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Title

DYNAMIC-MODEL FOR MULTIRESERVOIR OPERATION

Permalink

Journal

ISSN

Authors

MARINO, MA
LOAICIGA, HA

Publication Date

1985

DOI

Dynamic Model for Multireservoir Operation

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This paper presents a methodology to obtain optimal reservoir operation policies for a large-scale reservoir system. The model yields medium-term (one-year-ahead) optimal release policies that allow the planning of activities within the current water year, with the possibility of updating preplanned activities to account for uncertain events that affect the state of the system. River flows are characterized as a multivariate autoregressive process and are forecasted using maximum likelihood estimators. The solution method is a sequential dynamic decomposition algorithm that keeps computational requirements and dimensionality problems at low levels. The model maximizes the system annual energy generation while satisfying constraints imposed on the operation of the reservoir network. Several alternative versions of the model are also presented, which can be used under different assumptions. The model is applied to a large-scale multireservoir system, the northern portion of the California Central Valley Project. The optimal release policies show a potential increase in the system total annual energy with respect to heuristic schedules currently in use.

INTRODUCTION

Reservoir operation is a multistage dynamic stochastic control problem. The key to successful implementation of any model rests on the ability to take advantage of system features that lead to simpler mathematical models and of the proper choice of solution algorithms that overcome dimensionality problems and are numerically stable. In the water resources literature, several approaches to reservoir operation have appeared since the mid-1960's. Previous work can be classified as either deterministic or stochastic, although the mixed nature of some methods may result in an arbitrary classification.

Concerning deterministic models, dynamic programming (DP) or some algorithms based on it have been used extensively in reservoir operation models. *Hall et al.* [1969], *Heidari et al.* [1971], *Fults and Hancock* [1972], and *Fults et al.* [1976] presented applications of state increment dynamic programming (SIDP). *Larson and Keckler* [1969], *Larson and Korsak* [1970], *Yeh et al.* [1978], and *Turgeon* [1982] provided applications of dynamic programming successive approximations (DPSA). *Liu and Tedrow* [1973] applied DP to reservoir operation. *Becker and Yeh* [1974] and *Yeh and Becker* [1982] gave examples of linear programming (LP)-DP models. *Murray and Yakowitz* [1979] utilized differential dynamic programming (DDP). *Turgeon* [1981] applied the progressive optimality algorithm (POA). *Hicks et al.* [1974] presented a large-scale nonlinear programming (NLP) model. *Yazicigil et al.* [1983] solved a reservoir operation problem by using LP.

In the stochastic environment, *ReVelle et al.* [1969] applied chance constraints (CC) in conjunction with the linear decision rule (LDR). Many papers in the 1970's exploited the CC-LDR or some modifications of it to reservoir design and mangement problems. *Takeuchi and Moreau* [1974] and *Bras et al.* [1983] presented applications of stochastic dynamic programming (SDP). *Colorni and Fronza* [1976], *Simonovic and Mariño* [1982], and *Mariño and Mohammadi* [1983] reported applications of reliability programming (RP). The review of deterministic and stochastic methods in reservoir operation given above does not pretend to have mentioned all the interesting publications on the subject, but rather, references

those papers more closely related to the model presented herein.

Previous research on reservoir operation models point out two main difficulties associated with the computation of release schedules for multiunit systems. First, the decision space is usually vast (dimensionality) and, second, the incorporation of the stochastic nature of some elements (mainly inflows) introduces statistical complexities. Effective reservoir operation requires detail (i.e., that release schedules be computed for each reservoir) and validity (i.e., that policies be consistent with the actual realization of uncertain events) of any proposed release schedule. This paper develops an optimization model to compute release policies for large-scale reservoir systems. The model yields policies for every component of the network and keeps these policies up to date with the actual realization of streamflows, thus providing detailed and valid operation policies. The remainder of this paper contains a system and problem description, a formulation of several alternative models, and an application of one of the models to the northern portion of the California Central Valley Project (NCVP). A set of conclusions summarizes the findings of this work.

SYSTEM AND PROBLEM DESCRIPTION

The system under analysis is composed of the following reservoirs: (1) Clair Engle, (2) Lewiston, (3) Whiskeytown, (4) Shasta, (5) Keswick, (6) Folsom, (7) Natoma, (8) New Melones, and (9) Tullock. Figure 1 shows a schematic representation of the NCVP system and the points at which accretions and/or diversions occur. The NCVP is managed jointly by the U.S. Bureau of Reclamation (USBR) and the California Department of Water Resources. The main purposes of the NCVP are provision of water for irrigation (I), municipal and industrial uses (MI), environmental control and enhancement (E), fish and wildlife requirements (F), river navigation (N), water quality control (WQ), flood regulation (FC), hydropower (HP), recreation (R), and control of ocean intrusion and erosion.

Table 1 shows basic data for the NCVP system. The system release policy is subject to physical and technical constraints that arise from the capacity and technological features of the facilities, as well as institutional and environmental regulations. *Mariño and Loaiciga* [1983] provided a detailed quanti-
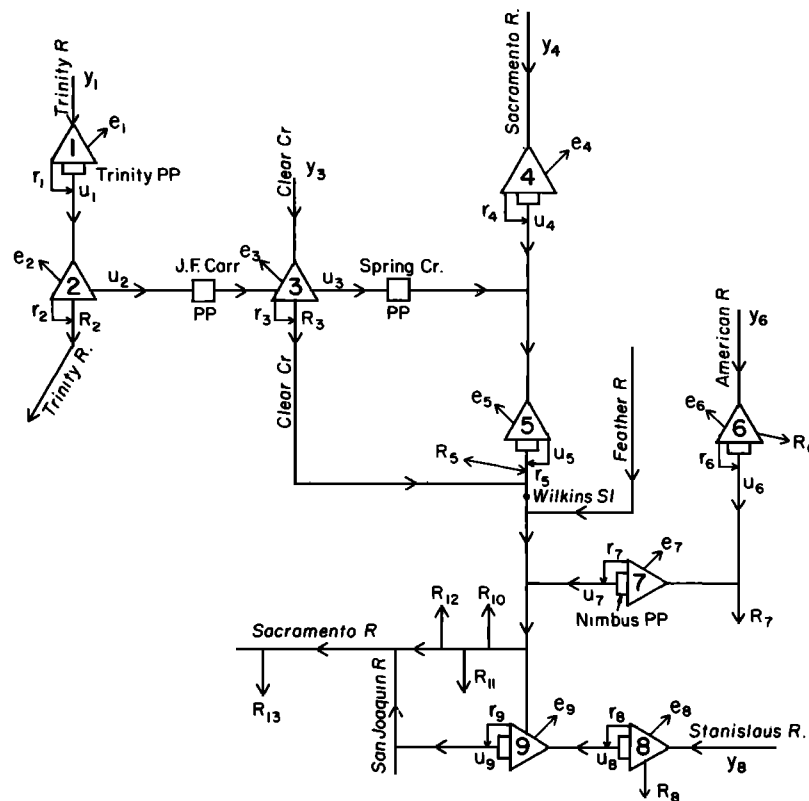
Fig. 1. Schematic representation of NCVP diversions (R), net losses (e), releases through penstocks (u), spills (r), and streamflows (y).

tative description of constraints imposed on the operation of the NCVP system that arise from the existence of the multiple functions outlined earlier. One of the important constraints imposed on the NCVP operation arises from flood control regulations. The flood control pool of the reservoirs is governed by regulations established by the U.S. Corps of Engineers. Those regulations stipulate that a certain amount of space be held empty in a reservoir during October 1 to May 31. Typical flood regulations are described by charts as the one shown in Figure 2 for New Melones reservoir. Similar time-dependent flood control regulations exist for Clair Engle, Shasta, and Folsom reservoirs. Such regulations do not exist

for the other reservoirs in the system. Other unique considerations affecting the NCVP system operation are related to the need of extensive fishery operations and salinity control requirements at the Sacramento-San Joaquin Delta.

The total power output of the system is delivered mostly to Pacific Gas and Electricity (a public utility), which uses it as peaking capacity to satisfy its power demand. The dependable capacity of the system (the power generation which under the "most adverse" flow conditions of record can be relied upon to its share of the power load) has been established at 860 MW.

The longest-term operation activities of the NCVP are planned for each water year. On October 1, the USBR esti-

TABLE 1. Basic NCVP Data

Reservoir	Managing Institution	First Year of Operation	Capacity, KAF	Installed Capacity,* MW	Functions Served
Shasta	USBR	1944	4552.0	559	I, FC, HP, MI, WQ, N, R, F
Clair Engle	USBR	1960	2448.0	128	I, FC, HP, MI, WQ, N, R, F
Lewiston	USBR	1962	14.7	···	Regulation, HP, F, R
Whiskeytown	USBR	1963	241.0	154 (J. F. Carr) 190 (Spring Creek)	Regulation, I, HP, MI, R, F
Keswick	USBR	1948	23.8	90	Regulation, HP, R, F
Folsom	USBR	1955	1010.0	198	I, FC, HP, MI, WQ, R, F
Natoma	USBR	1955	8.8	15	Regulation, HP, R, F
New Melones	USBR	1978	2600.0	383	I, WQ, FC, F, HP, R
Tullock	Oakdale Irrigation District	1958	67.0	17	I, HP, Regulation

1 KAF = 1.233 × 10^6 m^3. Functions are discussed in text.
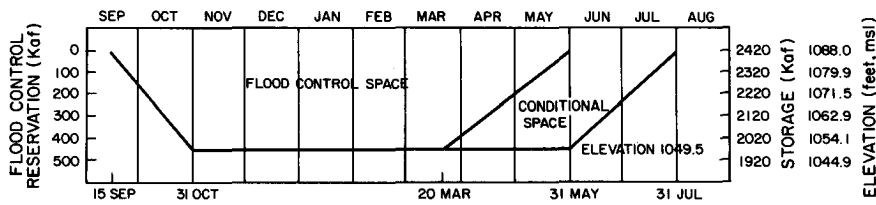*Indicates data as of July 1982.

Fig. 2. New Melones flood control diagram (Central Valley Operations Office, U.S. Bureau of Reclamation, Sacramento, California).

mates future streamflows for the next 12 months. Based on that forecast, a tentative release policy is proposed for the 12-month period. Because actual streamflows deviate from their expected values and institutional and/or technical conditions may vary from month to month, streamflow forecasts are updated at the beginning of each month, and the release policy is revised for the remaining months of the year. The proposed optimization model of the NCVP system is developed to fit this recurrent scheme for release policies. The updating scheme takes into consideration the most recent streamflow information and the actual system storage evolution.

By far, the stochasticity of flows is the major source for needing to adjust previous policies. The quality of release policies, computed at the beginnning of each month and followed strictly for the current month only, depends on the accuracy of the streamflow forecasts. H. A. Loaiciga and M. A. Mariño (unpublished manuscript, 1982) developed a multivariate autoregressive (AR) streamflow forecasting technique that takes into account the cross correlations between different streamflow stations and permits the introduction of multiple lags in the AR process. They estimated the AR parameters via maximum likelihood and utilized the estimated AR process for recursive streamflow forecasting. Their approach permits statistical testing of the order of the AR process as well as of some of the assumptions on the noise term and of the time invariance properties of the AR parameters. The application of the AR forecasting technique to the rivers of the NCVP system yielded accurate monthly flow estimates that ranged between $\pm 10\%$ from actual observed values.

The updating scheme utilized in this study consists of a sequential solution of deterministic problems in which forecasted flows enter in the formulation of each new problem, and initial storages are set equal to the actual values at every beginning of month. Such an updating scheme has been successfully applied in control engineering and is usually referred to as the certainty-equivalence controller (CEC) or under some circumstances as open-loop feedback controller [Bertsekas, 1976]. The solution of each (consecutive) problem can be obtained by deterministic optimal control or mathematical programming. For this study it is sufficient to point out that the adopted CEC is quasi-adaptive, i.e., if the value of the objective function achieved by applying a completely deterministic control is denoted by J^0, the value obtained by the CEC as J^1, and that for the true (unknown) optimal control as J^*, then (under maximization) $J^* \geq J^1 \geq J^0$ [Bertsekas, 1976, p. 199]. This inequality indicates that the use of the CEC control scheme would result in a performance that is at least as good as if a purely deterministic control were used (say, DPSA or SIDP).

FORMULATION OF ALTERNATIVE OPTIMIZATION MODELS

To adopt a 1-year planning horizon with monthly decisions, the following variables are defined

where

$\mathbf{u}_t$ n-dimensional decision vector of penstock releases at the beginning of month t; its components are u_t^i, where i refers to the ith reservoir;

$\mathbf{r}_t$ n-dimensional decision vector of spillages at the beginning of month t; its components are r_t^i;

$\mathbf{x}_t$ n-dimensional state vector of storages at the beginning of month t; its components are x_t^i.

The time index t goes from $t = 1$ to $t = N + 1 = 13$, and n is the number of reservoirs in the system. Figure 3 shows the relationship between the time index t and the vectors $\mathbf{x}_t$, $\mathbf{u}_t$, and $\mathbf{r}_t$. The objective is to compute decision policies $\{\mathbf{u}_t^*\}$ and $\{\mathbf{r}_t^*\}$ and their corresponding trajectory $\{\mathbf{x}_t^*\}$. It is important to notice the existence of more decisions ($\mathbf{u}_t$ and $\mathbf{r}_t$) than states ($\mathbf{x}_t$), and this fact must be considered in the solution algorithm. In the application of this paper, some simplifying assumptions are made to overcome such difficulty. Mariño and Loaiciga [this issue], in a generalized version of the method proposed herein, have shown that the difficulty can also be handled via restrictions on the spillages.

Mariño and Loaiciga [1983] made a comparative study of algorithms such as DPSA, SIDP, DP, LP, NLP, DDP, and the POA and concluded that for the NCVP system the POA [Howson and Sancho, 1975] had several attractive features well suited for the sequential optimization scheme. The reader is referred to the original work by Howson and Sancho [1975] or that by Turgeon [1981] for elaborated descriptions of the POA. The description of the POA is limited to the essentials necessary to make this work sufficiently self-contained. An algorithmic modification of the POA that has resulted in the improvement of its convergence rate (which has been reported as low by Turgeon [1981]) is presented in this paper. It must be emphasized that the POA is a modification of DP and its validity rests on the principle of optimality, and hence the POA is not applicable in control problems where such principle does not hold (as is the case in differential games).

The solution algorithm (POA) consists of the sequential solution of two-stage problems. Figure 4 illustrates the solution process for the first 3 months during the kth iteration. The first two-stage problem involves months 1 and 2. The beginning and ending storage vectors, $\mathbf{x}_1$ and $\mathbf{x}_{13}$, respectively, are fixed (the application below discusses the choice of $\mathbf{x}_{13}$). To solve the first two-stage problem, the current value of $\mathbf{x}_3^{(k)}$ is held fixed, and the optimization is carried out with respect to $\mathbf{x}_2$ to obtain $\mathbf{x}_2^*$, which, in turn, becomes the $(k + 1)$th
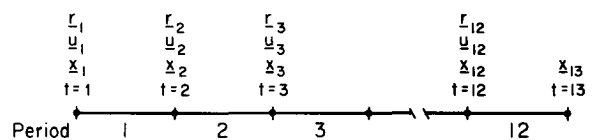

Fig. 3. Relation between t and vectors $\mathbf{x}_t$, $\mathbf{u}_t$, and $\mathbf{r}_t$.

Fig. 4. Sequential optimization scheme.

iterate value of x_2. Advancing to the second two-stage problem, $x_2^{(k+1)} = x_2^*$ and $x_4^{(k)}$ are held fixed, and x_3 is optimized to yield x_3^*. A sweep from months 1 to 12 completes the kth iteration. Several iterations are performed until an adequate user-specified criterion is satisfied. We have introduced some modifications in the POA such that it is not necessary to advance in each iteration from months 1 to 12. The POA has been modified in such a way that successive local optimizations are made at some periods in which relatively higher improvements in the objective function are observed, as is depicted in Figure 5, where the modified advancing scheme for the POA is illustrated.

Development of Two-Stage Problems

The mathematical structure of several alternative formulations of the two-stage problems are developed next. The different formulations arise from a variety of assumptions that permit the planner to choose among the alternative formulations, depending on the validity of particular assumptions in a given reservoir system. Notice that the structure of the two-stage problems needs to be characterized for one period t only ($t = 2, 3, \ldots, 12$), and state the boundary conditions x_1 and x_{13}, to completely specify the model structure when using the POA. Time-dependent information (e.g., flows, constraints, etc.) is handled as a simple data retrieval problem when solving the two-stage problem for any period t. Initial state and release sequences are provided to start the first iteration, and their developments are explained in the application section below.

The continuity equation for the NCVP system for months t

Fig. 5. Modified POA (to achieve state $x_2^{(5)}$: (1) x_1 and $x_3^{(1)}$ yield $x_2^{(2)}$; (2) $x_2^{(2)}$ and $x_4^{(1)}$ yield $x_3^{(2)}$; (3) x_1 and $x_3^{(2)}$ yield $x_2^{(3)}$; (4) $x_2^{(3)}$ and $x_5^{(1)}$ yield $x_3^{(3)}$; (5) x_1 and $x_3^{(3)}$ yield $x_2^{(4)}$; (6) $x_2^{(4)}$ and $x_6^{(1)}$ yield $x_3^{(4)}$; and (7) x_1 and $x_3^{(4)}$ yield $x_2^{(5)}$). This scheme should be used when significant improvements (e.g., 5% improvement in the objective function with respect to the previous iteration) arise from the two-stage problem involving periods 1 and 2).

and $t + 1$ is

$$x_{t+1} = x_t + \Gamma_1 u_t + \Gamma_2 r_t + z_t \qquad (1)$$

in which

$$\Gamma_1 = \begin{bmatrix} -1 & & & & & & & & \\ 1 & -1 & & & & & & & \\ & 1 & -1 & & & & & & \\ & & & -1 & & & & & \\ & & 1 & 1 & -1 & & & & \\ & & & & & -1 & & & \\ & & & & & 1 & -1 & & \\ & & & & & & & -1 & \\ & & & & & & & 1 & -1 \end{bmatrix} \qquad (2)$$

$$\Gamma_2 = \begin{bmatrix} -1 & & & & & & & & \\ 1 & -1 & & & & & & & \\ & & -1 & & & & & & \\ & & & -1 & & & & & \\ & & & 1 & -1 & & & & \\ & & & & & -1 & & & \\ & & & & & 1 & -1 & & \\ & & & & & & & -1 & \\ & & & & & & & 1 & -1 \end{bmatrix} \qquad (3)$$

$$z_t = y_t - e_t - R_t \qquad (4)$$

where y_t, e_t, and R_t are the forecast inflow [i.e., $y_t^T = (y_t^1, 0, y_t^3, y_t^4, 0, y_t^6, 0, y_t^8, 0)$, where the superscript denotes reservoir number], net loss, and diversion vectors, respectively.

The lower triangularity of Γ_1 and Γ_2 arises from a proper numbering of the network, as is done in Figure 1. Net losses from reservoir i during month t are given by

$$e_t^i = d_t^i + c_t^i(x_t^i + x_{t+1}^i) \qquad (5)$$

in which d_t^i and c_t^i are coefficients determined via multiple regression analysis [Mariño and Loaiciga, 1983] and $(x_t^i + x_{t+1}^i)$ denotes the use of average storage. Substitution of (5) into (1) yields

$$\Gamma_1 u_t + \Gamma_2 r_t = A_{t+1} x_{t+1} - B_t x_t - v_t \qquad (6)$$

in which $A_{t+1} =$ diagonal $(1 + c_t^i)$, $i = 1, 2, \ldots, 9$; $B_t =$ diagonal $(1 - c_t^i)$, $i = 1, 2, \ldots, 9$; and $v_t^T = [y_t^1 - d_t^1, -R_t^2 - d_t^2, y_t^3 - R_t^3 - d_t^3, y_t^4 - d_t^4, -d_t^5, y_t^6 - R_t^6 - d_t^6, -R_t^7 - d_t^7, y_t^8 - R_t^8 - d_t^8, -d_t^9]$. Equation (6) can be rewritten as

$$x_{t+1} = \phi_t x_t + P_{1t} u_t + P_{2t} r_t + Q_t v_t \qquad (7)$$

in which $\phi_t = A_{t+1}^{-1} B_t$, $P_{1t} = A_{t+1}^{-1} \Gamma_1$, $P_{2t} = A_{t+1}^{-1} \Gamma_2$, and $Q_t = A_{t+1}^{-1}$. Vectors u_t and r_t represent deterministic control terms, and $Q_t v_t$ contains stochastic inflows as well as demands R_t^i, and the d_t^i coefficients appearing in (5). Equation (7) represents the linear continuity equation for the NCVP system and plays a central role in arriving at the structure of the alternative two-stage problems.

The objective function of the optimization model consists of maximization of the energy generated during each year. A reservoir operation model that considers maximization of energy revenues is presented in the work by Mariño and Loaiciga [1985]. The NCVP management aims at maximizing its cash revenues accruing from power sales only while operating the system so as to provide adequate fulfillment of other func-

tions by satisfying contractual agreements or specified storages and releases ranges (e.g., for recreational or fisheries needs). Therefore the multiple functions of the NCVP system can be handled by obtaining a release schedule that maximizes energy while providing adequate service for other purposes via constraints on releases and storages.

The energy generated at reservoir i during month t (in megawatt-hours, MWh) is

$$E_t{}^i = \psi_t \xi_t{}^i u_t{}^i = \psi_t[\hat{a}^i + \hat{b}^i(x_t{}^i + x_{t+1}{}^i)]u_t{}^i$$
$$= [a^i + b^i(x_t{}^i + x_{t+1}{}^i)]u_t{}^i \qquad (8)$$

in which $\xi_t{}^i$ is the linear energy rate in MWh/kilo acre feet (KAF) (1 KAF $= 1.233 \times 10^6$ m^3); $u_t{}^i$ is the penstock release from reservoir i during month t; and $\hat{a}^i$ and $\hat{b}^i$ are coefficients determined from energy generation records by multiple regression [*Mariño and Loaiciga*, 1983]; ψ_t is the estimated fraction of time that power plants are normally on-line for month t and set constant at a level of $\psi_t = \psi = 0.85$ (George Link, USBR, personal communication, 1982). The coefficients a^i and b^i are equal to $\hat{a}_i\psi$ and $\hat{b}_i\psi$, respectively. The energy generation rates $\xi_t{}^i$ have been found to provide accurate energy production estimates, within $\pm 2\%$ of actual values for given average storages and releases. The basis for deriving the $\xi_t{}^i$ equations are well analyzed performance data of the power plants, of which an example is given in Figure 6. The $\xi_t{}^i$ equations are fits to curves like that shown in Figure 6 for each power plant. The accuracy of predictions by the $\xi_t{}^i$ rates shows that for planning studies it is quite adequate to utilize equations like (8) to estimate power production.

For the whole system the total energy generated during any month t can be expressed as

$$E_t = [\mathbf{a} + B(\mathbf{x}_t + \mathbf{x}_{t+1})]^T \mathbf{u}_t \qquad (9)$$

in which $\mathbf{a}^T$ contains the constant terms a^i in (8); B is a diagonal matrix whose (diagonal) terms are the b^i's in (8), and $\mathbf{u}_t$ is a nine-dimensional vector of penstock releases. A similar expression can be written for month $t - 1$.

Recalling that the POA maximizes a sequence of two-stage problems, i.e., maximize $E_{t-1} + E_t$ for $t = 2, 3, \ldots, 12$ subject to a set of constraints, it follows that the two-stage objective function is

$$E_{t-1} + E_t = [\mathbf{a} + B(\mathbf{x}_{t-1} + \mathbf{x}_t)]^T \mathbf{u}_{t-1}$$
$$+ [\mathbf{a} + B(\mathbf{x}_t + \mathbf{x}_{t+1})]^T \mathbf{u}_t \qquad (10)$$

Also, from (6),

$$\mathbf{u}_t = -\Gamma_1{}^{-1}\Gamma_2 \mathbf{r}_t + \Gamma_1{}^{-1}A_{t+1}\mathbf{x}_{t+1} - \Gamma_1{}^{-1}B_t\mathbf{x}_t - \Gamma_1{}^{-1}\mathbf{v}_t$$
$$= -\Gamma \mathbf{r}_t + C_{t+1}\mathbf{x}_{t+1} - D_t\mathbf{x}_t - F\mathbf{v}_t \qquad (11)$$

A similar expression can be developed for $\mathbf{u}_{t-1}$. Substitution of the expressions for $\mathbf{u}_{t-1}$ and $\mathbf{u}_t$ into (10) yields

$$E_{t-1} + E_t = \mathbf{q}_t{}^T \mathbf{r}_t + \mathbf{p}_t{}^T \mathbf{r}_{t-1} + \mathbf{g}_t{}^T \mathbf{x}_t - \mathbf{x}_t{}^T B\Gamma \mathbf{r}_t$$
$$- \mathbf{x}_t{}^T B\Gamma \mathbf{r}_{t-1} + \mathbf{x}_t{}^T G_t \mathbf{x}_t + k_t \qquad (12)$$

in which the vectors $\mathbf{q}_t$, $\mathbf{p}_t$, $\mathbf{g}_t$, the matrix G_t, and the scalar term k_t are uniquely defined from the terms in (10) and (11), and whose lengthy algebraic expressions have been omitted to save space. By defining the augmented vector

$$\boldsymbol{\theta}_t{}^T = [\mathbf{x}_t, \mathbf{r}_t, \mathbf{r}_{t-1}]^T \qquad (13)$$

(12) can now be rewritten as (deleting the term k_t)

$$E_{t-1} + E_t = \boldsymbol{\theta}_t{}^T H_t{}^1 \boldsymbol{\theta}_t + \mathbf{s}_t{}^T \boldsymbol{\theta}_t \qquad (14)$$
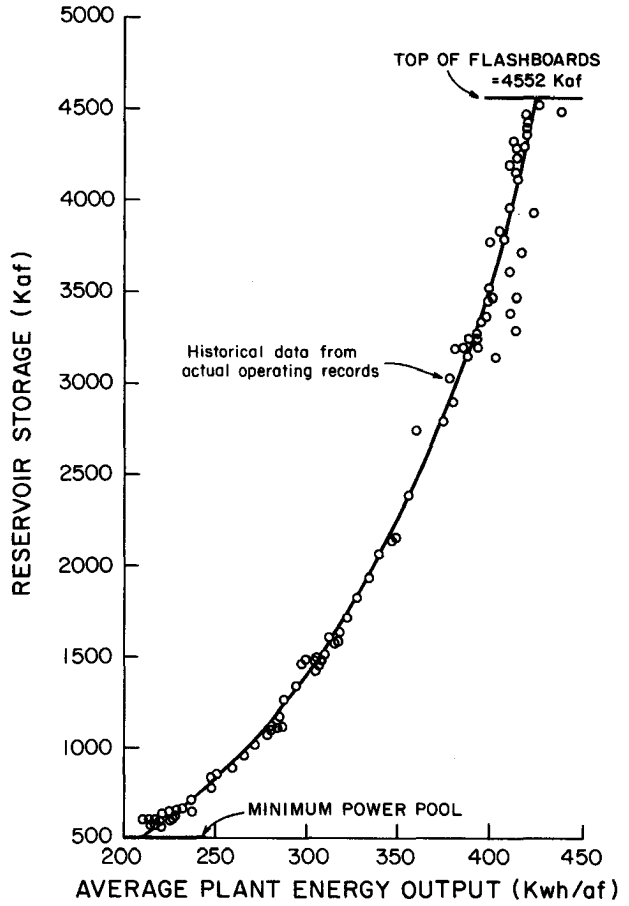
Fig. 6. Shasta power plant gross generation curve.

in which

$$\mathbf{s}_t{}^T = [\mathbf{g}_t{}^T, \mathbf{q}_t{}^T, \mathbf{p}_t{}^T] \qquad (15)$$

and

$$H_t{}^1 = \frac{1}{2}\begin{bmatrix} G_t + G_t{}^T & -B\Gamma & -B\Gamma \\ -\Gamma^T B & 0 & 0 \\ -\Gamma^T B & 0 & 0 \end{bmatrix} \qquad (16)$$

where 0 is a 9×9 null matrix. Notice that the continuity equation was used in developing (14), and thus it will not appear as a constraint in the constraint set associated with (14).

By using (11) to express $\mathbf{u}_t$ for any period t and using the augmented vector $\boldsymbol{\theta}_t$, the constraint set associated with (14) for month t consists of the following constraints.

Constraints on total releases, $\mathbf{u}_t + \mathbf{r}_t$,

$$C_{t+1}\mathbf{x}_{t+1} + [-D_t, (-\Gamma + I), 0]\boldsymbol{\theta}_t - F\mathbf{v}_t \in W_t \qquad (17)$$

where W_t is a feasible set for $\mathbf{u}_t + \mathbf{r}_t$ (and 0 and I are the 9×9 null and identity matrices, respectively).

Water requirements (arising from any of the system functions) at any control point can be expressed as a linear combination of total releases,

$$\mathbf{c}^T C_{t+1}\mathbf{x}_{t+1} + \mathbf{c}^T[-D_t, (-\Gamma + I), 0]\boldsymbol{\theta}_t - \mathbf{c}^T F\mathbf{v}_t \in De_t \qquad (18)$$

where $\mathbf{c}^T$ is the appropriate linear combination vector, and De_t is a set of feasible values for water requirements.

Constraints on penstock releases,

$$[-D_t, -\Gamma, 0]\theta_t + C_{t+1}x_{t+1} - Fv_t \in U_t \tag{19}$$

where U_t is a feasible set.

Spillage constraints imply

$$[0, I, 0]\theta_t \in R_t \tag{20}$$

in which R_t is a feasible set.

Constraints on storages imply

$$[I, 0, 0]\theta_t \in X_t \tag{21}$$

where X_t is a feasible set.

The appropriate constraints for month $t-1$ are obtained in a similar manner. All the constraints are linear, and constraints on power generation, which are of a complicated nonlinear form, have been avoided because they are implicitly considered by the energy rate equations introduced in (8). The two-stage problem for any t ($t = 2, 3, \cdots, 12$) can be expressed as a quadratic linearly constrained problem (QP), namely,

$$\underset{\theta_t}{\text{maximize}} \quad \theta_t^T H_t^1 \theta_t + s_t^T \theta_t \tag{22}$$

subject to

$$A_t^1 \theta_t \le b_t^1 \tag{23}$$

$$[I, 0, 0]\theta_{t-1} \qquad [I, 0, 0]\theta_{t+1} \text{ fixed}$$

where the matrix A_t^1 and the vector b_t^1 contain as their row components all the (linear) constraints detailed above (for month t and $t-1$). For the NCVP system, A_t^1 and b_t^1 are of dimensions 92×27 and 92×1, respectively [Mariño and Loaiciga, 1983]. The solution of (22)–(23) can be approached by Fletcher's [1981] active set method, which consists of finding an initial feasible vertex, and then work in a series of consecutive active (or binding) set of constraints. The active set method has been modified by Gill and Murray [1977] to handle indefinite Hessian matrices in a numerically stable manner, and in this regard is the only quadratic programming algorithm recommended to solve (22)–(23) [Mariño and Loaiciga, this issue].

There are two ways of reducing the dimensionality of (22)–(23): (1) to introduce assumptions on net losses and/or spillages as done below or (2) via equality constraints on spillages, as is shown in the work by Mariño and Loaiciga [this issue]. It has been found both in the application of this paper, and in that given in the preceding reference by the authors, that it is highly advantageous to recourse to simplifications 1 or 2, as adequate in a given application, in order to simplify the solution of the two-stage problems.

It is possible to reduce the computational work by introducing assumptions on net losses and/or spillages (simplification 1 above). In many practical cases, net losses can be ignored, which implies that the matrices A_{t+1} and B_t in (6) can be set equal to the identity matrix. Simplification 1 requires the continuity equation (7) to have stationary parameters, i.e.,

$$x_{t+1} = x_t + \Gamma_1 u_t + \Gamma_2 r_t + v_t \tag{24}$$

By using (24), the quadratic objective function of the model becomes linear, as will be shown later.

The second assumption relates to spillages. Spillages can be assumed to be zero whenever reservoir storage is below the spillway crest. This assumption is valid for average (normal) and below-average (dry) inflow years, as is shown in the application section. For above-average (wet) inflow years, spillages

will occur. This is because inflows will exceed penstock releases and spillages must take place to avoid dam overtopping. Under this (second) assumption, spillages can be handled by maximizing the objective function with respect to u_t. When u_t reaches its upper bound (determined by the constraint set), the value of u_t is set equal to that upper bound (in our case, penstock capacity). Any excess release necessary to maintain feasibility of the constraints is called spillage. Clearly, r_t is not handled explicitly as a decision variable; however, that has no effect on the optimal solution. This is because r_t will be zero whenever storages are within permissible levels, since no benefits arise from spilling water that can be routed through penstocks. By treating spillages as described above, the dimension of the decision vector θ_t, (22), is reduced from 27×1 to 9×1.

Simplified Linear Model

By using the simplifying assumptions on net losses and spillages it is possible to solve for u_t in (24), set r_t equal to zero, and use u_t (and a similarly derived expression for u_{t-1}) into (10) and into the constraint equations (17), (18), (19), and (21) (constraint equation (20) on spillages is not written as part of the rows of the constraint matrix), to obtain the following linear programming (LP) model for the two-stage problems (dropping constant terms irrelevant to the optimization in the objective function):

$$\underset{x_t}{\text{maximize}} \quad h_t^T x_t \tag{25}$$

subject to

$$Ax_t \le b_t \tag{26}$$

$$x_{t+1} \qquad x_{t-1} \text{ fixed}$$

Matrix A and vector b_t are of dimensions 74×9 and 74×1, respectively, and their row components contain the constraints for months t and $t-1$. The elements of h_t, b_t, and A are functions of the relevant components appearing in (24), (10), (17), (18), (19), and (21) [Mariño and Loaiciga, 1983]. Due to their lengthy algebraic form, their exact mathematical expressions have been omitted. Of conceptual importance are the facts that one needs to maximize with respect to x_t only (a 9×1 vector) and that the constraint matrix A is time invariant and needs to be computed once only.

Simplified Quadratic Model 1

Consider the case in which net losses are taken into account but the spillage assumption is maintained (use equation (7) with r_t set equal to zero). Under these conditions, the two-stage maximization problem becomes a quadratic maximization problem with a nine-dimensional unknown vector (dropping constant terms in the objective function):

$$\underset{x_t}{\text{maximize}} \quad g_t^T x_t + x_t^T G_t x_t \tag{27}$$

subject to

$$A_t^2 x_t \le b_t^2 \tag{28}$$

$$x_{t+1} \qquad x_{t-1} \text{ fixed}$$

where the vector g_t and the matrix G_t are the same as those appearing in (15) and (16), and the vector b_t^2 and matrix A_t^2 are of dimensions 74×1 and 74×9, respectively, containing the constraint set associated with (27) [Mariño and Loaiciga, 1983]. Notice that maintaining the assumption on spillages

results in a substantial reduction of the unknown vector (from 27×1 to 9×1) but makes the constraint matrix $A_t{}^2$ time variant again, forcing its recomputation for every period t, $t = 2, 3, \ldots, 12$.

Simplified Quadratic Model 2

If net losses are not considered but spillages are included explicitly in the continuity equation (see equation (24)), the two-stage problem becomes (dropping constant terms in the objective function):

$$\underset{\theta_t}{\text{maximize}} \ \theta_t{}^T H \theta_t + \hat{s}_t{}^T \theta_t \qquad (29)$$

subject to

$$A^3 \theta_t \leq b_t{}^3 \qquad (30)$$

$$[I, 0, 0]\theta_{t-1} \qquad [I, 0, 0]\theta_{t+1} \ \text{fixed}$$

with θ_t as defined in (13). A^3 and $b_t{}^3$ are of dimensions 92×27 and 92×1, respectively. Notice that relaxing the assumptions on spillages increases the problem dimensionality to its original size (27×1), and the introduction of the net loss assumption makes the constraint set matrix A^3 time invariant. Vectors $b_t{}^3$ and $\hat{s}_t$, and matrices H and A^3, as well as all the terms in models (22)–(23), (25)–(26), and (27)–(28) can be found in full detail in the work by *Mariño and Loaiciga* [1983].

Problem (29)–(30) is quadratic-linearly constrained, similar to (22)–(23), and the comments made earlier with respect to the choice of an adequate QP algorithm apply again.

APPLICATION

Analysis of net loss data for the NCVP [*Mariño and Loaiciga*, 1983] showed that the assumption with regard to ignoring net losses is quite appropriate for this system. Hence for the application of this study it was decided to use the linear model (25)–(26), which also treats spillages in a simplified form. A generalized quadratic model that includes net losses and spillages via constraints is given in the work by *Mariño and Loaiciga* [this issue]. The LP two-stage problems were solved by using a revised simplex method [*Dantzig*, 1963]. This variation of the standard simplex method resulted in a more efficient solution of (25)–(26) for $t = 2, 3, \cdots, 12$. It can be shown that the storage requirements to solve the two-stage problems (25)–(26) are limited to storing the lower triangular matrix Γ_1 (see equation (2)), the diagonal elements of B, and the vector a in (10). Recurrent formation of the vectors h_t and b_t involved elementary matrix-vector operations, and there were no numerically ill-conditioned problems. In fact, the repetitive appearance of terms in both h_t and b_t can be exploited to reduce the number of operations required to structure the LP problem (25)–(26).

The sequential optimization scheme used in this study requires an initial feasible state trajectory to start the iteration sweeps. Initial operation policies for the NCVP were developed by using a trial-and-error procedure that considers some heuristic criteria used by NCVP managers to set up their release policies. In essence, desired reservoir storages at the end of the water year are selected, and a feasible (initial) release policy that achieves those targets is chosen. As the system operation progresses through the year, actual flow conditions may lead to a revision of the ending storages selected initially. The overall philosophy is that reservoir storages must be kept high at the beginning of the dry season (usually, May) to meet increasing agricultural and Delta water requirements during the summer. Also, the operation during the rainy season (November to March) is conservative in the sense that a substantial flood storage volume is allocated to store eventual large runoff events. Clearly, there is a trade-off between the desire to maintain the reservoir levels below some specified elevation during the rainy season and the desire to have as large a storage volume as possible at the beginning of the dry season. A general rule would be to maintain reservoir storages at high but permissible levels during the rainy season and to make large releases during the dry season. Interestingly, because most power installations in the NCVP are of the high-head type, a greater generation of power will not result from the largest releases but from some optimal reservoir elevation associated with moderate releases. The largest releases would drive reservoir levels below the range at which turbines can operate efficiently.

Two initial policies were selected for storage and corresponding releases. This was done to determine if each initial policy yields the same optimal release policy. As will be discussed later, different initial policies generally yield different optimal release policies. However, all those optimal release policies give the same value for the objective function of the model (the same annual energy generation), thus indicating the existence of multiple optimal solutions. That is a common phenomenon in linear programming and convex (as opposed to strictly convex) quadratic problems.

In the POA, the beginning and ending storage vectors x_1 and x_{13}, respectively, are fixed. Vector x_1 is specified at the beginning of period 1, and vector x_{13} ($= x_{N+1}$) must be a value ranging from one half to two thirds of the capacities of the reservoirs. This range has been established by the NCVP managers as satisfactory from past experience acquired from the operation of the system. If desired, the value of x_{N+1} can be updated at every end of the month. This study adopted a value of $\frac{7}{12}$ of reservoir capacity, which was not updated because it proved satisfactory, as is shown in the analysis of results below.

The sequential solution procedure can be summarized as follows. (1) The initial and final states x_I and x_{13} are fixed. Subindex I can take values 1 through 11, depending on which month the future release policy is being computed. Forecast flows for the remaining ($13 - I$) months, develop (or adjust) an initial feasible state trajectory $\{x_t{}^{(k)}\}$, and set the iteration counter k equal to 1 and $t = I$. (2) Set the time index $t = t + 1$ and solve the LP two-stage problem (25)–(26). (3) Denote the solution of step 2 by $x_t{}^*$. Set $x_t{}^{(k+1)} = x_t{}^*$ and go to step 2. Repeat steps 2 and 3 until a complete iteration sweep is performed ($t = I, \cdots, N$). This ends the kth iteration. (4) Perform a convergence test, e.g., is

$$|[(x_t{}^i)^{(k+1)} - (x_t{}^i)^{(k)}]/(x_t{}^i)^{(k)}| \leq 0.01$$

for all i and t ($t = I, \cdots, 12$)? If yes, go to step 5. Otherwise, set $k = k + 1$ (provided that a maximum iteration limit is not exceeded), $t = I$, and go to step 2, and (5) apply the optimal policy for current month I. Set $I = I + 1$ and go to step 1.

Analysis of Results

The optimization model was tested with different streamflow conditions: below-average inflow volumes (1975–1976, total inflow of 6,057 KAF); average inflow conditions (1974–1975 and 1979–1980, total inflows of 12,198 and 13,936 KAF, respectively); and above-average conditions (1973–1974, total inflow of 20,146 KAF). This was done to determine if different initial policies under various streamflow scenarios result in distinct optimal policies. Two initial policies were developed

TABLE 2. Number of Iterations to Attain Convergence and CPU Time Requirements

Inflow Condition	Policy	Number of Iterations to Attain Convergence	Burroughs B7800 CPU Time, min
Average	1	6	6.01
1974–1975	2	9	8.94
Average	1	8	8.51
1979–1980	2	10	10.32
Below Average	1	3	2.98
1975–1976	2	3	2.79
Above Average 1973–1974	1	8	7.28

for 1974–1975, 1975–1976, and 1979–1980, while a single initial policy was considered for 1973–1974. Due to space limitations, initial policies are not included herein. A complete set of initial policies (policies 1 and 2) for the various years under consideration is available from the authors. Development of the initial policies indicated that for below-average streamflow conditions there is little opportunity for optimizing the operation of the system, because prevailing low inflows barely meet the system's demands by releasing flows near their minimum permissible values. For both average and above-average streamflow conditions there is a larger feasible region, and the gains from the optimization model can be significant. During the winter months of an extremely wet year such as 1973–1974 the initial policy is nearly optimal because the reservoirs are at near capacity during those months, and total releases are set equal to maximum permissible flows. Under those circumstances, the optimization model allows the determination of the best feasible release policy that simultaneously minimizes the spillage and maximizes the power generation. Because the reservoirs are at a high stage after the winter, substantial improvements in energy generation can be obtained during the subsequent summer months.

Some of the initial policies were refined so as to make them near possible optimal releases whereas others were deliberately set to be poor (but feasible) initial estimates. This was done to estimate the number of iterations and CPU time needed by the POA to reach optimality. Initial policies 1 for average (1974–1975, 1979–1980) and below-average (1975–1976) inflow years were carefully refined, attempting to be near their respective optimal policies. In those cases, convergence to the optimum was attained in six to eight iterations.

In contrast, initial policies 2 for average-inflow years 1974–1975 and 1979–1980 were purposely developed to be far from good initial policies. That was accomplished by releasing heavily during wet (winter) months to maintain a year-round low head and a corresponding decrease in power generation. That is also suboptimal from the standpoint of agricultural and Delta requirements, because those demands are low in the winter and thus larger than necessary flows will be of no use. This strategy forces summer releases to be at minimum permissible levels, when an additional acre foot of water during this season has a greater marginal value than in the rainy season. Those deliberately poor initial policies resulted in an increase in the number of iterations needed to attain convergence, ranging now from eight to ten iterations. Table 2 summarizes the required iterations and CPU times for the specified initial policies and inflow conditions. It is evident that the CPU time increases as the inflow conditions vary from below average to average. That is because for below-average flow conditions, the feasible region becomes so tight that there is no freedom to optimize any policy. Any feasible initial policy will be very close to an optimal release policy. As flow volumes increase to average-flow conditions, there is a corresponding increase in CPU time. Notice that policies 2 for average flow years 1974–1975 and 1979–1980 (which were deliberately chosen to be inferior to their counterparts, policies 1) also required more CPU time. For extremely wet conditions such as water year 1973–1974, the feasible region becomes very tight during the winter and that implies a reduction in CPU time as shown in Table 2.

Optimal state trajectories (i.e., end-of-month storages) and their corresponding release policies were obtained by applying the POA to the initial policies. Tables 3 and 4 show the optimal strategies for average-flow conditions (1979–1980) corresponding to the first initial policy (policy 1). Table 4 also shows the energy produced by the optimal policy as well as the monthly water deliveries to the Delta. For Clair Engle, Shasta, and New Melones reservoirs, the optimal state policies (end-of-month storages and releases) resulting from initial policy 1 were different from those resulting from initial policy 2. For Folsom reservoir the optimal end-of-month storages and release policies were the same for initial policies 1 and 2. For the remaining five smaller reservoirs the end-of-month storages were the same for initial policies 1 and 2, but their release policies were different. The results also showed that the value of the objective function of the model (total energy gen-

TABLE 3. Optimal State Trajectory Corresponding to Initial Policy 1, 1979–1980

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct.	1632.0	14.7	241.0	3035.0	23.8	673.0	8.8	1600.0	60.0
Nov.	1577.2	14.7	241.0	3008.0	23.8	567.6	8.8	1462.0	57.0
Dec.	1547.8	14.7	241.0	3046.0	23.8	364.7	8.8	1337.0	57.0
Jan.	1543.7	14.7	178.1	2729.0	23.8	206.6	8.8	1185.0	57.0
Feb.	1690.8	14.7	183.4	3184.0	23.8	733.5	8.8	1253.0	57.0
Mar.	1907.6	14.7	201.7	3800.0	23.8	985.1	8.8	1303.0	60.0
Apr.	1974.7	14.7	241.0	3900.0	23.8	1000.0	8.8	1230.0	61.0
May	2080.4	14.7	241.0	4168.0	23.8	1010.0	8.8	1098.0	67.0
June	2072.7	14.7	241.0	4294.0	23.8	1010.0	8.8	1068.0	67.0
July	1969.1	14.7	241.0	4134.0	23.8	1010.0	8.8	992.0	67.0
Aug.	1808.5	14.7	241.0	3691.0	23.8	992.5	8.8	800.0	67.0
Sept.	1637.1	14.7	241.0	3187.0	23.8	798.6	8.8	536.0	67.0
Oct.	1542.6	14.7	241.0	2622.0	22.8	624.8	8.8	323.0	57.0

Storages are in kilo acre feet (1 KAF = 1.233 × 10⁶ m³).

TABLE 4. Optimal Energy Production, Release Policy, and Delta Releases Corresponding to Policy 1
1979–1980

Month	Clair Engle Energy	Release	Lewiston Energy	Release	Whiskeytown Energy	Release	Shasta Energy	Release	Keswick Energy	Release
Oct.	33356.7	93.0	44117.3	75.0	43473.7	77.0	112078.0	300.0	35664.2	377.0
Nov.	38985.9	109.8	53999.5	91.8	53523.5	94.8	112154.2	300.0	37348.1	394.8
Dec.	31578.7	89.3	41940.8	71.3	77793.4	141.2	243955.3	664.0	76171.9	805.2
Jan.	32126.6	89.3	41940.8	71.3	41460.3	77.0	291284.0	786.0	81639.8	863.0
Feb.	33520.7	89.3	41940.8	71.3	41854.6	77.0	310729.7	786.0	81639.8	863.0
Mar.	34608.3	89.3	41940.8	71.3	42817.1	77.0	323729.8	786.0	81639.8	863.0
Apr.	35941.7	91.0	41940.8	73.0	54201.0	96.0	126111.2	300.0	37461.6	396.0
May	83426.1	209.0	112351.8	191.0	116871.0	207.0	128841.7	300.0	47962.2	507.0
June	82428.2	209.0	107057.9	182.0	105014.5	186.0	184764.0	431.0	58368.2	617.0
July	80059.3	209.0	105293.2	179.0	103885.3	184.0	281620.3	679.0	81639.8	863.0
Aug.	66386.9	180.0	88234.5	150.0	86947.5	154.0	278553.2	709.0	81639.8	863.0
Sept.	36455.7	102.0	44117.3	75.0	43473.7	77.0	289027.5	785.0	81639.8	863.0
Total	588874.9		764875.5		811315.6		2682848.9		782815.0	

Month	Folsom Energy	Release	Natoma Energy	Release	New Melones Energy	Release	Tullock Energy	Release	Delta Release
Oct.	53544.8	194.0	6373.6	191.4	71641.1	157.0	7165.3	60.0	628.4
Nov.	77852.2	303.0	9990.0	300.0	70422.2	160.0	7081.4	60.0	754.8
Dec.	71289.2	303.0	9990.0	300.0	85889.9	203.0	12156.5	103.0	1208.2
Jan.	118406.8	460.0	9990.0	300.0	87766.8	210.0	12982.6	110.0	1273.0
Feb.	139893.4	460.0	9990.0	300.0	90566.3	213.0	13136.4	110.0	1273.0
Mar.	147248.8	460.0	9990.0	300.0	89417.5	211.0	13341.5	110.0	1273.0
Apr.	126260.2	392.6	7745.6	232.6	88813.2	216.0	13700.3	110.0	738.6
May	131038.7	406.7	8215.1	246.7	84253.9	210.0	14007.8	110.0	863.7
June	76361.4	237.0	7792.2	234.0	82884.9	210.0	14007.8	110.0	961.0
July	63362.9	197.3	6470.2	194.3	79423.7	210.0	14007.8	110.0	1167.3
Aug.	93465.2	303.0	9990.0	300.0	70032.8	200.0	12734.4	100.0	1263.0
Sept.	86780.4	303.0	9990.0	300.0	48124.3	150.0	7361.0	60.0	1223.0
Total	1185503.9		106526.7		949236.5		141683.0		12627.0

Energy is in megawatt-hour and releases are in kilo acre feet (1 KAF = 1.233 × 10⁶ m³). Delta releases are the sum of Keswick, Natoma, and Tullock releases. In addition to the above penstock releases, spillages are 160 KAF at Lake Natoma during January to April ant 100 KAF at Tullock during October to September. Total annual energy equals 8,013,680 MWh.

erated during the year) is practically the same for release policies 1 and 2, with a difference of less than 0.17% due essentially to the effect of roundoff in the convergence test. This implies that there are multiple ways of achieving the optimum performance index.

It was found that an optimal release policy is achieved by releasing less water than the maximum possible penstock capacity. Because hydropower production depends on the storage level (the larger the head, the greater the energy production for a given discharge), an optimal release policy is a feasible trade-off point between a high head and a small release and a low head and a large release. Such a trade-off point is the optimal solution given by the POA. Because the power installations in the NCVP are of the high-head type, except Nimbus (at Natoma) and Keswick, the trade-off point is shifted towards a relatively high head with a moderate discharge.

Optimal release policies for average-flow year 1974–1975 essentially led to the same findings for 1979–1980, except that due to the slightly lower annual inflow, no spillage occurred and the annual system energy also decreased. For water year 1975–1976 with below-average streamflows, initial policies 1 and 2 were near-optimal policies. Because of tight feasible region conditions, the benefits from using the optimization model were marginal. Initial policies 1 and 2 yielded the same

optimal state and release policies. The gain in energy production (as obtained from the model) associated with both initial policies 1 and 2 was about 1%. For above-average flow year 1973–1974, with almost twice as much inflow as in 1974–1975 or 1979–1980, substantial spillage occurred. Also, the higher storage levels and greater releases that occurred in this year resulted in an increased total energy production. Figure 7 shows the relationship between total annual energy and total annual inflow, obtained from the values of the objective function computed for the water years under consideration. The energy versus volume of inflow curve, a fairly straight curve, is applicable to the range of inflow volumes depicted in Figure 7, for the given initial and final storages considered in the water years under study.

It was found from the computed results (e.g., Table 3) that the optimal state trajectories for the smaller reservoirs were to keep them full all year. That stems from the ratio of the capacities of the major reservoirs to their corresponding downstream regulating reservoirs. The largest capacity ratio of the system is 241/2448 = 10%, corresponding to Clair Engle and Whiskeytown reservoirs. When a capacity ratio becomes less than the largest capacity ratio of the system, all the state variables corresponding to downstream, smaller, regulating reservoirs can be treated as constant and equal to the maximum capacity of the regulating reservoirs. Those nodes in the

INFLOW TO RESERVOIRS   **TOTALS**

YEAR	NEW MELONES	SHASTA	FOLSOM	CLAIR ENGLE	WHISKEYTOWN	INFLOW	ENERGY
1973-74	1498	10 796	4408	2672	771	20146	11 14
1974-75	1206	6405	2786	1408	394	12198	7 35
1975-76	470	3611	1142	695	139	6057	4 53
1979-80	1734	6415	3972	1471	344	13936	8 01

INFLOWS IN 10³ ACRE FEET
ENERGY IN 10⁶ MWH
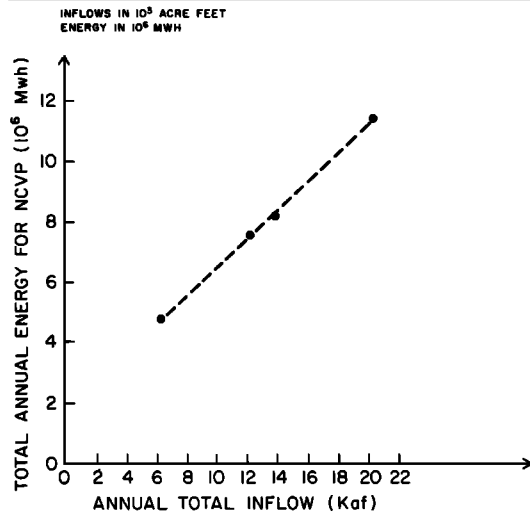
Fig. 7. Total annual energy versus total annual inflow for the NCVP.

network can thus be treated as transmission points only. The number of state variables in the NCVP system would be reduced from nine to four: Clair Engle, Shasta, Folsom, and New Melones. Care must be taken in reformulating the model in terms of the reduced number of state variables because the constraints that hold for the operation of the smaller reservoirs must still be satisfied. For example, if constraints representing penstock and spillage capacities are not observed, releases from Shasta reservoir could cause overtopping of Keswick reservoir. Mariño and Loaiciga [this issue] developed a model that sets the storages at the regulating reservoirs equal to the values computed in this study.

Table 5 summarizes the energy production levels obtained for water year 1979–1980 by using the optimization model and actual operation schedules. The ratio of actual energy (E_a) and maximized energy (E_m) varied from 29% at New Melones power plant to 72% at Shasta power plant. Those ratios

should be interpreted as an indication of the potential that exists to improve energy generation levels. For example, at New Melones, legal battles of environmental nature kept the reservoir from being filled completely and also the power plants were in complete halt during 3 months, which affected the actual power production adversely. The authors consider that the ψ_t factor (see equation (8)) as estimated by the NCVP management is probably somewhat optimistic, leading to overestimation of energy production by the optimization model. From the ψ_t factor estimate of 0.85 it can be estimated that for water year 1979–1980 there would have been a potential increase of up to 30% in power production. The actual realizable increase is definitely lower but not estimable with the available information.

Further insight into the differences between actual operation policies and those resulting from the optimization model can be gained from Figure 8, which shows actual and optimal state trajectories (for policies 1 and 2, 1979–1980) for Shasta reservoir. It is evident that substantially smaller storages are maintained from November to February in the optimal policies. That is accomplished by releasing large volumes of water through the penstocks, resulting in greater available flood control storages than in the actual operation. Thus the level of energy generation during November to February is higher with the optimal trajectory because the releases are routed through the penstocks at a larger magnitude relative to the actual operation. Also, when the high inflows of January to April occur, the actual operation follows the flow control regulations by spilling large volumes of water because the empty volume in the flood control pool is not as large as that attained with the optimal state trajectories. In March to June the optimal state trajectories maintain higher storage elevations than in the actual operation. That also results in increased energy production because energy is linearly dependent on storages, as is indicated in (8). The lower storages during March to June in the actual operation are due to water spillages that drive the reservoir level to lower stages. Those spillages reflect the conservativeness of the actual operation policy. Because they bypass the power plant, those spillages do not generate energy. In contrast, the reliance of the optimal trajectories on greater penstock outflows and smaller spillages reflects (1) the foreknowledge of future inflows (within a certain range of error) that arises from streamflow forecast and

TABLE 5. Actual (E_a) and Maximized (E_m) Energy Production Corresponding to Policy 2, 1979–1980

Month	Trinity Power Plant at Clair Engle		Judge Francis Carr Power Plant		Spring Creek Power Plant		Shasta Power Plant		Keswick Power Plant		Folsom Power Plant		Nimbus Power Plant at Lake Natoma		New Melones Power Plant	
	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.
Oct.	30.4	73.9	36.8	112.4	42.3	109.0	76.5	112.1	19.0	46.6	37.7	53.5	4.6	6.4	⋯	71.6
Nov.	9.6	71.2	4.8	112.4	19.5	109.5	89.6	126.2	20.7	50.3	41.3	77.9	4.7	10.0	⋯	70.4
Dec.	17.1	61.4	15.4	98.5	19.2	98.5	111.1	251.3	23.2	81.6	37.2	71.3	4.7	10.0	⋯	68.1
Jan.	6.2	29.7	⋯	41.9	23.9	77.2	237.4	266.9	47.6	81.6	107.0	118.4	8.7	10.0	19.6	81.9
Feb.	17.7	31.1	3.4	41.9	55.3	41.8	209.9	310.7	39.2	81.6	84.3	139.9	6.7	10.0	23.6	92.1
Mar.	73.4	32.2	78.2	41.9	99.5	42.8	228.3	323.7	49.9	81.6	130.0	147.2	10.6	10.0	47.8	90.9
Apr.	44.9	32.9	53.5	41.9	54.7	53.2	112.9	126.1	29.0	37.3	99.8	126.3	9.7	7.7	55.8	90.4
May	21.2	33.7	21.2	41.9	18.6	49.3	164.4	128.8	33.7	36.6	82.0	131.0	9.1	8.2	39.5	85.8
June	51.5	75.3	55.2	101.2	59.6	99.4	212.9	129.5	47.3	45.0	66.6	76.4	7.6	7.8	23.3	84.4
July	54.3	39.4	57.2	44.7	57.2	45.7	237.9	307.5	52.2	77.0	84.4	63.4	9.7	6.5	38.5	80.9
Aug.	75.7	37.6	87.2	42.9	85.9	43.5	165.7	310.2	43.6	81.6	35.6	93.5	4.3	10.0	22.1	74.9
Sept.	71.1	36.5	84.7	44.2	88.6	43.5	86.8	289.0	29.3	81.6	40.5	86.8	4.6	10.0	9.0	64.8
Total	473.1	817.5	497.6	765.9	624.3	813.7	1933.4	2682.2	434.7	782.8	846.4	1185.5	85.0	106.5	279.2	956.5
E_a/E_m	0.58		0.65		0.77		0.72		0.56		0.71		0.80		0.29	

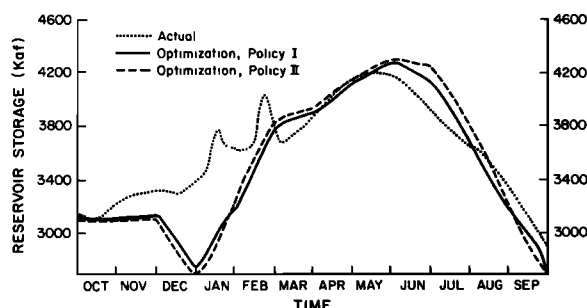Power plant was not in operation where no values are shown. Values are in megawatt-hour.

Fig. 8. Operation of Shasta reservoir, water year 1979–1980.

(2) the knowledge that for a given release, the higher the storage level the higher the energy generation rate. During July to September there is a steady drawdown of the reservoir storage level in the actual and optimal policies, reflecting increased demands for water and energy during the summer. Because the optimal state trajectories start at a higher elevation in July and end at a slightly lower level in September than the actual policy, the rate of water release during this period is higher for the optimal policies. That results in a greater generation of energy for the optimal policies in the summer as is evident in Table 5. Optimal policies 1 and 2 follow a similar pattern throughout the year and result in the same total energy production. The actual state trajectory shows high peaks in January and February that are due to short-term floods that raise the reservoir level for a few days. Those floods are partly spilled and do not contribute to energy generation at the reservoir power plant. Those short-term high flood events are not well captured within the monthly period framework, resulting in an overestimation of energy production of approximately 2% during such high-inflow months. These findings are also applicable to the other major reservoirs.

Benefits of the optimization model can also be measured in terms of increased water deliveries to downstream users. For example, the Delta requires a delivery of 3,850 KAF of water per year. Optimal release policies indicated a total annual release of 12,627 KAF (for 1979–1980), more than 3 times the required amount. For May to August, when most agricultural activities take place, additional water could be supplied for leaching and crop growing purposes. The Delta requirements for May to August are about 2,698 KAF. For the same period, optimal releases indicated that 4,813 KAF were delivered in 1979–1980. This suggests the possibility of a conjunctive use of surface water and groundwater reservoirs. Also, with increased deliveries, cultivated areas could be expanded or better leaching of salts might be achieved, resulting in an expanded economic output. Fish spawning, water quality, and navigation would also benefit from increased water deliveries.

CONCLUSIONS

Several conclusions can be drawn from this study.

1. It is possible to increase the annual energy production of the system for below-average, average, and above-average inflow conditions by using the proposed model relative to current operational criteria.

2. Delta and agricultural water deliveries can be increased by adopting the optimal release policies. This suggests the possibility of increasing irrigated areas, providing better leaching of agricultural fields, and improving conjunctive management of surface water and groundwater reservoirs.

3. Much of the improvement achieved by the optimal op-

eration policies developed in this study relative to the actual implemented operation schedules is due to (1) an accurate river inflow forecasting technique, (2) a highly conservative set of flood control provisions currently enforced in the operation of the NCVP, and (3) an integrative analysis, intrinsic in the optimization model, that allows to represent all the links and constraints that act simultaneously and interactively within the system. Clearly, this integral conceptualization cannot be achieved by a heuristic approach based solely on experience.

4. The improved performance reported by the use of the optimization model should be viewed as an upper bound to the possible gains that could be derived from the use of mathematical models. Also, the more knowledgeable the system managers become with reservoir optimization models, the closer the performance of the system will be to the upper bounds obtained under the conditions assumed by the models. Clearly, the use of mathematical models and the better understanding that emanates from their use should result in a feedback to the models, with their probable reformulation and modification that would bring closer the unpleasant difficulties of any real world system and the simplifications inherent to any mathematical model.

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