In search of time crystalline behavior in Kerr optical frequency combs

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ABSTRACT

In this invited article, we report the experimental demonstration of the simultaneous coherent locking of two independent lasers with arbitrary multi-FSR (free spectral range) frequency separation to a Kerr microcomb soliton, resulting in the creation of *synthetic* microcomb soliton crystals. Each of the two pumps is self-injection-locked to its neighboring microcavity mode and acts as an arbiter linking the microcomb to the cavity. We show that the beating of the two pumps creates a manifest discrete symmetry and that certain microcomb states generated in this pumping scheme spontaneously break this symmetry, thereby realizing *dissipative* discrete time crystals. Apart from introducing a powerful platform leveraging advanced photonics for the creation and scientific exploration of various types of dissipative time crystals and their properties, our results constitute a decisive step towards the two-point locking of Kerr microcombs with moderate bandwidths much smaller than an octave which cannot be self-referenced through standard approaches such as the $f-2f$ technique.[*]

Keywords: Dissipative Kerr Solitons, High-Q Resonators, Optical Frequency Combs, Soliton Crystals, Discrete Time Crystals, Dissipative Time Crystals

1. INTRODUCTION

Time crystals (TC) extend the familiar notions of crystallinity and order from space to the time domain.[1] The idea and word coinage appeared in the literature in 2012[2] and fall along the temporal evolution of the concept of time from Newton to Einstein. Newton believed that time was absolute, while Einstein suggested that it was relative and should be treated on the same footing as space, hence his introduction of "*spacetime*." In a sense, the notion of time crystallinity extends the equitable treatment of time and space one step further. TCs are also closely related to symmetry, a prevalent concept in physics underpinning conservation laws, and to spontaneous symmetry breaking (SSB), a universal theme appearing in various branches of physics,[3] which was suggested by Landau as a criterion for classifying various states of matter.[4] TCs are hence being studied as a new state of matter about which much is yet to be discovered.

While the initial proposal for observing TCs and continuous time SSB in equilibrium systems was faced with criticism and had to be abandoned,[5,6] research on discrete time-translation symmetry (TTS) and its spontaneous breaking in periodically-forced systems was shown to be a profitable endeavor leading to the rich study of *discrete* time crystals (DTCs).[7,8] In a DTC, the drive of the system (the Hamiltonian describing it) introduces a discrete TTS which is spontaneously broken by the system response (a measured observable) into another discrete TTS with a temporal period which is an integer ($n > 1$) multiple of the drive periodicity.[1] This period multiplications signature of DTCs can also be traced by sub-harmonic generation in the systems. The SSB resulting in DTC formation is enabled by the interaction of many entities in the system and is robustly observed over a range of system parameters with respect to their fluctuations.

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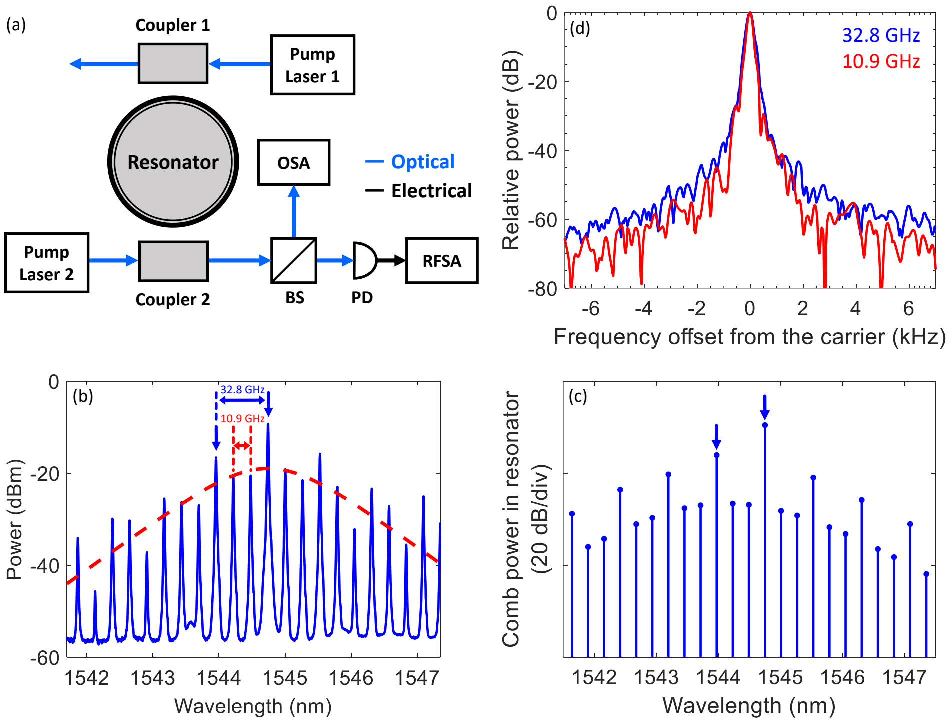

Figure 1. Experimental and matching numerical data. (a) Setup; OSA: optical spectrum analyzer, BS: beam splitter, PD: photodetector, RFSA: radio frequency spectrum analyzer. (b) Experimental microcomb spectrum. Red dashed line is the hyperbolic secant envelope. The two pumps are marked by blue arrows. (c) Numerical data with experimental parameters, showing excellent agreement. The blue arrows point at the pumps. (d) Demodulated RF signal (beatnote) of the two pumps (blue, carrier 32.8 GHz, without soliton formation) and of adjacent comb harmonics (red, carrier 10.9 GHz, with solitons).

Thus far, the study of DTCs has mainly focused on isolated, so-called *closed*, systems in which coupling with the environment is vanishingly small.[9] This appeal is partially because of the curious and creative ways some closed physical systems find to avoid thermalization, i.e., heat death and loss of temporal crystalline order resulting from the continuous supply of energy from the periodic drive. Yet, to unleash the full potential of TCs, particularly for real-world applications, *dissipative* DTCs need to be explored as well. In this work we show that dissipative DTCs can be realized in a group of Kerr-nonlinear resonators with certain pumping schemes.

Kerr-nonlinear optical systems offer a prime test-bed for the exploration of SSB, whose appeal has grown stronger over the past decade in light of milestone demonstrations of frequency combs in high-Q (quality factor) optical microcavities in various crystalline and integrated material platforms.[10,11] In these experiments, a significant practical step was soliton self-injection locking in which the pump laser is locked to the cavity mode it is pumping as a result of the Rayleigh back-scattering of the laser light from the microresonator back into the laser cavity. This approach was originally utilized in commercial microcomb-based products utilizing whispering-gallery-mode (WGM) resonators (see, e.g., Refs.[12,13]) and attracted much interest among academic researchers more recently, especially because it empowers turnkey generation of stable soliton microcombs.[14–16] While different aspects of self-injection-locked microcomb generation have been explored, simultaneous locking of independent lasers with arbitrarily large spectral separation has not been demonstrated thus far. Most recent theoretical and experimental studies hint at numerous explicit technological advantages for driving a resonator certain temporally structured pumps, benefits such as deterministic broadband soliton formation, enhanced stability, significantly reduced RF signal phase noise, and the possibility of two-point locking and stabilization (a necessity for precision metrology and timekeeping) *without* the exacting requirement of microcomb octave-spanning spectral coverage.[17–20] We note that available commercial modulator bandwidths do not allow generating sidebands multiple cavity free spectral ranges (FSRs) away from the pump in a microresonator,

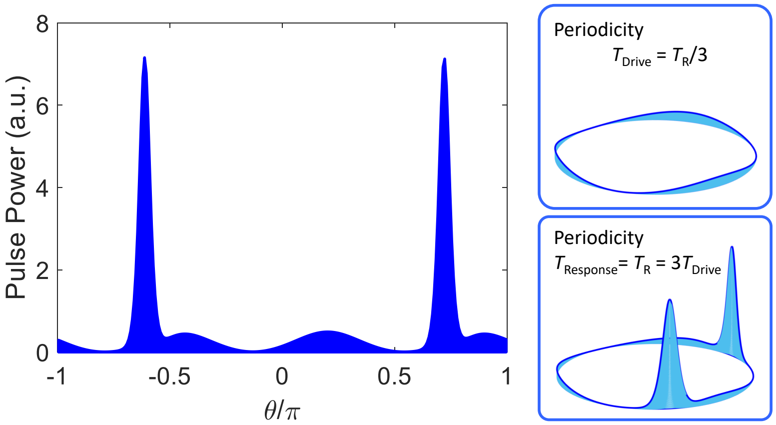

Figure 2. Left panel: Waveform found from the fast Fourier transform of the waveform shown in Figure 1. Top right panel: The 3D snapshot of the waveform when no soliton is formed in the microcavity. The waveform is periodic with $T_{\mathrm{R}}/3$, T_{R} being the round-trip time. This periodicity is defined by the drive of the system. Bottom right panel: The waveform in the left panel plotted on the ring-shaped resonator, showing its periodic with T_{R}, three times that of the drive.

especially in smaller resonators with hundreds of GHz to THz-level FSRs. Therefore, microcomb stabilization utilizing two independent lasers constitutes a decisive empirical leap. When paired with self-injection locking of the laser pump to its neighboring microresonator mode,[21] dual-pump excitation will result in more robust yet simpler experiments, where compact laser sources (e.g., DFB laser diodes) can pump a stable microcomb without an isolator sitting between the pumps and the resonator. This approach is hence readily conducive to chip-scale integration, even in frequency windows where Kerr microcombs are currently not available, and can replace complex designs which have been proposed to extend the reach of microcombs to currently 'uncharted' frequency windows.[22]

Besides the merits enumerated above, simultaneous driving of a microcomb with two independent lasers separated by multiple cavity free-spectral ranges (FSRs) defines a manifest discrete symmetry through the beating frequency of the two pumps. This symmetry can spontaneously be broken by pulse formation resulting from the nonlinear interaction of a large number of photons in the Kerr cavity. Certain soliton distributions in the potential lattice defined by the beating of the pump lasers create period-multiplying states which qualify as DTCs. Our photonic platform readily supports realizing DTCs with dramatic TTS breaking[23] and even novel DTC types carrying defects (the temporal equivalent of solid-state spatial crystal point defects, such as vacancies, dislocations, and interstitials).[24] It will also make possible the demonstration of unobserved and unexplored aspects of time crystalline states such as DTC interactions and phase transitions. Given the stable and dissipative nature of microcomb formation in this Kerr-nonlinear system and the capacity of time crystals for overcoming coherence time limitations of some current quantum technologies, experimental realization of such dissipative DTCs adds a new platform to the few ones supporting dissipative DTC formation which have been identified thus far[25,26] and has important far-reaching ramifications. Here, we report achieving this milestone by combining dual-pump excitation and self-injection locking in a high-Q WGM resonator. The coherent nature of the locking replicates optical Cherenkov dispersive wave (DW) radiation arising from higher-order resonator dispersion and avoided mode crossing. It therefore also gives rise to *synthetic* soliton crystals.[19,27–29]

Figure 1 illustrates our experimental setup schematic and shows a typical set of experimental and matching theoretical data. The resonator was fabricated out of a magnesium fluoride ($\mathrm{MgF_2}$)[21] cylindrical preform by mechanical polishing. The preform rim was shaped into an oblate spheroid optimized for evanescent field coupling with a free space beam. Resonator radius was approximately 1.06 mm, while the radius of the vertical curvature was 0.2 mm. The resonator had an FSR of 32.8 GHz and loaded resonance bandwidth of approximately 200 kHz. Resonator group velocity dispersion at the pumping frequency was nearly -4.9 $\mathrm{ps^2/km}$. The resonator generates microcombs soliton trains if pumped with mW level optical power at around 1545 nm. The criteria for soliton pulse train generation was the observation of a stable sech-shaped frequency comb spectral envelope as well as a low-noise radio frequency (RF) signal on a fast photodiode.[30] A representative spectra corresponding to SSB and DTC formation is plotted in Fig. 1(b), for 3 FSRs separating the driving lasers. The pumps are marked by blue arrows and the red dashed curve shows the hyperbolic secant soliton envelope. The stronger pump was kept nearly at 1545 nm. The matching numerical modeling spectra is depicted in Fig. 1(c); excellent agreement between experiment and theory is observed. Sub-harmonic generation (harmonics between the pumps) is clear in both theoretical and experimental data. The corresponding time-domain pulse train per round-trip time is shown in Fig. 2 (left panel), with a snapshot of the pulse train traveling around the resonator circumference (bottom right panel). The waveform in the absence of any soliton formation is depicted in the top right panel of Figure 2. Comparing the top and bottom right panels, it is obvious that Figs. 1(b,c) represent a period-tripling DTC. Elsewhere, we have shown that other frequency separations between the pumps can also generate the same effect, and by changing this beating frequency between the pumps, realizing various period-multiplying DTC states and DTCs carrying defects is possible.[24] Additionally, besides the soliton formation regime explored here, we have shown theoretically that DTC formation can also be observed in the modulational instability regime, in a reliable and repeatable manner.[31]

2. CONCLUSION

We demonstrated the simultaneous coherent locking of two independent lasers with arbitrary multi-FSR frequency separation to a Kerr microcomb soliton, resulting in the creation of *synthetic* microcomb soliton crystals and dissipative DTCs. Besides a powerful system leveraging the well-developed platform and techniques of nonlinear photonics for the creation and investigation of various DTCs and germane effects, our results constitute an important step towards the experimental demonstration of two-point locking of Kerr microcombs with moderate bandwidths below an octave which cannot be self-referenced through standard approaches such as the $f-2f$ technique.

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