Title
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Publication Date
1984-04-01

LBL-17652
Preprint c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics Division

Submitted for publication

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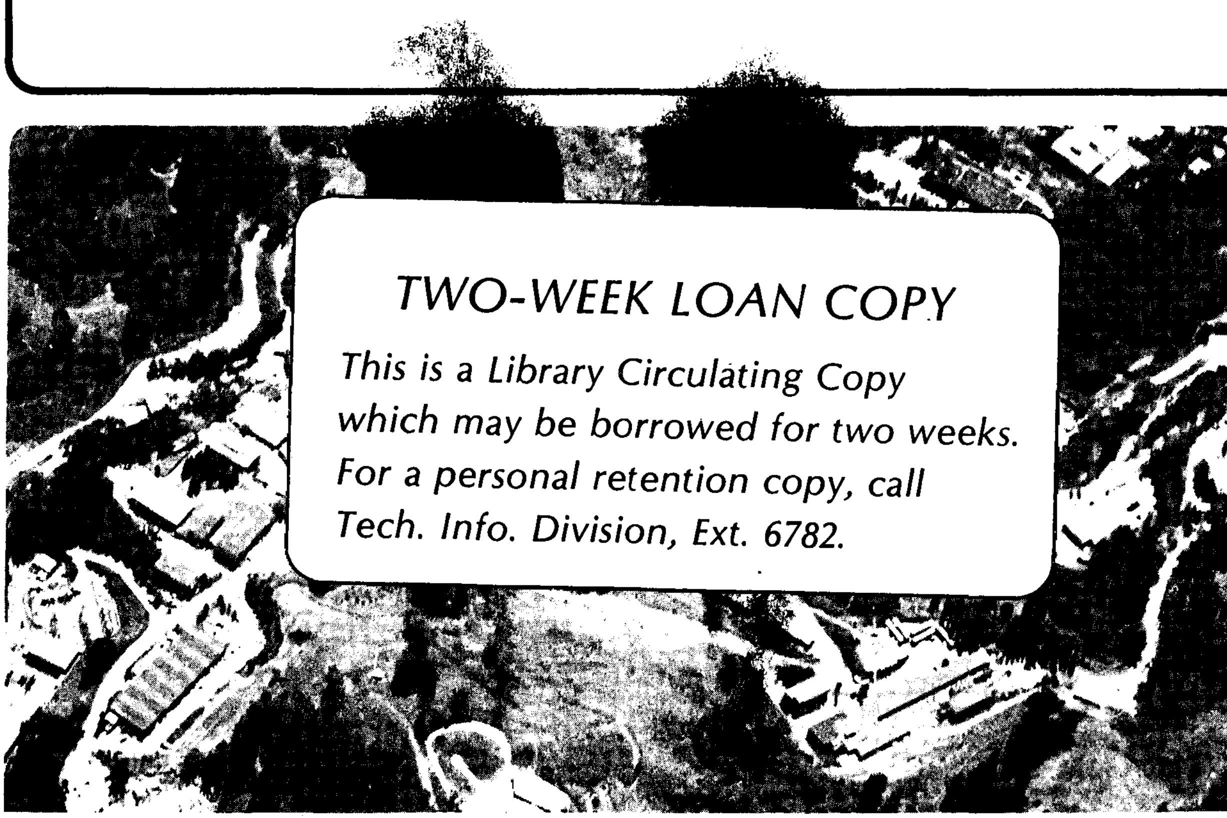

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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April 1984 LBL-17652

On the Possibility of Deeply Bound Di-Baryon Resonances*

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*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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ABSTRACT

The question of the existence of deeply bound positive parity non-strange di-baryon resonances is addressed in the context of a QCD-like potential model. Hyperfine effects are found to produce strong binding in certain non-NN channels, notably the I=0 S=3 channel which is predicted to lie well below $N\Delta\pi$ threshold.

1. Introduction

The presence of a confined, SU(3) valued color degree of freedom in ordinary hadrons makes possible the existence of exotic multi-quark bound states as part of the physical hadronic spectrum. Among these states are the six-quark or di-baryon states, which, owing to the absence of annihilation effects and the existence of successful models of baryon structure based on QCD, are expected to be especially amenable to phenomenological study. Recently such states have been the subject of considerable experimental interest[1-16)*]. Experimental backgrounds are generally held to be well-understood, although it is not clear that this is actually the case since the relative importance of soft QCD effects and meson exchange in the short and intermediate range regions of the NN force is not yet fully clarified. While it seems likely that these regions are dominated by quark and gluon degrees of freedom[17-23)], this view is not universally accepted[24)] and, at any rate, the lack of a solid quantitative framework within which to include both quark and meson exchange effects means that non-traditional contributions to backgrounds are difficult to estimate reliably. In addition the experimental situation itself remains somewhat unclear. Interesting effects have been seen (and in some cases then not seen) in a number of different experiments and in several channels, but as yet the identification of these effects with dibaryon resonances, with the possible

*References to pre-1982 experimental results may be found in Refs. 1-3).

exceptions of the I=1 $^1D_2$ and $^3F_3$ states, remains controversial. Theoretically, most of the work on di-baryon resonances has been performed in the context of the bag model[25-29)]. Apart from phenomenological extensions to orbitally excited configurations in hypothetical stretched bags[27)] these calculations have been restricted to the spherical static cavity approximation. While the usual difficulties of the bag model--the CM motion in the bag and the artificial confinement of color singlet sub-units--are understood and reasonably well under control[30,31)], the restriction to a permutationally symmetric, (6), spatial configuration is less satisfactory since it is known that the color hyperfine interaction tends to favor configurations of lower spatial symmetries[32)]. QCD-like potential models, the commonly used alternative to the bag in low energy phenomenology, have been applied only to the I=0 S=3 and I=3 S=0 channels[33,34)], with not completely compatible results. Nonetheless, such models have certain advantages, primarily that the lower spatial symmetry (42) configuration may be included with no particular technical complications. This is expected to be a significant advantage in studying channels in which there is potentially deep binding due to the short range hyperfine forces. In addition, owing to the absence of any artificial confinement of the lowest lying asymptotic state with given quantum numbers*, potential

*Note that this is not true for higher lying states, e.g., a $\Delta\Delta$ state in the deuteron channel bound with respect to $\Delta\Delta$ will in general still have super-allowed rearrangement decays into free NN states.

models provide a useful framework for bridging the gap between channels in which two baryons are bound by short range forces and those in which they are not.

In this paper we apply a QCD-like potential model to the study of possible deeply bound positive parity di-baryon states. Before proceding, a few comments are in order regarding motivation and limitations. Our aim here is to attempt to identify only those channels in which deeply bound states might exist. The reasons for this are two-fold. First, such states should be more accessible to experiment than corresponding states above hadronic thresholds, for which fall-apart decay modes exist, and second, the nature of the physics of the s-wave NN channels suggests that such states might actually exist and be unambiguous predictions of the model*. As we shall see, such states are indeed predicted, in particular an interesting state in the I=0, S=3 channel which lies below NΔπ threshold. It is important, however, to temper

*Recall that the NN channels are dominated by the repulsive nature of the NN exchange hyperfine interaction. This repulsion inhibits mixing with orthogonal hidden color configurations which, owing to confinement, must be localized at short distances. Since the hyperfine interaction is sensitive to the spin-color-isospin couplings of a state, one expects that there may exist channels in which the induced exchange interaction is strongly attractive and in which, therefore, significant mixing may also occur. In terms of theoretical underpinnings the hyperfine potential is the most firmly established of the quark-quark interactions so that effects arising from it are expected to be accurately predicted in the model.

the conclusions of these calculations with a realistic evaluation of the uncertainties. First, we do not attempt to include the quark-quark tensor interactions which, while negligible in the deuteron, may be significant in a tightly bound system. Second, we do not include mixings with p-wave color octet states, states analogous to those which appear responsible for much of the intermediate range attraction in the NN channels[23]. Third, since the two body confinement potentials employed are purely phenomenological, and since baryon spectroscopy constrains only interactions between quarks in an antisymmetric color state, the energies of hidden color configurations are subject to unknown uncertainties. While there are some indications that such two body confinement forces may be equivalent to string effects in multi-quark systems[35], there is no clear reason, for example, to expect the strength of the interaction to be identical to that determined in the baryon sector. This means that, while the qualitative picture of hidden color states lying above corresponding hadronic thresholds is satisfied, the magnitudes of the splittings predicted by the model and the consequences of these splittings for mixing should be viewed with some caution. Finally, note that we make no attempt to include pionic corrections. In chiral extensions of the bag model, such as the cloudy bag[29,36], such corrections may be made in a reasonably quantitative manner. For di-baryon states they are generally of order 10 to 40 MeV, and may be either binding or anti-binding. Since potential models do not incorporate chiral symmetry in any obvious way one cannot expect to make quantitative estimates of these effects in the present calculation. Cloudy bag estimates, however, should provide reasonable guidelines, especially regarding the sign of the corrections.

2. The Model and Methodology

In Table 1 we display all non-strange six quark states which can be constructed from two three quark clusters in spatial ground states and in a relative s-wave. The three quark clusters are taken to be completely antisymmetrized and are coupled to the appropriate total spin, isospin and zero net color in the usual manner. The form of the full six quark antisymmetrizer is considerably simplified if these states are also antisymmetrized with respect to cluster interchange, where required. This categorization of states provides a natural framework for investigating the possibility of deep binding in various hadronic channels. We will proceed as follows, motivated by the physics of the NN channels, as studied previously by many authors.* The first step is to evaluate the "diagonal" hadronic potentials, i.e. the induced exchange potential between two (color singlet) baryons in the channel in question. If this potential is

*See Refs. 17-23 and further references therein.

repulsive, i.e., if the trial bound state wavefunction is depleted at short distances, then the situation is as in the NN channels and there will be no deep binding. Note that, as in the NN case, there is still the possibility of weak binding but this may depend critically on a reliable evaluation of mixing and pionic effects.* If, on the other hand, the diagonal potential is attractive, one must expect significant mixing with available hidden color excitations and strong binding. The degree of binding is then a detailed dynamical question. Many of the channels contain a large number of such available states so that, in general, this program might require many calculations of a complexity comparable to that required in the two NN channels--a rather unpalatable prospect. As it turns out, however, the majority of the channels have clearly repulsive short range diagonal potentials so that this is not a problem.

The model used in these calculations is one that has been applied previously, with considerable success, to both baryon spectro-

*In channels involving one or more Δ's there is an additional uncertainty due to ambiguities regarding the correct mass parameter to associate with the inter-cluster motion. In potential models the naturally occurring mass is $3m_q \cong M_N$ and not, for example m_Δ as would be appropriate for weakly bound Δ's. Although the potential model value may be reasonable for tightly bound $\Delta\Delta$ systems, at least in the spirit of viewing the constituent quark model as an effective theory below a chiral symmetry breaking scale, T_χ, little of a quantitative nature can be said in this regard so that, at least in potential models, predictions regarding weakly bound di-baryon states in non-NN channels, are likely intrinsically unreliable.

scopy[37] and decays[38], as well as to the NN problem[23]. For a more detailed discussion of the model, especially with regard to its use in the six quark sector, the reader is referred to Ref. 23. We record here only the model Hamiltonian.

$$H = \sum_{i=1}^{6}(m_i + p_i^{\,2}/2m_i) + \sum_{i<j}(H^{ij}_{conf} + H^{ij}_{hyp}) \tag{1}$$

where, with $\underset{\sim}{r}_{ij} = \underset{\sim}{r}_i - \underset{\sim}{r}_j$ and

$$S_{ij} = 3\underset{\sim}{S}_i\cdot\underset{\sim}{r}_{ij}\underset{\sim}{S}_j\cdot\underset{\sim}{r}_{ij}/r_{ij}^{\,2} - \underset{\sim}{S}_i\cdot\underset{\sim}{S}_j$$

the two body confining potential H^{ij}_{conf} is given by

$$H^{ij}_{conf} = -(e_o + \tfrac{1}{2}kr_{ij}^{\,2} + U(\underset{\sim}{r}_{ij}))(\tfrac{1}{2}\vec{\lambda}_i)\cdot(\tfrac{1}{2}\vec{\lambda}_j) \tag{2}$$

and the two body color hyperfine interaction, H^{ij}_{hyp}, arising from one gluon exchange, by

$$H^{ij}_{hyp} = -\left(\frac{\alpha_s}{m_i m_j}\right)\left(\frac{8\pi}{3}\underset{\sim}{S}_i\cdot\underset{\sim}{S}_j\delta^3(r_{ij}) + S_{ij}r_{ij}^{\,-3}\right)(\tfrac{1}{2}\vec{\lambda}_i)\cdot(\tfrac{1}{2}\vec{\lambda}_j). \tag{3}$$

The anharmonicity, U, in (2) is meant to include, as well as departures from the harmonic limit, spin independent contributions due to one gluon exchange, notably the attractive short range color Coulomb inter-

action.* The parameters appearing in (2), (3) are determined from baryon spectroscopy and given in Ref. 23.

Consider a given channel in Table 1 and let $|I\rangle, |J\rangle, \ldots$ be the available partially antisymmetrized states contained therein. We indicate the quark content of the two three quark clusters (ijk) and (rst) in $|I\rangle$ as follows

$$|I(ijk;rst)\rangle\,. \tag{4}$$

Restricted to states of this structure the six quark antisymmetrizer takes the simple form

$$\mathcal{A} = 1 - \sum_{\substack{m\in\{i,j,k\}\\ n\in\{r,s,t\}}}\pi_{mn} \tag{5}$$

where π_{mn} is the transposition operator $m\leftrightarrow n$. The normalized, fully antisymmetrized state $|I_A\rangle$ corresponding to $|I\rangle$ in (4) is then

$$|I_A\rangle = \frac{1}{N_I}\mathcal{A}|I(123;456)\rangle\,. \tag{6}$$

Using permutational symmetries one can readily show that

---

*In practice U is taken to be a δ-function in order to simplify calculations. Since cluster sizes are fixed and the interaction is smeared over clusters, the sensitivity to this choice is greatly reduced.

$$N_I = \sqrt{10(1-9\langle I(123;456)|I(126;453)\rangle)}\,. \tag{7}$$

Note that $|I_A\rangle$, $|J_A\rangle$ ... are not, in general, orthogonal. An orthonormal set is constructed from the states (6) in the usual manner. Using permutation symmetries one readily obtains, for any permutationally symmetric operator such as H

$$\langle I_A|H|J_A\rangle = \frac{10}{N_I N_J}\langle I(123;456)|H(1-9\pi_{36})|J(123;456)\rangle \tag{8}$$

from which the Hamiltonian matrix in the orthonormal basis may be constructed once the values of $\langle I_A|J_A\rangle$ are known.

In order to evaluate the matrix elements in (8) we require the spatial wavefunctions of the incompletely antimsymmetrized six quark states $|I(123;456)\rangle$. We choose the form

$$\Phi(123;456) = \psi(\underset{\sim}{R}_{123;456})\phi(123)\phi(456) \tag{9}$$

where

$$\phi(123) = \frac{\alpha^3}{\pi^{3/2}}\exp\left(-\alpha^2(\rho_{123}{}^2+\lambda_{123}{}^2)/2\right) \tag{10}$$

$$\text{with}\quad \underset{\sim}{\rho}_{123} = (\underset{\sim}{r}_1 - \underset{\sim}{r}_2)/\sqrt{2}, \qquad \underset{\sim}{\lambda}_{123} = (\underset{\sim}{r}_1 + \underset{\sim}{r}_2 - 2\underset{\sim}{r}_3)/\sqrt{6} \tag{11}$$

is the ground state three quark cluster wavefunction taken from baryon spectroscopy and $\psi(\underset{\sim}{R}_{123;456})$ is a variational wavefunction for the intercluster coordinate $\underset{\sim}{R}_{123;456} = (\underset{\sim}{r}_1+\underset{\sim}{r}_2+\underset{\sim}{r}_3)/3-(\underset{\sim}{r}_4+\underset{\sim}{r}_5+\underset{\sim}{r}_6)/3$, chosen to be of the form

$$\psi(\underset{\sim}{R}) = \frac{1}{N}\sum_i \xi_i \exp(-\beta_i{}^2R^2/2) \tag{12}$$

with ξ_i, β_i the variational parameters and N a normalization constant such that Φ of (9) is normalized with respect to the measure $d\tau = d^3R_{123;456}d^3\rho_{123}d^3\lambda_{123}d^3\rho_{456}d^3\lambda_{456}$. The variational parameters are allowed to vary independently for each state in a given channel.

From (8) we see that, given the form of the states $|I(123;456)\rangle$, it is convenient to evaluate the spin, color and isospin matrix elements entering the calculation in a basis characterized by specific $S_3^{123}\times S_3^{456}$ permutational symmetries. The required color matrix elements have been evaluated previously[23)] and are not reproduced here. Similarly the spin matrix elements in the total spin S=0,1 channels. In the Appendix we briefly sketch the decomposition of the states $|I(123;456)\rangle$ into products of spin, color and isospin states with well-defined permutational symmetries and list the required matrix elements of the operator π_{36}. Those matrix elements required to generate all desired expectations in the S=2,3 sectors are also presented.

3. Diagonal Hadronic Potentials

In what follows the spatial matrix elements k_D, k_O, B, b^c_{sn}, b^h_{sn}, etc. are as defined in the Appendix. Evaluating spin, isospin and color expectations we obtain Hamiltonian matrix elements as linear combinations of these quantities. Note that for color singlet hadrons, direct cross-cluster interactions vanish for color reasons so that, apart from the effect of the normalizing constants, N_I, the direct two body terms in (8), with I=J, are given by the sum of the interquark interactions in the corresponding isolated hadrons. Let us write

$$N_I^2 = 10(1+B/\nu_I). \tag{13}$$

The values of ν_I for NN, NΔ, and ΔΔ in the various spin-isospin channels are given in Table 2. In terms of the ν_I the exchange confinement term in (8) can then be shown to reduce to

$$\frac{1}{3\nu_I}\,\frac{4}{(1+B/\nu_I)}\left(198\text{MeV}(B+4b^c_{sn}-b^c_{nc}-b^c_{ni}) - 176\text{MeV}(B+4b^h_{sn}-b^h_{nc}-b^h_{ni})\right) \tag{14}$$

where the 198 MeV term arises from the quadratic piece of the confinement interaction and the 176 MeV term from the U perturbation. The constant term in the confinement potential, while contributing to (8), not only vanishes between orthogonal states, but also adds the same

constant energy to all states in the problem and so has not been included in (14). Similarly, the exchange hyperfine contribution in (8) can be written

$$\frac{260\text{ MeV}}{9\nu_I(1+B/\nu_I)}\left(c_1B+c_2b^h_{sn}+c_3b^h_{nc}+c_4b^h_{ni}\right) \tag{15}$$

with the values of the c_i as given in Table 2. The kinetic energy, in terms of k_D, k_O is

$$(k_D+k_O/\nu_I)/(1+B/\nu_I) \tag{16}$$

where the direct term k_D contains the internal kinetic energy of the three quark clusters as well as that associated with the intercluster motion. One can now minimize the energy of the state $|I_A\rangle$ with respect to the variational parameters. In the event that no binding is found, the system is put in a weak harmonic box in order to ascertain whether or not the lack of binding results from an effective short range repulsive interaction. The results are given in Table 3, where we list only those states for which the short range behavior is not strongly repulsive. Note that the results for the I=0 S=3 and I=3 S=0 channels are in agreement with those found previously by Oka and Yazaki[33]. The I=0 S=1 ΔΔ state is of interest, being the only one in which two ordinary baryons are strongly bound by diagonal exchange forces. This may have some bearing on the physics of the deuteron channel, especially since, treated as independent states and not as part of an orthonormalized basis, the hidden color states in that channel also turn out to relatively low lying as a result of the hyperfine interaction. However, any di-baryon resonances in either of the two NN channels are likely to be very broad owing to the existence of super-allowed (rearrangement) decays to NN, so that, practically speaking, we focus our attention on the I=0 S=3 and I=3 S=0 channels. In the next section we allow the hidden color states to mix with the $\Delta\Delta$ states in these channels. We also consider mixing in the I=S=2 channel, since it contains only one hidden color state and is, therefore, readily studied, as a test of the overall strategy. Finally we evaluate the energies of the hidden color states in the I=2 S=0 and I=0 S=2 channels, as these channels contain no states consisting of only two ground state color singlet baryons, so that, if the hidden color states lie low enough in energy, they will be expected to dominate the composition of the lowest lying di-baryon states in these channels.

4. Effects of the Hidden Color Configurations

Constructing the hidden color states as indicated in the Appendix and evaluating the necessary spin, color and isospin matrix ele-

ments one obtains for the direct part of the expectation of the Hamiltonian in these states

$$\left(k_D+198\text{ MeV}(1+3n^c)-176\text{ MeV}(1+3n^h)+260\text{ MeV}(a_1+a_2n^h)/4\right)/(1+B/\nu) \tag{17}$$

and for the exchange part

$$\Big(6k_0/\nu+198\text{ MeV}(c_1^cB+c_2^cb_{sn}^c+c_3^cb_{nc}^c+c_4^cb_{ni}^c) -176\text{ MeV}(c_1^cB+c_2^cb_{sn}^h+c_3^cb_{nc}^h+c_4^cb_{ni}^h) +260\cdot\text{ MeV}(c_1^hB+c_2^hb_{sn}^h+c_3^hb_{nc}^h+c_4^hb_{ni}^h)/4\Big)/6(1+B/\nu)\ . \tag{18}$$

As before we omit the contributions due to the constant piece of the confinement potential in (17), (18). The values of ν, a_i, c_i^c, and c_i^h for the channels of interest are given in Table 4.

The expressions (17), (18) allow us to complete the calculation in the I=2 S=0 and I=0 S=2 channels. The result is that the hidden color states lie at 3600 MeV and 3350 MeV respectively, well above thresholds for available states consisting of baryons with internal spatial excitations. As a result these channels are not of physical interest.

Two further elements are required to perform the mixing calculation in the remaining channels: first the off-diagonal Hamiltonian matrix elements and second, the overlap between states in the channel in question. Let us write the overlap between the states of a given two-state channel as

$$EB_T = \frac{e}{N_g N_{(c)}} B_T \tag{19}$$

where N_g, $N_{(c)}$ are the normalization factors for the ground and hidden color states respectively. The values of e are given in Table 5. Explicit evaluation of the spin, color and isospin matrix elements than yields, for the off-diagonal Hamiltonian matrix element, the expression

$$\frac{2}{3N_g N_{(c)}}\Big(198\ \text{MeV}(c^c_{T1}B_T + c^c_{T2}b^c_{Tsn_{(c)}} + c^c_{T3}b^c_{Tns_{(c)}} + c^c_{T4}b^c_{Tnc} + c^c_{T5}b^c_{Tni})$$
$$-176\ \text{MeV}(c^c_{T1}B_T + c^c_{T2}b^h_{Tsn_{(c)}} + c^c_{T3}b^h_{Tns_{(c)}} + c^c_{T4}b^h_{Tnc} + c^c_{T5}b^h_{Tni})$$
$$+260\ \text{MeV}(c^h_{T0}n^h_T + c^h_{T1}B^h_T + c^h_{T2}b^h_{Tsn_{(c)}} + c^h_{T3}b^h_{Tns_{(c)}} + c^h_{T4}b^h_{Tnc} + c^h_{T5}b^h_{Tni})/4$$
$$+3ek_{OT}/2\Big) \tag{20}$$

where the spatial matrix elements B_T, b^c_T, b^h_T and k_{OT} are as defined in the Appendix and the coefficients c^c_T, c^h_T for the channels of interest

are given in Table 5. The results of the mixing calculations are presented in Table 6.

5. Discussion

We see that the only channels which exhibit clear binding with respect to available s-wave two baryon thresholds are the I=0 S=3 and I=3 S=0 channels. The former is of interest since its ground state lies 260 MeV below ΔΔ threshold and hence also well below NΔπ threshold. The decay modes will thus not be simply those of a bound Δ. The ground state of the latter channel, on the other hand, is open with respect to NΔπ, although this channel, in the cloudy bag, is subject to an unusually large 100 MeV downward shift in energy due to pionic corrections[29]. While one cannot simply carry over cloudy bag results to potential models, it is likely that non-trivial pionic corrections to binding will be present in this channel.

It is instructive to isolate the physical origin of the binding effects in the two channels under consideration. In both cases the deep binding with respect to ΔΔ is due, not to diagonal exchange interactions, but to mixing, although the mixing is facilitated by the non-repulsive nature of the diagonal potentials. The difference between the two channels results from the hyperfine interaction. In the I=0 S=3 channel the couplings required to form a state with the correct quantum numbers are such that the state orthogonal to the bound ΔΔ configuration may lie as low in energy as 120 MeV above ΔΔ threshold. Mixing is thus strong and this produces considerable additional binding. Note that, here, the phenomenological uncertainties in treating hidden color configurations are undoubtedly significant. However, the nature of the effect of the hyperfine interaction in this channel, especially with regard to significantly lowering the energy of the orthogonal hidden color state, is independent of this uncertainty. In the I=3 S=0 channel, on the other hand, the orthogonal state lies at least 650 MeV above ΔΔ threshold, with the result that the mixing effects are much less significant. In the "color only" channels I=2 S=0 and I=0 S=2 it is again the hyperfine interaction which is responsible for pushing the optimized hidden color state energies well above excited two baryon thresholds.

The above observations make clear the dominant role of the hyperfine interaction in determining the qualitative physics of the six quark sector. Note the importance of including both the symmetric (6) and mixed (42) spatial symmetries in this regard, as well as the necessity of configuration mixing. While potential and bag model calculations are not strictly equivalent, hyperfine effects are expected to be similar in both cases so that the static spherical cavity approximation to the bag is likely to be an inadequate one for calculations in general multi-quark channels.

Finally let us note that, apart from the intrinsic interest in di-baryon resonances as consequences of the color degree of freedom within hadrons, such resonances, if isolated, may provide useful constraints on multi-quark phenomenology. Such constraints may be necessary in order to successfully untangle quark and meson effects and obtain a real understanding of the nuclear physics of few nucleon systems.

Table 1: Allowed States of the Six Quark System Consisting of Two Non-Strange Three Quark Clusters in Spatial Ground States

I \ S	3	2	1	0
3		Δ3/2Δ3/2		Δ3/2Δ3/2 Δ_c1/2Δ_c1/2
2	Δ3/2Δ3/2	N1/2Δ3/2 Δ_c1/2N_c3/2	N1/2Δ3/2 Δ3/2Δ3/2 Δ_c1/2Δ_c1/2 Δ_c1/2N_c1/2 Δ_c1/2N_c3/2	Δ_c1/2N_c1/2
1		N1/2Δ3/2 Δ3/2Δ3/2 N_c3/2N_c3/2 Δ_c1/2N_c3/2 N_c1/2N_c3/2	N1/2 3/2 N_c1/2Δ_c1/2 Δ_c1/2N_c3/2 N_c1/2N_c3/2	N1/2N1/2 Δ3/2Δ3/2 Δ_c1/2Δ_c1/2 N_c1/2N_c1/2 N_c3/2N_c3/2 N_c1/2Δ_c1/2
0	Δ3/2Δ3/2 N_c3/2N_c3/2	N_c1/2N_c3/2	N1/2N1/2 Δ3/2Δ3/2 Δ_c1/2Δ_c1/2 N_c1/2N_c1/2 N_c3/2N_c3/2 N_c1/2N_c3/2	

Table 2: Normalization Constants and Coefficients for the Exchange Hyperfine Contribution to Diagonal Hadronic Potentials

Asymptotic State	I	S	ν	c_1	c_2	c_3	c_4
NN	1	0	9	-51	84	0	93
	0	1	9	-51	84	6	66
NΔ	2	2	-1	-6	12	3	-9
	2	1	-9	30	-60	15	-105
	1	2	-9	30	-60	-9	-33
	1	1	-1	2	-4	3	-1
$\Delta\Delta$	3	2	-1	3	12	9	-15
	3	0	1	3	12	9	21
	2	3	-1	3	12	-3	-3
	2	1	9	3	12	-3	57
	1	2	9	3	12	9	-15
	1	0	-9	3	12	9	21
	0	3	1	3	12	-3	-3
	0	1	-9	3	12	-3	57

Table 3: Di-Baryon Channels With Non-Repulsive or Weakly-Repulsive Induced Diagonal Potentials

Asymptotic State	I	S	comment
$\Delta\Delta$	3	0	unbound, weak short range repulsion
	1	2	unbound, weak short range repulsion
	1	0	unbound, weak short range repulsion
	0	3	bound by 3 MeV relative to $\Delta\Delta$
	0	1	bound by 30 MeV relative to $\Delta\Delta$

Table 4: Coefficients for the Energies and Normalizations of Hidden Color States

I	S	ν	a_1	a_2	c_1^c	c_2^c	c_3^c	c_4^c	c_1^h	c_2^h	c_3^h	c_4^h
0	3	1/7	1	3	2	80	70	16	2	80	70	16
3	0	1/7	5	5	2	80	70	16	74	272	114	44
2	2	-1	3	0	10	-32	-10	8	-2	-32	-14	30
2	0	-1	2	0	1	-14	-10	-1	7	-38	-18	37
0	2	-1	0	-1	1	-14	-10	-1	7	-14	-10	23

Table 5: Coefficients For Off-Diagonal Hamiltonian Matrix Elements in the Non-Orthogonal Bases

I	S	e	c_{T1}^{c}	c_{T2}^{c}	c_{T3}^{c}	c_{T4}^{c}	c_{T5}^{c}	c_{T0}^{h}	c_{T1}^{h}	c_{T2}^{h}	c_{T3}^{h}	c_{T4}^{h}	c_{T5}^{h}
0	3	4	8	16	-2	1	1	0	8	16	-2	1	1
3	0	4	8	16	-2	1	1	6	8	16	22	21	-1
2	2	0	0	0	0	0	0	-2	0	0	0	0	2

Table 6: Results of Mixing Calculations for Two State Channels

ordinary baryonic content	I	S	comment
ΔΔ	0	3	bound by 260 MeV relative to ΔΔ
ΔΔ	3	0	bound by 30 MeV relative to ΔΔ
NΔ	2	2	unbound relative to NΔ

Appendix

The possible configurations of a spatially symmetric three quark cluster in an overal color singlet six quark state are

$$N1/2,\ \Delta 3/2,\ \Delta_c 1/2,\ N_c 3/2,\ N_c 1/2 \qquad \text{(A1)}$$

where by the subscript 'c' we mean that the state transforms as a color octet. These configurations represent the totally antisymmetrized states which can be constructed from basis states in the spin, color and isospin sectors having well-defined permutational symmetries with respect to the permutation group S_3. The appropriate basis states and the Clebsch-Gordan decomposition corresponding to the choice of a (ρ, λ) basis for the mixed representation of S_3 have been given previously in Ref. 23 and are not reproduced here. Pairs of three quark states from (A1) are coupled to definite total spin, isospin and zero net color in the standard manner, and antisymmetrized, where required, with respect to cluster interchange. Such states then consist of a sum of terms, each with specific joint $S_3^{123} \times S_3^{456}$ symmetries in the spin, isospin and color sectors, where the superscripts '123' and '456' are particle labels and represent a particular choice of the partitioning of quarks between clusters. All calculations are then performed using $S_3^{123} \times S_3^{456}$ symmetry bases for each sector. This requires only the evaluation of the matrix elements of the permutation operator, π_{36}, relative

to these bases. We order the symmetry basis in the spin (isospin) sector as follows:

$$\begin{array}{ll} S=0 & \rho\rho,\ \rho\lambda,\ \lambda\rho,\ \lambda\lambda,\ SS \\ S=1 & \rho\rho,\ \rho\lambda,\ \lambda\rho,\ \lambda\lambda,\ S\rho,\ \rho S,\ S\lambda,\ \lambda S,\ SS \\ S=2 & S\rho,\ \rho S,\ S\lambda,\ \lambda S,\ SS \\ S=3 & SS \end{array} \qquad \text{(A2)}$$

where S is the symmetric (spin 3/2) three quark spin wavefunction and the couplings to the desired total spin are suppressed. Similarly we order the symmetry basis for the (zero net) color sector as

$$AA,\ \rho\rho,\ \rho\lambda,\ \lambda\rho,\ \lambda\lambda \qquad \text{(A3)}$$

where A is the antisymmetric three quark color state. The matrix elements of π_{36} with respect to these bases are given in Table A1. Spin and isospin matrix elements are identical so we display only the former.

Of the spin, color and isospin matrix elements required in (8), only those of the spin operators $\underset{\sim}{S}_i \cdot \underset{\sim}{S}_j$ in the S=2,3 channels have not been previously evaluated. These may be obtained, using the transformation properties of S, A, ρ and λ under S_3, from the set listed in Table A2.

Owing to the symmetries of the spatial wavefunction, Φ in (9), with respect to both cluster interchange and quark interchange within clusters, relations exist between the spatial expectations of two body operators for different particle labels i,j. These relations are listed in Table A3, any choice of i,j in a given set serving to define the quantity s_T, n_T, B_T, $b_{Tsn_{(J)}}$, $b_{Tns_{(J)}}$, b_{Tnc}, or b_{Tni} which occurs on the right hand side. The notation therein is as follows. (ij) represents either of the possible two body operators, $\delta^3(r_{ij})$ or $r_{ij}^{\ 2}$, with $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. Matrix elements of the former are denoted by a superscript 'h', those of the latter by a superscript 'c'. $\langle f \rangle^{IJ}_D$ represents the direct spatial matrix element of an operator f, namely,

$$\int d\tau\, \Phi_I(123;456)^* f \Phi_J(123;456) \qquad \text{(A4)}$$

with $d\tau$ as in the text, and $\langle f \rangle^{IJ}_E$ the corresponding exchange matrix element,

$$\int d\tau \Phi_I(123;456)^* f \Phi_J(126;453). \qquad \text{(A5)}$$

In exactly the same way we define

$$k_{TD} = \langle K \rangle^{IJ}_D$$
$$k_{TO} = \langle K \rangle^{IJ}_E \qquad \text{(A6)}$$

where K is the kinetic energy operator which, in terms of the natural coordinates of the 123;456 clustering, can be written

$$-\frac{1}{2m}\left(\nabla^2_{\rho_{123}} + \nabla^2_{\lambda_{123}} + \nabla^2_{\rho_{456}} + \nabla^2_{\lambda_{456}}\right) - \frac{1}{3m}\nabla^2_{R_{123;456}} \qquad \text{(A7)}$$

and

$$B_T = \langle 1 \rangle^{IJ}_E. \qquad \text{(A8)}$$

The subscripts I,J in (A4)-(A8) refer to the fact that the variational parameters for the states $|I\rangle$, $|J\rangle$ will, in general, be different. In the event that either the states are the same or the variational parameters for them are the same, we have the additional relation

$$b_{Tsn_{(J)}} = b_{Tns_{(J)}} \equiv b_{sn}. \qquad \text{(A9)}$$

In this case we drop the subscript 'T' on the spatial matrix elements of (A6) and Table A3. Note that all spatial matrix elements may be evaluated by Gaussian integration. Although not terribly complicated we do not record the resulting expressions here. In the case I=J they reduce to those previously given in Ref. 23.

Table A1: Matrix Elements of the Permutation Operator, π_{36}*

Spin

S=0

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 \\ & & 1 & 0 & 0 \\ & & & 1/3 & -2\sqrt{2}/3 \\ & & & & -1/3 \end{bmatrix}$$

S=1

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 1/3 & 0 & 0 & 0 & -2\sqrt{2}/3 & 0 & 0 & 0 \\ & & 1/3 & 0 & 2\sqrt{2}/3 & 0 & 0 & 0 & 0 \\ & & & 5/9 & 0 & 0 & -2\sqrt{2}/9 & 2\sqrt{2}/9 & -2\sqrt{10}/9 \\ & & & & -1/3 & 0 & 0 & 0 & 0 \\ & & & & & -1/3 & 0 & 0 & 0 \\ & & & & & & 7/9 & 2/9 & -\sqrt{20}/9 \\ & & & & & & & 7/9 & \sqrt{20}/9 \\ & & & & & & & & -1/9 \end{bmatrix}$$

S=2

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 \\ & & 1/3 & 2/3 & -2/3 \\ & & & 1/3 & 2/3 \\ & & & & 1/3 \end{bmatrix}$$

S=3 $\begin{bmatrix} 1 \end{bmatrix}$

Table A1 (continued)

Color

$$\begin{bmatrix} 1/3 & 2\sqrt{2}/3 & 0 & 0 & 0 \\ & -1/3 & 0 & 0 & 0 \\ & & -1 & 0 & 0 \\ & & & -1 & 0 \\ & & & & 1 \end{bmatrix}$$

*All matrices are symmetric and defined relative to the bases given in the Appendix. Elements not explicitly shown may be obtained by transposition.

Table A2: A Generating Set for the Matrix Elements of the Spin Operators $\underset{\sim}{S}_i \cdot \underset{\sim}{S}_j$ in the S=2,3 Sectors*

$\langle SS(33) \vert \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j \vert SS(33) \rangle$	$= 1/4$	all ij
$\langle SS(22) \vert \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j \vert SS(22) \rangle$	$= \begin{cases} 1/4 \\ -1/12 \end{cases}$	ij=12,13,23,45,46,56 otherwise
$\langle SS(22) \vert \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j \vert S\rho(22) \rangle$	$= \begin{cases} \pm 1/2\sqrt{3} \\ 0 \end{cases}$	$ij = \begin{cases} 14,24,34 \\ 15,25,35 \end{cases}$ otherwise
$\langle S\rho(22) \vert \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j \vert S\rho(22) \rangle$	$= \begin{cases} -3/4 \\ 1/4 \\ 0 \end{cases}$	ij=45 ij=12,13,23,26,26,36 otherwise
$\langle S\rho(22) \vert \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j \vert \rho S(22) \rangle$	$= \begin{cases} \pm 1/4 \\ 0 \end{cases}$	$ij = \begin{cases} 14,25 \\ 24,15 \end{cases}$ otherwise

*$\langle SS(S,S_z)\vert = \langle S_{123}S_{456}(S,S_z)\vert$ etc.. In applying permutational arguments note that $\vert SS(22)\rangle$ is antisymmetric with respect to cluster interchange.

Table A3: Symmetries Among Spatial Matrix Elements

$\langle (ij)^{c,h} \rangle^{IJ}_{D}$	$= \begin{cases} s_T^{c,h} \\ s_T^{c,h} n_T^{c,h} \end{cases}$	ij=12,13,23,45,46,56 otherwise
$\langle (ij)^{c,h} \rangle^{IJ}_{E}$	$= \begin{cases} s_T^{c,h} B_T^{c,h} \\ s_T^{c,h} b_{Tsn_{(J)}} \\ s_T^{c,h} b_{Tns_{(J)}} \\ s_T^{c,h} b_{Tnc} \\ s_T^{c,h} b_{Tni} \end{cases}$	ij=12,45 ij=13,23,46,56 ij=16,26,34,35 ij=14,15,24,25 ij=36

ACKNOWLEDGEMENTS

The author would like to thank the Theoretical Physics Group at LBL for their hospitality during the course of this work and the Natural Sciences and Engineering Council of Canada for its financial support.

References

1) T. Kamae, Nucl. Phys. A374(1982)25c

2) D. Bugg, Nucl. Phys. A374(1982)95c

3) M.M. Makarov, Sov. Phys. Usp. 25(1982)83

4) A.B. Wicklund, AIP Conf. Proc. 95 "High Energy Spin Physics--1982", Brookhaven National Laboratory, ed. Gerry M. Bunce (American Institute of Physics, New York, 1983)p.168; N. Hiroshige and W. Watari, ibid., p.181; F. Lehar, ibid., p.187; Akira Masaike, ibid., p.199; Akihiro Yokosawa, ibid., p.223; R. Bertini et al., ibid., p.231; W. Grüebler et al., ibid., p.234; F.V. Adamyan, ibid., p.252

5) L.G. Dakhnov et al., Sov. J. Nucl. Phys. 36(1982)83

6) Susumu Furuichi et al., Prog. Theor. Phys. 67(1982)547

7) Mitsuo Akemoto et al., Prog. Theor. Phys. 67(1982)554

8) Noburo Hiroshige et al., Prog. Theor. Phys. 68(1982)2074

9) M.M. Makarov et al., Phys. Lett. 122B(1983)343

10) T. Siemiarczuk et al., Phys. Lett. 128B(1983)367

11) A. Saha et al., Phys. Rev. Lett. 51(1983)759

12) V. Konig et al., J. Phys. G 9(1983)L211

13) I. Auer et al., Phys. Rev. Lett. 51(1983)1411

14) K.H. Althoff et al., Bonn University Preprint Bonn-HE-83-15(1983)

15) N. Katayama et al., University of Tokyo Preprint UT-HE-83/14(1983)

16) V.G. Vovchenko et al., Sov. J. Nucl. Phys. 37(1983)90

17) M. Harvey, in "Proceedings of the C.A.P. Summer School on Progress in Nuclear Dynamics", Vancouver Island, 1982 (unpublished)

18) I. Bender and H.G. Dosch, Fortsch. Phys. 30(1982)633

19) D. Robson, Progress in Part. and Nucl. Phys. 8(1982)257

20) Y. Suzuki and K.T. Hecht, Phys. Rev. C27(1983)299; Phys. Rev. C28 (1983)1458

21) M. Cvetič et al., Nucl. Phys. A395(1983)349

22) A. Faessler et al., Nucl. Phys. A402(1983)555

23) Kim Maltman and Nathan Isgur, Phys. Rev. D29(1984)952

24) M. Rho, AIP Conf. Proc. 97 "The Interaction Between Medium Energy Nucleons in Nuclei--1982", Indiana University Cyclotron Facility, ed. H.O. Meyer (American Institute of Physics, New York, 1983)p.350; G.E. Brown, ibid., p.389

25) R.L. Jaffe, Phys. Rev. Lett. 38(1977)195

26) A.Th.M. Aerts et al., Phys. Rev. D17(1978)260

27) P.J. Mulders, Phys. Rev. D25(1982)1269; Phys. Rev. D26(1982)3039

28) P.J. Mulders, A.T. Aerts and J.J. DeSwart, Phys. Rev. D21(1980)2653

29) P.J. Mulders and A.W. Thomas, J. Phys. G 9(1983)1159

30) R.L. Jaffe and F.E. Low, Phys. Rev. D19(1979)2105

31) K.F. Liu and C.W. Wong, Phys. Lett. 113B(1982)1

32) Yu. F. Smirnov et al., Sov. J. Nucl. Phys. 27(1978)456; I.T. Obukhovsky et al., Sov. J. Nucl. Phys. 31(1980)269

33) M. Oka and K. Yazaki, Phys. Lett. 90B(1980)41

34) M. Cvetič et al., Phys. Lett. 93B(1980)489

35) Jack Paton and Nathan Isgur, Phys. Lett. 124B(1983)247

36) S. Theberge, A.W. Thomas and Gerald A. Miller, Phys. Rev. D22 (1980)2838

37) Nathan Isgur and Gabriel Karl, Phys. Rev. D18(1978)4187; Phys. Rev. D19(1979)2653; Phys. Rev. D20(1979)1191; Phys. Rev. D21(1980)3175

38) Roman Koniuk and Nathan Isgur, Phys. Rev. D21(1980)1868; Roman Koniuk, Nucl. Phys. B195(1982)452

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