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<b>ENGINEERING NOTE</b>		MD1111	M 5584	1 OF 15
AUTHOR	DEPARTMENT	LOCATION	DATE	
R. B. Meuser	Mechanical Engineering	Berkeley	September 24, 1980	
PROGRAM - PROJECT - JOB				
High-Field Magnet Development				
Analysis				
TITLE				
Circumferential Displacements of Thin Cosine-Theta Dipole Coils.				

## INTRODUCTION

Circumferential displacements and the resulting field-quality aberrations are investigated for the simplest possible mathematical model of a superconducting dipole: the winding is thin, has a  $\cos \theta$  current distribution, and has a linear stress-strain curve. There is no friction between the coil and the surrounding rigid structure.

## NOMENCLATURE

$f$	Local circumferential Lorentz body force component per unit circumference (+)	
$f_0$	Max. value of $f$ .	
$F$	Internal circumferential force (+ for compression)	
$F_0$	$F$ at $\theta = 0$	
$F_1$	$F$ at $\theta = 90^\circ$	} Everything is per unit length $L$ to $r\theta$ plane
$a$	Radius of coil	
$\Delta$	Cross-sectional area of coil	
$E$	Elastic modulus of coil	
$\theta$	Azimuthal coordinate of point on coil	
$\epsilon$	Local strain	
$\Delta''$	Local circumferential displacement, total. (+)	
$\Delta'$	Circ. disp. for initial prestress	
$\Delta$	Additional circ. disp. caused by body forces.	
$\Delta_1$	$\Delta$ at $\theta = 90^\circ$	
$N, M$	See Eq. 22, 23	
$j$	Local lineal current density, current per unit circum	
$j_0$	$j$ at $\theta = 0$	
$j', j'_0$	$j, j_0$ after displacement	
$C_n$	See Eq. 29	
$b$	Iron inside radius	
$\rho$	Normalizing reference radius	
$B_0$	Magnetic field in aperture.	

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circumferential

We apply local Lorentz body forces (force per unit circumference) of

$$f = f_0 \sin 2\theta$$

In terms of the magnet parameters,  $f_0$  can be expressed as

$$f_0 = \frac{1}{4} \mu_0 J_0^2 \left[ 1 + \left( \frac{a}{b} \right)^2 \right] = 2 \left( \frac{B_0^2}{2\mu_0} \right) \frac{1}{1 + \left( \frac{a}{b} \right)^2}$$

There is initially a pre-stress circumferential <sup>compressive</sup> force  $F'$

Case I: For  $\frac{1}{2} f_0 a \leq F'$ ,  $\Delta = 0$  at  $\theta = 90^\circ$   
After application of the body forces  
the local internal circum. force is

$$F = F' + \frac{1}{2} f_0 a \cos 2\theta$$

$$\text{at } \theta = 0^\circ \quad F = F_0 = F' + \frac{1}{2} f_0 a$$

$$\text{at } \theta = 90^\circ \quad F = F_1 = F' - \frac{1}{2} f_0 a$$

The circumferential displacement is

$$\Delta = \frac{1}{4} \frac{f_0 a^2}{AE} \sin 2\theta \quad \rightarrow$$

For  $\theta = 0$  and  $90^\circ$ ,  $\Delta = 0$

Case II: For  $\frac{1}{2} f_0 a \geq F'$ ,  $F = 0$  at  $\theta = 90^\circ$

$$F = \frac{1}{2} f_0 a (1 + \cos 2\theta)$$

$$\Delta = \frac{a}{AE} \left[ \left( \frac{1}{2} f_0 a - F' \right) \theta + \frac{1}{4} f_0 a \sin 2\theta \right]$$

## FIELD ABERRATIONS

$$\text{LET } \Delta = -aN\theta - aM \sin 2\theta$$

$$\text{or } \theta' - \theta = N\theta + M \sin 2\theta$$

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The relative multipole coefficients of the field are

$$\text{For } M: \quad \frac{C_3}{C_1} = -K_3 \frac{3}{2} M$$

$$\frac{C_n}{C_1} = 0 \quad \text{for } n \neq 3$$

where  $K_n = \left(\frac{\rho}{a}\right)^{n-1} \frac{1 + (a/b)^{2n}}{1 + (a/b)^2}$

$$\text{For } N: \quad \frac{C_n}{C_1} = -K_n \frac{2n}{n^2-1} (-1)^{(n+1)/2} N \quad \text{for } n=3,5,7,\dots$$

and zero for  $n \neq 3,5,7,\dots$

In terms of the body forces and pre-stress force:

Case I:

$$C_3/C_1 = K_3 \cdot \frac{3}{8} \frac{f_0 a}{AE}$$

$$C_n/C_1 = 0 \quad \text{for } n \neq 3$$

Case II:

$$C_3/C_1 = K_3 \frac{3}{4} \frac{1}{AE} (f_0 a - F')$$

$$C_n/C_1 = K_n (-1)^{(n+1)/2} \frac{2n}{n^2-1} \frac{1}{AE} \left(\frac{1}{2} f_0 a - F'\right)$$

$$\text{for } n=5,7,9,\dots$$

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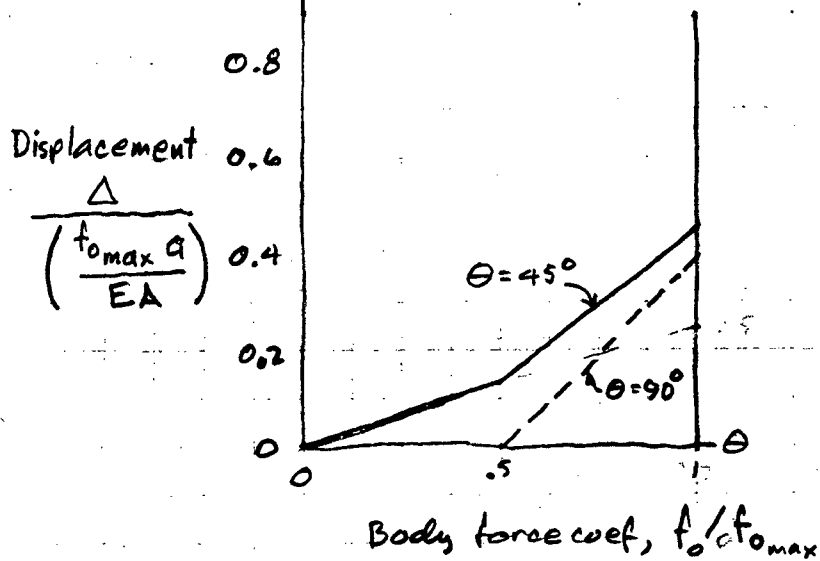
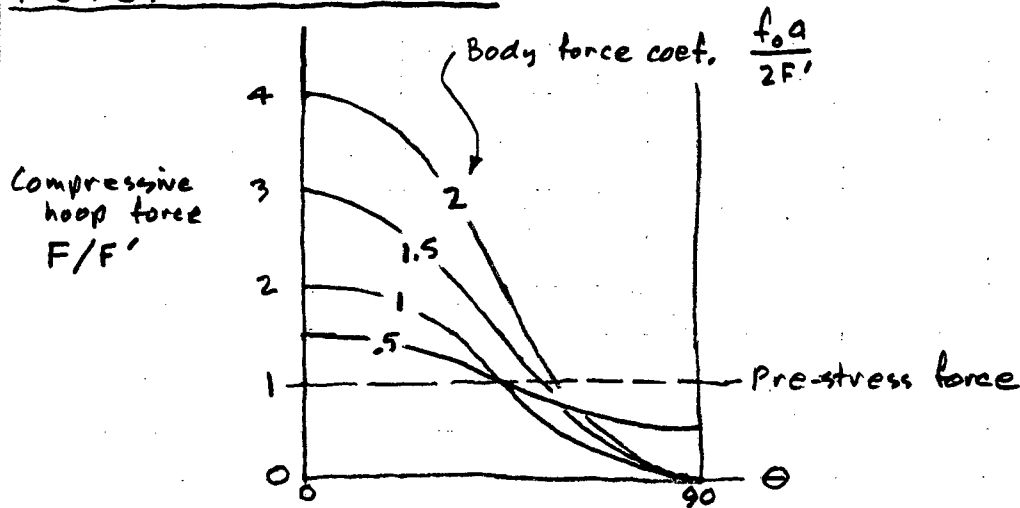
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INTERPRETATION

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DISPLACEMENTS

For a dipole magnet having a thin  $\cos \theta$  winding the circumferential body force on an element of the winding is  $f a d\theta$  where

$$f = f_0 \sin 2\theta \quad (1)$$

For equilibrium, the internal hoop force  $F(\theta)$  is

$$F = F_0 - \int_0^\theta f a d\theta \quad (2)$$

Upon inserting  $f$  from eq. 1 and integrating we obtain

$$F = F_0 - \frac{1}{2} f_0 a (1 - \cos 2\theta) \quad (3)$$

At  $\theta = 90^\circ$ ,  $F = F_1$ , or

$$F_1 = F_0 - f_0 a \quad (4)$$

The local strain is

$$\epsilon = \frac{F}{AE} \quad (5)$$

The change in length of an element is  $\epsilon a d\theta$ , so the local displacement is

$$\Delta'' = \int_0^\theta \epsilon a d\theta \quad (6)$$

Upon substituting  $F$  from Eq 3 into Eq 5, and  $\epsilon$  from that into Eq 6 and integrating we get

$$\Delta'' = \frac{a}{AE} \left[ (F_0 - \frac{1}{2} f_0 a) \theta + \frac{1}{4} f_0 a \sin 2\theta \right] \quad (7)$$

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or using Eq 4

$$\Delta'' = \frac{a}{AE} \left[ (F_1 + \frac{1}{2} f_0 a) \theta + \frac{1}{4} f_0 a \sin 2\theta \right] \quad (8)$$

Initially there are displacements  $\Delta'(\theta)$  resulting from the pre-stress circum. force  $F_1$ , uniform in  $\theta$ , of

$$\Delta' = \int_0^\theta \frac{F_1 a}{AE} d\theta = \frac{F_1 a}{AE} \theta \quad (9)$$

which has a value at  $\theta = 90^\circ$  of

$$\Delta'_1 = \frac{F_1 a}{AE} \frac{\pi}{2} \quad (10)$$

Case I: No displacement at  $\theta = 90^\circ$ ,  $F_1 \geq 0$

The local change in displacement with application of the body forces - which is what we are after - is

$$\Delta = \Delta'' - \Delta' \quad (11)$$

At  $\theta = 90^\circ$ , then

$$\Delta_1 = \Delta''_1 - \Delta'_1 = 0 \quad (12)$$

Substituting  $\Delta'_1$  from Eq 10 and  $\Delta''_1$  from Eq. 8 for  $\theta = \pi/2$  into Eq 12 and solving for  $F_1$  we obtain

$$F_1 = F' - \frac{1}{2} f_0 a \quad (13)$$



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Substituting  $F_1$  from Eq. 13 into Eq. 8 we obtain

$$\Delta'' = \frac{a}{AE} \left[ F' \theta + \frac{1}{4} f_0 a \sin 2\theta \right] \quad (14)$$

Substituting Eq. 14 and Eq. 9 into Eq. 11 we get for the change in displacement  $\Delta$ ,

$$\Delta = \frac{1}{4} \frac{f_0 a^2}{AE} \sin 2\theta \quad (15)$$

The local strain  $\epsilon$  is

$$\epsilon = \frac{1}{a} \frac{d\Delta''}{d\theta}$$

and the local internal force  $F$  is

$$F = AE\epsilon$$

Upon differentiating Eq. 14, etc., we get

$$F = F' + \frac{1}{2} f_0 a \cos 2\theta \quad (16a)$$

$$F_0 = F' + \frac{1}{2} f_0 a \quad (16b)$$

$$F_1 = F' - \frac{1}{2} f_0 a \quad (16c)$$

For incipient separation at  $\theta = 90^\circ$  - the limit of validity of Case I - we have  $F_1 = 0$ , so

$$F'_{\min} = \frac{1}{2} f_0 a = \frac{1}{2} (F_1 - F_0) \quad (17)$$

That is, the minimum prestress hoop force,  $F'_{\min}$ , is one-half the integrated circumferential body force,  $f_0 a$  or  $F_1 - F_0$ .

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Case II: No internal force  $F$  at  $\theta = 90^\circ$ .

For this case the change in displacement  $\Delta_1 \geq 0$  at  $\theta = 90^\circ$ .

For  $F_1 = 0$ , Eq. 8 gives:

$$\Delta'' = \frac{a}{AE} f_0 a \left( \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right) \quad (18)$$

So using Eqs 18, 9, and 11 (Eq. 11 is valid for Case II) we get for the change in displacement:

$$\Delta = \frac{a}{AE} \left[ \left( \frac{1}{2} f_0 a - F' \right) \theta + \frac{1}{4} f_0 a \sin 2\theta \right] \quad (19)$$

Since  $\Delta_1$  cannot be negative, this case is valid only if the bracketed term is positive;

$$F'_{\max} = \frac{1}{2} f_0 a \quad (20)$$

which jibes with Eq 17.

The local force is, as before,

$$F = AE \frac{1}{a} \frac{d\Delta''}{d\theta}$$

$$F = \frac{1}{2} f_0 a (1 + \cos 2\theta) \quad (21a)$$

$$F_0 = f_0 a \quad (21b)$$

$$F_1 = 0 \quad (21c)$$

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## EFFECT OF DISPLACEMENTS UPON FIELD QUALITY

The current density (current/circum) is originally

$$J = J_0 \cos \theta \quad (20)$$

A coil element originally situated between  $\theta$  and  $\theta + d\theta$  moves to a new position between  $\theta'$  and  $\theta' + d\theta'$ . The position angle becomes  $\theta'$  and the current density is changed by a factor  $d\theta/d\theta'$ . So the new current density is

$$J' = J_0 \cos \theta' \frac{d\theta}{d\theta'} \quad (21)$$

We express the displacement  $\Delta$  (see Eq. 19)

$$\Delta = -a N \theta - a M \sin 2\theta \quad (22)$$

so

$$\theta' - \theta = N \theta + M \sin 2\theta \quad (23)$$

( $\Delta$  is the CW displacement;  $\theta' - \theta$  is positive for CCW displacement.)

We consider the effects of the two terms separately.

### M term

$$\theta' = \theta + M \sin 2\theta \quad (24)$$

$$\frac{d\theta}{d\theta'} = 1 - 2M \cos 2\theta \quad (25)$$

Eq. 21 then yields,

$$J' = J_0 \cos(\theta + M \sin 2\theta) \times (1 - 2M \cos 2\theta) \quad (26)$$

Let's look at the factor  
 $\cos(\theta + M \sin 2\theta)$

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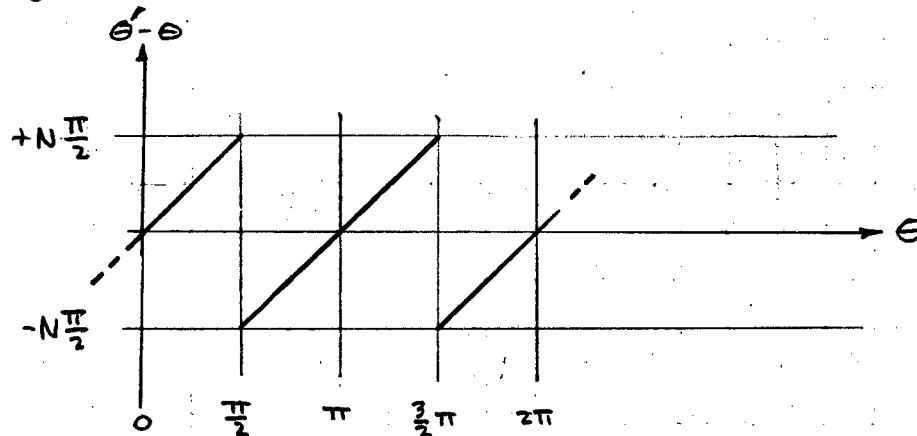
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N term

In the first quadrant the displacement is

$$\theta' - \theta = N\theta$$

The character of the displacement  $\theta' - \theta$  in the other quadrants is as shown



This can be represented by the Fourier series

$$\theta' - \theta = -2N \sum_{m=1,2,3,\dots} \frac{1}{m} \cos \frac{m\pi}{2} \sin m\theta \quad (35)$$

As before, we wish to evaluate  $J'$  given by

$$J' = \int_0 \cos \theta' \frac{d\theta}{d\theta'} \quad (36)$$

To first order in  $N$

$$\frac{d\theta}{d\theta'} = 1 + 2N \sum_{m=1,2,3,\dots} \cos \frac{m\pi}{2} \cos m\theta \quad (37)$$

The equation for  $J'$  becomes

$$J' = \int_0 \left\{ \cos \left[ \theta - 2N \sum_{m=1,2,3,\dots} \frac{1}{m} \cos \frac{m\pi}{2} \sin m\theta \right] \right\} \left[ 1 + 2N \sum_{m=1,2,3,\dots} \cos \frac{m\pi}{2} \cos m\theta \right] \quad (38)$$

We can simplify the term in  $\{ \}$  by using

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$$\begin{aligned}\cos(x-\delta) &= \cos x \cos \delta + \sin x \sin \delta \\ &\approx \cos x + \delta \sin x \quad \text{for } \delta \ll x\end{aligned}$$

That results in a term  $\sin m\theta \sin \theta$  within the summation which is replaced by  $\frac{1}{2} [\cos(m-1)\theta - \cos(m+1)\theta]$ .

The term  $\cos \frac{m\pi}{2}$  for  $m=1,2,3$  is replaced by  $(-1)^{m/2}$  for  $m=2,4,6,\dots$  and zero for  $m \neq 2,4,6,\dots$

The equation for  $J'$  becomes

$$J' = \int_0^1 \left\{ \cos \theta - N \sum_{m=2,4,6,\dots} \frac{1}{m} (-1)^{m/2} [\cos(m-1)\theta - \cos(m+1)\theta] \right\} \left[ 1 + 2N \sum_{m=2,4,6,\dots} (-1)^{m/2} \cos m\theta \right] \quad (39)$$

We write out the summations term by term, collect like terms, discard terms in  $N^2$  and higher order, and let  $m = n+1$ . Finally we get

$$J' = \int_0^1 \left\{ \left(1 - \frac{1}{2}N\right) \cos \theta - N \sum_{n=3,5,7} (-1)^{(n+1)/2} \frac{2n}{n^2-1} \cos n\theta \right\} \quad (40)$$

which has the desired form

$$J' = \sum_{n=1,3,5} J_{0n} \cos n\theta$$

The  $n=1$  term is

$$J'_{01} = \int_0^1 \left(1 - \frac{N}{2}\right) \approx J_0$$

and the higher-order terms are

$$J'_{0n} = -N (-1)^{(n+1)/2} \frac{2n}{n^2-1} \quad n=3,5,7,\dots$$

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As before

$$\frac{C_n}{C_1} = \frac{J_{0n}}{J_{0,1}} \left(\frac{\rho}{a}\right)^{n-1} \frac{1+(a/b)^{2n}}{1+(a/b)^2} \quad \text{for } n=3,5,7,\dots$$

$$\text{and } \frac{C_n}{C_1} = 0 \quad \text{for } n=2,4,6,\dots$$

so for  $n=3,5,7,\dots$ 

$$\frac{C_n}{C_1} = -N (-1)^{(n+1)/2} \frac{2n}{n^2-1} \left(\frac{\rho}{a}\right)^{n-1} \frac{1+(a/b)^{2n}}{1+(a/b)^2} \quad (41)$$

Recall that the displacement change is given by

$$\Delta = \frac{1}{4} \frac{f_0 a^2}{AE} \sin 2\theta \quad \text{for Case I and}$$

$$\Delta = \frac{a}{AE} \left[ \left( \frac{1}{2} f_0 a - F' \right) \theta + \frac{1}{4} f_0 a^2 \sin 2\theta \right] \quad \text{for Case II}$$

and that we have, in this section, expressed the

$$\Delta = -aN\theta - aM \sin 2\theta$$

So M and N are as follows

Case I

$$M = -\frac{1}{4} \frac{f_0 a}{AE}, \quad N = 0 \quad (42)$$

Case II

$$M = -\frac{1}{4} \frac{f_0 a}{AE},$$

$$N = -\frac{1}{AE} \left( \frac{1}{2} f_0 a - F' \right)$$

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Upon substituting these into the formulas for  $C_n/C_1$ , we obtain the following:

Case I

$$\frac{C_3}{C_1} = \left(\frac{\rho}{a}\right)^2 \frac{1+(a/b)^6}{1+(a/b)^2} \cdot \frac{3}{8} \frac{f_0 a}{AE} \quad \left. \vphantom{\frac{C_3}{C_1}} \right\} (44)$$

and  $\frac{C_n}{C_1} = 0$  for all others

Case II

$$\frac{C_3}{C_1} = \left(\frac{\rho}{a}\right)^2 \frac{1+(a/b)^6}{1+(a/b)^2} \cdot \frac{3}{4} \frac{1}{AE} (f_0 a - F') \quad \left. \vphantom{\frac{C_3}{C_1}} \right\} (45)$$

$$\text{and } \frac{C_n}{C_1} = \left(\frac{\rho}{a}\right)^{n-1} \frac{1+(a/b)^{2n}}{(1+a/b)^2} (-1)^{(n+1)/2} \frac{2n}{n^2-1} \frac{1}{AE} \left(\frac{1}{2} f_0 a - F'\right)$$

for  $n = 5, 7, 9, \dots$

(Phew!)

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