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MULTIPLE NUCLEON TRANSFER IN DAMPED NUCLEAR COLLISIONS*

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Abstract

This lecture discusses a theory for the transport of mass, charge, linear and angular momentum and energy in damped nuclear collisions, as induced by multiple transfer of individual nucleons.

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1. Introduction

Modern accelerator technology has presented us with intense beams of heavy nuclei with energies of several MeV per nucleon. Such beams offer a powerful and flexible tool for studying large-scale nuclear dynamics. One of the central aspects in this undertaking is the interplay between nuclear dynamics and nuclear structure. How does the specific nuclear structure affect the nuclear dynamics? Or, conversely: How can nuclear collisions be used to probe new aspects of nuclear structure? These questions are now receiving increased attention and it is therefore most appropriate at the present time to devote a school to the theme: nuclear structure and heavy-ion collisions.

As a first step towards understanding the role of nuclear structure in nuclear dynamics one might consider the question: How would nuclei behave if there were no special structure effects? The underlying motivation for this is the wish to establish a general reference dynamics against which to interpret the experimental data on actual nuclei and relative to which the nuclear-structure effects can be discussed in an instructive manner.

The philosophy is here very similar to the one associated with the macroscopic ("liquid-drop") model of nuclear statics. We do of course not believe that the nuclear properties vary smoothly as a function of particle number, deformation etc. But nevertheless the actual nuclear properties do exhibit regularities and systematic trends which can be understood on rather general grounds without reference to the specific structure of the individual nuclei. The identification of this smooth background part would enable us to understand the gross features of the experimental data. Moreover, by being able to eliminate the macroscopic part from our considerations we could enhance the features arising from the specific microscope structure.

When two heavy nuclei collide with energies of several MeV per nucleon a large fraction of their initial translational energy is converted into intrinsic excitation (hence the term "damped collisions"). However, although strongly perturbed and highly excited, the two final fragments do show a large resemblance with the initial nuclei; this fact indicates that a binary configuration has been maintained throughout the entire collision process. The dynamics of the intermediate complex can therefore be discussed in terms of the degrees of freedom associated with a dinuclear system.

Although the final fragments resemble the initial nuclei they are by no means identical to them: typical mass and charge widths amount to several units. This implies that a substantial number of nucleons are transferred in such a collision. The transfer of a nucleon is generally associated with a dissipation of energy and momentum and in fact simple estimates suggest that this mechanism is an important, if not dominant, agency for the damping of the dinuclear motion.

In this lecture I shall indicate how the effects of nucleon transfer in nuclear collisions may be explored in a simple model. The aim is to obtain an impression of the general features associated with the transfer-induced transport in nuclear collisions. It should be emphasized from the outset that although we focus our attention on the transfer mechanism we do not wish to preclude the coexistence of other important mechanisms, such as the excitation of various collective modes in the nuclei. In fact, the dynamical interplay between the different coexisting mechanisms forms a fascinating subject for future study.

2. The Model

In the present discussion we take advantage of the approximate validity of the independent-particle model of nuclei. We thus assume that the nucleons move nearly independently in the nuclear one-body mean field. The occupation probabilities of the single-particle states in the nucleus are then taken to be as in Fermi-Dirac gases.

$$\begin{aligned} f^A(\varepsilon_a) &= (1 + e^{(\varepsilon_a-\varepsilon_A)/\tau})^{-1} \\ f^B(\varepsilon_b) &= (1 + e^{(\varepsilon_b-\varepsilon_B)/\tau})^{-1}. \end{aligned} \tag{1}$$

For simplicity we consider here only one type of nucleon - the generalization is straightforward. In (1) ε_a is the energy of the nucleon in the projectile-like nucleus A and ε_b is the energy of the nucleon in the target-like nucleus B; the corresponding Fermi energies are denoted by ε_A and ε_B. The above assumption need not be good at the very early stages of the collision but this is less important. At the time when good communication is established between the two nucleides they are typically excited by several MeV and (1) appears a reasonable description. Furthermore, due to the good communication, we assume that the (time-dependent) temperature τ is the same in the two collision partners.

The two nuclei move with velocities $\vec{U}_A$ and $\vec{U}_B$. These velocities refer to those parts of the nuclei which are in the interaction zone rather than to the nuclear centers.

When the communication between A and B has been established the transfer of nucleons is possible. In line with the mean-field independent-particle description we shall assume that the nucleons are transferred individually rather than as correlated clusters (although we do not wish to preclude that such transfers might also occur and even play an important role). The driving force for the nucleon transfer arises partly from the difference in the Fermi levels, $F = \varepsilon_B - \varepsilon_A$, and partly from the relative velocity of the two gases, $\vec{U} = \vec{U}_A - \vec{U}_B$. The transfer of a nucleon creates a one-particle one-hole type excitation of the intrinsic nuclear system. It can be shown[1)] that the energy of this exciton amounts to

$$\omega = F - \vec{U}\cdot\vec{p} \ . \tag{2}$$

Here $p = \frac{1}{2}(\vec{p}_a + \vec{p}_b)$ where $\vec{p}_a$ is the momentum of the nucleon relative to A and $\vec{p}_b$ is its momentum relative to B. It is an important assumption that once a nucleon has been transferred it is quickly accepted as an equal member of the recipient nucleus and no memory of its heritage remains. By this assumption the multiple transfers can be considered as markovian and the process can be treated by standard transport theory. The relevance of transport theory to damped nuclear collisions was first recognized by Nörenberg.[2)]

Consider for the moment a flat contact geometry with the area σ. The rates of transfer between the two nuclei are then given by

$$\dot{A}^{+} = \sigma \int \frac{d\vec{p}}{h^3} \left|\frac{v_z}{2}\right| \bar{f}^A(\vec{p}) f^B(\vec{p})$$
$$\dot{A}^{-} = \sigma \int \frac{dp}{h^3} \left|\frac{v_z}{2}\right| f^A(\vec{p}) \bar{f}^B(\vec{p}) \qquad (3)$$

for transfer into and out of A, respectively. Here the occupation factors f give the probability that a nucleon is initially present in the donor nucleus with the momentum $\vec{p}$ and the blocking factors $\bar{f} = 1 - f$ give the probability that such a state is available in the recipient nucleus. The transfer rate is proportional to the magnitude of the velocity in the direction normal to the contact surface as simple classical arguments would suggest.[3)]

The above expressions for the basic transfer rates from the core of the model. If the geometry of the interaction zone is gently curved the proper generalization of (3) can be accomplished by application of the proximity method.[4)] The ensuing formulas make it relatively simple to study the dynamical role of nucleon transfer in nuclear collisions. Such studies, based on direct numerical simulation, are presently under way.[5)]

For the general discussion of the transport problem it is convenient to reduce the master equation implied by (3) to its Fokker-Planck approximation. The characteristic quantities are then the transport coefficients, which govern the rate of change of the mean values of the macroscopic variables and their covariances. They can be determined by following the short-term evolution of a system which has been prepared with specified sharp values of the macroscopic variables. If we were to consider only the particle number A we would have

$$V_A\ (A) = \frac{d}{dt} A = \dot{A}^+ - \dot{A}^-$$
$$2D_{AA}(A) = \frac{d}{dt}\sigma_A{}^2 = \dot{A}^+ + \dot{A}^- \tag{4}$$

for the drift and diffusion coefficients, respectively, and the dynamical evolution of the probability distribution for a given mass partition, $P(A;t)$, would be governed by

$$\frac{\partial}{\partial t} P = - \frac{\partial}{\partial A} V_A P + \frac{\partial}{\partial A^2} D_{AA} P \ . \tag{5}$$

This only serves as a simple illustration. In reality we wish to consider the simultaneous evolution of several interrelated macroscopic variables.

3. Nearly Degenerate Limit

Until now, most of the experimental studies of damped nuclear collisions have been carried out at relatively low energies, with the nuclei meeting each other with kinetic energies of a few MeV per nucleon. Since this energy is small in comparison with the intrinsic kinetic energies of the nucleons the collective motion is relatively slow and the nuclei acquire only modest excitation. Under these circumstances only the nucleons near the Fermi surface take part in the exchange and the entire treatment simplifies considerably. In the following we shall specialize to this limit and thus assume

$$U << V_F, \quad F, \tau, \omega << T_F. \tag{6}$$

Then the occupation factors can be written in the approximate form[1)]

$$\bar{f}^A f^B \approx \iota(\omega)\ \delta(\varepsilon - \varepsilon_F)$$
$$f^A \bar{f}^B \approx \iota(-\omega)\ \delta(\varepsilon - \varepsilon_F) \tag{7}$$

where

$$\iota(\omega) = \omega(1 - e^{-\omega/\tau})^{-1}. \tag{8}$$

Due to the appearance of the δ-function in (7) the energy integration in (3) is trivial and only the directional integration remains. It is useful to introduce the flux-weighted directional average of a function $g(\Omega)$ by

$$\langle g\rangle \equiv \frac{1}{2\pi}\int d\Omega\ |\cos\Theta|\ g(\Omega) \tag{9}$$

where the polar axis is perpendicular to the interaction surface between A and B.[6)]

With these simplifications the transfer rates (3) reduce to

$$\dot{A}^{\pm} \approx N'(\varepsilon_F)\ \langle\ \iota(\pm\ \omega)\rangle_F \tag{10}$$

where the subscript F has been attached to the flux average to indicate that only the particles in the Fermi surface should be considered. The overall transfer rate is governed by the quantity $N'(\varepsilon_F)$ which is the differential one-body current of nucleons transferred at the Fermi surface:[4)]

$$N'(\varepsilon_F) = \frac{\partial}{\partial T_F} N(\varepsilon_F),\quad N(\varepsilon_F) = \frac{1}{4}\rho\bar{v}\sigma. \tag{11}$$

Here $\frac{1}{4}\rho\bar{v}$ is the one-way flux in standard nuclear matter. The expression for $N(\varepsilon_F)$ holds for a flat, fully open contact surface; in general the current $N(\varepsilon_F)$ depends sensitively on the geometry of the interaction zone and may be difficult to calculate. For a certain family of dinuclear configurations simple estimates can be obtained by use of the proximity method.[4)]

From the knowledge of the basic transfer rates it is straightforward to derive the appropriate expressions for the transport coefficients for a given set of macroscopic variables $\{\zeta\}$ (assumed to be additive such as e.g. the particle number A and the momentum $\vec{P}$). The drift coefficient vector $\vec{V}$ represents net rate of change of the variables, hence

$$\begin{aligned} V_\zeta &= N'(\varepsilon_F)\,\langle\,[\,\imath(\omega) - \imath(-\omega)]\,\zeta(\vec{p})\,\rangle_F \\ &= N'(\varepsilon_F)\,\langle\,\omega\,\zeta(\vec{p})\rangle_F\,. \end{aligned} \tag{12}$$

The corresponding diffusion coefficient tensor $\overleftrightarrow{D}$ represents the rate of increase in the covariances, hence

$$\begin{aligned} D_{\zeta_1\zeta_2} &= N'(\varepsilon_F)\left\langle\tfrac{1}{2}\{\,\imath(\omega)+\imath(-\omega)\}\zeta_1(\vec{p})\,\zeta_2(\vec{p})\right\rangle_F \\ &= N'(\varepsilon_F)\left\langle\tfrac{\omega}{2}\coth\left(\tfrac{\omega}{2\tau}\right)\zeta_1(\vec{p})\,\zeta_2(\vec{p})\right\rangle_F. \end{aligned} \tag{13}$$

These expressions appear immediately plausible when one considers the fact that the differential transition rate from B to A, at the energy ε, is given by $N'(\varepsilon)\,\bar{f}^A f^B$ and the rate for the opposite direction is given by $N'(\varepsilon)\,f^A \bar{f}^B$. (The situation corresponds to a random walk where the net gain is the difference in the number of steps taken and the variance is the total number of steps.) The cancellation of the blocking factors $\bar{f}$ in the expression for the drift coefficients V is a general reflection of the fact that this quantity can be represented in terms of one-body operators and hence is insensitive to correlations among the particles.

4. The Dinucleus

The expressions (12) and (13) have been written for arbitrary additive observables $\{\mathbb{C}\}$. Let us now consider the case of actual interest, the dinucleus. For simplicity we shall restrict our attention to the following variables

$$\mathbb{C} = \{ Z, N, P, \vec{L}, \vec{S}_A, \vec{S}_B \} \tag{14}$$

where Z and N are the proton and neutron numbers of the projectile-like partner A, P and $\vec{L}$ are the radial and angular momenta at the relative dinuclear motion, and $\vec{S}_A$ and $\vec{S}_B$ are the individual angular momenta carried by the dinuclear partners A and B.

It is simplest to consider the particle numbers Z and N. From (12) and (13) we obtain

$$\begin{aligned} V_Z &= N'_Z F_Z \quad , \quad V_N = N'_N F_N \\ D_{ZZ} &= N'_Z \tau^* \quad , \quad D_{NN} = N'_N \tau^* \end{aligned} \tag{15}$$

and $D_{ZN} = 0$. Here N'_Z and N'_N are the differential one-body currents of protons and neutrons, respectively, and $F_Z = -\partial\mathcal{H}/\partial Z$ and $F_N = -\partial\mathcal{H}/\partial N$ are the corresponding driving forces. The "effective temperature" τ^* is given by

$$\tau^* \equiv \langle \frac{\omega}{2} \coth \left(\frac{\omega}{2\tau}\right) \rangle_F \,. \tag{16}$$

It is the average energy stored in the elementary transfer modes of excitation. The appearance of τ^* is a characteristic feature of the model. It should be noted that the transport coefficients (15) satisfy a generalized Einstein relation DF = V τ^* in accordance with the fluctuation-dissipation theorem. A general discussion of the implications of the fluctuation-dissipation theorem on low-energy nuclear dynamics has been made by Hofmann and Siemens.[7] The transport coefficients (15) make it possible to study the simultaneous transport of charge and mass in nuclear collisions.[8]

By elementary but somewhat more complicated calculation it can be shown that the transport coefficients for the various momentum variables are approximately

$$\begin{aligned}
V_{PP} &= -\ 2m\ N\ \dot{R} \\
D_{PP} &\approx 2m\ N\ \tau^* \\
\vec{V}_L &= -\ m\ N\ \vec{R} \times \vec{U} \\
\vec{V}_{S_A} &= -\ \frac{a}{R}\vec{V}_L, \quad \vec{V}_{S_B} = -\ \frac{b}{R}\vec{V}_L \\
\overleftrightarrow{D}_{LL} &\approx m\ N\ R^2 \tau^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
\overleftrightarrow{D}_{LS_A} &= -\ \frac{a}{R}\overleftrightarrow{D}_{LL}, \quad \overleftrightarrow{D}_{LS_B} = \frac{b}{R}\overleftrightarrow{D}_{LL} \\
\overleftrightarrow{D}_{S_A S_B} &\approx -\ \frac{ab}{R^2}\overleftrightarrow{D}_{LL} + m\ N\ \rho_{eff}{}^2\ \tau^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\overleftrightarrow{D}_{S_A S_A} &= \overleftrightarrow{D}_{S_A S_B} \quad (b \rightarrow a) \\
\overleftrightarrow{D}_{S_B S_B} &= D_{S_A S_B} \quad (a \rightarrow b).
\end{aligned} \tag{18}$$

Here $N = N_Z + N_N$ denotes the average one-body nucleon current from one partner to the other, $\vec{R}$ is the position of A relative to B, and a and b are the distances from (the centers of) A and B to the interaction zone where the transfers occur. The off-diagonal diffusion coefficients coupling the above momentum variables to the particle numbers have been omitted here since they are often (but not always) negligible. Likewise, the explicit appearance of τ^* is only approximate and may not always be quantitatively accurate.

The drift coefficients for P and $\vec{L}$ are recognized as the radial and tangential components of the window friction[3,9] and can be derived on simple classical grounds due to their one-body character. The vanishing of the zz-component of D_{LL} is a trivial reflection of angular momentum conservation ($\vec{L}$ is always perpendicular to $\vec{R}$). The diffusion tensors for the intrinsic angular momenta contain an additional term resulting from off-axis transfers (such transfers may change the K quantum numbers in A and B and hence $\vec{S}_A$ and $\vec{S}_B$ need not remain perpendicular to $\vec{R}$); this term depends explicitly on the geometrical size of the interaction zone as measured by the effective neck radius ρ_{eff} (and it is therefore typically smaller than the first term by a factor ρ_{eff}^2/R^2).

The degradation of the initial macroscopic energy leads to excitation of the microscopic degrees of freedom in the two nucleides. The generated heat Q is of primary interest since it characterizes the state of the intrinsic system (through the temperature $\tau = (Q/a)^{1/2}$, $a \approx (A + B)/8$ MeV). It is therefore convenient to include Q in the set of macroscopic variables considered. This can be done although the intrinsic energy is not an additive variable. The drift coefficient, equal to the energy dissipation rate, can be calculated from the loss of macroscopic energy,

$$\begin{aligned} V_Q &= \dot{Q} \\ &= F_Z V_Z + F_N V_N + mN(2\dot{R}^2 + U_t)^2 . \end{aligned} \tag{19}$$

The various diffusion coefficients involving Q can be obtained approximately by use of the generalized Einstein relation (with the driving force $F_Q = -\partial\mathcal{H}/\partial Q$ equal to unity):

$$\begin{aligned} D_{QQ} &\approx V_a \, \tau^* \\ D_{\ Q} &\approx V \quad \tau^* . \end{aligned} \tag{20}$$

This completes the derivation of the transport coefficients for the disphere. No arbitrary parameters enter in the expressions (although the form factors depend delicately on the details of the interaction zone and therefore are difficult to estimate accurately). The theory thus implies that certain specific relations exist between the different macroscopic variables. This feature may be particularly useful when trying to determine from experiment the relative importance of the particle-transfer mechanism in damped nuclear collisions.

5. Confrontation with Experiment

In order to determine the relative importance of the nucleon transfer mechanism in damped nuclear collisions, and verify the specific structure implied by the above formulas, it is necessary to confront the theory with experiment. This task is made difficult by the fact that the transport process depends delicately on the details of the interaction zone whose dynamics is still only poorly understood. One may try to circumvent this complication, which presents an interesting problem by itself, by correlating a number of different observables. We shall discuss here one example of such an approach, namely the relation between the energy loss and the mass dispersion. It was first suggested by Huizenga et al.[10] that this relation may be used to elucidate the basic nature of the transfer mechanism. (This subject has also been addressed by Gobbi in a lecture at this school.)

The basic idea ia the following: Each nucleon transfer induces an energy loss ω given by (2). The transfer rate is equal to the rate of increase in the dispersion σ_A^2, by ordinary random-walk theory. Hence the rate of energy loss can be written

$$\frac{d}{dt} T \approx - \dot{Q} = - \omega_{ave} \frac{d}{dt} \sigma_A^2 \tag{21}$$

where ω_{ave} is the appropriate average exciton energy and $T = \frac{1}{2} \mu U^2$ is the kinetic energy of the relative motion. We restrict our attention to nearly peripheral, partially damped collisions so that the relative velocity $\vec{U}$ is predominantly tangential.

If there were no intrinsic motion of the nucleous prior to their transfer they would contribute an excitation energy $\omega = \frac{1}{2} m U^2 = \frac{m}{\mu} T$ (neglecting the relatively small contribution from driving force F acting on asymmetric systems). Therefore, Huizenga et al.[10] have introduced the quantity

$$\alpha \equiv - \frac{\mu}{m} \frac{1}{T} \frac{dT}{d\sigma_A^2} \tag{22}$$

which can be extracted empirically from the relation between the kinetic energy loss and the mass dispersion obtained in a given experiment. (It is here important that α indeed turns out to be experimentally well-defined.) It follows that if all the dissipation were induced by transfer of particles initially at rest then α would be unity.

In the present theory where the dissipation is produced by transfer of Fermi-Dirac particles (which have an initial motion and are subject to the blocking effect) we have

$$\frac{d}{dt} T = - \langle \omega^2 \rangle_F N'(\varepsilon_F) \tag{23}$$

and

$$\frac{dT}{dt} \sigma_A^2 \approx 2\tau^* N'(\varepsilon_F) \tag{24}$$

as follows from the general expressions (12) and (13) respectively. Hence it follows that

$$\frac{d}{d\sigma_A^2} \approx - \frac{\langle \omega^2 \rangle_F}{2\tau^*} . \tag{25}$$

For a nearly peripheral collision (where $\vec{U}$ is almost tangential) between nearly symmetric systems (so that F_A can be neglected) we have

$$\langle \omega^2 \rangle_F \approx \frac{1}{4} U^2 p_F^2 = \frac{1}{2} mU^2 T_F . \tag{26}$$

Consequently we arrive at the following simple estimate

$$\alpha \approx \frac{T_F}{2\tau^*} . \tag{27}$$

It should be noted that if the blocking effect were ignored in the calculation one would arrive at $\alpha \approx 1$.

The above simplistic estimate (27) indicates that α should typically be substantially larger than unity (since usually $\tau^* << T_F$). Moreover the formula suggests that α should decrease as the bombarding energy E_{cm} (and hence τ^*) is increased. Both of these features are indeed present in the experimental data where α-values of 2 to 16 have been found and where a clear decrease with E_{cm} has been established in all cases explored.[11)] We wish to emphasize that these are both features which would not find an explanation within a classical transfer model.

For nearly symmetric systems the value of τ^* is essentially determined by $\frac{1}{2}$ m U^2. The formula (27) therefore suggests that the α-values for many different systems should fall on the same "universal" curve when plotted against $\frac{1}{2}$ m U^2. Such a behavior is indeed borne out by experiment. This fact lends support to the employed model, and provides evidence that the mechanism considered, namely the transfer of individual nucleons, plays an essential role as a damping mechanism. However, it must be stressed that the perturbative estimate (27) relies on a number of idealizations and a more refined treatment is called for before a definite comparison can be made.

6. Concluding Remarks

We have explored the consequences of the independent-particle idealization for the dynamical properties of the dinucleus; no additional physical assumptions have been introduced. In this way clear and relatively simple results have been derived for the dinuclear transport coefficients. Although the approach treats only uncorrelated microscopic modes of excitation, it is not meant to preclude the possible coexistence of additional mechanisms, such as the interplay with collective dinuclear modes and the special dynamics of the interaction zone. The high specificity of the present results holds promise that a careful confrontation with experiment might indicate conclusively to what extent the predicted behavior is borne out by real nuclei. At the same time the relative importance of other agencies might be established; should they prove important, an appropriate extension of the theory is called for.

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