Title
Essays on Simultaneous Innovation and Macroeconomics

Author
Gabrovski, Miroslav Nikolov

Publication Date
2018

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA
RIVERSIDE

Essays on Simultaneous Innovation and Macroeconomics

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Miroslav Nikolov Gabrovski

June 2018

Dissertation Committee:

Professor Jang-Ting Guo, Co-Chairperson
Professor Victor Ortego-Marti, Co-Chairperson
Professor Guillaume Rocheteau
Professor Hiroki Nishimura
Professor David Malueg

The Dissertation of Miroslav Nikolov Gabrovski is approved:

Committee Co-Chairperson

Committee Co-Chairperson

University of California, Riverside

Acknowledgments

I am immensely grateful to my advisors, Dr. Jang-Ting Guo and Dr. Victor Ortego-Marti, for their patience and their continued support throughout the last five years. I am especially thankful for all the time and effort they have put into helping me develop as a researcher; for all their help with my writing, communication, and presentation skills; for all their comments and suggestions on my research. I would not have been able to achieve what I have without their guidance, mentorship, and encouragement.

I am deeply grateful to my dissertation committee members for their helpful comments and guidance on my research. Their help has been invaluable. In particular, I am thankful to:

Dr. Guillaume Rocheteau, especially for his insightful suggestions and comments during our many meetings and during the many presentations of mine he attended; for his invaluable help with my development as a researcher.

Dr. Hiroki Nishimura, especially for his instructions on economic theory.

Dr. David Malueg, especially for his instructions on game theory; for his helpful comments and suggestions on my research; for his continued involvement as part of my dissertation committee even after his retirement.

I am thankful to my classmates and friends at the Department of Economics at UC Riverside and at the Department of Economics at UC Irvine for their help and support during the last five years. Graduate school has been a very happy and memorable experience because of them. I would also like to express gratitude towards Ms. Damaris Carols, Mr. Gary Kuzas, and Ms. Tanya Wine for their exceptional administrative support.

I am immensely grateful to all my family and friends. Their continued support and encouragement made it possible for me to be where I am today.

Lastly, I would like to thank my parents Nikolai and Marina. For their love, support, and encouragement. It would have been impossible to obtain a doctoral degree without their endless sacrifices.

To my parents for all the support.

ABSTRACT OF THE DISSERTATION

Essays on Simultaneous Innovation and Macroeconomics

by

Miroslav Nikolov Gabrovski

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, June 2018
Professor Jang-Ting Guo, Co-Chairperson
Professor Victor Ortego-Marti, Co-Chairperson

This dissertation consists of three essays which explore the impact of simultaneous innovation on growth, macroeconomic fluctuations, and welfare, as well as their implications for the patent system. Chapter one provides an overview. Chapter two analyses the impact of the coordination frictions implicit in the presence of simultaneous innovation on growth and welfare. The coordination failure generates a mass of foregone innovation and reduces the economy-wide research intensity. Both effects decrease the growth rate. Because of this, the frictions also amplify the fraction of wasteful simultaneous innovation. A calibration suggests the impact of coordination frictions on the growth rate and welfare is substantial.

Chapter three studies the impact of simultaneous innovation on macroeconomic fluctuations in a stochastic expanding-variety endogenous growth model. Research projects innovated by many firms simultaneously are of higher quality, on average, and contribute relatively more to the expansion of the knowledge stock in the economy. This delivers an endogenous mechanism that amplifies the volatility of R&D investment by a factor of two. Furthermore, due to this endogenous mechanism, the model produces mildly pro-cyclical R&D — a well-documented feature of the data. Whereas the existing literature has proposed several mechanisms that explain the positive correlation between investment in R&D and output, the moderate strength of the relationship has remained under-explored.

The conventional viewpoint on the patent system is that it allocates market power in order to stimulate disclosure of information and create incentives for firms to innovate. Chapter four develops a dynamic equilibrium search model to show that, in sharp contrast to this traditional view, the patent system can erode, rather than allocate market power. This result is obtained, regardless of whether or not it provides prior user rights, by incentivizing firms to patent and, at the same time, delivering a sufficiently weak patent protection. The patent system delivers incentives by either punishing firms that choose not to patent or by providing a strategic advantage to firms that patent. Analysis of the welfare properties of the patent system suggests its ability to erode market power may be central to its capability to increase welfare.

Contents

List of Figures **xi**

List of Tables **xii**

1 Introduction **1**

2 Coordination Frictions and Economic Growth **4**

2.1 Introduction . . . 4
2.2 The Economy . . . 10
2.2.1 Final Good Sector . . . 10
2.2.2 R&D Sector . . . 11
2.2.3 Households . . . 13
2.2.4 Equilibrium . . . 13
2.2.5 Balanced Growth Path . . . 16
2.3 Second-Best Allocation . . . 18
2.4 The Impact of Coordination Frictions . . . 24
2.4.1 Decentralized Economy . . . 24
2.4.2 Planner's Allocation . . . 28
2.5 Numerical Exercise . . . 29
2.6 Conclusion . . . 39

3 Simultaneous Innovation and the Cyclicality of R&D **40**

3.1 Introduction . . . 40
3.2 The Economy . . . 44
3.2.1 Final Good Sector . . . 44
3.2.2 R&D Sector . . . 45
3.2.3 Households . . . 49
3.2.4 Equilibrium . . . 50
3.3 Calibration and Numerical Results . . . 51
3.3.1 Calibration . . . 51
3.3.2 Impulse Response Functions . . . 55
3.3.3 Cyclicality and Second Moments . . . 59

3.4 Conclusion 61

4 The Patent System as a Tool for Eroding Market Power 63
4.1 Introduction 63
4.2 The Model 69
4.2.1 The Innovation Race 75
4.3 Equilibrium Behavior 77
4.3.1 Equilibrium Behavior with a Patent System 78
4.3.2 Equilibrium Behavior without a Patent System 83
4.4 Competition 83
4.4.1 No Prior User Rights 84
4.4.2 Prior User Rights 88
4.5 Welfare and the Optimal Patent Strength 92
4.5.1 Welfare 94
4.5.2 The Optimal Patent Strength 97
4.6 Conclusion 103

5 Conclusions 106

Bibliography 108

A Appendix for Chapter 2 113
A.1 Welfare Comparison 113
A.2 Augmented Model 114
A.2.1 Decentralized Economy 115
A.2.2 Coordination Economy 116
A.2.3 Second-Best Allocation 118
A.2.4 First-Best Allocation 119
A.2.5 Numerical Exercise 120
A.3 Proofs Omitted from the Text 122

B Appendix for Chapter 3 132
B.1 The Deterministic Balanced Growth Path 132

C Appendix for Chapter 4 136
C.1 Proofs Omitted from the Text 136
C.2 Alternative Timing 149

List of Figures

2.1 Difference in the Market Tightness 27
2.2 Welfare Gain: Comparative Statics 31
2.3 Welfare Gain: Comparative Statics 31
2.4 Welfare Gain: Comparative Statics 32
2.5 Welfare Gain: Comparative Statics 32
2.6 Wasteful Innovation 34
2.7 Foregone Innovation 34
2.8 Growth Rate 35
2.9 Welfare Gain 35
2.10 Welfare Gain: Decentralized Economy 36
2.11 Welfare Gain: Planner's Allocation 36

3.1 Impulse Response Function: Output 53
3.2 Impulse Response Function: Consumption 53
3.3 Impulse Response Function: R&D 54
3.4 Impulse Response Function: Market Tightness 54
3.5 Impulse Response Function: Pool of Ideas 54
3.6 Impulse Response Function: Varieties 55
3.7 Impulse Response Function: Entry 55
3.8 Impulse Response Function: Profits 56
3.9 Impulse Response Function: Hours in Research 56
3.10 Impulse Response Function: Hours in Production 56
3.11 Impulse Response Function: Hours 57
3.12 Impulse Response Function: Investment 57

4.1 Timing 70

A.1 Welfare Gain in the Augmented Model 121

List of Tables

3.1 Moments for Data and Model 59
3.2 Autocorrelation for Data and Model 61

Chapter 1

Introduction

When inventors engage in research and development activities, it is often the case that several of them make the same innovation simultaneously. This pattern is observed with both major and non-major innovations, in different industries, and in both recent and historical cases.[1] Since innovation has been a major cause of the observed increase in productivity in the last century and a half, the phenomenon of simultaneous innovation can potentially have important economic implications. This dissertation examines the economic consequences of this phenomenon, putting a particular emphasis on its macroeconomic impacts.

Chapter two examines the coordination frictions implied by the presence of simultaneous innovation and their effect on economic growth and welfare. It develops an expanding-variety endogenous growth model in which firms cannot coordinate their research efforts. The chapter analyses the aggregate consequences of these frictions, as well

[1] For an overview of the evidence see chapters two, three, and four of this dissertation and the sources cited therein.

as their impact on firms' decisions to engage in innovative activities. The implications for the social planner's allocation are analyzed as well. Moreover, the chapter gauges the quantitative importance of these frictions for growth and welfare.

The third chapter of the dissertation extends the analysis to a stochastic environment by introducing aggregate uncertainty. The theoretical framework therein features endogenous innovation quality. Moreover, the quality of inventions is interlinked with the simultaneous innovation present in the model — the more firms simultaneously make the same innovation, the higher is its expected quality. This relationship delivers an endogenous mechanism which amplifies the volatility of the cyclical component of R&D investment. Furthermore, this mechanism allows the model to deliver mildly pro-cyclical R&D — a well-established feature of the data that previous theoretical research has struggled to reproduce.

Chapter four sheds new light on the role of the patent system and its welfare-enhancing capabilities in the context of simultaneous innovation. It develops a theoretical model in which firms can choose between patenting their innovations or keeping them secret. We find that, in sharp contrast to the traditional wisdom, the patent system can erode rather than allocate market power. This is the case because in the presence of simultaneous innovation, patents provide a strategic advantage to firms that choose to use them. Given the importance of lead-time advantage firms' investment decisions are tightly linked with the strategic aspect of patents. In particular, due to this strategic advantage firms that choose to patent can secure an initial monopoly position with higher chance and at a lower expected cost than firms which opt for secrecy. Because of this, when patent protection is relatively weaker than secrecy, the loss in appropriability from choosing weaker protection may be outweighed by the benefit of the strategic advantage. If this is the case some firms would choose to patent even if this results in a shorter expected duration of monopoly. Furthermore, if the strategic aspect of patents is strong enough a social planner may find it optimal to set weak patent protection so as to induce only some firms to patent in equilibrium. This result suggests that the patent system's ability to erode market power may be central to its capability to increase welfare.

Chapter 2

Coordination Frictions and Economic Growth

2.1 Introduction

Innovators have technological access to many distinct research avenues (ideas).[1] However, it is often the case that several firms engage in an innovation race for the exact same idea, i.e. research avenues are scarce. In particular, [52] details anecdotal evidence that virtually every major historical innovation (such as the cotton gin, the steam engine, the computer, and the laser) has been simultaneously innovated by several groups of researchers. Perhaps the most famous example is that of the Alexander Bell and Elisha Gray telephone controversy. On February 14, 1876 Bell filed a patent application for the telephone and only hours later Gray submitted a similar application for the exact same innovation.

[1] For example, during 2015 the U.S. Patent and Trademark Office granted more than quarter of a million patents.

Furthermore, the same empirical regularity is observed for non-major innovations. [21] find that a positive fraction of patents for the period between 1988 and 1996 were declared in interference.[2] More recent examples of simultaneous innovation include companies such as Siemens, Philips, Google Inc., Microsoft Corporation, and Yahoo! Inc.[3]

Furthermore, coordination of research efforts by firms (firm A directs its effort towards project 1, firm B towards project 2, and so on) is very unlikely in this setting because of two main reasons. First, the size of the "market" for ideas makes coordination very hard to achieve. Second, such coordination requires each firm to know the portfolio of research projects of all of its rivals. This is particularly implausible in the current context given that firms actively employ secrecy as an intellectual property protection mechanism.[4]

Motivated by these observations, we develop an expanding-variety endogenous growth model that features scarce research avenues and lack of research effort coordination. The paper examines the impact of these coordination frictions on firms' decision to undertake R&D activities as well as their aggregate consequences. We also study the implications of the frictions for the planner's allocation. Furthermore, we gauge the importance of the coordination problems for growth and welfare in a numerical exercise.

In our model, R&D firms direct their research efforts towards a particular project

[2] Patents are declared in interference if two innovators file for the same patent within three months of each other (six months for major innovations).

[3] Siemens applied for a patent for a positron emission tomography scanner on April 23, 2013 (application number 13/868,256). Most claims are rejected because Philips (application number 14/009,666 filed on March 29, 2012 and application number 14/378,203 filed on February 25, 2013) had simultaneously made similar innovations. Google Inc. filed a patent application on November 1 2012 (number 13/666,391) for methods, systems, and apparatus that provide content to multiple linked devices. All twelve claims contained in the application are rejected because of simultaneous innovations made by Yahoo! Inc. (application number 13/282,180 with filing date October 26, 2011), Microsoft Corporation (application number 13/164,681 with filing date June 20, 2011), and Comscore Inc. (application number 13/481,474 with filing date May 25, 2012). The information on the patent applications is taken from the U.S. Patent and Trademark Office Patent Application Information Retrieval.

[4] For a survey of the evidence see, for example, [37].

out of an endogenously determined mass of ideas. If innovated, each idea is transformed into one new variety. Firms which secure a patent over a variety produce. We focus on the symmetric equilibrium where firms use identical mixed strategies when directing their R&D efforts, so as to highlight their inability to coordinate. Thus, each idea is innovated by a random number of firms with mean equal to the tightness in the market for ideas (the ratio of firms to ideas). Knowledge is cumulative — each innovated idea allows firms to "stand on the shoulders of giants" and gain technological access to a number of new research projects. This inter-temporal spillover effect is the ultimate source of growth in our economy — an expanding mass of ideas permanently alleviates future congestion problems, thus reducing the cost of discovering new varieties. Along the balanced growth path (BGP henceforth), the growth rate of the economy is determined by the growth rate of the mass of ideas, which is in turn endogenously determined by the market tightness and the coordination problems.

The frictions in our model have a direct impact on the growth rate. Firms cannot coordinate their efforts, so they unintentionally gravitate towards the same research projects. This leaves a mass of profitable ideas uninnovated each period. As a result, the growth rate of the decentralized frictional economy (DE henceforth) is lower, as compared to a hypothetical economy in which firms can coordinate their efforts (CE henceforth). At the same time, due to a general equilibrium effect, the frictions amplify the fraction of wasteful simultaneous innovation.[5] Due to the lower growth rate firms discount future profit streams at a lower rate. This increases the value of holding a patent and, in equilibrium, induces more congestion in the market for ideas. This higher congestion, in turn translates to a

[5]Since only one firm can obtain a patent over a particular variety, the R&D investment by all other rivals who innovate simultaneously represents wasteful duplication of effort.

higher fraction of wasteful innovation. Furthermore, for any market tightness, the coordination frictions reduce firms' probability of securing a monopoly position. Given a market tightness, the ratio of innovations to ideas is the same for both the DE and the CE. In the DE, however, there is a mass of foregone innovation. Hence, a lower fraction of these innovations are distinct which leads to a lower number of patents to be distributed among firms. This reduced probability of securing a patent induces firms to decrease their entry into the R&D sector, leaving the DE with a lower R&D intensity (market tightness). As a result, the DE growth rate is decreased even further.

The planner's second-best allocation (SB henceforth) also features positive fractions of foregone innovation and wasteful simultaneous innovation. The planner can choose the mass of R&D entrants, but she cannot assign firms to projects. When the market tightness is low, so is the fraction of wasteful innovation, but many innovations are foregone. When the tightness is high, foregone innovation is low, but many firms make a wasteful duplication of effort. Thus, at the margin the planner chooses a tightness that strikes a balance between these two effects. The SB research intensity may be higher or lower than the one in the first-best allocation (FB henceforth). This is the case because in the FB, the planner can assign firms to projects and as a consequence she does not face the same trade-off. Thus, she sets the first-best tightness to unity.

The frictions in our model impact welfare negatively through two channels: they (i) generate a mass of foregone innovation and (ii) amplify the fraction of wasteful innovation. In the benchmark calibration, eliminating the frictions in the DE leads to a 13% welfare gain (in consumption equivalent terms). The DE growth rate is only 2/3 of the CE one, so the welfare cost of foregone innovation is 10.35%. Coordination problems increase the fraction of wasteful innovation by 8pp (to 39%), which translates to a 2.65% welfare cost. Moreover, if the planner could eliminate the frictions and assign the FB, she would achieve welfare 16.15% higher than that in the SB. However, only 5.66pp of the gain is due to eliminating foregone innovation. This is because of two reasons. First, the SB features a much smaller fraction of foregone innovation than the DE. Second, removing the frictions in the SB reduces the fraction of wasteful duplication of effort from 52% to 0 since the FB does not suffer from the over-investment present in CE.

Relationship to the literature. Our paper models firms' choice of direction for their R&D efforts and the coordination problems inherent in this decision. As such, it is related to a recent literature on economic growth which emphasizes matching and other frictions in the innovation process (see, for example, [57], [54], [11], [20], and [5]). The work here complements that literature by examining a different source of friction. In particular, to the best of my knowledge, this is the first growth paper to emphasize search frictions in the market for ideas which take the form of a coordination failure. Previous growth models have focused instead on a search process which takes the form of arrival rate of innovations, a McCall-type search for innovations, or frictions in the market for innovations.[6]

The theoretical model in this paper differs from the existing literature on economic growth in a number of additional dimensions. First, our analysis emphasizes firms' choice of research avenues by explicitly modeling the mass of available ideas. In particular, we make

[6] Papers which feature search as arrival rate of innovations include [2], [36], and [47]. [48], [57], and [54], among others, feature a McCall-type search for heterogeneous technologies. For papers which focus on frictions in the market for innovations see, for example, [20] and [5]. It is worth noting that [20] and [5] do not make a distinction between ideas and innovations. In particular, the market for ideas in our paper (firms searching for a potential R&D project) is different from the "market for ideas" in [20] and [5] where firms search for opportunities to trade the property rights over an innovation.

a distinction between potential innovations (ideas) and actual innovations.[7] Second, our model features a scarce mass of potential research projects such as, for example, [36] and [47].[8] Unlike those studies, our paper explicitly models the decision of firms to direct their R&D activities and emphasizes the coordination frictions inherent in this problem.[9] Third, in contrast to the previous literature, this paper features an endogenously determined mass of ideas. Fourth, in our paper firms compete for ideas through their choice of research avenue. This competition is different than the competition firms face in the product market or the innovation race which the previous literature has examined.[10]

Within the literature on industrial organization the two closest papers are [51] and [49] which also feature search frictions in the market for ideas. In these papers there is the possibility of simultaneous innovation due to a matching technology which is the same as the equilibrium one in our paper. [51] and [49] focus on intellectual property rights in a partial equilibrium framework with a fixed mass of ideas and without free entry into the innovation sector. In contrast, our model focuses on a general equilibrium framework with growth, an endogenously determined mass of ideas, and an endogenously determined market tightness through free entry in the R&D sector.

The rest of the paper is organized as follows. Section two introduces the environment and characterizes the decentralized equilibrium. Section three examines the social planner's second-best allocation. Section four highlights the impact of coordination frictions

[7]This is in contrast to the previous literature on economic growth [41], [42], [43], [20], [5], [13] which has used ideas and innovations interchangeably.

[8]In contrast, some previous studies [61, 24, 25, 48] have examined models which feature an abundance of research avenues, whereas others [2, 62] have examined models where a single avenue of research is available. For a recent review of the literature see, for example, [3].

[9]In contrast, these papers do not focus on this decision and assume that firms can either perfectly coordinate their efforts [36] or cannot choose the direction of their research altogether [47].

[10]See, for example, [62], [2], [24], [25], [4], and [1]

in our model. Section five presents a numerical exercise. Section six concludes.

2.2 The Economy

The environment is an augmented, discrete time version of the textbook model in [10] Chapter 6 (BSM henceforth). There are three types of agents — a final good producer, a unit measure of consumers, and a continuum of R&D firms. The only point of departure from BSM is in the R&D sector, so as to emphasize the novel features of the model. In particular, R&D projects are scarce and R&D entrants can direct their efforts towards a particular project, but they cannot coordinate their research activities.

2.2.1 Final Good Sector

The final good is produced by a single price taker, using the following technology

$$Y_t = AL^{1-\lambda} \int_0^{N_t} X_t^{\lambda}(n) dn, \quad 0 < \lambda < 1 \tag{2.1}$$

where Y_t is output, L is the fixed labor supply of households, N_t is the mass of intermediate varieties, and $X_t(n)$ is the amount of a particular variety n employed in production. The price of the final good is normalized to unity. The final good firm faces a competitive market for labor, which is hired at the wage w_t, and a monopolistically competitive market for varieties, where a unit of each variety n is bought at the price $P_t(n)$. As in BSM, the firm's maximizing behavior yields the wage $w_t = (1-\lambda)Y_t/L$ and the inverse demand function for varieties $P_t(n) = \lambda AL^{1-\lambda} X_t^{\lambda-1}(n)$.

2.2.2 R&D Sector

The novel features of our model are contained in the R&D sector of the economy. The innovation process has three stages and makes a distinction between potential innovations (ideas) and actual innovations (new varieties). At stage one, firms enter the R&D sector at a cost $\eta > 0$ units of the final good. The mass of R&D entrants is denoted by μ_t and is to be determined in equilibrium. At stage two firms direct their innovative effort towards a particular R&D project from a finite mass ν_t of ideas.[11] The choice is private knowledge and firms cannot coordinate their efforts. To capture this coordination failure, we follow the previous literature on coordination frictions and focus on a symmetric equilibrium where firms use identical mixed strategies.[12] Ideas are identical and, if innovated, transform into exactly one new variety. Innovation takes one period — a firm which enters at time t innovates the chosen project at time $t+1$. Thus, the only source of uncertainty in our model is the random realization of firms' equilibrium mixed strategies — some ideas may be innovated by many firms simultaneously, while others may not be innovated at all. Innovators apply for a patent which grants monopoly rights over the variety. Each innovation is protected by exactly one patent — if several firms simultaneously apply for the same patent, then each has an equal chance of receiving it. Patents never expire — the

[11] It should be noted that the assumption of a continuum of ideas is made for ease of exposition. Nothing is lost if, instead, one assumes the initial number of ideas in the economy is finite. In particular, we follow the previous literature on coordination frictions (see, for example, [63]) and treat our economy with a continuum of ideas as the limiting case of a finite-idea economy. The equilibrium outcome in a finite economy is that the number of firms which simultaneously direct their efforts towards a particular idea follows a binomial distribution. As the number of ideas increases, this distribution converges to Poisson. We focus on an economy with a continuum of ideas so as to abstract the analysis away from any approximation errors associated with using the limiting Poisson distribution. Of course, one could just as easily analyze an economy with a finite number of ideas. Because of growth, the number of ideas eventually becomes "large" and the approximation error from focusing on the limiting case becomes negligible.

[12] See, for example, [45], [19], and [63].

firm retains its monopoly position forever. Stage three is as in BSM. Patent holders supply their variety in a monopolistically competitive market. Both the average and marginal costs of production are normalized to unity so profits are given by $\pi_t(n) = (P_t(n)-1)X_t(n)$. Furthermore, the value of holding a monopoly over a variety n at time t, V_t, is given by

$$V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi_i(n), \tag{2.2}$$

where d_{it} is the stochastic discount factor.

A necessary condition for positive long term growth in our model is that the mass of ideas, ν_t, grows at a positive rate. We follow [48] and [61], among others, and assume that knowledge is cumulative. Patenting an idea at time t allows firms to "stand on the shoulders of giants" and gain access to $M > 1$ new research avenues at $t + 1$. Thus, unlike previous growth models, in ours the mass of ideas is endogenously determined. Once an idea is innovated, it is no longer a potential R&D project and so it is removed from the pool.[13] Thus, the net increase in the pool of ideas from innovating one new variety is $M-1$. Due to the frictions in our model, there is a chance that an idea is not innovated, i.e. no firm directs its research efforts towards the idea in question. Let us denote this probability by ζ_t, then the law of motion for ideas is given by

$$\nu_{t+1} = \nu_t + (1-\zeta_t)(M-1)\nu_t. \tag{2.3}$$

As each innovated idea is transformed into a new variety, it follows that

$$N_{t+1} = N_t + (1-\zeta_t)\nu_t. \tag{2.4}$$

[13]Each innovation is protected by a patent, so no firm has an incentive to imitate at a later date. Thus, the idea no longer represents a profitable R&D project and as a consequence it is no longer in ν_{t+1}.

2.2.3 Households

Consumers are endowed with a discount factor β and a per-period utility function $U(C_t) = \ln C_t$. They can save by accumulating assets, which in this economy are claims on intermediate firms' profits. In particular, households have access to a mutual fund that covers all intermediate good firms. Let a_t denote the amount of shares held by the representative household at the beginning of period t. Each period all profits are redistributed as dividends, thus the total assets of the household entering period t are $a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn$. At time t households decide on the shares they would like to hold at $t+1$, a_{t+1}. The mutual fund at that time covers all firms which exist at time $t+1$, N_{t+1}. Hence, the household's budget constraint is given by

$$a_{t+1} \int_0^{N_{t+1}} V_t(n)dn = a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn + w_t L - C_t. \tag{2.5}$$

The household's first order conditions imply the Euler equation below

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \Big(\int_0^{N_{t+1}} (\pi_{t+1}(n) + V_{t+1}(n))dn \Big) \Big(\int_0^{N_{t+1}} V_t(n)dn \Big)^{-1}. \tag{2.6}$$

The intuition is standard — consumers equate the marginal utility at time t with the discounted marginal utility at time $t+1$, times the gross rate of return on their assets.

2.2.4 Equilibrium

We restrict the analysis to a set of parameter values which ensures that firms have an incentive to enter the R&D sector, i.e. $\eta \leq (1-\lambda)\beta(\lambda^2 A)^{1/(1-\lambda)} L / [\lambda(M - \beta)]$. The usual profit maximization of intermediate good firms along with the demand function imply that $P_t(n) = 1/\lambda$ and $X := X_t(n) = (\lambda^2 A)^{1/(1-\lambda)} L$. Thus, every intermediate good firm yields the same per period profits of $\pi := \pi_t(n) = X(1-\lambda)/\lambda$. This implies that $V_t := V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi$ — every firm is equally valuable. Since each variety carries the same amount of profits, the stage two equilibrium strategy of firms is to direct their R&D effort towards each idea with equal probability.[14] This implies the following equilibrium outcome.

Proposition 1 *The number of firms which direct their R&D effort towards a particular idea follows a Poisson distribution with mean θ_t, where $\theta_t \equiv \mu_t/\nu_t$.*

A proof is in Appendix A.3. The random realization of firms' equilibrium strategies gives rise to the standard urn-ball matching technology.[15] The ratio of firms to ideas, θ_t, represents the tightness in the market for ideas and captures the level of congestion in the economy. An R&D firm becomes a monopolist with probability $\sum_{m=0}^{\infty} Pr($ exactly m rival firms direct their research effort towards the particular idea$)/(m+1) = \sum_{m=0}^{\infty} e^{-\theta_t}\theta_t^m/(m+1)! = (1-e^{-\theta_t})/\theta_t$. This probability captures the business-stealing effect in the model. An innovator faces the threat that a rival directs its research efforts towards the exact same idea. If that is the case, then the rival has a chance of securing a patent over the innovation, effectively stealing the innovator's monopoly rents. Thus, higher congestion increases the expected number of rivals, which lowers each firm's chance of securing a patent. Given free entry, it follows that

$$\eta = \frac{1-e^{-\theta_t}}{\theta_t} V_t. \tag{2.7}$$

The level of congestion firms are willing to tolerate is governed by the net present value of profits and the entry cost. Higher profits (or lower costs) induce firms to tolerate a

[14] See the proof of Proposition 16 for details.

[15] See, for example, [65], [53], [45], [19], and [63].

lower chance of securing a monopoly position and as a consequence higher tightness. The matching technology implies that $\zeta_t = e^{-\theta_t}$. Hence,

$$\nu_{t+1} = \nu_t + (1 - e^{-\theta_t})(M - 1)\nu_t, \tag{2.8}$$

$$N_{t+1} = N_t + (1 - e^{-\theta_t})\nu_t. \tag{2.9}$$

Furthermore, the frictions in our model induce an economy-wide varieties production function (New Varieties $= (1 - e^{-R_t/(\eta\nu_t)})\nu_t$) which is concave in the aggregate research effort, $R_t \equiv \eta\mu_t$. A higher aggregate research effort is associated with higher mass of firms which, in turn, increases the congestion in the market. Thus, the marginal entrant has a higher chance of duplicating an innovation, rather than innovating a distinct new variety. In particular, the higher level of congestion increases the fraction of wasteful duplicative innovation, $\omega \equiv 1 - (1 - e^{-\theta_t})/\theta_t$.[16]

Since all firms receive the same profits, the Euler equation simplifies to

$$V_t = \beta \frac{C_t}{C_{t+1}} \Big(\pi + V_{t+1} \Big). \tag{2.10}$$

Hence, the stochastic discount factor is $d_{it} = \beta^i C_t / C_{t+i}$. Given consumers' budget constraint, free entry, and the law of motion for varieties it is straightforward to derive the economy-wide resource constraint which takes the usual form — output is distributed towards consumption, production of intermediate inputs, and investment in R&D

$$Y_t = C_t + N_t X + \mu_t \eta. \tag{2.11}$$

[16] Only one firm can hold a patent over a certain variety. Hence, whenever $m \geq 1$ firms innovate the same idea, $m - 1$ of them make a wasteful duplicative innovation. Each entrant makes an innovation, so the total number of innovations is μ_t. The total number of useful innovations equals the total number of new varieties, $(1 - e^{-\theta_t})\nu_t$. Thus, the fraction of innovations which represent wasteful duplication of effort is simply $1 - (1 - e^{-\theta_t})/\theta_t$.

2.2.5 Balanced Growth Path

Our analysis focuses on the BGP of the economy, where output, consumption, varieties, ideas, and the mass of entrants all grow at constant (but possibly different) rates. Denote the growth rate of any variable x along the BGP by g_x. It is straightforward to establish that output, varieties, consumption, entry into R&D, and the stock of ideas all grow at the same rate along the BGP. Namely, $g \equiv g_Y = g_C = g_N = g_\mu = g_\nu = (1 - e^{-\theta})(M - 1)$, where θ is the value of the market tightness along the BGP.[17] As in BSM Y_t, C_t, N_t, and μ_t all grow at the same rate. In our model, the mass of ideas, ν_t, also grows at this rate. In fact, the expansion of ν_t is the ultimate source of growth in the economy. Due to learning, innovation today increases the mass of ideas in the future. This permanently reduces the severity of the coordination problems and subsequently the cost of securing a monopoly position.[18] This lower cost in turn induces higher entry into R&D up to the point where congestion reaches its BGP level. Furthermore, the fraction of foregone innovation, $e^{-\theta}$, directly impacts the growth rate, as only innovated ideas at time t contribute to the expansion of ν_{t+1}.

It is convenient to solve the model by looking at the stable ratios θ, $\frac{\nu}{N}$ and $\frac{C}{N}$. From the law of motion of ideas and varieties, and from $g_N = g_\nu$, it follows that $\frac{\nu}{N} = M - 1$. Next, the resource constraint implies that

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \eta\theta(M-1). \qquad (2.12)$$

Lastly, we can use the fact that $g_C = g_\nu$, the Euler equation, the law of motion for ν_t, and

[17] A proof is available upon request.

[18] The average cost of securing a monopoly position is $\eta/Pr(\text{monopoly}) = \eta\theta/(1-e^{-\theta})$, which is decreasing in ν_t.

the free entry condition to find an implicit solution for the market tightness

$$\eta = \Big(\frac{1-e^{-\theta}}{\theta}\Big)\frac{\beta\pi}{1+(1-e^{-\theta})(M-1)-\beta}. \tag{2.13}$$

Even though we cannot explicitly solve for θ, it is straightforward to establish that the solution is unique. Intuitively, as θ increases the market for ideas gets more congested and each firm's chance of becoming a monopolist decreases. At the same time, higher market tightness implies a higher growth rate. This, in turn, increases the rate with which firms discount future profit streams and as a consequence decreases the value of holding a patent. Both of these effects decrease the incentives to enter the R&D sector when the market tightness is high and vice versa.

Proposition 2 *The equilibrium market tightness, θ, is:*

- *increasing in π and β*
- *decreasing in η and M*

A proof is included in Appendix A.3. Intuitively, an increase in profits raises the value of being a monopolist, V_t. This increases firms' incentives to innovate, which leads to a higher mass of R&D entrants and subsequently to a higher market tightness. Similarly, a higher entry cost, η, discourages entry into R&D, which decreases the market tightness. An increase in β or a decrease in M both lead to an increase in the effective discount factor, $\beta C_t/C_{t+1}$, along the BGP. Thus, firms value future profits more, which increases the value of a patent, V_t, and ultimately the market tightness.

2.3 Second-Best Allocation

This section examines the planner's second best allocation — the planner chooses the optimal BGP allocations subject to the coordination frictions in the market for ideas. Without loss of generality, we impose symmetry in the intermediate varieties, i.e. $X_t(n) = X_t(n')$ for any varieties n and n'. Thus, the planner faces the problem of choosing production of varieties, consumption, a mass of varieties, a mass of ideas, and the market tightness in order to maximize welfare subject to the resource constraint, the laws of motion for ideas and varieties, and the coordination frictions

$$\max_{\{C_t,X_t,\theta_t,N_t,\nu_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t,$$

$$AL^{1-\lambda} N_t X_t^{\lambda} = N_t X_t + C_t + \eta\theta_t\nu_t, \tag{2.14}$$

$$N_{t+1} = N_t + (1 - e^{-\theta_t})\nu_t, \tag{2.15}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\theta_t})(M - 1)\nu_t. \tag{2.16}$$

Maximizing with respect to X_t yields the usual solution for varieties $X^* := X_t = (\lambda A)^{1/(1-\lambda)} L$. As in BSM the difference between the planner's solution and the decentralized outcome comes from the monopoly pricing of intermediate goods. Let $\pi^* = X^*(1 - \lambda)/\lambda$ denote the implied per period monopoly profits at efficient level of intermediate varieties.

Then, the rest of the first order conditions are:

$$[C_t]: \quad \beta\frac{C_t}{C_{t+1}} = \frac{\phi_{t+1}}{\phi_t}, \tag{2.17}$$

$$[N_{t+1}]: \quad h_t = h_{t+1} + \phi_{t+1}\pi^*, \tag{2.18}$$

$$[\nu_{t+1}]: \quad \lambda_t = \lambda_{t+1}\Big(e^{-\theta_{t+1}} + (1-e^{-\theta_{t+1}})M\Big) + h_{t+1}(1-e^{-\theta_{t+1}}) - \phi_{t+1}\eta\theta_{t+1}, \tag{2.19}$$

$$[\theta_t]: \quad \eta = e^{-\theta_t}\Big(\frac{h_t}{\phi_t} + \frac{\lambda_t}{\phi_t}(M-1)\Big), \tag{2.20}$$

where ϕ_t, h_t, λ_t are the multipliers associated with (2.14), (2.15), and (2.16), respectively. From (2.17) and (2.18), it follows that

$$\frac{h_t}{\phi_t} = \beta\frac{C_t}{C_{t+1}}\Big(\pi^* + \frac{h_{t+1}}{\phi_{t+1}}\Big). \tag{2.21}$$

The above equation characterizes the planner's valuation of varieties: the value of a variety equals the discounted sum of per period profits, π^*, and the continuation value h_{t+1}/ϕ_{t+1}. There are only two differences as compared to the DE — the level of profits is higher and the planner chooses a different tightness.

The value of an idea is the discounted sum of several terms

$$\frac{\lambda_t}{\phi_t} = \beta\frac{C_t}{C_{t+1}}\Big(-\eta\theta_{t+1} + (1-e^{-\theta_{t+1}})\Big(\frac{h_{t+1}}{\phi_{t+1}} + \frac{\lambda_{t+1}}{\phi_{t+1}}(M-1)\Big) + \frac{\lambda_{t+1}}{\phi_{t+1}}\Big). \tag{2.22}$$

First, there is the dividend, $-\eta\theta_{t+1}$, which represents the average cost of R&D per idea. It captures the intuition that unlike other assets, which carry positive returns, an idea is only valuable if it is innovated. Hence, the planner finds it costly to keep a stock of ideas because it diverts resources away from consumption and into R&D. The second term represents the capital gain from innovation — the probability an idea is innovated, $(1-e^{-\theta_{t+1}})$, times the social benefit from innovating. This benefit is the value of the extra variety, h_{t+1}/ϕ_{t+1}, plus the value of the extra ideas that would be added to the pool because of innovation, $\lambda_{t+1}/\phi_{t+1}(M-1)$. Lastly, the idea carries its continuation value λ_{t+1}/ϕ_{t+1}.

Apart from the monopoly pricing of intermediate goods, there are two additional externalities in the model, which are illustrated in equation (2.20). First, the congestion externality manifests through the difference in the fraction of socially and privately beneficial innovations. The planner finds the marginal entry beneficial only if the firm is the sole inventor, i.e. with probability $e^{-\theta_t}$. Firms, on the other hand, value entry even if they duplicate an innovation, as long as they receive the patent for it. In particular, due to the business-stealing effect, the probability of a privately beneficial innovation is $(1-e^{-\theta_t})/\theta_t > e^{-\theta_t}$. Hence, the congestion externality induces firms to over-invest in R&D as compared to the SB. This business-stealing effect is similar to the one examined in the previous literature.[19] Unlike in previous papers, in ours the magnitude of the effect is affected by the coordination frictions. Second, there is the learning externality — firms cannot appropriate the benefit of any ideas that come about from their innovations, so they do not value them. The planner, on the other hand, does because they permanently alleviate future coordination problems. Specifically, more innovation today increases the amount of future research avenues, which allows the economy to innovate more varieties without increasing the congestion problems. Thus, the extra ideas permanently reduce the cost of discovering new varieties.[20] As a result the learning externality creates incentives for firms to under-invest as compared to the SB. This externality is similar in spirit to the inter-temporal

[19] See, for example, [24] and [25].

[20] The average cost of discovering one new variety is $\eta/Pr(\text{sole inventor}) = \eta e^{\theta_t}$, which is decreasing in the mass of ideas.

spillover effects present in previous models.[21] In ours, the externality operates through the market for ideas — the planner values ideas because they alleviate the coordination problems in the economy.

It is straightforward to establish that along the BGP the SB allocations are characterized by[22]

$$\Big(\frac{\nu}{N}\Big)^{SB} = M-1, \quad (2.23)$$

$$\Big(\frac{C}{N}\Big)^{SB} = \pi^* - \eta\theta^{SB}(M-1), \quad (2.24)$$

$$1+(1-e^{-\theta^{SB}})(M-1) = \beta\Big(1+\frac{\pi^*}{\eta}e^{-\theta^{SB}} + (1-e^{-\theta^{SB}} - \theta^{SB}e^{-\theta^{SB}})(M-1)\Big). \quad (2.25)$$

The difference between the SB solution for the market tightness, (2.25), and the DE one, (2.13), comes from the aforementioned externalities. To see their impact clearly, let us define the implied rate of return in the DE by

$$r := \frac{C_{t+1}}{\beta C_t} - 1 = \frac{\pi}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big), \quad (2.26)$$

which is nothing but the rate of return on a unit investment in R&D — π is the flow of profits and $(1-e^{-\theta})/\theta$ is the probability of securing a monopoly position. The implied rate of return in the SB represents the social rate of return on a unit of investment on R&D and is defined by

$$r^{SB} := \frac{C_{t+1}^{SB}}{\beta C_t^{SB}} - 1 = e^{-\theta^{SB}}\Big(\frac{\pi^*}{\eta} - \theta^{SB}(M-1)\Big) + (1-e^{-\theta^{SB}})(M-1). \quad (2.27)$$

First, the planner eliminates the monopoly distortion, so the flow of profits is π^*. Second, she values the marginal innovation only when the firm is the sole inventor, which occurs

[21] See, for example, [61], [2], and [36].

[22] A proof is available upon request.

with probability $e^{-\theta^{SB}}$. In that event, the net return is given by the normalized profits, π^*/η, less the normalized "storage cost" of the new research avenues, $\theta^{SB}(M-1)$. Third, each innovation increases the mass of ideas, so the permanent decrease in future congestion yields the return of $(1-e^{-\theta^{SB}})(M-1)$.

To implement the SB, the planner needs to impose a tax on the entry into R&D. This is because the congestion externality is larger than the learning one, so the over-investment effect of the former dominates the under-investment effect of the latter. In particular, suppose that the government imposes a subsidy on the purchases of intermediate varieties at a rate s and a tax on R&D activities at a rate τ. Furthermore, if the government keeps a balanced budget through the means of lump-sum transfers, then the optimal policy is summarized below.

Proposition 3 *The optimal subsidy on the purchase of intermediate varieties is given by* $s^* = 1 - \lambda$. *The optimal tax rate on R&D entry is given by*

$$\tau^* = \frac{\beta\pi^*(1-e^{-\theta^{SB}})}{\eta\theta^{SB}(e^{-\theta^{SB}} + (1-e^{-\theta^{SB}})M - \beta)} - 1. \tag{2.28}$$

Furthermore, $\tau^* > 0$ *because the magnitude of the congestion externality is larger than that of the learning externality.*

A proof is included in Appendix A.3. Even though it is optimal to impose a tax on R&D spending, it may be the case that the decentralized economy suffers from under-investment, i.e. $\theta < \theta^{SB}$. This is due to the monopoly distortion induced by patents. Whether or not there will be under-investment in equilibrium depends on parameter values.

Proposition 4 *The second best market tightness,* θ^*, *is:*

- *increasing in π^* and β*
- *decreasing in η and M*

A proof is included Appendix A.3. Intuitively, an increase in the implied profits, π^*, increases the planner's valuation of each variety and each idea. Hence, each entry is now more beneficial, so the planner increases the market tightness. This increases congestion and as a consequence decreases the value of the marginal entry. The planner increases the tightness until the value of the marginal entry reaches the entry cost η. Similarly, an increase in the entry cost, η, requires the planner to extract more benefit from the marginal entry. Thus, she finds it optimal to reduce the market tightness and decrease congestion. At the same time, an increase in η leads to an increase in the storage cost of ideas and subsequently reduces their value. This induces the planner to decrease the tightness further.

An increase in β increases the discount factor, so the value of varieties and ideas increases because the stream of future profits is now more valuable. This increases the value of the marginal entry and induces the planner to increase the market tightness. An increase in M leads to two opposing effects. First, a higher M is associated with a higher growth rate. This decreases the value of future consumption and induces the planner to set a lower tightness. At the same time, a higher M implies each innovation increases the mass of ideas next period, ν_{t+1}, by a higher amount. Thus, the value of the marginal entry into R&D is higher and the planner has an incentive to set a higher market tightness. At the optimum, the former effect dominates the latter and the planner decreases θ^{SB}.

2.4 The Impact of Coordination Frictions

2.4.1 Decentralized Economy

A goal of the analysis is to study the impact of coordination frictions in our economy. To this end we compare the DE's BGP to the BGP of a hypothetical CE. In particular, the only difference between the latter economy and the DE is that firms can coordinate their research efforts at stage two of the innovation process.[23] Let superscript c denote the value of any variable in the CE along the BGP. Evidently, when firms can coordinate their research efforts, all research avenues are undertaken and subsequently all ideas are innovated. At the same time, the CE may feature a positive fraction of wasteful duplication of effort due to the usual "over-grazing" problem.[24] However, this waste, ω^c, is smaller than the one in the DE. Furthermore, this is the case, even though the CE features a higher market tightness.

Proposition 5 *In the coordination economy all ideas are innovated and the growth rate equals $M-1$. Furthermore, $\theta < \theta^c$ and*

$$\omega - \omega^c = \frac{e^{-\theta}(M-1)\eta}{\beta\pi} > 0. \tag{2.29}$$

A proof is included in Appendix A.3. Intuitively, when firms can coordinate their R&D activities all ideas are innovated because each of them represents an opportunity to gain a profitable monopoly position. Thus, in the CE there is no foregone innovation. This results in a higher growth rate as compared to the DE. Because of this the foregone innovation in the DE generates a general equilibrium effect which induces firms to tolerate

[23] The proof of Proposition 5 explicitly defines the process of coordination.
[24] For a survey of the literature see, for example, [60].

a higher congestion than firms in the CE. In particular, the lower growth rate increases the effective discount factor, which in turn raises the value of holding a patent. Since, in both economies, the probability of making a wasteful innovation is simply the probability of not receiving a patent, it follows that $\omega > \omega^c$. In particular, the level of amplification, $\omega - \omega^c$, equals the difference in the growth rates, $g^c - g = e^{-\theta}(M-1)$, divided by the discounted normalized profits, $\beta\pi/\eta$.

Moreover, $\theta < \theta^c$, even though the DE features a higher fraction of wasteful simultaneous innovation. This is the case because, for a given market tightness, the coordination frictions reduce an entrant's chance of securing a monopoly position. In particular, the probability of securing a patent in the DE for a given tightness $\tilde{\theta}$, $Pr(\text{patent}|\tilde{\theta}) = (1-e^{-\tilde{\theta}})/\tilde{\theta}$, is only a fraction $1-e^{-\tilde{\theta}}$ of the one in the CE, $Pr(\text{patent}|\tilde{\theta})^c = 1/\tilde{\theta}$. As firms cannot coordinate their efforts, in the DE only a fraction $1-e^{-\tilde{\theta}}$ of ideas are patented. Thus, even though the number of patent applications per idea, $\tilde{\theta}$, is the same in both economies, in the DE there are relatively fewer patents to be distributed among innovators. This decreases each entrant's chance of securing a monopoly position and subsequently reduces the incentives to enter the R&D sector. This is true even though the DE features a higher value of holding a patent. In other words, the decrease in the probability of securing a patent dominates the increase in the net present value of profits, ultimately reducing incentives to enter the R&D sector and decreasing the market tightness. Furthermore, the effect on the market tightness provides an indirect channel through which the presence of foregone innovation reduces the growth rate in the DE. A lower tightness decreases each idea's chance of being innovated which results in a lower aggregate mass of innovation.

The impact of coordination frictions is higher when profits are low, consumers are more inpatient, and the entry cost is high. The next proposition states the result.

Proposition 6 *The fraction of foregone innovation, $e^{-\theta}$, the level of amplification of wasteful innovation, $\omega - \omega^c$, and the amount by which the tightness is reduced, $\theta^c - \theta$ are*

- *decreasing in π and β*

- *increasing in η*

$e^{-\theta}$ and $\omega - \omega^c$ are increasing in M.

A proof is included in Appendix A.3. The fraction of foregone innovation depends only on the market tightness in the DE. When the tightness is low, the probability that an idea is not matched with any firm is high, which leads to a high fraction of foregone innovation and vice versa. The level of the amplification of wasteful innovation moves in the same direction as $e^{-\theta}$. This is because firms in the DE are willing to tolerate lower probability of securing a monopoly position only due to the higher value of holding a patent induced by $g < g^c$. As the fraction of foregone innovation decreases, the difference in the growth rates decreases as well. This reduces the incentives for firms to tolerate extra congestion, which decreases the amplification.

When the fraction of foregone innovation is low, the incentives for firms in the DE to over-invest (as compared to the CE) induced by the difference in the growth rates is low as well. This generates an upward pressure on the difference in research intensities, $\theta^c - \theta$. At the same time, a smaller fraction of forgone innovation implies that, for a given market tightness, there are relatively more patents to be distributed among firms in the DE.

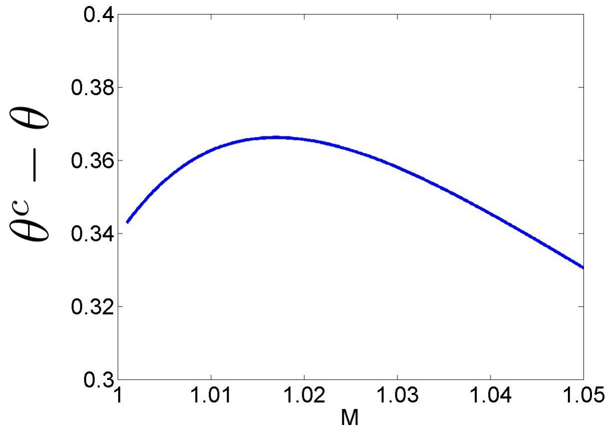

Figure 2.1: Difference in the Market Tightness

Hence, $Pr(\text{patent}|\tilde{\theta})^c - Pr(\text{patent}|\tilde{\theta})$ decreases, which increases the incentives for firms in the DE (relative to CE) to enter the R&D sector. Consequently, this generates a downward pressure on $\theta^c - \theta$. For a decrease in the fraction of foregone innovation induced by changes in π, β, or η the latter effect dominates the former and $\theta^c - \theta$ decreases.

A decrease in the fraction of foregone innovation due to lower M can lead to either an increase or decrease in $\theta^c - \theta$, depending on the relative size of M. This is because changes in M directly impact the difference in the growth rates and consequently increases the relative size of the former effect. For low M this increase is small, so $\theta^c - \theta$ moves in the same direction as the fraction of foregone innovation. For large M, however, the increase in the size of the effect in question is large and so $\theta^c - \theta$ is decreasing in M (Figure 2.1).[25]

[25] All parameter values used in Figure 2.1, except for M are set at their calibrated values detailed in section 5.

2.4.2 Planner's Allocation

To highlight the impact of coordination frictions in the planner's allocation we compare the BGP in the SB to that in the FB. In the FB the planner can directly assign firms to projects. Thus, it is straightforward to establish that $\theta^{FB} = 1$ and that the FB does not feature any foregone innovation, nor any wasteful duplication of effort.[26] Thus, it is readily observable that the frictions amplify both the fractions of foregone and wasteful innovation. Unlike in the decentralized case, however, the coordination failure does necessarily reduce the research intensity in the economy. This is so because in the SB the planner faces a trade-off when deciding on the market tightness (as depicted in equation (2.20)). On the one hand, a higher tightness increases congestion and subsequently the cost of wasteful innovation, $\eta \times Pr(\text{duplication of effort}) = \eta(1-e^{-\theta_t})$. On the other hand, a higher tightness decreases the fraction of foregone innovation. The benefit from this decrease is given by the probability the marginal firm is the sole inventor, $e^{-\theta_t}$, times the the social benefit of the innovation net of the entry cost, η. Thus, the planner chooses θ^{SB} that, at the margin, strikes a balance between these two opposing effects. In the FB, however, she faces no such trade-off so the decision of setting the market tightness is independent of the parameters which govern the welfare costs of wasteful duplication of effort and foregone innovation.

Proposition 7 *The fraction of wasteful (foregone) innovation, ω^{SB} ($e^{-\theta^{SB}}$), is*

- *increasing (decreasing) in π^* and β*
- *decreasing (increasing) in η and M*

[26] Furthermore, $(\nu/N)^{FB} = M - 1$, $(C/N)^{FB} = \pi^* - \eta(M - 1)$, and $g^{FB} = M - 1$. A proof is available upon request.

The proof is immediate from Proposition 4 and $\omega^{SB} = 1 - (1 - e^{-\theta^{SB}})/\theta^{SB}$. The intuition behind the result is straightforward. High π^*, β or low η, M induce the planner to set high θ^{SB}. This leads to more congestion, which subsequently increases ω^{SB} and decreases $e^{-\theta^{SB}}$.

2.5 Numerical Exercise

We gauge the importance of the frictions in our model for growth and welfare through the means of a numerical exercise. Our calibration matches key moments of the U.S. economy and is set at annual frequency. The discount factor, β, is set to 0.95, the productivity parameter, A, and labor supply, L, are both normalized to unity. We set the markup to 17.43% ($\lambda = 0.8516$) to match the average R&D share of non-farm GDP, $\eta\mu_t/Y_t = 3.1194\%$, for the period between 1966 and 2011.[27] To calibrate η and M we use two additional moments. First, we match the average growth rate of non-farm GDP for the same period of 1.7546%. Second, in our model the ratio of patent grants to patent applications is $(1 - e^{-\theta})/\theta$. Matching this fraction to its empirical counterpart, 0.60957, results in a market tightness $\theta = 1.0876$.[28] Together these two moments yield $\eta = 0.1715$ and $M = 1.0265$.

The calibrated DE features a fraction of wasteful innovation $\omega = 39\%$. This is about 25% larger than that in the CE, $\omega^c = 31\%$, even though θ is about 25% smaller than $\theta^c = 1.4491$. At the same time, the DE features a large fraction of research avenues

[27]The data on non-farm GDP is in 2009 chained dollars and taken from NIPA table 1.3.6. The data on nominal R&D expenditures is from NIPA table 5.6.5 and includes private fixed investment in R&D (including software). To obtain the series on real R&D investment, we deflate the nominal series using the implicit GDP price deflator from NIPA table 1.1.9.

[28]The data is taken from the U.S. Patent and Trademark Office. The data on patent grants is by year of application.

which are not undertaken — 33.7%. This implies that the growth rate is about 2/3 of the CE growth rate $g^c = 2.65\%$. Eliminating the frictions generates a welfare gain of 13% in consumption equivalent terms.[29] About 10.35pp of the gain is due to the increased growth rate and the rest is due to the reduction in the fraction of wasteful innovation.

The DE exhibits too little innovation — the SB market tightness, θ^{SB}, is 1.7154. Thus, in the SB the percentage of innovations which represent a wasteful duplication of effort, ω^{SB}, is 52%. The SB features a fraction of uninnovated research avenues of 18%. While this is still quite sizable, it is about half of that in the DE. As a consequence, the SB growth rate (of 2.17%) is considerably larger than the one in the DE. Eliminating the frictions in the planner's allocation results in a 16.15% welfare gain. Of this 5.6pp is the gain due to eliminating the fraction of foregone innovation and the rest is due to eliminating the fraction of wasteful innovation.

The relative welfare costs of foregone and wasteful innovation are different in the decentralized equilibrium and the planner's allocation. This is the case because of two reasons. First, the planner chooses θ^{SB} which, at the margin, strikes a balance between these two welfare costs. As a result the fraction of foregone innovation in the SB is much smaller. Thus, eliminating this fraction leads to a relatively smaller welfare gain. Second, eliminating the frictions in the DE does not fully eliminate wasteful innovation. In particular, since the CE features $w^c = 31\%$, the reduction in the waste is only 8pp. On the other hand, if the planner could achieve the first-best, then all of the waste would be eliminated, leading to a reduction of 52pp.

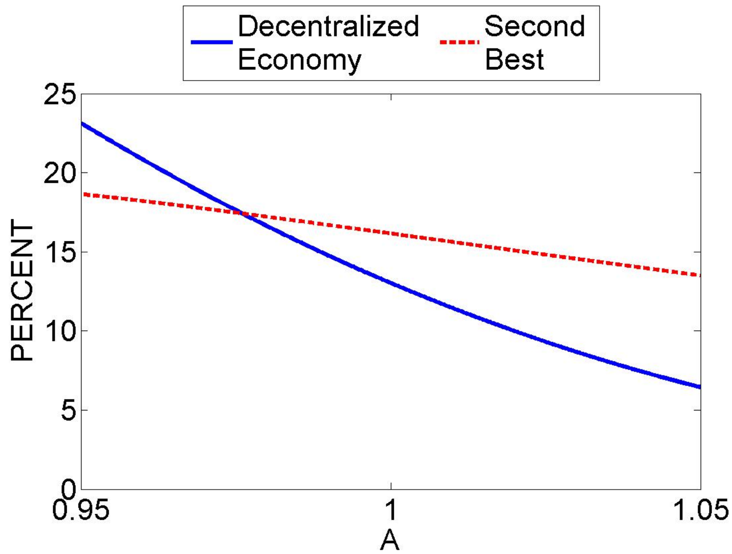

Figure 2.2: Welfare Gain: Comparative Statics

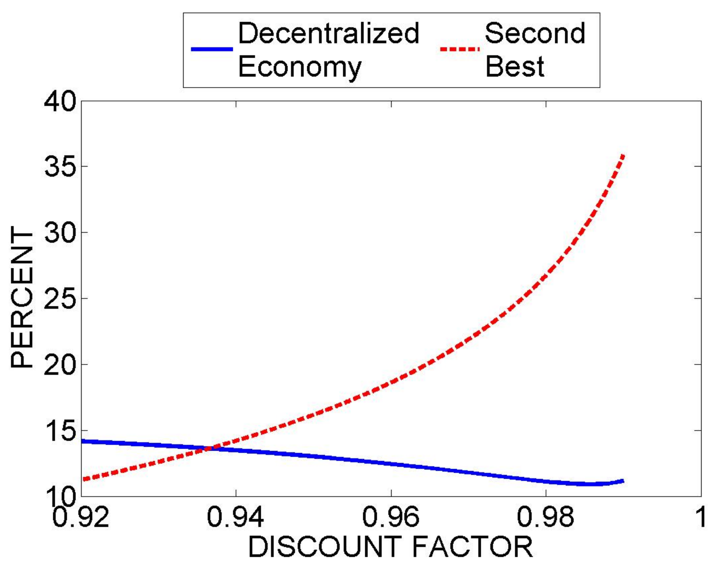

Figure 2.3: Welfare Gain: Comparative Statics

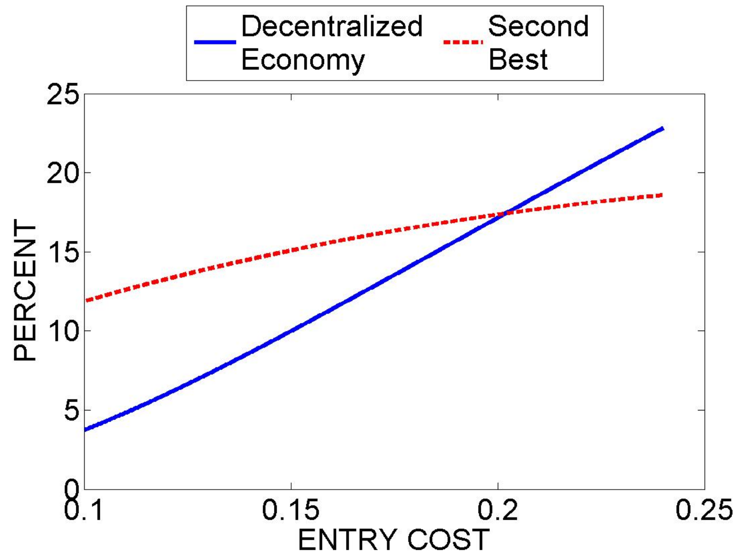

Figure 2.4: Welfare Gain: Comparative Statics

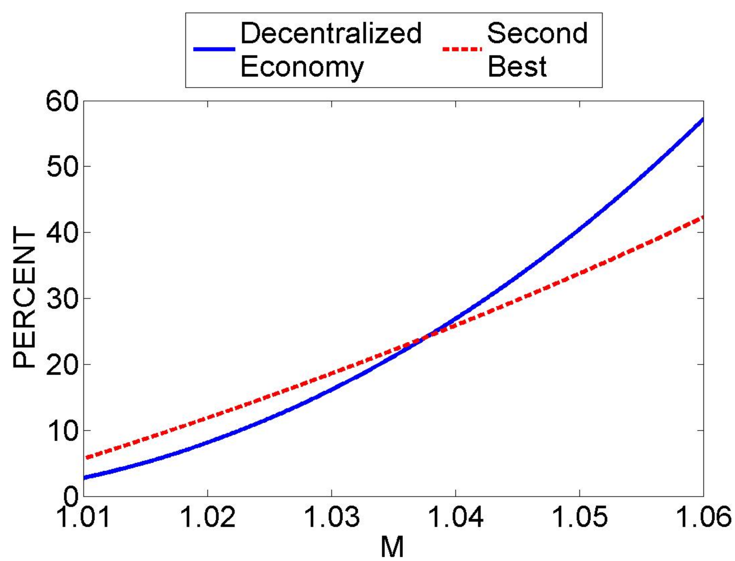

Figure 2.5: Welfare Gain: Comparative Statics

An increase in productivity (or a decrease in η, M) leads to a decrease in the welfare cost of frictions in both the DE and the planner's allocation (Figure 2.2, Figure 2.4, Figure 2.5).[30] In the DE, this is because high π (or low η, M) leads to a smaller fraction of foregone innovation and a lower level of amplification in the waste. In the planner's allocation, an increase in π^* (or a decrease in η, M) decreases the relative social cost of wasteful innovations. The planner, thus finds it optimal to increase the tightness. As a result the welfare costs of both foregone and wasteful innovation decrease.

An increase in the discount factor has two effects on the welfare gain. The first is the same as that of an increase in productivity. The second effect is directly related to consumers' impatience — high β makes agents more patient and as a result the same reduction in the growth rate has a higher welfare impact. At the extreme, as $\beta \to 1$, any reduction in the growth rate due to frictions generates infinite welfare losses. For the planner's allocation the latter effect always dominates, whereas for the DE it dominates only when β is sufficiently high (Figure 2.3).

Two potential shortcomings of our calibration strategy are that, in practice, not all firms use patents and not all patents are rejected because of simultaneous innovation.[31] To eliminate these potential issues, we turn to an alternative calibration strategy. Rather than matching the fraction of successful patent applications, we calibrate the market tightness,

[29] A detailed explanation of the welfare calculations is included in Appendix A.1.

[30] Unless on the horizontal axis, all parameter are set at their calibrated values. Different levels of productivity capture different levels of π and π^*.

[31] For a recent survey of the literature on the choice between formal and informal intellectual property protection mechanisms see, for example, [37].

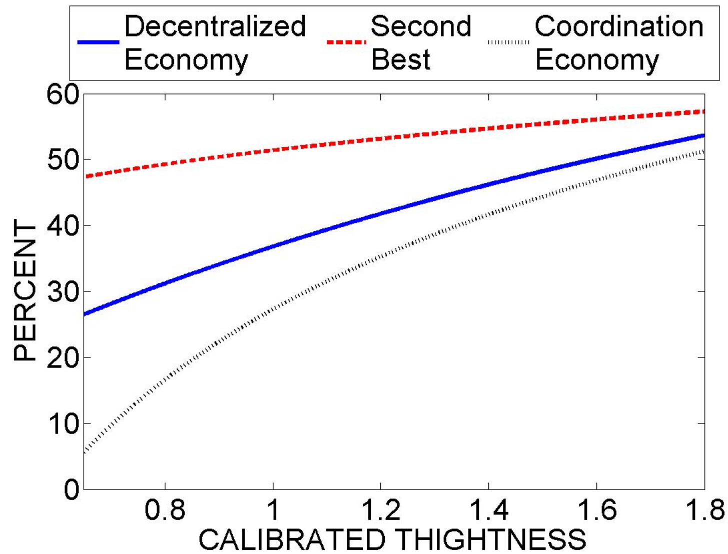

Figure 2.6: Wasteful Innovation

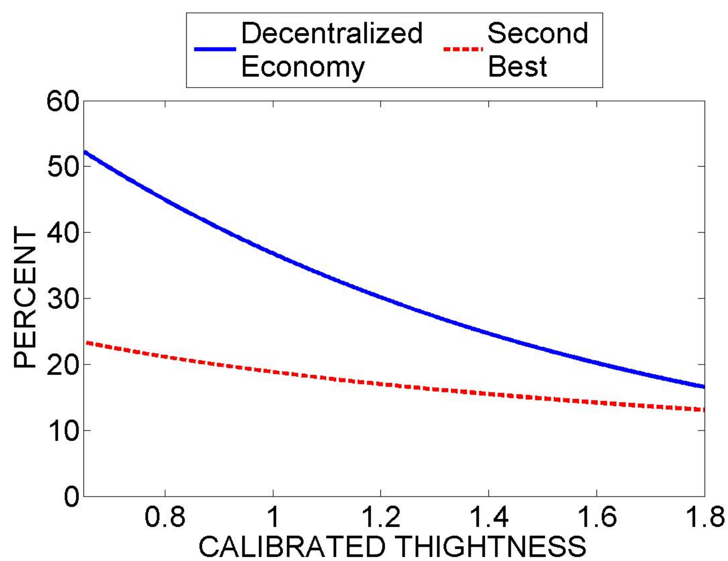

Figure 2.7: Foregone Innovation

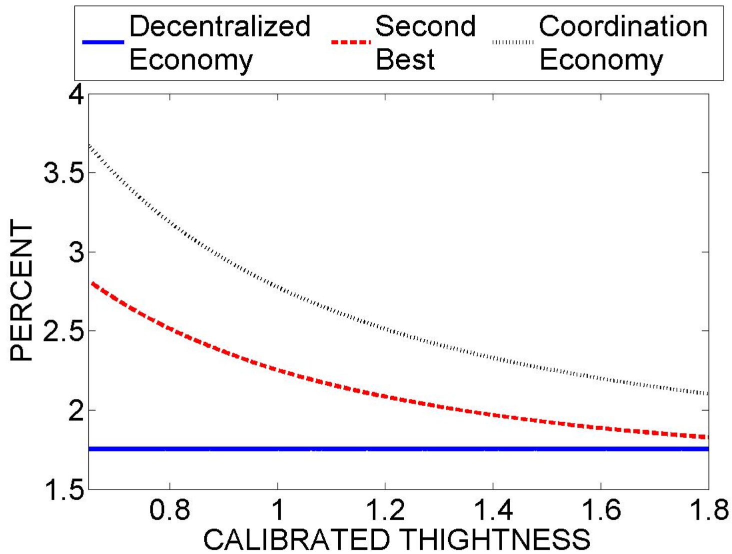

Figure 2.8: Growth Rate

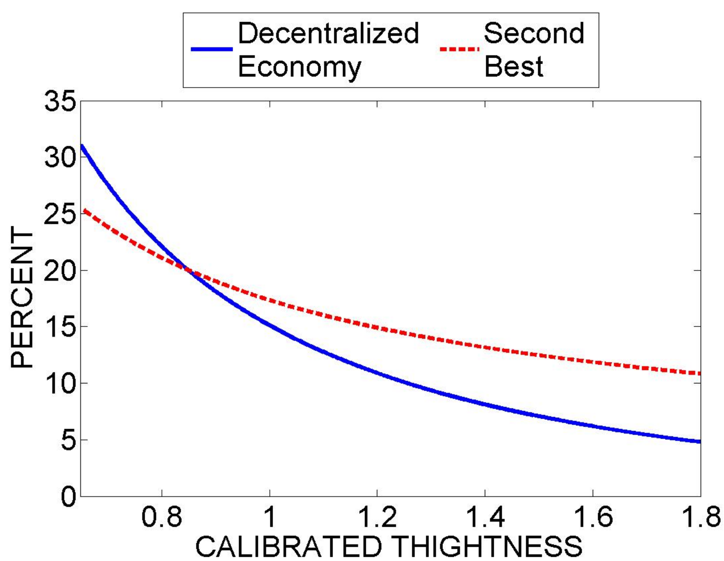

Figure 2.9: Welfare Gain

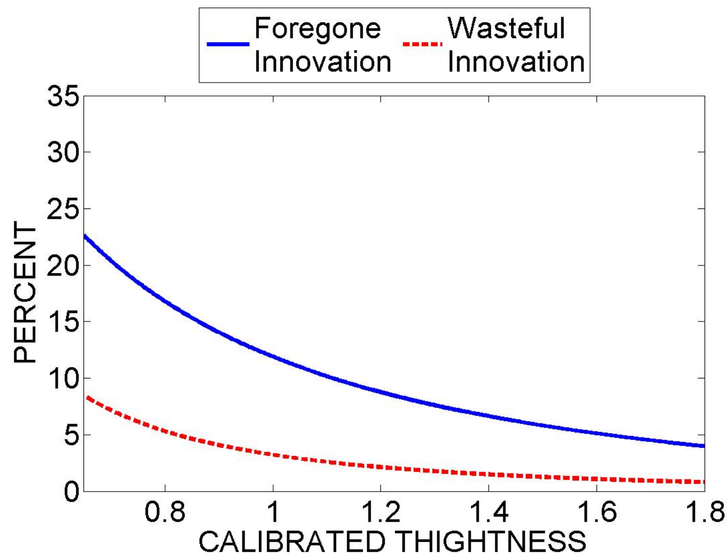

Figure 2.10: Welfare Gain: Decentralized Economy

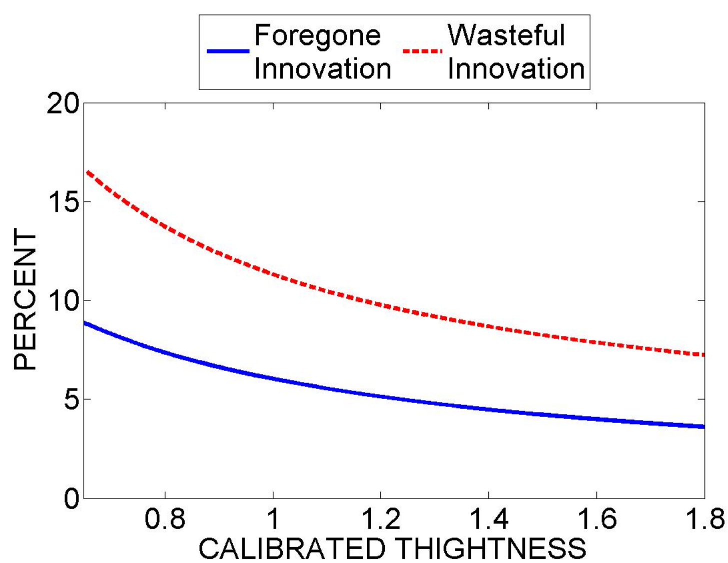

Figure 2.11: Welfare Gain: Planner's Allocation

θ, so as to be consistent with estimates on the return to R&D expenditure from the existing literature.[32] The majority of these estimates suggest the rate of return for the U.S. economy is between 20% to 40%. This yields a market tightness $\theta \in [0.6471, 1.8039]$.

Figure 2.6, Figure 2.7, Figure 2.8, Figure 2.9, Figure 2.10, Figure 2.11 illustrate the quantities of interest for the different values of the calibrated market tightness. The welfare gain is substantial for all considered values — it is at least 4.7% for the decentralized economy and 10.8% for the planner's allocation (Figure 2.9). Furthermore, the gain is decreasing in the calibrated value of the tightness. Intuitively, higher θ implies a lower fraction of foregone innovation, $e^{-\theta}$, which in turn decreases the difference between the CE and DE growth rates, $g^c - g = e^{-\theta}(M-1)$. At the same time, the reduced growth rate gap implies that the amplification in the fraction of wasteful innovation, $(\omega - \omega^c)$ is smaller. Both of these effects serve to mitigate the impact of the coordination frictions and as a consequence the welfare gain from eliminating these frictions. The intuition for the case of the planner's allocation is slightly different and it serves to explain why the welfare gain decreases relatively less than the gain in the decentralized case. Firstly, a higher calibrated market tightness implies a lower parameter value for the entry cost, η, and for the number of new ideas generated per new variety, M. Thus, the planner finds it optimal to set a higher SB market tightness, since innovation is cheaper and the net present value of implied profits higher. However, this increase is relatively smaller than the corresponding increase in θ. Thus, the response in $e^{-\theta^{SB}}$ and the SB growth rate is relatively smaller. At the same time a higher θ^{SB} leads to a larger fraction of wasteful duplication of effort, ω^{SB}. This effect puts an upward pressure on the welfare gain as θ^{SB} increases. Nonetheless, the

[32] For a survey see, for example, [38].

welfare cost of wasteful innovation decreases because of the lower entry cost and ratio of ideas to varieties, $(\nu_t/N_t)^{SB}$. The resulting net effect on the welfare gain due to eliminating wasteful innovation is negative. Yet, this effect is smaller than the one in the DE and as a result the welfare gain in question is larger for all considered values of θ.

We explore the robustness of the quantitative results in an extension of our baseline model. In particular, our augmented model features uncertainty in the innovation process and endogenous firm-level research intensity. Upon choosing a direction for their effort, R&D firms decide on an intensity i and incur the cost ϕi ($\phi > 0$). The amount of effort devoted affects their probability of successfully innovating the idea according to $Pr(\text{success}) = 1 - e^{-\gamma i}$ ($\gamma > 0$). The welfare cost of frictions in this extension is virtually the same as in the baseline model, so the results are presented in Appendix A.2.

The numerical exercise suggests that firms' failure to coordinate is likely to have a substantial impact on welfare. Thus, it may be a worthwhile endeavor to devise and analyze policies aimed at mitigating the coordination problems. One such policy might be for the government to allocate project-specific grants to firms. In our current model, for example, the planner could set a high enough tax rate on R&D investment so that no firm finds it worthwhile to engage in innovative activity. She can then allow firms to apply for a subsidy to work on a specific idea prior to entering the R&D sector. Each firm chooses one project and applies for a subsidy to innovate that particular idea. Since applications are costless, all ideas receive at least one application, so there would be no forgone innovation in equilibrium. The planner can then eliminate wasteful duplication of effort by granting a subsidy to a single firm per idea. A careful examination of such policies and their practical feasibility, however, is left for future work.

2.6 Conclusion

We develop an expanding-variety endogenous growth model in which firms direct their investment towards a specific research avenue (out of a scarce mass of potential R&D projects), but cannot coordinate their efforts. Due to the coordination frictions, the equilibrium number of firms which innovate the exact same idea is a random variable with mean given by the tightness in the market for ideas. Because of the frictions in our model, a fraction of research avenues remain uninnovated. This foregone innovation reduces the growth rate which in turn generates a general equilibrium effect that amplifies the fraction of wasteful simultaneous innovation. Furthermore, these frictions reduce the equilibrium level of research intensity. In the planner's allocation, the frictions amplify both the fraction of foregone and wasteful innovation. Whether or not they reduce the level of the optimal research intensity, however, depends on parameter values.

Our paper gauges the impact of coordination frictions on the growth rate and welfare. In the benchmark calibration, eliminating the coordination failure in the decentralized economy results in a 13% welfare gain, whereas the gain in the planner's allocation is 16.15%. Furthermore, the majority of the welfare gain is due to eliminating the welfare cost of foregone innovation in the decentralized case and due to eliminating the welfare cost of wasteful simultaneous innovation in the planner's allocation.

Chapter 3

Simultaneous Innovation and the Cyclicality of R&D

3.1 Introduction

Economic activities that promote long-term growth are affected by macroeconomic fluctuations. Specifically, the pro-cyclical nature of innovation, as measured by investment in R&D, is a well-established empirical regularity.[1] Whereas the existing literature has proposed several explanations behind this positive relationship, the theoretical models predict near-perfect correlation between output and investment in R&D.[2] This strong relationship is at odds with the empirically observed mild correlation between output and R&D.[3]

This paper fills this gap in the literature by developing an expanding-variety en-

[1] See, for example, [35], [31], [64], [23], [9], [56], and [30].

[2] See, for example, [31], [23], [9], and [32].

[3] The correlation between the cyclical components of output and R&D in the data is between 0.3 and 0.5, depending on the data source. For further details see, for example, [23], [32], and section 3 of this paper.

dogenous growth model that can resolve the discrepancy. The model delivers an endogenous mechanism that can break the near-perfect correlation between R&D investment and output. This mechanism relies on two key features of our model: there is the possibility that several firms make the same innovation simultaneously and the quality of innovations is endogenous. A calibrated version of the model closely matches the pro-cyclicality and volatility of R&D observed in the data

The source of technological progress in our model is the invention and adoption of new intermediate varieties. As in [51], the innovation process makes the distinction between potential innovations (ideas) and actual innovations (new varieties). Upon entry into the R&D sector each firm is randomly matched with a particular idea from a pool of feasible research avenues. In particular, there is the possibility that some ideas are simultaneously innovated by many firms while others are not innovated at all. This assumption is motivated by the fact that the phenomenon of simultaneous innovation is commonly observed in practice. For example, on February 14, 1876 Alexander Bell and Elisha Gray applied for a patent over the telephone within hours of each other. This same phenomenon is observed with virtually every major innovation from history, such as the cotton gin, the steam engine, the laser, and the computer.[4] Furthermore, instances of simultaneous innovation have also been documented in many cases for non-major innovations as well.[5]

Our model allows for the quality of innovations to be endogenous. All varieties are equally productive but innovations differ in their level of pervasiveness. Specifically, firms can develop new varieties only when there are available avenues for research. Because

[4]See, for example, [52].
[5]See, for example, [21] and Chapter 2 of this dissertation.

of this, positive long-run growth in our model necessitates an expanding pool of ideas. Hence, we follow [61] and [48], among others, and assume that knowledge is cumulative — developing a single idea into a new variety allows firms to "stand on the shoulders of giants" and gain technological access to a number of new avenues of research. The quality level of an innovation is captured by this number of new ideas — the higher the number, the more pervasive the technology. Thus, the quality of innovations in our model is similar in spirit to the notion of general-purpose-technology (GPT henceforth).[6] In particular, the more new ideas enter the pool from a single innovation the more pervasive it is and the higher its chance of being a GPT. To capture the intuition that the quality of innovation depends on the level of effort devoted to it, we allow for the pervasiveness of the technology to depend on the number of firms that make the same innovation simultaneously.

The main contribution of our paper is the model's ability to reproduce the mild pro-cyclicality of R&D observed in the data. In particular, in our model the correlation between output and R&D is 0.43 and R&D is 1.77 times as volatile as output. In the data, the correlation is 0.43 and the relative volatility of R&D is 1.79 times that of output. The relationship between the model-generated series of output and R&D is not strong because the model features endogenous oscillations driven by simultaneous innovation and the endogenous quality of innovation. In particular, following a positive technology shock both R&D and output have similar convergent paths that take them to their new balanced growth paths (BGP henceforth). However, during this transition both series oscillate around their convergent paths in such a way that whenever output overshoots its convergent path, R&D investment undershoots it and vice versa.

[6] See, for example, [18], [39], [44], and [40].

To see the intuition behind why the economy exhibits these oscillations, suppose that at a given period τ the economy features relatively more varieties and relatively less available research avenues. This scarcity of ideas implies that there will be few innovations made in the economy and as a result relatively fewer varieties the next period. This relatively low number of varieties at $\tau + 1$ increases the expected profitability of innovation. Hence, firms have more incentives to enter the R&D sector. This in turn leads to higher congestion in the "market" for ideas (i.e. relatively higher ratio of firms to ideas) and to a higher average number of firms that simultaneously innovate the same idea. Because of this, the average quality of innovations is higher, which leads to more feasible research avenues at period $\tau + 1$. Thus, at period $\tau + 1$ the economy features relatively more ideas and fewer varieties. As a result, at that period, there is relatively more innovation which leads to more varieties at $\tau + 2$. This then leads to lower expected profits and lower incentives for firms to enter the R&D sector, which ultimately leads to lower mass of ideas at $\tau + 2$. Furthermore, in periods when there are more varieties output is relatively high, whereas R&D investment is relatively low because research avenues are scarce. Conversely, when ideas are relatively abundant R&D investment is high and output is low because such periods feature a relatively lower mass of varieties.

The rest of the paper is organized as follows. Section two introduces the environment and characterizes the equilibrium. Section three presents the main results of the paper. Section four concludes.

3.2 The Economy

There are three types of agents in the economy — a final good producer, a unit measure of consumers, and a continuum of R&D firms. Time is discrete and infinite. The final good firm employs capital, labor, and intermediate varieties, which it uses to produce a single final good. Consumers supply labor, own the capital stock and the R&D firms, and consume the final good. R&D firms employ labor and engage in innovative activities, upon successful innovation they produce the intermediate varieties.

3.2.1 Final Good Sector

The final good, Y_t, is produced by a single price taker. We follow [23] and endow the firm with the following technology:

$$Y_t = A_t(K_t^{\alpha} LP_t^{1-\alpha})^{1-\sigma}\Big(\int_0^{N_t} X_t^{\lambda}(n)dn\Big)^{\frac{\sigma}{\lambda}}, \quad \alpha, \sigma, \lambda \in (0,1) \tag{3.1}$$

The price of the final good is normalized to unity. The firm rents capital, K_t, from households at the rate $r_t+\delta^K$, where δ^K is the depreciation rate of capital and r_t is the households' rate of return. It faces a competitive market for labor in production, LP_t, which is hired at the wage w_t. $X_t(n)$ is the amount of a particular variety n employed in production and N_t is the mass of intermediate varieties. The final good firm faces a monopolistically competitive market for these varieties, where a unit of each variety n is bought at the price $P_t(n)$.

We follow the RBC literature and assume the only source of aggregate uncertainty in the model is a productivity shock. In particular, the productivity parameter, A_t, follows an AR(1) process in logs:

$$A_{t+1} = A_t^{\rho} u_{t+1}, \tag{3.2}$$

where $\rho \in (0, 1)$ is a persistence parameter and u_{t+1} is a unit mean shock with variance σ_u.

The usual profit maximization of the final good firm implies the following demand functions for labor in production, capital, and intermediate varieties:

$$w_t = (1-\alpha)(1-\sigma)\frac{Y_t}{LP_t}, \tag{3.3}$$

$$r_t = \alpha(1-\sigma)\frac{Y_t}{K_t} - \delta^K, \tag{3.4}$$

$$P_t(n) = \sigma X_t^{\lambda-1}\frac{Y_t}{\int_0^{N_t} X_t^{\lambda}(n)dn}. \tag{3.5}$$

3.2.2 R&D Sector

The innovation process consists of three stages and makes the distinction between potential innovations (ideas) and actual innovations (new varieties). At stage one, firms enter the R&D sector at a cost $\eta/(N_t A_t)$ units of labor. The entry cost depends on both the knowledge stock of the economy and the aggregate productivity, A_t. As in [61], among others, the cost of innovation decreases as the mass of varieties expands, which allows the model to exhibit positive growth in the long-run. In the spirit of the RBC literature we consider an economy-wide productivity shock. To this end, we follow [12] and assume that the entry cost is decreasing in A_t.[7] Let μ_t be the mass of R&D entrants and LR_t be the total amount of labor employed in R&D. Then, the economy-wide research production function is given by

$$\mu_t = A_t N_t LR_t \eta^{-1}. \tag{3.6}$$

At stage two, firms are matched a particular idea from a finite mass ν_t of feasible research avenues. Given that, in practice, there are many possible research avenues and

[7] If the entry cost were not dependent on A_t, then it would have been a sector specific shock.

firms often rely on secrecy as an intellectual property protection mechanism, the "market" for ideas is likely to suffer from coordination frictions.[8] To capture these frictions in the economy we follow the previous literature and assume the number of firms matched with a particular idea is a random variable that follows a Poisson distribution with a mean equal to the tightness in the market for ideas, $\theta_t = \mu_t/\nu_t$.[9] The random realization of these draws is what gives rise to the possibility of simultaneous innovation in our economy — some research avenues are innovated by many firms simultaneously while others are not innovated at all. The likelihood of simultaneous innovation is captured by the market tightness — a higher θ_t implies the market is relatively more congested and as a result the average number of firms which innovate the same idea is higher. Ideas are ex-ante identical and transform into exactly one new variety if innovated. Innovation is uncertain and takes one period to complete. A firm which enters at time t is successful in innovating its project at time $t+1$ with probability p. With probability $1-p$ the firm fails at innovating the idea and exits the innovation sector. This implies that the number of firms which successfully innovate a particular idea follows a Poisson distribution with mean $p\theta_t$.[10] If innovation is successful, firms apply for a patent over the variety. Each variety is protected by a single patent — in the event that several firms innovate the same idea simultaneously, each receives the patent with equal probability.

At stage three, firms that have secured a patent over a variety produce it in a monopolistically competitive market. We normalize the average and marginal costs of production to unity, so profits are given by $\pi_t(n) = (P_t(n) - 1)X_t(n)$. Patents grant

[8] For further details see, for example, Chapter 2 of this dissertation.
[9] See, for example, [45], [19], [63].
[10] See, for example, [51].

perpetual monopoly, but varieties become obsolete with probability δ. Thus, the value of holding a monopoly of a variety n at time t, $V_t(n)$, is given by

$$V_t(n) = E_t \sum_{i=t+1}^{\infty} (1-\delta)^{i-t} d_{it} \pi_i(n), \tag{3.7}$$

where d_{it} is the stochastic discount factor.

A necessary condition for positive long term growth in the model is that the mass of ideas, ν_t, grows at a positive rate. We follow [48] and [61], among others, and assume that knowledge is cumulative. In particular, the act of innovating an idea and introducing it on the market for intermediate goods allows firms to "stand on the shoulders of giants" and gain access to new avenues for research. Thus, innovating an idea at time t allows, on average, $M(\theta_t)$ new ideas to enter the pool at time $t+1$. The function $M(\theta_t)$ is assumed to be increasing in the market tightness, θ_t, and $M(\theta_t) > 1$ for all positive θ_t.

In our model varieties are equally productive, however, the quality of innovations (as captured by $M(\theta_t)$) is endogenous. In particular, the average number of new research projects which enter the pool from the innovation of a single idea, $M(\theta_t)$, to depend on the market tightness. This captures the intuition that firms would make higher "quality" innovations if they invest more, on average, in R&D. As all innovations are equally productive, the quality of innovations in our model is measured by how many new ideas stem from that innovation. This is in the spirit of the literature on GPT.[11] One of the main features of GPT is that it is pervasive and allows for innovative complementarities — its innovation allows the development of many other R&D projects. In the context of this theory, our model allows for GPTs to arise endogenously due to increased effort in innovation — the

[11] See, for example, [18], [39], [44], and [40].

higher the average spending on a particular idea, the higher the chance it will be innovated into a GPT.

To understand the intuition clearly, let us consider an example. Suppose that whenever a firm innovates a particular idea, say a new computer microchip, the number of new research avenues that stem from the innovation is a random variable with CDF $F(q)$. The number of new ideas, q, captures the level of pervasiveness of the innovation. This level would, of course, depend on the technological characteristics of the microchip. The size of the microchip and the temperature at which it runs determines the set of devices with which it can be integrated — a relatively large microchip may only be feasibly used in desktop computers, if the chip is smaller it may also be feasibly integrated with laptops, if it is smaller still it may be feasibly integrated with smartphones and/or smartwatches. Then, a new avenue of research, into say smartwatches, will become available only when at least one firm draws a sufficiently high quality, q. Thus, if exactly m firms successfully innovate the microchip the number of new feasible R&D projects due to the invention of the chip, k, would be the sample maximum, i.e. $k \sim F(k)^m$. Since the number of innovators, m, follows a Poisson distribution with mean $p\theta_t$, then the average number of new ideas which become feasible from the development of a single innovation is given by $M(\theta_t) = \sum_{k=0}^{\infty} k(e^{F(k)p\theta_t} - e^{F(k-1)p\theta_t})e^{-p\theta_t}/(1 - e^{-p\theta_t})$. Thus, the higher the expected R&D effort devoted to the microchip, $\eta\theta_t/(N_t A_t)$, the higher the expected number of new avenues for future research that become available from the innovation of the microchip. This is because a higher economy-wide R&D investment leads to more congestion in the market for ideas, which in turn implies a higher expected number of firms simultaneously innovate the microchip.

Once a variety is innovated, it is no longer a potential R&D project, so the corresponding idea is removed from the pool. As a result the average net increase in the stock of research avenues from innovating one new variety is $M(\theta_t) - 1$. Furthermore, the matching technology implies that only a fraction $1 - e^{-p\theta_t}$ of ideas are innovated each period. Then, the law of motion for ideas is given by

$$\nu_{t+1} = (1 - \delta^{\nu})\nu_t + (1 - \delta^{\nu})(1 - e^{-p\theta_t})(M(\theta_t) - 1)\nu_t, \tag{3.8}$$

where δ^{ν} is the probability an idea becomes obsolete.

As each innovated idea is transformed into a new variety, it follows that varieties have the following law of motion

$$N_{t+1} = (1 - \delta)N_t + (1 - \delta)(1 - e^{-p\theta_t})\nu_t. \tag{3.9}$$

3.2.3 Households

There is a unit measure of infinitely lived identical consumers. They discount the future with a factor β and have the per-period utility function $U(C_t, L_t) = \ln C_t - \chi L_t^{1+1/\phi}/(1 + 1/\phi)$, where C_t is consumption, L_t is labor hours, ϕ is the Frisch elasticity of labor supply, and χ governs the disutility of labor. Since labor can be devoted to production or R&D, it follows that

$$L_t = LR_t + LP_t. \tag{3.10}$$

Households own capital and have access to a mutual fund that covers all R&D firms. Let a_t denote the amount of shares held by the representative household at the beginning of period t. Firms distribute all profits as dividends, so the total assets of households in the beginning of t are $a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn + (1 + r_t)K_t$. At time t households choose shares a_{t+1} of the mutual fund which covers all R&D firms even though a fraction δ of varieties become obsolete next period. Thus, the household budget constraint is given by:

$$K_{t+1} + a_{t+1} \int_0^{N_{t+1}} V_t(n)dn = (1 + r_t)K_t + a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn + w_t L_t - C_t. \quad (3.11)$$

3.2.4 Equilibrium

Intermediate good producers maximize per period profits subject to the inverse demand function given by equation (3.5). This yields $P_t = 1/\lambda$ and

$$X_t = \lambda\sigma \frac{Y_t}{N_t}, \quad (3.12)$$

$$\pi_t = (1 - \lambda)\sigma \frac{Y_t}{N_t}, \quad (3.13)$$

$$Y_t = (A_t(\sigma\lambda)^\sigma)^{\frac{1}{1-\sigma}} K_t^\alpha L P_t^{1-\alpha,} N_t^{\frac{\sigma(1-\lambda)}{\lambda(1-\sigma)}}. \quad (3.14)$$

Specifically, each monopoly position is equally profitable. Furthermore, profits depend on the amount of intermediate varieties, N_t, and on the concavity of the production function. If $\sigma/\lambda > 1$, then the production function exhibits increasing returns to scale and as a result profits are increasing in N_t. If, on the other hand, $\sigma/\lambda < 1$, then there are decreasing marginal returns to the extra variety and profits are decreasing in N_t.

At stage one of the innovation process, free entry implies that

$$\frac{\eta w_t}{A_t N_t} = \frac{1 - e^{-p\theta_t}}{\theta_t} V_t. \quad (3.15)$$

Each entrant must higher $\eta/(A_t N_t)$ units of labor at the market wage w_t, so the left hand side of (3.15) captures the cost of engaging in R&D activities. Firms are successful in innovating with probability p. If they do innovate, then they receive the patent with probability $\sum_{m=0}^{\infty} Pr(\text{exactly } m \text{ rivals successfully innovate the same idea})/(m+1) = \sum_{m=0}^{\infty} e^{-p\theta_t}(p\theta_t)^m/(m+1)! = (1-e^{-p\theta_t})/(p\theta_t)$. In particular, the more congested the market is the higher the expected number of rivals who innovate the same idea simultaneously, $p\theta_t$, and as a result the lower the probability of receiving the patent. So the expected benefit from entering the R&D sector is given by the right hand side of (3.15).

The first-order conditions of the representative household yield

$$w_t = \chi C_t L_t^{\frac{1}{\phi}}, \tag{3.16}$$

$$\frac{1}{C_t} = \beta E_t\left(\frac{1}{C_{t+1}}(1+r_{t+1})\right), \tag{3.17}$$

$$V_t = (1-\delta)\beta E_t\left(\frac{C_t}{C_{t+1}}(\pi_{t+1}+V_{t+1})\right), \tag{3.18}$$

where (3.18) makes use of the symmetry in varieties and their law of motion. Furthermore, the stochastic discount factor is $d_{it} = (\beta(1-\delta))^i C_t/C_{t+i}$.

Lastly, we can combine the consumer's budget constraint, (3.11), with the demand for capital, (3.4), for labor in production, (3.3), and the free entry condition, (3.15), to get the law of motion for capital

$$K_{t+1} = (1-\delta^K)K_t + Y_t - X_t N_t - C_t. \tag{3.19}$$

3.3 Calibration and Numerical Results

3.3.1 Calibration

We follow the RBC literature and examine a first-order approximation of the model in order to study its dynamical properties. Specifically, we examine a log-linearized version of our model around the deterministic BGP. A detailed description of the deterministic BGP is in the appendix.

We follow the previous literature (see, for example, [9]) and calibrate the model at annual frequency. Hence, the discount rate is set at $\beta = 0.95$. The capital's share of output, α, is 0.33 and its depreciation rate, δ^K, is 0.08. Following [23] we set the materials' share of output, σ to 0.5 and the persistence parameter, ρ, to 0.88. Set the Frisch elasticity of labor supply, ϕ, to 4 and normalize $L_t = 1$. This yields $\chi = 0.8472$. Following [12], the obsolescence rate of varieties, δ, is set to 0.1. To the best of my knowledge, there is no empirically established value of the obsolescence rate for ideas, δ^ν. Since ideas are simply uninnovated varieties, we set $\delta^\nu = 0.1$ as well. The probability of successfully innovating a research project, p, turns out to be a scaling parameter, so we normalize it to unity.

To pin down the entry cost, η, the gross markup, $1/\lambda$, and the value of $M(\theta_t)$ along its deterministic balanced growth path, $M(\theta)$, we use three balanced growth path restrictions. First, we use data on the fraction of approved patent applications in the U.S. for the period from 1966 to 2011.[12] We match its empirical average of 0.60957 to its model counterpart, $(1 - e^{-p\theta})/(p\theta)$. This yields $\theta = 1.0876$. Second, we set the R&D share of output, $\eta w_t \mu_t/(N_t A_t Y_t)$, to the average in the U.S. — 3.1194%.[13] Third, we calibrate the growth rate of output to its empirical counterpart for the period of 1.7546%. This yields $\eta = 0.1879$, $1/\lambda = 1.0841$, and $M(\theta) = 1.4168$.

Lastly, to calibrate the volatility of the technology shock, σ_u, and the elasticity of

[12]The data on both patents and patent applications is taken from the U.S. Patent and Trademark Office. The data on patent grants is by year of patent applications.

[13]The data is taken from the U.S. Bureau of Economic Analysis. The data on non-farm GDP is in chained 2009 dollars and is taken from NIPA table 1.3.6. The data for R&D expenditures is from NIPA table 5.6.5 and includes software expenditures. To deflate the series for R&D we use the implicit GDP price deflator from NIPA table 1.1.9.

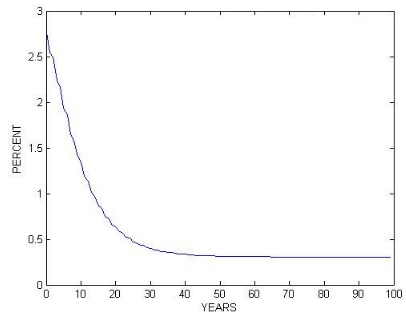

Figure 3.1: Impulse Response Function: Output

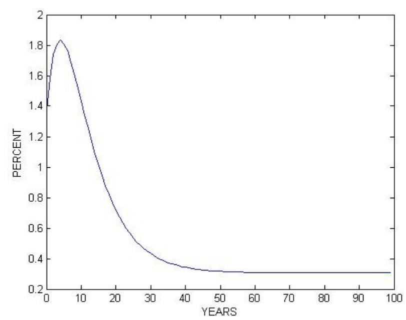

Figure 3.2: Impulse Response Function: Consumption

$M(\theta_t)$ along the balanced growth path, $\varepsilon_{M,\theta}$, we use two second moment conditions. We set $\sigma_u = 0.01074$ to match the standard deviation of per capita non-farm GDP in the data of 2.7279%.[14] To match the standard deviation of per capita patent applications, $p\mu_t$, to its empirical counterpart of 3.9786%, calibrate $\varepsilon_{M,\theta} = 8.97$.

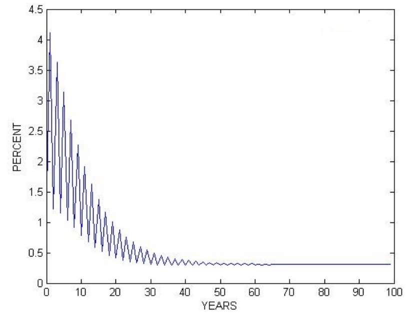

Figure 3.3: Impulse Response Function: R&D

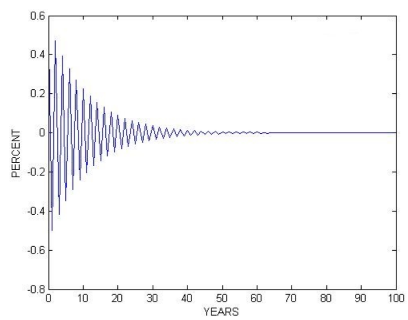

Figure 3.4: Impulse Response Function: Market Tightness

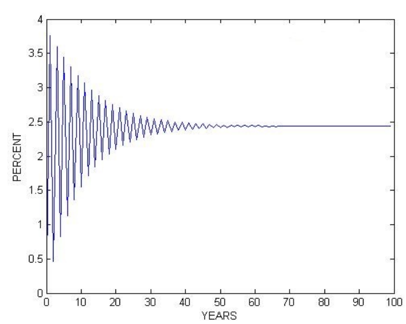

Figure 3.5: Impulse Response Function: Pool of Ideas

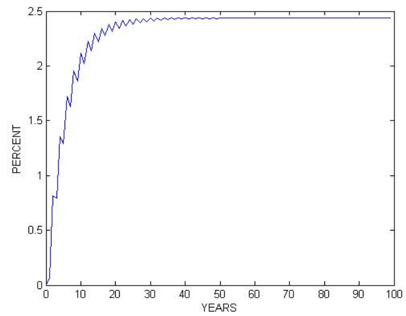

Figure 3.6: Impulse Response Function: Varieties

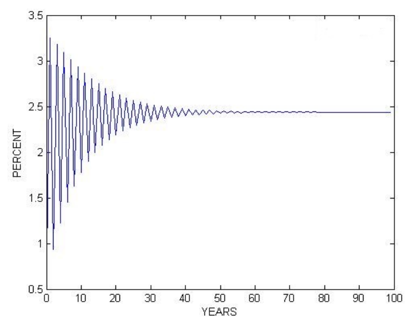

Figure 3.7: Impulse Response Function: Entry

3.3.2 Impulse Response Functions

Because the model features endogenous growth, we construct the impulse response functions as percentage deviations from the deterministic balanced growth path of the economy. Following a positive technology shock, the economy converges to a new, higher BGP.

[14]All per capita variables are normalized by the civilian non-institutionalized population. The data on this is taken from the Bureau of Labor Statistics' Employment Situation Release.

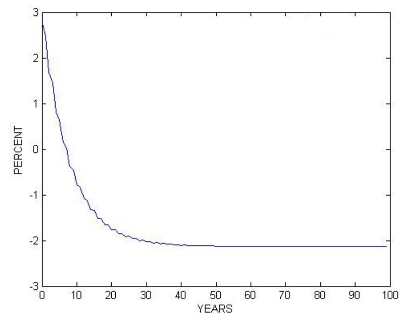

Figure 3.8: Impulse Response Function: Profits

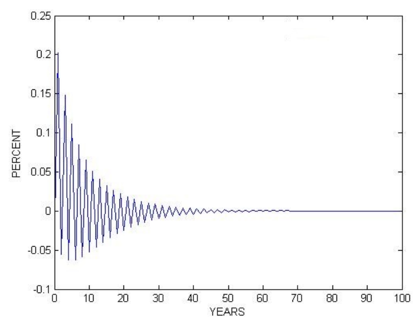

Figure 3.9: Impulse Response Function: Hours in Research

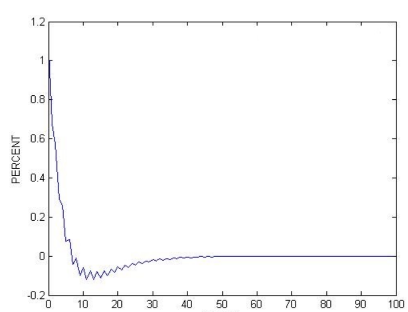

Figure 3.10: Impulse Response Function: Hours in Production

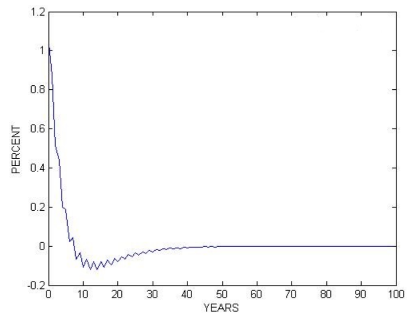

Figure 3.11: Impulse Response Function: Hours

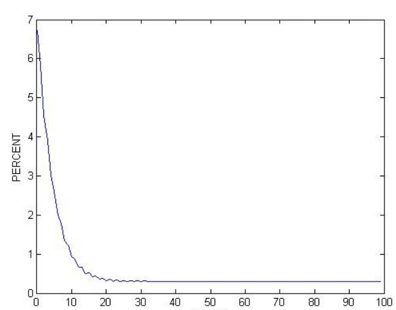

Figure 3.12: Impulse Response Function: Investment

Output, consumption, investment in R&D, labor hours, investment, and profits initially increase and overshoot their new BGPs. In subsequent periods, they gradually converge to the higher BGP. Varieties, on the other hand, converge to the new BGP without initially overshooting it.

As is evident from the impulse response functions, the model exhibits endogenous oscillations. Whereas all variables feature these oscillating behavior, it is more pronounced in the ones that describe the R&D sector. In particular, the pool of ideas, the mass of entrants, the market tightness, and labor hours in research all oscillate around their new BGP following a positive technology shock. Since the tightness and LR_t are constant along the BGP, they converge to their old levels. These endogenous oscillations are the mechanism which breaks the near-perfect correlation between output and R&D that is featured in the previous literature.[15] In particular, in periods when output overshoots its convergent path, investment in R&D undershoots it and vice versa.

The main reason why the oscillations are more pronounced in the R&D sector is twofold. First, households want to smooth consumption and leisure, but the volatility of the variables which affect the market for ideas does not directly impact utility. Thus, the oscillation is least apparent in consumption, labor hours, output, total investment, spending on varieties, and profits. Second, the main driver behind the oscillations in the economy is the market for ideas. To see the intuition behind this, suppose that at period τ there are relatively more varieties, N_τ, and relatively fewer ideas, ν_τ. The relative scarcity of ideas implies that the number of innovations, $(1 - e^{-p\theta_\tau})\nu_\tau$, is relatively low as well. This in turn leads to a low number of varieties the next period, $N_{\tau+1}$. Then, by equation (3.13), it follows

[15] See, for example, [23] and [32].

that expected profits next period, $E_\tau \pi_{\tau+1}$, is high since there is relatively less competition between intermediate good producers.[16] At the same time, higher mass of varieties, N_τ, decreases the cost of innovation. Both the lower entry cost and higher expected profits create incentives for firms to enter the R&D sector. As a consequence the market tightness, θ_τ, increases. The resulting higher congestion implies that more firms, on average, innovate the same idea simultaneously. This leads to a higher average quality of innovation, $M(\theta_\tau)$, and as a consequence to relatively high pool of available research projects the next period, $\nu_{\tau+1}$. Thus, at $\tau+1$ there are relatively low number of varieties, $N_{\tau+1}$, and relatively high number of ideas, $\nu_{\tau+1}$. Because of this, the number of innovations at time $\tau+1$ and subsequently the number of varieties $N_{\tau+2}$ are relatively high. This leads to relatively low expected profits, $E_{\tau+1}\pi_{\tau+2}$. Because of this and because of the relatively high entry cost at time $\tau+1$, the market tightness, $\theta_{\tau+1}$, is low. This leads to low congestion and lower average quality of innovations, which ultimately leads to low future pool of ideas, $\nu_{\tau+2}$. At time $\tau+2$ the cycle repeats.

3.3.3 Cyclicality and Second Moments

Table 3.1: Moments for Data and Model

	σ_X/σ_Y			Corr(X,Y)		
Variable X	Data	Benchmark Model	Exogenous Quality	Data	Benchmark Model	Exogenous Quality
R&D	1.79	1.77	0.81	0.43	0.43	0.96
Consumption	0.69	0.54	0.53	0.90	0.93	0.93
Hours	0.70	0.41	0.42	0.83	0.92	0.93
Investment	2.39	2.62	2.67	0.91	0.97	0.97

[16]Expected profits are decreasing in the number of varieties next period because in our calibration $\sigma < \lambda$.

The mechanism in our model that breaks the perfect correlation between output and R&D investment is the endogenous oscillations. Specifically, in periods when varieties are relatively abundant, output is relatively high because the final good firm can employ a wider range of intermediaries in production. These periods also feature a low mass of available research projects. This means that the market for ideas gets congested relatively easy and as a result fewer firms enter the R&D sector. This leads to low aggregate R&D investment. Thus, although both output and R&D investment initially overshoot their new BGPs and then converge to them gradually, they oscillate around their convergent paths in such a way that when output overshoots its path, R&D undershoots it and vice versa.

Table 3.1 reports the standard deviations and cyclical properties in the data and in our economy.[17] The model matches the moments in the data remarkably well. In particular, it is able to reproduce the mild procyclicality of R&D and its relative volatility. Alternative data sources for R&D yield a correlation between it and output between 0.3 and 0.5, and relative standard deviation of R&D around 1.9 times that of output.[18] The model does well against these alternative measures as well. The third and last columns of Table 3.1 highlight the importance of the endogenous quality in innovation for the model's ability to match the data. These columns report the moments for the model when innovation quality is exogenous, as captured by $M(\theta_t)$ being a constant. In that specification, there is no oscillating behavior in the economy since higher congestion this period does not imply more ideas next period. Because of this, output and R&D move very closely together and their correlation is almost perfect. Furthermore, the absence of oscillations decreases the relative

[17] The data is obtained from the U.S. Bureau of Economic Analysis. Investment in the model corresponds to investment in physical capital and in R&D.

[18] [32] use data from Compustat and find a correlation of 0.5 and relative standard deviation of 1.9. [23] find a correlation of 0.3 and standard deviation of 1.89 using data from the National Science Foundation.

volatility of R&D by about one half.

Table 3.2: Autocorrelation for Data and Model

	Autocorrelation		
Variable X	Data	Benchmark Model	Exogenous Quality
Output	0.55	0.41	0.43
R&D	0.76	-0.40	0.47
Consumption	0.67	0.59	0.60
Hours	0.62	0.38	0.36
Investment	0.57	0.39	0.37

The model is also able to match the autocorrelation of most variables relatively closely (Table 3.2). The only shortcoming is with respect to investment in R&D. This is because the endogenous oscillations in the model put a pressure on the autocorrelation coefficient to become negative. Since the oscillations in the economy are more pronounced in the innovation sector, the rest of the variables reported in Table 3.2 have positive autocorrelation coefficients. R&D investment in turn has a negative autocorrelation coefficient because the oscillations are very pronounced there. The version of the model with exogenous quality of ideas does better in this regard precisely because it features no endogenous oscillations.

3.4 Conclusion

We develop an expanding-variety endogenous growth model that can reproduce the empirically observed mild pro-cyclicality of R&D investment. The model is able to match the data because it features an endogenous mechanism that breaks the near-perfect correlation between output and R&D present in the previous theoretical literature. The mechanism relies on two features of our model — there is the possibility several firms simultaneously innovate the same research project and the quality of innovations is endogenous.

In periods when varieties are abundant and ideas scarce the economy exhibits a relatively low level of innovation and relatively high output. This low investment in R&D leads to a scarce mass of varieties next period, which increases the expected profitability of research. This creates extra incentives for firms to enter the R&D sector, which in turn increases the congestion in the market for ideas. This higher congestion then implies a higher number of firms, on average, innovate the same research project simultaneously. This ultimately leads to a higher average innovation quality and an abundant mass of ideas next period. The endogenously generated cyclical behavior of the economy implies that, following a technology shock, whenever output overshoots its convergent path, investment in R&D undershoots it and vice versa.

Chapter 4

The Patent System as a Tool for Eroding Market Power

4.1 Introduction

The prevailing view on the patent system is that it creates temporary monopolies in order to incentivize firms to innovate (the reward theory of patents) and disclose information (the contract theory of patents). According to the reward theory, on the one hand, patents allocate market power (appropriability) to firms, allowing them to yield profits from their innovative activity, which creates incentives to innovate. On the other hand, the generated monopoly produces a dead weight loss ([7], [55]).[1] The contract theory of patents claims that, by patenting, firms disclose useful information which allows others to "avoid duplication of research, possibly acquire useful knowledge and, when the patent expires,

[1] The resulting trade off between these two effects has been a major focus for a large body of literature (for surveys see [26] among others).

quickly imitate the innovation" — [37, p. 3]. The benefit to society from disclosure is then a justification for bearing the social cost of the dead weight loss due to monopoly.[2]

Both the reward and contract theory of patents have the salient feature that the patent system has to allocate market power (reduce ex-post competition).[3] The rationale behind this traditional view goes as follows. In the absence of a patent system firms can secure a certain level of appropriability through informal intellectual property protection mechanisms (IPPMs henceforth) such as secrecy and complexity. With a patent system, they have an extra mechanism at their disposal — a patent. If patenting reduces their appropriability, then firms opt for the outside option of informal IPPMs and the patent system does not affect the degree of competition. If, on the other hand, patenting increases appropriability, firms choose to patent and the patent system reduces competition. Hence, the patent system must (weakly) reduce competition. This rationale, however, ignores two important features of the patent system. First, an innovator's outside option of not patenting has different values with and without a patent system — with a patent system a potential duplicator can patent the innovation, whereas she cannot do so when there is no patent system. Second, patents provide a strategic advantage — a patent legally precludes rivals who have innovated simultaneously from commercially exploiting the innovation. In the US, for example, the patent holder can block all rivals that have began commercially exploiting the innovation less than one year prior to the patent filling date. Hence, it is easier to secure an initial monopoly position for innovators that patent, as compared to

[2]See, for example, [27] and [29].

[3]Throughout the paper we refer to competition as a measure of the ex-post number of innovating firms. In particular, we say that the patent system increases competition if the number of firms which can commercially exploit an innovation is larger when there is a patent system as compared to the case when a patent system is absent. Given the stochastic nature of the model, the criterion we use for the comparison of the degree of competition is first order stochastic dominance that we define formally in section four.

those who do not.

To capture the importance of these two aspects of the patent system, we develop a dynamic equilibrium model of innovation with features that are consistent with the empirical findings. First, firms try to secure a lead time advantage and can choose to protect their innovations with either a patent or secrecy.[4] Second, firms search simultaneously for ideas (potential R&D projects) as in [50] and [51].[5] Third, the dynamic nature of the model allows for duplicative innovation — there is a chance that a firm will independently innovate a previously developed innovation.[6]

We find that, in sharp contrast to the traditional view, the patent system can erode, rather than allocate market power to firms, i.e. it can increase competition, under a very general market structure. Moreover, the patent system can achieve this outcome, regardless of whether or not it provides prior user rights (PUR henceforth). To see the intuition behind this result, notice that if the patent system is to increase competition, then it has to provide weak patent protection so that it decreases firms' appropriability. At the same time, it also needs to deliver incentives for firms to patent, even though secrecy would give them stronger protection. When the patent system provides no PUR, the result is driven by its ability to punish those firms which keep their innovations secret. If an innovator chooses to opt for secrecy, then a duplicator can patent the innovation at a later

[4]For a discussion of the evidence see, for example, [22] and [8], for a recent survey see [37]. For other papers that allow firms to choose between patenting and secrecy see, for example, [6], [27], [28], [29], and [51].

[5]For example, [33] and [52] point out strong evidence that most significant innovations have been (nearly) simultaneously innovated. This assumption is in part motivated by the famous example of Alexander Bell and Elisha Gray telephone controversy — on February 14, 1876 Bell filed a patent application for the telephone and two hours later Gray filed a similar application for the same innovation.

[6]For models where firms innovate sequentially (duplicate previous innovations) see, for example [27], [28], [34], [66], [58] and [59].

time and block the original innovator from receiving any revenue from the innovation. In the absence of a patent system, however, the duplicator cannot patent and subsequently block the innovator. Thus, the innovator can earn positive profits even if the innovation is duplicated. Hence, by allowing the duplicator to patent and not providing PUR to the innovator, the patent system redistributes the profits of the original innovator, effectively punishing it for not patenting the innovation. By reducing the option value of secrecy, the patent system can induce firms to use patent protection, even when it results in a lower expected duration of monopoly than what they would have had in its absence, effectively increasing competition. In the presence of PUR, the patent system can increase competition by providing a strategic advantage to firms that patent. Given the assumption of lead time advantage, firms' investment decisions are tightly linked to the strategic aspect of patents. Most importantly, the strategic aspect allows firms that patent to not only have a higher chance of securing an initial monopoly position, but also to do so with lower investment in R&D, on average, as compared to secrecy using firms. Because of this, it is possible that the benefit due to the strategic advantage outweighs the loss in appropriability due to weak patent protection. If this is the case, some firms choose to patent, which leads to lower market power on average, i.e. increased competition.

To the best of our knowledge, this is the first paper which finds that the patent system can increase competition. There are a number of previous studies which feature a patent system with the potential of increasing competition (due to duplicative innovation and no PUR), yet they have not commented on this possibility.[7] However, none of the pre-

[7] For example, [66] develops a model of sequential innovation where the patent system may increase competition for some parameter values, but the author does not examine this feature of the model.

vious papers in the literature examine a model with the property that a patent system can increase competition when it provides PUR. Most previous studies abstract from simultaneous innovation, which is a necessary condition for both the strategic aspect of patents and the result when the patent system provides PUR. Even previous papers that have allowed firms to innovate simultaneously, however, find that the patent system increases market power. For example, [51] develop an equilibrium search model of simultaneous innovation, but do not take into account that firms will "race" against each other. In their model, investment decisions can only affect the probability of successfully innovating — they do not give firms lead time advantage. As the strategic aspect of patents is not linked to investment in R&D, it cannot deliver large enough benefits to firms for them to choose patenting when it yields lower appropriability than secrecy. Hence, their model underestimates the degree of the strategic benefit.

The ability of the patent system to increase competition has important practical applications. This paper analyses the planner's problem of choosing a patent protection strength in order to maximize welfare in the case of Bertrand competition and a patent system which provides PUR. We find that the patent system is always welfare improving — it can induce all firms to patent and subsequently disclose their innovations without increasing the expected duration of monopoly.[8] Depending on the size of the strategic advantage, however, the planner may find it optimal to provide weak patent protection so as to induce only a fraction of all firms to patent. When the protection is weak, patented innovations have a lower expected duration of monopoly as compared to innovations developed

[8]This paper abstracts from firms' incentives to innovate. Incorporating firms' incentives to innovate might be an interesting avenue for future research.

under secrecy. This reduces the average expected duration of monopoly in the economy and, hence, increases competition and welfare. At the same time, however, weak patent protection implies that some firms have an incentive to not disclose their innovations and resort to secrecy, instead. In the model, disclosing an innovation allows firms to acquire useful knowledge about new potential R&D projects. Thus, lower overall disclosure implies a lower steady state mass of ideas and innovations, which decreases welfare. The planner chooses the strength of patent protection which strikes a balance between these two opposing effects. Larger strategic benefits from patents induces a higher fraction of firms to use patent protection, for a given protection strength, which leads to higher competition and disclosure overall in the economy. Thus, if the strategic benefit is small, reducing the market power of firms is too costly, so the planner sets a patent protection strength which induces all firms to patent. If, on the other hand, it is large enough the welfare gains due to higher competition outweigh the costs from reduced disclosure and the planner induces an equilibrium in which patent protection is weak and only some firms use patents. This result suggests that the patent system's ability to increase competition may be a key driver of its capability to increase welfare.

The result that the planner may find it optimal to induce only a fraction of all firms to patent is novel to this paper. Most previous studies do not feature an equilibrium where some firms patent and others protect identical innovations with a secret. Even previous papers that have this feature, however, find it optimal for the planner to induce all firms to patent (see, for example, [51]). The reason is that the planner has an incentive to provide such weak patent protection only if the patent system can increase competition.

Since previous studies do not feature such a patent system, they find that it is not socially optimal to induce some firms to use secrecy protection.

The rest of the paper is structured as follows. Section two explains the timing and assumptions of the model. Section three studies the equilibrium behavior of firms. Section four presents the main results by comparing the degree of competition with and without a patent system. Section five studies welfare and the optimal patent protection strength. Section six concludes.

4.2 The Model

The model is a dynamic innovation game, $\mathcal{G}_P$. Time is discrete, runs from 1 to infinity, and every period has three stages. At stage one firms are matched with ideas. At stage two firms observe which ideas they are matched with and decide on a protection strategy of patenting or secrecy. At stage three, they "race" by choosing an investment strategy (or a bid). The winner of the race introduces the innovation as a new product on the consumer market and yields a certain per period profit.[9] The assumption that firms commit to a protection strategy prior to innovating is not unreasonable. In practice, firms may need to take specific steps, prior to innovating, to ensure that the protection strategy is effective. For example, patenting firms may need to hire a team of lawyers to help the firm navigate through the patent system and represent it in potential lawsuits. Secrecy using firms, on the other hand, may need to make the innovation as complex as possible to deter competitors from reverse engineering. They may also need to pay their workers

[9] An alternative interpretation is that the innovation constitutes a quality improvement of an existing product or a cost reducing technology.

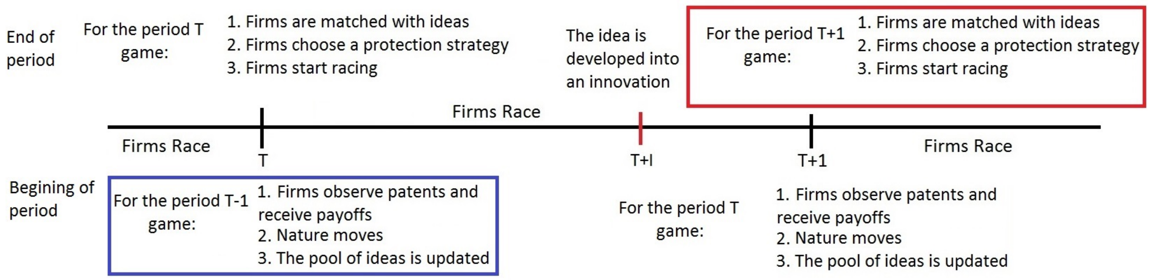

Figure 4.1: Timing

higher wages in order to provide incentives for them to protect the secret. Nevertheless, the main results in the paper would hold even if firms are to chose a protection strategy after they innovate. In particular, in Appendix C.2 we show that both theorems in this chapter hold even if firms choose bids at stage two and a protection strategy at stage three. Figure 4.1 illustrates the timing of the model.[10]

Let us consider an arbitrary period T. There is a fixed measure μ of ex-ante identical firms which, at stage one, are randomly matched with ideas. The measure of ideas is ν (we also refer to ν as the "pool" of ideas) and each idea is equally productive if developed into an innovation. As in [51], the innovation process distinguishes between ideas (potential innovations) and actual innovations (ideas which have been brought to fruition through costly investment in R&D). Each firm is paired with exactly one idea, but the number of firms matched with a given idea is a random variable that follows a Poisson distribution with parameter $\theta := \mu/\nu$.[11] This matching technology would arise if, for example, firms could direct their search towards ideas. In particular, we interpret ν as the pool of all

[10] In this paper, both secrecy and patent protection have a chance to fail at the end of each period. Nature's move is to choose which patents remain valid and which secrets do not leak.

[11] For a derivation in the context of the labor market see, for example, [45]. In this model, the economic interpretation of θ is that it is a measure of the market tightness in the market for ideas.

possible R&D projects that firms might choose to undertake and a match as the particular project each firm chooses to devote effort to. The ratio of firms to ideas, θ, captures the congestion in the "market" for ideas: higher θ means, on average, a higher number of firms race for the same idea. Matches are private knowledge and, in particular, firms cannot observe how many rivals (if any) are matched with the same idea. They observe, however, exactly how many competitors have previously innovated the idea they are matched with. Furthermore, firms can exert R&D effort only towards developing ideas they are matched with in the current period.

At stage two firms choose between patenting or secrecy and the choice is private knowledge.[12] Both IPPMs provide imperfect protection. Each period $T + l$, $l \geq 1$, the patent fails with probability $1 - \alpha$ and the secret leaks with probability $1 - \beta$. In the event that the patent fails (secret leaks) all firms can costlessly imitate the innovation which is then produced under perfect competition.[13] Firms cannot switch protection strategies, i.e. a firm which innovated under secrecy cannot apply for a patent at a later point in time. However, this assumption is innocuous — in Appendix C.1 we prove that, in equilibrium, no firm has an incentive to switch protection strategies.

When a firm chooses to patent it goes to the patent office and discloses all of the relevant information, which then becomes public knowledge. An innovation protected by secrecy, on the other hand, becomes public knowledge only if the secret leaks. This difference in disclosure has important effects on duplicative innovation that takes place in the model, as well as on the welfare analysis in section five. We assume that whenever an inno-

[12]Wefollow the majority of the existing literature by assuming an innovation can be protected by either a patent or a secret, but not both (see, for example, [6] and [51]).

[13]We follow previous research (see, for example [51]) and assume that the leakage of the secret and a failure of the patent have the same effect.

vation becomes public knowledge, the corresponding idea is removed from the pool. This assumption captures the intuition that no firm will engage in costly innovative activity if it has the know-how to imitate the innovation. Since ideas protected by a valid secret remain in ν, there is the potential of duplicative innovation. We refer to duplicative innovation as the possibility that at time T a firm is matched with and independently innovates an idea previously developed by a rival at some $T' < T$.

The potential of duplicative innovation requires us to make two further assumptions. First, we assume the patent system does not provide PUR. The important implication for this model is that a patent holder excludes from the market all previous innovators. Thus, a patented innovation is produced under either monopoly or perfect competition, whereas innovations protected with a secret may be produced by any number of firms. We refer to this assumption as A1 and relax it later on.

Assumption A1 *The patent system provides no prior user rights. That is, if a firm duplicates an innovation and patents it, then all previous innovators receive zero profits from then on.*

Second, we must make an assumption on the structure of the consumer market. To keep it as general as possible, the only two restrictions are that firms' profits are not strictly increasing in the number of producing firms and that for a large enough number of firms (strictly larger than some $\bar{n}$), no firm makes positive profits. In particular, the assumption allows for an arbitrary degree of product differentiation. Formally, the assumption is given by A2 below.

Assumption A2 *Take a sequence* $(d_n)_{n \in \mathbb{N}}$ *such that* $d_n \leq d_{n'}$ *for all* $n > n'$, $0 \leq d_n \leq 1$ *for all* n, $d_1 = 1$, *and* $d_n = 0$ *for* $n > \bar{n} \geq 1$. *If exactly* $n \in \mathbb{N}$ *firms produce an innovation, each receives a fraction* d_n *of the monopoly profits. Otherwise, if the protection mechanism has failed, the innovation is produced under perfect competition and all firms earn zero profits.*

We assume exactly one new idea enters in ν whenever an innovation becomes public knowledge, i.e. it is patented or its secret leaks. Since the idea corresponding to such innovation is removed from the pool, the measure ν is constant. This leads to constant θ which makes the model much simpler without sacrificing any of the insights. Alternatively, if there were convergent dynamics the model's implications would still hold at the steady state.[14]

At stage three firms compete in a winner-take-all race to develop the idea into an innovation. The race is a generalized version of the symmetric game in [46]. However, it is worth noting that the specific structure of the race is not important for the main results in this paper. What is key is lead time advantage and the strategic aspect of patents that we introduce in assumption A3.

Firms race by choosing an investment strategy, or a bid, that specifies at what time the firm will innovate. Investment is sunk, only the winner receives a positive reward, and the reward and cost structures, which we explicitly discuss in the next subsection, ensure that all races end by time $T + 1$. Every firm innovates for sure at the chosen time and bids are private knowledge. The winner in the race is granted the right to produce the innovation

[14] However, it is key to have a positive θ at the steady state. If $\theta \longrightarrow 0$, then there would be no simultaneous search asymptotically.

which is protected by the chosen IPPM — patenting or secrecy. To capture an important feature of the patent system (a patent holder can, in practice, legally block all rivals who have innovated nearly simultaneously from commercially exploiting the innovation), the winner is not necessarily the firm which innovates first. In particular, we make the following assumption:

Assumption A3 *The firm which innovates first among all firms that use patent protection wins the race. In the event that all firms use secrecy, the firm which innovates first is the winner.*

The assumption captures what we refer to as the *strategic aspect of patents*. Whenever a firm chooses patent protection, it can block all rivals that have innovated simultaneously under secrecy from producing.[15] Most importantly, patents allow firms to block rivals even if the rival has innovated sooner than the patent holder. For the purposes of the model, the most important implication of this assumption is that firms which use patent protection are guaranteed to win against rivals that use secrecy, regardless of the chosen bids. The assumption also captures the importance of lead time advantage which, as a myriad of studies suggests, is a key IPPM for firms.[16] When a firm uses secrecy protection it "races" to be the first innovator. Innovating first allows the firm to secure a monopoly position through lead time advantage. If a firm decides to patent, then it must (in a first-to-file patent system) "race" to the patent office to ensure it is the first one to do so.

The model allows for lead time advantage to die out over time. In particular,

[15]Firms that participate in the same race are said to innovate simultaneously.

[16]For a survey of the evidence see, for example, [37]. We implicitly assume that lead time advantage is so strong as to discourage firms other than the first innovator from producing (provided, of course, the other firms cannot block the first innovator by patenting).

several firms can have an innovation based on the same idea, but only if these firms innovate in different periods. The rationale behind this is that later innovators could potentially differentiate their innovation from the earlier ones. This product differentiation allows the later innovators to circumvent the lead time advantage and gain a certain share of the market. If several firms innovate at the same period, then they cannot observe each other's innovations and, hence, cannot differentiate them. However, the model allows for the lead time advantage to persist over time, as well. In particular, in the case of Bertrand competition ($\bar{n} = 1$) no firm is willing to duplicate an innovation under secrecy. Then, the lead time advantage persists until the secret leaks or a competitor duplicates the innovation using patent protection.

At the beginning of $T + 1$, when all matched ideas have been developed into innovations the following events happen. First, firms receive their profits and observe all patents filed between dates T and $T + 1$. Second, a fraction $1 - \alpha$ of all patents fail and a fraction $1 - \beta$ of all secrets leak. Firms observe which patents fail and all secrets that leak. Third, the pool of ideas is updated in the aforementioned way. After all these events have occurred, at the end of $T + 1$, the three stages repeat.

4.2.1 The Innovation Race

Consider an innovation race that starts at T and suppose that the idea has already been developed by $n - 1$ firms in previous periods. Participants in the race observe how many firms have previously innovated the idea, i.e. how many firms are currently selling the corresponding product. However, prior to innovating, they do not know the details on how to produce and commercialize the product in question. Each firm that participates in the race bids a time $t \in [0, 1]$ which means the firm will innovate at time $T+t$.[17] Innovating is costly. At time T firms pay a cost of $c(t)$, with $c'(t) < 0$ for $t \in [0, 1)$.

The reward from the race, which also depends on t (due to discounting), is comprised of all expected future profits. In particular, let $\pi(t)$ be the per period monopoly profit, where t stands or the innovation time and $\pi'(t) < 0$ on $t \in [0, 1)$.[18] Then, let us consider a firm that chooses patent protection. Define $R_P(t)$ to be the reward from the race, conditional on the firm under study winning, and V_P the value of holding a valid patent.[19] Both are independent of how many firms have already innovated, by assumption A1. Then

$$R_P(t) = \pi(t) + \alpha\gamma V_P, \tag{4.1}$$

$$V_P = \pi(0) + \alpha\gamma V_P, \tag{4.2}$$

where γ is the discount factor. When a firm innovates at time $T + t$, the T period profits are $\pi(t)$ because the firm could not produce between times T and $T + t$. As is evident from (4.2), the profits for any period $T' > T$ are given by $\pi(0)$, since innovation took place at a previous period. The patent is valid next period with probability α and has a discounted value of γV_P.

Analogously, let $R_S^n(t)$ be the reward from the race when the firm shares a secret with $n - 1$ others and V_S^n be the value of sharing that secret. Then

$$R_S^n(t) = d_n\pi(t) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1 - e^{-\theta})\beta\gamma V_S^{n+1}. \tag{4.3}$$

The period T profits are shared with the rivals who also know the secret, so the firm

[17]Firms can bid any $t \in [0, \infty)$, however, a normalizing assumption on the profit function ensures that in equilibrium no firm will bid $t > 1$.

[18]We further impose that $\pi(t)$ is strictly decreasing over $[0, 1]$.

[19]Firms receive their profits for period T at the beginning of $T + 1$. As a result, $\pi(t)$, $R_P(t)$ and V_P are time T discounted quantities.

receives only a fraction d_n of the monopoly profits. It keeps the value V_S^n next period if the secret does not leak and no one duplicates the idea at $T+1$, i.e. with probability $\beta \times Pr(\text{the idea is matched with no firms}) = \beta e^{-\theta}$. The firm receives the value V_S^{n+1} if one more rival begins commercially exploiting the innovation under secrecy protection next period.[20] This happens with probability $Pr(\text{at least one firm is matched with the idea}) \times Pr(\text{the } n+1\text{-st innovator has chosen secrecy}) = (1-e^{-\theta})\zeta_{n+1}$.[21] In the events that the secret leaks or the next innovator patents, the information about the innovation becomes public knowledge and the value of knowing the secret is 0. V_S^n is defined similarly

$$V_S^n = d_n\pi(0) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1-e^{-\theta})\beta\gamma V_S^{n+1}. \tag{4.4}$$

Lastly, we make three technical assumptions: i) $\pi(t) - c(t)$ is increasing over $[0,1]$ to ensure the support of the equilibrium CDF is connected; ii) $\pi(t) = 0$ for $t > 1$ to ensure all races will end by $T+1$; iii) for all n either $e^{-\theta}d_n\pi(1) - c(1) \geq 0$ or $d_n = 0$, which implies that either all firms choose to innovate or none of them do. We make this assumption to abstract the analysis from firms' incentives to innovate.

4.3 Equilibrium Behavior

The equilibrium is an infinite sequence of matches, protection strategies, and bids (investment strategies). At stage one firms make no decisions and the equilibrium outcome

[20] By assumption A3 at most one new firm can begin commercially exploiting the innovation each period. Also, at $T+1$, the $n+1$-st innovator receives $d_{n+1}\pi(t)$ while the other n firms each receive $d_{n+1}\pi(0)$, as they have innovated previously. Hence, V_S^{n+1} is independent of t.

[21] The n-th innovator develops the innovation under secrecy if all firms at the innovation race choose secrecy protection, due to assumption A3. Let s_n be the probability firms place on playing secrecy in a symmetric equilibrium, when they are the n-th innovator. Then, $\zeta_n := P(\text{all matched firms choose secrecy}|\text{the idea is matched with at least 1 firm}) = \sum_{i=1}^{\infty} \frac{1}{1-e^{-\theta}} \frac{e^{-\theta}\theta^i}{i!} s_n^i = \frac{e^{-\theta}}{1-e^{-\theta}}\left(e^{\theta s_n} - 1\right)$.

in this stage plays no role in the analysis of the main results. Thus, for the sake of clarity in exposition, the characterization of the equilibrium stage one outcome is placed in the appendix. In general the protection and investment strategies firms decide on at time T will depend on all past decisions and outcomes. However, one should observe that the payoffs from each innovation race are independent of any previous decisions or outcomes. Hence, the focus in this paper is on stationary equilibria where strategies are independent of the history. Moreover, firms are ex-ante identical and, as is customary in the search and matching literature, we will focus on symmetric equilibria in which firms do not collude. The equilibrium concept we use is subgame perfect Nash equilibrium, and we solve for the equilibrium using backward induction.

4.3.1 Equilibrium Behavior with a Patent System

Let us first characterize the equilibrium behavior at stage three. Take an innovation race which starts at time T and is associated with an idea that has been developed by $n-1$ firms in previous periods. Define the equilibrium CDFs to be $F_j^n(t)$ on $[\underline{S}_j^n, \bar{S}_j^n]$, where $j = P, S$ stands for the protection strategy of patenting or secrecy, respectively.[22] In anticipation of the stage two results, let s_n denote the probability firms place on using secrecy protection. We follow [51] and, also, refer to s_n as the fraction of firms which choose secrecy protection. Then, the equilibrium behavior in stage three is given by the following lemma.

[22] In equilibrium, firms use a mixed strategy when they choose an innovation time. This can be seen by applying a standard argument.

Lemma 8 *At stage 3 firms randomize their innovation time using the CDFs*

$$F_P^n(t) = \frac{1}{\theta(1-s_n)} ln\Big(\frac{R_P(t)}{c(t) + e^{-\theta(1-s_n)} R_P(1) - c(1)}\Big),$$
$$F_S^n(t) = \frac{1}{\theta s_n} ln\Big(\frac{R_S^n(t)}{c(t) + e^{-\theta} R_S^n(1) - c(1)}\Big) - \frac{1-s_n}{s_n}.$$

The proof is fairly standard and, therefore, in the appendix.

Firms use a different investment strategy, depending on the IPPM, because there is a substitution effect between investment in R&D and patents' strategic advantage. To see the intuition behind this result, first observe that as a straightforward result of the assumptions, it follows that $\bar{S}_j^n = 1$. Second, firms choose to innovate sooner ($t < 1$) only for the sake of winning the race, as $\pi(t) - c(t)$ is increasing in t. Third, the strategic advantage of patents depends on how many firms use patent protection. The more firms use patents, the lower the fraction of rivals that a patenting firm can block, hence, the lower the strategic advantage. The result in Lemma 8 implies that when patenting provides a higher strategic advantage (s_n increases) a firm that patents decreases its investment (chooses a higher t) because it faces less competition from rivals. In the extreme case, as $s_n \longrightarrow 1$ the firm can block all rivals, so there is no incentive to bid any $t < 1$, which leads to $\underline{S}_P^n \longrightarrow 1$. Firms that use secrecy condition their investment strategies on the level of the strategic advantage as well. A firm that uses secrecy protection cannot win against a rival that patents, so lower s_n implies that secrecy using firms get "discouraged" and bid less aggressively. In particular, as $s_n \longrightarrow 0$ innovating sooner is not beneficial because all rivals can block the firm, hence, $\underline{S}_S^n \longrightarrow 1$.

This result differs from what previous research has found, because we explicitly account for the innovation race, as well as simultaneous innovation. For example, in [51] firms cannot exclude rivals from commercially exploiting the innovation by innovating sooner — they invest solely to increase their chance of successfully developing an idea into an innovation. Because of this the level of investment, in their model, does not react to changes in the level of strategic advantage of patents. More importantly, the strategic benefit is smaller than that in our model. To see this clearly, let $s_n \to 1$. Then a secrecy-using firm expects to win the race with probability $(1-e^{-\theta})/\theta$ and makes, on average, an investment of $c(t^*) > c(1)$.[23] A firm that patents, on the other hand, wins the race for sure and makes the minimum investment of $c(1)$. Hence, the benefit from the strategic advantage consists of both a higher chance to win the race and a lower investment in R&D. In [51], however, investment decisions are independent of the chosen IPPM and the strategic advantage affects only the probability of securing a monopoly position. Thus, [51] underestimate the importance of patents to firms because, in their model, it does not include the benefit of lower average investment.

At stage two, firms choose between patenting and secrecy in the following way.

Lemma 9 *At stage 2 firms choose secrecy with probability s_n, where*

$$s_n = \begin{cases} 0 & \text{if } \alpha \geq \bar{\alpha}_n, \\ \frac{1}{\theta} ln\left(\frac{R_S^n(1)}{R_P(1)}\right) & \text{if } \alpha \in (\max\{0, \underline{\alpha}_n\}, \bar{\alpha}_n), \\ 1 & \text{if } \alpha \leq \underline{\alpha}_n. \end{cases}$$

where $\bar{\alpha}_n$ is such that $V_P - V_S^n = \pi(0)(1-\gamma)(1-d_n)$ and $\underline{\alpha}_n$ is such that $V_P = e^{-\theta}V_S^n + (1-\gamma)\pi(0)(1-e^{-\theta}d_n)$.

Proof is included in the appendix. The intuition behind the result is the same as in [51]. When $\alpha \geq \bar{\alpha}_n$ patenting provides higher appropriability ($R_P(t) \geq R_S^n(t)$), so all firms choose

[23] Here t^* is such that $c(t^*)$ equals the expected investment of secrecy using firms in equilibrium.

patent protection in equilibrium. For intermediate patent strengths there is a randomizing behavior because of patents' strategic advantage. In particular, the expected payoff for a firm when it uses patenting is $e^{-\theta(1-s_n)}R_P(1) - c(1)$, which is simply the chance that no other patenting firm participates in the innovation race ($e^{-\theta(1-s_n)}$) times the expected profits ($R_P(1)$) minus the cost of innovating ($c(1)$) when the firm bids $t = 1$. Analogously, when the firm uses secrecy protection, its expected payoff is $e^{-\theta}R_S^n(1) - c(1)$. When $\alpha \in (\max\{0, \underline{\alpha}_n\}, \bar{\alpha}_n)$ there is a trade-off between the reward and strategic aspect of patents. As patent protection decreases, the reward $R_P(t)$ decreases as well and more firms opt for secrecy. However, as s_n increases, firms that still use patent protection have a higher chance of winning the race ($e^{-\theta(1-s_n)}$) because they can block a higher fraction of their rivals. Thus, as α decreases patents provide lower appropriability, but higher benefit from the strategic advantage. This randomizing behavior persists until α is so low that the strategic gain from patenting cannot compensate for the loss in appropriability, i.e. $\alpha < \underline{\alpha}_n$. In particular, $\underline{\alpha}_n$ is the protection strength at which the net gain from the strategic advantage of blocking all rivals, $R_S^n(1) - e^{-\theta}R_S^n(1) = (1-e^{-\theta})(V_S^n - d_n(1-\gamma)\pi(0))$, equals the loss in appropriability from patenting, $R_S^n(1) - R_P(1) = V_S^n - V_P + (1-\gamma)(1-d_n)\pi(0)$.[24] Unlike in [51], however, in the present paper an equilibrium where $s_n = 0$ may not be achievable, regardless of how low is α. This is due to the assumption of lead time advantage — first, the strategic benefit of patents is higher in the present paper than it is in [51] and, second, firms that patent can

[24]From equations (4.1), (4.2), (4.3), (4.4) we have that $R_P(t) = V_P - (\pi(0) - \pi(t))$ and $R_S^n(t) = V_S^n - d_n(\pi(0) - \pi(t))$. When the firm can block all rivals, it wins for sure and its expected payoff from the race is $R_S^n(1) - c(1)$. If the firm does not have the strategic advantage of blocking rivals, then it receives positive payoffs only when it faces no rivals (i.e. with probability $e^{-\theta}$) and the expected payoff is $e^{-\theta}R_S^n(1) - c(1)$. Thus, the net gain due to the strategic advantage is simply the difference given in the text. The loss in appropriability is simply the difference of the values of sharing the secret with $n-1$ other firms and the value of holding a valid patent, adjusted for the difference in per period profits under secrecy, $d_n\pi(0)$, and under patent protection, $\pi(0)$.

receive some profits even if $\alpha = 0$. If it is the case that $\underline{\alpha}_n < 0$, then the strategic aspect of patents is strong enough to always induce at least some firms to patent, regardless of the strength of patent protection.

It is important to note that the cutoff values $\underline{\alpha}_n$ and $\bar{\alpha}_n$ depend on how many times the innovation has been duplicated. In particular, firms require less of an incentive to patent the larger the number of previous innovators. Corollary 10 establishes the result.

Corollary 10 $e^{-\theta}\beta = \bar{\alpha}_1 \geq \bar{\alpha}_2 \geq \cdots \geq \bar{\alpha}_{\bar{n}} \geq \bar{\alpha}_{\bar{n}+1} = 0.$

The proof is in the appendix. As in [51], firms patent even if patent protection is lower than secrecy protection. In particular, all firms patent as long as the patent strength, α, is weakly higher than the effective protection under secrecy, $e^{-\theta}\beta$. This is key to establishing the main results in this paper and is due to the patent system's ability to punish innovators that choose secrecy. Intuitively, at $\alpha = e^{-\theta}\beta$ the expected duration of monopoly under patenting and secrecy is the same, but a firm that uses secrecy protection can potentially receive positive profits even if it looses its monopoly position (whenever a firm duplicates under secrecy). However, this potential is never realized in equilibrium, because the patent system incentivizes the duplicator to always patent by redistributing the original innovator's profits.[25] Since the innovator always receives those potential profits in its absence, the patent system effectively punishes the firm.

[25] We say the patent system redistributes the profits, because if the duplicator uses secrecy, then both it and the original innovator receive $d_2 \leq 1$ of the monopoly profits. If the duplicator patents, however it receives the monopoly profits and the original innovator receives 0. Moreover, the patent system provides an additional incentive for the n-th innovator to patent — the threat that the $n+1$-st innovator will patent.

4.3.2 Equilibrium Behavior without a Patent System

Without a patent system firms play the dynamic game $\mathcal{G}_N$, which is the same as $\mathcal{G}_P$ with the only exception that now firms cannot use patent protection. We again present the equilibrium outcome of stage one in the appendix as it plays no role in the analysis of the main results. The behavior of firms at stages two and three is summarized in the proposition below.

Proposition 11 $\mathcal{G}_N$ *has a unique stationary symmetric equilibrium, where*

- *At stage 2: Firms trivially choose secrecy protection*
- *At stage 3: Firms randomize their innovation time using the CDF*

$$F_s^n(t) = \frac{1}{\theta} ln\Big(\frac{R_S^n(t)}{c(t) + e^{-\theta} R_S^n(1) - c(1)}\Big),$$

for $n \leq \bar{n}$, and they stay out of the race, if $n \geq \bar{n} + 1$.

The proof is analogous to that of Lemma 8 when $s_n = 1$, for all n.

4.4 Competition

This section presents the main results in the paper: the patent system can increase competition, regardless of whether or not it provides PUR. It is natural to think of competition in terms of how many firms are producing the innovation.[26] Given the stochastic nature of the model, an unambiguous criterion for comparing the degree of competition

[26]For example, under Cournot competition with no fixed costs the market output and price converge to their perfect competition levels as the number of firms increases. In this sense, a higher degree of competition translates to a market outcome which is "closer" to perfect competition.

between different equilibria is the notion of first order stochastic dominance. This paper abstracts from uncertainty in the innovation process and incentives to innovate. Because of this, we compare the distribution of firms conditional on the innovation having been innovated. By symmetry, these distributions will be the same for all innovations regardless of which period the idea was originally innovated. Thus, without loss of generality we will only focus on ideas innovated at period one.

Let $P_i(n)$ be the probability an innovation is produced by exactly n firms in equilibrium i, given that the corresponding idea was originally developed in period 1, where $i = P$ stands for the equilibrium with a patent system and $i = N$ for the equilibrium without one. Notice that $P_i(n)$ will, in general, depend on T, but we suppress it from the notation to simplify the exposition. Also, we slightly abuse notation by letting $n = \infty$ denote the case of perfect competition.

Next, let us define the distribution of firms producing the innovation by $G_i^T(k) := \sum_{n=1}^{k} P_i(n)$. Then, the following definition formally introduces the criterion for comparing the degree in competition across equilibria.

Definition 12 *We say that equilibrium i provides higher competition than equilibrium i' if G_i^T first order stochastically dominates $G_{i'}^T$. Formally, $G_i^T(k) \leq G_{i'}^T(k)$ for all $k \geq 1$, $T \geq 1$ and the inequality is strict for at least some k for all $T \geq \max\{2, k\}$.*

4.4.1 No Prior User Rights

Intuitively, the patent system would increase competition if patents provide weaker protection than secrecy. Weaker protection increases competition and reduces firms' appropriability — the more firms produce, the higher the degree of competition and the lower the per firm profits. Yet, this raises a question: Why would firms use patent protection if this reduces appropriability? The answer is that when the patent system does not provide PUR, it can punish firms that opt for secrecy by incentivizing duplicators to patent, effectively reducing the option value of secrecy. In particular, for high enough patent strengths ($\alpha \geq \bar{\alpha}_1$) a firm that chooses secrecy protection yields positive profits next period with probability $e^{-\theta}\beta$. Since in the absence of a patent system a secrecy using firm receives positive profits next period with probability β, it follows that the presence of a patent system reduces the attractiveness of secrecy. Thus, in order for firms to patent, patents must provide stronger protection than secrecy in the presence of a patent system, which might still be weaker than the protection secrecy provides in the absence of a patent system. The theorem below formalizes this intuition.

Theorem 13 *For all $\gamma, \beta \in (0,1)$ and all $\theta > 0$, there exists a patent strength α such that the patent system increases competition.*

The proof is straightforward and carries much of the intuition behind the result, so we provide it in the text below.

Proof. Let us fix $\alpha = \bar{\alpha}_1 = e^{-\theta}\beta$, so that the equilibrium with a patent system is such that all firms patent for sure. If $T = 1$ the innovation is produced under monopoly with certainty regardless of which equilibrium we are in, so $P_P(n) = P_N(n)$ for all n. Then, fix $T \geq 2$ for the remainder of the proof.

Let us first consider the case when there is a patent system. With a patent, an innovator protects its monopoly position by blocking all potential duplicators from securing any market power. Hence, the innovation is produced under monopoly or perfect competition, i.e. $P_P(n) = 0$ for $n \neq 1, \infty$. Moreover, the innovation is produced under monopoly at time T, if the patent has held for all previous periods, i.e. $P_P(1) = (e^{-\theta}\beta)^{T-1}$ and $P_P(\infty) = 1 - (e^{-\theta}\beta)^{T-1}$.

Next, consider the case without a patent system. It will be useful to distinguish between two possibilities: i) $\bar{n} \geq 2$ and ii) $\bar{n} = 1$. From corollary 10, the patent system punishes secrecy-using firms by redistributing their profits when the innovation is duplicated. The difference between the two possibilities is that under $\bar{n} \geq 2$ the patent system exacerbates this threat, while under $\bar{n} = 1$ — creates it.

If $\bar{n} = 1$ no firm is willing to become a duopolist, so the innovator does not face the threat of duplication when there is no patent system. In particular, any firm matched with a previously developed idea chooses to opt out of the race, so $P_N(n) = 0$ for all $n \geq 2$. Then it follows that, a monopolist loses its market power only when the secret leaks, i.e. $P_N(1) = \beta^{T-1}$. This implies that there is a strictly higher expected duration of monopoly when there is no patent system, $1/(1-\beta)$, as compared to the case with a patent system, $1/(1-e^{-\theta}\beta)$. The patent system can achieve this and at the same time induce all firms to patent because the punishment for firms that opt for secrecy is much stronger when $\bar{n} = 1$. Thus, we have that $G_N^T(k) = P_N(1) = \beta^{T-1} > (e^{-\theta}\beta)^{T-1} = P_P(1) = G_P^T(k)$ for all $1 \leq k < \infty$. Moreover, under both scenarios (with and without a patent system) an innovation can only be produced under perfect competition or monopoly, hence, $P_N(\infty) = 1 - \beta^{T-1} < 1 - (e^{-\theta}\beta)^{T-1} = P_P(\infty)$.

If $\bar{n} \geq 2$ firms find it profitable to share a secret with at least one rival, so they will duplicate whenever given the chance. Thus, $P_N(n) > 0$ for at least one $n \geq 2$. Moreover, an innovator would lose its monopoly position if the innovation is duplicated, hence $P_N(1) = Pr(\text{the secret has not leaked}) \times Pr(\text{no rival has duplicated}) = (e^{-\theta}\beta)^{T-1}$. Then, it follows that $G^T_P(1) = G^T_N(1)$ and $G^T_P(k) < G^T_N(k)$ for any $2 \leq k < \infty$. This concludes the proof.

■

To the best of our knowledge, this is the first paper which finds that the patent system can increase competition, even though some previous studies may have that feature under no PUR. For example, [66] looks at the optimal patent protection strength in a model of sequential innovation that allows for duplicative innovation and no PUR. Yet, he does not compare the degree of competition between the presence and absence of a patent system. One can show that, unlike this paper, in the framework of [66] the patent system can increase competition only for a certain range of parameter values. Furthermore, in [66] the patent system cannot increase competition under PUR since the paper abstracts away patents' strategic aspect.

It is worth noting that the conclusion in Theorem 13 is not unique to the particular matching technology and innovation race. It is straightforward to see that the only requirement we have placed on the matching function is that $e^{-\theta} < 1$, i.e. there is a possibility of duplication. Moreover, the assumption on the existence of a pool of ideas, ν, can be easily relaxed. In particular, a model where there is exactly one idea that can be innovated and firms enter the innovation race each period stochastically according to $Pr(\text{exactly } n \text{ firms enter}) = \frac{e^{-\theta}\theta^n}{n!}$ leads to the exact same results. Furthermore, the assumptions behind the innovation race are not critical either. The key aspect is that

$\bar{\alpha}_1 = e^{-\theta}\beta$, which is mainly a feature of the assumption on lead time advantage.

4.4.2 Prior User Rights

The result in Theorem 13 (and its intuition) relies on assumption of no PUR. In practice however, the patent laws of most countries provide some PUR.[27] Then, it is relevant to ask if the patent system can increase competition, given that it provides PUR. In what follows we relax the assumption of no PUR and show that the patent system can, indeed, increase competition.

Assume that if a later innovator patents, then all firms that benefit from the prior user defense make the same profits as the patent holder.[28] Formally,

Assumption A4 *The patent system provides prior user rights. That is, suppose that $n-1$ firms have independently developed an innovation and kept it secret. If the n-th innovator develops the innovation and patents it, then all n firms receive the fraction d_n of monopoly profits.*

From now on, we replace assumption A1 with assumption A4. Since the patent system cannot exacerbate the threat of sequential innovation, it can only increase competition by providing a strategic advantage to firms which patent. Theorem 14 gives the

[27]The US patent system did not provide PUR for most patents until 2011. The America Invents Act, however, increased the scope of "prior user rights" defense to infringement.

[28]Even with PUR, there are some restrictions on what can the first innovator do. For example, in the US the original innovator cannot license, assign, or transfer the prior user defense. Moreover, the defense is geographically limited to sights where the innovation has been commercially exploited for at least one year prior to the patent filling date. Hence, in reality, the second innovator could "partially" exclude (or block from expanding) the original one, even if the patent system provides PUR. From the anteceding analysis it will become clear that the results hold for any arbitrary degree of partial exclusion. Intuitively, the threat of duplication depends on the degree of exclusion –the higher the degree of exclusion, the more the patent system exacerbates the threat. By the logic of Theorem 13, it will be "easier" for the patent system to increase competition under partial exclusion than under full exclusion.

result.

Theorem 14 *If*

$$\frac{1}{\gamma} > \Big(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\Big)\Big(\frac{\beta}{1-\beta\gamma}\Big), \tag{4.5}$$

then the patent system can increase competition.

The proof is included in the appendix. The intuition behind the result goes as follows. Take an innovation developed by $n-1$ innovators, who all chose secrecy protection. With a patent system the n-th innovator can choose to patent. In this case, the innovation will be produced by n firms next period with probability α. With the complimentary probability, $1-\alpha$, it will be produced under perfect competition. When there is no patent system, the n-th innovator has no choice but to keep the innovation secret. Then, with probability $e^{-\theta}\beta$ the innovation will be produced by n firms next period, with probability $(1-e^{-\theta})\beta$ by $n+1$ firms, and with probability $1-\beta$ under perfect competition. Hence, a necessary condition for the patent system to increase competition is $\alpha \leq e^{-\theta}\beta$, otherwise it will increase the chance an innovation is produced by n firms next period. With PUR, however, the patent holder receives only a fraction $d_n \leq 1$ of monopoly profits, as opposed to $d_1 = 1$ without PUR. Thus, the option of patenting with PUR is not as attractive as the one without PUR. Hence, with PUR, it is no longer true that $\bar{\alpha}_1 = e^{-\theta}\beta$. When $\alpha \leq e^{-\theta}\beta$ patenting provides strictly lower expected reward than secrecy does, and the only way to incentivize firms to patent is by providing them with strategic benefits.

Then, we need to see under what conditions this patent strength ($\alpha = e^{-\theta}\beta$) is consistent with $s_n \in (0,1)$, that is, the gain in the expected payoff from the race due to the strategic aspect of patents can exceed the loss in appropriability. The strategic advantage is largest when all other firms use secrecy protection, i.e. $s_n = 1$, hence, a sufficient condition for $s_n \in (0,1)$ is $R_P^n(1) > e^{-\theta} R_S^n(1)$.[29] The inequality can be rewritten to separate the strategic and reward aspects of patents

$$V_S^n - V_P^n < (1 - e^{-\theta})(V_S^n - d_n \pi(0)(1-\gamma)). \tag{4.6}$$

On the left hand side we have the net loss in appropriability due to patenting — the difference between the value of sharing the secret with $n-1$ other firms and the value of patenting when the firm is the n-th innovator. The right hand side captures the strategic benefit of patenting. If the firm could block rivals (when it uses secrecy) in the innovation race, then its reward would be $R_S^n(1)$, which is nothing but $V_S^n - d_n \pi(0)(1-\gamma)$. Because secrecy protection does not aid the firm in blocking its rivals, its expected reward is $e^{-\theta} R_S^n(1)$. The difference between the two is the net gain in expected profits from the ability to block rivals.

Theorem 14 provides a sufficient condition for equation (4.6) to hold for at least one n for any market structure (captured by $(d_n)_{n \in \mathbb{N}}$).[30] To see the intuition behind (4.5) we can interpret $1 - \gamma$ as the rate with which innovations become obsolete. Then, the right hand side of (4.5) is the product of Pr(Keeping a monopoly position next period, given that the innovation does not become obsolete) $\times$ (Expected duration of monopoly) when firms are willing to become a duopolist, $e^{-\theta}\beta/(1 - e^{-\theta}\beta\gamma)$, and when firms are not willing to become a duopolist, $\beta/(1-\beta\gamma)$. Then, the theorem says that if the inverse of the probability the innovation does not become obsolete next period, $1/\gamma$, is larger than the right hand side, the benefit from the strategic aspect of patents (when no other firm patents) would dominate the loss in appropriability for $\alpha = e^{-\theta}\beta$.

[29]Since the patent system provides PUR, the expected profits when patenting depend on the number of innovators. Also, $R_P^n(1) \leq e^{-\theta} R_S^n(1)$ whenever $s_n = 1$.

[30]In particular, $\bar{n}$ could be any positive integer, so the theorem provides a sufficient condition for $s_1 \in (0,1)$.

The condition is more likely to hold when θ is higher. Larger congestion implies that a patenting firm can block, on average, a higher number of rivals, which directly translates into a higher strategic benefit from patenting. Similarly, the patent system is more likely to have the ability to increase competition when β is lower. The reason is that low secrecy protection strength implies a lower loss in appropriability due to patenting.

This intuition helps explain why, to the best of our knowledge, no previous studies were able to find that with PUR the patent system can increase competition: it can only do this if patenting provides enough strategic benefits. [27], for example, develop a two-firm game without simultaneous innovation. Because of this, patenting does not grant a strategic advantage and cannot increase competition.

Even in previous models, with simultaneous innovation, the patent system cannot increase competition, as they do not explicitly consider the firms' incentive to secure lead time advantage. For example, in [51] patents can only affect the probability of becoming a monopolist. In their model, the patent system cannot incentivize firms to patent when this provides lower appropriability, because the strategic aspect of patents and investment in R&D are not linked, i.e. in the absence of lead time advantage, the strategic benefit is too low.[31]

The patent system can increase competition even if (4.5) does not hold — the condition is only sufficient. For example, if $\bar{n} = 1$ (Bertrand competition), no firm is willing to duplicate the innovation. Then, without a patent system, a developed innovation will be produced under monopoly next period with probability β and under perfect competi-

[31] It should be noted that the assumption of lead time advantage affects the market structure as well. For example, in this model if two secrecy using firms innovate simultaneously, only the one which innovated first would begin commercially exploiting the innovation. On the other hand, in [51] both of these firms would exploit the innovation.

tion with probability $1 - \beta$. In an equilibrium where all firms patent the corresponding probabilities are α and $1 - \alpha$. Thus, the patent system increases competition whenever $s_1 < 1$ and $\alpha < \beta$. This is the case for all patent strengths consistent with $s_1 \in (0, 1)$. Hence, the patent system can always increase competition when $\bar{n} = 1$. The reason is that secrecy cannot allocate market power to more than one firm under Bertrand competition. Thus, the patent system will increase competition whenever it decreases the chance that an innovation is produced under monopoly.

The analysis so far has only looked at a single industry. In practice, however, there are many different industries, say $j \in [1, 2, \ldots, J]$, which have different strengths of secrecy protection, $\beta(j)$, market tightness, $\theta(j)$, and discount factors $\gamma(j)$. It is, however, easy to see that the results can be generalized, so that the patent system may increase the degree of competition in some industries without adversely affecting other industries. If one follows the logic behind Theorem 14, this will be true if (4.5) holds for at least one industry j, such that $j \in \text{argmin}\{e^{-\theta(j')}\beta(j') \mid j' \in [1, 2, \ldots, J]\}$. The intuition behind the condition is analogous to the single industry case, with the only exception that now the necessary condition for the patent system to increase competition is not $\alpha \leq e^{-\theta}\beta$, but rather $\alpha \leq e^{-\theta(j')}\beta(j')$ for all $j' \in [1, 2, \ldots, J]$.

4.5 Welfare and the Optimal Patent Strength

The main results of the paper are not only of theoretical interest, but of practical as well. This section applies the insights of the preceding analysis in the context of the planner's problem to choose a patent strength which maximizes welfare. We find that the patent system is always welfare improving — it can induce all firms to disclose their innovations without increasing their market power. Depending on parameter values, however, it may be optimal for the planner to induce only some firms to patent. She can increase competition and, hence, welfare by setting a low patent protection strength. At the same time, however, this may decrease welfare because some firms might have incentives to not disclose their innovations and opt for secrecy, instead. Ideas which correspond to innovations protected by a secret remain in the pool, thus, lower disclosure implies a lower mass of new products is introduced to the consumer market each period, which leads to a decrease in welfare. Because of this the planner sets an optimal patent strength to strike a balance between increasing competition and inducing disclosure. If congestion is low, then the welfare loss due to reduced disclosure outweighs the gain due to higher competition and the planner chooses a patent strength that induces all firms to patent. If it is high, on the other hand, she finds it optimal to induce some firms to use secrecy protection.

To highlight the features of this trade off, we abstract the analysis from the impact of firms' equilibrium investment choices on welfare. Formally, $\pi(t) = \pi$ for $t \in [0, 1]$. Furthermore, for the purposes of this section, we follow [51], among others, and assume Bertrand competition in the consumer market. Formally, we set $d_n = 0$ for all $n \geq 2$, which implies that no firm is willing to share a secret with someone else. Hence, in equilibrium, all innovations are produced under monopoly or perfect competition. We denote the corresponding per period consumer surplus by S_M and S_C, where $S_C > S_M + \pi$.[32] Also, the analysis focuses on a patent system which provides prior user rights, as this is the empiri-

[32]For consistency we assume that the per period consumer surplus is received at the same time firms receive their profits — after all races in a given period have ended. Hence, the surplus for period T is received at $T + 1$ and, similarly to π, S_M and S_C are period T discounted quantities.

cally more relevant case. Since profits are time independent and no firm has an incentive to duplicate an innovation, we write R_S, R_P, s, and ζ instead of $R_S^n(t)$, $R_P^n(t)$, s_n, and ζ_n.

4.5.1 Welfare

When there is a patent system, depending on the chosen patent strength, the planner can induce three equilibria — patenting equilibrium ($s = 0$), secrecy equilibrium ($s = 1$), and mixed equilibrium ($s \in (0,1)$). It will be useful to consider each of these equilibria separately when we look at the planner's problem, so we denote $i = P, S, M, N$ for $s = 0$, $s = 1$, $s \in (0,1)$, and the equilibrium without a patent system, receptively.

The planner's problem is the maximize the steady state value of welfare, which is equivalent to maximizing the expected net present value (ENPV henceforth) of welfare generated by all innovations made in a given period. We denote this quantity by $\bar{W}^i$, where i stands for the equilibrium under study. By symmetry, $\bar{W}^i$ is the ENPV of welfare generated by a single innovation, denoted by W^i, times the mass of innovations made in a given period. As firms do not find it profitable to duplicate innovations, they innovate only if matched with a new (previously not innovated) idea. We also refer to such ideas as profitable and denote their steady state fraction in the pool by $\bar{N}^i$. Given the matching technology, it follows that each period the mass of innovations is $\nu(1 - e^{-\theta})\bar{N}^i$ (the mass of new ideas matched with at least one firm). If we denote the ENPV of per innovation profits and consumer surplus by Π^i and $\mathbb{S}^i$, it follows that

$$\bar{W}^i = \nu(1 - e^{-\theta})\bar{N}^i W^i = \nu(1 - e^{-\theta})\bar{N}^i \Big(\Pi^i + \mathbb{S}^i\Big).$$

The above representation of welfare helps illustrate the intuition behind the analysis. W^i

captures the patent system's effect on welfare due to competition. Lower(higher) competition implies longer(shorter) expected duration of monopoly, which leads to lower(higher) ENPV of per innovation welfare, and ultimately to lower(higher) $\bar{W}^i$. At the same time, $\bar{N}^i$ captures the patent system's effect on welfare due to disclosure. Lower(higher) disclosure implies lower(higher) steady state fraction of new ideas, which leads to a lower(higher) mass of innovations made each period, and ultimately to lower(higher) $\bar{W}^i$.

The structure of welfare under $i = N, S, P$ is very similar, so it is useful to characterize it separately from the welfare under the mixed equilibrium. First, it is easy to see that the reward firms receive conditional on winning the race is $R_S = \pi/(1 - \beta\gamma)$ for $i = N, S$, whereas it is $R_P = \pi/(1 - \alpha\gamma)$ in the patenting equilibrium. Then, the ENPV of profits from participating in a race is given by $e^{-\theta}\pi/(1 - p_i\gamma) - c(1)$, where p_i stands for the strength of the chosen protection strategy, i.e. $p_P = \alpha$ and $p_S = p_N = \beta$. Given the matching technology, only a fraction $\bar{N}^i$ of firms choose to participate in a race, so the ENPV of profits from all innovations made in a given period is $\mu\bar{N}^i(e^{-\theta}\pi/(1-p_i\gamma) - c(1))$. Hence, the ENPV of profits per innovation is given by

$$\Pi^i = \frac{\theta}{1 - e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1 - p_i\gamma} - c(1)\Big).$$

An innovation is produced under monopoly for as long as the protection strategy holds and under perfect competition every period after that. So, if an innovation is developed in period T and its protection strategy fails in period $T+j$ the NPV of consumer surplus generated by that innovation is $((1-\gamma^j)S_M+\gamma^j S_C)/(1-\gamma)$. The probability that the protection strategy fails in period $T+j$ is simply $Pr($the protection strategy has held for all previous periods) $\times$ Pr(the protection strategy fails in $T+j$) $= p_i^{j-1}(1-p_i)$. Then, we have that

$$\mathbb{S}^i = \sum_{j=1}^{\infty} p_i^{j-1}(1-p_i)\Big(\frac{S_M + \gamma^j(S_C - S_M)}{1-\gamma}\Big) = \frac{S_M}{1-p_i\gamma} + \frac{\gamma(1-p_i)}{(1-\gamma)(1-p_i\gamma)}S_C.$$

Whenever a firm patents an innovation its specifications are disclosed and it becomes public knowledge. In that case, the corresponding idea is replaced with a new one in the pool. Thus, in the patenting equilibrium all ideas are profitable, i.e. $\bar{N}^P = 1$. In the secrecy and no patent system equilibria, however, firms do not disclose their innovations, so they become public knowledge only when the secret leaks. Thus, for $i = S, N$ we have the following law of motion

$$N_T^i = e^{-\theta}N_{T-1}^i + (1-\beta)\Big((1-e^{-\theta})N_{T-1}^i + 1 - N_{T-1}^i\Big).$$

An idea is new in some period T in one of two cases. First, if it was new at $T-1$ and no firm was matched with it, as captured by the term $e^{-\theta}N_{T-1}^i$. Second, if it was produced under monopoly during $T-1$ and secrecy protection failed, as captured by the second term. Thus, $\bar{N}^S = \bar{N}^N = (1-\beta)/(1-\beta+(1-e^{-\theta})\beta)$.

Then, $\bar{W}^i$ under the three equilibria is given by

$$\begin{aligned}\bar{W}^N = \bar{W}^S =& \nu(1-e^{-\theta})\frac{1-\beta}{1-\beta+(1-e^{-\theta})\beta}\left(\frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\beta\gamma} - c(1)\Big) + \right.\\ &\left. + \frac{S_M}{1-\beta\gamma} + \frac{\gamma(1-\beta)S_C}{(1-\gamma)(1-\beta\gamma)}\right),\end{aligned}$$

$$\bar{W}^P = \nu(1-e^{-\theta})\left(\frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\alpha\gamma} - c(1)\Big) + \frac{S_M}{1-\alpha\gamma} + \frac{\gamma(1-\alpha)S_C}{(1-\gamma)(1-\alpha\gamma)}\right).$$

Next, let us characterize the structure of welfare for $i = M$. In a mixed equilibrium firms make the same expected profits from participating in a race, regardless of which protection strategy they use. Hence, similarly to the other equilibria

$$\Pi^M = \frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\beta\gamma} - c(1)\Big).$$

The NPV of the consumer surplus for an innovation which was made in period T and whose protection strategy fails at $T+j$ is still given by $((1-\gamma^j)S_M + \gamma^j S_C)/(1-\gamma)$. In the mixed equilibrium, however, some innovations are protected by a secret and some by a patent. Hence, the ex-ante probability that an innovation's protection fails in period $T+j$ is given by Pr(the innovation is protected by a secret) $\times$ Pr(secrecy protection fails in $T+j$) $+Pr$(the innovation is protected by a patent) $\times$ Pr(patent protection fails in $T+j$) $= \zeta\beta^{j-1}(1-\beta) + (1-\zeta)\alpha^{j-1}(1-\alpha)$. Thus, $\mathbb{S}^M = \zeta\mathbb{S}^S + (1-\zeta)\mathbb{S}^P$. This implies that the ENPV of welfare generated by a single innovation in the mixed equilibrium is given by

$$\begin{aligned} W^M =& \frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\beta\gamma} - c(1)\Big) + (1-\zeta)\frac{S_M}{1-\alpha\gamma} + \zeta\frac{S_M}{1-\beta\gamma} + \\ &+ (1-\zeta)\frac{\gamma(1-\alpha)S_C}{(1-\gamma)(1-\alpha\gamma)} + \zeta\frac{\gamma(1-\beta)S_C}{(1-\gamma)(1-\beta\gamma)}. \end{aligned}$$

The law of motion for $\bar{N}^M$ is similar to the one in the secrecy and no patent system equilibria. The only difference is that in the mixed equilibrium a fraction $1-\zeta$ of all innovations are patented and become public knowledge right away. Hence,

$$N_T^M = e^{-\theta}N_{T-1}^M + (1-\zeta)(1-e^{-\theta})N_{T-1}^M + (1-\beta)\Big(\zeta(1-e^{-\theta})N_{T-1}^M + 1 - N_{T-1}^M\Big).$$

Then, the steady state fraction is given by $\bar{N}^M = (1-\beta)/(1-\beta+(1-e^{-\theta})\beta\zeta)$.

4.5.2 The Optimal Patent Strength

It is instructive to approach the planner's problem in the following manner. First, we find the optimal patent strength consistent with each equilibrium. After that, we compare the resulting optimized welfare. To this end, one first need to find the range of patent strengths consistent with each equilibrium. The next lemma gives the result.

Lemma 15 *At stage 2 firms choose secrecy with probability s, where*

$$s = \begin{cases} 0 & \text{if } \alpha > \beta, \\ \frac{1}{\theta} ln\left(\frac{1-\alpha\gamma}{1-\beta\gamma}\right) & \text{if } \alpha \in [\max\{0, \frac{\beta\gamma+e^{-\theta}-1}{e^{-\theta}\gamma}\}, \beta], \\ 1 & \text{if } \alpha < \frac{\beta\gamma+e^{-\theta}-1}{e^{-\theta}\gamma}. \end{cases}$$

The proof is immediate from the proof of lemma 23 in the appendix. As in the general model from the previous sections, all firms patent if patent protection is at least as strong as the effective protection under secrecy. Since no firm is willing to duplicate an innovation, this effective protection is simply β. Furthermore, if the strategic advantage of patenting is large enough, a secrecy equilibrium may not be achievable: when $e^{-\theta} < 1 - \beta\gamma$ congestion is very high, so the strategic aspect of patenting is always large enough to induce at least some firms to use patent protection.

Next, we turn to the planner's problem. Proposition 16 establishes that it is never optimal to have a patent system which does not provide incentives for any firm to patent.

Proposition 16 *The optimal patent strength consistent with a patenting equilibrium is $\alpha_P = \beta$. Moreover, a secrecy equilibrium is never optimal.*

The proof is in the appendix. The intuition behind the result is straightforward. In the patenting equilibrium all firms disclose their innovations, so the planner can increase competition at no cost. Thus, she sets $\alpha = \beta$ — the patent strength which maximizes competition subject to the equilibrium being patenting. At α_P both the patenting and secrecy equilibria provide the same expected duration of monopoly, namely $1/(1 - \beta)$. Hence, the ENPV of per innovation welfare is the same in both equilibria, $W^P(\alpha_P) = W^S$.[33] Welfare in the patenting equilibrium, however, can be strictly larger than that in the secrecy equilibrium because of disclosure. In the secrecy equilibrium not all innovations are public knowledge, because of this only a fraction $\bar{N}^S < 1$ of ideas are new. This lower mass of profitable R&D projects effectively reduces the total mass of innovations made each period, as compared to the patenting equilibrium.

Corollary 17 *The maximum welfare when there is a patent system is always strictly larger than the welfare when there is no patent system.*

Proof. This is immediate from the fact that the maximum welfare with a patent system is at least as large as $\bar{W}^P(\alpha_P)$, proposition 16, and $\bar{W}^N = \bar{W}^S$.

■

As in most previous works (see for example, [27] and [51]) the patent system is welfare improving. The intuition behind the result is the same as in the previous paragraph. At $\alpha = \beta$ the expected duration of monopoly is the same under the patenting equilibrium and the equilibrium without a patent system, hence, $W^P(\alpha_P) = W^N$. However, disclosure and, hence, the mass of innovations made each period is lower in the equilibrium without a patent system.

The intuition behind the result differs somewhat from previous papers, however. For example, [27] find that the patent system is welfare improving because of two reasons. First, in their model the dead weight loss under patenting may be smaller than the dead

[33] $\bar{W}^N$ and $\bar{W}^S$ are independent of the patent strength. However, to be explicit, we use $\bar{W}^M(\alpha)$ and $\bar{W}^P(\alpha)$ when the ENPV of welfare is evaluated at a particular patent strength in the mixed and patenting equilibria.

weight loss under secrecy (even though the expected duration of monopoly is longer under patenting than under secrecy). Second, because of disclosure, patents allow society to avoid wasteful duplication of R&D effort. In contrast, in this paper the result is driven by the diffusion aspect of disclosure: whenever an innovation is patented, it is disclosed and a new idea, which can potentially be innovated and generate welfare, enters the pool. When an idea is developed in secrecy, however, the innovation is not disclosed and society bares the cost of foregone welfare.

The present paper features the novel result that the optimal patent strength may lie in the region consistent with a mixed equilibrium. In that region the planner faces a trade off between increasing competition and providing incentives for firms to disclose their innovations. As the patent strength decreases there is the potential of reducing the expected duration of monopoly and, hence, increasing the ENPV of per innovation welfare. At the same time, however, more firms use secrecy protection, which reduces disclosure and, hence, the mass of profitable innovations. Thus, the planner aims to strike a balance between these two opposing effects when she decides on the optimal patent strength.

The welfare loss from reduced disclosure depends on the speed with which firms switch from patenting to secrecy as the patent strength decreases ($\partial s/\partial\alpha$). If the firms' marginal response is large, so is the impact of α on $\bar{N}^M$. The gain from competition, on the other hand, depends on both this speed and the dead weight loss from monopoly. The welfare gain from reduced patent strength is higher for larger dead weight loss, but more importantly, it is smaller for large $\partial s/\partial\alpha$. To see the intuition clearly, observe that in the mixed equilibrium the ex-ante expected duration of monopoly is $\zeta/(1-\beta)+(1-\zeta)/(1-\alpha)$.

As α decreases, so does the expected duration of monopoly of patented innovations. At the same time, however, more firms switch to secrecy protection which decreases the ex-ante expected duration of monopoly (because $\alpha < \beta$ for $s \in (0,1)$). The larger $\partial s/\partial\alpha$ is, the larger $\partial\zeta/\partial\alpha$ is, and the larger is the marginal decrease in ex-ante expected duration of monopoly.

Thus, if firms switch too fast, the planner may find it optimal to induce a patenting equilibrium and not sacrifice disclosure. If they switch slow enough, however, she may find it optimal to provide weak patent protection and induce the mixed equilibrium, as the welfare gains from higher competition are larger than the costs of reduced disclosure. The next proposition gives a sufficient condition for the gains from competition to outweigh the loss from reduced disclosure for at least some patent strengths in the mixed equilibrium as compared to the patenting equilibrium.

Proposition 18 *If*

$$\frac{S_C - S_M}{S_C} \geq \frac{e^{-\theta}\beta}{1-\beta}\Big(1 + \frac{\gamma(1-\beta)}{1-\gamma}\Big), \tag{4.7}$$

then the optimal patent strength is such that the equilibrium is mixed.

The proof is in the appendix. The left hand side of (4.7) captures the monopoly dead weight loss in per period consumer surplus. The right hand side represents the relative cost of of destroying that monopoly. The term in the brackets represents an upper bound on the welfare loss from decreasing disclosure: lower disclosure implies a lower mass of profitable ideas and, hence, a lower mass of innovations made each period. The term $e^{-\theta}\beta/(1-\beta)$ captures the speed with which firms switch protection strategies. When secrecy protection is low, firms are slow to switch because secrecy allows for only minor increases in appropriability. Similarly, firms are slow to switch to secrecy when there is higher congestion. This is due to the strategic aspect of patents. When θ is higher, so is the expected number of competitors any given firm would face at the race. This implies a higher strategic benefit because an innovator that patents can block a higher number of rivals, on average. Thus, the strategic aspect of patents is important not only for the patent system's ability to increase competition, but also for its ability to allow the planner to turn higher competition into welfare gains. More precisely, in the current setting the patent system provides a strictly higher degree of competition (as compared to $i = N$) when $s \in (0, 1)$. Thus, the strategic advantage is always large enough to allow the patent system to erode market power. If (4.7) does not hold, however, the strategic benefit from patenting is relatively low and firms switch protection strategies relatively fast. So, the planner finds it too costly to increase competition. Only when congestion is high enough, the strategic advantage of patents is sufficient to allow the planner to exploit the patent system's ability to increase competition for welfare gains.

The result is interesting because of a couple of reasons. First, it suggests that the analysis in the preceding sections may have important practical applications for policy makers. Second, the result suggests that it may be optimal to provide weak incentives to firms, so that only a fraction of them use patent protection. To the best of our knowledge, this is the first paper which finds a mixed equilibrium might be optimal. The majority of previous studies do not feature an equilibrium where some firms would patent and some would keep identical innovations secret. Even studies which feature such an equilibrium, for example [51], have found that it is socially optimal to incentivize all firms to use patent protection.[34] This is the case because, to the best of our knowledge, previous studies do not feature a patent system that provides prior user rights and can increase competition and, as the preceding analysis suggests, the patent system's ability to increase competition is the key reason why the planner might want to induce a mixed equilibrium.

4.6 Conclusion

The traditional view of the patent system is that it creates temporary monopolies in order to stimulate disclosure of information and/or create incentives for firms to innovate. This paper develops a dynamic equilibrium search model of innovation which aims to show that this traditional view does not hold when one takes into account the possibility of duplication, simultaneous innovation, and the importance of lead time advantage. In fact, the patent system can reduce the market power of innovators, i.e. increase competition, while providing incentives for at least some firms to patent.

The patent system can increase competition, regardless of whether or not it provides PUR. This is achieved by setting weak enough patent protection and providing incentives for firms to patent given the reduced appropriability. Without PUR, the patent system can incentivize firms by reducing the option value of secrecy through the threat of duplicative innovation. With a patent system, a firm can independently duplicate and patent the innovation, thus, excluding the original innovator from any profits. In contrast,

[34] In their model the dead weight loss due to monopolies is the same in the equilibrium without a patent system, the secrecy equilibrium, and the mixed equilibrium. Moreover, in the patenting equilibrium it is at least as large as in the other equilibria. Hence, the planner has no welfare gains from eliciting the mixed equilibrium. [51] also consider firms' incentives to innovate. In their model, however, firms make the same equilibrium investment in the equilibrium without a patent system, the secrecy equilibrium, and the mixed equilibrium. Moreover, this level of investment is achievable in the patenting equilibrium as well. Thus, the planner cannot induce any gains in welfare due to changing incentives to innovate from eliciting a mixed equilibrium.

without a patent system, the duplicator uses secrecy and both the original innovator and the duplicator yield duopoly profits. With PUR, the patent system can provide incentives for firms to patent, because patents have a strategic aspect (a firm that patents can block all rivals who innovate simultaneously and opt for secrecy from commercially exploiting the innovation). When firms want to secure a lead time advantage, the benefit due to the strategic advantage of patents could outweigh the loss in appropriability. Then, at least some firms choose patent protection even if this provides them with lower market power as compared to secrecy.

The results of this paper are not only of theoretical interest, but of practical as well. In a series of works Boldrin and Levine make a case against the patent system by pointing out the "huge" social costs due to temporary monopolies induced by patents.[35] In light of the results of this paper, however, this argument can be viewed in favor of the patent system, as it can erode temporary monopolies and, in fact, reduce the aforementioned social costs.

Furthermore, this paper analyses the welfare implications of a patent system which can increase competition. We find that the patent system is always welfare improving and that it can induce all firms to patent (and disclose their innovations) without imposing any additional social costs due to temporary monopolies. We also find that, depending on parameter values, the patent system can, in fact, improve welfare by directly reducing these costs. If the strategic advantage of patenting is large enough, then the planner finds it optimal to induce an equilibrium in which only a fraction of all firms use patent protection.

[35] See, for example, [14], [15], [17], and [16]. More precisely, they argue that the rationale behind the traditional view of patents implies that the presence of a patent system decreases welfare in practice. This is the case since the social loss due to monopolies is too large and cannot be compensated for by the diffusion of information and increased incentives to innovate (which the authors argue yield very low social benefit).

If this is the case, then the patent system induces disclosure of at least some innovations and at the same time increases competition. This result suggests that the patent system's ability to erode market power may be a key factor when it comes to its capacity to improve welfare.

Chapter 5

Conclusions

This dissertation consists of several chapters, each analyzing the economic impact of simultaneous innovation. Whereas, the topics covered therein fit under this common theme, each of them highlights a different channel through which the phenomenon of simultaneous innovation impacts economic growth, macroeconomic fluctuations, and welfare.

Chapter two of the dissertation focuses on the inherent coordination frictions implied by the presence of simultaneous innovation. It details the three impacts of these frictions on the economy — they generate a mass of foregone innovation, amplify the fraction of wasteful innovation, and reduce the economy-wide R&D intensity. As a result, the growth rate and welfare in the economy are depressed. A numerical exercise suggests the frictions are likely to have quantitatively sizable impact on both economic growth and welfare.

Chapter three focuses on the interplay between simultaneous innovation and innovation quality. It develops a theoretical model in which innovations made simultaneously by more firms have a higher expected quality. This interaction delivers an endogenous mechanism that amplifies the model-generated volatility of R&D investment and, at the same time, produces a mildly pro-cyclical R&D. The analysis, thus, provides a rationale behind the magnitude of the correlation between R&D and output observed in the data and, at the same time, emphasizes the consequences of simultaneous innovation for macroeconomic fluctuations.

Chapter four focuses on the role of the patent system in the context of simultaneous innovation. It shows that in this context the traditional view of the patent system which states that it must allocate market power to firms in order to incentivize them to patent their innovations does not necessarily hold. On the contrary, the patent system can induce firms to patent through a strategic advantage even if patenting erodes their market power. As a result the patent system can increase the level of competition in the economy. Moreover, the analysis suggests that the patent system's ability to erode market power may be central to its capability to improve welfare.

Bibliography

[1] Daron Acemoglu and Ufuk Akcigit. Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association*, 10(1):1–42, 2012.

[2] P Aghion and P Howitt. A model of growth through creative destruction. *Econometrica*, 60(2), 1992.

[3] Philippe Aghion, Ufuk Akcigit, and Peter Howitt. Chapter 1 - what do we learn from schumpeterian growth theory? In Philippe Aghion and Steven N. Durlauf, editors, *Handbook of Economic Growth*, volume 2 of *Handbook of Economic Growth*, pages 515 – 563. Elsevier, 2014.

[4] Philippe Aghion, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. Competition and innovation: An inverted-u relationship. *The Quarterly Journal of Economics*, 120(2):701–728, 2005.

[5] Ufuk Akcigit, Murat Alp Celik, and Jeremy Greenwood. Buy, keep, or sell: Economic growth and the market for ideas. *Econometrica*, 84(3):943–984, 2016.

[6] James J Anton and Dennis A Yao. Little patents and big secrets: managing intellectual property. *RAND Journal of Economics*, pages 1–22, 2004.

[7] Kenneth Arrow. Economic welfare and the allocation of resources for invention. In *The rate and direction of inventive activity: Economic and social factors*, pages 609–626. Nber, 1962.

[8] Anthony Arundel. The relative effectiveness of patents and secrecy for appropriation. *Research policy*, 30(4):611–624, 2001.

[9] Gadi Barlevy. On the cyclicality of research and development. *The American Economic Review*, pages 1131–1164, 2007.

[10] Robert J. Barro and Xavier I. Sala-i Martin. *Economic Growth*. The MIT Press, 2nd edition, 2003.

[11] Jess Benhabib, Jesse Perla, and Christopher Tonetti. Catch-up and fall-back through innovation and imitation. *Journal of Economic Growth*, 19(1):1–35, 2014.

[12] Florin O Bilbiie, Fabio Ghironi, and Marc J Melitz. Endogenous entry, product variety, and business cycles. *Journal of Political Economy*, 120(2), 2012.

[13] Nicholas Bloom, Charles I Jones, John Van Reenen, and Michael Webb. Are ideas getting harder to find? *Manuscript, Stanford University, Palo Alto*, 2016.

[14] Michele Boldrin and David K Levine. The economics of ideas and intellectual property. *Proceedings of the National Academy of Sciences of the United States of America*, 102(4):1252–1256, 2005.

[15] Michele Boldrin and David K Levine. Perfectly competitive innovation. *Journal of Monetary Economics*, 55(3):435–453, 2008.

[16] Michele Boldrin and David K Levine. The case against patents. *The journal of economic perspectives*, 27(1):3–22, 2013.

[17] Michele Boldrin, David K Levine, et al. *Against intellectual monopoly*. Cambridge University Press Cambridge, 2008.

[18] Timothy F Bresnahan and Manuel Trajtenberg. General purpose technologies engines of growth? *Journal of econometrics*, 65(1):83–108, 1995.

[19] Kenneth Burdett, Shouyong Shi, and Randall Wright. Pricing and matching with frictions. *Journal of Political Economy*, 109(5):1060–1085, 2001.

[20] Jonathan Chiu, Cesaire Meh, and Randall Wright. Innovation and growth with financial, and other, frictions. Working Paper Series 4732, Victoria University of Wellington, School of Economics and Finance, 2015.

[21] Linda R Cohen and Jun Ishii. Competition, innovation and racing for priority at the us patent and trademark office. *USC CLEO Research Paper*, (C05-13):05–22, 2005.

[22] Wesley M Cohen, Richard R Nelson, and John P Walsh. Protecting their intellectual assets: Appropriability conditions and why us manufacturing firms patent (or not). Technical report, National Bureau of Economic Research, 2000.

[23] Diego Comin and Mark Gertler. Medium-term business cycles. *The American Economic Review*, 96(3):523, 2006.

[24] Louis Corriveau. Entrepreneurs, growth and cycles. *Economica*, pages 1–15, 1994.

[25] Louis Corriveau. Innovation races, strategic externalities and endogenous growth. *Economica*, 65(259):303–325, 1998.

[26] Vincenzo Denicolo. Patent races and optimal patent breadth and length. *The Journal of Industrial Economics*, pages 249–265, 1996.

[27] Vincenzo Denicolo and Luigi Alberto Franzoni. The contract theory of patents. *International Review of Law and Economics*, 23(4):365–380, 2003.

[28] Vincenzo Denicolo and Luigi Alberto Franzoni. Patents, secrets, and the first-inventor defense. *Journal of Economics & Management Strategy*, 13(3):517–538, 2004.

[29] Vincenzo Denicolò and Luigi Alberto Franzoni. Innovation, duplication, and the contract theory of patents. *Contributions to Economic Analysis*, 286:15–32, 2008.

[30] Kira R Fabrizio and Ulya Tsolmon. An empirical examination of the procyclicality of r&d investment and innovation. *Review of Economics and Statistics*, 96(4):662–675, 2014.

[31] Antonio Fatas. Do business cycles cast long shadows? short-run persistence and economic growth. *Journal of Economic Growth*, 5(2):147–162, 2000.

[32] Patrick Francois and Huw Lloyd-Ellis. Schumpeterian cycles with pro-cyclical r&d. *Review of Economic Dynamics*, 12(4):567–591, 2009.

[33] Ove Granstrand. *The economics and management of intellectual property towards intellectual capitalism*. External organization, 1999.

[34] Jerry R Green and Suzanne Scotchmer. On the division of profit in sequential innovation. *The RAND Journal of Economics*, pages 20–33, 1995.

[35] Zvi Griliches. Patent statistics as economic indicators: A survey. *Journal of Economic Literature*, 28(4):1661–1707, 1990.

[36] Gene M Grossman and Elhanan Helpman. Quality ladders in the theory of growth. *The Review of Economic Studies*, 58(1):43–61, 1991.

[37] Bronwyn Hall, Christian Helmers, Mark Rogers, and Vania Sena. The choice between formal and informal intellectual property: a review. *Journal of Economic Literature*, 52(2):375–423, 2014.

[38] Bronwyn H Hall, Jacques Mairesse, and Pierre Mohnen. Measuring the returns to r&d. *Handbook of the Economics of Innovation*, 2:1033–1082, 2010.

[39] Elhanan Helpman. *General purpose technologies and economic growth*. MIT press, 1998.

[40] Andreas Hornstein, Per Krusell, and Giovanni L Violante. The effects of technical change on labor market inequalities. *Handbook of economic growth*, 1:1275–1370, 2005.

[41] Charles I Jones. R&D-Based Models of Economic Growth. *Journal of Political Economy*, 103(4):759–784, August 1995.

[42] Charles I Jones. Sources of us economic growth in a world of ideas. *The American Economic Review*, 92(1):220–239, 2002.

[43] Charles I Jones and Jihee Kim. A schumpeterian model of top income inequality. Technical report, National Bureau of Economic Research, 2014.

[44] Boyan Jovanovic and Peter L Rousseau. General purpose technologies. *Handbook of economic growth*, 1:1181–1224, 2005.

[45] Benoıt Julien, John Kennes, and Ian King. Bidding for labor. *Review of Economic Dynamics*, 3(4):619–649, 2000.

[46] Todd R Kaplan, Israel Luski, and David Wettstein. Innovative activity and sunk cost. *International Journal of Industrial Organization*, 21(8):1111–1133, 2003.

[47] Tor Jakob Klette and Samuel Kortum. Innovating firms and aggregate innovation. *Journal of Political Economy*, 112(5), 2004.

[48] Samuel S Kortum. Research, patenting, and technological change. *Econometrica: Journal of the Econometric Society*, pages 1389–1419, 1997.

[49] Klaus Kultti and Tuomas Takalo. Optimal fragmentation of intellectual property rights. *International Journal of Industrial Organization*, 26(1):137–149, 2008.

[50] Klaus Kultti, Tuomas Takalo, and Juuso Toikka. Simultaneous model of innovation, secrecy, and patent policy. *The American economic review*, 96(2):82–86, 2006.

[51] Klaus Kultti, Tuomas Takalo, and Juuso Toikka. Secrecy versus patenting. *The RAND Journal of Economics*, 38(1):22–42, 2007.

[52] Mark A Lemley. Myth of the sole inventor, the. *Mich. L. Rev.*, 110:709, 2011.

[53] Xiaohua Lu and R Preston McAfee. The evolutionary stability of auctions over bargaining. *Games and Economic Behavior*, 15(2):228–254, 1996.

[54] Robert E. Jr. Lucas and Benjamin Moll. Knowledge Growth and the Allocation of Time. *Journal of Political Economy*, 122(1):1 – 51, 2014.

[55] William D Nordhaus. *Invention, growth, and welfare: A theoretical treatment of technological change*. MIT press Cambridge, MA, 1969.

[56] Min Ouyang. On the cyclicality of r&d. *Review of Economics and Statistics*, 93(2):542–553, 2011.

[57] Jesse Perla and Christopher Tonetti. Equilibrium imitation and growth. *Journal of Political Economy*, 122(1):52–76, 2014.

[58] Jennifer F Reinganum. A dynamic game of r and d: Patent protection and competitive behavior. *Econometrica: Journal of the Econometric Society*, pages 671–688, 1982.

[59] Jennifer F Reinganum. Uncertain innovation and the persistence of monopoly. *The American Economic Review*, pages 741–748, 1983.

[60] Jennifer F Reinganum. The timing of innovation: Research, development, and diffusion. *Handbook of industrial organization*, 1:849–908, 1989.

[61] Paul M Romer. Endogenous technological change. *Journal of Political Economy*, 98(5 pt 2), 1990.

[62] Paul S Segerstrom, Thirumalai CA Anant, and Elias Dinopoulos. A schumpeterian model of the product life cycle. *The American Economic Review*, pages 1077–1091, 1990.

[63] Robert Shimer. The assignment of workers to jobs in an economy with coordination frictions. *Journal of Political Economy*, 113(5):996–1025, 2005.

[64] Klaus Wälde and Ulrich Woitek. R&d expenditure in g7 countries and the implications for endogenous fluctuations and growth. *Economics Letters*, 82(1):91–97, 2004.

[65] Asher Wolinsky. Dynamic markets with competitive bidding. *The Review of Economic Studies*, 55(1):71–84, 1988.

[66] Tianle Zhang. Patenting in the shadow of independent discoveries by rivals. *International Journal of Industrial Organization*, 30(1):41–49, 2012.

Appendix A

Appendix for Chapter 2

A.1 Welfare Comparison

We follow [5] and compare the welfare difference between any two economies A and B in consumption equivalent terms. In particular, consider the welfare in economy A, W^A, and economy B, W^B, along their BGPs. Suppose at time $t = 0$, both economies start at the same initial position with $N_0^A = N_0^B$. Now, welfare in economy i is given by

$$W^i = \sum_{t=0}^{\infty} \beta^t \ln C_t^i = \ln\left((1+g^i)^{\frac{\beta}{(1-\beta)^2}} C_0^{i\,\frac{1}{1-\beta}} \right). \tag{A.1}$$

Then, let $\alpha^{A,B}$ measure the fraction with which initial consumption in economy A, C_0^A, must be increased for consumers to have the same welfare as people in economy B. Thus, α is given by

$$\alpha^{A,B} = e^{(1-\beta)(W^B - W^A)} - 1. \tag{A.2}$$

This measure of welfare is used throughout the text. In particular, the welfare gain from eliminating frictions in the DE is given by $\alpha^{DE,CE}$ and the gain from eliminating frictions in the planner's allocation is given by $\alpha^{SB,FB}$.

We decompose the welfare gain from eliminating frictions into the gain from eliminating foregone innovation and the gain from eliminating wasteful innovation. The welfare gain from eliminating foregone innovation in the DE is given by $\alpha^{DE,DEF}$, where DEF is a hypothetical decentralized economy that features no foregone innovation but the same level of wasteful innovation as the DE. In particular, $g^{DEF} = g^c$, $C_0^{DEF}/N_0 = \pi(1+\lambda)/\lambda - \eta\theta^{DEF}(M-1)$, and $\theta^{DEF} = \beta\pi/(\eta(1+g^{DE}-\beta)$. Thus, the welfare cost of wasteful innovation in the decentralized economy is given by $\alpha^{DE,CE} - \alpha^{DE,DEF}$. Similarly, the welfare cost of foregone innovation in the SB is given by $\alpha^{SB,SBF}$, where SBF is a hypothetical allocation in which the planner can assign firms to projects but has to keep the fraction of wasteful innovation as in the SB. In particular, $g^{SBF} = g^{FB}$, $C_0^{SBF}/N_0 = \pi^* - \eta\theta^{SBF}(M-1)$, and $\theta^{SBF} = \theta^{SB}/(1-e^{-\theta^{SB}})$. Thus, the welfare cost of wasteful duplication of effort in the SB is given by $\alpha^{SB,FB} - \alpha^{SB,SBF}$.

A.2 Augmented Model

We explore the robustness of the quantitative results from section five in an extension of our baseline model. The economy in this extension features uncertainty in the innovation process and endogenous research effort intensity. In the interest of consistency, the only difference with the baseline model is in the innovation sector. At stage one firms still enter at a cost $\eta > 0$ and at stage two firms still choose a direction for their R&D effort. However, now at stage three entrants choose a research intensity i which affects their probability of successfully innovating. In particular, the cost of exerting effort i is ϕi and the probability of successfully innovating the chosen project is $1 - e^{-\gamma i}$, where $\phi, \gamma > 0$. Stage four is as in the baseline model.

A.2.1 Decentralized Economy

The final good sector and the final stage of the innovation process are as in the baseline model. Hence, $P_t(n) = 1/\lambda$ and $X_t(n) = X$. At stage three, firms choose effort i that maximizes the expected reward from the R&D stage, $R_t(i) \equiv Pr(\text{patent})V_t - \phi i$. Since $Pr(\text{patent}) = Pr(\text{success})Pr(\text{patent}|\text{success}) = (1-e^{-\gamma i})Pr(\text{patent}|\text{success})$, it follows that the optimal research effort solves

$$Pr(\text{patent}|\text{success})V_t = \frac{\phi}{\gamma}e^{\gamma j}, \tag{A.3}$$

where j is the level of research effort in a symmetric equilibrium. The second stage is analogous to the one in the baseline model, except now there is a chance firms are not successful in innovating. Let the effective market tightness be denoted by $\tilde{\theta}_t \equiv (1 - e^{-\gamma j})\theta_t$. Then, it is straightforward to establish that the number of firms that successfully innovate a particular idea follows a Poisson distribution with mean $\tilde{\theta}_t$.[1] Thus, the probability of receiving a patent conditional on innovating is given by $Pr(\text{patent}|\text{success}) = (1 - e^{-\tilde{\theta}_t})/\tilde{\theta}_t$. Hence,

$$\frac{1 - e^{-\tilde{\theta}_t}}{\tilde{\theta}_t}V_t = \frac{\phi}{\gamma}e^{\gamma j}. \tag{A.4}$$

Free entry implies that $\eta = R(j)$. Thus,

$$\eta + \phi j = \frac{\phi}{\gamma}(e^{\gamma j} - 1), \tag{A.5}$$

which yields an implicit solution for the equilibrium research intensity j.

[1] A proof is available upon request.

The laws of motion for varieties and ideas are analogous to the baseline model, with the only exception that now the probability an idea is innovated is given by $1 - e^{-\tilde{\theta}_t}$. Hence, $\nu/N = M - 1$ and $g = (1 - e^{-\tilde{\theta}})(M - 1)$. Furthermore, consumers face the same problem as in the baseline model. Thus, $V_t = \beta\pi/(1 + g - \beta)$. Using the economy's resource constraint and (A.5) it follows that along the BGP

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \tilde{\theta}(M-1)\frac{\phi}{\gamma}e^{\gamma j}. \tag{A.6}$$

Finally, using (A.4) and the expression for V_t, it follows that the effective market tightness solves

$$\frac{\beta\pi}{1 + (1 - e^{-\tilde{\theta}})(M-1) - \beta} = \frac{\phi}{\gamma}e^{\gamma j}\frac{\tilde{\theta}}{1 - e^{-\tilde{\theta}}}. \tag{A.7}$$

A.2.2 Coordination Economy

As in the baseline version of the model, the only difference between the DE and the CE is that at stage two of the innovation process — in the CE, a Walrasian auctioneer coordinates firm's research efforts. Thus, $P_t(n) = 1/\lambda$ and $X_t(n) = X$. Next, as in the DE, the optimal research effort in equilibrium solves

$$Pr(\text{patent}|\text{success})V_t^c = \frac{\phi}{\gamma}e^{\gamma j}. \tag{A.8}$$

Then, let us focus on stage two. Whenever there are $\mu_t < \nu_t$ firms in the R&D sector, the auctioneer assigns a unique idea to each firm and $Pr(\text{patent}|\text{success}) = 1$. When $\theta^c \geq 1$, however, the auctioneer distributes firms to ideas as equally as she can, subject to assigning integer number of firms to each research avenue. In the event that $l \geq 1$ firms successfully innovate the same idea, they each receive the patent with probability $1/l$.[2] For example,

[2] This process of coordination is different from the one in the baseline model where innovation is certain, so the auctioneer can effectively assign patents to entrants. This is because firms do not innovate for sure

if $\theta^c = 8.2$, then a fraction 0.2 of ideas are matched with 9 firms and a fraction 0.8 of ideas are matched with 8 firms. Thus, a fraction 1.8/8.2 of firms face 8 rivals and a fraction 6.4/8.2 face 7. In general, a fraction $\lceil\theta^c\rceil(\theta^c - \lfloor\theta^c\rfloor)/\theta^c$ of firms face $\lfloor\theta^c\rfloor$ rivals and a fraction $\lfloor\theta^c\rfloor(\lceil\theta^c\rceil - \theta^c)/\theta^c$ face $\lfloor\theta^c\rfloor - 1$, where $\lfloor x\rfloor$ is the largest integer less than x and $\lceil x\rceil$ is the smallest integer larger than x. Hence,

$$\begin{aligned} Pr(\text{patent}|\text{success}) =& \frac{\lfloor\theta^c\rfloor(\lceil\theta^c\rceil - \theta^c)}{\theta^c} \sum_{l=0}^{\lfloor\theta^c\rfloor-1} \binom{\lfloor\theta^c\rfloor - 1}{l} (1 - e^{-\gamma j})^l e^{-\gamma j(\lfloor\theta^c\rfloor-1-l)} \frac{1}{l+1} \\ &+ \frac{\lceil\theta^c\rceil(\theta^c - \lfloor\theta^c\rfloor)}{\theta^c} \sum_{l=0}^{\lfloor\theta^c\rfloor} \binom{\lfloor\theta^c\rfloor}{l} (1 - e^{-\gamma j})^l e^{-\gamma j(\lfloor\theta^c\rfloor-l)} \frac{1}{l+1} \\ =& \frac{\lceil\theta^c\rceil - \theta^c}{(1 - e^{-\gamma j})\theta^c}(1 - e^{-\gamma j\lfloor\theta^c\rfloor}) + \frac{\theta^c - \lfloor\theta^c\rfloor}{(1 - e^{-\gamma j})\theta^c}(1 - e^{-\gamma j\lceil\theta^c\rceil}). \end{aligned} \tag{A.9}$$

The case relevant for out numerical exercise is $\theta^c \geq 1$, so we restrict our attention to it.

Next, free entry and (A.8) imply that

$$\eta + \phi j^c = \frac{\phi}{\gamma}(e^{\gamma j^c} - 1), \tag{A.10}$$

which yields the same equilibrium research effort as in the DE.

The laws of motion for varieties and ideas is the same as in the DE with the exception that now the probability an idea is innovated is given by $(\theta^c - \lfloor\theta^c\rfloor)(1 - e^{-\gamma j^c\lceil\theta^c\rceil}) + (\lceil\theta^c\rceil - \theta^c)(1 - e^{-\gamma j^c\lfloor\theta^c\rfloor})$. The consumer's optimization problem yields $V_t^c = \beta\pi/(1 - \beta + g^c)$, where $g^c = (\theta^c - \lfloor\theta^c\rfloor)(1 - e^{-\gamma j^c\lceil\theta^c\rceil}) + (\lceil\theta^c\rceil - \theta^c)(1 - e^{-\gamma j^c\lfloor\theta^c\rfloor})(M - 1)$. Lastly, the resource constraint and the expression for the value of holding a patent yield

in our augmented model.

$$\left(\frac{C}{N}\right)^c = \frac{1+\lambda}{\lambda}\pi - \theta^c(M-1)(\eta+\phi j^c), \quad (A.11)$$

$$\left(\frac{\lceil\theta^c\rceil - \theta^c}{(1-e^{-\gamma j})\theta^c}(1-e^{-\gamma j\lfloor\theta^c\rfloor}) + \frac{\theta^c - \lfloor\theta^c\rfloor}{(1-e^{-\gamma j})\theta^c}(1-e^{-\gamma j\lceil\theta^c\rceil})\right)^{-1}\frac{\phi}{\gamma}e^{\gamma j^c} = \frac{\beta\pi}{1-\beta+g^c}. \quad (A.12)$$

A.2.3 Second-Best Allocation

Analogously to the baseline model, the planner solves

$$\max_{\{C_t,X_t,\tilde{\theta}_t,N_t,\nu_t,j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t \ln C_t,$$

$$AL^{1-\lambda}N_tX_t^{\lambda} = N_tX_t + C_t + \tilde{\theta}_t\nu_t\frac{\eta+\phi j}{1-e^{-\gamma j}}, \quad (A.13)$$

$$N_{t+1} = N_t + (1-e^{-\tilde{\theta}_t})\nu_t, \quad (A.14)$$

$$\nu_{t+1} = \nu_t + (1-e^{-\tilde{\theta}_t})(M-1)\nu_t. \quad (A.15)$$

The first order condition with respect to X_t yields $X_t = X^*$ as in the baseline model. Furthermore, the first order condition with respect to the research effort, j, yields

$$\eta + \phi j^{SB} = \frac{\phi}{\gamma}(e^{\gamma j^{SB}} - 1), \quad (A.16)$$

which is the same level of research effort as in the DE. Let $\tilde{\eta} \equiv \phi e^{\gamma j^{SB}}/\gamma$, hence, the rest of the first order conditions are

$$[C_t]: \quad \beta\frac{C_t}{C_{t+1}} = \frac{\tilde{\phi}_{t+1}}{\tilde{\phi}_t}, \quad (A.17)$$

$$[N_{t+1}]: \quad h_t = h_{t+1} + \tilde{\phi}_{t+1}\pi^*, \quad (A.18)$$

$$[\nu_{t+1}]: \quad \lambda_t = \lambda_{t+1}\left(e^{-\tilde{\theta}_{t+1}} + (1-e^{-\tilde{\theta}_{t+1}})M\right) + h_{t+1}(1-e^{-\tilde{\theta}_{t+1}}) - \tilde{\phi}_{t+1}\tilde{\eta}\tilde{\theta}_{t+1}, \quad (A.19)$$

$$[\tilde{\theta}_t]: \quad \tilde{\eta} = e^{-\tilde{\theta}_t}\left(\frac{h_t}{\tilde{\phi}_t} + \frac{\lambda_t}{\tilde{\phi}_t}(M-1)\right), \quad (A.20)$$

where $\tilde{\phi}_t$, h_t, and λ_t and the multipliers associated with (A.13), (A.14), and (A.15), respectively. Thus, the planner's problem reduces to the one in the baseline model. Hence,

$$\left(\frac{\nu}{N}\right)^{SB} = M - 1, \quad \text{(A.21)}$$

$$\left(\frac{C}{N}\right)^{SB} = \pi^* - \tilde{\eta}\tilde{\theta}^{SB}(M-1), \quad \text{(A.22)}$$

$$1 + (1 - e^{-\tilde{\theta}^{SB}})(M-1) = \beta\Big(1 + \frac{\pi^*}{\eta}e^{-\tilde{\theta}^{SB}} + (1 - e^{-\tilde{\theta}^{SB}} - \tilde{\theta}^{SB}e^{-\tilde{\theta}^{SB}})(M-1)\Big). \quad \text{(A.23)}$$

A.2.4 First-Best Allocation

Without loss of generality we impose symmetry in the production of varieties and the research effort intensity. Observe that by symmetry the planner assigns the same number of firms per idea. Hence, the probability an idea is innovated is given by $1 - e^{-\gamma\tilde{j}_t}$, where $\tilde{j}_t = j\theta_t$ is the effective research effort per idea. Now, the planner can achieve an additional unit of effective research by either increasing θ_t by $1/j$ units or increasing j by $1/\theta_t$ units. Furthermore, the cost of the former is $\nu_t\phi + \nu_t\eta/j$ units of the final good and the cost of the latter is $\nu_t\phi$. Thus, it is always cheaper to induce higher effective research effort by increasing the research intensity, j. Hence, $\theta^{FB} = 1$. Then, the planner's problem reduces to

$$\max_{\{C_t,X_t,N_t,\nu_t,j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t,$$

$$AL^{1-\lambda}N_tX_t^{\lambda} = N_tX_t + C_t + \nu_t\eta + \nu_t\phi j, \quad \text{(A.24)}$$

$$N_{t+1} = N_t + (1 - e^{-\gamma j})\nu_t, \quad \text{(A.25)}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\gamma j})(M-1)\nu_t. \quad \text{(A.26)}$$

The first order condition for X_t implies that the level of intermediate varieties is still given by X^*. Taking the rest of the first order conditions and applying straightforward algebra yields

$$\left(\frac{C}{N}\right)^{FB} = \pi^* - (\eta + \phi j^{FB})(M-1), \tag{A.27}$$

$$\frac{\phi}{\gamma} e^{\gamma j^{FB}} = \frac{\beta\pi^*}{1-\beta+g^{FB}} + \frac{\beta^2\pi^*(1-e^{-\gamma j^{FB}})(M-1)}{(1-\beta+g^{FB})(1+g^{FB})(1-\beta)} - \frac{\beta(\eta+\phi j^{FB})(M-1)}{(1+g^{FB})(1-\beta)}, \tag{A.28}$$

where $g^{FB} = (1-e^{-\gamma j^{FB}})(M-1)$.

A.2.5 Numerical Exercise

As in the baseline case, we calibrate the model at annual frequency, so the discount factor is set at $\beta = 0.95$. Furthermore, we normalize $\gamma = L = A = 1$. To calibrate η, M, and λ we use the same three moments as in the baseline case. In addition, we set the elasticity of firm-level output with respect to R&D investment at 0.05. This value is consistent with most firm-level estimates for the U.S.[3] The elasticity in our model is given by $\gamma j e^{-\gamma j}/(1-e^{-\gamma j})$, hence, the equilibrium research effort of firms is $j = 4.5139$.[4] Setting the R&D share of GDP to its empirical value yields $\lambda = 0.8516$, as in the baseline model. Next, the fraction of patents to patent applications is $(1-e^{-\tilde{\theta}})/\tilde{\theta}$. Matching this expression to its empirical counterpart yields $\tilde{\theta} = 1.0876$. Hence, $\theta = 1.0997$. Lastly, setting

[3] For a survey see [38].

[4] In our model firm-level output corresponds to $O(\tilde{c}) \equiv (1-e^{-\gamma\tilde{c}/\phi})(1-e^{-\tilde{\theta}})V_t/\tilde{\theta}$, where $\tilde{c} \equiv \phi j$ is the firm's R&D investment.

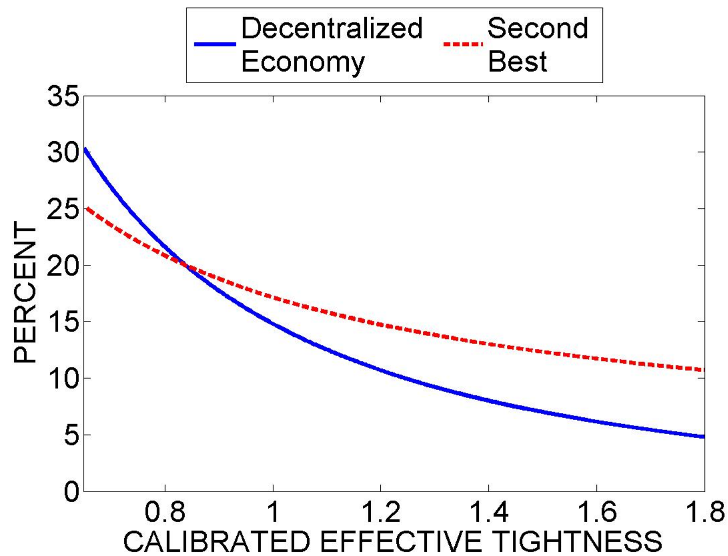

Figure A.1: Welfare Gain in the Augmented Model

$g = (1 - e^{-\tilde{\theta}})(M - 1) = 1.7546\%$ and using (A.5), (A.7) yields $M = 1.0265$, $\eta = 0.1611$, and $\phi = 0.0019$. The resulting welfare cost of coordination frictions in the DE is 12.76% and in the SB is 15.97%.

As in the baseline model, we explore the robustness of our quantitative results by varying the effective market tightness in the interval $\tilde{\theta} \in [0.65, 1.8]$.[5] The magnitude of the welfare costs is virtually the same as in the baseline model for all considered values of $\tilde{\theta}$ (Figure A.1).

[5]In our augmented model the two alternative calibration strategy sets the bounds on $\tilde{\theta}$. In particular the return of R&D is now given by $\partial Y_{t+1}/\partial R_t = g\tilde{\theta}e^{-\tilde{\theta}}/((1 - e^{-\tilde{\theta}})\eta\mu_t/Y_t)$.

A.3 Proofs Omitted from the Text

Proof of Proposition 16:

Proof. We follow the literature on coordination frictions in the labor market and treat our economy with a continuum of ideas as the limiting case of a finite-idea economy (see, for example, [63]). In particular (as in [45] and [19]), we characterize the equilibrium when the number of ideas and firms is finite and then evaluate the resulting equilibrium outcome in the limit as the number of projects tends to infinity (keeping the market tightness constant). Because we assume the economy features a continuum of ideas, we can abstract the analysis away from any approximation errors associated with focusing on the limiting equilibrium outcome.[6]

First, by assumption, the firm's probability of securing a monopoly position given that there are exactly n rivals, $Pr(\text{monopoly}|n) = 1/(n+1)$. In a symmetric equilibrium all firms place the same probability s_i of directing their effort towards a particular idea i. Then, the chance that a firm would face exactly n rivals is

$$Pr(n) = \binom{\mu_t - 1}{n} s_i^n (1 - s_i)^{\mu_t - 1 - n}.$$

[6]If one were to instead focus on an economy with a finite number of ideas, nothing in the analysis is lost. Since the economy exhibits positive long-run growth, the number of ideas eventually becomes "large" and the approximation error associated with focusing on the limiting equilibrium outcome negligible.

Hence, the probability of securing a monopoly position is given by

$$Pr(\text{monopoly}) = \sum_{n=0}^{\mu_t-1} Pr(\text{monopoly}|n)P(n) = \sum_{n=0}^{\mu_t-1} \binom{\mu_t-1}{n} s_i^n(1-s_i)^{\mu_t-1-n}\frac{1}{n+1} =$$
$$= \frac{1}{\mu_t}\sum_{n=0}^{\mu_t-1}\binom{\mu_t}{n+1} s_i^n(1-s_i)^{\mu_t-1-n} = \frac{1}{\mu_t s_i}\left(\sum_{n=0}^{\mu_t}\binom{\mu_t}{n} s_i^n(1-s_i)^{\mu_t-n} - (1-s_i)^{\mu_t}\right) =$$
$$= \frac{1-(1-s_i)^{\mu_t}}{\mu_t s_i}. \tag{A.29}$$

Next, we show that $s_k = s_j$ for all $k, j \in \nu_t$. Suppose not. Then, there exists some k, j such that $s_k > s_j$. But for any $i \in \nu_t$, we have that

$$\frac{\partial Pr(\text{monopoly})}{\partial s_i} = \frac{\mu_t^2 s_i(1-s_i)^{\mu_t-1} - \mu_t[1-(1-s_i)^{\mu_t}]}{(\mu_t s_i)^2}. \tag{A.30}$$

For any $s_i \in (0,1)$, it follows that $Pr(\text{monopoly})$ is decreasing in s_i if and only if $(1-s_i)^{\mu_t-1} < Pr(\text{monopoly})$ which clearly holds since $\mu_t \geq 2$. Now, for $s_i = 1$, we have that $\partial Pr(\text{monopoly})/\partial s_i = -1/\mu_t < 0$. Furthermore, $\lim_{s_i\to 0} \partial Pr(\text{monopoly})/\partial s_i = -(\mu_t-1)/2 < 0$. Hence, $Pr(\text{monopoly})$ is decreasing in s_i everywhere in its domain. Then, $s_k > s_j$ implies that $Pr_k(\text{monopoly}) < Pr(\text{monopoly})_j$, which then implies that $Pr_k(\text{monopoly})V_{k,t} < Pr_j(\text{monopoly})V_{j,t}$ since all varieties are equally profitable. Thus, $s_k > s_j$ cannot be an equilibrium. Hence, we must have $s_i = s_j$ for all $i, j \in \nu_t$. Thus, $s_i = 1/\nu_t$.

Then, it follows that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) = \binom{\mu_t}{n}\left(\frac{1}{\nu_t}\right)^n\left(1-\frac{1}{\nu_t}\right)^{\mu_t-n}. \tag{A.31}$$

Taking the limit as $\mu_t, \nu_t \to \infty$ (keeping the ratio θ_t constant) we get that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) \to \frac{\theta_t^n e^{-\theta_t}}{n!}. \tag{A.32}$$

■

Proof of Proposition 2

Proof. Totally differentiating both sides of (2.13) with respect to π yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}\pi} = \frac{\beta}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} > 0, \tag{A.33}$$

which is positive since $1 - e^{-\theta} - \theta e^{-\theta} > 0$. Similarly, totally differentiating (2.13) with respect to β, η, and M yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}\beta} = \Big[1 + \frac{\pi}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big]\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} > 0, \tag{A.34}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = -\frac{\beta\pi}{\eta^2}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big]\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} < 0, \tag{A.35}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}M} = -(1-e^{-\theta})\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} < 0. \tag{A.36}$$

■

Proof of Proposition 3:

Proof. First, let us prove the following lemma

Lemma 19 *The magnitude of the congestion externality is larger than that of the learning externality.*

Proof. First, we can decompose the difference between the planner's valuation of the benefit of entry and the firm's valuation of this benefit. At the SB this difference is given by

$$\mathcal{A} + \mathcal{L} + \mathcal{C} = \eta - \Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big)V^{SB}, \tag{A.37}$$

where $\mathcal{A}$, $\mathcal{L}$, and $\mathcal{C}$ denote the appropriability, learning, and congestion externalities; $V^{SB} :=$ $\beta\pi/(e^{-\theta^{SB}} + (1-e^{-\theta^{SB}})M - \beta)$ is the value of having a monopoly position at the second best level of the market tightness. The right hand side of (A.37) gives the difference between the planner's valuation of the benefit of entry, η, and the firm's, V^{SB} times the probability of securing a patent. Then, one can decompose the sum of the three externalities in the following manner

$$\mathcal{A} := \left(\left(\frac{h}{\phi}\right)^{SB} - V^{SB}\right)\left(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\right), \tag{A.38}$$

$$\mathcal{L} := \left(\frac{\lambda}{\phi}\right)^{SB}\left(e^{-\theta^{SB}}(M-1)\right), \tag{A.39}$$

$$\mathcal{C} := -\left(\frac{h}{\phi}\right)^{SB}\left(\left(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\right) - e^{-\theta^{SB}}\right). \tag{A.40}$$

Thus, $\mathcal{A}$ is the measure of how much more would the planner value entry than the firm if the appropriability externality was the only one in the model. $\mathcal{L}$ and $\mathcal{C}$ measure the same difference if the only externality in the model was learning and congestion, respectively.

From equations (A.39) and (A.40), it follows that the magnitude of the congestion externality is larger than that of the learning externality if and only if

$$\left(\frac{h}{\phi}\right)^{SB}\left(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\right) > e^{-\theta^{SB}}\left(\left(\frac{h}{\phi}\right)^{SB} + \left(\frac{\lambda}{\phi}\right)^{SB}(M-1)\right). \tag{A.41}$$

From equations (2.20) and (2.21), it then follows that (A.41) holds if and only if

$$\frac{(1-e^{-\theta^{SB}})\beta\pi^*}{\theta^{SB}\eta} > e^{-\theta^{SB}} + (1-e^{-\theta^{SB}})M - \beta. \tag{A.42}$$

Next, from the planner's solution, (2.25), it follows that $|\mathcal{C}| > \mathcal{L}$ if and only if $\pi^* - \eta\theta^{SB}(M-1) > 0$. But this has to hold, from equation (2.24), as the SB must feature $C_t > 0$. ■

Now, let us turn back to the problem of implementing the SB. The government imposes a tax on R&D activities at a rate τ and subsidizes the purchase of intermediate varieties at a rate s. Furthermore, it keeps a balanced budget through the means of lump-sum transfers to households in the amount T_t. Thus, the government's budget constraint is given by

$$T_t = \int_0^{N_t} sP_t(n)X_t(n)dn - \tau\eta\mu_t. \tag{A.43}$$

The final good firm chooses labor and intermediate inputs to maximize profits, now given by $Y_t - w_t L - \int_0^{N_t}(1-s)P_t(n)X_t(n)dn$. The first order conditions yield the same labor demand equation as in the DE, $w_t = (1-\lambda)Y_t/L$, and an inverse demand function for intermediaries given by $P_t(n) = \lambda A L^{1-\lambda}X_t^{\lambda-1}(n)/(1-s)$.

At stage three of the innovation process, the monopolist faces an analogous problem as in the DE. The only difference now is in the inverse demand function. Hence, in equilibrium, $P = 1/\lambda$, $X = [A\lambda^2/(1-s)]^{1/(1-\lambda)}L$, $\pi = (1-\lambda)X/\lambda$, $Y_t = [A(\lambda^2/(1-s))^{\lambda}]^{1/(1-\lambda)}LN_t$.

As in the economy without government intervention, all ideas are equally profitable, so the matching technology is as in the DE. The free entry condition is now given by

$$\eta(1+\tau) = \frac{1-e^{-\theta_t}}{\theta_t}V_t. \tag{A.44}$$

where the value of the monopoly position, V_t, is defined as in the DE.

The laws of motion for ideas and varieties, and the Euler equation are as in the DE. Hence, the value of the monopoly position is still given by (2.10). Furthermore, the resource constraint is still given by (2.11).

Along the BGP, we still have that $\nu_t/N_t = M-1$, as the laws of motion for ideas and varieties are as in the DE. Thus, from the resource constraint, (2.11) it follows that

$$\frac{C}{N} = \frac{1-s-\lambda^2}{(1-\lambda)\lambda}\pi - \eta\theta(M-1). \tag{A.45}$$

Next, (2.10), the law of motion for ideas, and the free entry condition imply that

$$1 + (1 - e^{-\theta})(M - 1) = \beta\Big(1 + \frac{\pi}{\eta(1+\tau)}\Big(\frac{1 - e^{-\theta}}{\theta}\Big)\Big). \tag{A.46}$$

Then, setting $s = s^{SB}$ implies that $\pi = \pi^*$ and setting $\tau = \tau^*$ implies that $\theta = \theta^{SB}$. Thus, $C/N = (C/N)^{SB}$. Furthermore, τ^* is given by

$$\tau^* = \frac{\beta\pi^*(1 - e^{-\theta^{SB}})}{\eta\theta^{SB}(e^{-\theta^{SB}} + (1 - e^{-\theta^{SB}})M - \beta)} - 1. \tag{A.47}$$

To see that the optimal tax rate is positive because the congestion externality dominates the learning one, observe that

$$\begin{aligned} -\mathcal{C} - \mathcal{L} = & \Big(\frac{h}{\phi}\Big)^{SB}\Big(\frac{1 - e^{-\theta^{SB}}}{\theta^{SB}}\Big) - e^{-\theta^{SB}}\left(\Big(\frac{h}{\phi}\Big)^{SB} + \Big(\frac{\lambda}{\phi}\Big)^{SB}(M-1)\right) \\ = & \Big(\frac{h}{\phi}\Big)^{SB}\Big(\frac{1 - e^{-\theta^{SB}}}{\theta^{SB}}\Big) - \eta \\ = & \eta\tau^*. \end{aligned} \tag{A.48}$$

where the first equality follows from (2.20) and the second equality from (2.21) and the fact that the SB growth rate is given by $(1 - e^{-\theta^{SB}})(M - 1)$. Hence, $|\mathcal{C}| > |\mathcal{L}| \Rightarrow \tau^* > 0$. ■

Proof of Proposition 4

Proof. Totally differentiating (2.25) with respect to π^*, β, η, and M respectively and applying some algebra yields

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\pi^*} = \frac{\beta e^{-\theta^*}/\eta}{e^{-\theta^*}(M-1) + (1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)} > 0, \tag{A.49}$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\beta} = \frac{1 + \pi^* e^{-\theta^*}/\eta + (1 - e^{-\theta^*} - \theta^* e^{-\theta^*})(M-1)}{e^{-\theta^*}(M-1) + (1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)} > 0, \tag{A.50}$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\eta} = -\frac{\beta\pi^* e^{-\theta^*}/\eta^2}{e^{-\theta^*}(M-1) + (1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)} < 0, \tag{A.51}$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}M} = -\frac{(1-\beta)(1-e^{-\theta^*}) + \beta\theta^* e^{-\theta^*}}{e^{-\theta^*}(M-1) + (1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)} < 0. \tag{A.52}$$

■

Proof of Proposition 5:

Proof. First, let us explicitly characterize the environment in the CE. The only difference to the DE is at the second stage in the innovation process. Coordination is achieved through the means of a centralized allocation of firms to ideas. In particular, upon entry, a Walrasian auctioneer directs firms' research efforts and assigns patents in the following way. If $\mu_t \leq v_t$, then each firm is directed towards a distinct project and each firm receives a patent. If $\mu_t > \nu_t$, the auctioneer chooses ν_t firms at random, assigns each a distinct project, and grants each a patent over the corresponding variety. The rest $\mu_t - \nu_t$ firms are randomly assigned a project, but none of them receives a patent.

The assumption we have placed on the parameter vales ensures that firms find all research avenues profitable. Hence, in equilibrium, all ideas are innovated, i.e. $\mu_t \geq \nu_t$, and each firm secures a patent with probability $Pr(\text{monopoly}) = 1/\theta_t$. Hence, the laws of motion for ideas and varieties are given by

$$\nu_{t+1} = M\nu_t, \tag{A.53}$$

$$N_{t+1} = N_t + \nu_t. \tag{A.54}$$

Since the final good sector and the intermediate varieties production technology are as in the DE, it follows that in equilibrium it is still the case that $P_t(n) = 1/\lambda$, $X = (\lambda^2 A)^{1/(1-\lambda)}L$, $Y_t = (\lambda^{2\lambda}A)^{1/(1-\lambda)}LN_t$, $\pi = X(1-\lambda)/\lambda$, $V_t^c = \sum_{i=t+1}^{\infty} d_{it}\pi$. As all ideas are equally productive, the free entry condition is now given by

$$\eta = \frac{1}{\theta_t}V_t^c. \tag{A.55}$$

Moreover, consumers face the same problem as in the DE, so the Euler equation is analogous to (2.10):

$$V_t^c = \beta\frac{C_t}{C_{t+1}}\Big(\pi + V_{t+1}^c\Big). \tag{A.56}$$

Furthermore, the resource constraint is still given by (2.11).

One can establish in a manner analogous to that in the DE case that have $g_Y = g_C = g_N = g_\mu = g_\nu$. However, now from the law of motion for ideas, it follows that $g_\nu = M - 1$.

Next, using the laws of motion for ideas and varieties, it follows that along the BGP we still have, $\nu/N = M - 1$. Furthermore, from the resource constraint, it follows that

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \eta\theta^c(M-1). \tag{A.57}$$

Lastly, using the free entry condition and the Euler equation, it follows that the market tightness is given by

$$\theta^c = \frac{\beta\pi}{\eta(M-\beta)}. \tag{A.58}$$

Next, we can compare the percent of wasteful innovations in the two economies. In the CE there are μ_t innovations and ν_t of those are beneficial. Hence, $\omega^c = 1 - 1/\theta^c$. Then, it follows that

$$\omega - \omega^c = \frac{\eta(M-\beta)}{\beta\pi} - \frac{\eta(1+g-\beta)}{\beta\pi} = \frac{e^{-\theta}(M-1)\eta}{\beta\pi} > 0. \tag{A.59}$$

Next, from (A.58) it follows that

$$\frac{\theta^c}{1-e^{-\theta}} = \frac{\beta\pi}{\eta(M-\beta)(1-e^{-\theta})} > \frac{\beta\pi}{\eta(1+(1-e^{-\theta})(M-1)-\beta)} = \frac{\theta}{1-e^{-\theta}}, \tag{A.60}$$

where the inequality follows because $\beta < 1 \Rightarrow 1+(1-e^{-\theta})(M-1)-\beta > (M-\beta)(1-e^{-\theta})$. Hence, $\theta^c > \theta$. ■

Proof of Proposition 6:

Proof.

The results for the fraction of foregone innovation are immediate from Proposition 2. Next, let us look at the difference in the fraction of wasteful simultaneous innovation. Totally differentiating equation (2.29) with respect to π, β, η, and M yields

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}\pi} = -\frac{\omega-\omega^c}{\pi} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}\pi} < 0, \tag{A.61}$$

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}\beta} = -\frac{\omega-\omega^c}{\beta} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}\beta} < 0, \tag{A.62}$$

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}\eta} = \frac{\omega-\omega^c}{\eta} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}\eta} > 0, \tag{A.63}$$

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}M} = \frac{\eta e^{-\theta}}{\beta\pi} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}M} > 0. \tag{A.64}$$

Next, let us look in the difference in the market tightness. Given $\theta^c = \beta\pi/(\eta(M-\beta))$, it follows that

$$\frac{\mathrm{d}\theta^c}{\mathrm{d}\pi} = \frac{\beta}{\eta(M-\beta)}, \tag{A.65}$$

$$\frac{\mathrm{d}\theta^c}{\mathrm{d}\eta} = -\frac{\beta\pi}{\eta^2(M-\beta)} = -\Big(\frac{\pi}{\eta}\Big)\frac{\mathrm{d}\theta^c}{\mathrm{d}\pi}, \tag{A.66}$$

$$\frac{\mathrm{d}\theta^c}{\mathrm{d}\beta} = \frac{M\pi}{\eta(M-\beta)^2} = \Big(\frac{M\pi}{\beta(M-\beta)}\Big)\frac{\mathrm{d}\theta^c}{\mathrm{d}\pi}. \tag{A.67}$$

Then, using equations (A.33) and (A.65) and straightforward algebra, it follows that

$$\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\pi} < 0. \tag{A.68}$$

Similarly, equations (A.35) and (A.66) imply that

$$\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\eta} = -\Big(\frac{\pi}{\eta}\Big)\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\pi} > 0. \tag{A.69}$$

Lastly, equation (A.34) implies that

$$\begin{aligned}
\frac{\mathrm{d}\theta}{\mathrm{d}\beta} &= \Big(\frac{\eta\theta}{\beta(1-e^{-\theta})}\Big)\Big(1+\frac{\pi}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big)\frac{\mathrm{d}\theta}{\mathrm{d}\pi},\\
\frac{\mathrm{d}\theta}{\mathrm{d}\beta} &= \Big(\frac{\pi}{\beta}+\frac{\pi}{1-\beta+g}\Big)\frac{\mathrm{d}\theta}{\mathrm{d}\pi},\\
\frac{\mathrm{d}\theta}{\mathrm{d}\beta} &= \Big(\frac{(1+g)\pi}{\beta(1-\beta+g)}\Big)\frac{\mathrm{d}\theta}{\mathrm{d}\pi}.
\end{aligned} \tag{A.70}$$

Since $(1+g)\pi/(\beta(1-\beta+g)) > M\pi/(\beta(M-\beta))$, it follows that

$$\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\beta} < \Big(\frac{M\pi}{\beta(M-\beta)}\Big)\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\pi} < 0. \tag{A.71}$$

■

Appendix B

Appendix for Chapter 3

B.1 The Deterministic Balanced Growth Path

Along the deterministic balanced growth path variables grow at the following rates

Proposition 20 *Along the deterministic balanced growth path θ_t, LP_t, LR_t, L_t, r_t are all constant. The growth rates of N_t, ν_t, and μ_t are given by $(1-\delta^{\nu})(e^{-p\theta}+(1-e^{-p\theta})M(\theta))-1$, where θ is the value of θ_t along the deterministic BGP. The growth rates of Y_t, C_t, K_t, w_t are given by $(1+g_N)^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))}-1$. Moreover, the growth rate of profits is given by $(1+g_Y)/(1+g_N)-1$.*

Proof.

First, it is clear that $g_{LP}=g_{LR}=g_L=0$. Then, from (3.17) a constant growth rate of consumption implies that the return on capital, r_t, must be constant. Thus, from the final producer's first order condition with respect to capital, (3.4), we have that $g_Y=g_K$. Moreover, from the first order condition with respect to labor in production, (3.3), it follows that $g_w = g_Y$. Next, from the solution for intermediate goods, (3.12), and the law of motion for capital, (3.19), it follows that $g_K = 1 - \delta^K + ((1-\lambda\sigma)Y_t - C_t)/K_t$. As g_K is constant along the BGP, it follows that $g_K = g_C$.

As $g_K = g_Y$ and $g_{LP} = 0$, from the production function in equilibrium, (3.14), it follows that $g_Y = (1+g_N)^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))} - 1$. Next, from the solution for profits, (3.13), it must be the case that $g_\pi = (1+g_Y)/(1+g_N) - 1$. Thus, from the Bellman equation for the value of a monopoly, (3.18), and from the fact that g_C is constant, it follows that $g_\pi = g_V$. Then, free entry, (3.15), implies that $g_\theta = 0$. Hence, the growth rates of varieties and ideas must equal to each other, $g_\mu = g_\nu$. Thus, the law of motion for varieties, (3.9), implies that $g_N = g_\nu$. Lastly, the law of motion for ideas, (3.8), implies that $g_\nu = (1-\delta^\nu)(e^{-p\theta} + (1-e^{-p\theta})M(\theta)) - 1$. ■

Characterizing the deterministic balanced growth path. We cannot explicitly solve for all variables along the deterministic balanced growth path. Instead we reduce the model to a system of eleven equations and unknowns in order to provide an implicit solution. Let $x_t := \nu_t/N_t$; $z_t := Y_t/C_t$; $n_t := K_t/C_t$; $\gamma_t := N_t^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))}/K_t^{1-\alpha}$. Furthermore, omitted time subscripts denote variables along the deterministic balanced growth path.

From the laws of motion for varieties and ideas, (3.9) and (3.8), it follows that

$$(1-\delta)(1+(1-e^{-p\theta})x) = (1-\delta^\nu)(e^{-p\theta} + (1-e^{-p\theta})M(\theta)).$$

Hence,

$$x = \Big(\frac{1-\delta^\nu}{1-\delta}\Big)\Big(\frac{e^{-p\theta}}{1-e^{-p\theta}} + M(\theta)\Big) - \frac{1}{1-e^{-p\theta}}. \tag{B.1}$$

From the law of motion for capital, (3.19), it follows that

$$g_Y = -\delta^K + (1-\lambda\sigma)\frac{z}{n} - \frac{1}{n}. \tag{B.2}$$

Next, from the Euler equation, (3.17), and the fact that $g_Y = g_C$ it is immediate to see that

$$g_Y = \beta(1 + r) - 1. \tag{B.3}$$

The equation for demand for capital, (3.4), gives us

$$r = \alpha(1 - \sigma)\frac{z}{n} - \delta^K. \tag{B.4}$$

Next, from the research production function, (3.6), we have that

$$x\theta = ALR\eta^{-1}. \tag{B.5}$$

Combining the demand for labor in production, (3.3), and the households' supply decision for labor, (3.16), we get

$$(1 - \alpha)(1 - \sigma)z = \chi L^{\frac{1}{\phi}} LP. \tag{B.6}$$

Next, using (3.18), (3.15), and (3.13)

$$\frac{\eta\theta}{A(1 - e^{-p\theta})LP} = \frac{(1 - \delta)\beta(1 - \lambda)\sigma}{(1 - \alpha)(1 - \delta)(g_N - (1 - \delta)\beta)}. \tag{B.7}$$

From the resource constraint for labor, it is clear that

$$L = LP + LR. \tag{B.8}$$

Next, from the definition of z_t and the production function in equilibrium, (3.14), one gets

$$z = (A(\sigma\lambda)^{\sigma})^{\frac{1}{1-\sigma}} nLP^{1-\alpha}\gamma. \tag{B.9}$$

Lastly, from proposition 20, it follows that

$$g_y = (1 + g_N)^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))} - 1, \tag{B.10}$$

$$g_N = (1 - \delta^{\nu})(e^{-p\theta} + (1 - e^{-p\theta})M(\theta)) - 1. \tag{B.11}$$

Equations (B.1) through (B.11) for a system of eleven equations and eleven unknowns (x, z, γ, n, r, L, LP, LR, θ, g_Y, and g_N) that characterizes the deterministic balanced growth path.

Appendix C

Appendix for Chapter 4

C.1 Proofs Omitted from the Text

Proof of Lemma 8:

Proof. Consider firms matched with an idea that has been previously developed by $n-1$ innovators and look at stage three. First, notice that there is no pure equilibrium where firms bid some $t \in [0,1]$ with certainty.[1] Observe that $F_j^n(t)$ has no atoms and the support is a connected interval.[2] Let $k_n(p)$ be the firm's expected payoff from the race if it has chosen to patent and $k_n(s)$ if it has chosen secrecy. Also, let N represent the number of competitors a firm faces in the race. Now, consider a firm that chooses to patent. In equilibrium, a firm bidding any t in the support of $F_P^n(t)$ receives an expected payoff of

$$P(\text{win}|t)R_P(t) - c(t) = k_n(p).$$

The probability a firm will win the race when there are exactly m competitors is the chance

[1] If this were the case, then a firm can do strictly better by bidding t' just before t, as this means it will win the race for sure.

[2] This can be seen easily by applying standard arguments.

that exactly i competitors choose patent protection and the firm bid a lower innovation time than all of them, summed across $0 \leq i \leq m$. Then one can solve for $F_P^n(t)$:

$$\sum_{m=0}^{\infty} P(N=m) \sum_{l=0}^{m} \binom{m}{l} (1-s_n)^l (1-F_P^n(t))^l s_n^{m-l} R_P(t) - c(t) = k_n(p),$$

$$F_P^n(t) = \frac{1}{\theta(1-s_n)} \ln\Big(\frac{R_P(t)}{c(t)+k_n(p)}\Big). \tag{C.1}$$

where the second equation follows by applying the binomial theorem, substituting for the probabilities using the matching function, and using the fact that $e^\theta = \sum_{m=0}^{\infty} \theta^m/m!$. Now, suppose $\bar{S}_P^n$ is the upper bound of the support of $F_P^n(t)$. At $t = \bar{S}_P^n$ the firm will win the race only when it does not face a competitor that patents (which happens with probability $e^{-\theta(1-s_n)}$). Then, it follows that, from (C.1)

$$e^{-\theta(1-s_n)} R_P(\bar{S}_P^n) - c(\bar{S}_P^n) = k_n(p). \tag{C.2}$$

Also, let the lower bound of the support be $\underline{S}_P^n$, then at $t = \underline{S}_P^n$ the firm will always win the race and, hence,

$$k_n(p) = R_P(\underline{S}_P^n) - c(\underline{S}_P^n). \tag{C.3}$$

(C.2) and (C.3) uniquely determine $k_n(p)$ and $\underline{S}_P^n$ as a function of $\bar{S}_P^n$. One can determine $\bar{S}_P^n$ from firms' maximization behavior:

Claim 21 *The upper bound of the support of $F_P^n(t)$ is given by*

$$\bar{S}_P^n = 1 = \underset{t}{\operatorname{argmax}}\, e^{-\theta(1-s_n)} R(\bar{S}_P^n) - c(\bar{S}_P^n)$$

.

Proof. Let us prove the claim by contradiction. Suppose $\bar{S}_P^n < t_m$ where t_m is the unique solution to the maximization problem. Then, a firm can deviate by choosing t_m as the upper bound of the support instead and yield strictly higher expected profits since the chance of winning will be unaffected (there are no atoms), but the net reward will be higher. On the other hand if $\bar{S}_P^n > t_m$, a deviant can choose to place the upper bound of the support at t_m and then she will earn strictly higher expected profits as

$$\begin{aligned} P(\text{win}|t_m)R_P(t_m) - c(t_m) \geq & \sum_{m=0}^{\infty} P(N=m)(s_n)^m R_P(t_m) - c(t_m), \\ > & \sum_{m=0}^{\infty} P(N=m)(s_n)^m R_P(\bar{S}_P^n) - c(\bar{S}_P^n). \end{aligned}$$

The first inequality follows by the fact that the chance of winning cannot be less when the deviant bids a lower t and the second inequality by the fact that t_m is the unique maximizer. Thus, the upper bound must solve the maximization problem. Moreover, by assumption the unique maximizer is 1 since

$$e^{-\theta(1-s_n)}R_P'(\bar{S}_P^n) - c'(\bar{S}_P^n) = e^{-\theta(1-s_n)}\pi'(\bar{S}_P^n) - c'(\bar{S}_P^n) \geq \pi'(\bar{S}_P^n) - c'(\bar{S}_P^n) > 0,$$

for all $\bar{S}_P^n < 1$ and the payoff is negative for all $\bar{S}_P^n > 1$. ■

Then, equations (C.1), (C.2), (C.3), and Claim 21 uniquely characterize the expected payoff in equilibrium, the equilibrium CDF, and its bounds, when the firm patents.

Analogously, when the firm chooses secrecy to protect its innovation the equilibrium CDF could be solved for by observing that

$$\sum_{m=0}^{\infty} P(N=m)(1-F_S^n(t))^m s_n^m R_S^n(t) - c(t) = k_n(s),$$

$$F_S^n(t) = \frac{1}{\theta s_n}\ln\Big(\frac{R_S^n(t)}{c(t) + e^{-\theta}R_S^n(\bar{S}_S^n) - c(\bar{S}_S^n)}\Big) - \frac{1-s_n}{s_n}. \quad \text{(C.4)}$$

Since the firm will win the race only if all competitors in the race choose secrecy as well and it bid a lower time than all of them. At $t = \bar{S}_S^n$ the firm will win only if it faces no competitors (which happens with probability $e^{-\theta}$), hence,

$$e^{-\theta} R_S^n(\bar{S}_S^n) - c(\bar{S}_S^n) = k_n(s). \tag{C.5}$$

Similarly, if $\underline{S}_S^n$ is the lower bound of the support, at $t = \underline{S}_S^n$ the firm will win the race as long as no competitor chooses patent protection. Then,

$$e^{-\theta(1-s_n)} R_S^n(\underline{S}_S^n) - c(\underline{S}_S^n) = k_n(s). \tag{C.6}$$

Thus, analogously to the case when the firm patents, the following claim holds.

Claim 22 *The upper bound of the support of $F_p(t)$ is given by*

$$\bar{S}_s^n = 1 = \underset{t}{\operatorname{argmax}}\, e^{-\theta} R_S^n(\bar{S}_s^n) - c(\bar{S}_s^n)$$

.

We omit the proof as it is analogous to that of Claim 1. Then, equations (C.4), (C.5), (C.6), and Claim 2 uniquely characterize the expected payoff in equilibrium, the equilibrium CDF, and its bounds, when the firm chose secrecy. At this point one needs to show the proposed strategies follow valid CDFs and are indeed an equilibrium. However, we omit this as it is straightforward. ■

Proof of Lemma 9:

Proof. Let us now derive s_n, $\bar{\alpha}_n$, and $\underline{\alpha}_n$. From Claim 1 and Claim 2, the upper bounds of the support for $F_S^n(t)$ and $F_P^n(t)$ are the same. Then, using (C.2) and (C.5), the probability firms will place on using secrecy protection, s_n, in the mixed equilibrium is given by

$$s_n = \frac{1}{\theta}\ln\Big(\frac{R_S^n(1)}{R_P(1)}\Big). \tag{C.7}$$

Next, we will show that for high enough patent strength, $\alpha \geq \bar{\alpha}_n$, the equilibrium is such that $s_n = 0$ and for low enough patent strength, $\alpha \leq \underline{\alpha}_n$, it is such that $s_n = 1$. The argument follows by induction. First, take $n = \bar{n}$ and observe that by equations (4.1) and (4.2), it follows that $R_P(1)$ is strictly increasing in α. Second, by assumption $d_{\bar{n}+1} = 0$, so the $\bar{n}+1$-st innovator chooses patenting regardless of the patent strength. Hence, $s_{\bar{n}+1} = 0$ and $\zeta_{\bar{n}+1} = 0$. Thus, from equations (4.3) and (4.4), it follows that $R_S^{\bar{n}}(1)$ is independent of α. Hence, $R_S^{\bar{n}}(1)/R_P(1)$ is strictly decreasing in α. Thus, for $\alpha \geq \bar{\alpha}_{\bar{n}}$ we have that $s_{\bar{n}} = 0$ and for $\alpha \leq \underline{\alpha}_{\bar{n}}$ we have that $s_{\bar{n}} = 1$. Moreover, $s_{\bar{n}}$ is weakly decreasing in α and so is $\zeta_{\bar{n}}$.

Next, take any n such that $1 \leq n < \bar{n}$ and suppose that ζ_{n+i} is weakly decreasing in α for $i > 0$. Then, by equations (4.3) and (4.4), it follows that $R_S^n(1)$ is weakly decreasing in α as well. Hence, $R_S^n(1)/R_P(1)$ is strictly decreasing in α. Thus, for $\alpha \geq \bar{\alpha}_n$ we have that $s_n = 0$ and for $\alpha \leq \underline{\alpha}_n$ we have that $s_n = 1$. Moreover, s_n is weakly decreasing in α and so is ζ_n.

From (C.7) we can implicitly solve for the upper and lower bounds $\bar{\alpha}_n$ and $\underline{\alpha}_n$. In particular, $\bar{\alpha}_n$ is such that[3]

$$V_P - V_S^n = \pi(0)(1-\gamma)(1-d_n). \tag{C.8}$$

And $\underline{\alpha}_n$ is such that

$$V_P = e^{-\theta}V_S^n + \pi(0)(1-e^{-\theta}d_n)(1-\gamma). \tag{C.9}$$

[3]Since $\pi(1) = \gamma\pi(0)$.

The three equations — (C.7),(C.8), and (C.9) characterize the equilibrium protection strategies and the bounds of the patent strength consistent with each equilibrium. ■

Equilibrium stage one outcome:

Here we characterize the equilibrium outcome in stage one. First, suppose that there is a patent system. Let N_T^n be the fraction of ideas innovated by n firms at time T, just before matches are made. Then, recall that nature matches firms with ideas at random. Thus, it follows that the probability of matching a firm with an idea previously developed by n innovators is just the fraction of these ideas in the pool of readily available ideas. Then, for all T in the patenting equilibrium $N_T^0 = 1$ and $N_T^n = 0$ for all $n \neq 0$, because all ideas in the pool are new ideas.

Next, let us look at the case where some firms might use secrecy protection, i.e. $s_n \in (0, 1]$ for at least one n. Take any T and $n \neq 0$ and observe that ideas are developed by n firms at period T in one of two cases. First, if at the beginning of period $T - 1$ they were developed by n firms, at $T - 1$ no firm was matched with them, and secrecy held at the beginning of T. Second, if at the beginning $T - 1$ they had been developed by $n - 1$ firms, at $T - 1$ they were matched with at least one firm which innovated under secrecy, and at the beginning of T secrecy protection held.

Then, for any $n \neq 0$ we have that

$$N_T^n = e^{-\theta}\beta N_{T-1}^n + (1 - e^{-\theta})\beta\zeta_n N_{T-1}^{n-1}. \tag{C.10}$$

Similarly, new ideas ($n = 0$) at time T come from two sources — ideas which are new and stay new, and ideas which replace previously developed ones in the pool. If an idea has not been previously developed and no firm was matched with it at time $T - 1$, it is still new at T. On the other hand, new ideas replace old ones whenever the secret has leaked or the idea is patented. It is worth nothing that if an idea was new in the beginning of $T-1$ and it was matched with at least one firm that innovated under patent protection at $T-1$, then it is replaced by a new idea at T. Similarly, if an idea was new in the beginning of $T-1$, it was matched with at least one firm that innovated under secrecy at $T-1$, and secrecy protection failed at T, then the idea is replaced by a new one at T. Hence,

$$\begin{aligned} N_T^0 =& e^{-\theta} N_{T-1}^0 + (1-e^{-\theta}) \sum_{i=0}^{\bar{n}} (1-\zeta_{i+1}) N_{T-1}^i + \\ &+ (1-e^{-\theta}) \sum_{i=0}^{\bar{n}} (1-\beta)\zeta_{i+1} N_{T-1}^i + (1-\beta) \sum_{i=1}^{\bar{n}} N_{T-1}^i . \end{aligned} \tag{C.11}$$

Equations (C.10) and (C.11) characterize nature's moves for all T.

When there is no patent system, the laws of motion are given by

$$N_T^n = e^{-\theta}\beta N_{T-1}^n + (1-e^{-\theta})\beta N_{T-1}^{n-1} \quad \text{for } n \neq 0, \bar{n}, \tag{C.12}$$

$$N_T^{\bar{n}} = \beta N_{T-1}^{\bar{n}} + (1-e^{-\theta})\beta N_{T-1}^{\bar{n}-1}, \tag{C.13}$$

$$N_T^0 = e^{-\theta} N_{T-1}^0 + (1-e^{-\theta}) \sum_{i=0}^{\bar{n}-1} (1-\beta) N_{T-1}^i + (1-\beta) \sum_{i=1}^{\bar{n}} N_{T-1}^i . \tag{C.14}$$

The intuition and derivation is similar to the case when there is a patent system, so we omit them.

Proof of Corollary 10:

Proof. First, observe that $\bar{\alpha}_{\bar{n}+1} = s_{\bar{n}+1} = \zeta_{\bar{n}+1} = 0$ as an immediate consequence of assumption A2. Also, it must be the case that $\bar{\alpha}_n \geq 0$ for all $n \leq \bar{n}$. Hence, $\bar{\alpha}_n \geq \bar{\alpha}_{\bar{n}+1}, \forall n \leq \bar{n}$. Thus, what is left to show is that $e^{-\theta}\beta = \bar{\alpha}_1 \geq \bar{\alpha}_2 \geq \bar{\alpha}_3 \geq \cdots \geq \bar{\alpha}_{\bar{n}}$. We will first show the inequalities hold and then prove that $e^{-\theta}\beta = \bar{\alpha}_1$.

First, observe that by equation (4.4), it follows that

$$\begin{aligned} V_S^n = & \frac{d_n\pi(0)}{1-e^{-\theta}\beta\gamma} + \sum_{i=1}^{\infty} \frac{d_{n+i}\pi(0)(1-e^{-\theta})^i\beta^i\gamma^i}{(1-e^{-\theta}\beta\gamma)^{i+1}} \prod_{k=1}^{i} \zeta_{n+k}, \\ \leq & \frac{d_n\pi(0)}{1-e^{-\theta}\beta\gamma} + \sum_{i=1}^{\infty} \frac{d_n\pi(0)(1-e^{-\theta})^i\beta^i\gamma^i}{(1-e^{-\theta}\beta\gamma)^{i+1}}, \\ \leq & \frac{d_n\pi(0)}{1-e^{-\theta}\beta\gamma}\Big[1+\sum_{i=1}^{\infty}\Big(\frac{(1-e^{-\theta})\beta\gamma}{1-e^{-\theta}\beta\gamma}\Big)^i\Big], \\ \leq & \frac{d_n\pi(0)}{1-\beta\gamma}. \end{aligned}$$

Hence, by equation (4.3), it follows that

$$R_S^n(1) \leq d_n\Big(\frac{\pi(0)}{1-\beta\gamma} - \pi(0)(1-\gamma)\Big).$$

Thus, by equations (4.1) and (4.2), it follows that at $\alpha = 1$, $R_P(1) > R_S^n(1)$, for all n. Hence, by Lemma 9, it follows that $1 > \max_n\{\bar{\alpha}_n | n \leq \bar{n}\}$. Now fix any m such that $1 \leq m \leq \bar{n}$, we will show that $\bar{\alpha}_m = \max_n\{\bar{\alpha}_n | m \leq n \leq \bar{n}\}$. For $m = \bar{n}$, the claim holds trivially. So, take $m < \bar{n}$ and proceed the proof by contradiction. Suppose, to the contrary, that there exists some k, such that $m < k \leq \bar{n}$ with $\bar{\alpha}_k = \max_n\{\bar{\alpha}_n | m \leq n \leq \bar{n}\}$ and $\bar{\alpha}_k > \bar{\alpha}_m$. Then, equations (4.1) and (4.2) together with Lemma 9 imply that $R_S^k(1; \bar{\alpha}_k) = R_P(1; \bar{\alpha}_k) > R_P(1; \bar{\alpha}_m) = R_S^m(1; \bar{\alpha}_m)$, where we have explicitly denoted the values of α at which the reward functions are evaluated. But, from Lemma 9 it follows that at $\alpha = \bar{\alpha}_k$, it is the case that $s_n = 0$ for all n such that $m \leq n \leq \bar{n}$. Then, from equations (4.3) and (4.4) it follows that $R_S^n(1; \bar{\alpha}_k) = d_n\gamma\pi(0) + d_n e^{-\theta}\beta\gamma\pi(0)/(1-e^{-\theta}\beta\gamma)$, for n such that $m \leq n \leq \bar{n}$. Thus, $R_S^m(1; \bar{\alpha}_k) \geq R_S^k(1; \bar{\alpha}_k)$, since $d_m = \max_n\{d_n | m \leq n \leq \bar{n}\}$ by assumption A2. Hence, $R_S^m(1; \bar{\alpha}_k) > R_S^m(1; \bar{\alpha}_m)$. But this is a contradiction since $R_S^m(1)$ is weakly decreasing in α by the arguments in the proof of Lemma 9. Thus, it must be the case that $\bar{\alpha}_m = \max_n\{\bar{\alpha}_n | m \leq n \leq \bar{n}\}$. Hence, $\bar{\alpha}_1 \geq \bar{\alpha}_2 \geq \bar{\alpha}_3 \geq \cdots \geq \bar{\alpha}_{\bar{n}}$.

Lastly, we need to show that $e^{-\theta}\beta = \bar{\alpha}_1$. By Lemma 9, it follows that, $R_P(1;\bar{\alpha}_1) = R_S^1(1;\bar{\alpha}_1)$. Since, at $\alpha = \bar{\alpha}_1 = \max_n\{\bar{\alpha}_n | n \leq \bar{n}\}$, we have that $s_n = \zeta_n = 0, \forall n \leq \bar{n}$, it follows that $R_S^1(1;\bar{\alpha}_1) = d_1\gamma\pi(0) + d_1 e^{-\theta}\beta\gamma\pi(0)/(1 - e^{-\theta}\beta\gamma)$. As $d_1 = 1$ by assumption A2, straightforward algebra implies that $e^{-\theta}\beta = \bar{\alpha}_1$. ■

Proof of Theorem 14:

Proof. First, we need the following lemma.

Lemma 23 *If* $\frac{1}{\gamma} > \left(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\right)\left(\frac{\beta}{1-\beta\gamma}\right)$, *then* $\alpha = e^{-\theta}\beta$ *implies that* $s_1 \in (0,1)$.

Proof. With PUR the repeated game firms place is exactly the same with the only difference that now $R_P^n(t)$ depends on n. Thus, it is straightforward to establish that the investment decisions at stage three will follow the CDFs

$$F_p^n(t) = \frac{1}{\theta(1-s_n)}\ln\Big(\frac{R_P^n(t)}{c(t)+k^n(p)}\Big),$$
$$F_s^n(t) = \frac{1}{\theta s_n}\ln\Big(\frac{R_S^n(t)}{c(t)+k^n(s)}\Big) - \frac{1-s_n}{s_n}.$$

Where $k^n(p) = e^{-\theta(1-s)}R_P^n(\bar{S}) - c(\bar{S})$, $k^n(s) = e^{-\theta}R_S^n(\bar{S}) - c(\bar{S})$ and $\bar{S} = 1$.

Similarly, at stage two if firms place a probability of $s_n \in (0,1)$ on playing secrecy, then

$$s_n = \frac{1}{\theta}\ln\Big(\frac{R_S^n(1)}{R_P^n(1)}\Big).$$

The difference with PUR comes from the reward structure. In particular, $R_P^n(t)$ has the following form

$$R_P^n(t) = d_n\pi(t) + \alpha\gamma V_P^n, \tag{C.15}$$

where $V_P^n = d_n\pi(0) + \alpha\gamma V_P^n$. If a firm chooses secrecy, for $n < \bar{n}$, the reward will be

$$R_S^n(t) = d_n\pi(t) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1-e^{-\theta})\beta\gamma V_S^{n+1} + \\ + (1-\zeta_{n+1})(1-e^{-\theta})\beta\gamma V_P^{n+1}, \tag{C.16}$$

where $V_S^n = d_n\pi(0) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1-e^{-\theta})\beta\gamma V_S^{n+1} + (1-\zeta_{n+1})(1-e^{-\theta})\beta\gamma V_P^{n+1}$. For $n = \bar{n}$, the reward is given by

$$R_S^{\bar{n}}(t) = d_{\bar{n}}\pi(t) + \beta\gamma V_S^{\bar{n}}, \tag{C.17}$$

where $V_S^{\bar{n}} = d_{\bar{n}}\pi(0) + \beta\gamma V_S^{\bar{n}}$.

Next, observe that if $\alpha = e^{-\theta}\beta$, then $R_P^1(1) < R_S^1(1)$. Hence, $s_1 > 0$. Also, $s_1 < 1$ if and only if

$$V_S^1 - V_P^1 < (1-e^{-\theta})(V_S^1 - \pi(0)(1-\gamma)). \tag{C.18}$$

From the reward structure, it is fairly straightforward to establish that $V_S^1 \leq \frac{\pi(0)}{1-\beta\gamma}$. Hence, a sufficient condition for $s_1 < 1$ is

$$\frac{\pi(0)}{1-\beta\gamma} - V_P^1 < (1-e^{-\theta})\Big(\frac{\pi(0)}{1-\beta\gamma} - \pi(0)(1-\gamma)\Big),$$

which simplifies to the condition in the lemma. ■

Let us now prove Theorem 14. Again, if $T = 1$ the innovation is produced under monopoly for sure and $P_P(n) = P_N(n)$ for all n. So, fix $T \geq 2$.

When $\bar{n} = 1$, it follows that $P_N(1) = \beta^{T-1}$ and $P_N(n) = 0$ for $n \neq 1, \infty$. Since no firm is willing to duplicate the innovation (even if it has the option to patent), it follows that $\bar{\alpha}_1 = \beta$ and $P_P(n) = P_N(n)$ for $n \neq 1, \infty$. Then, for any $\underline{\alpha}_1 < \alpha < \beta$, we have that

$P_P(1) = \zeta_1 \beta^{T-1} + (1-\zeta_1)\alpha^{T-1} < P_N(1)$. Thus, the patent system increases the degree of competition when $\bar{n} = 1$.

Now, suppose that $\bar{n} \geq 2$ and fix $\alpha = e^{-\theta}\beta$. When there is no patent system we have that $P_N(1) = e^{-\theta}\beta$ and also $P_N(n) > 0$ if and only if $n \leq \min\{T, \bar{n}\}$ or $n = \infty$. Next, consider the equilibrium with a patent system. By Lemma 23, $s_1 < 1$, hence, $\zeta_1 < 1$. With $\alpha = e^{-\theta}\beta$ a firm has the same chance of keeping its monopoly position next period, regardless of whether it has chosen secrecy or patent protection, i.e. $P_P(1) = (e^{-\theta}\beta)^{T-1}$. Moreover, $P_P(n) = 0$ for $n > \min\{T, \bar{n}\}$ and $n \neq \infty$.

Then, take any n such that $1 < n \leq \min\{T, \bar{n}\}$, $n \neq \bar{n}$ and look at $P_N(n)$. An innovation originally developed in $T = 1$ in an equilibrium without a patent system is produced by exactly n firms in period T if and only if it has been matched with at least one firm n times and secrecy has held for all T periods. Hence, $P_N(n) = \binom{T-1}{n-1}(1 - e^{-\theta})^{n-1} e^{-\theta(T-n)} \beta^{T-1}$. For $n = \bar{n} \leq T$, the innovation is produced by exactly $\bar{n}$ firms if the idea has been matched at least $\bar{n}$ times and secrecy has held for all T periods. Thus, $P_N(n) = P_N(\bar{n}) = \sum_{i=0}^{T-\bar{n}} \binom{T-1}{i+\bar{n}-1}(1 - e^{-\theta})^{i+\bar{n}-1} e^{-\theta(T-\bar{n})} \beta^{T-1}$.

In an equilibrium with a patent system, an innovation can be produced under n firms only if the first $n-1$ innovators have chosen secrecy. Hence, for n such that $1 < n \leq \min\{T, \bar{n}\}$, $n \neq \bar{n}$ we have that $P_P(n) \leq \binom{T-1}{n-1}(1 - e^{-\theta})^{n-1} e^{-\theta(T-n)} \beta^{T-1} \prod_{i=1}^{n-1} \zeta_i = P_N(n) \prod_{i=1}^{n-1} \zeta_i$, where the inequality holds because a secret has a lower chance to leak than a patent has to fail, i.e. $\beta > \alpha = e^{-\theta}\beta$. Since $\zeta_1 < 1$, it follows that $P_P(n) < P_N(n)$ for all such n. Similarly, for $n = \bar{n} \leq T$ we have that $P_P(n) = P_P(\bar{n}) \leq \sum_{i=0}^{T-\bar{n}} \binom{T-1}{i+\bar{n}-1}(1 - e^{-\theta})^{i+\bar{n}-1} e^{-\theta(T-\bar{n})} \beta^{T-1} \prod_{i=1}^{\bar{n}-1} \zeta_i = P_N(\bar{n}) \prod_{i=1}^{\bar{n}-1} \zeta_i$. Hence, $P_P(\bar{n}) < P_N(\bar{n})$.

Thus, for all finite $n \geq 2$, it follows that $G_P^T(n) < G_N^T(n)$. Hence, the equilibrium with a patent system provides higher competition than the equilibrium without, when $\bar{n} \geq 2$. This concludes the proof. ■

Claim 24 *Suppose that in the beginning of ever period, just before firms observe which patents fail and which secrets leak, innovators are given the choice whether or not to switch their protection strategies. Then, in equilibrium, no firm has an incentive to switch protection strategies.*

Proof. Whenever an innovation is patented all the relevant information becomes public knowledge. Hence, no firm has an incentive to give up the patent and switch to secrecy. Thus, we only need to show that no firm has an incentive to switch from secrecy to patenting.

First, suppose that the patent system provides no PUR. Take an idea developed by n firms at the beginning of period T, just before firms observe which patents fail and which secrets leak. Each of the n firms faces the following decision. It can either keep using secrecy protection, which yields an expected payoff of $e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1-e^{-\theta})\beta\gamma V_S^{n+1} = R_S^n(1) - d_n\pi(1)$, or it can switch to patenting which yields an expected payoff of $\alpha\gamma V_P = R_P(1) - \pi(1)$. An innovator would have an incentive to switch protection strategies if and only if $R_P(1) - \pi(1) > R_S^n(1) - d_n\pi(1) \Leftrightarrow R_P(1) - R_S^n(1) > (1-d_n)\pi(1)$. This inequality, however, can never hold. To see this, observe that if there are n innovators that produce the innovation under secrecy, it must be the case that $s_n > 0$. Hence, by Lemma 9, it follows that $R_S^n(1) > R_P(1)$. Thus, $R_P(1) - R_S^n(1) < 0 \leq (1-d_n)\pi(1)$. Then, no firm has an incentive to switch from secrecy to patent protection.

Next, suppose that the patent system provides PUR. Again, take an idea developed under secrecy by n firms at the beginning of period T, just before firms observe which patents fail and which secrets leak. Observe that we do not need to consider an idea where the n-th innovator has patented, because no two firms can hold a patent over the same innovation. Hence, the $n-1$ firms that have innovated under secrecy do not have the opportunity to switch to patent protection. So, consider an idea developed by n firms all of which use secrecy protection. Analogously to the case with no PUR, if an innovator keeps using secrecy they payoff is given by $R_S^n(1) - d_n\pi(1)$, and if she chooses to switch to patent protection it is $R_P^n(1) - d_n\pi(1)$. Thus, a firm would have an incentive to switch protection strategies if and only if $R_P^n(1) > R_S^n(1)$. As all firms have developed the idea under secrecy, it must be the case that $s_n > 0$. Hence, $R_P^n(1) < R_S^n(1)$ and no firm has an incentive to switch protection strategies. ∎

Proof of Proposition 16:

Proof. First, observe that at $\alpha = \beta$ we have that $W^P(\beta) = W^S$. Next, look at the optimal patent strength consistent with a patenting equilibrium.

$$\begin{aligned} \frac{\partial W^P(\alpha)}{\partial \alpha} &= \frac{\theta e^{-\theta}\pi\gamma}{(1-e^{\theta})(1-\alpha\gamma)^2} + \frac{\gamma S_M}{(1-\alpha\gamma)^2} - \frac{\gamma S_C}{(1-\alpha\gamma)^2}, \\ &= -\frac{\gamma}{(1-\alpha\gamma)^2}\Big(S_C - S_M - \frac{\theta e^{-\theta}}{1-e^{-\theta}}\pi\Big), \\ &< 0. \end{aligned}$$

where the inequality holds because $S_C > S_M + \pi$ and $\theta e^{-\theta}/(1-e^{-\theta}) < 1$. Since $\bar{W}^P(\alpha) = \nu(1-e^{-\theta})W^P(\alpha)$, it follows that the optimal patent strength consistent with a patenting equilibrium is $\alpha_P := \beta$. As $W^P(\beta) = W^S$, it follows that $\bar{W}^P(\alpha_P) > \bar{W}^S$, since $\bar{N}^S < 1$. Thus, a secrecy equilibrium is never optimal. ∎

Proof of Proposition 18:

Proof. First, straightforward algebra implies that

$$\frac{\partial W^M(\alpha)}{\partial \alpha} = (S_C - S_M)\frac{\gamma}{1-\alpha\gamma}\Big(\frac{\zeta}{1-\beta\gamma} - \frac{1-\zeta}{1-\alpha\gamma}\Big).$$

Then, suppose that the condition in the statement of proposition 18 holds. Hence,

$$\begin{aligned}
\frac{\partial \bar{W}^M(\alpha)}{\partial \alpha}|_{\alpha=\beta} = & \quad \nu(1-e^{-\theta})\bar{N}^M\frac{\partial W^M(\alpha)}{\partial \alpha}|_{\alpha=\beta}+ \\
& +\nu(1-e^{-\theta})W^M(\alpha)\frac{\partial \bar{N}^M}{\partial \alpha}|_{\alpha=\beta}, \\
= & \quad \frac{\nu(1-e^{-\theta})\gamma}{1-\beta\gamma}\Bigg(-\frac{S_C-S_M}{1-\beta\gamma} + \frac{e^{-\theta}\beta}{1-\beta}W^P(\beta)\Bigg), \\
< & \quad \frac{\nu(1-e^{-\theta})\gamma}{(1-\beta\gamma)^2}\Bigg(-(S_C-S_M)+ \\
& +\frac{e^{-\theta}\beta}{1-\beta}\Big(\frac{\theta e^{-\theta}\pi}{1-e^{-\theta}} + S_M + \frac{\gamma(1-\beta)S_C}{1-\gamma}\Big)\Bigg), \\
< & \quad \frac{\nu(1-e^{-\theta})\gamma}{(1-\beta\gamma)^2}\Bigg(-(S_C-S_M) + \frac{e^{-\theta}\beta}{1-\beta}\Big(S_C + \frac{\gamma(1-\beta)S_C}{1-\gamma}\Big)\Bigg), \\
< & \quad 0.
\end{aligned}$$

Hence, by continuity, $\bar{W}^M(\alpha)$ is decreasing in α in the neighborhood of β. Thus, $\max_\alpha\{\bar{W}^M(\alpha)\}$ $> \bar{W}^M(\beta) = \bar{W}^P(\alpha_P)$. Thus, by propoition 16, it follows that the optimal patent strength lies in the region consistent with a mixed equilibrium. ■

C.2 Alternative Timing

In this section we prove that the main results in the paper hold under an alternative timing of stages two and three. In particular, we prove that the statements in theorems one and two hold even if firms choose investment strategies and stage two and protection strategies at stage three.

Assumption A4 *Assume that the order of stages two and three is reversed. That is, at stage one firms are matched with ideas, at stage two firms choose an investment strategy, at stage three firms choose a protection strategy.*

Equilibrium Behavior

First, observe that when there is no patent system the game effectively consists of only two stages: first firms are matched with ideas, second — firms choose an investment strategy. Hence, the equilibrium behavior without a patent system is the same, regardless of whether or not we use the alternative timing. In what follows we focus on the equilibrium with a patent system. Since the goal is to show that the main results still hold under the alternative timing, we do not fully characterize the equilibrium. Instead, we characterize only what is needed to prove the results.

Stage 3: At stage three firms choose the protection strategy which gives the higher expected payoff. Let $\text{Payoff}^n(i|t) = P_i^n(\text{win}|t)R_i^n(t)$ be the expected payoff from choosing a protection strategy $i = S, P$ given that the firm has innovated at time t, where $P_i^n(\text{win}|t)$ is the probability of winning the race, $R_i^n(t)$ is the reward conditional on winning (as defined in the text), and $n \geq 1$ means that $n-1$ firms have already innovated the idea at previous periods.

Given assumption A3, a firm that chooses secrecy wins the race only if no other firm innovates before the firm under study and no firm innovates at a later time under patent protection. Hence, $P_S^n(\text{win}|t) = Pr(\text{no firm has innovated at } t' < t \cap$ no firm will innovate and choose to patent at $t' > t|n)$. Similarly, a firm that chooses patent protection wins whenever no firm patents at an earlier time. Thus, $P_P^n(\text{win}|t) = Pr(\text{no firm has innovated at } t' <$

t under patent protection $|n)$. Notice that we have left out the possibility that more than one firm chooses the same t, which is the case in equilibrium. However, if more than one firm innovate at the same time and choose the same protection strategy, the usual tie braking rule is applied: whenever $m > 1$ firms innovate at the same time under the same protection strategy, each firm has a $1/m$ chance of winning the race.

Stage 2: At stage two firms choose t to maximize $\Pi^n(t) - c(t)$, where $\Pi^n(t) = \max\{\text{Payoff}^n(S|t), \text{Payoff}^n(P|t)\}$. The following four lemmas establish some useful properties of the equilibrium behavior. These are rather standard, nonetheless we provide the proofs for completeness.

Lemma 25 *For any $1 \leq n \leq \bar{n}+1$, there is no equilibrium where firms choose some $\bar{t}_n$ for sure at stage two.*

Proof. First, suppose that $n = \bar{n} + 1$. If the patent system provides PUR, then all firms choose to stay out of the race. If, on the other hand, the patent system provides no PUR, then firms choose patenting for sure at stage three. Since, under no PUR the reward under patenting is independent of how many firms have previously innovated, it follows that the behavior of firms when $n = \bar{n}+1$ is the same as the behavior when $1 \leq n \leq \bar{n}$ and firms patent for sure at stage three. Hence, it is enough to consider $1 \leq n \leq \bar{n}$.

Next, Fix $1 \leq n \leq \bar{n}$ and suppose, to the contrary, that there exists some $\bar{t}_n$ that all firms choose when matched with an idea previously developed by $n-1$ firms. Look at a deviant who considers bidding $t' \in (\bar{t}_n - \epsilon, \bar{t}_n)$ for some small $\epsilon > 0$. We will distinguish between two cases.

Case 1: Suppose that it is optimal to choose patent protection with positive probability at stage three, i.e. $\Pi^n(\bar{t}_n) = \text{Payoff}^n(P|\bar{t}_n) \geq \text{Payoff}^n(S|\bar{t}_n)$. Let $1 - s_n$ be the probability a firm places on patenting at stage three in a symmetric equilibrium. Then, the probability that there are exactly m firms that innovate at time $\bar{t}_n$ and choose to patent is given by

$$P(N = m) = \sum_{i=m}^{\infty} \frac{e^{-\theta}\theta^i}{i!} \binom{i}{m} (1 - s_n)^m s_n^{i-m} = \frac{e^{-(1-s_n)\theta}[(1 - s_n)\theta]^m}{m!}.$$

Hence, $P_P^n(\text{win}|t') = 1 > P_P^n(\text{win}|\bar{t}_n) = \sum_{i=0}^{\infty} e^{-(1-s_n)\theta}[(1-s_n)\theta]^i/(i+1)! = (1-e^{-(1-s_n)\theta})/[(1-s_n)\theta]$. Then, since $R_P^n(t)$ and $c(t)$ are continuous in t, it follows that for small enough ϵ, $\Pi^n(t') - c(t') > \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation.

Case 2: Suppose that firms find it optimal to choose secrecy protection, i.e. $\Pi^n(\bar{t}_n) = \text{Payoff}^n(S|\bar{t}_n) > \text{Payoff}^n(P|\bar{t}_n)$ and $s_n = 1$. Analogously, if a firm chooses $t' \in (\bar{t}_n - \epsilon, \bar{t}_n)$, then $P_S^n(\text{win}|t') = 1 > P_S^n(\text{win}|\bar{t}_n) = \sum_{i=0}^{\infty} e^{-\theta}\theta^i/(i + 1)! = (1 - e^{-\theta})/\theta$. Hence, by continuity, for small enough ϵ, $\Pi^n(t') - c(t') > \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation. ■

Lemma 26 *With no PUR, for any $1 \leq n \leq \bar{n} + 1$, the upper bound of the support for the equilibrium CDF is $\bar{S}_n = 1$. With PUR, for any $1 \leq n \leq \bar{n}$, the upper bound of the support for the equilibrium CDF is $\bar{S}_n = 1$.*

Proof. First, from the argument in lemma 25, to show that the statement holds when the patent system provides no PUR and $n = \bar{n} + 1$, it is enough to show that it holds under no PUR when $1 \leq n \leq \bar{n}$.

Then, fix any $1 \leq n \leq \bar{n}$ and suppose to the contrary that $\bar{S}_n < 1$. Hence, $P_i^n(\text{win}|\bar{S}_n) = P_i^n(\text{win}|1)$, for $i = S, P$. Since $\pi(t) - c(t)$ is increasing over $[0, 1]$, it follows that $\pi(1) - c(1) > \pi(\bar{S}_n) - c(\bar{S}_n)$. Hence, $c(\bar{S}_n) - c(1) > \pi(\bar{S}_n) - \pi(1) > 0$, which implies that $c(\bar{S}_n) - c(1) > A\pi(\bar{S}_n) - A\pi(1) > 0$, for any $A \in [0, 1]$. Hence, $\text{Payoff}^n(i|1) - c(1) > \text{Payoff}^n(i|\bar{S}_n) - c(\bar{S})$. Thus, there exists a profitable deviation.

If, on the other hand, $\bar{S}_n > 1$, then $P_i^n(\text{win}|\bar{S}_n) \leq P_i^n(\text{win}|1)$. As $\pi(t) = 0$ for $t > 1$, it follows that $\pi(1) - c(1) > \pi(\bar{S}_n) - c(\bar{S}_n)$. Hence, by the arguments in the preceding paragraph, it follows that there exists a profitable deviation. ■

Lemma 27 *With no PUR, for any $1 \leq n \leq \bar{n} + 1$, the support of the equilibrium CDF is a connected interval. With PUR, for any $1 \leq n \leq \bar{n}$, the support of the equilibrium CDF is a connected interval.*

Proof. First, from the argument in lemma 25, to show that the statement holds when the patent system provides no PUR and $n = \bar{n} + 1$, it is enough to show that it holds under no PUR when $1 \leq n \leq \bar{n}$.

Let the equilibrium CDF be $F^n(t)$. Fix $1 \leq n \leq \bar{n}$ and suppose, to the contrary that there is a gap between some t_1 and t_2. Hence, for any $t' \in (t_1, t_2)$, it is the case that $P_i^n(\text{win}|t') = P_i^n(\text{win}|t_1)$, for $i = S, P$. By assumption, $\pi(t) - c(t)$ is increasing in over $[0, 1]$. Hence, $c(t_1) - c(t') > \pi(t_1) - \pi(t') > 0$. Then, $c(t_1) - c(t') > A\pi(t_1) - A\pi(t')$, for any $A \in [0, 1]$. Thus, $\text{Payoff}^n(i|t') - c(t') > \text{Payoff}^n(i|t_1) - c(t_1)$, for $i = S, P$. Hence, $\Pi^n(t') - c(t') > \Pi(t_1) - c(t_1) = \Pi(t) - c(t)$ for all t in the support. Thus, there exists a profitable deviation. ■

Lemma 28 *With no PUR, for any $1 \leq n \leq \bar{n} + 1$, the support of the equilibrium CDF has no atoms. With PUR, for any $1 \leq n \leq \bar{n}$, the support of the equilibrium CDF has no atoms.*

Proof. First, from the argument in lemma 25, to show that the statement holds when the patent system provides no PUR and $n = \bar{n} + 1$, it is enough to show that it holds under no PUR when $1 \leq n \leq \bar{n}$.

Then, fix any $1 \leq n \leq \bar{n}$ and suppose to the contrary that there is an atom at some $\bar{t}_n$. Again, we would need to distinguish between two cases.

Case 1: Suppose that firms place some positive probability $1 - s_n$ of playing patenting, in a symmetric equilibrium, given that they have innovated at $\bar{t}_n$. Hence, $\text{Payoff}^n(P|\bar{t}_n) \geq \text{Payoff}^n(S|\bar{t}_n)$. Then, look at a firm that considers deviating to $t' \in (\bar{t}_n - \epsilon, \bar{t}_n)$. Because there is a strictly positive probability a firm would innovate at $\bar{t}_n$ under patenting, it follows that $P_P^n(\text{win}|t') > P_P^n(\text{win}|\bar{t}_n)$, $\forall \epsilon > 0$. Thus, by continuity, it follows that for small enough ϵ, $\text{Payoff}^n(P|t') - c(t') > \text{Payoff}^n(P|\bar{t}_n) - c(\bar{t}_n) \geq \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation.

Case 2: Suppose that firms that innovate at $\bar{t}_n$ use secrecy for sure. Then $\text{Payoff}^n(S|\bar{t}_n) > \text{Payoff}^n(P|\bar{t}_n)$. Moreover, since there is a positive probability a firm would innovate at $\bar{t}_n$ under secrecy, it follows that $P_S^n(\text{win}|t') > P_S^n(\text{win}|\bar{t}_n)$, $\forall \epsilon > 0$. Hence, by continuity, for small enough ϵ, it follows that $\text{Payoff}^n(S|t') - c(t') > \text{Payoff}^n(S|\bar{t}_n) - c(\bar{t}_n) \geq \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation. ■

Competition

Given the results in the previous four lemmas, we can now state and prove the results under the alternative timing.

Theorem 29 *Suppose that the patent system provides no prior user rights. For all* $\gamma, \beta \in$

$(0,1)$ *and all* $\theta > 0$*, there exists a patent strength* α *such that the patent system increases competition.*

Proof. First, observe that the key feature of the equilibrium with a patent system that we have used in the proof of theorem 13 is that when $\alpha = e^{-\theta}\beta$, the first innovator patents for sure. We proceed by establishing the analogous result under the alternative timing. More specifically, the following two lemmas prove that if $\alpha = e^{-\theta}\beta$, then it is an equilibrium strategy for firms to patent following any innovation time $t \in [0,1]$.

Lemma 30 *If* $\alpha = \tilde{\alpha}_n$*, then it is a symmetric equilibrium strategy for firms to use patent protection following any innovation time* $t \in [0,1]$*, where* $\tilde{\alpha}_n$ *is such that* $R_S^n(1) = R_P^n(1)$*.*

Proof. Suppose that firms choose patent protection following any innovation time $t \in [0,1]$. Look at a deviant that considers choosing secrecy, instead. She has an incentive to deviate if and only if $\text{Payoff}^n(S|t) > \text{Payoff}^n(P|t)$. When all firms patent, it follows that $P_S^n(\text{win}|t) = Pr^n(\text{no other firm is matched with the idea }|n) = e^{-\theta}$ and $P_P^n(\text{win}|t) = Pr^n(\text{all other firms matched with the same innovation choose } t' > t|n) = \sum_{k=0}^{\infty} \theta^k e^{-\theta}(1 - F^n(t))^k/(k!) = e^{-\theta F^n(t)}$, where $F^n(t)$ is the equilibrium CDF. Hence, the deviant would have no incentive to choose secrecy if and only if

$$e^{-\theta} R_S^n(t) \leq e^{-\theta F^n(t)} R_P(t). \tag{C.19}$$

From the definition of $R_S^n(t)$, it follows that (C.19) is equivalent to

$$e^{-\theta}(d_n\pi(t) + V_S^n - d_n\pi(0)) \leq e^{-\theta F^n(t)} R_P(t), \quad \text{iff}$$

$$e^{-\theta F^n(t)} R_P(t) + e^{-\theta} d_n(\pi(0) - \pi(t)) \geq e^{-\theta} V_S^n. \tag{C.20}$$

Now, look at the left hand side of (C.20). Let $t_1 < t_2$, then

$$e^{-\theta F^n(t_2)} R_P(t_2) + e^{-\theta} d_n(\pi(0) - \pi(t_2)) - \Big(e^{-\theta F^n(t_1)} R_P(t_1) +$$

$$+ e^{-\theta} d_n(\pi(0) - \pi(t_1))\Big) = e^{-\theta F^n(t_2)}(R_P(t_2) - R_P(t_1)) +$$

$$+ R_P(t_1)(e^{-\theta F^n(t_2)} - e^{-\theta F^n(t_1)}) - e^{-\theta} d_n(\pi(t_2) - \pi(t_1)) =$$

$$= R_P(t_1)(e^{-\theta F^n(t_2)} - e^{-\theta F^n(t_1)}) + e^{-\theta}(\pi(t_2) - \pi(t_1))(e^{\theta(1-F^n(t_2))} - d_n) < 0.$$

The inequality follows because, by lemmas 27 and 28, $F^n(t)$ is strictly increasing in t. Also, by assumption $\pi(t)$ is strictly decreasing over $[0, 1]$. Moreover, $d_n \leq 1$ and $F^n(t) \leq 1$, hence, the left hand side of (C.20) is strictly decreasing in t. Thus, if (C.20) holds for $t = 1$, it holds for all $t \in [0, 1]$. Thus, if $R_S^n(1) \leq R_P(1)$, then $e^{-\theta} R_S^n(t) \leq e^{-\theta F^n(t)} R_P(t)$ for all $t \in [0, 1]$. Hence, if $R_S^n(1) = R_P(1)$, then following any innovation time t, firms have no incentive to deviate and choose secrecy protection. ■

Lemma 31 $e^{-\theta}\beta = \tilde{\alpha}_1 \geq \tilde{\alpha}_2 \geq \cdots \geq \tilde{\alpha}_{\bar{n}} \geq \tilde{\alpha}_{\bar{n}+1} = 0.$

Proof. The proof is completely analogous to the proof of corollary 1. ■

By lemmas 30 and 31, it follows that at $\alpha = e^{-\theta}\beta$, $P_P(1) = (e^{-\theta}\beta)^{T-1}$, $P_P(\infty) = 1 - (e^{-\theta}\beta)^{T-1}$, and $P_P(n) = 0$, for $n \neq 1, \infty$, $\forall T \geq 2$.Thus, the rest of the proof is completely analogous to the proof of theorem 13. ■

Theorem 32 *Suppose the patent system provides prior user rights. If*

$$\frac{1}{\gamma} > \Big(\frac{e^{-\theta}\beta}{1 - e^{-\theta}\beta\gamma}\Big)\Big(\frac{\beta}{1 - \beta\gamma}\Big),$$

then the patent system can increase competition.

Proof. Recall that the key feature in the proof of Theorem 14 was that when $\alpha = e^{-\theta}\beta$, there is a strictly positive chance that the first innovator would use patent protection. Thus, if we can prove that the same is true under the alternative timing the proof of the theorem would follow directly from the proof of Theorem 14. The next lemma establishes just that.

Lemma 33 *If* $\frac{1}{\gamma} > \left(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\right)\left(\frac{\beta}{1-\beta\gamma}\right)$, *then, in a symmetric equilibrium, a positive fraction of firms that innovate an idea which has not been previously innovated do so under patent protection, i.e.* $\zeta_1 < 1$.

Proof. By lemma 23,

$$\frac{1}{\gamma} > \Big(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\Big)\Big(\frac{\beta}{1-\beta\gamma}\Big),$$

holds if and only if

$$e^{-\theta}R_S^1(1) < R_P^1(1). \tag{C.21}$$

Hence, we need to prove that if equation (C.21) holds, then in equilibrium a positive fraction of firms that innovate a previously undeveloped idea would do so under patent protection.

Suppose that firms use secrecy protection following any $t \in supp(F)$ (except possibly over a set with measure 0). We will now show that this can never be an equilibrium because firms have a profitable deviation.

Since all firms use secrecy, it follows that if a deviant is to patent her payoff is $R_P^1(t)$, as she wins the race for sure. On the other hand, if she is to use secrecy protection, then $P_S^n(\text{win}|t) = \sum_{k=0}^{\infty} \theta^k e^{-\theta}(1-F^n(t))^k/(k!) = e^{-\theta F^n(t)}$ and her payoff is $e^{-\theta F^n(t)}R_S^1(t)$. By lemmas 27 and 28 and by continuity of $R_P^n(t)$ and $R_S^n(t)$, since (C.21) holds, it follows that there exists some $\delta > 0$ such that for all $t \in (1-\delta, 1)$, $R_P^1(t) > e^{-\theta F^n(t)} R_S^1(t)$. Hence, firms that innovate at such t would have an incentive to deviate to patenting. Since there is a strictly positive fraction of such firms, we have a contradiction. ■

Hence, if $\alpha = e^{-\theta}\beta$, it follows that $\zeta_1 < 1$.Thus, the rest of the proof is analogous to the proof of Theorem 14. ■