Sn. It is in fact rather constant at least for the lower eigenstates, and a second check is obtained by looking at the actual coefficients of the expansion (3.2), which reveals a remarkable concentration on the leading order term as can be seen in table 5. This feature we found

quite generally, namely that the boson expansion of the quadrupole moment converges much more rapidly than that of the Hamiltonian.

The relation (3.6) can be inserted into (2.14) whereby we obtain

$$H = \rho^{00} + \rho^{22}\,(PP)_0 + \rho^{20}(QQ)_0 + \rho^{32}(PPQ)_0 + \rho^{30}(QQQ)_0 + \rho^{44}(PP)_0(PP)_0$$
$$+ \sum_J \rho_J^{42}\{((PP)_J(QQ)_J)_0 + ((QQ)_J(PP)_J)_0\} + \rho^{40}(QQ)_0(QQ)_0 \qquad (3.8)$$

where again[†]

$$\rho^{00} = 2w^{00} - \sqrt{5}\,w^{21} - \frac{7}{2}\,w^{40} + \frac{5}{2}\,w_0^{42} + \sum_J \hat{J}\,w_J^{42}\,,$$

$$\rho^{22} = q^2(-2w^{20} + 2w^{21} + \sqrt{5}\,w_0^{41} - \frac{2}{\sqrt{5}}\sum_J \hat{J}\,w_J^{41} - \frac{4}{\sqrt{5}}\sum_J \hat{J}\,w_J^{42})\,,$$

$$\rho^{20} = \frac{1}{4q^2}(2w^{20} + 2w^{21} - \sqrt{5}\,w_0^{41} + \frac{2}{\sqrt{5}}\sum_J \hat{J}\,w_J^{41} - \frac{4}{\sqrt{5}}\sum_J \hat{J}\,w_J^{42})\,,$$

$$\rho^{32} = q(-3w^{30} + w^{31})\,,$$

$$\rho^{30} = \frac{1}{4q^3}(w^{30} + w^{31})\,, \qquad (3.9)$$

$$\rho^{44} = 2q^4(w^{40} + \sum_J \frac{\hat{J}}{5}(w_J^{42} - w_J^{41}))\,,$$

$$\rho_J^{42} = -\frac{\hat{J}}{5}\,w^{40} + \sum_{J'} \hat{J}\hat{J}'\,\begin{pmatrix} 22J' \\ 22J \end{pmatrix} w_{J'}^{42}\,,$$

[†] In $\rho^{\lambda\mu}$ μ indicates the power of $\underline{P}$'s and λ is μ plus the power of $\underline{Q}$'s.

$$\rho^{40} = \frac{1}{8q^4}\left(w^{40} + \sum_J \frac{\hat{J}}{5}\left(w_J^{42} + w_J^{41}\right)\right) ,$$

$$\hat{J} = (2J+1)^{1/2} .$$

The principal axes of the quadrupole moment tensor define an intrinsic coordinate system rotated by the angles ω from the laboratory frame

$$Q_{2M} = \sum_N \mathcal{D}^{2}_{MN}(\omega)\, Q'_{2N} . \tag{3.10}$$

In the intrinsic frame we use the conventional parametrization[8])

$$Q'_{20} = Q\cos\gamma \quad , \quad Q'_{22} = Q'_{2-2} = \frac{1}{\sqrt{2}}\, Q \sin\gamma , \tag{3.11}$$

$$Q'_{21} = Q'_{2-1} = 0 .$$

By these definitions, the potential energy part of the Hamiltonian (3.8), defined as all terms which do not depend on the momentum $\underline{P}$, can be expressed

$$V = \rho^{00} + \frac{1}{\sqrt{5}}\rho^{20}\, Q^2 - \sqrt{\frac{2}{35}}\, \rho^{30}\, Q^3 \cos 3\gamma + \frac{1}{5}\rho^{40}\, Q^4 . \tag{3.12}$$

The remaining kinetic energy comprises terms in (3.8) characterized by coefficients ρ^{22}, ρ^{32}, ρ^{44} and ρ_J^{42}. It depends on $\underline{P}$ as well as on $\underline{Q}$ and is in general highly anharmonic, involving even the fourth power $\underline{P}$, which is always neglected in the adiabatic approach [9,10,15]) aiming at the low-lying excitations of well-deformed nuclei.

In table 6 we give the coefficients of (3.8) for ^{116}Sn and ^{152}Sm, two nuclei believed to be of different equilibrium shape. In order to estimate the relative importance of each term in (3.8) we have omitted the scale factors involved in the definition of $\underline{P}$ and $\underline{Q}$, thus quoting approximately the expectation value of each individual term in (3.8). The scale factor q arising from (3.6) is in the figures shown here (and in IV) fixed at the value of the ratio (3.7) taken for the first excited 2+ state. It is seen that ρ^{44}/q^4 is extremely small, meaning that the adiabatic approach presumably would be allowed in these cases. The negative sign of ρ^{20} means that the system is not stable at zero deformation.[†]) As deformation increases, the fourth order terms soon raise in magnitude and assures stability. We refer to figs. 16 and 5 of IV for the ^{116}Sn and ^{152}Sm equipotential surfaces of the potential V calculated from (3.12). Despite an oblate minimum ^{116}Sn is seen to be rather spherical, as its spectrum also reveals (cfr. IV). ^{152}Sm on the other hand is strongly deformed, of prolate shape. It should be mentioned that the fourth order boson expansions can not describe systems with asymmetric equilibria, for which up to sixth order terms are required. The radial scale Q defined by (3.11) is a unit of intrinsic quadrupole moment. In addition another scale, denoted β, is indicated on the figures in IV, giving the more conventional deformation parameter defined by Bohr and Mottelson[11]). This and that of Nilsson[12]) are related to ours by the approximate relations

$$Q \simeq \sqrt{\frac{5}{16\pi}} \left(\frac{3}{2}\right)^{1/3} A^{4/3}\, \delta^{SGN} \approx \frac{15}{16\pi} \left(\frac{3}{2}\right)^{1/3} A^{4/3} \beta \quad . \tag{3.13}$$

[†]However $H^{(2)}$ is stable, so this example illustrates the discussion in section 2.2 concerning definitions of phase transition.

A widely used method of determining equilibrium deformations (see e.g. refs. [9-10]) uses the requirement of self-consistency, that the deformation of the system must equal that of a property determined average of the interaction assumed. Using the quadrupole force there will be a single value of the quadrupole force strength χ, for which the deformation of the average field equals that of the wave-function with which it was calculated. When the kinetic energy contains anharmonicities as we have seen it generally does, there is, however, no reason to expect such selfconsistency. This means that the system may not prefer to be close to the shape of the potential energy minimum, because of details in the kinetic energy. Our model in principle makes it possible to investigate how close the system is to selfconsistency. From the calculated expectation values of the mass quadrupole operator in low lying states, e.g. in the first 2+ state, one may extract a value of the intrinsic quadrupole moment by the relation

$$\langle 2_1 M=2 | Q_{20} | 2_1 M=2 \rangle = -\frac{2}{7} \langle Q'_{20} \rangle_{\text{intr.}} \quad , \tag{3.14}$$

valid if the wave-function has the product form $|22\rangle = Y_{22}(\omega)|\text{intr.}\rangle$. The mass quadrupole moments of the first 2+ state for ^{116}Sn and ^{152}Sm are 1.92 and -9.4, and the intrinsic ones thus -6.72 and 32.9, compared to the minimima of the potential energy functions shown in IV which are -17.0 and 57.0, respectively. It should be remarked, that the ratio of wave-function and potential deformations is independent of the effective mass used, since it enters as a scale factor both on Q_{20} and on the unit q used in the expansion (3.8) of H. On the other hand the relation (3.14) is not

likely to be even approximately fulfilled for ^{116}Sn, but maybe for a rather well-deformed nucleus as ^{152}Sm. One might have anticipated that self-consistency is not fulfilled for ^{116}Sn, already by looking at the actual potential and kinetic energies. For ^{152}Sm a large part of the deviation from selfconsistency is probably caused by inaccuracy in the boson calculation. This is estimated from the fact that both the energy of the first 2+ state and the transition from this state to the ground state are wrong by roughly a factor of two (cfr. IV). We blame this on the structure of the collective boson, which i) is that of a spherical TD calculation which may not represent the collective effects necessary at large deformation and ii) is built from only one major proton and one major neutron shell, presumably being again not a fair representation of the collective branch. The enlargement of the configuration space is no serious problem, but for the objection i), which means that the non-collective TD branches will admix strongly with the collective one for a well-deformed nucleus, we see no practical way forth except starting from a deformed basis.

4. Conclusion

We have tried in this paper to exhibit some of the important features of the boson method, emphasizing in particular its relation to other methods and to conventional concepts, pointing out the possibility of obtaining independent clarification on the concepts of phase transition and spherical stability, and finally to check assumptions like adiabaticity and self-consistency, which of course could not be discussed within the methods using these approximations. Doing so one should of course not forget the approximations and limitations present in the boson method, in particular the question of the validity of the so-called collectivity approximation, which is the fundamental point barring the method from providing accurate quantitative results when treating nuclei with an equilibrium shape very different from that of the representation in which the structure of the collective boson is determined.

Most of the numerical calculations were performed at the GIER computer center of the Niels Bohr Institute and on the NEUCC facilities in Copenhagen. Numerous discussions with Professors A. Bohr, B. Mottelson and Drs. K. Kumar and T. Udagawa are highly appreciated.

References

1) B. Sørensen, Nucl. Phys. A97 (1967) 1

2) B. Sørensen, Nucl. Phys. A119 (1968) 65

3) B. Sørensen, Progr. Theor. Phys. 39 (1968) 1468

4) B. Sørensen, Nucl. Phys.

5) B. Sørensen, Phys. Letters 21 (1966) 683

6) L. Kisslinger and R. Sorensen, Rev. Mod. Phys. 35 (1963) 853

7) R. Broglia and B. Sørensen, Nucl. Phys. A110 (1968) 241

8) A. Bohr, Mat. Fys. Medd., Dan. Vid. Selsk. 26 No. 14 (1952)

9) M. Baranger and K. Kumar, Nucl. Phys. 62 (1965) 113

10) K. Kumar and M. Baranger, Nucl. Phys. A110 (1968) 529

11) A. Bohr and B. Mottelson, Nuclear Structure, Benjamin, New York, to be published

12) S. G. Nilsson, Mat. Fys. Medd., Dan. Vid. Selsk. 29 No. 16 (1955)

13) D. Thouless, Nucl. Phys. 22 (1961) 78

14) K. Kumar and M. Baranger, Nucl. Phys. A92 (1967) 608

15) K. Kishimoto and K. Ikeda, Progr. Theor. Phys. 40 (1968) 499

Table Captions

Table 1. Diagonalization of a model boson Hamiltonian with one parameter, z^{20}, starting from various representations. Besides the coefficients of the boson expanded Hamiltonian and the 6 lowest eigenvalues some characteristic components of the wave-function of the second excited level are given.

Table 2. Diagonalization of the model Hamiltonian (2.10) with (z^{20}, z^{21}, z^{42}) = (0.95, 1.0, 0.2) for two values of w^{41}. For $w^{41} = 0.211$ the exact NM equations do not have a solution and NMi's are divergent. A look at the TD wave-function for the state E_2 seems to indicate more accuracy than the corresponding one for $w^{41} = 0.199$, a very peculiar value for which RPA happens to be extremely good and at the same time TD poorer than ordinarily.

Table 3. ^{116}Sn with pairing plus quadrupole force. Only 5 neutron levels were included. The corresponding spectra are shown in fig. 1. Here coefficients of the boson Hamiltonian are given and components of the wave-function of the second 2+ state from the 4th order calculation. The indices on ϕ give boson number and an alphabetic label if there are more than one basis state of a given boson number.

Table 4. Test of state independence of the unit q of quadrupole moment defined by eq. (3.7). The parameters for this and the following figures are identical to those used for the calculation in IV (table 2 of ref. 4), and in the case of ^{116}Sn thus different from those of table 3 and fig. 1, where only neutrons were considered.

Table 5. Coefficients of the TD boson expansion of the mass quadrupole operator, eq. (3.2). The effective mass $\underline{m}$ entering into (3.1) is 1.5 for Sn and 2.12 for Sm. All coefficients are in units of b^2.

Table 6. Magnitudes of the different terms in an expansion of the Hamiltonian for ^{116}Sn and ^{152}Sm in terms of $\underline{P}$ and $\underline{Q}$ (cfr. eq. (3.8)).

Table 1a

$Z^{20} = 0.8$

	NMO(RPA)	NMi	NM_{exact}	TD
$\varepsilon^{(i)}, i=1$		0.11		
s	-0.5774	-0.4568	-0.4161	0
w^{00}	-0.27	-0.31	-0.32	0
w^{20}	-0.37	-0.08	0	0.8
w^{21}	1.87	1.55	1.49	2.0
w^{40}	0.07	0.03	0.02	0
w^{41}	-0.43	-0.27	-0.22	0.1
w^{42}	0.80	0.58	0.52	0.4
E_0	-0.3174	-0.3174	-0.3174	-0.3174
E_1	1.1257	1.1257	1.1257	1.1257
E_2	3.4175	3.4175	3.4175	3.4175
E_3	6.4447	6.4447	6.4447	6.4447
E_4	10.1982	10.1982	10.1982	10.1982
E_5	14.6742	14.6740	14.6740	14.6740
$\phi_0(E_2)$	-0.0913	-0.0205	0.0052	0.2787
ϕ_2	0.9006	0.9659	0.9806	0.8418
ϕ_6	0.1379	0.0507	0.0286	0.1659
ϕ_{12}	0.0034	0.0001	0.0000	-0.0061
ϕ_{18}	0.0000	0.0000	0.0000	0.0002
ϕ_{24}	0.0000	0.0000	0.0000	0.0000

Table 1b

$z^{20} = 0.95$

	NMO(RPA)	NMi	NM_{exact}	TD
$\varepsilon^{(i)}$, i=1		0.44		
s	-1.0494	-0.3032	-0.5238	0
w^{00}	0.20	-0.39	-0.47	0
w^{20}	-2.18	0.39	0	0.95
w^{21}	5.04	1.24	1.35	2.0
w^{40}	0.44	0.00	0.05	0
w^{41}	-1.95	-0.12	-0.35	0.1
w^{42}	3.03	0.42	0.69	0.2
E_0	-0.4774	-0.4774	-0.4774	-0.4774
E_1	0.7650	0.7649	0.7649	0.7649
E_2	2.9690	2.9688	2.9688	2.9688
E_3	5.9239	5.9044	5.9044	5.9045
E_4	9.7151	9.5704	9.5704	9.5704
E_5	14.9623	13.9604	13.9604	13.9604
$\phi_0(E_2)$	-0.2155	0.1602	0.0183	0.3503
ϕ_2	0.5892	0.9840	0.9623	0.7740
ϕ_6	0.4132	-0.0054	0.0501	0.2131
ϕ_{12}	0.0951	0.0000	-0.0007	-0.0108
ϕ_{18}	0.0148	0.0000	0.0000	0.0004
ϕ_{24}	0.0009	0.0000	0.0000	0.0000

Table 1c

$z^{20} = 0.999$

	NMO(RPA)	NMi	NM_{exact}	TD
$\varepsilon^{(i)}$, i=1		0.40		
s	-3.2685	-2.5612	-0.5611	0
w^{00}	68.66	25.59	-0.53	0
w^{20}	-144.34	-56.12	0	0.999
w^{21}	288.95	112.54	1.30	2.0
w^{40}	24.94	9.89	0.06	0
w^{41}	-99.95	-39.77	-0.41	0.1
w^{42}	150.03	59.76	0.76	0.2
E_0	-0.1613	-0.4906	-0.5412	-0.5412
E_1	6.0634	2.0337	0.6288	0.6288
E_2	26.2355	10.0411	2.8083	2.8083
E_3	83.0105	32.3100	5.7134	5.7134
E_4	177.7791	69.8174	9.3506	9.3506
E_5	383.9658	151.2519	13.7125	13.7125
$\phi_0(E_2)$	-0.5020	-0.4749	0.0258	0.3761
ϕ_2	0.0676	0.0959	0.9548	0.7477
ϕ_6	0.3759	0.3900	0.0590	0.2291
ϕ_{12}	0.3172	0.3143	-0.0012	-0.0129
ϕ_{18}	0.1348	0.1306	-0.0001	0.0005
ϕ_{24}	0.0145	0.0139	0.0000	0.0000

Table 1d

	$z^{20} = 1.2$		$z^{20} = 2.0$	
	NM_{exact}	TD	NM_{exact}	TD
s	-0.7200	0	-1.2921	0
w^{00}	-0.84	0	-3.2	0
w^{20}	0	1.2	0	2.0
w^{21}	1.13	2.0	0.78	2.0
w^{40}	0.13	0	0.87	0
w^{41}	-0.72	0.1	-3.66	0.1
w^{42}	1.21	0.2	5.60	0.4
E_0	-0.8850	-0.8850	-5.3498	-5.3505
E_1	-0.0391	-0.0391	-5.3179	-5.3238
E_2	2.0857	2.0857	-0.9556	-0.9590
E_3	4.8557	4.8557	0.3778	0.3386
E_4	8.3737	8.3737	3.5439	3.4763
E_5	12.6196	12.6195	7.2779	7.1631
$\phi_0(E_2)$	0.0839	0.4980	0.5977	0.8322
ϕ_2	0.9185	0.6121	0.7070	-0.2822
ϕ_6	0.0991	0.2932	0.0371	0.2637
ϕ_{12}	-0.0062	-0.0249	-0.1031	-0.1400
ϕ_{18}	-0.0006	0.0013	-0.0235	0.0228
ϕ_{24}	0.0000	0.0000	-0.0007	-0.0018

Table 2a

$z^{41} = 0.199$

	NMO(RPA)	NMi	NM_{exact}	TD
$\varepsilon^{(i)}$, i=3		-0.30		
s	-1.0494	-2.7607	-2.8745	0
w^{00}	-0.7921	-1.1713	-1.1739	0
w^{20}	0.2394	0.0653	0	0.95
w^{21}	0.1526	0.1477	0.2724	2.0
w^{40}	-0.0438	0.0817	0.1034	0
w^{41}	-0.0144	-0.5265	-0.6131	0.199
w^{42}	0.1369	0.8903	1.0202	0.4
E_0	-1.2905	-1.2903	-1.2902	-0.8814
E_1	-1.2577	-1.2578	-1.2577	-0.5986
E_2	-0.5225	-0.5200	-0.5191	-0.2100
E_3	0.0634	0.1116	0.1259	1.5023
E_4	0.9197	1.1436	1.2019	3.1309
E_5	1.9908	2.7820	2.9600	5.5710
$\phi_0(E_2)$	0.7980	0.3758	0.3550	0.6688
ϕ_2	0.4428	0.7693	0.7510	0.1045
ϕ_6	0.2230	0.2286	0.2549	0.3376
ϕ_{12}	-0.0427	0.0487	0.0560	-0.2463
ϕ_{18}	0.0075	0.0144	0.0163	0.1197
ϕ_{24}	-0.0011	0.0020	0.0022	-0.0244

Table 2b

$z^{41} = 0.211$

	NMO(RPA)	TD
s	-1.0494	0
w^{00}	-0.9128	0
w^{20}	0.5330	0.95
w^{21}	-0.4394	2.00
w^{40}	-0.1023	0
w^{41}	0.2198	0.211
w^{42}	-0.2138	0.4
E_0	-331.3	-2.7941
E_1	-301.8	-2.4849
E_2	-182.0	-0.5675
E_3	-162.4	0.3422
E_4	-99.2	1.6455
E_5	-86.2	3.9748
$\phi_0(E_2)$	0.0000	0.8805
ϕ_2	0.0001	-0.3719
ϕ_6	0.0058	-0.0529
ϕ_{12}	-0.2138	-0.0977
ϕ_{18}	0.4833	0.0920
ϕ_{24}	0.4280	-0.0237

Table 3

	NMO(RPA)	NM1	NM3	TD
$\varepsilon^{(i)}$		-0.27	-0.27,-0.1,-0.06	
$\sqrt{\sum_{ij}\{s(ij)\}^2}$	0.427	0.233	0.334	0
w^{20}	1.67	-0.39	-0.03	-0.75
w^{21}	2.80	1.50	1.13	1.17
w^{30}	0.13	0.04	0.04	0
w^{31}	0.38	0.08	0.06	-0.06
w^{40}	1.20	0.16	-0.13	0
w_0^{41}	1.75	0.62	-0.12	0.40
w_2^{41}	1.50	0.69	0.74	0.22
w_4^{41}	1.82	0.77	0.41	0.22
w_0^{42}	1.73	0.84	0.27	0.29
w_2^{42}	0.89	0.30	-0.03	0.22
w_4^{42}	1.29	0.50	0.06	0.31
$\phi_1 (2_2^+)$	0.073	0.028	-0.017	-0.031
ϕ_2	0.887	0.975	-0.896	0.999
ϕ_3	-0.036	-0.008	0.016	0.008
ϕ_{4a}	-0.442	-0.216	0.382	0.036
ϕ_{4b}	-0.002	-0.001	0.001	0.000
ϕ_{5a}	0.007	0.000	-0.008	0.000
ϕ_{5b}	-0.019	-0.022	0.071	0.001

(continued)

Table 3 Continued

	NMO(RPA)	NM1	NM3	TD
ϕ_{6a}	0.102	0.031	-0.211	-0.004
ϕ_{6b}	0.001	0.000	-0.001	0.000
ϕ_{7a}	-0.002	-0.001	-0.005	0.000
ϕ_{7b}	0.010	0.009	-0.035	0.000
ϕ_{7c}	0.000	0.000	0.000	0.000

Table 4

	J	q (eq. (3.7))
^{116}Sn	2_1	5.31
	2_2	5.03
	4_1	5.42
^{152}Sm	2_1	-8.97
	2_2	-7.99
	2_3	-9.97
	4_1	-9.21
	4_2	-16.26
	4_3	-10.54
	4_4	-8.06

Table 5

	^{116}Sn	^{152}Sm
Q^{10}	5.18	-8.30
Q^{21}	0.33	-1.77
Q_0^{31}	-0.35	0.37
Q_2^{31}	-0.24	0.25
Q_4^{31}	-0.35	0.42

Table 6

	^{116}Sn	^{152}Sm
q	5.306	8.97
ρ^{00}	0.242	-0.163
ρ^{22}/q^2	2.716	2.680
$4q^2\ \rho^{20}$	-3.152	-2.389
$2\rho^{32}/q$	-0.578	1.368
$8q^3\ \rho^{30}$	-0.580	1.367
ρ^{44}/q^4	0.052	0.042
$4\rho_0^{42}$	1.342	0.644
$4\rho_0^{42}$	0.820	0.380
$4\rho_4^{42}$	1.672	0.780
$16q^4\ \rho^{40}$	0.128	0.608

Figure Captions

Fig. 1. Comparison of spectra for ^{116}Sn obtained by diagonalizing boson Hamiltonians, truncated at second or fourth order, for various choices of normal mode representation. The original fermion Hamiltonian was the same in all cases. Only 5 neutron shells were included corresponding exactly to table 3.

Fig. 2. Selected coefficients of boson Hamiltonian for ^{122}Te. The RPA solution, which is real for $y^0 > 0.07$, is used for the iterative process NMi, which is stopped when a minimum in w^{20} is reached.

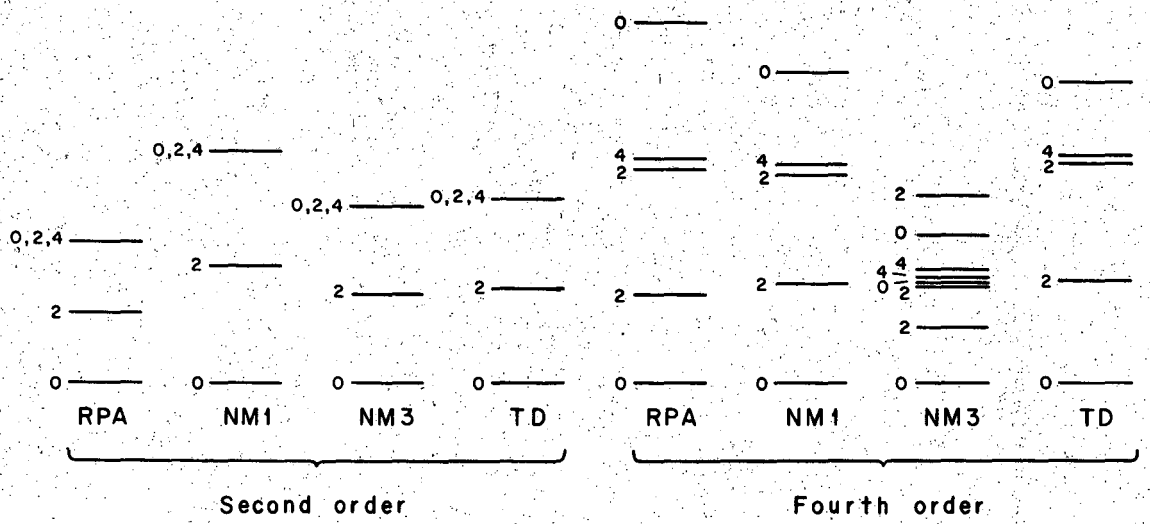
0,2,4
2
0
0,2,4
2
0
0,2,4
2
0
0,2,4
2
0
RPA
NM1
NM3
TD
Second order
0
4
2
2
0
0
4
2
2
0
0
4
2
2
0
4
4
2
0
2
0
RPA
NM1
NM3
TD
Fourth order
XBL694-2555

Fig. 1.

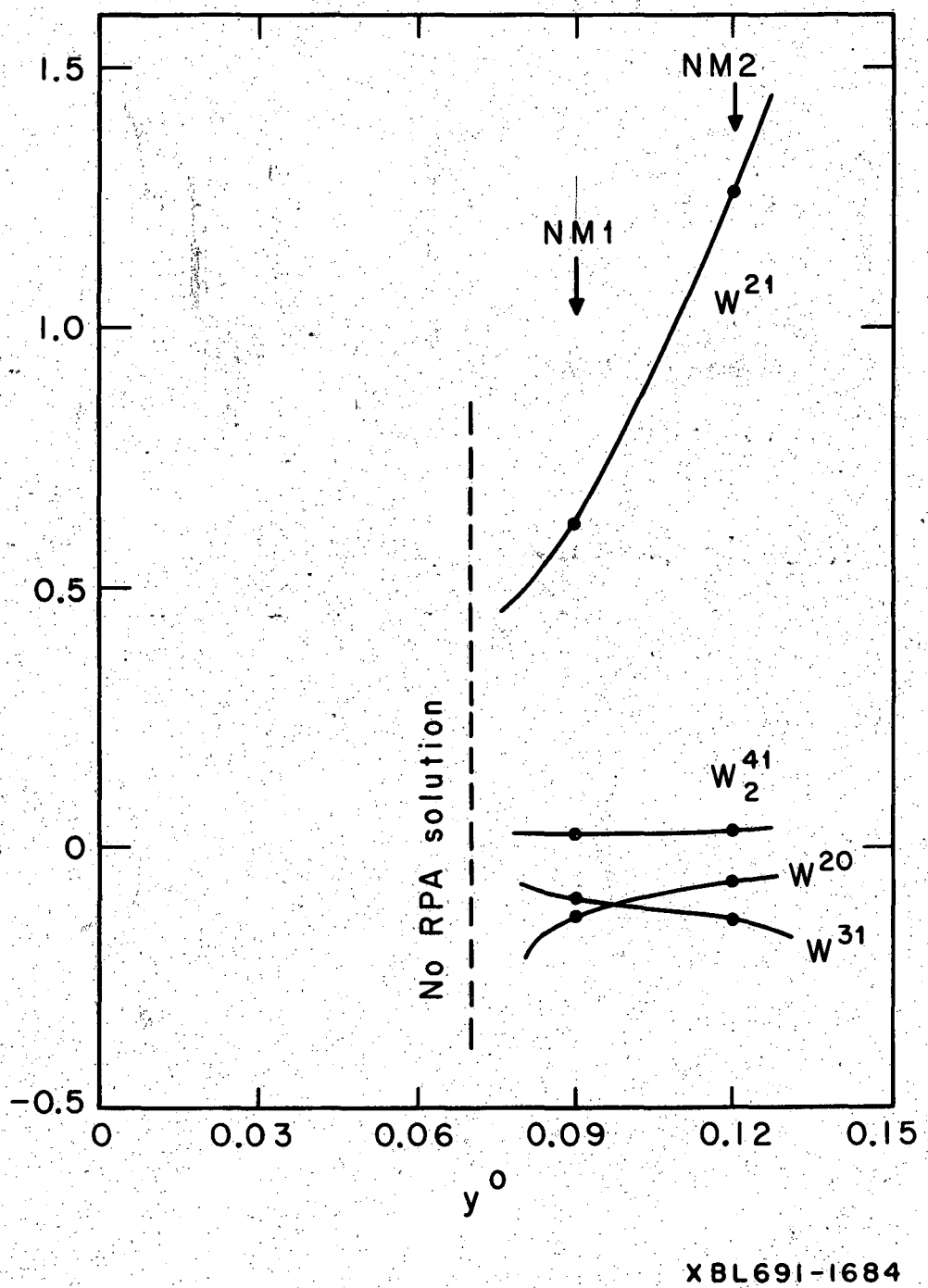
1.5
1.0
0.5
0
-0.5
0
0.03
0.06
0.09
0.12
0.15
y°
NM2
NM1
W21
W241
W20
W31
No RPA solution
XBL691-1684

Fig. 2.

LEGAL NOTICE

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720