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Authors
Sikk, Allan
Taagepera, Rein

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Peer reviewed
How population size affects party systems and cabinet duration

Allan Sikk
University College London, UK

Rein Taagepera
University of Tartu, Estonia, USA; University of California, Irvine, USA

Abstract
This study develops and tests theoretical formulas for linking country size and party system characteristics. For countries using one-seat electoral districts or nationwide districts, the averages of the largest seat-share, effective number of assembly parties and mean duration of cabinets can be predicted based solely on population. For countries allocating seats by PR in multi-seat districts, the averages of these characteristics can be predicted based on population and district magnitude. We show that first-past-the-post countries of less than one million tend to have highly dominant largest parties and one-and-a-half party assemblies, rather than a balance of two parties. For larger countries, and PR countries of any size, population is not destiny, as far as party system is concerned.

Keywords
country size, electoral change, electoral systems, party systems, predictive models

Introduction
Do party system characteristics differ in small and large countries, all other conditions being the same? If they do, then what is the specific average relationship to the population, and why? One may expect that tiny countries would not have room for many parties. With fewer parties to divide the pie, the largest party’s share might be larger than is the case in larger countries. Also, the governmental cabinets might have fewer parties, which might make them more stable.

All these guesses are merely directional. This means they attempt to predict the direction of change in party system when population \( (P) \) changes (i.e. the sign of \( dy/dP \)). They do not specify how large the number of parties, etc., might be, for a given population. A more specific prediction would be offered by a quantitative model that also stipulates the expected functional relationship (i.e. the form of \( y = f(P) \)), all other conditions having medium values.

It is usually easier to test the direction of change than to establish the functional relationship. However, the reverse could be the case when both of the following two conditions apply: (a) The rate of change is so low that even small random fluctuations could blur the empirical trend; and (b) we have a logical model that predicts the functional form of the relationship.

Under these conditions, research emphasis changes from measuring the degree of fit of data to an empirical best fit curve, based on the same data, to testing the agreement between predicted logical relationship and data that did not enter into construction of the model.

It will be seen that this is so in the present case. The impact of population can be expected to be so mild that it is hard to prove its existence by statistical means. Yet a logical model predicts the zone of occurrence of data largely within a factor of 1.5 (meaning multiplying or dividing the mean prediction by 1.5).

We focus on three characteristics of party systems: the effective number of legislative parties \( (N) \) and largest seat-share \( (s_1) \) in the first or only chamber of the
Previous work

The size ($S$) of the first or only chamber of representative assemblies (i.e., the number of their members) was connected to the population represented ($P$) by Taagepera (1972) both empirically and through a logical model. The simplest expression for this cube root law of assembly sizes is

$$S = P^{1/3}.$$  

For a population of one million, it predicts $S = 100$, while for a billion it would be $S = 1000$. Marked exceptions are relatively small island nations, whose assemblies fall short of the cube root of their populations. A number of other relationships, with predicted direction empirically confirmed, were added in the seminal work by Robert Dahl and Edward Tufte, *Size and Democracy* (1973).

A recent overview of ‘Size and Politics’ (Taagepera 2007: ch. 12) includes more recent advantages regarding party systems. Its substance is given here very briefly, omitting most of the specific formulas. Smaller countries seem to have a lower number of parties registered (Anckar, 1998, 2000). Smaller PR countries seem to have higher party memberships, per capita (Weldon, 2006). Smaller first-past-the-post (FPTP) countries may have a lower share of seats going to third parties (Gerring, 2005). What is of most interest for the present purposes is a string of models that connects several party system indicators to assembly size. This is so because assembly size itself connects to population through the cube root law.

The largest seat-share ($s_1$) has been shown to logically connect to the product of mean district magnitude ($M$) and assembly size ($S$) as:

$$s_1 = (MS)^{-1/8}.$$  

For a set of 46 systems, the correlation coefficient of logarithms of $s_1$ and $MS$ is $R^2 = 0.51$ (Taagepera, 2007: 129; Taagepera and Ensch, 2006). The model applies in principle only to ‘simple’ electoral systems where all seats are allocated in districts of roughly comparable magnitude, with no further complexities such as thresholds, second tiers or second rounds. Only basic list PR and FPTP qualify as simple, given that all other electoral rules add features that are not indispensable. In order to expand the sample, the set of 46 also included some Two-Rounds (TR) and Alternative Vote (AV) systems. Graphs of $s_1$ vs. $S$ for $M = 1$ systems in Taagepera and Ensch (2006) and Taagepera (2007: 126) show that TR systems involve wild randomness and reduce the overall $R^2$, while the single AV system (Australia) fits into the general picture.

The effective number of parties ($N$) also logically connects to the same ‘seat product’ $MS$:

$$N = (MS)^{1/6}.$$  

For a set of 25 systems, the coefficient of determination between the logarithms of $N$ and $MS$ is again $R^2 = 0.51$ (Taagepera, 2007: 153).

Finally, the mean duration of cabinets ($C$) logically connects to the number of parties and thus, more indirectly, to the seat product $MS$:

$$C = \frac{42 \text{ years}}{(MS)^{1/3}}.$$  

For the previous set of 25 systems, the correlation coefficient of logarithms of $C$ and $MS$ is $R^2 = 0.24$ (Taagepera, 2007: 171; Taagepera and Sikk, 2010). Increased scatter, compared to $N$, reflects the more indirect nature of the connection. The dataset includes all those systems for which Lijphart (1999) offers cabinet duration figures and which have simple electoral systems: FPTP, simple list PR, and also Single Transferable Vote (STV), Single Non-Transferable Vote (SNTV), and TR. The constant value $k = 42$ years is the one that best fits the more direct model $C = k/N^2$. No $M$ or $S$ data enter into the determination of $k$.

Party and electoral systems interact of course in both directions. Existing party constellations are a most important factor in the choice of electoral rules (Colomer, 2005). Once fixed, the electoral rules continue exerting pressure on the number of parties both directly through the mechanical and indirectly through the psychological effect (Benoit, 2002; Duverger, 1951). Our objective here is to assess the aggregate relationship between country size/electoral rules and party system characteristics without trying to distinguish between the two effects.

Taagepera (2007: 191) points out that combining these models with the cube root law of assembly sizes leads to expecting a connection between the party system characteristics and population:

$$s_1 = M^{-1/8}P^{-1/24},$$

$$N = M^{1/6}P^{1/18}; \text{ and}$$

$$C = 42\text{years}(M^{-1/3}P^{-1/9}).$$

Population enters with horrendously low exponents, however. On logarithmic graphs of $s_1$, $N$ and $C$ vs. $P$, with equal scales on both axes, one would expect straight lines with absolute values of slopes $1/24 = 0.0417$, $1/18 = 0.0555$ and $1/9 = 0.111$, respectively, for a given value of $M$. These slopes are so close to zero that even minor random fluctuation could reverse the apparent direction of the linear regression slope. Given such indeterminacy, Taagepera (2007: 191–192) does not even try to test the equations for $s_1$ and $N$. For $C$, the geometric means for 8 smaller and 6 larger systems using one-seat districts are compared. The larger countries tend indeed to have shorter
cabinets (5.8 vs. 7.6 years), but the ratio is only 1.3, as compared to the expected 1.6. In the following, an attempt is made to go further.

Many studies in political science aim at empirical accounting for outputs, trying to include all variables that conceivably could affect the output. Visibly, our aim is quite different and should not be mistaken for empirical data fit. We aim at testing the impact of a single factor, population, as predicted by a logical quantitative model. The model used indicates that population interacts with district magnitude in a specific way. Thus we have to include the latter – but, giving more than lip service to Occam’s Razor, we include no more variables. Factors such as the number of social conflicts, including ethnic fractionalization (Amorim Neto and Cox, 1997; Clark and Golder, 2006; Mozaffar et al., 2003; Ordeshook and Shvetsova, 1994; Singer and Stephenson, 2009; Stoll, 2008, 2011; Taagepera, 1999; Taagepera and Grofman, 1985) federalism and the extent to which district-level parties aggregate on national level (Chhibber and Kollman, 1998, 2004; Chhibber and Murali, 2006; Hicken, 2009) and parliamentarism/presidentialism (Filippov et al., 1999; Golder, 2006; Hicken and Stoll, 2011; Jones, 1994; Lipphart, 1994; Shugart and Carey, 1992) may also affect the outputs considered, but the issue here is not maximal accounting for these outputs but testing (1) whether population has any impact in the predicted direction (all other factors being more or less random), and more demandingly, (2) whether it has the degree of impact predicted by the model.

The model and data

To the extent that the cube root law is valid, the seat product $MS$ at the core of the previous equations in $s_1$, $N$ and $C$ becomes

$$MS = M(P^{1/3}) = (M^3P)^{1/3}.$$

Hence

$$s_1 = (M^3P)^{-1/24},$$

$$N = (M^3P)^{1/18},$$

and

$$C = 42 \text{ years}(M^3P)^{-1/9}.$$  

If so, then the logarithms of $s_1$, $N$ and $C$ should have linear relationships to the logarithm of $(M^3P)^{1/3}$.

In the special case of $M = 1$, the equations above simplify to

$$s_1 = P^{-1/24},$$

$$N = P^{1/18},$$

and

$$C = 42 \text{ years}P^{-1/9}.$$  

We first test this simple case by comparing the logarithms of output variables to log $P$. The advantage of this approach is that no assumptions regarding the impact of $M$ have to be made. The disadvantage is that the range of $P$ with the appropriate exponent is very narrow. For populations ranging from 10,000 to 1 billion, $P^{1/18}$ ranges only from 1.67 to 3.16. Even minor random fluctuations in $N$ can blur out trends when the range of the input variable is so narrow. It is worse for $s_1$ and only slightly more promising for $C$.

Next, we test the more general case that includes $M > 1$, by comparing the logarithms of output variables to log $(M^3P)$. The advantage here is that the range of log $(M^3P)$ is much wider than that of log $P$. Hence, with the same degree of random fluctuations in $N$, trends can become more visible. The disadvantage is that success depends on the correctness of the assumption that multi-seat districts magnify the impact of population by a factor $M^3$. If this assumption does not hold, we risk with false negatives. Note, however, that false positives are extremely unlikely, as it would require that the error on the impact of $M$ exactly cancels out the error on the impact of $P$.

Included in the analysis are democracies with simple electoral systems (those where all seats are allocated in districts, in a single round, with no legal thresholds) for which the relevant data for $s_1$, $N$ and $C$ are available (tabulated in Taagepera and Sikk, 2010), adding mean population figures for the relevant periods as estimated from Atlas of World Population History (McEvedy and Jones, 1978) and World Almanac (1993). For $N$ and $s_1$, we also consider further countries tabulated in Taagepera and Enschede (2006), which extends the population range toward lower $P$. We exclude the two rounds systems, because they are unpredictable in votes–seats transformations (Taagepera, 2007: 54, 72) so that the models for simple electoral systems clearly do not apply to them. This leaves us with a main list (Table 1) of 24 systems with $M = 1$ (one of them Alternative Vote = STV) and 11 with $M > 1$ (including two STV and one SNTV).

The values shown for $S$, $s_1$, $N$ and $C$ are geometric means of values for individual elections. Populations are approximate geometric means of the extremes over the period. The impact of minor changes in population is muted by the exponents applied in the models, ranging from 1/9 to 1/24. The largest variation in population occurred in Costa Rica – from 1 to 3.7 million, with a geometric mean at roughly 2 million. Even if the mean should be off by half a million, i.e. by a factor of 2.5/2 = 1.25, it would alter our estimate of mean cabinet duration by a factor of $1.25^{1/9} = 1.025$; the estimates of the effective number of parties and the largest seat-share would be affected much less.

Results

**Party system predicted from population alone:**

$M = 1$ systems

We proceed in two stages, first assessing the mean predictions and thereafter looking at the fit of actual data with...
lines proposed by the theoretical models. The logical models above predict the average levels around which the fluctuation should take place. For the 24 $M = 1$ systems in Table 1, mean populations for the periods considered range from 0.017 million for Cook Islands to 800 million for India, with a geometric mean of 1.14 million. Table 2 shows the values calculated using the equations above, the actual geometric means, and the percent difference.

The geometric mean values of the largest seat-share, effective number of parties, and mean cabinet duration are predicted by the logical model within 9 percent or better, based on nothing but population. The logical causal concatenation on which the model in Taagepera (2007) is built is $P \rightarrow S \rightarrow s_1 \rightarrow N \rightarrow C$. Thus the error would be expected to increase in that order. Surprisingly, this is not the case. The mean prediction for assembly size is strongly off. This is due to 16 of the countries being small island nations that tend to have disproportionately small parliaments (see Taagepera, 1972). Yet the later predictions further down in the causal chain are off by much less, suggesting that population may additionally have a direct impact on party systems, bypassing assembly size.

Table 1. Population, seat allocation rule, mean district magnitude ($M$), assembly size ($S$), largest seat share effective number of legislative parties ($N$), and cabinet duration ($C$) for 25 one-seat and 11 multi-seat systems.

<table>
<thead>
<tr>
<th>Population (million)</th>
<th>$M^P$ (billion)</th>
<th>Rule</th>
<th>$S$</th>
<th>$s_1$</th>
<th>$N$</th>
<th>$C$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cook Islands 1965-99</td>
<td>0.017</td>
<td>1</td>
<td>$1.7 \times 10^5$</td>
<td>FPTP</td>
<td>23.2</td>
<td>0.609</td>
</tr>
<tr>
<td>St. Kitts 1980-2000</td>
<td>0.04</td>
<td>1</td>
<td>$4.0 \times 10^5$</td>
<td>FPTP</td>
<td>10.5</td>
<td>0.539</td>
</tr>
<tr>
<td>Antigua 1980-99</td>
<td>0.07</td>
<td>1</td>
<td>$7.0 \times 10^5$</td>
<td>FPTP</td>
<td>17.0</td>
<td>0.781</td>
</tr>
<tr>
<td>Dominica 1975-2000</td>
<td>0.07</td>
<td>1</td>
<td>$7.0 \times 10^5$</td>
<td>FPTP</td>
<td>21.0</td>
<td>0.621</td>
</tr>
<tr>
<td>Grenada 1972-2003</td>
<td>0.1</td>
<td>1</td>
<td>$1.0 \times 10^4$</td>
<td>FPTP</td>
<td>15.0</td>
<td>0.676</td>
</tr>
<tr>
<td>St. Vincent 1974-2001</td>
<td>0.1</td>
<td>1</td>
<td>$1.0 \times 10^4$</td>
<td>FPTP</td>
<td>14.1</td>
<td>0.765</td>
</tr>
<tr>
<td>St. Lucia 1974-2001</td>
<td>0.13</td>
<td>1</td>
<td>$1.3 \times 10^4$</td>
<td>FPTP</td>
<td>17.0</td>
<td>0.696</td>
</tr>
<tr>
<td>Samoa 1979-2001</td>
<td>0.19</td>
<td>1</td>
<td>$1.9 \times 10^4$</td>
<td>FPTP</td>
<td>47.6</td>
<td>0.531</td>
</tr>
<tr>
<td>Belize 1979-2003</td>
<td>0.2</td>
<td>1</td>
<td>$2.0 \times 10^4$</td>
<td>FPTP</td>
<td>26.5</td>
<td>0.691</td>
</tr>
<tr>
<td>Bahamas 1972-2002</td>
<td>0.25</td>
<td>1</td>
<td>$2.5 \times 10^4$</td>
<td>FPTP</td>
<td>42.0</td>
<td>0.732</td>
</tr>
<tr>
<td>Barbados 1966-94</td>
<td>0.25</td>
<td>1</td>
<td>$2.5 \times 10^4$</td>
<td>FPTP</td>
<td>26.0</td>
<td>0.700</td>
</tr>
<tr>
<td>Trinidad 1961-2001</td>
<td>1.2</td>
<td>1</td>
<td>0.0012</td>
<td>FPTP</td>
<td>36.0</td>
<td>0.746</td>
</tr>
<tr>
<td>Botswana 1965-2004</td>
<td>1.3</td>
<td>1</td>
<td>0.0013</td>
<td>FPTP</td>
<td>37.0</td>
<td>0.749</td>
</tr>
<tr>
<td>Norway 1882-1903</td>
<td>2.1</td>
<td>1</td>
<td>0.0021</td>
<td>FPTP</td>
<td>114.4</td>
<td>0.585</td>
</tr>
<tr>
<td>Jamaica 1962-69</td>
<td>2.5</td>
<td>1</td>
<td>0.0025</td>
<td>FPTP</td>
<td>55.0</td>
<td>0.755</td>
</tr>
<tr>
<td>Denmark 1884-1913</td>
<td>2.5</td>
<td>1</td>
<td>0.0025</td>
<td>FPTP</td>
<td>109.9</td>
<td>0.539</td>
</tr>
<tr>
<td>New Zealand 1946-96</td>
<td>3</td>
<td>1</td>
<td>0.0003</td>
<td>FPTP</td>
<td>85.0</td>
<td>0.569</td>
</tr>
<tr>
<td>Papua New Guinea 1977-97</td>
<td>4</td>
<td>1</td>
<td>0.004</td>
<td>FPTP</td>
<td>108.0</td>
<td>0.397</td>
</tr>
<tr>
<td>Sweden 1887-1905</td>
<td>5</td>
<td>1</td>
<td>0.005</td>
<td>FPTP</td>
<td>226.3</td>
<td>0.511</td>
</tr>
<tr>
<td>Australia 1946-96</td>
<td>12</td>
<td>1</td>
<td>0.012</td>
<td>STV</td>
<td>128.0</td>
<td>0.507</td>
</tr>
<tr>
<td>Canada 1945-93</td>
<td>18</td>
<td>1</td>
<td>0.018</td>
<td>FPTP</td>
<td>270.0</td>
<td>0.555</td>
</tr>
<tr>
<td>United Kingdom 1945-97</td>
<td>50</td>
<td>1</td>
<td>0.05</td>
<td>FPTP</td>
<td>635.0</td>
<td>0.534</td>
</tr>
<tr>
<td>United States 1947-2001</td>
<td>230</td>
<td>1</td>
<td>0.23</td>
<td>FPTP</td>
<td>435.0</td>
<td>0.619</td>
</tr>
<tr>
<td>India 1977-96</td>
<td>800</td>
<td>0.8</td>
<td>0.502</td>
<td>FPTP</td>
<td>542.0</td>
<td>0.550</td>
</tr>
</tbody>
</table>

Geometric mean for $M = 1$. 1.14 1.14 0.597 0.615 2.08 7.99

Malta 1966-87 0.3 5 0.038 STV 59.0 0.529 1.99 10.6

Luxembourg 1945-99 0.35 14.2 1 L 56.4 0.411 3.36 6.0

Costa Rica 1953-98 2 7.8 0.95 L 55.0 0.524 2.41 4.9

Ireland 1948-97 3 3.5 1.3 STV 154.0 0.482 2.84 3.8

Israel 1949-96 4 120 6900 L 120.0 0.379 4.55 1.8

Norway 1945-97 4 7.7 1.8 L 154.0 0.466 3.35 4.3

Finland 1945-2003 4.5 14 12.3 L 200.0 0.268 5.03 1.5

Portugal 1976-2002 9 11.3 13 L 249.0 0.430 3.33 3.2

Netherlands 1946-2002 13 140 35700 L 137.7 0.344 4.65 3.3

Spain 1977-2004 35 6.7 10.5 L 350.0 0.501 2.76 9.0

Japan 1946-96 110 4 7 SNTV 486.0 0.540 3.71 3.9

Geometric mean for $M > 1$. 10.85 145.6 0.434 3.33 4.06


Notes: Seat allocation rules: FPTP – First-Past-The-Post; STV – single transferable vote; L – list PR; SNTV – single non-transferable vote.

* Multi-member districts in five largest towns (Mackie & Rose 1974: 341)
Yet graphing the largest seat-share against population shows only a faint trend. Figure 1 graphs $s_1$ against $P$, both on logarithmic scales. Also shown are the lines corresponding to the prediction $s_1 = P^{-1/24}$ and to deviations from it by a factor of 1.5, i.e. $s_1 = (3/2)P^{-1/24}$ and $s_1 = (2/3)P^{-1/24}$. All the data points are within this range of error, and the geometric means of both sets are close to the predicted line. Yet the linear regression line is much gentler than the prediction and the fit is poor. The picture is similar for the effective number and mean cabinet duration (graphs not shown). What does it mean?

This is another instance where a purely statistical approach cannot substitute for logical models (cf. Taagepera, 2008) and would lead us astray. Compared to the range of random fluctuation, the range of the input variable is just too narrow to test for the expected slope. We have to find ways to extend this range – and this is where multi-seat systems come in.

### Table 2. Actual geometric means of system characteristics and predictions based on mean population, 24 $M = 1$ countries.

<table>
<thead>
<tr>
<th>Actual Geometric Mean</th>
<th>Calculated from Mean Population</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>$1.14 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Assembly size</td>
<td>59.7</td>
<td>104.6</td>
</tr>
<tr>
<td>Largest seat share</td>
<td>0.615</td>
<td>0.559</td>
</tr>
<tr>
<td>Effective number of parties</td>
<td>2.08</td>
<td>2.17</td>
</tr>
<tr>
<td>Mean cabinet duration</td>
<td>7.99 yrs</td>
<td>7.35 yrs</td>
</tr>
</tbody>
</table>

* Based on the 12 countries for which the data are available (geometric mean of population $6.51 \times 10^6$).

### Table 3. Actual geometric means of system characteristics and predictions based on mean population, 11 $M > 1$ countries.

<table>
<thead>
<tr>
<th>Actual Geometric Mean</th>
<th>Calculated from Mean Population</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>$4.62 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Assembly size</td>
<td>145.3</td>
<td>166.6</td>
</tr>
<tr>
<td>Magnitude cubed $P$</td>
<td>$8.8 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Largest seat share</td>
<td>0.434</td>
<td>0.385</td>
</tr>
<tr>
<td>Effective number of parties</td>
<td>3.33</td>
<td>3.57</td>
</tr>
<tr>
<td>Mean cabinet duration</td>
<td>4.06 yrs</td>
<td>3.30 yrs</td>
</tr>
</tbody>
</table>

Note: Assembly size is calculated from $P$, the other outputs from $M^3P$.

### Figure 1. The largest seat-share versus population for $M = 1$ countries.

Notes: Bold line: theoretically based prediction ($R^2 = 0.22$, negative as the sum of squared distances from mean $s_1$ is smaller than the sum of squared distances from values predicted by the model.) Thin line: OLS best fit between logarithms ($R^2 = 0.17$).

### Figure 2. The largest seat-share versus the product of population and district magnitude cubed.

Notes: Bold line: theoretically based prediction ($R^2 = 0.35$). Thin line: OLS best fit between logarithms ($R^2 = 0.55$).

**Party system predicted from population and district magnitude**

Table 3 shows the geometric means for multi-seat electoral systems listed in Table 1. Here the mean for assembly size is close to the cube root of mean population, as only 2 of the 11 countries are relatively small. The basis for calculating the mean $s_1$, $N$ and $C$ now shifts from mean $P$ to mean $M^3P$. The actual means are predicted within 19 percent. Here we faced a double risk. The model for the effect of population as such could be off, or the model for combining population and district magnitude could be off. The quality of the prediction offers some confidence on both accounts.

Figure 2 graphs the largest seat-share against the product $M^3P$, both on logarithmic scales. Different symbols are used for $M = 1$ and $M > 1$ systems and the mean points for the two sets are shown with bold symbols. Also shown are the lines corresponding to the prediction $s_1 = (M^3P)^{-1/24}$ and to deviations from it by a factor of 1.5. All the data points are within this range.
of error, and the geometric means of the three sets are close to the predicted line.

For $M = 1$ systems, the data cloud exactly replicates Figure 1. Yet the overall picture is quite different, as the inclusion of $M > 1$ systems doubles the range of the input variable. The linear regression line now approaches the expected line: $s_1 = 0.981(M^3P)^{0.034} - 0.034 \log s_1 = -0.019 - 0.034 \log (M^3P)$, as compared to the expected $s_1 = M^3P^{1/24} \log s_1 = 0 - 0.0416 \log (M^3P)$. The precise location of the regression line is still at the mercy of a few outliers – Israel and The Netherlands at the very high end of the scale. The more important question is to what degree the logical model-based line accounts for the variation in data, compared to the statistical best-fit line. The standard error of estimate (SEE) is 0.17 for the OLS best fit and 0.20 for the theoretical model – not appreciably less given the mean of $\log s_1 = -0.60$ ($R^2$ correspondingly 0.55 and 0.35).\(^5\) The model’s goodness of fit is strongly affected by the outliers of Finland, Israel and The Netherlands. The upper end of the $M^3P$ scale is sparsely populated as we are limited to simple electoral systems here, while systems with $M >> 1$ systems tend to use legal thresholds or multiple tiers of seat allocation. Rough approximations of predicted largest party’s seat-share in five European democracies with $M^3P$ ranging from 47 to 270 billion presented in Appendix B conform reasonably well to the theoretical model.

Similarly, Figure 3 graphs the effective number of parties against the product $M^3P$ for data in Table 1. The geometric means of the two sets are close to the predicted line. Only two countries deviate from expectation by more than a factor of 1.5, in opposite directions. Papua New Guinea had a profusion of tribally based parties, while Botswana had only one large and one small party, both based on ethnic identity. SEE and $R^2$ are heavily affected by Papua New Guinea. High levels of ethnic fractionalization have been shown to increase $N$ (Singer and Stephenson, 2009) – Papua New Guinea ‘approximately a perfectly fractionalized state’ (Fearon, 2003: 205) and as it deviates by more than two standard deviations from the best-fit line, its exclusion is justified because of potentially massive impact of ethnic heterogeneity. Without Papua New Guinea, the regression line is $N = 0.928(M^3P)^{0.055} \log N = -0.0075 + 0.055 \log (M^3P)$, as compared to the expected $N = (M^3P)^{1/18} \log N = 0 - 0.056 \log (M^3P)$. Here, too, the observations do not deviate from theoretically based predictions appreciably more than from those based on OLS estimation (see Figure 4) and the model-based line has a SEE (0.21) very close to that of OLS regression (0.19) (correspondingly, $R^2 = 0.66$ versus $R^2 = 0.72$).\(^6\)

Finally, Figure 5 graphs the mean duration of cabinets against the product $M^3P$ for the 24 systems for which the data are available. Once more, the geometric means of the two sets
are close to the predicted line, but nine systems deviate from expectation by more than a factor of 1.5, in opposite directions. For Papua New Guinea and Botswana, the deviations in cabinet duration are logical consequences of their unusual effective numbers of parties. While these cases can be explained, there is no obvious way to account for the opposite deviations of stable democracies such as Finland and The Netherlands, except for acknowledging their deviant party systems (see Figures 2 and 3). In the logical causal concatenation $P \rightarrow S \rightarrow s_{1} \rightarrow N \rightarrow C$, cabinet duration is simply so far removed from population that the latter’s impact becomes blurred.

Statistically speaking, exclusion of no country is justified here. The regression line is log $C = \log(36.3\text{yrs}) - 0.096\log(M^3P)$. It is still not far off the expected $\log C = \log(42\text{yrs}) - 0.11\log(M^3P)$, particularly in the middle of the data cloud. Once again, the model-based line fits the data as well as the OLS line: for theoretical model, $\text{SEE} = 0.65$ and for OLS model $\text{SEE} = 0.63$ ($R^2 = 0.33$ versus $R^2 = 0.38$, respectively). What matters is that the predicted zone still includes two-thirds of the data points and the rest is positioned evenly around it.

**Nationwide PR elections**

For nationwide PR elections, district magnitude equals assembly size: $M = S$. Then, to the extent that the cube root law is valid, $M^3 = S^3 = P$, so that the core expression $M^3P$ becomes $P^2$. The previous equations in $s_{1}$, $N$ and $C$ simplify into

$$s_{1} = P^{-1/12},$$
$$N = P^{1/3}; \text{and}$$
$$C = 42 \text{years}P^{-1/4.5}.$$

With increasing population, these values change much more rapidly than is the case with one-seat districts.

Testing is difficult because most countries with nationwide PR balance it with appreciable legal thresholds on votes, so as to keep the number of parties down. Table 1 includes only two systems with nationwide PR, and even these have mild thresholds: 1 percent for Israel and 0.67 percent for The Netherlands. Table 4 shows the degree of agreement. The actual values are off in the direction expected when legal thresholds are applied. They are off by up to 35 percent, except for cabinet duration for The Netherlands, which is much higher than expected (cf. Figure 4).

**Logically predictable effect, largely blurred by other factors**

A look at Figures 2 to 4 confirms that population does affect the largest seat-share, the effective number of parties and mean duration of cabinets, conjointly with district magnitude, for simple electoral systems. All other factors (and district magnitude in particular) being the same, larger countries have smaller seat-shares going to the top party. Larger countries tend to have more parties and shorter cabinets. The effect of log $M^3P$ remains robust when controlling for other factors that may affect party systems and cabinet durations, such as ethnic fractionalization, presidentialism and federalism. When introduced in OLS models on their own or in interaction with district size, only the effect of fractionalization is statistically significant ($p < 0.05$) on log $s_{1}$ and log $N$, with Papua New Guinea having high leverage.

It is important to go beyond these directional conclusions. The party system characteristics of most systems can be predicted within a factor of 1.5 using a logical model that combines population and district magnitude. For countries using one-seat districts, this boils down to predicting party system characteristics based solely on population.
Over large differences in population, these logical predictions vary appreciably. For an FPTP country of 100,000, an effective number of parties of 1.9 would be predicted, while it would be 2.8 for an FPTP country of 100 million (see Figure 6), and 3.6 for a country of 100,000 that applies nationwide PR. The respective largest seat-shares would be 62, 46 and 39 percent. The expected cabinet durations would be 11.5, 5.3 and 3.2 years, respectively.

The catch is that the largest seat-shares of individual countries are observed to vary within a factor of 1.4 of these averages (cf. Figures 1 and 2). Deviations are even larger for the effective number of parties (Figure 3) and especially for cabinet duration (Figure 4). Within such a range, countries have appreciable leeway. This leeway is more limited if countries wish to adhere to a definite brand of democracy, as discussed next.

**Population dependency of the majoritarian and consensus models of democracy**

Consider first countries determined to hold on to FPTP. Such countries probably have in mind the majoritarian ideal of a single-party cabinet and a mostly single-party opposition of an almost equal strength, such as in 52/48, with regular alternation in power at every election but no cabinet change in between. This would mean \( s_1 = 0.52, N = 2.00 \) and \( C = 4 \) years (assuming elections every 4 years). Does population place limitations on reaching such an average outcome?

Data scatter in Figure 1 shows that a largest seat-share of 52 percent has been obtained with populations as low as 40,000 and as high as 70 million. This covers most of the entire range of sovereign countries. However, most countries of less than one million tend to have an overly dominant largest party, facing a combined opposition reduced to less than one-third of the seats. The majoritarian ideal of a strong opposition seems hard to reach in very small countries.

For the effective number of parties to range around 2 (say, from 1.8 to 2.2), populations in Figure 3 are observed to range from 2 to 50 million. Below 1 million, the preponderance of the largest party reduces \( N \) to appreciably less than 2. On the high side, countries above 10 million population tend to have \( N \) larger than 2.2, reflecting third-party incidence.

As for duration of cabinets, no \( M = 1 \) systems in Figure 4 happen to have mean durations around 4 years. Durations are either below 3 years, denoting frequent cabinet changes in between elections, or above 6 years, denoting the same party remaining in power for a second or even third term in a row. Our model predicts that regular alternation every 4 years would be the likeliest for very large countries. The only actual case (India), however, has had appreciably more frequent changes.

In sum, for \( M = 1 \) countries our model predicts that the majoritarian ideal could be best satisfied by populations around 10 million on the account of largest seat-share, around 0.5 million on the account of effective number of parties, and rather incongruously, around 1 billion for cabinet duration. In this light, the ideal of two-party alternation may involve an internal contradiction.

Actual data suggest that countries below one million have major difficulties in holding up the majoritarian ideal. Their largest party tends to dominate the assembly excessively, reducing the 2-party system to a one-and-a-half party system. Moreover, the same party tends to be in power for several electoral periods in a row. The mechanism behind this outcome is rooted in an excessive largest party bonus in conversion of vote-shares to seat-shares, due to small assembly sizes (see Lijphart, 1990). At the other extreme, very populous FPTP countries might be expected to have difficulty avoiding strong third parties, but actual evidence is inconclusive.

Shifting now to multi-seat PR systems, many of them conform to the consensus model as described by Lijphart (1999), with a large effective number of parties, small largest seat-share and frequently reshuffled coalition cabinets. However, these are not explicit goals, such as single party majorities and two-party alternation are for the majoritarian model. They are merely the outcomes, once the electoral system allows for small party representation. Maximal small party representation is achieved with nationwide PR. In the absence of legal thresholds, the model predicts an effective number of 2.8 parties even for a population of 10,000. The largest party would have 47 percent of the seats and cabinets would last 5.5 years on the average. Consensual countries usually have a more fractionalized party system than that.

A population of 10 million could expect an effective number of parties of \( N = 6 \). The largest party would have only 26 percent of the seats and cabinets would be reshuffled once every 1.2 years on the average. This is about the most fractionalized party system that we observe among those that still have some stability. Larger countries than 10 million would have to avoid nationwide PR or add restrictive legal thresholds.

**Conclusions**

Scientists going beyond pure empirics should find it exciting that the means of some party system characteristics at given population can be predicted without any data input, solely on the basis of population, using a set of logical models. These characteristics are the seat-share of the largest party and the effective number of assembly parties. The mean duration of cabinets can be added to the list. The solely population-based prediction applies to FPTP and possibly to other single-round one-seat electoral systems.
By multiplying the population by the cube of district magnitude – another logically motivated step – the prediction can be extended to those multi-seat electoral systems where all seats are allocated within districts. In all this, political science proceeds beyond empirical description to quantitative prediction on logical grounds.

The theoretical model presented and tested in this article is concerned with an indirect link between country size and party system, i.e. working through assembly size. There is no evidence for an additional direct effect on the effective number of parties and cabinet duration, as the residuals (i.e. ratios of actual to theoretically expected \( N \)-s and \( C \)-s) are not correlated with country size. When it comes to the size of the largest party, increasing population reduces it somewhat less than predicted by our model (see Figure 7). Yet, the effect is mild at best and we currently lack a theoretical model to explain the empirical discrepancy. This is also well visible in Figure 2, where the slopes for theoretical and empirical fit lines differ much more than for \( N \) and \( C \). Therefore, we broadly agree with Dahl & Tufte that all modern democracies are too populous for there to be any discernible direct effects – these may appear on the level of municipal party systems (1973: 94–97). The proposed and tested model can be used as a baseline for studies on any direct effects or the impact of other societal factors that may affect party systems and cabinet duration (including analysis of individual cabinets rather than aggregate measures as is the case here).

While population has a definite overall effect on size characteristics of party systems, for individual countries this effect is heavily blurred by other factors. Only FPTP countries of less than one million are markedly hemmed in; they could expand their one-and-a-half party systems only by shifting to PR. Thus population is not destiny, as far as party system is concerned.

What are the practical implications? Countries are not prisoners of their population sizes. They are not locked in regarding their choice of party systems as the limits that population size puts on party systems are mediated by the choice of electoral rules.

Figure 7. Residuals of largest seat-share versus population.

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Notes
1. We measured cabinet duration used ‘average cabinet life I’ (see Dodd, 1976 and Lijphart, 1999: 132 f.). This index best fits the stipulated logical relationship.
2. We analyse means rather than individual elections for several reasons. First, we believe that party systems are durable characteristics of political systems. Secondly, there is very little variation over time in our independent variables and in most dependent variables. Geometric means are less sensitive than arithmetic means to untypically high values of the variables.
3. Other factors mentioned above that may affect party systems have in most cases been fairly constant in political systems under study here. Analysing means rather than levels of dependent variables over time is justified here as country size and mean district magnitude stay constant over long periods and could not possibly explain fluctuations in party systems and cabinet duration over time.
4. Coefficients from OLS models and indicators of fit of theoretical and OLS models are given in Appendix A. Here, the coefficient of determination for the predicted line is negative. This may look erroneous since the common range for \( R^2 \) is from 0 to 1. However, that strictly applies to OLS best fit lines only. For other lines, the \( R^2 \) – as based on the most frequently used formula – can be negative. Other fitting methods that are less sensitive to outliers (such as robust regression, quantile regression and deepest regression, see Wilcox, 2005) would result in similar best-fit lines in Figure 1 and in the Figures below. For an extensive discussion on the properties and variants of \( R^2 \) see Kvalseth (1985).
5. SEE reflects the standard deviation of model residuals. While the widely used \( R^2 \) shows predictive capability of the model (Lewis-Beck and Skalaban, 1990), SEE has certain advantages over \( R^2 \) (see Achen, 1990), especially as it has the same units as the dependent variable. As we use logarithms of our dependent variables throughout the study, SEE has to be transformed to make the units meaningful. If \( \text{SEE} = 0.20 \) as in the case of \( \log Y \), \( \exp(0.20) = 1.22 \) is the standard (multiplicative) factor of deviation in \( s_Y \).
6. With Papua New Guinea included, \( \text{SEE} = 0.26 \) for both theoretical and OLS fit (\( R^2 = 0.54 \) versus \( R^2 = 0.56 \)). From the perspective of empirical model fitting it might be tempting to include logarithms of \( M \) and \( P \) independently – that not only unites the coefficients but also frees up the exponent of \( M \). The resulting coefficients (\( \log M: 0.153 \) and \( \log P: 0.059 \)) are strikingly close to the theoretical model (\( \log M: 0.167 = 3 \times 0.056 \) and \( \log P: 0.056 \)); a trivial improvement in model fit would be accompanied by an increase in the degrees of freedom and thus a lower F-statistic. The same applies for the other two models discussed in this study, see Appendix A for models with \( \log P \) and \( \log M \) included separately.
7. We used operationalizations from Singer and Stephenson (2009), adding fractionalization data from Alesina et al. (2003) for small island states. Resulting models are available on request.

8. Cabinet duration involves a parameter (42 years) determined empirically from the relationship between $N$ and $C$. No data on $P$ or $M$ enter this determination.

References


**Author biographies**

**Allan Sikk** is a Lecturer at the University College London School of Slavonic and East European Studies. His main research interests are new political parties, electoral politics and quantitative methods in political science. He has previously published in *Party Politics, The European Journal of Political Research* and the *Journal of Communist Studies and Transition Politics*.

### Appendix A. Summary of models.

#### Table 5. Comparison of theoretical and OLS coefficients and model fits.

<table>
<thead>
<tr>
<th></th>
<th>$s_1$ (M=1)</th>
<th>$s_1$</th>
<th>$N$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>theor. model</td>
<td>OLS1</td>
<td>theor. model</td>
<td>OLS1</td>
</tr>
<tr>
<td>$\log(M^3P)$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.056</td>
</tr>
<tr>
<td>$\log M$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\log P$</td>
<td>–0.042</td>
<td>–0.025**</td>
<td>–</td>
<td>0.056*</td>
</tr>
<tr>
<td>constant</td>
<td>0</td>
<td>–0.136</td>
<td>0</td>
<td>–0.019</td>
</tr>
</tbody>
</table>

**Coefficients in exponential model**

| $M^3P$ | – | – | – | 0.056 | 0.054 | – | –1.111 | –0.096 | – |
| $M$ | – | – | – | – | – | – | – | 0.153 | – | – | 0.270 |
| $P$ | –0.042 | –0.025 | – | – | – | – | – | 0.059 | – | – | 0.126 |
| Multiplier* | 1 | 0.873 | 1 | 0.981 | 0.821 | 1 | 0.983 | 0.19 | 42yrs | 36.3yrs | 56.9yrs |
| SEE | 0.19 | 0.16 | 0.20 | 0.17 | 0.17 | 0.26 | 0.26 | 0.26 | 0.65 | 0.63 | 0.64 |
| $R^2$ | –0.22 | 0.17 | 0.35 | 0.55 | 0.57 | 0.54 | 0.56 | 0.56 | 0.33 | 0.38 | 0.39 |
| $N$ | 24 | 24 | 35 | 35 | 35 | 35 | 23 | 23 | 23

Notes: Standard errors of OLS models are in parenthesis. OLS1 – models with $\log(M^3P)$ as an independent variable, OLS2 – models with $\log M$ and $\log P$ entered individually.

* $p < .1$, ** $p < .05$, *** $p < .01$.

* Applies for OLS2 only.

* See page 7.

* multiplier = $\exp(\text{constant})$. 
Appendix B: Testing the theoretical model in complex electoral systems

Most countries that employ proportional representation use legal thresholds or multi-tier seat allocation. In such systems, $M$ is impossible to determine exactly. However, approximations can in many cases be calculated based on a premise that if a legal threshold is used the electoral system functions roughly as a single nationwide constituency with estimated effective magnitude $M_{\text{eff}}=\frac{75\%}{T_{\text{legal}}} - 1$ (Taagepera 2007: 246).

While such estimates are rough, great precision is not of paramount importance here. Even if the estimated effective magnitude is off the ‘real’ $M_{\text{eff}}$ by a factor of 2, the prediction of $s_1$ is only off by less than 10 percent, and by about 5 percent for an error within a faction of 1.5 in $M_{\text{eff}}$.

The estimates of $M$ for complex systems are interesting as they enable us to add data to the sparsely populated higher end of $M^P$. Table 6 shows the estimates together with predicted and actual values of $s_1$ in five European democracies where we would expect high values of $M^P$. The predicted values for $s_1$ are remarkably close to actual $s_1$ in most cases. Adding the approximations to the dataset would bring the OLS coefficient for $\log M^P$ closer to theoretical $-0.042 (-0.034 \rightarrow -0.037)$ and enhance the fit with the theoretical model (SEE: 0.20 $\rightarrow$ 0.19, $R^2$: 0.35 $\rightarrow$ 0.49). It suggests that the countries with high $M^P$ included in our main dataset may in fact be outliers, and if more countries could be added the goodness of fit for the model would be improved (see Figure 8).

Table 6. Estimates of predicted and actual $S_j$ in complex electoral systems.

<table>
<thead>
<tr>
<th>Legal threshold</th>
<th>Estimated $T_{\text{legal}}$</th>
<th>Estimated $M_{\text{eff}}$</th>
<th>Estimated $M^P$</th>
<th>Expected $s_j$</th>
<th>Actual $s_j$</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>5%</td>
<td>14</td>
<td>$270 \times 10^6$</td>
<td>0.33</td>
<td>0.33</td>
<td>1989-2005</td>
</tr>
<tr>
<td>Denmark</td>
<td>2%</td>
<td>37</td>
<td>$253 \times 10^6$</td>
<td>0.33</td>
<td>0.35</td>
<td>1945-2007</td>
</tr>
<tr>
<td>Poland</td>
<td>5%</td>
<td>14</td>
<td>$135 \times 10^6$</td>
<td>0.34</td>
<td>0.32</td>
<td>1991-2007</td>
</tr>
<tr>
<td>Sweden</td>
<td>4%*</td>
<td>9</td>
<td>$52.5 \times 10^6$</td>
<td>0.39</td>
<td>0.44</td>
<td>1948-2006</td>
</tr>
<tr>
<td>Austria</td>
<td>4%</td>
<td>18</td>
<td>$47 \times 10^6$</td>
<td>0.36</td>
<td>0.44</td>
<td>1945-2008</td>
</tr>
</tbody>
</table>


Notes: * Corresponding to $M_{\text{eff}} = 18$. Yet, Sweden has a very small number of national compensation mandates (39 out of the total 349) and in most districts $M < 18$. The 4% threshold applies nationally (disregarding the alternative 12% district level threshold that has never been put into practice). Hence, our estimate of $M_{\text{eff}}$ is based on mean $M$ with an upper constraint of $M = 18 = (75\% / 4\%) - 1$. 

Figure 8. The largest seat-share versus the product of population and district magnitude cubed, approximations for complex electoral systems included.