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Publication Date

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May 1982

LBL-13874
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A Saturating Chiral Field Theory of Nuclear Matter

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Abstract:

Catastrophe Theory analysis is used to construct a chiral theory of pions which leads to a saturating nuclear matter equation of state. This is achieved by introducing a vector meson field via the Higgs mechanism. The equation of state and the nuclear optical potential is computed. A metamorphosis of the nuclear force is suggested.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Relativistic quantum field theory approach to the study of nuclear matter and finite nuclear structure can account for a large number of data in terms of a few phenomenologically determined parameters. These parameters are adjusted so that relevant nuclear matter properties, such as nuclear matter saturation, its compressibility and the effective nucleon mass in the media are reasonable ad initio. That is, in the mean field approximation, the model is constructed to describe these essential properties reasonably well. Clearly, such an approach must be phenomenological in its very nature, since quantum corrections, pion contribution, two body correlations are known to be important.

The above mentioned method has dealt rather cavalierly with some essential properties of a relativistic field theory. The phenomenological approach yields a Lagrangian that in fact does not have a lower bound for the energy. The best fits are obtained when the scalar meson quartic self interactions are attractive and not repulsive, as they should be. A related problem is that pions, in the linear realization of chiral symmetry, cannot be introduced into the theory without spoiling nuclear matter saturation in an essential way. It was shown by A.K.Kerman and L.D. Miller² and independently by the author³ that the usual chiral theory of pions leads to a theory of nuclear matter that does not have a saturating normal matter ground state if the theory is solved in the mean field approximation. The reason for this is that the spontaneous breaking of chiral symmetry requires the potential functional $U(\phi^2 + \pi^2)$ to have a double minimum as a function of the fields. This leads to the well known cusp catastrophe⁴. The field solutions bifurcate. A typical energy per particle curve

is shown in Fig. 1b. The aim of this work is to show a simple and elegant solution to this problem. We will explicitly show that if a isoscalar vector field ω_μ is introduced into the theory via the Higgs mechanism⁵, these bifurcations disappear and a saturating equation of state can be obtained for a potential functional having a double minimum. Physically, the Higgs mechanism in our case amounts to the statement that the mass of the vector field is not static, but dynamically determined. Thus we will construct a chiral theory of pions in which not only the mass of the nucleon and the scalar meson are spontaneously produced, but also the mass of the vector field.

The bifurcation problem encountered in the linear chiral model is a general one, shared by all theories that have a potential functional $U(\phi)$ possessing a double minimum. To illustrate this, let's consider a model of nuclear matter in which the attraction between nucleons is generated by a scalar meson field ϕ and the repulsion by an isoscalar vector field ω_μ . The Lagrangian is taken to be

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\psi} \left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_N \right) \psi - \frac{1}{2} \left(\frac{\partial}{\partial x_\mu} \phi \right)^2 - U(\phi) \\ & - g_S \bar{\psi} \psi \phi + i g_V \bar{\psi} \gamma_\mu \psi \omega_\mu - \frac{1}{2} m_V^2 \omega_\mu \omega_\mu \end{aligned} \quad \text{.....(1a)}$$

where

$$F_{\mu\nu} = \frac{\partial}{\partial x_\nu} \omega_\mu - \frac{\partial}{\partial x_\mu} \omega_\nu \quad \text{.....(1b)}$$

and

$$U(\phi) = \frac{a}{2} \phi^2 + \frac{b}{3} \phi^3 + \frac{c}{4} \phi^4 \quad \text{.....(1c)}$$

The coefficients $\bar{a} = a/g_s^2$, $\bar{b} = b/(m_N g_s^3)$ and $\bar{c} = c/g_s^4$ are chosen to yield a potential functional having a double minimum shown in Fig. 1a. The values are $\bar{b} = 7.71$ and $\bar{c} = 55.94$ and $C_s = g_s m_N / m_\sigma = 2.0$ and $C_v = g_v m_N / m_\omega = 4.0$. These values for the parameters will yield a potential functional with a double minimum and a saturating nuclear matter state at a saturating density of $\rho_0 = 0.19 / f m^3$ and a binding energy per particle of -16 MeV . We will show that this cannot be a normal nuclear matter state.

The field equations for infinite symmetric nuclear matter are

$$\frac{\partial U}{\partial \phi} = -g_s \rho_s \quad \dots\dots\dots (2a)$$

$$m_v^2 \omega_0 = g_v \rho_v \quad \dots\dots\dots (2b)$$

$$\vec{\omega} = 0 \quad \dots\dots\dots (2c)$$

where the source terms ρ_s and ρ_v are given by

$$\rho_s = \frac{4}{(2\pi)^3} \int^{k_F} d^3k \frac{m_N^*}{\sqrt{k^2 + m_N^{*2}}} \quad \dots\dots\dots (3a)$$

$$\rho_v = \frac{2}{3\pi^2} k_F^3 \quad \dots\dots\dots (3b)$$

$$m_N^* = m_N + g_s \phi \quad \dots\dots\dots (3c)$$

The energy density is

$$\epsilon = \frac{1}{2} \left(\frac{g_v}{m_v} \right)^2 \rho_v^2 + U(\phi) + \frac{4}{(2\pi)^3} \int d^3k \sqrt{k^2 + m_N^{*2}} \quad \dots\dots\dots(4)$$

and the pressure is given by

$$P = \rho_v^2 \frac{d}{d\rho_v} \left(\frac{\epsilon}{\rho_v} \right) \quad \dots\dots\dots(5)$$

We solve the classical field equations numerically. In Fig. 1b the corresponding energy per particle as a function of density is shown. Indeed there is a saturating equation of state at a density of $\rho_0 = .19/fm^3$. But it is not the normal state of nuclear matter since the equation of state does not vanish for vanishing density. The state that should have corresponded to the lowest minimum of $U(\phi)$ is not realized, since it bifurcated with the solution corresponding to the maximum of U . In general, bifurcation will occur when the point $\frac{\partial^3 U}{\partial \phi^3} \approx 0$ is reached. As was explicitly shown by A.K.Kerman and L.D.Miller, this is exactly what happens in the chiral model, where the potential functional is symmetric under the interchange $m_N^* \rightarrow -m_N^*$.

A more judicious choice of the potential functional will not evade the bifurcation problem. We will just get more complicated bifurcations. In fact, the bifurcations will exist even if the potential functional does not have a double minimum. For bifurcations to appear it is necessary that the double derivative of the U , $\partial^2 U / \partial \phi^2$, vanish. The single derivative need not vanish at all. The resolution of this problem lies in the observation that the field equations Eq. (2a, 2b) are decoupled. That is, the value of the omega field does not depend on the value of the sigma field and vice versa. This means that the bifurcations are completely determined by U . To avoid the bifurcations, one should allow for a coupling between the meson fields. For example a term like $\phi^2 \omega_\mu \omega_\mu$ is quite adequate. In the language of Catastrophe Theory, this term will place the Lagrangian in a different catastrophe class. By an explicit calculation we will show that to avoid the cusp catastrophe encountered in the chiral model it is sufficient to spontaneously generate the mass of the nucleon, the scalar and vector fields all at once.

We take the chiral Lagrangian to be

$$\begin{aligned} \mathcal{L} = & - \bar{\psi} (\gamma_\mu \frac{\partial}{\partial x_\mu}) \psi - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{\lambda}{4} (\phi^2 + \hat{\pi}^2)^2 - \frac{\mu^2}{2} (\phi^2 + \hat{\pi}^2) \\ & - g_S \bar{\psi} (\phi + i \hat{\tau} \cdot \hat{\pi} \gamma_5) \psi + i g_V \bar{\psi} \gamma_\mu \psi \omega_\mu - \frac{g_V^2}{2} (\phi^2 + \hat{\pi}^2) \omega_\mu \omega_\mu \\ & - \frac{1}{2} \left(\frac{\partial}{\partial x_\mu} \phi \right)^2 - \frac{1}{2} \left(\frac{\partial}{\partial x_\mu} \hat{\pi} \right)^2 \end{aligned} \quad \dots\dots\dots(6a)$$

with

$$\mu^2 < 0$$

$\dots\dots\dots(6b)$

By shifting to the minimum of the potential functional and spontaneously breaking chiral symmetry we get (setting $\hat{\pi} = 0$ for convenience)

$$\phi = \sigma_0 + \sigma \quad (7)$$

$$\begin{aligned} \mathcal{L} = & -\bar{\psi} \left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_N \right) \psi - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - U(\sigma) - \frac{1}{2} \left(\frac{\partial \sigma}{\partial x_\mu} \right)^2 \\ & - g_S \bar{\psi} \psi \sigma + i g_V \bar{\psi} \gamma_\mu \psi \omega_\mu - \frac{m_V^{*2}}{2} \omega_\mu \omega_\mu \end{aligned} \quad (8)$$

where

$$g_S \sigma_0 = m_N \quad (9a)$$

$$g_V \sigma_0 = m_V \quad (9b)$$

$$m_N^* = m_N + g_S \sigma \quad (9c)$$

$$m_V^* = m_V + g_V \sigma \quad (9d)$$

$$m_\sigma^2 = 2 \lambda^2 \sigma_0^2 \quad (9e)$$

and

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 \left\{ 1 + \frac{1}{2} \frac{g_S}{m_N} \sigma^2 \right\}^2 \quad (10)$$

The nucleon, the scalar meson and the vector meson acquire a mass given by Eq. (9a-9e). The effective mass of the nucleon and the vector meson depend explicitly on the value of the scalar field. They have become density dependent. We note that when the effective mass of the nucleon vanishes, so does the effective mass of the vector meson. In the Lee-Wick state, the vector field becomes Coulomb like.

The field equations for infinite nuclear matter now become

$$\frac{\partial U}{\partial \sigma} - g_V^2 m_{V0}^{*2} = -g_S \rho_S \quad \dots\dots\dots(11a)$$

$$m_{V0}^{*2} = g_V \rho_V \quad \dots\dots\dots(11b)$$

The source terms ρ_S and ρ_V remain the same. These equations are to be compared with those of Eq. (2). There are additional terms due to the self-coupling of the mesonic fields. This is just sufficient, as we will immediately show, to avoid the cusp catastrophe. The above field equations have been studied by C.-G. Källman in a different context.⁶ He showed that they lead to an asymptotically free equation of state.

Given the values of the nucleon and vector meson masses m_N and m_V respectively, there are two additional parameters, g_S and m_S to be determined. The value of the vector coupling constant g_V is determined by

$$g_V = g_S \frac{m_V}{m_N} \quad \dots\dots\dots(12)$$

The energy density $\mathcal{E}$ is now given by

$$\mathcal{E} = \frac{1}{2} \left(\frac{g_v}{m_v} \right)^2 \left(\frac{m_N}{m_N^*} \right)^2 \rho_v^2 + U(\sigma) + \frac{4}{(2\pi)^3} \int d^3k \sqrt{k^2 + m_N^{*2}} \quad \dots\dots\dots(13)$$

The values of g_s and m_σ are chosen so that there is a saturating nuclear matter state at a saturating density of $\rho_0 = .145/\text{fm}^3$, with a binding energy of -16 MeV/particle. This is achieved by taking $g_s = 8.35$ and $m_\sigma = 650$ MeV. In Fig.2a we show the energy per particle as a function of density. There are no bifurcations. The reason for this can be seen from Fig. 2b, where the nucleon effective mass m_N^* is shown as a function of nuclear matter density. The solution $m_N^* = 0$ is an isolated solution, existing only for vanishing nuclear density. The mechanism that caused the bifurcations in the previous calculation is now avoided. The isolation of the Lee-Wick solution, $m_N^* = m_V^* = 0$ follows from the physical fact that Coulomb like solutions cannot be realized in infinite nuclear matter. They have infinite energy density. By allowing the nuclear repulsion to undergo a metamorphosis, that is, to take on a Coulomb like nature, the normal solution is protected from the unwanted bifurcation. This has interesting consequences for finite systems. We know that Coulomb force is realizable in finite systems. Thus we expect that there should be a new class of nuclei in whose interior the vector repulsion becomes long ranged.

From Fig.2b we see another interesting aspect of the model. In previous relativistic field theory models of nuclear matter, the nucleon effective mass vanished for large densities. That is $m_N^* \rightarrow 0$ for $\rho_v \rightarrow \infty$. In the present model that is not the case. In fact $m_N^* \rightarrow \infty$ for $\rho_v \rightarrow \infty$. Thus chiral symmetry is not restored for asymptotic densities. At normal nuclear density the nucleon effective mass is $m_N^*/m_N = .78$. This is a reasonable value for the

effective mass, leading to a reasonable energy and density dependence of the nucleon optical potential]. If we define the optical potential to be

$$E = \sqrt{k^2 + m_N^2} + U_{\text{eff}} \quad \dots\dots\dots(14a)$$

where

$$U_{\text{eff}} = E - \left[(E - g_V \omega_0)^2 + m_N^2 - m_N^{*2} \right]^{1/2} \quad \dots\dots\dots(14b)$$

then in Fig. 3 we show the energy dependence of this potential. The dotted lines correspond to the recent many body calculations of Friedman and Pandharipande.⁷

The low density behavior of the energy per particle shown in Fig. 2a also in good agreement with many body calculations.⁸ Since it is this part of the equation of state that determines the essential features of the single particle properties of nuclear structure, we expect that the present model should reproduce the good predictions made by the relativistic field theory models for matter distribution in finite nuclei.^{1,9} We must note though that the equation of state deviates from a parabolic form very rapidly as the density varies from saturation. At saturation, the predicted nuclear matter compressibility is quite large, $K=650\text{MeV}$ and becomes about 300MeV at a density of $.8\rho_0$.

Such a large compressibility is a common feature of relativistic field theory models that attempt to be theoretically acceptable in the mean field approximation.¹⁰

We have shown phenomenologically, that to obtain a reasonable compressibility ($K=210$ MeV) in the mean field approximation one is forced into a model¹¹ where the quartic self-interactions are attractive. This in theory leads to a Lagrangian that does not have a lower bound in energy. Phenomenologically this is of no great concern, but if a consistent theoretical model is to be entertained, the mean field approximation cannot be the whole story.¹² This observation is strengthened by the fact that in our chiral model the determined value of the pion-nucleon coupling constant g_π is 8.35 instead of $g_\pi \sim 13.5$ as determined from pion-nucleon scattering.

The aim of this work was to show that the bifurcation problem encountered in the linear chiral theory of pions can be avoided by introducing the ω_μ field dynamically. This not only eliminated the bifurcations, but allowed the vector field to become Coulomb like. Thus the Lee-Wick solution of the chiral model will not be realized in infinite nuclear matter.¹³ In the mean field approximation, the model predicts a number of reasonable nuclear properties, whose significance is valuable phenomenologically. There will be very important corrections to be taken into account though. From chiral theory itself we know that the axial coupling constant g_A is not unity but $g_A \approx 1.26$. Second, the nucleon, scalar and vector meson masses were determined only at the tree approximation level. There will be significant changes here also. Furthermore, there will be quantum corrections to the potential functional itself. Thus one of the outstanding problems of relativistic quantum field theory of nuclear matter and nuclear structure is to show that the inclusion of these corrections will lead to a reasonable theory of pions and reasonable properties of nuclear matter and nuclear structure. The suggestion that nuclear forces can undergo a metamorphosis must be answered experimentally.

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Figure Captions

- Fig. 1a. Potential functional $U(\sigma)$ having a double minimum leading to bifurcations of the equation of state.
- Fig. 1b The binding energy per particle as a function of the Fermi momentum. The cusp is clearly visible at $k_F \approx 1.5 \text{ fm}^{-1}$.
- Fig. 2a The binding energy per particle as a function of density for the dynamical chiral model. Here $g_s = 8.35$ and $m_\sigma = 650 \text{ MeV}$.
- Fig. 2b. Nucleon effective mass as a function of density for the dynamical chiral model. Note that $m_N^* = 0$ implies $m_V^* = 0$. The vector field becomes long ranged.
- Fig. 3 . The optical potential as a function of incident energy at normal nuclear density. FP is the Friedman-Pandharipande results of ref.7

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Fig.1a

Fig.1b

Fig.2a

Fig.2b

Fig. 3

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