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T Self-Dual Transverse Space and Gravity Trapping

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Abstract

We advocate that the orbifold $\mathbb{Z}_2$ symmetry of the gravity trapping model proposed by Randall and Sundrum can be seen, in appropriate coordinates, as a symmetry that exchanges the short distances with the large ones. Using diffeomorphism invariance, we construct extensions defined by patch glued together. A singularity occurs at the junction and it is interpreted as a brane, the jump brane, of codimension one. We give explicit realization in ten and eleven dimensional supergravity and show that the lower dimensional Planck scale on the brane is finite. The standard model would be trapped on a supersymmetric brane located at the origin whereas the jump brane would surround it at a finite distance. The bulk interactions could transmit the supersymmetry breaking from the jump brane to the SM brane.

1 Introduction

Since the works of Kaluza and Klein [1], we know that, if there exists some extra-dimensions to our universe, an infinity of massive states will be associated to each usual 4D field. Because these KK modes have not yet been observed, necessarily their masses must be beyond the experimental range of energies resolved in accelerators ($\sim 1\,\mathrm{TeV}$). That is why the size of extra-dimensions cannot exceed such a ridiculously tiny scale ($\sim 1\,\mathrm{TeV}^{-1} \sim 10^{-19}\,\mathrm{m}$). However recent progresses in string theories [2] have corrected this old scenario suggesting that the Standard Model gauge interactions are confined to a four dimensional hypersurface while gravity can still propagate in the whole *bulk* space-time. Since the gravity has not yet been tested for energy beyond $10^{-4}\,\mathrm{eV}$ [3], the bounds on the size of extra-dimensions are now much lower ($\sim 1\,\mathrm{mm}$). This lack of experimental data allows for a modification of gravitational interactions at submillimetric distances *i.e.* far away from the Planck scale ($\sim 10^{19}\,\mathrm{GeV}$) where quantum gravity were usually thought to take place. This proposal, in a sense, nullifies[1] the gauge hierarchy problem [4]. However this analysis was not yet complete essentially because it assumes a particular factorizable geometry associated to the higher-dimensional space-time being a direct product of a 4D space-time with a compact space. Recently this last assumption has been overcome [5] unwarping a very rich potential of physical effects. The most exciting one reveals the non-incompatibility between non-compact extra-dimensions and experimental gravity [6]. The crucial point is the existence, in some curved background, of a normalizable bound state for the metric fluctuations which can be interpreted as the usual 4D graviton. Of course, there still exists an infinite tower of KK modes, even a continuum spectrum without gap, but the shape of their wave functions is such that they almost do not overlap with the 4D graviton and thus maintain the deviations to the Newton's law in limits which are still very far from experimental bounds. Subsequent to studies of thin shells in general relativity [7] and their revival in a low-energy M-theory context [8], a toy model was constructed by Randall and Sundrum that exhibits the previous properties (see [9] for previous related works). The question whether this scenario reproduces the usual 4D gravity beyond the Newton's law has been addressed in [10]. The cosmological aspects have been intensively studied [11]. This model brings new approach to tackle the more severe hierarchy encountered in physics, namely the cosmological constant problem [12, 13]. Whereas phenomenological aspects of *warped* compactifications with one compact extra-dimension have raised some interesting works [14], the case of one and many infinite extra-dimensions still waits for further investigations.

The initial model by Randall–Sundrum involves only one extra-dimension. Several subsequent works [15] extend it by considering many intersecting codimension one branes. Three papers [16] consider branes of codimension two. In a previous publication [13], we have proposed an effective action inspired from the brane construction in supergravity that leads to warped compactification with many extra-dimensions, however it fails to localize gravity. The present paper gives a generic method that leads to warped geometry trapping gravity:

[1] More precisely, this proposal translates the gauge hierarchy problem in energy into its Fourier dual, namely a hierarchy between the size of extra-dimensions and the electroweak scale.

the idea consists in taking a solution of the equations of motion in the bulk and using the diffeomorphism invariance in the transverse space to construct a new solution, defined by patch, gluing together two slices of the initial solution. By applying a transformation that exchanges the radial distance to the brane with its inverse, namely imposing a kind of T duality *i.e.* a symmetry between the short and the long distances, we can keep the region of the space-time that naturally confines gravity and we throw away the domain where the lower dimensional Planck mass diverges. The next section is devoted to enlighten our method while reproducing the RS scenario. In section 3, the Ramond–Ramond fields of low energy effective action of superstring theories are introduced in the bulk and our method is used to T dualize the usual p-brane solutions of supergravity.

2 Randall–Sundrum scenario as a T dualization of the transverse space

In this section we would like to present our method on a simple example which leads us to the Randall–Sundrum scenario of gravity trapping. We will be mainly interested here in the dynamics of the gravitational fields assuming that the dynamics of the other fields results in an effective cosmological constant in the *bulk* — the next section will be devoted to a more elaborated scenario taking into full account the massless modes of the low energy effective action of superstring theories. Thus the space-time physics is governed by the following action[2]:

$$\mathcal{S}^{\text{bulk}}_{\text{gravity}} = \int d^D x \sqrt{|g|} \left(\frac{\mathcal{R}}{2\kappa^2} - \Lambda_{bk} \right) , \tag{1}$$

It is well known for a long time that, when the cosmological constant is negative, $\Lambda_{bk} < 0$, the solution of Einstein equations corresponds to an Anti–de–Sitter space-time:

$$ds^2 = \left(\frac{r}{R_{AdS}} \right)^2 \eta_{\mu\nu}\, dx^\mu \otimes dx^\nu + \left(\frac{R_{AdS}}{r} \right)^2 dr \otimes dr \quad \mu = 0 \ldots D-1. \tag{2}$$

where the radius R_{AdS} is related to the bulk cosmological constant by:

$$R_{AdS}^{-2} = -\frac{2\kappa^2 \Lambda_{bk}}{(D-2)(D-1)} . \tag{3}$$

The aim of this section is to describe a method to construct new solutions to Einstein equations using a regular solution such as the previous one. These new solutions will develop, on hypersurfaces, some discontinuities in the first derivatives of the metric which will be interpreted as branes. In the vain of the works of extra-dimensions, we are looking for solutions that preserve a Poincaré invariance in some space-time directions hereafter called

[2]Our conventions correspond to a mostly plus Lorentzian signature $(- + \ldots +)$ and the definition of the curvature in terms of the metrics is such that an Euclidean sphere has a positive curvature.D is the space-time dimension.

longitudinal directions and that could be identified as the dimensions associated to our world; the remaining dimensions will be extra-dimensions transverse to us. Notice that the *AdS* solution already exhibits a Poincaré invariance in $D-1$ dimensions. The most general D dimensional metric that preserves a $Poincaré_{D-1}$ symmetry can be written as:

$$ds^2 = A^2(r)\,\eta_{\mu\nu}\,dx^\mu \otimes dx^\nu + B^2(r)\,dr \otimes dr\ . \tag{4}$$

In terms of the two functions A and B, the Einstein equations read:

$$(D-2)\,\frac{A''}{A} + \frac{(D-2)(D-3)}{2}\left(\frac{A'}{A}\right)^2 - (D-2)\,\frac{A'}{A}\frac{B'}{B} = -\kappa^2\Lambda_{bk}B^2\ ; \tag{5}$$

$$\frac{(D-2)(D-1)}{2}\left(\frac{A'}{A}\right)^2 = -\kappa^2\Lambda_{bk}\,B^2\ , \tag{6}$$

where primes denote derivatives with respect to the transverse coordinate r. The *AdS* solutions simply corresponds to $A_{AdS}(r) = r/R_{AdS}$ and $B_{AdS}(r) = R_{AdS}/r$.

The key observation is that, even after requiring a Poincaré invariance, the equations of motion still possess a reparametrization invariance in the transverse space. In our one extra-dimension example, this invariance is associated to diffeomorphism in the coordinate r and insures that, if $A_\circ(r)$ and $B_\circ(r)$ are a solution of the Einstein equations (5)-(6), thus $\tilde{A}(\tilde{r}) = A_\circ(f(\tilde{r}))$ and $\tilde{B}(\tilde{r}) = \pm B_\circ(f(\tilde{r}))f'(\tilde{r})$ are also a solution, as it can be explicitly checked, for any diffeomorphism f whose image falls in the support of the original $A_\circ$ and $B_\circ$. Of course these two solutions correspond to the same physical space if they are used for covering the whole space-time. However they can be used separately in order to construct new solution defined by patch on two non-overlapping regions: this construction mimics the procedure used by Randall and Sundrum and consists in taking two identical slices of space-time and gluing them together. Explicitly, the solution can be defined, starting from any solution $A_\circ(r)$ and $B_\circ(r)$, as:

$$\text{for}\quad r \le r_\circ \qquad A(r) = A_\circ(r)\ \ B(r) = B_\circ(r) \tag{7}$$

$$\text{for}\quad r \ge r_\circ \qquad A(r) = A_\circ(r_\circ f(r)/f(r_\circ))\ \ B(r) = \pm B_\circ(r_\circ f(r)/f(r_\circ))\frac{r_\circ f'(r)}{f(r_\circ)} \tag{8}$$

The requirement that the metric remains continuous at $r_\circ$ gives a constraint between $r_\circ$ and the function f, namely:

$$\frac{r_\circ f'(r_\circ)}{f(r_\circ)} = \pm 1\ . \tag{9}$$

In the present case where r represents a radial distance that remains positive, there is a particular change of coordinate that fulfills the constraint (9) whatever the value of $r_\circ$ is: $f(r) = r_\circ^2/r$. The significance of this change of coordinate is very clear: the original solution near infinity is cut and replaced by a copy of the origin and the long distance solution becomes just a mirror of the short distance region. In that sense the new solution is *T self-dual*. Notice that the $r \leftrightarrow r_\circ^2/r$ symmetry is just a $\mathbb{Z}_2$ symmetry in the Randall–Sundrum

coordinate, $y = -R\ln(r/R_{AdS})$, when $r_\circ = R_{AdS}$. In the RS coordinate, the full AdS metric (2) reads:

$$ds^2 = e^{-2y/R_{AdS}}\eta_{\mu\nu}\,dx^\mu \otimes dx^\nu + dy \otimes dy. \tag{10}$$

Randall and Sundrum have looked for a $\mathbb{Z}_2$ symmetric configuration and have obtained:

$$ds^2 = e^{-2|y|/R_{AdS}}\eta_{\mu\nu}\,dx^\mu \otimes dx^\nu + dy \otimes dy. \tag{11}$$

when a brane with a positive and fine-tuned cosmological constant is placed at $y = 0$ *i.e.* $r = R_{AdS}$. This $\mathbb{Z}_2$ symmetrization is nothing but the procedure of T dualization of the transverse space described above: the region of negative y, that would correspond to $r \geq R_{AdS}$, is cut and replaced by a copy of the region of positive y *i.e.* $r \leq R_{AdS}$. This procedure of cutting and pasting is not specific to the T dualization of transverse space, for instance it has been used in the first reference in [15] to generalize the RS construction with cosmological constants on a setup of intersecting codimension one branes. The notion of T transformation will be important for trapping gravity on higher codimension branes like those appearing in supergravity theories.

The diffeomorphism invariance insures that (7)-(8) is a solution of the Einstein equations in the bulk. However, even if the metric is continuous at $r = r_\circ$, its first derivatives are usually not and thus a Dirac singularity appears in the left hand side of (5) which has to be associated to a singular stress-energy tensor. In our example it is not difficult to see that this singular stress-energy tensor can be derived from a term interpreted as a cosmological constant on the hypersurface $r = r_\circ$:

$$\mathcal{S}_{eff}^{\text{brane}} = -\int d^D x\,\sqrt{|g|}\,\Lambda_{br}\,\delta\left(\sqrt{|g_{rr}|}(r - r_\circ)\right) \tag{12}$$

where Λ_{br} is given by:

$$\Lambda_{br} = \sqrt{-\frac{8(D-2)}{(D-1)\kappa^2}\Lambda_{bk}}\,. \tag{13}$$

The above procedure of T dualization has the nice property to lead to a finite $D-1$ dimensional Planck mass. Indeed, whereas in the original solution it would be:

$${M_P}^{D-3} = \frac{1}{\kappa^2}\int_0^\infty dr\,A^{D-3}(r)\,B(r) = \int_0^\infty dr\,\left(\frac{r}{R_{AdS}}\right)^{D-4} \tag{14}$$

which diverges near infinity, our solution that throws away the region near infinity and paste a copy of the region near the origin, naturally gives a finite $D-1$ dimensional Planck mass[3]:

$${M_P}^{D-3} = \frac{1}{\kappa^2}\left(\int_0^{r_\circ} dr\,\left(\frac{r}{R_{AdS}}\right)^{D-4} + \int_{r_\circ}^\infty dr\,\frac{r_\circ^{2D-6}}{r^{D-2}R_{AdS}^{D-4}}\right) = \frac{2}{(D-3)\kappa^2}\,r_\circ\left(\frac{r_\circ}{R_{AdS}}\right)^{D-4} \tag{15}$$

[3]Notice that we could have cut the horizon of AdS and kept two copies of the infinite boundary but this geometry would not lead to a finite lower dimensional Planck scale.

At this stage, the position $r_\circ$ of the fixed point under the T symmetry is arbitrary. It is believed that a dynamical description of the brane beyond its effective description in terms of a cosmological constant (12) should allow to stabilize the value of $r_\circ$. Notice that, when $r_\circ = R_{AdS}$, the expression (15) coincides with the Planck mass on the brane computed in the RS model.

In the next section, we extend our procedure of T dualization to solutions of the equations of motion of supergravity.

3 Gravity trapping from the branes of supergravity

In this section, we apply our procedure of T dualization to the brane solutions of supergravity theories. An electric p-brane couples to a $(p+1)$ differential form. While preserving a Poincaré invariance in the dimensions parallel to the brane, the electric field strength curves the transverse space. Nice solutions of the equations of motion have been constructed [17]. Their remarkable supersymmetric and BPS properties insure their stability. They are interpreted as collective excitations of perturbative string theories and they become the elementary objects of dual theories capturing part of the non-perturbative aspects. In [13], it was argued that the brane configurations can be seen as warped geometry of space-time. Unfortunately, the shape of the warp factor along the infinite extra-dimension associated to the radial distance to the brane in the transverse space does not localize gravity as in the RS scenario. The origin of such a discrepancy is due to the geometry of space-time far away from the brane. For example in the particular case of vanishing dilaton coupling, the geometry corresponds[4] to a product $AdS_{d_\parallel+1} \times S^{d_\perp-1}$, where $d_\parallel$ is the dimension of the longitudinal space and $d_\perp$, the dimension of the transverse space, $d_\parallel + d_\perp = D$. Whereas the region near the brane is the horizon of AdS, the region near infinity is associated to the conformal boundary of AdS which is precisely the part of space-time cut in the RS configuration. Our solution will consist in T dualizing the AdS horizon in the region near infinity. A new singularity will appear where the two slices are glued and we will describe this singularity.

We begin with a review of the brane construction in supergravity theories. A p-brane is coupled to the low-energy effective theory of superstrings. Below the fundamental energy scale, identified as the energy of the first massive excitations of the string, the theory can be described by supergravity theories whose bosonic spectrum contains the metric, a scalar field (the dilaton) and numerous differential forms. The bosonic effective action, in supergravity units where the curvature term is canonically normalized, takes the general form (we will

[4]Usually the solution constructed in supergravity theories asymptotes $AdS_{d_\parallel+1} \times S^{d_\perp-1}$ near the brane only and it is normalized such as to recover a D dimensional Minkowskian flat space at infinity. As we will see in eq. (28), this normalization corresponds to a particular choice of constant of integration. $AdS_{d_\parallel+1} \times S^{d_\perp-1}$ provides also a solution in the full space. The physical relevance of this solution is suggested by the fact that the dynamics of a brane becomes free near the conformal boundary of AdS [18]. Our argument concerning the gravity localization is unchanged if the solution with a flat space at infinity is considered since the $d_\parallel$ Planck mass also diverges in that case.

use $\kappa^2 = M^{2-D}$ where M is the Planck mass in ten or eleven dimensions):

$$\mathcal{S}_{eff}^{\text{sugra}} = \int d^D x \sqrt{|g|} \left(\frac{1}{2\kappa^2} \mathcal{R} - \frac{1}{2} \partial_{\hat{\mu}} \Phi \partial^{\hat{\mu}} \Phi - \sum_n \frac{1}{(n+2)!} e^{\alpha_n \Phi} F_{\hat{\sigma}_1 \dots \hat{\sigma}_{n+2}} F^{\hat{\sigma}_1 \dots \hat{\sigma}_{n+2}} \right) , \quad (16)$$

where $F_{\hat{\mu}_1 \dots \hat{\mu}_{n+2}} = (n+2)\, \partial_{[\hat{\mu}_1} C_{\hat{\mu}_2 \dots \hat{\mu}_{n+2}]}$ is the field strength of the $(n+1)$-differential form C, whose coupling to the dilaton is measured by the coefficient α_n. The allowed values of n depends on the theory we consider. The coefficients α_n are explicitly determined by a string computation: the coupling of the dilaton to differential forms from the Ramond-Ramond sector appears at one loop and thus $\alpha_n^{RR}/\sqrt{2\kappa^2} = (3-n)/2$ in supergravity units, while the Neveu-Schwarz–Neveu-Schwarz two-form couples at tree level, so $\alpha_1^{NS}/\sqrt{2\kappa^2} = -1$. In some cases, we can also add a Chern–Simons term ($C \wedge F \wedge F$) to the action, but it does not have any effect on the classical solutions and we will neglect it in our analysis.

This *bulk* effective action can couple to some *branes*. And the total action is:

$$\mathcal{S} = \mathcal{S}_{eff}^{\text{sugra}} + \mathcal{S}_{eff}^{\text{branes}} \quad (17)$$

The equations of motion are derived form this action and can be read ($\hat{\mu}, \hat{\nu} = 0 \dots D-1$):

$$G_{\hat{\mu}\hat{\nu}} = \kappa^2\, \partial_{\hat{\mu}} \Phi \partial_{\hat{\nu}} \Phi + \sum_n \frac{2\kappa^2}{(n+1)!} e^{\alpha_n \Phi} F_{\hat{\mu}\hat{\sigma}_1 \dots \hat{\sigma}_{n+1}} F_{\hat{\nu}}{}^{\hat{\sigma}_1 \dots \hat{\sigma}_{n+1}}$$
$$+ \frac{1}{2} \left(-\kappa^2\, \partial_{\hat{\sigma}} \Phi \partial^{\hat{\sigma}} \Phi - \sum_n \frac{2\kappa^2}{(n+2)!} e^{\alpha_n \Phi} F_{\hat{\sigma}_1 \dots \hat{\sigma}_{n+2}} F^{\hat{\sigma}_1 \dots \hat{\sigma}_{n+2}} \right) g_{\hat{\mu}\hat{\nu}} + T^{br}_{\hat{\mu}\hat{\nu}} \; ; \quad (18)$$

$$D_{\hat{\mu}} D^{\hat{\mu}} \Phi = \sum_n \frac{\alpha_n}{(n+2)!} e^{\alpha_n \Phi} F_{\hat{\sigma}_1 \dots \hat{\sigma}_{n+2}} F^{\hat{\sigma}_1 \dots \hat{\sigma}_{n+2}} + T^{br}_{\Phi} \; ; \quad (19)$$

$$\partial_{\hat{\mu}_0} \left(\sqrt{|g|}\, e^{\alpha_n \Phi}\, F^{\hat{\mu}_0 \dots \hat{\mu}_{n+1}} \right) = J^{\hat{\mu}_1 \dots \hat{\mu}_{n+1}}_{br} \; . \quad (20)$$

The *brane* stress-energy tensor $T^{br}_{\hat{\mu}\hat{\nu}}$, the electric currents J_{br} and the dilatonic current T^{br}_{Φ} are formally given by:

$$T^{br}_{\hat{\mu}\hat{\nu}} = -\frac{2\kappa^2}{\sqrt{|g|}} \frac{\delta \mathcal{S}^{\text{branes}}_{eff}}{\delta g^{\hat{\mu}\hat{\nu}}} \; ; \quad J^{\hat{\mu}_1 \dots \hat{\mu}_{n+1}}_{br} = -\frac{(n+1)!}{2} \frac{\delta \mathcal{S}^{\text{branes}}_{eff}}{\delta A_{\hat{\mu}_1 \dots \hat{\mu}_{n+1}}} \; ; \quad T^{br}_{\Phi} = -\frac{\delta \mathcal{S}^{\text{branes}}_{eff}}{\delta \Phi} \; , \quad (21)$$

and can be derived whenever the effective action describing the dynamics of the branes is known.

We would like now to construct a solution of these equations of motion with particular symmetries namely a Poincaré invariance in $d_{\parallel} = p+1$ dimensions that will be identified as longitudinal dimensions and also a rotational invariance in the $d_{\perp}$ dimensional transverse space. Such a solution has been known for a long time in supergravity theories and it is expressed as ($\mu, \nu = 0 \dots d_{\parallel} - 1$ and $i, j = 1 \dots d_{\perp}$):

$$ds^2 = H^{2n_x}\, \eta_{\mu\nu}\, dx^{\mu} \otimes dx^{\nu} + H^{2n_y}\, \eta_{ij}\, dy^i \otimes dy^j \; ; \quad (22)$$

$$e^{\Phi} = H^{n_\Phi}\, e^{\phi_\circ} \quad (\phi_\circ \text{ is a constant}) \; ; \quad (23)$$

$$C_{\mu_1 \dots \mu_{p+1}} = -\epsilon_{\mu_1 \dots \mu_{p+1}} \frac{1}{\mathcal{A}_{WZ}}\, e^{-\alpha_p \phi_\infty / 2}\, H^{-1} \; ; \quad (24)$$

where H is a function of the radial distance in the transverse space only. Notice that the $Poincaré_{d_\parallel}$ symmetry allows only for a non-vanishing $d_\parallel$ differential form and furthermore all the fermionic fields have to vanish. The consistency of the whole set of equations of motion determines the powers n_x, n_y and n_Φ:

$$n_x = -\frac{2(d_\perp - 2)\kappa^2}{(d_\parallel + d_\perp - 2)\,\mathcal{A}_{WZ}^2} \qquad n_y = \frac{2d_\parallel \kappa^2}{(d_\parallel + d_\perp - 2)\,\mathcal{A}_{WZ}^2} \qquad n_\Phi = \frac{\alpha_p}{\mathcal{A}_{WZ}^2}\,, \tag{25}$$

and the coefficient $\mathcal{A}_{WZ}$ which has to be related to the dilaton coupling by:

$$\mathcal{A}_{WZ}^2 = 2\kappa^2 \frac{d_\parallel (d_\perp - 2)}{d_\parallel + d_\perp - 2} + \frac{\alpha_p^2}{2}\,. \tag{26}$$

In supergravity theories, according to the particular values of the dilaton coupling previously given, the coefficient $\mathcal{A}_{WZ}$ is a constant independent of the dimension of the brane:

$$\mathcal{A}_{WZ}^2 = 4\kappa^2\,. \tag{27}$$

The function H is harmonic in the transverse space:

$$H = l + \frac{Q}{r^{d_\perp - 2}} \tag{28}$$

where l is an arbitrary dimensionless constant and Q is a constant with a dimension $d_\perp - 2$ in length.

This solution solves the bulk equations of motion everywhere except at the origin where occurs a singularity, interpreted as a *p-brane*. We know exactly the brane action generating such a singularity:

$$\mathcal{S}_{eff}^{\text{brane}} = T_{p+1} \int d^{p+1}\xi \left(-\frac{1}{2}\sqrt{|\gamma|}\gamma^{ab}\partial_a X^{\hat{\mu}} \partial_b X^{\hat{\nu}} g_{\hat{\mu}\hat{\nu}}(X)\, e^{\beta_p \Phi} + \frac{p-1}{2}\sqrt{|\gamma|} \right.$$
$$\left. + \frac{\mathcal{A}_{WZ}}{(p+1)!}\, \epsilon^{a_1 \ldots a_{p+1}}\, \partial_{a_1} X^{\hat{\mu}_1} \ldots \partial_{a_{p+1}} X^{\hat{\mu}_{p+1}} C_{\hat{\mu}_1 \ldots \hat{\mu}_{p+1}} \right)\,. \tag{29}$$

And the corresponding constant Q in the expression (28) of the harmonic function H is related to the tension T_{p+1} of the brane by:

$$Q = \frac{\mathcal{A}_{WZ}^2 T_{p+1}}{2(d_\perp - 2)\Omega_{d_\perp - 1}}\, e^{-\alpha_p \phi_\circ / 2} \tag{30}$$

where $\Omega_{d_\perp - 1}$ is the volume of $S^{d_\perp - 1}$, the sphere with $d_\perp - 1$ angles.

Using this solution, we will now construct a new solution by patch. The most general solution that respects $Poincaré_{d_\parallel} \times SO(d_\perp)$ can be written as:

$$ds^2 = A^2(r)\, \eta_{\mu\nu}\, dx^\mu \otimes dx^\nu + B^2(r)\, dr \otimes dr + D^2(r) d^2\Omega_{d_\perp - 1}\,; \tag{31}$$
$$\Phi = \Phi(r); \tag{32}$$
$$C_{\mu_1 \ldots \mu_{p+1}} = -\epsilon_{\mu_1 \ldots \mu_{p+1}} \mathcal{A}_{WZ}^{-1}\, C(r)\,; \tag{33}$$

where $d^2\Omega_{d_\perp-1} = \tilde{g}_{\alpha\beta} d\theta^\alpha \otimes d\theta^\beta$ is the metric on the $S^{d_\perp-1}$ described by the angles θ^α, $\alpha = 1 \ldots d_\perp - 1$. In terms of these functions, the equations of motion (18)-(20) in the bulk, *i.e.* dropping any singularity, read:

$$\begin{aligned}
&\tfrac{(d_\parallel-1)(d_\parallel-2)}{2}\left(\frac{A'}{A}\right)^2 + (d_\parallel-1)\frac{A''}{A} - (d_\parallel-1)\frac{A'}{A}\frac{B'}{B} + (d_\parallel-1)(d_\perp-1)\frac{A'}{A}\frac{D'}{D} \\
&\quad \tfrac{(d_\perp-1)(d_\perp-2)}{2}\left(\frac{D'}{D}\right)^2 + (d_\perp-1)\frac{D''}{D} - (d_\perp-1)\frac{B'}{B}\frac{D'}{D} - \tfrac{(d_\perp-1)(d_\perp-2)}{2}\frac{B^2}{D^2} \\
&\quad = -\kappa^2(\Phi')^2 - \kappa^2 \mathcal{A}_{WZ}^{-2} A^{-2d_\parallel} (C')^2 \qquad (34) \\
&\tfrac{d_\parallel(d_\parallel-1)}{2}\left(\frac{A'}{A}\right)^2 - d_\parallel(d_\perp-1)\frac{A'}{A}\frac{B'}{B} - \tfrac{(d_\perp-1)(d_\perp-2)}{2}\left(\frac{D'}{D}\right)^2 - \tfrac{(d_\perp-1)(d_\perp-2)}{2}\frac{B^2}{D^2} \\
&\quad = \kappa^2(\Phi')^2 - \kappa^2 \mathcal{A}_{WZ}^{-2} A^{-2d_\parallel} (C')^2 \qquad (35) \\
&\tfrac{d_\parallel(d_\parallel-1)}{2}\left(\frac{A'}{A}\right)^2 + d_\parallel\frac{A''}{A} - d_\parallel\frac{A'}{A}\frac{B'}{B} + d_\parallel(d_\perp-2)\frac{A'}{A}\frac{D'}{D} \\
&\quad \tfrac{(d_\perp-2)(d_\perp-3)}{2}\left(\frac{D'}{D}\right)^2 + (d_\perp-2)\frac{D''}{D} - (d_\perp-2)\frac{B'}{B}\frac{D'}{D} - \tfrac{(d_\perp-2)(d_\perp-3)}{2}\frac{B^2}{D^2} \\
&\quad = -\kappa^2(\Phi')^2 + \kappa^2 \mathcal{A}_{WZ}^{-2} A^{-2d_\parallel} (C')^2 \qquad (36) \\
&\left(A^{d_\parallel} B^{-1} D^{d_\perp-1} \Phi'\right)' = -\alpha_p \mathcal{A}_{WZ}^{-2} A^{d_\parallel} B^{-1} D^{d_\perp-1} (C')^2 \qquad (37) \\
&\left(A^{-d_\parallel} B^{-1} D^{d_\perp-1} C'\right)' = 0 \qquad (38)
\end{aligned}$$

where the primes denote derivative with respect to the radial distance r.

The brane solution of supergravity corresponds to $A_{sg} = H^{n_x}$, $B_{sg} = H^{n_y}$, $D_{sg} = r\, H^{n_y}$, $e^{\Phi_{sg}} = H^{n_\Phi}\, e^{\phi_\circ}$ and $C_{sg} = H^{-1}\, e^{\phi_\circ/2}$ where $\phi_\circ$ is an arbitrary constant and H is given in (28). The diffeomorphism invariance insures that

$$\begin{aligned}
\text{for} \quad r \le r_\circ \qquad & A(r) = H^{n_x}(r) \;\; B(r) = H^{n_y}(r) \;\; D(r) = r\, H^{n_y}(r) \\
& \Phi(r) = \phi_\circ + n_\Phi \ln H(r) \;\; C(r) = e^{\phi_\circ/2}\, H^{-1}(r) \qquad (39) \\
\text{for} \quad r \ge r_\circ \qquad & A(r) = H^{n_x}(r_\circ^2/r) \;\; B(r) = r_\circ^2/r^2 H^{n_y}(r_\circ^2/r) \;\; D(r) = r_\circ^2/r\, H^{n_y}(r_\circ^2/r) \\
& \Phi(r) = \phi_\circ + n_\Phi \ln H(r_\circ^2/r) \;\; C(r) = e^{\phi_\circ/2}\, H^{-1}(r_\circ^2/r) \qquad (40)
\end{aligned}$$

is also a solution of the equations of motion in the bulk, as it can be checked explicitly. Whereas this T dualization of the transverse space provides a continuous junction between the two patches, the first derivatives of the fields have a jump and thus lead to a Dirac singularity. We can interpret this singularity as a brane of codimension one located at the junction region: we will call it the *jump brane*[5]. Among the dimensions on this brane, $d_\parallel$ ones are non-compact and are parallel to the dimensions of the brane at the origin, while the $d_\perp - 1$ remaining directions are compact and describe a sphere of radius $D(r_\circ)$ and thus at energies below $D^{-1}(r_\circ)$, the second brane will also appear as a $(d_\parallel - 1)$-brane. From the

[5]This geometry reminds some aspects of the model of concentric branes constructed in the third reference in [16] with a discontinuous cosmological constant in a 6D bulk.

explicit expression of the equations of motion, we can derive the singularity associated to the second brane in terms of the original supergravity solution:

$$T^{br}_{\mu\nu} = -2\left((d_\parallel - 1)\frac{A'_{sg}}{A_{sg}} + (d_\perp - 1)\frac{D'_{sg}}{D_{sg}}\right)\frac{A^2_{sg}}{B^2_{sg}}\eta_{\mu\nu}\,\delta(r - r_\circ) \tag{41}$$

$$T^{br}_{\alpha\beta} = -2\left(d_\parallel\frac{A'_{sg}}{A_{sg}} + (d_\perp - 2)\frac{D'_{sg}}{D_{sg}}\right)\frac{D^2_{sg}}{B^2_{sg}}\tilde{g}_{\alpha\beta}\,\delta(r - r_\circ) \tag{42}$$

$$T^{br}_{\Phi} = -2\Phi'_{sg}B^{-2}_{sg}\,\delta(r - r_\circ) \tag{43}$$

$$J^{\mu_1\cdots\mu_{p+1}}_{br} = 2\epsilon^{\mu_1\cdots\mu_{p+1}}A^{-1}_{WZ}A^{-d_\parallel}_{sg}B^{-1}_{sg}D^{d_\perp-1}_{sg}C'_{sg}\,\delta(r - r_\circ) \tag{44}$$

where we remind that $\tilde{g}_{\alpha\beta}$ has been defined as the metric on $S^{d_\perp-1}$. An interesting and opening problem that we will not address in this letter is to determine the effective action $\mathcal{S}^{\text{brane}}_{eff}$ describing the second brane that would lead to the currents (41)-(44). In the particular case where the integration constant, l, of the supergravity solution (28) has been chosen to vanish, the stress-energy tensor on the brane can be parametrized by two cosmological constants along the compact and non-compact directions:

$$T^{br}_{\mu\nu} = -\kappa^2\Lambda^{br}_\parallel\, g_{\mu\nu}\,\delta(\sqrt{g_{rr}}(r - r_\circ)) \quad T^{br}_{\alpha\beta} = -\kappa^2\Lambda^{br}_\perp\, g_{\mu\nu}\,\delta(\sqrt{g_{rr}}(r - r_\circ)) \tag{45}$$

where

$$\Lambda^{br}_\parallel \propto \left(d_\perp - 1 - 2(d_\perp - 2)\mathcal{A}^{-2}_{WZ}\,\kappa^2\right) r_\circ^{-\alpha_p^2\mathcal{A}^{-2}_{WZ}/2} \tag{46}$$

$$\Lambda^{br}_\perp \propto (d_\perp - 2) r_\circ^{-\alpha_p^2\mathcal{A}^{-2}_{WZ}/2} \tag{47}$$

Notice that the cosmological constants in the two directions are equal only when the Wess–Zumino coupling is given by:

$$\mathcal{A}^2_{WZ} = 2(d_\perp - 2)\kappa^2 \quad i.e. \quad \alpha_p^2 = 4\kappa^2\frac{(d_\perp - 2)^2}{D - 2}\,. \tag{48}$$

This is never the case in supergravity.

The most attractive feature of the solution we have just constructed is that, just as in the RS model, it provides a finite $d_\parallel$ dimensional Planck mass in spite of the infinite extra-dimension. Indeed, this scale is now given by:

$$M_P{}^{d_\parallel-2} = \kappa^{-2}\int d^{d_\perp}y\, A^{d_\parallel-2}BD^{d_\perp-1} = 2\kappa^{-2}\Omega_{d_\perp-1}\int_0^{r_\circ} dr\, r^{d_\perp-1}H^{4\kappa^2\mathcal{A}^{-2}_{WZ}} = \kappa^{-2}\Omega_{d_\perp-1}Qr_\circ^2 \tag{49}$$

In the last equality, we have used the supersymmetric value of $\mathcal{A}_{WZ}$ and set the constant of integration, l, to zero. The fact that M_P converges is a good indication that the gravitational force will mainly follow a Newton's law in $d_\parallel$ dimensions, up to deviations beyond experimental bounds, and suggests the existence of a normalizable bound state for the metric fluctuations that will be interpreted as a 4D graviton [19, 20]. The deviations to the

Newton's law can be obtained form the KK spectrum of the 4D graviton. The equations of motion for the fluctuations will be greatly simplified by noticing that, in the RS gauge, the stress-energy tensor in the bulk derived from (16) satisfies, at the first order in perturbation:

$$T^{(1)}_{\mu\nu} = \left(T^{(0)}_{\mu\sigma} h_{\rho\nu} + T^{(0)}_{\sigma\nu} h_{\rho\mu}\right) \eta^{\rho\sigma} \tag{50}$$

where

$$ds^2 = A^2(r)(\eta_{\mu\nu} + h_{\mu\nu}(x, r, \theta))dx^\mu \otimes dx^\nu + B^2(r)\, dr \otimes dr + D^2(r) d^2\Omega_{d_\perp - 1} \tag{51}$$

As noticed in [19], this relation, that is here rather non-trivial since the stress-energy tensor is non-linear in the metric, is what is needed to cancel all the non-derivative terms in h in the Einstein equations. However, without knowing the effective action for the second brane, it is impossible to derive the full equations of motion for the fluctuations near the second brane. We leave this question for further investigations.

Since the supersymmetric extension of the RS model has been debated with rather confusion [13, 21], it is important to comment about this issue regarding the solutions we have constructed. Concerning the brane located at the fixed point of the T symmetry, we are missing some elements to make any statement. However the conclusions are positive for the bulk and the p-brane located at the origin: since locally they correspond to the usual solutions encountered in supergravity theories, they preserve eight supercharges. If the second brane breaks part of these supercharges, it would be interesting to study the transmission of this breaking to the first brane.

In conclusion, we hope that our construction has shed light on the geometrical origin of the gravity trapping scenario proposed by Randall and Sundrum. It provides insight on how to extend it to higher-codimensional brane worlds. We have studied an explicit realization in supergravity models exhibiting finite lower dimensional Planck mass on the brane despite the non-compact transverse space. Our solution is invariant under a T symmetry that exchanges the short distances with the large ones in the transverse space. Two singularities occur that are interpreted as a p-brane at the origin and, at a finite distance, the jump brane, a $(D-2)$-brane with $d_\perp - 1$ compact dimensions. The bulk, as well as the brane at the origin where the standard model can propagate, preserves half of the sixteen supercharges. The warp factor is maximum on the jump brane that can be taken as a *Planck brane*, *i.e.* a brane where the energy scale would be of the order of the Planck scale ($\sim 10^{19}$ GeV). In this case, the natural energy scale on the brane at the origin would be suppressed by a factor $A(r_\circ)/A(0)$ which can lead to the electroweak scale depending on the location of the T self-dual point $r_\circ$. A dynamical description of the jump brane should help to address some interesting questions like the stabilization of its position, the supersymmetry breaking transmission to the first brane and the determination of the KK spectrum associated to the 4D graviton which would allow to compute the deviations to the Newton's law.

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