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Boundary integral calculations of scattered fields: Application to a spacecraft launcher

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The objective of this article is to illustrate the possibilities of the boundary integral method in the analysis of acoustic scattering by the structure of a space launcher. Such an analysis is important in situations where the sound field reaches critical levels at takeoff. In these circumstances an increase in amplitude associated with reflection and scattering must be estimated with precision. The method is based on integral representations of the wave field and only requires a surface mesh of the solid structure. The number of nodes of such surface meshes is much smaller than the 3-D finite difference or finite element equivalents. Another important advantage of the method is that it automatically incorporates the farfield radiation condition. This condition is only approximated in field methods. The calculations are performed with a code originally developed by Hamdi [Doctoral thesis, Université de Compiègne (1982)]. It is here validated for the scattering of a plane wave by a hard sphere. The convergence properties of the method are examined and are found to be acceptable for most practical purposes. Scattering by the Ariane IV fairings (the payload housing) is then specifically considered. It is found that the geometrical shape of the fairings causes local increases of the sound field. This tendency is confirmed by measurements performed on a $\frac{1}{20}$ model of the launcher. Effects of geometrical modifications are also discussed.

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INTRODUCTION

Modern space vehicles such as the Space Shuttle and spacecraft launchers such as Ariane rockets generate intense sound fields at liftoff and during the first instants of atmospheric flight. Acoustic loads induced on the structure and on the payload are particularly important when the vehicle is on the launch pad with a propulsion system operating at full power. Extreme acoustic loads are also generated in certain cases at a moderate elevation of the vehicle, corresponding to a strong interaction of the propulsive jets with the structure of the launch pad. The prediction of these loads constitutes an important technological problem. This prediction is commonly performed with semiempirical models based on spectral and spatial decompositions of the sound power radiated by the propulsion system. Simple concepts are used to describe the propagation from the noise sources and take into account ground effects and the lateral deflection of the jets exhausted by the rocket engines. Standard prediction methods and recommended practices are discussed in Ref. 1. A typical application to the Ariane I launcher is given in Ref. 2.

Theoretical calculations based on these concepts require a fair amount of experimental data and their accuracy is limited. As a consequence subscale testing is extensively used to obtain this experimental input, complement the theoretical evaluation of the acoustic loads, and tune the prediction codes.

One of the shortcomings of the prediction methods is that they do not account for the complicated wave reflections and diffractions on the launch stand and on the vehicle. The geometry is so complex that until recently it has discour-

aged any detailed analysis.

Recent progress in computational methods now allows a direct treatment of such problems. An example of this approach is provided by Buell³ in a study of the Space Shuttle ignition overpressure (IOP). The treatment relies on a three-dimensional finite difference simulation. A 3-D mesh comprising 47 671 cells is used and the time-dependent Euler equations are solved with the Godunov scheme. Field calculations of this type require a very large mesh and the solution accuracy strongly depends on the numerical dispersion and dissipation characteristics of the finite difference scheme.

Now if a linear analysis of the wave motion is acceptable, then most of these difficulties may be avoided with boundary integral methods (BIM). These methods are based on integral representations of the wave field and appear under various names in the technical literature, such as the boundary integral equation method (BIE), the Helmholtz integral equation method (HIE), and boundary element method (BEM). All these formulations rely on the solution of integral equations on the solid boundaries of the propagation domain. Thus, for a 3-D problem, the calculation only requires the discretization of 2-D solid surfaces. The number of nodes of the surface meshes involved is therefore much smaller than that of 3-D finite difference or finite element meshes required in field methods. It is also much easier to generate surface meshes. Another important advantage of the BIM is that they automatically incorporate radiation conditions. These conditions are only approximated in field methods.

The literature on the BIM is already extensive. Initial

applications of these methods to acoustic problems have been made by Chen and Schweikert,⁴ Chertock,⁵ Copley,⁶ Schenck,⁷ Burton and Miller,⁸ Kleinmann and Roach,⁹ Jones,¹⁰ and Fillipi.¹¹ Various integral formulations are proposed in these studies. It is also shown that for certain boundary integral equations the solution of the exterior problem is perturbed by the eigensolutions of the interior problem for the same geometry. This nonuniqueness problem is extensively discussed in these early studies. Improved formulations and numerical calculations have been presented by Meyer *et al.*,¹² Terai,¹³ Sayhi *et al.*,¹⁴ Koopmann and Benner,¹⁵ and Hamdi.¹⁶

Recent additions to the literature have been made by Tobocman,^{17,18} Schuster and Smith,¹⁹ Schuster,²⁰ and Seybert *et al.*²¹ References 17–20 deal with wave scattering problems, while most earlier work was concerned with radiation problems. While it is indicated in the recent literature (see, for example, the conclusion of Ref. 17) that BIM are well suited to calculations of wave scattering by complex structures, such applications have not been reported.

It is one of our objectives to illustrate this aspect and show that BIM may be used to analyze scattering by complex shapes and multibody configurations. Our study specifically concerns the scattering of incident acoustic waves by the structure of the Ariane IV space launcher. This analysis is important in situations where the sound field reaches critical levels. In these circumstances an increase of the field amplitudes associated with reflections and diffractions must be estimated with precision.

A detailed examination of the field in the vicinity of the launcher may be used to evaluate the acoustic impact of some modifications of the external shape of the fairings.

In this study, we use a boundary integral equation formulation originally developed by Hamdi.¹⁶ The calculations are performed with the REDSTAR code (REFlection and DIffraction on the STRucture of ARIane launchers). This code contains a boundary integral computational kernel also implemented by Hamdi.¹⁶

We begin by giving a brief theoretical formulation of the method (Sec. I). Limits associated with resources available on current computers are discussed in Sec. II. A surface mesh generation procedure based on CAD for "Computer Aided Design" software is described in Sec. III. Test calculations are examined in Sec. IV and acoustic wave scattering by the Ariane IV launcher is considered in Sec. V.

I. THEORETICAL BACKGROUND

This section gives a brief description of the boundary integral formulation used in the present study. Further details and an application to radiation problems may be found in Ref. 16.

Consider a diffracting object B bounded by a closed surface S (Fig. 1). Let p_i designate the incident acoustic field radiated by an ensemble of harmonic sources located outside S in the domain Ω . The angular frequency is ω and all waves have a common factor $\exp(-i\omega t)$. Let p_s represent the field scattered by the surface S ; then the total pressure field in the external domain Ω is

FIG. 1. Scattering problem geometry.

$$p_t = p_i + p_s. \quad (1)$$

If the surface S is rigid then, the acoustic velocity normal to S vanishes and the boundary condition on S is

$$\frac{\partial p_t}{\partial n} = 0, \quad \text{on } S. \quad (2)$$

The boundary value problem for the scattered field p_s is then defined by

$$\nabla^2 p_s + k^2 p_s = 0, \quad \text{in } \Omega, \quad (3)$$

$$\frac{\partial p_s}{\partial n} = -\frac{\partial p_i}{\partial n}, \quad \text{on } S, \quad (4)$$

$$\lim_{r \rightarrow \infty} r \left(\frac{\partial p_s}{\partial n} - ikp_s \right) = 0, \quad (5)$$

where $k = \omega/c$ is the wavenumber.

The field p_s is the solution of the Helmholtz equation in Ω , its normal derivative on S is fixed by (4), and it satisfies Sommerfeld's radiation condition at infinity.

This problem may be replaced by equivalent integral relations. Among the many possible representations we adopt a double-layer potential formulation. The scattered field is cast in the form

$$p_s(\mathbf{r}_M) = \int_S \mu(\mathbf{r}_P) \frac{\partial G(\mathbf{r}_M, \mathbf{r}_P)}{\partial n_P} dS_P, \quad (6)$$

where

$$G(\mathbf{r}_M, \mathbf{r}_P) = \frac{1}{4\pi} \frac{\exp(ik|\mathbf{r}_M - \mathbf{r}_P|)}{|\mathbf{r}_M - \mathbf{r}_P|} \quad (7)$$

is the free-space Green's function and $\mu(\mathbf{r}_P)$ is a *double-layer* potential.

It is a simple matter to show that expression (6) satisfies the Helmholtz equation (3) and the radiation condition (5).

Applying the boundary condition (4) then yields an integral equation for the potential μ :

$$-\frac{\partial p_i}{\partial n_M} = \int_S \mu(\mathbf{r}_P) \frac{\partial^2 G(\mathbf{r}_M, \mathbf{r}_P)}{\partial n_M \partial n_P} dS_P, \quad (8)$$

where M belongs to S and f designates the principal value of the surface integral. This principal value may be defined by the following limit:

$$\begin{aligned} & \int_S \mu(\mathbf{r}_P) \frac{\partial^2 G(\mathbf{r}_M, \mathbf{r}_P)}{\partial n_M \partial n_P} dS_P \\ &= \lim_{\substack{Q \rightarrow M \\ M \in S}} \frac{\partial}{\partial n_Q} \int_S \mu(\mathbf{r}_P) \frac{\partial G(\mathbf{r}_Q, \mathbf{r}_P)}{\partial n_P} dS_P. \end{aligned} \quad (9)$$

Equation (8) may be solved directly, but it involves the determination of principle value integrals. Various methods may be used to bypass these difficult calculations. In the method proposed by Hamdi,¹⁶ Eq. (8) is replaced by an equivalent variational formulation. The potential μ is obtained as the solution of the variational equation

$$\delta \mathfrak{L}(\mu) = 0, \quad (10)$$

where

$$\begin{aligned} \mathfrak{L}(\mu) = & \frac{1}{2} \int_S \int_S \{ k^2 (\mathbf{n}_M \cdot \mathbf{n}_P) \mu(\mathbf{r}_M) \mu(\mathbf{r}_P) [\mathbf{n}_M \times \nabla_M \mu(\mathbf{r}_M)] \\ & \cdot [\mathbf{n}_P \times \nabla_P \mu(\mathbf{r}_P)] \} G(\mathbf{r}_M, \mathbf{r}_P) dS_M dS_P \\ & - \int_S \frac{\partial p_i}{\partial n_M} \mu(\mathbf{r}_M) dS_M, \end{aligned} \quad (11)$$

where ∇_M and ∇_P are the gradient operators at points $\mathbf{r}_M$ and $\mathbf{r}_P$, respectively.

Once μ is determined, the scattered field $p_s(\mathbf{r}_M)$ in the exterior domain Ω may be obtained from Eq. (6).

At points belonging to S this expression must be replaced by

$$p_s(\mathbf{r}_P) = \frac{1}{2} \mu(\mathbf{r}_M) + \int_S \mu(\mathbf{r}_P) \frac{\partial G}{\partial n_P}(\mathbf{r}_M, \mathbf{r}_P) dS_P. \quad (12)$$

In practice, the search of μ is conducted in discrete form. A surface mesh of M triangular panels comprising N nodes is first defined and the potential μ is evaluated at these nodes. Second-order interpolating functions are used to approximate the integrands appearing in the variational principle (10) and (11).

The functional $\mathfrak{L}$ then takes the discrete form

$$\mathfrak{L}(\mu) = \frac{1}{2} \{\mu\}^T D \{\mu\} - \{\mu\}^T \{V\}, \quad (13)$$

where D is an $(N \times N)$ complex symmetric influence matrix and $\{\mu\}$ and $\{V\}$ are $(N \times 1)$ potential and source column vectors.

The potential $\{\mu\}$ that makes $\mathfrak{L}$ stationary is the solution of the linear system

$$D \{\mu\} = \{V\}. \quad (14)$$

The nodal values of the potential are obtained by solving this linear system of complex symmetric equations.

II. COMPUTATIONAL LIMITATIONS

The computational resources available on current computers set upper limits to the size of the problems that may be solved with the BIM described above.

A first limitation comes from the high-speed memory size. If MSIZE designates the maximum accessible size, then the number of nodes N in the surface mesh is approximately fixed by

$$N^2 \simeq \text{MSIZE}. \quad (15)$$

Now consider a diffracting solid and let A designate surface area. If h is the typical dimension of the triangular panels of the mesh, the total number of nodes N will be

$$N \simeq A/h^2. \quad (16)$$

Numerical experiments indicate that the panel charac-

teristic dimension h must be chosen less than or equal to a quarter of the wavelength

$$h \leq \lambda/4. \quad (17)$$

This sets a lower bound to the wavelength:

$$16A/(\text{MSIZE})^{1/2} \leq \lambda^2. \quad (18)$$

For instance, consider the Ariane IV fairings of radius $a = 2$ m and length $l = 8.6$ m, including a portion of the third stage of length $l' = 14.4$ m, and let $\text{MSIZE} \simeq 1\,440\,000$ (the maximum memory size available for the user on the CRAY-1 computer operating at ONERA). In this situation $\lambda \geq 2$ m, which corresponds to a frequency limitation of 170 Hz.

It is difficult to give a definite statement about the degree of precision of the results obtained. However, the test calculations performed for known configurations indicate that an acceptable accuracy is reached if condition (17) is satisfied. Finally, it is worth noting that large-scale calculation (more than 1000 nodes) requires the use of 64-bit words.

Another limitation to the problem size is set by the CPU time required. Test calculations carried out on the CRAY-1 computer indicate that this time is essentially determined by the node number N and by the number of points NP where the pressure field is evaluated:

$$\begin{aligned} T_{\text{CPU}} \simeq & 1.2 \times 10^{-7} N^3 + 5.4 \times 10^{-4} N^2 \\ & + 1.5 \times 10^{-3} \text{NP} \times N. \end{aligned} \quad (19)$$

If NP is moderately large and N is large it is obvious that the first term becomes rapidly dominant and that as a consequence the computation time increases as the sixth power of the frequency ($T_{\text{CPU}} \propto f^6$). The calculations pertaining to the Ariane IV launcher typically required the maximum memory size available and about 45 min of CPU time on the CRAY-1 computer.

III. SURFACE MESH GENERATION

A considerable amount of time is usually spent on the generation of the surface meshes. Specific procedures may be devised for simple geometrical configurations, but it is more interesting to develop general methods applicable to moderately complex 3-D configurations.

One approach that is particularly well suited to technical applications consists of using a general purpose computer-aided design (CAD) software to define the solid surface geometry and mesh this surface automatically. From the stored graphic model one then retrieves a data file containing the nodal coordinates and nodal connectivities (i.e., the couples of nodes that define the branches of the surface mesh). This procedure is schematically described in Fig. 2.

Further details are given in the remainder of this section. For simplicity we will only consider the case of an axisymmetric solid body.

The first operation is performed with the CAD software CATIA[®]. This operation consists of defining the exact geometry of the solid object by drawing this object on the graphic terminal. In the case of an axisymmetric body a meridional line is first determined and this curve is then rotated [Fig. 3(a)–(c)]. A polyhedral approximating the surface is then

FIG. 2. Surface mesh generation based on computer-aided design (CAD) software.

created with the POLYHEDR function of the CATIA system. The degree of approximation is fixed by an azimuthal and an axial parameter selected by the user. The polyhedral surface obtained is formed by a set of flat quadrilateral facets and the number of boundary edges at each level is constant [Fig. 3(d)].

The nodal points corresponding to the facets are then automatically generated by means of the CATIA function POINT. Node numbering is performed by the CAD system, which scans the polyhedral surface in the clockwise direction starting from the top [Fig. 3(e)]. Each nodal point is stored in memory in the form of several numerical blocks including an identification number and three spatial coordinates. The file containing this information is not directly accessible to the user.

The generation operation is terminated. The next step consists of retrieving the nodal information. This is achieved with the CATGEO system (see Fig. 2). This system manages the relations between CATIA and external applications. It is,

FIG. 3. Main operations performed in the mesh generation process.

Incident plane wave

FIG. 4. Plane-wave scattering by a rigid sphere.

for example, possible to create or scan CATIA graphic models by associating this *manager* with an external FORTRAN code IAMLECT. The output file obtained contains the nodal information and the nodal identifiers. Another external code IAMFICH then generates the connectivity matrix. The output file obtained contains all the necessary information required by an external application such as the boundary integral computer code REDSTAR. The operations described above are interactive and the user may step in the process, for example, to modify certain parts of the mesh.

IV. TEST CALCULATIONS

From among the many test calculations used to validate the REDSTAR code we will only examine the results obtained in the case of scattering of a plane wave by a rigid sphere (Fig. 4). This standard problem has a well-known analytic solution, which may be expressed as a series of spherical harmonics (see, for example, Junger and Feit²²). If the incident plane wave progresses in the positive x direction with an amplitude B ,

$$p_i = B \exp(ikr \cos \theta), \quad (20)$$

then the field scattered by the rigid sphere may be written in the form

$$p_s(r, \theta) = -B \sum_{n=0}^{\infty} (2n+1) i^n b_n h_n(kr) P_n(\cos \theta), \quad (21)$$

where $b_n = j'_n(ka)/h'_n(ka)$.

In these expressions, $h_n = j_n + iy_n$ designates the spherical Hankel function of order n ; P_n is the Legendre polynomial of order n ; and j'_n, y'_n are the derivatives of the n th-order spherical Bessel functions. This solution only depends on the dimensionless parameters ka and kr and on the polar angle θ .

FIG. 5. Surface mesh used to represent the sphere. The mesh comprises 80 triangular facets.

A recent article by Waterhouse²³ contains an extended set of solutions for this problem displayed as contour plots. These results will be used as a reference in the present test. In distinction with most of the previous work, the comparison will be conducted in the nearfield of the scattering object.

For the numerical calculations, the sphere is replaced by the mesh of 80 nearly equilateral triangles displayed in Fig. 5. With this mesh the criterion $h \leq \lambda/4$ is satisfied if $ka \leq 2$.

Exact and numerical solutions are displayed in Figs. 6 and 7. The normalized pressure modulus $|p_i|/|p_i|$ is plotted as constant contour lines for $ka = 2$ and 6. Thick contours correspond to a value of 1 and the difference between successive contours is 0.1.

For $ka = 2$, the numerical and analytic solutions closely agree. Accurate results are obtained if $h \leq \lambda/4$. For $ka = 6$, the field distributions retain a similar aspect, but there are also some notable differences. In this case, the typical mesh size h is of the order of the wavelength λ and accuracy of the numerical solution is not assured. (Note, however, that the wiggles appearing in the contours near $r/a = 10$ are only due to the contour plotting routine operating over an undersampled data set.)

FIG. 6. Diffraction of a plane wave by a hard sphere. The incident wave propagates from left to right and its wave vector is parallel to the x axis. The normalized pressure amplitude $|p_i|/|p_i|$ is displayed in the form of constant contour lines. Thick contours correspond to a value of 1. The increment between successive contours is 0.1, for $ka = 2$. (a) Numerical results and (b) analytic results reproduced from Ref. 23 (by permission of R. V. Waterhouse).

FIG. 7. Same legend as in Fig. 6, for $ka = 6$. (a) Numerical result and (b) analytic results reproduced from Ref. 23 (by permission of R. V. Waterhouse).

In the immediate vicinity of the sphere, the numerical evaluation of the field produces unreliable results for all values of ka . This is so because the integral appearing in expression (6) becomes singular when the field point r_M approaches the scattering surface S . This difficulty only arises in a thin *boundary layer* adjacent to the body. To obtain accurate values of the field in this layer it is, for example, possible to extrapolate these values from the field estimates obtained outside this region.

It is now worth examining the convergence properties of the BIM used in this study. To this purpose let us again consider the scattering of plane acoustic waves by a rigid sphere.

To estimate the numerical error it is convenient and also more appropriate to compare the calculated and exact scattered fields p_s^n and p_s^a . The exact scattered field p_s^a is obtained by summing 90 terms of the infinite spherical harmonic series appearing in expression (21). Figure 8 shows a typical plot of the exact amplitude of this field corresponding to $ka = 2$. Thick contours represent an amplitude of 0.5 and the difference between successive contours is 0.1. The maximum amplitude equals 1.18 and is reached at $r/a = 1$ and $\theta = 0$. When the incident plane wave p_i is added to p_s^a , the results of Waterhouse²³ are retrieved precisely.

Exact and numerical scattered fields are now compared for different values of the reduced wavenumber $ka = 1.5, 2, 3, 4$, and 4.4934 . The computational mesh is fixed and comprises 80 triangles (as shown in Fig. 5). The characteristic size of each panel is $h = 0.524 a$. The relative error defined by

$$e = \sum_{ij} |p_s^a(r_i, \theta_j) - p_s^n(r_i, \theta_j)|^2 / \sum_{ij} |p_s^a(r_i, \theta_j)|^2 \quad (22)$$

is calculated by taking 96 equidistributed values of r_i in the range $(1.25a, 5a)$ and 51 values of θ_j in the range $(0, \pi)$.

FIG. 8. The exact scattered pressure amplitude $|p_s^e|$ is displayed in the form of constant contour lines. Thick contours correspond to a value of 0.5, the increment between successive contours is 0.1, and $ka = 2$.

Figure 9 indicates that the relative error decreases rapidly with the ratio h/λ . When h/λ is less than $\frac{1}{6}$ the error falls below 9% and the accuracy is sufficient for most engineering purposes.

Another evaluation of the rate of convergence of the method consists of changing the mesh size while keeping ka fixed. This is done in Fig. 10 for $ka = 2$ and for six different mesh sizes corresponding to $\lambda/h = 3, 6, 9, 12, 16$, and 24. The relative error decreases when h decreases (i.e., when the number of nodes increases). However, it is also found that the error cannot be made infinitely small by augmenting the number of nodes because round-off errors become non-negligible and also because the precision of the linear elements used in the formulation becomes insufficient. This second aspect may be improved if the linear surface elements are replaced by quadratic elements.

As a final point let us consider the evolution of the relative error with the radial distance. The reduced wavenumber is fixed at $ka = 2$ and the error is calculated for five mesh sizes and five values of the radius $r/a = 1.2, 2, 3, 4$, and 5. For a given mesh the error remains essentially constant (Fig. 11). This indicates that the choice of the mesh size essentially determines the accuracy.

Another difficulty which is classically encountered in BIM is the nonuniqueness of the solution at certain *irregular* frequencies. At these frequencies, the interior Helmholtz problem has a standing wave solution consistent with the boundary conditions imposed on the surface of the scatterer. This well-known difficulty is extensively studied in the literature.^{7,8} It may be overcome in various ways. One method⁷

FIG. 9. Relative quadratic error calculated on an area for a fixed mesh size ($h = 0.524a$) and for different reduced wavenumbers ka .

FIG. 10. Relative quadratic error calculated on an area for a fixed reduced wavenumber ($ka = 2$) and for different mesh size.

consists of complementing the BIM with a few null field relations and stating that field vanishes inside the scattering body. This method,⁷ which usually eliminates the non-uniqueness problem, is not implemented in the present code.

V. SCATTERING BY THE ARIANE IV STRUCTURE

We now describe some typical results of calculations relating to the Ariane IV launcher. We will first consider the complete launcher and then examine in some detail the field structure in the vicinity of the equipment bay and fairings of the rocket. In all calculations it is assumed that the outer structure of the launcher is acoustically rigid.

A. Geometrical configuration

The geometry of the Ariane IV launcher is shown in Fig. 12; the definitions of the different elements of this vehicle and a plan view of the launch pad and gas trenches are also given. Among the different types of fairings that may be mounted on the launcher we will only consider the short version corresponding to a single payload. In this case, the fairings length is 8.6 m.

For a frequency f less than or equal to 63 Hz, it is possible to discretize the complete launcher, including the four lateral boosters, and obtain a mesh that satisfies condition (17). Beyond this frequency the analysis must be restricted to a portion of the launcher. The most sensitive ensemble is

FIG. 11. Relative quadratic error calculated on a circle as a function of the radius and plotted for different mesh sizes: $\diamond$, $h/\lambda = \frac{1}{3}$; $\times$, $h/\lambda = \frac{1}{6}$; $\blacksquare$, $h/\lambda = \frac{1}{12}$; $\bullet$, $h/\lambda = \frac{1}{16}$; $\blacktriangledown$, $h/\lambda = \frac{1}{24}$.

FIG. 12. Geometry of the Ariane IV launcher and schematic representation of the launch pad and gas trenches.

the fairings and equipment bay. To perform calculations of the field in the vicinity of these elements it is necessary to represent a portion of the third stage. The mesh used in the calculations is displayed in Fig. 13(a). A comparison between results obtained at $f = 63$ Hz for the complete launcher and for the upper part indicates that the truncation effects only weakly perturb the sound field near the fairings if the length of the third stage included in the calculation exceeds 5 m. The length of 14.6 m used here appears sufficient to avoid these effects.

Because the geometry of the equipment bay base plate has a notable influence on the sound field, we will also consider a modified version of this element. In this version the base plate is replaced by a toric panel, which assures a smooth transition between the fairings and the third stage. The mesh corresponding to this modified configuration is shown in Fig. 13(b). The mesh used in the complete launcher calculation is displayed in Fig. 13(c).

B. Results of calculations for the complete launcher

The results presented here correspond to a frequency $f = 63$ Hz. For this frequency, the Strouhal number based on the nozzle diameter and jet exhaust velocity is $St = fd / u_j \approx 0.025$. This number is below that corresponding to the maximum of the spectral density of the acoustic power radiated by the launcher ($St_{\max} \approx 0.1$). From model scale experiments it is known that noise at this frequency is mainly radiated from a point located at about 40 nozzle diameters. When the rocket is on the launch pad, the noise sources corresponding to $f = 63$ Hz are located at $x_s \approx 26.5$ m,

$y_s \approx 11$ m, and $z_s \approx -7$ m and $x_s \approx -21.7$ m, $y_s \approx 17.2$ m, and $z_s \approx -7$ m. We only consider the first source and we use a point source to represent the noise radiation. This is justified since we are not trying to predict the acoustic environment of the launcher, but only wish to examine scattering effects associated with the presence of the vehicle in its own sound field. For this study, it is also natural to normalize the total pressure field $p_t = p_i + p_s$ by the incident field p_i . The amplitude of the reduced pressure field is then evaluated in decibels, thus yielding a *reflection-diffraction index*:

$$RDI(\text{dB}) = 20 \log_{10} |p_t| / |p_i|. \quad (23)$$

This index is plotted in Figs. 14 and 15 on a ten-level grey scale. The darkest symbol of this scale corresponds to the largest values of the RDI, while the lightest symbol represents the lowest values of this index.

Figure 14 shows the distribution of the RDI in the $\phi = 45^\circ$ vertical plane passing through the launcher axis and containing the axis of the P1 and P3 lateral boosters. To obtain this plot the pressure field was calculated at $51 \times 303 = 15\,453$ points of this plane.

The RDI takes large values on the right of the launcher. This side faces the source. These values of the RDI result from the constructive interference of the direct and scattered acoustic fields. A shadow region is formed on the left of the launcher.

Another view of the pressure field is obtained by plotting the RDI in a horizontal plane (Fig. 15). This plane, located at $z = 8$ m, cuts the lateral boosters near their middle. The diffraction index reaches local maximums on the booster P1, which directly faces the noise source, and in the regions situated between the central body and boosters P2 and P4. The shadow region formed behind the vehicle is also well defined.

FIG. 13. Surface meshes used to represent the upper part of the Ariane IV launcher and the complete launcher. (a) Standard configuration (short fairings), (b) modified configuration, and (c) complete launcher.

C. Results of calculations for the launcher fairings

We now consider the acoustic environment of the rocket fairings. We will only examine results obtained at a frequency $f = 125$ Hz, which corresponds to a Strouhal number $St \approx 0.05$. This number is close to that corresponding to the maximum radiated power. When the launcher is on the ground this frequency is mainly radiated by points located in the gas trenches at a radial distance $r_s \approx 29$ m and in a plane $z_s \approx -7$ m.

The calculations are now performed with a truncated third stage, as indicated in the beginning of this section. Figure 16 displays the field structure in a vertical plane containing the sound source. The diffraction index takes large values on the source side and a shadow region is formed on the other side of the fairings. A strong diffracted wave is produced in the region of the base. This wave interferes with the

FIG. 14. Field structure in the vicinity of the Ariane IV launcher. The plot corresponds to the vertical plane $\phi = 45^\circ$. This plane contains the axis of the main body and of the lateral boosters P1 and P3. The reflection-diffraction index $RDI = 20 \log_{10} |p_r| / |p_i|$ is displayed on a scale of grey levels. Frequency $f = 63$ Hz. Source location: $x_s = 26$ m, $y_s = 11$ m, and $z_s = -7$ m.

FIG. 15. Field structure in the vicinity of the Ariane IV launcher. The plot corresponds to the horizontal plane $z = 8$ m. Other captions are similar to those of Fig. 10.

FIG. 16. Field structure in the vicinity of the fairings of the Ariane IV launcher. The plot corresponds to an axial plane containing the source. Frequency $f = 125$ Hz. Source location: $r_s = 29$ m and $z_s = -7$ m.

incident field and local maximums are formed below the equipment bay and on the upper part of the third stage.

A detailed view of the field in the vicinity of the base plate is given in Fig. 17. The extrapolation method described in Sec. IV was used to calculate the field in the thin surface layer, where the direct evaluation of the field breaks down (at points located at less than 0.1 m from the structure). The diffraction index is about 4 dB along the fairings and exceeds 6 dB, reaching 8 dB near the base and on the top portion of the third stage. The acoustic loads on the base plate are significantly more intense than those imposed to the fairings.

FIG. 17. Detailed view of the equipment bay neighborhood. Same conditions as in Fig. 12.

For acoustic radiation at 125 Hz, the difference between the field amplitudes on the fairings and under the base is of the order of 4 dB. This difference becomes more important at higher frequencies because the sound sources get closer to the rocket axis. The sound waves emitted in that case are weakly diffracted by surfaces that are nearly parallel to their direction of propagation. On the other hand, diffraction by the base remains important.

Large pressure field differences are indeed observed in model scale experiments performed on a $\frac{1}{20}$ model of the Ariane IV launcher. In these experiments, the model is equipped with a propulsion system that closely simulates the real motors and the main geometrical characteristics of the launch pad are represented. The model rocket is fixed in position at various heights above the ground to simulate the successive elevations of the launcher during liftoff. Acoustic pressure data are obtained from $\frac{1}{4}$ -in. microphones flush mounted on the pad and on the vehicle. Further information on the experimental procedure and additional data may be found in Ref. 24.

To allow a qualitative comparison with the calculations we have plotted in Fig. 18 some of the experimental data obtained for different elevations of the launcher and a single propulsion configuration.

The amplitude of the sound field below the equipment bay is found to exceed the general level observed on the fairings and on the third stage by 5–6 dB.

Another interesting view of the sound field structure near the base is shown in Fig. 19. The plot corresponds to a plane perpendicular to the rocket axis located at a close distance from the base of the equipment bay. Figure 19 indicates that the field is enhanced on the incidence side and in the two quadrants adjacent to this direction. The extension of the region of increased diffraction index is strongly influenced by the presence of the base plate.

FIG. 18. Sound pressure level variations on the upper part of the Ariane IV launcher. These results are obtained from experiments performed on a $\frac{1}{20}$ scale model of the rocket at three elevations corresponding to the launcher altitudes $z_{alt} = 0, 13.5$, and 15 m.

FIG. 19. Field structure in the vicinity of the base of the equipment bay. The plot corresponds to a horizontal plane located at $z = 44.52$ m at a distance of 0.3 m from the base of the equipment bay. Same conditions as in Fig. 12.

From a technical point of view it is interesting to see if a modification of the geometry of the base may induce a decrease of the field amplitude near the equipment bay. The modification considered consists of placing an inclined torus below the base and in this way assuring a smooth transition between the fairings and the third stage. Figure 20 shows the RDI distribution corresponding to this new geometry. The general structure of the sound field is not profoundly modified in this configuration. The most important changes are observed near the transition element. The diffracted wave that was generated in the base region has also nearly vanished.

FIG. 20. Field structure in the vicinity of a modified version of the fairings of the Ariane IV launcher. Same conditions as in Fig. 12.

FIG. 21. Field structure in the vicinity of a modified version of the fairings of the Ariane IV launcher. The plot corresponds to a horizontal plane located at $z = 44.52$ m. Same conditions as in Fig. 12.

A view of the field structure plotted in the same horizontal plane as Fig. 15 shows some significant changes in the RDI distribution (Fig. 21). The maximum of this index is about 4 dB. On the shadow side, the RDI index passes below -3 dB.

These results indicate that the modified geometry leads to a 3-dB decrease of the sound amplitude in the vicinity of the equipment bay.

VI. CONCLUSION

The results presented in this article illustrate the possibilities of the BIM in the analysis of acoustic scattering by complicated structures. At present the main limitations of the computational method used are related to the maximum high-speed memory space available in the computer and to the CPU time necessary for executing the calculation. Despite these limitations some important technological problems may be solved with this method. In particular, this method has been used to evaluate the effect of the launcher Ariane IV on its own sound field and obtain a better estimate of the acoustic loads imposed on the structure.

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