Lawrence Berkeley National Laboratory

Recent Work

Title

RADIATIVE CORRECTIONS TO MUON-ELECTRON UNIVERSALITY IN A GAUGE THEORY OF CHARGED PION DECAY

Permalink

<https://escholarship.org/uc/item/8z51138q>

Author

Goldman, Terrence J.

Publication Date

1975-11-01

Submitted to Physics Letters B

LBL-4618 c.
LA-UR-75-2301
Preprint

RADIATIVE CORRECTIONS TO MUON-ELECTRON UNIVERSALITY
IN A GAUGE THEORY OF CHARGED PION DECAY

Terrence J. Goldman and Warren J. Wilson

November 19, 1975

For Reference

Not to be taken from this room

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

IN A GAUGE THEORY OF CHARGED PION DECAY*

November 19, 1975

One of the most important principles in lepton physics is muon electron universality--except for their differing masses, the electron and the muon appear identical. An important test of this principle is the measurement of the electronic branching ratio in charged pion decay. In a recent paper Bryman and Picciotto (1) have recomputed the branching ratio

* Supported by the U. S. Energy Research and Development Administration.

In order to remove this ambiguity we will attempt to extend the $SU(2) \times U(1)$ model of weak and electromagnetic interactions invented by Weinberg (5) and Salam (6) (WS) to include the pions. In such gauge theories the renormalization procedure is well defined, with no arbitrariness. We also get the additional bonus that the lowest order

* A more precise measurement of R will soon be undertaken at the Tri-University Meson Facility, Vancouver, Canada.

μ -e universality is a natural consequence of the gauge symmetry and need not be put in by hand.

We consider two points of view in treating the pions. First, we can consider the pions as truly elementary particles which are explicitly contained in the Lagrangian. This is the most straightforward approach. Second, we can interpret the pions as composite particles; only their constituents (e.g., quarks) appear in the Lagrangian. This method is not as simple since we also require the proper description of the pion in terms of its constituents, but it may be closer to reality. We will now briefly describe our investigation into the first approach. The second approach will be described elsewhere.

The Model.

Our method of calculating radiative corrections will closely parallel the method used by Appelquist, Primack, and Quinn (7) (APQ) in calculating radiative corrections to W decay. We begin by writing down our starting Lagrangian:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} (\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g \vec{A}_\mu \times \vec{A}_\nu)^2 - \frac{1}{2} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\ & + \sum_\ell \bar{L}_\ell (i\not{\partial} + \frac{1}{2} g \vec{\tau} \cdot \vec{A} + \frac{1}{2} g' \not{B}) L_\ell + \sum_\ell \bar{R}_\ell (i\not{\partial} + g' \not{B}) R_\ell \\ & + \frac{1}{2} |(\partial_\mu + \frac{1}{2} i g \vec{\tau} \cdot \vec{A}_\mu - \frac{1}{2} g' B_\mu) \phi|^2 - \sum_\ell G_\ell (\bar{L}_\ell \phi R_\ell + \bar{R}_\ell \phi^\dagger L_\ell) \\ & + \frac{1}{2} |(\partial_\mu + \frac{1}{2} i g \vec{\tau} \cdot \vec{A}_\mu - \frac{1}{2} i g' B_\mu) \Sigma|^2 - V(\Sigma, \phi) \end{aligned}$$

The first three lines of this expression is just the WS Lagrangian written in the notation of APQ. We have taken the liberty of redefining the ϕ field as

$$\phi = \begin{pmatrix} i\sqrt{2} \eta_+ \\ \phi - i\eta_0 \end{pmatrix}$$

with ϕ and η_0 real and η_+ complex. The Σ field is defined to be

$$\Sigma = \begin{pmatrix} i\sqrt{2} \tau_+ \\ \sigma - i\tau_0 \end{pmatrix}$$

with σ and τ_0 real and τ_+ complex. The τ 's will later be associated with the pions and σ is an additional scalar field.

$V(\Sigma, \phi)$ is the "potential" in our model and includes all possible $\Sigma - \phi$ interactions which are $SU(2) \times U(1)$ symmetric and at most quartic in the fields. We write it as

$$\begin{aligned} V(\Sigma, \phi) = & -A(\Sigma^\dagger \Sigma) + B(\Sigma^\dagger \Sigma)^2 - C(\phi^\dagger \phi) + D(\phi^\dagger \phi)^2 + E(\Sigma^\dagger \Sigma)(\phi^\dagger \phi) \\ & - F \left(\frac{\Sigma^\dagger \phi + \phi^\dagger \Sigma}{2} \right)^2 + H \left(\frac{\Sigma^\dagger \phi - \phi^\dagger \Sigma}{2} \right)^2. \end{aligned}$$

Notice that we have eliminated a possible $\Sigma^\dagger \phi + \phi^\dagger \Sigma$ term by requiring the potential to have an additional symmetry $\Sigma \leftrightarrow -\Sigma$. To insure stability we require B and D to be positive and $E - F > -2BD$.

We now find the vacuum expectation values of ϕ and σ by minimizing V . The minimum occurs at

$$\phi_0^2 = \frac{2BC - A(E - F)}{4BD - (E - F)^2}$$

$$\sigma_0^2 = \frac{2AD - C(E - F)}{4BD - (E - F)^2}$$

After shifting the ϕ and σ fields by these amounts we find a $\eta - \tau$ mass mixing term in the Lagrangian which we can remove by a rotation in $\eta - \tau$ space. The particles with definite masses are just

$$\eta' = \eta \cos \gamma + \tau \sin \gamma$$

$$\pi = -\eta \sin \gamma + \tau \cos \gamma$$

where $\tan \gamma = \sigma_0/\phi_0$. The masses of these new states are

$$m_{\eta'}^2 = 0$$

$$m_{\pi^\pm}^2 = 2F(\sigma_0^2 + \phi_0^2)$$

$$m_{\pi_0}^2 = 2(F + H)(\sigma_0^2 + \phi_0^2)$$

We notice that the factor H just represents an isospin breaking in the pion mass spectrum, and we henceforth set $H = 0$ for convenience.

We perform our calculation in the U-gauge formalism, as in APQ. In this gauge the massless fields η' are absorbed into the definitions of the massive vector fields and do not appear in the

Lagrangian. The Lagrangian then becomes

$$\begin{aligned} \mathcal{L} = & \text{kinetic terms} - F(\sigma_0^2 + \phi_0^2)\pi^2 - 4B\sigma_0^2\sigma^2 - 4(E - F)\sigma_0\phi_0\sigma\phi \\ & - 4D\phi_0^2\phi^2 + \frac{1}{4}g^2(\sigma_0^2 + \phi_0^2)|W_\mu|^2 + \frac{1}{8}\frac{g^2}{\cos^2\theta}(\sigma_0^2 + \phi_0^2)Z_\mu^2 \\ & - \sum_\ell G_\ell\phi_0\bar{\ell}\ell - \sum_\ell G_\ell\phi\bar{\ell}\ell \\ & + \sum_\ell \frac{iG_\ell \sin \gamma}{\sqrt{2}} \left\{ \bar{\nu}_\ell(1 - \gamma_5)\ell\pi_+ + \sqrt{2}\bar{\ell}\gamma_5\ell\pi_0 - \bar{\ell}(1 + \gamma_5)\nu_\ell\pi_- \right\} \\ & + \text{additional interaction terms} \end{aligned}$$

where we have introduced the Weinberg angle, θ . Among the additional interactions are the usual pion and lepton electromagnetic couplings. However, there are no pion-vector couplings like $\partial_\mu\pi V^\mu$ or $\pi V_\mu V^\mu$. The Lagrangian also contains a σ, ϕ mixing term proportional to $E - F$. As we will see below, F is a very small parameter, so that if E is also small then the mixing is exceedingly small; and we will use the approximation that $E - F = 0$ in subsequent calculations. We can now read off the masses of the remaining particles directly from the Lagrangian:

$$m_W^2 = \frac{1}{4}g^2(\sigma_0^2 + \phi_0^2) = \cos^2\theta M_Z^2$$

$$m_\chi = \phi_0 G_\ell$$

$$m_\sigma^2 = 8B\sigma_0^2$$

$$m_\phi^2 = 8D\phi_0^2$$

The most important term in the Lagrangian for our considerations is the fourth line: In our theory there is a direct, nonderivative coupling between the pions and leptons of strength $G_L \sin \gamma / \sqrt{2}$. By equating this coupling with the known strength of pion decay we find that $\sin^2 \gamma = O(G_F m_\pi^2)$. In addition, by relating the pion mass and the W mass we find that $F = G_F m_\pi^2$. Thus we see that the $\eta - \tau$ mixing and the pion mass are indeed both weak effects, i.e., if G_F were zero they would both vanish.

One-loop Calculation

We are now able to calculate radiative corrections in our model. While doing so we, of course, encounter the typical ultra-violet (U.V.) divergences which must be renormalized away. In a gauge theory, however, the renormalization constants (or counter-terms) are not all independent. In our case, if we define the $\mu\nu$ counterterm to be

$$(Z_S - 1) \frac{iG_L \sin \gamma}{\sqrt{2}} \bar{\nu}'(1 - \gamma_5)\mu\pi^+$$

then the counterterm for $\mu\nu$ must necessarily be

$$\left(Z_S \frac{Z_{Ge}}{Z_{Gu}} \left(\frac{Z_{\nu} Z_{\mu L}}{Z_{\nu'} Z_{eL}} \right)^{\frac{1}{2}} - 1 \right) \frac{iG_e \sin \gamma}{\sqrt{2}} \bar{\nu}(1 - \gamma_5)e\pi^+$$

where $Z_{\mu L}$, $Z_{\nu'}$, Z_{eL} , and Z_{ν} are wave function renormalization constants for the left-handed muon, the muon neutrino, the left-hand electron, and the electron neutrino, respectively. The Z_{GL} are essentially the mass renormalization constants for the leptons. Except for these known factors the single factor, Z_S , must be sufficient to render both the muon and electron integrals finite.

To check this point, we have calculated the corrections due to one loop graphs. On performing the integrals we find three types of divergences: quadratic and logarithmic divergences which are independent of the lepton mass, and, in addition, logarithmic divergences proportional to m_ℓ^2/M_W^2 . The first two types are clearly the same for both decay processes and cause no problem. We have also checked that the third type of divergences exactly cancel when combined according to the recipe given above. To put it another way: the combination $Z_S - Z_{Gu} - \frac{1}{2} Z_{\nu'} + \frac{1}{2} Z_{\mu L}$ contains no U.V. divergences which depend on the muon mass. Having successfully removed the U.V. divergent pieces, we can now find the universality breakdown in the remaining U.V. finite parts of the integrals.

We divide the graphs into two classes--those with a virtual photon and those without. The graphs with no photon in them can be separated into a part independent of the lepton mass and a part of order $\alpha m_\ell^2/M_W^2$. They make a negligible contribution to universality breakdown. The only significant contribution comes from the U.V.

finite parts of graphs involving photons. Thus, by considering only graphs with virtual photons, we only make errors of order $\alpha m_\ell^2/M_W^2$ (or $G_F m_\ell^2$) provided we use the same cutoff for both decay modes--by neglecting higher order weak effects we make no significant error. Furthermore, Kinoshita (4) and others have pointed out that to lowest order in the weak interactions, a derivative coupling model and a scalar coupling are equivalent to all orders in α . Thus, ^{+ O($\alpha m_\ell^2/M_W^2$)} we have shown that our model leads to the same predictions for R as the previous models with the additional requirement that the two cutoffs

be the same. We have removed the ambiguity in the original calculations; however, we have also reaffirmed the apparent two standard deviation discrepancy between theory and experiment.

Discussion

Since we have calculated only one loop graphs we have not taken into account any strong interaction corrections. This defect we have in common with the previous work of Berman and Kinoshita. In effect, we have ignored any structure which the pion might have and which might make an appreciable contribution to R . To rectify this we might begin to calculate the contributions of two loops, three loops, and so on. However, we would then become involved in doing perturbation theory with a large coupling constant--a fruitless endeavor. To get some idea of the effect of strong interactions we have decided to follow an alternative course and to consider the pion as a composite particle. This procedure leads us to an estimate for R which is significantly different from the result derived here and is closer to the new experimental number.^[1] A discussion of this calculation and a more detailed account of the calculation of this paper will appear in a future publication.

REFERENCES

1. D. Bryman and C. Picciotto, A Revised Value for the $\pi \rightarrow e \nu$ Branching Ratio, TRIUMF-Victoria preprint (1975).
2. E. DiCapua et al., Phys. Rev. 133B (1964) 1333.
3. S. Berman, Phys. Rev. Lett. 1 (1958) 468.
4. T. Kinoshita, Phys. Rev. Lett. 2 (1959) 477.
5. S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; Phys. Rev. Lett. 27 (1971) 1688.
6. A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium, No. 8) ed. by N. Svartholm (Wiley, N.Y., 1969).
7. T. Appelquist, J. Primack, and H. Quinn, Phys. Rev. D7 (1973) 2998. This decay was also discussed by W. Marciano and A. Sirlin, Phys. Rev. D8 (1973) 3612.

0000440341

0 0 1 3 4 4 5 5 5 4 2

LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720