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PROPAGATOR IN A THEORY WITH CONFINEMENT

Robert Roth

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PROPAGATOR IN A THEORY WITH CONFINEMENT*†

Robert Roth

Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

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ABSTRACT

We study the propagator in a model theory with confinement and attempt to show that, when summed to all orders, the propagator is free of singularities in the finite momentum plane. We find that Bethe-Salpeter ladder-like diagrams alone are insufficient to exhibit this behavior. However, in a nonrelativistic approximation in the crossed channel, confinement is obtained and all poles disappear.

I. INTRODUCTION

Because quarks have not been seen (or if they have they must still be strongly bound), physicists have been interested in field theories with confined particles for some time. If particles are permanently confined, they cannot appear as asymptotic states. Therefore, no singularities at energies equal to their mass should appear in the S-matrix. One manifestation of this should be in the behavior of the propagator for the confined particle. All singularities at energies equal to its mass should vanish.[1] Thus, for example, the pole that appears in the propagator in the lowest order of perturbation theory must somehow be cancelled by higher order corrections. Also because of the confinement, we would expect that for large space-like separations, the propagator should fall off very rapidly. If

the propagator falls off as $r^n e^{-mr}$, then in momentum space there will be a singularity at $p^2 = m^2$. This can be seen by deforming the r integral of the Fourier transform into the upper half plane. Thus if we want all p plane singularities absent, we must demand a large $|\bar{x}|$ behavior that falls off faster than e^{-mr} for all m. This behavior is in fact sufficient.[6]

We start by considering the theory with an electromagnetic field A_μ interacting with a scalar Higgs particle ϕ and a spin-$\frac{1}{2}$ monopole ψ. The Hamiltonian is

$$\mathcal{H} = \frac{1}{2}(\bar{E}^{T^2} + \bar{H}^2) + |\partial_0\phi|^2 + e^2 A_0^2 |\phi|^2$$
$$+ |(\partial_i - ie\, A_i)\phi|^2 - \mu^2|\phi|^2 + \lambda|\phi|^4$$
$$+ \bar{\psi}\,\bar{\gamma}\cdot(-i\bar{\nabla} - g\bar{B} - m)\psi$$

where $\bar{A} = \bar{A}^T + \bar{A}_g$, $\qquad \bar{B} = \bar{B}^T + \bar{B}_g$,

and we have used the formalism of Schwinger.[2] It can be shown that the monopoles in this theory are confined.[3]

Because of technical reasons,[3] however, it is hard to obtain meaningful results with this theory as it stands. We therefore replace the photon and the Higgs particle with an effective potential, which we choose to be linear by analogy with the vortex solutions of Nielsen and Oelson.[4] Our Lagrangian is now

$$\mathcal{L} = \bar{\psi}(x)(i\not{\partial} - m)\psi(x) + \lambda^2 \int d^4y\; j^0(x) j^0(y) |\bar{x} - \bar{y}|\, \delta(t_x - t_y)$$

where $j^0(x) = \bar{\psi}(x)\gamma^0\psi(x)$.

The fact that the theory is nonrenormalizable should not matter, since the confinement in our theory is a large distance or infrared effect.

Ordinarily, we could look at the large $|\bar{x}|$ behavior of the propagator in any spacelike direction. However, since we now have a nonlocal interaction, we must confine ourselves to the $t = 0$ direction. Thus in the considerations that follow we shall examine the propagator integrated over p_o.

II. Bethe-Salpeter equation

The approximation to the propagator we shall use is the set of ladder-like diagrams in Fig. 1a, where we have written the nonlocal four-point interaction as two two-point interactions connected by a dashed line. The Bethe-Salpeter equation satisfied by these graphs $\psi(p)$ is then

$$\psi(p) = \frac{1}{\not{p} - m + i\varepsilon} + \frac{1}{\not{p} - m + i\varepsilon}\int \frac{i\, d^4p'}{(2\pi)^4}\left(-8\pi\lambda^2 \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3}\right)\gamma_o\psi(p') \times$$

$$\times\, \gamma_o \frac{1}{\not{p} - m + i\varepsilon} \quad . \qquad (1)$$

The quantity in the brackets in the above equation is the Fourier transform of our potential $\lambda^2|\bar{x}|\delta(t)$. Letting $G(p) = (\not{p} - m + i\varepsilon) \times \psi(p)(\not{p} - m + i\varepsilon)$, we get

$$G(p) = \not{p} - m + \int \frac{i\, d^4p'}{(2\pi)^4}\left(-8\pi\lambda^2 \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3}\right)\times$$

$$\times\, \gamma_o \frac{1}{\not{p}' - m + i\varepsilon}\, G(p')\frac{1}{\not{p}' - m + i\varepsilon}\gamma_o \; .$$

It is now clear that $G(p)$ must be of the form $G(p) = \gamma_o p_o + H(\bar{p})$ where $H(\bar{p})$ now depends only on the 3-vector $\bar{p}$. We then can do the p_o' integral to obtain

$$H(\bar{p}) = -\bar{\gamma}\cdot\bar{p} - m - \frac{i}{4}\int \frac{i\, d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2 \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3}\right)\times$$

$$\times\left[\frac{2(\bar{\gamma}\cdot\bar{p}' + m)}{(\bar{p}'^2 + m^2)^{\frac{1}{2}}} + \frac{H(\bar{p}')}{(\bar{p}'^2 + m^2)^{\frac{1}{2}}} - \frac{(\bar{\gamma}\cdot\bar{p}' + m)\gamma^o H(\bar{p}')\gamma^o(\bar{\gamma}\cdot\bar{p}' + m)}{(\bar{p}'^2 + m^2)^{3/2}}\right] . \qquad (2)$$

We now write $H(\bar{p})$ in the form

$$H(\bar{p}) = J_1(\bar{p}) + \gamma_o J_2(\bar{p}) + \bar{\gamma}\cdot\bar{p}\, J_3(\bar{p}) + \gamma_o\bar{\gamma}\cdot\bar{p}\, J_4(\bar{p}) \, ,$$

where the J's are numbers, not matrices. We have not included terms with γ_5 because they do not appear in lowest order and they are not generated in higher orders. After substitution of Eq. 3 into 2, we can separately equate the coefficients of the different γ's. This leads to

$$J_1(\bar{p}) = -m + \frac{1}{2}\int \frac{d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2 \frac{(p - p')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3}\right)\times$$

$$\times\left[\frac{m}{(\bar{p}'^2 + m^2)^{\frac{1}{2}}} + \frac{\bar{p}'^2(J_1(\bar{p}') - mJ_3(\bar{p}'))}{(\bar{p}'^2 + m^2)^{3/2}}\right] \qquad (4a)$$

$$J_2(\bar{p}) = 0 \qquad (4b)$$

$$\bar{p}J_3(\bar{p}) = -\bar{\gamma}\cdot\bar{p} + \frac{1}{2}\int\frac{d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2\,\frac{(\bar{p}-\bar{p}')^2-3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\times$$

$$\times\left[\frac{1}{(\bar{p}'^2+m^2)^{\frac{1}{2}}} - \frac{m(J_1(\bar{p}')-mJ_3(\bar{p}'))}{(\bar{p}'^2+m^2)^{3/2}}\right]\bar{p}' \qquad (4c)$$

$$\bar{p}\,J_4(\bar{p}) = \frac{1}{2}\int\frac{d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2\,\frac{(p-p')^2-3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\times$$

$$\times\;\frac{J_4(\bar{p}')}{(\bar{p}'^2+m^2)^{\frac{1}{2}}}\;\bar{p}'\;. \qquad (4d)$$

We can at this point set $J_4(\bar{p}) = 0$, since it does not appear in lowest order, and from Eq. 4d each succeeding term is zero. Equation 4a and c remain. We do not know how to solve them. However, we can show that no solution exists such that the singularity of the propagator at the mass of the monopole has disappeared. Since we are interested in large spacelike separations, we can set $t = 0$ and consider

$$\int dp_o\psi(p) = \int dp_o\,\frac{1}{\not{p}-m+i\varepsilon}\,(\gamma_o p_o + H(p))\,\frac{1}{\not{p}-m+i\varepsilon} =$$

$$= \int\frac{dp_o}{(\bar{p}^2-m^2+i\varepsilon)^2}\,(\not{p}+m)(\gamma_o p_o + J_1(\bar{p}) + \bar{\gamma}\cdot\bar{p}\,J_3(\bar{p}))(\not{p}+m) =$$

$$= -\frac{\pi i}{(\bar{p}^2+m^2)^{3/2}}\left\{\left[J_1(\bar{p}) - m\,J_3(\bar{p}) + \frac{m}{\bar{p}^2}(\bar{p}^2+m^2)\right]\right.$$

$$\left. + m\,\bar{\gamma}\cdot\bar{p}\left[J_1(\bar{p}) - m\,J_3(\bar{p}) - \frac{1}{m}(\bar{p}^2+m^2)\right]\right\}\;.$$

In doing the p_o integral, we have ignored a term, formally infinite, but odd in p_o. In order for the result in Eq. 5 to be regular at

$\bar{p}^2 = -m^2$, we must have both

$$\frac{1}{(\bar{p}^2+m^2)^{3/2}}\left[J_1(\bar{p}) - m\,J_3(\bar{p}) + \frac{m}{\bar{p}^2}(\bar{p}^2+m^2)\right]$$

and

$$\frac{1}{(\bar{p}^2+m^2)^{3/2}}\left[J_1(\bar{p}) - m\,J_3(\bar{p}) - \frac{1}{m}(\bar{p}^2+m^2)\right]$$

regular. However, this is impossible because their difference is singular. Therefore, if a solution exists at all, it contains a singularity at the mass of the monopole. Thus we have failed to exhibit confinement. We believe that this is due to our inclusion of ladder diagrams with pair production (Fig. 2b), but not the corresponding crossed diagram (Fig. 2c). These diagrams also contribute to the binding "forces" on the monopoles and should be important. Unfortunately, we know of no way to correctly take them into account. However, in the nonrelativistic approximation in the crossed channel (Fig. 1b) there is no pair creation, and the types of diagrams represented by both Figs. 2b and c are absent (remember we have an instantaneous interaction) and only those of Fig. 2a remain. Thus we might hope to obtain confinement in this approximation, and we examine this possibility next.

III. Non-relativistic Approximation

Starting with our Bethe-Salpeter Eq. 1, we can do the $p^{o'}$ integration on the right hand side. Defining

$$\phi(\bar{p}) = \int_{-\infty}^{\infty} dp^o \psi(p)$$

$$H_a = \gamma_o(\bar{\gamma}\cdot\bar{p} + m)$$

$$H_b = \gamma_o(-\bar{\gamma}\cdot\bar{p} + m)$$

we have

$$(p^o - H_a(\bar{p}))\psi(p)(p^o - H_b(\bar{p})) = \gamma^o p^o + \bar{\gamma}\cdot\bar{p}$$

$$+ \int \frac{i\, d^3p'}{(2\pi)^4} \left(-8\pi\lambda^2 \frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\phi(\bar{p}') . \qquad (6)$$

We can now treat the 4×4 matrix $\psi(p)$ as a wave function in the product space of two spinor particles. We proceed according to the method of Salpeter[5] for treating instantaneous interactions and make the following definitions

$$\Lambda^a_{\pm} = \frac{E_a(\bar{p}) + H_a(\bar{p})}{2E_a(\bar{p})}$$

where $E_a(\bar{p}) = (\bar{p}^2 + m^2)^{\frac{1}{2}}$

and similarly for particle b. In addition we define

$$\psi_{++}(p) = \Lambda^a_+(\bar{p})\psi(p)\Lambda^b_+(\bar{p})$$

$$\psi_{+-}(p) = \Lambda^a_+(\bar{p})\psi(p)\Lambda^b_-(\bar{p}), \text{ etc.}$$

Then we arrive at

$$F_{++}(p)\psi_{++}(p) = \Lambda^a_+(\bar{p})\Gamma(p)\Lambda^b_+(\bar{p})$$

$$F_{+-}(p)\psi_{+-}(p) = \Lambda^a_+(\bar{p})\Gamma(p)\Lambda^b_-(\bar{p}), \text{ etc.,} \qquad (7)$$

where $F_{++}(p) = (p^o - E_a(\bar{p}) + i\varepsilon)(p^o - E_b(\bar{p}) + i\varepsilon)$

$$F_{+-}(p) = (p^o - E_a(\bar{p}) + i\varepsilon)(p^o + E_b(\bar{p}) - i\varepsilon), \text{ etc.}$$

and $\Gamma(p)$ is the right hand side of Eq. 6. We now divide each of Eq. 7 by the appropriate $F(p)$ and integrate over p^o using

$$\int_{-\infty}^{\infty} dp^o \; (p^o + a \mp i\varepsilon)^{-1}(p^o + b \pm i\varepsilon)^{-1} = \pm 2\pi i\,(b-a)^{-1}$$

$$\int_{-\infty}^{\infty} dp^o \; (p^o + a \pm i\varepsilon)^{-1}(p^o + b \pm i\varepsilon)^{-1} = 0$$

$$\int_{-\infty}^{\infty} dp^o \; p^o\,(p^o - a + i\varepsilon)^{-1}(p^o - b + i\varepsilon)^{-1} = -\pi i$$

$$\int_{-\infty}^{\infty} dp^o \; p^o\,(p^o + a - i\varepsilon)^{-1}(p^o - b + i\varepsilon)^{-1} = \pi i\,\frac{a-b}{a+b} .$$

This leads to

$$-(E_a(\bar{p}) + E_b(\bar{p}))\phi_{-+}(\bar{p}) = \Lambda^a_-(\bar{p})\Big[\pi i(E_b(\bar{p}) - E_a(\bar{p}))\gamma_o + 2\pi i(\bar{\gamma}\cdot\bar{p} - m)$$

$$- \int \frac{i\, d^3p'}{(2\pi)^4}\left(-8\pi\lambda^2 \frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\phi(\bar{p}')\Big] \times \Lambda^b_+(\bar{p}) -$$

$$-(E_a(\bar{p}) + E_b(\bar{p}))\phi_{+-}(\bar{p}) = \Lambda^a_+(\bar{p})\Big[\pi i(E_a(\bar{p}) - E_b(\bar{p}))\gamma_o + 2\pi i(\bar{\gamma}\cdot\bar{p} - m) -$$

(Equation continued on the next page)

$$- \int \frac{i\, d^3p'}{(2\pi)^3} \left(-8\pi\lambda^2 \, \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - p')^2 + \varepsilon^2)^3} \right) \phi(\bar{p}') \Big] \Lambda_-^b(\bar{p}) -$$

$$-\phi_{++}(\bar{p}) = \pi i \Lambda_+^a(\bar{p})\gamma_o \Lambda_+^b(\bar{p}) \,, \qquad -\phi_{--}(\bar{p}) = \pi i \Lambda_-^a(\bar{p})\gamma_o \Lambda_-^b(\bar{p}) \,.$$

Since in our case, $E_a - E_b = 0$, we have

$$(H_a(\bar{p}) - H_b(\bar{p}))\, \phi(\bar{p}) = (\Lambda_-^a(\bar{p})\Lambda_+^b(\bar{p}) - \Lambda_+^a(\bar{p})$$

$$\times \; \Lambda_-^b(\bar{p})) \left[2\pi i(\bar{\gamma} \cdot \bar{p} - m) - \int \frac{i\, d^3p'}{(2\pi)^3} \left(-8\pi\lambda^2 \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((p - p')^2 + \varepsilon^2)^3} \right) \phi(\bar{p}') \right] \tag{8}$$

where for convenience we have written the b operators on the left, even though they really should appear on the right. Eq. 8 looks different than the corresponding equation in Salpeter's[5] article. This is due to the fact that our $\phi(\bar{p})$ is a wave function for a particle and antiparticle, whereas his is for two particles. To remedy this we multiply by the charge conjugation operator C on the right to obtain

$$-(H_a(\bar{p}) + H_b^{\,T}(\bar{p}))\, \phi^c(\bar{p}) = (\Lambda_+^a(\bar{p})\Lambda_+^{b\;T}(\bar{p}) - \Lambda_-^a(\bar{p})\Lambda_-^{b\;T}(\bar{p})) \;\times$$

$$\times \left[2\pi i(\bar{\gamma} \cdot \bar{p} - m)\, C \; - \int \frac{d^3k}{(2\pi)^3} \left(-8\pi\lambda^2 \, \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3} \right) \phi^c(\bar{p}') \right]$$

where $\phi^c(\bar{p}) = \phi(\bar{p})C$. In the nonrelativistic limit the factor involving the Λ's equals one, and all the homogeneous terms in ϕ reduce to the Schrödinger Hamiltonian operator acting on the "large" part of the wave function $\phi_{++}(\bar{p})$. In coordinate space we then have

$$H_s(\bar{x})\phi^c(\bar{x}) = 2\pi i(-i\bar{\gamma} \cdot \bar{\nabla} - m)\delta^3(\bar{x})\, C \tag{9}$$

where $H_s(\bar{x}) = -\frac{1}{2m} \bar{\nabla}^2 + \lambda^2|\bar{x}|$.

For $\bar{x} \neq 0$, the right hand side in Eq. 9 equals zero. The problem becomes simply that of finding the large $\bar{x}$ behavior of the Schrödinger wave function with energy $E = 0$. (Boundary conditions at the origin, which quantize the allowed energies, do not apply here). The angular part of the equation can be separated out in the standard way for a central potential. We are then left with the radial equation

$$\left(\frac{d^2}{dr^2} - 2m\lambda^2 r - \frac{\ell(\ell+1)}{r^2} \right) \left(rR_\ell(r) \right) = 0.$$

For large r, we can ignore the angular momentum term compared to the potential. Rescaling r, we arrive at the Airy differential equation. The solution has the asymptotic form

$$R_\ell(r) = \frac{1}{r} \, \mathrm{Ai}((2m\lambda^2)^{1/3} r) \xrightarrow{\; r \to \infty \;} \frac{\sqrt{\pi}}{2(2m\lambda^2)^{1/12}} \; \frac{\exp(-\frac{2}{3}\sqrt{2m\lambda^2}\, r^{3/2})}{r^{3/2}}$$

where, as usual, we have discarded the exponentially increasing solution. This shows that our wave function, and consequently the propagator, falls off much faster than the free propagator. In fact, since it falls off faster than e^{-mr} for any m, all singularities in momentum space must be absent[6] and we have finally exhibited confinement.

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REFERENCES

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† Address after September 1, 1977: Weizmann Institute of Science, Rehovot, Israel.

1. One might object that in two-dimensional QCD this does not happen. Rather the quarks decouple when put on mass shell. However, if this were the case here, we would expect the propagator to have a pole, not a cut, at the mass of the monopole. We find no such behavior.
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3. For more detail, see R. Roth, Ph. D. thesis, University of California, Berkeley (1977).
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5. E. E. Salpeter, Phys. Rev. 87, 328 (1952).
6. This follows from Theorem IX. 13, Page 18 in M. Reed and B. Simon, "Methods of Modern Mathematics, II: Fourier Analysis, Self-Adjointness, " Academic Press (1975).

Figure Captions

1. Ladder diagrams in physical and crossed channels.

2. Diagrams neglected in various approximations.

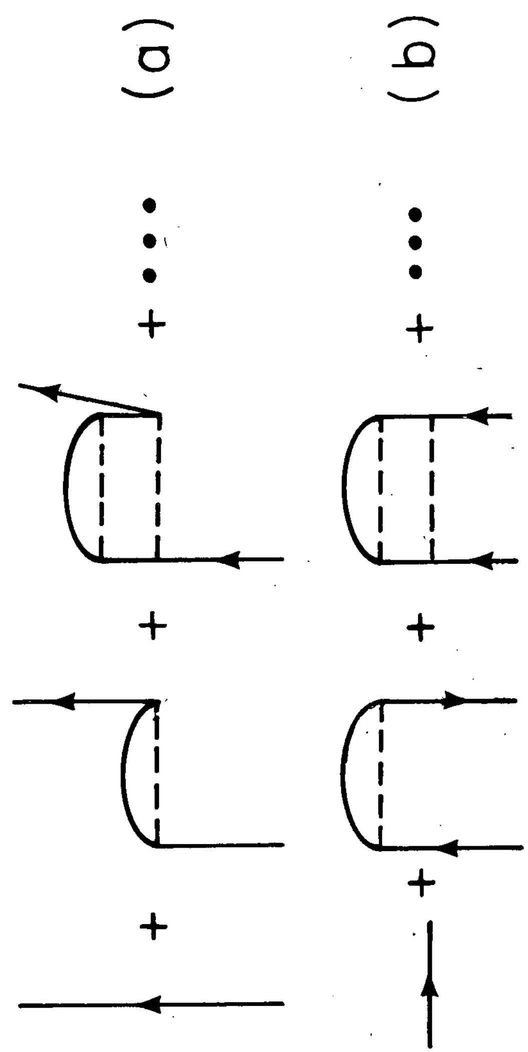

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Fig. 1

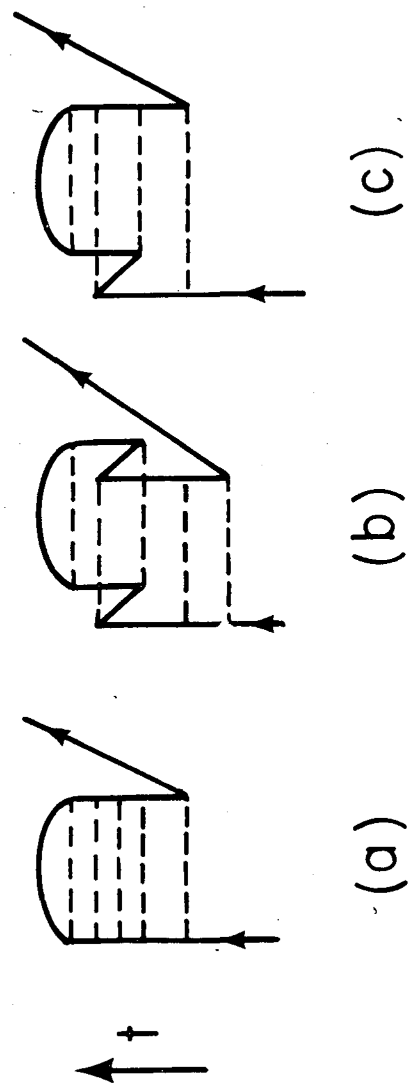

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Fig. 2

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