



Signal Recognition as Influenced by Information Feedback

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Eight human observers were tested on a signal recognition task involving two tones of different amplitudes. The independent variables were (a) three binomial schedules for presenting the two signals, with parameter values 0.2, 0.5, and 0.8, and (b) four conditions varying the information given to an observer about the signal presentation schedules. The information that an observer was given about the presentation schedules markedly influenced hit and false alarm rates (the probabilities of reporting a loud signal when a loud and soft signal, respectively, occurred). The influence of the preceding trial's signal and response on hits and false alarms also varied as a function of both the presentation schedule and the information given about the schedules. A mathematical model of signal recognition is shown to provide a fairly accurate account of the various conditions investigated.

Findings from experiments by Kinchla (1966) and Tanner, Haller, and Atkinson (1967) suggest that signal recognition is a function of both the signal presentation probabilities and the amount of information given observers about these probabilities. Both experiments involved the recognition of two amplitudes of a 1000 Hz tone; whenever a signal was presented, the observer was required to judge whether it was the louder or the softer of two tones. A major independent variable in both studies was the signal presentation schedule. The schedules were binomial sequences of loud and soft tones. For Kinchla the probability of the loud tone (γ) took on three values: 0.25, 0.50, and 0.75; for Tanner *et al.* five values were used: 0.1, 0.3, 0.5, 0.7, and 0.9. In order to compare their findings with research on signal detection (Green and Swets,

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1966), these authors (Kinchla, 1966; and Tanner *et al.*, 1967) defined a *hit* as a report that the loud signal occurred when in fact it did occur, and a *false alarm* as a report that the loud signal occurred when the soft signal was presented.

In Kinchla's experiment two types of information were given to observers about the presentation schedules. In one condition observers were told the signal presentation probabilities at the start of each experimental session and were given feedback identifying the correct response on every trial. In the other condition they were told the presentation probabilities, but were not given trial-by-trial feedback. For both of these conditions Kinchla reported results that were similar to those of signal detection studies (Green and Swets, 1966), *viz.*, that both hit and false alarm probabilities increased as γ increased, although the effect was less pronounced when feedback was omitted.

In the study of Tanner *et al.*, however, observers were not told the signal presentation probabilities and were not given trial-by-trial feedback, and both hit and false alarm rates decreased as γ increased. Under the no-feedback conditions of both studies, hit and false alarm rates were strongly influenced by the signal and response that occurred on the immediately preceding trial; these sequential effects were in sharp contrast with the relatively weak sequential effects that typically have been reported for signal detection experiments (Atkinson and Kinchla, 1965). Results similar to those obtained by Tanner *et al.* have been reported by Parducci and Sandusky (1965) for recognition of visual displacement.

In order to provide a theoretical account of their results, Tanner *et al.* presented a model which incorporates both memory and detection processes. This model will be referred to as the Memory-Recognition Model or more simply, the MR-Model. Tanner *et al.* applied the model to their data and indicated how it might be modified to account for the data obtained in Kinchla's feedback condition.

The study reported here had the following objectives: (a) to replicate the findings of Kinchla in his feedback condition and of Tanner *et al.* in their no-feedback condition; (b) to determine whether the inverse relationship between γ and the hit and false alarm rates, reported by Tanner *et al.*, depends on observers not being informed about changes in the presentation schedule; and (c) to test the ability of the MR-Model to predict performance both in a feedback condition and in three no-feedback conditions.

The MR-Model

The following notation will be used in discussing the experiment and model: S_1 = presentation of the loud signal; S_0 = presentation of the soft signal; A_1 = the response identifying a signal as loud; A_0 = the response identifying a signal as soft; and γ = the presentation probability of the loud signal. Thus on any trial of an experimental session, either S_1 is presented with probability γ , or S_0 with probability $1 - \gamma$. After each presentation, the observer is required to make either an A_1 or A_0 response identifying his judgment of which signal was presented. The observer may or may not be told the signal presentation probabilities, and may or may not be given

feedback after each response. As defined here, the feedback condition involves both trial-by-trial feedback and telling the observer the signal presentation probabilities at the start of a session.

The principal dependent variables are the hit and false alarm probabilities and the (first order) sequential probabilities, defined as follows: $\Pr(A_1 | S_1)$ = the probability of a hit; $\Pr(A_1 | S_0)$ = the probability of a false alarm; $\Pr(A_1 | S_1 A_j S_k)$ = the probability of a hit, given that an A_j was made to an S_k on the preceding trial ($j, k = 0$ or 1); and $\Pr(A_1 | S_0 A_j S_k)$ = the probability of a false alarm, given that an A_j was made to an S_k on the preceding trial ($j, k = 0$ or 1).

A graphic representation of the MR-Model is shown in Fig. 1.² The model assumes three processes: a *memory process* which maintains an image of the signal presented on

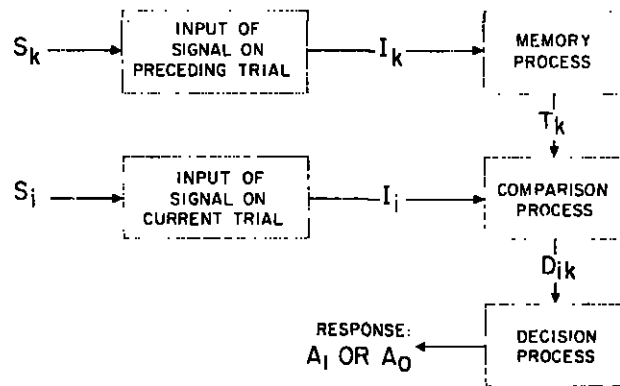


FIG. 1. Schematic representation of the MR-Model.

the preceding trial, a *comparison process* that calculates a difference function on the stored image and the incoming signal, and a *decision process* that selects a response on the basis of the comparing process. We assume that an observer has in memory an image of the signal presented on the immediately preceding trial. This stored image will be referred to as the *trace*. Due to the influence of various noise sources, the trace of signal S_i will take on different values from presentation to presentation and is best described as a random variable T_i . It is assumed that T_i is normally distributed with mean t_i and variance σ_T^2 . More specifically, the trace distributions for the signals S_1 and S_0 have different means, t_1 and t_0 , but a common variance σ_T^2 .

On each trial of the experiment, the observer processes both the presented signal and the trace of the last signal. We shall call the sensory event associated with the occurrence of S_i the *input*, a random variable denoted as I_i , which is normally distributed with mean s_i and variance σ_I^2 . Thus, the two signals S_1 and S_0 are characterized by two input distributions with means s_1 and s_0 but a common variance σ_I^2 .

² A similar model has been presented by Kinchla and Allan (1969) and Kinchla and Smyzer (1967).

The values of s_1 and s_0 are regarded as scaling parameters, and for mathematical convenience are set arbitrarily at $s_1 = 1$ and $s_0 = 0$. For reasons discussed by Haller (1969) it is assumed that t_1 and t_0 depend on γ . The postulated relationship is linear and is specified by the parameter α as follows:

$$t_1 = \alpha + (1 - \alpha)\gamma, \quad (1)$$

$$t_0 = (1 - \alpha)\gamma, \quad (2)$$

where $0 \leq \alpha \leq 1$. Thus, the more probable signal is remembered with the least amount of distortion, and the greater the value of α the more accurate the memory for both signals.

According to the model, on each trial the observer compares the trace from the preceding signal with the input of the current signal. He then computes the difference between the trace and the input on the relevant dimension (the dimension on which he is asked to base his judgment). If signal S_k was presented on the preceding trial and signal S_j is presented on the current trial, then the difference score d_{jk} is distributed as a random variable D_{jk} that is specified by the equation

$$D_{jk} = I_j - T_k. \quad (3)$$

To avoid confusion, it should be noted that whereas the trace on any trial is determined by the stimulus input on the preceding trial, the input on a trial is assumed to be independent of the trace active on that trial. Thus D_{jk} is normally distributed with mean $s_j - t_k$ and variance $\sigma_D^2 = \sigma_T^2 + \sigma_I^2$.

The decision process uses the output of the comparison process to generate a response as follows:

$$\text{If } \begin{cases} d_{jk} > \delta_0 \\ d_{jk} < \delta_1 \\ \text{otherwise} \end{cases} \text{ then } \begin{cases} \text{respond } A_1 \\ \text{respond } A_0 \\ \text{repeat response made on the preceding trial} \end{cases} \quad (4)$$

where $\delta_0 \geq \delta_1$. If the difference between the input and the trace is greater than some criterion value δ_0 , then A_1 occurs; if the difference is less than some criterion value δ_1 , then A_0 occurs; if the difference does not exceed either the lower or the upper criterion, then the response made on the preceding trial is repeated. In essence, when the observer subtracts the trace of the last signal from the image of the current signal and obtains a "large" positive difference, he calls the current signal loud; when he obtains a "large" negative difference, he calls the current signal soft; and when he obtains little or no difference, he identifies the current signal as a repetition of the preceding one and repeats his last response.

From the above assumptions, Haller (1969) and Tanner *et al.* (1967) have shown that

$$\Pr(A_1 | S_j A_j S_k) = \Phi \left(\frac{s_j - t_k - \delta_1}{\sigma_D} \right). \quad (5)$$

where i, j , and k can take on the values 0 or 1, and $\Phi(x)$ is the integral of the unit normal density function; i.e.,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-(1/2)y^2} dy. \quad (6)$$

If the predicted sequential probabilities are plotted as points in a receiver-operating-characteristic (ROC) space, then the points $[\Pr(A_1 | S_0 A_j S_k), \Pr(A_1 | S_1 A_j S_k)]$ fall on a symmetric, bow-shaped curve that is defined by the parameter σ_D , and is of the type predicted by signal detectability theory (Green and Swets, 1966).³

For the feedback condition, Tanner *et al.* (1967) proposed that, if d_{ik} falls between the lower criterion δ_i and the upper criterion δ_0 , then the observer reports that S_k , the signal presented on the preceding trial, has been repeated. More specifically, in the feedback condition the observer always knows on a given trial which signal occurred on the last trial, since the feedback event has given him that information. Therefore, when the trace from S_k and the input from S_j are perceived to be approximately the same (i.e., when d_{ik} falls between δ_i and δ_0), the observer makes the response that was designated by feedback as correct on the last trial. For this assumption,

$$\Pr(A_1 | S_j A_j S_k) = \Phi\left(\frac{s_j - t_k - \delta_k}{\sigma_D}\right). \quad (7)$$

A more general assumption when d_{ik} falls between δ_i and δ_0 is that the observer's response strategy is influenced by any information from the preceding trial. Specifically, we assume that the observer responds according to a weighted combination of two tendencies, a tendency to repeat the response he made on the preceding trial and a tendency to report a repetition of the signal (signified by the feedback event) that occurred on the preceding trial. Under this assumption,

$$\Pr(A_1 | S_j A_j S_k) = \omega \Phi\left(\frac{s_j - t_k - \delta_k}{\sigma_D}\right) + (1 - \omega) \Phi\left(\frac{s_j - t_k - \delta_j}{\sigma_D}\right) \quad (8)$$

where ω is the weighting parameter. Note that Eq. 8 is simply a weighted average of Eqs. 5 and 7. The no-feedback condition of Tanner *et al.*, is a special case when $\omega = 0$ and Eq. 8 reduces to Eq. 5. At the other extreme, if the observer ignores his last response, then $\omega = 1$ and Eq. 8 reduces to Eq. 7.

For the feedback condition, the point $[\Pr(A_1 | S_0 A_j S_k), \Pr(A_1 | S_1 A_j S_k)]$ generated by Eq. 8 lies on a smooth ROC curve defined by σ_D only when $j = k$, i.e., when the response on the preceding trial was correct. When $j \neq k$, the points generated by Eq. 8 fall below the ROC curve that passes through the points generated when $j = k$. In fact, the points $[\Pr(A_1 | S_0 A_0 S_1), \Pr(A_1 | S_1 A_0 S_1)]$ and $[\Pr(A_1 | S_0 A_1 S_0), \Pr(A_1 | S_1 A_1 S_0)]$

³ A point in the ROC space is represented by the ordered pair (x, y) , where x denotes the value on the abscissa and y denotes the value on the ordinate.

generated by Eq. 8, each lie on a straight line between the corresponding points generated by Eqs. 5 and 7, which lie on the ROC curve.

METHOD

The observers were two male college sophomores (Nos. 1 and 3) and five female housewives (Nos. 2, 4, 5, 6, and 7) ranging in age from 20 to 21 and 31 to 42, respectively. Audiometric tests established that all observers had normal hearing. The observers were paid at the rate of \$2.25 per hour plus \$0.75 per hour upon completion of the experiment. In addition, they received \$0.01 for every 4 correct responses.

The task required the observer to judge which of two auditory amplitudes occurred on each of a series of trials. Responses were recorded by having the observer press one of two buttons on a panel directly before him. The buttons were separated horizontally 4.25 inches from each other. For three observers (Nos. 3, 6, and 7), the buttons were labeled from left to right, *loud signal*, *soft signal*; for the other four observers, the order was reversed.

The sequence of events on each trial of the experiment was as follows: a 1-sec ready period, designated by the illumination of a small white light on the observer's panel; the presentation of one of the two signals for 0.1 sec; a 1.9-sec response period, designated by both response buttons being illuminated; a 2-sec interval, followed by the ready light for the next trial. Thus, a total of 5 sec elapsed between signal presentations. When trial-by-trial feedback was given, a red light illuminated the correct response button during the last 2-sec interval of the trial; otherwise the interval contained no information.

The signals were 1000-Hz sinusoidal tones, presented through earphones for a duration of 100 msec. The equipment and method of tone generation were the same as reported by Tanner *et al.* (1967). No background noise was presented. The amplitude of S_1 , the loud signal, was constant throughout the experiment at a sound pressure level of 70 dB. The amplitude of S_0 , the soft signal, was adjusted individually for each observer, contingent on his performance during four practice sessions. The adjustment was made after each block of 50 trials so that by the end of the fourth practice session the observer was responding correctly on about 70% of the trials. At that time the amplitude settings of S_0 were as follows (Nos. 1 to 7): 67.4, 63.0, 67.4, 67.1, 65.6, 67.6, and 66.4 (mean = 66.4); these amplitudes were held constant for the remainder of the experiment. During the practice sessions γ was set at 0.5.

The experiment involved 63 sessions plus 4 practice sessions for each observer, who was tested individually for 2 sessions a day with a 15-min break between sessions. Each session consisted of 350 trials. Within a session the proportion of S_1 trials was determined by one of three presentation schedules defined by γ : $\gamma = 0.2, 0.5$, and 0.8 . Within each block of 50 trials, γ defined a random sequence with the restriction that

there were $\gamma \times 50$ loud signals and $(1 - \gamma) \times 50$ soft signals. The order of presenting the schedules was randomly determined with two restrictions: in successive 3-session blocks the observer was tested for 1 session on each of the three schedules, and he was not tested on the same schedule in any 2 consecutive sessions.

Observers were not given information about the signal presentation probabilities either before the experiment or during the practice sessions. Following the practice sessions the experiment involved four major parts (conditions), described in their order of occurrence: NF/N, F, NF/E, NF/EL.

Condition NF/N (no feedback/naive). The observer was not given trial-by-trial feedback and was not told that the presentation schedule varied from one session to another. This condition lasted for 12 sessions, 4 sessions with each of the three presentation schedules. Condition NF/N was designed to be comparable to the no-feedback condition of Tanner *et al.* (1967).

Condition F (feedback). This condition followed NF/N and lasted for 12 sessions, 4 with each schedule. On each trial during Condition F, the observer was given feedback identifying the signal that had occurred on that trial. In addition, he was told the presentation probabilities at the start of each session. Condition F was designed to be comparable to Kinchla's (1966) feedback condition.

Condition NF/E (no feedback/experienced). This condition followed Condition F and lasted for 30 sessions, 10 with each presentation schedule. As in Condition NF/N, the observer did not receive trial-by-trial feedback and was not told the presentation probabilities. However, since the observer had participated in Condition F, he was now aware that the presentation probabilities might be varying from session to session. The extended duration of this condition was designed to allow investigation of possible changes in performance as a function of elapsed time following Condition F.

Condition NF/EL (no feedback/experienced, later). This condition started 1 month after the completion of Condition NF/E and lasted for 9 sessions, 3 with each presentation schedule. Observers were not told at the end of Condition NF/E that they would be asked to return (No. 3 did not participate in Condition NF/EL). As in Condition NF/N and NF/E, the observer was not given trial-by-trial feedback and was not told the presentation probabilities. Condition NF/EL was included to determine if the elapse of a fairly long period of time would dissipate any influence that Condition F might have on subsequent performance in a no-feedback condition.

RESULTS AND DISCUSSION

Table 1 presents the sequential probabilities, hit and false alarm probabilities, the probability of an A_1 , and the probability of a correct response, $\text{Pr}(C)$. The figures also

TABLE 1

Observed (Ob) and Theoretical (Th) Response Probabilities and Parameter Estimates

RESPONSE PROBABILITIES AND PARAMETER ESTIMATES	NO FEEDBACK MAIVE						FEEDBACK						NO FEEDBACK EXPERIENCED						NO FEEDBACK EXPERIENCED, LATER						
	$\gamma=0.2$		$\gamma=0.5$		$\gamma=0.8$		$\gamma=0.2$		$\gamma=0.5$		$\gamma=0.8$		$\gamma=0.2$		$\gamma=0.5$		$\gamma=0.8$		$\gamma=0.2$		$\gamma=0.5$		$\gamma=0.8$		
	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	Ob	Th	
$Pr(A_1 S_1 A_1 S_1)$.81	.85	.81	.79	.71	.71	.75	.72	.86	.87	.95	.94	.92	.94	.91	.92	.89	.89	.92	.92	.89	.89	.89	.89	.89
$Pr(A_1 S_0 A_1 S_1)$.43	.45	.34	.36	.27	.28	.08	.20	.20	.30	.35	.33	.36	.31	.30	.24	.25	.21	.26	.26	.22	.21	.17	.21	.21
$Pr(A_1 S_1 A_1 S_0)$.87	.92	.86	.88	.79	.82	.54	.71	.79	.83	.92	.94	.96	.99	.94	.99	.93	.98	.97	.98	.91	.97	.90	.97	.97
$Pr(A_1 S_0 A_1 S_0)$.61	.60	.55	.50	.38	.41	.10	.32	.26	.62	.43	.68	.67	.62	.61	.58	.55	.47	.54	.54	.45	.46	.46	.47	.47
$Pr(A_1 S_1 A_0 S_1)$.61	.69	.61	.60	.51	.51	.46	.57	.72	.71	.88	.83	.60	.56	.53	.47	.46	.55	.63	.65	.55	.57	.54	.55	.55
$Pr(A_1 S_0 A_0 S_1)$.32	.25	.26	.18	.14	.13	.08	.06	.20	.15	.31	.26	.14	.04	.13	.03	.08	.02	.15	.05	.12	.03	.11	.03	.03
$Pr(A_1 S_1 A_0 S_0)$.77	.81	.76	.73	.64	.65	.63	.62	.77	.77	.89	.92	.84	.83	.81	.81	.75	.76	.87	.87	.82	.82	.81	.80	.80
$Pr(A_1 S_0 A_0 S_0)$.38	.38	.25	.30	.17	.22	.06	.05	.13	.11	.19	.28	.15	.17	.14	.15	.11	.11	.18	.18	.14	.13	.14	.12	.12
$Pr(A_1 S_1)$.81	.85	.78	.76	.65	.65	.62	.63	.80	.81	.93	.93	.87	.88	.85	.86	.81	.81	.88	.88	.80	.83	.84	.83	.83
$Pr(A_1 S_0)$.47	.47	.34	.34	.22	.23	.06	.06	.18	.16	.31	.34	.31	.33	.28	.27	.21	.21	.27	.27	.20	.18	.17	.18	.18
$Pr(A_1)$.54	.55	.56	.55	.56	.57	.18	.17	.49	.49	.81	.81	.43	.44	.56	.56	.69	.69	.39	.40	.52	.51	.70	.70	.70
$Pr(C)$.59	.59	.72	.71	.67	.68	.87	.88	.81	.82	.89	.88	.72	.71	.79	.80	.80	.81	.76	.76	.81	.83	.83	.83	.83
σ_0	.12	.12					.78	.45	.02			.39	.25	.17				.32	.23	.07					.07
σ_1	-.35	-.35					-.01	-.39	-.73			.34	-.44	-.53				-.17	-.27	-.48					-.48
σ	.32	.32					.65	.65				.42	.42					.36	.36						.36
σ_D	.86	.86					.51	.51				.52	.52					.49	.49						.49
ω	—	—					.73	.73				—	—					—	—						—
min	9.57	9.57					10.75	10.75				11.01	11.01					3.97	3.97						3.97

present the sequential probabilities (Fig. 2), hit and false alarm rates (Fig. 3), and the A_1 probability (Fig. 4). These data were calculated for observers individually and for the group as a whole. To conserve space, data for individual observers are not presented; however, they are reasonably well represented by the group values. The adequacy of the representation is comparable to that displayed in Tanner *et al.* (1967).

The data represent performance over all of the sessions of a given schedule and a given condition. Data for single sessions were also examined to determine if there were systematic changes over sessions. Such changes were not observed, even for Condition NF/E where they were most expected. Thus the data presented are representative of individual sessions as well as individual observers.

To obtain predictions for the MR-Model, it is necessary to make estimates of α , σ_D , δ_0 , δ_1 , and ω . For the present study the parameters were estimated by minimizing the following function:

$$\xi(\alpha, \sigma_D, \delta_0, \delta_1, \omega) = \sum_{k=1}^2 \sum_{j=1}^2 \sum_{i=1}^2 \{[\hat{\Pr}(A_1 | S_i A_j S_k) - \Pr(A_1 | S_i A_j S_k)]^2 \text{fr}(A_1 S_i A_j S_k)\}, \quad (9)$$

where $\hat{\Pr}(A_1 | S_i A_j S_k)$ denotes the observed sequential probability and $\text{fr}(A_1 S_i A_j S_k)$ denotes the corresponding observed frequency. The parameter estimates were obtained using a high-speed computer to calculate the function $\xi(\alpha, \sigma_D, \delta_0, \delta_1, \omega)$ over a grid of possible values of the parameters, then selecting those values that approximated the minimum of the function (see Atkinson, Bower, and Crothers, 1965, p. 386).

Two estimation procedures, designated as Methods A and B, were employed. In Method A, which was used for Condition NF/N, the five parameter values were estimated simultaneously for all three presentation schedules. In Method B, δ_0 and δ_1 were estimated separately for each presentation schedule, while α , σ_D , and ω were estimated simultaneously over all three schedules. Thus in Method B, one value each for α , σ_D , and ω , but three values each for δ_0 and δ_1 (a total of nine parameters) were estimated. Method B was used for Conditions F, NF/E, and NF/EL, because it was assumed that δ_0 and δ_1 would vary with γ when observers were aware that the signal presentation probabilities were being varied from session to session. The parameter estimates and the minimum values of $\xi(\alpha, \sigma_D, \delta_0, \delta_1, \omega)$ are presented in Table 1.

The sequential probabilities are discussed first, since the principal predictions of the MR-Model are based on sequential relations. These probabilities are presented in Fig. 2. The columns of Fig. 2 correspond to the four experimental conditions, and the rows to the three presentation schedules. The circles and squares in each graph (see the figure legend) plot the observed points $[\Pr(A_1 | S_0 A_j S_k), \Pr(A_1 | S_1 A_j S_k)]$. The bow-shaped curves are the ROC functions predicted by the MR-Model. The curves are determined by the single parameter σ_D (Haller, 1969; Tanner *et al.*, 1967). Therefore, each condition (having its own estimated value of σ_D) has a different curve, but the three schedules of a condition (having the same value of σ_D) have the same curve.

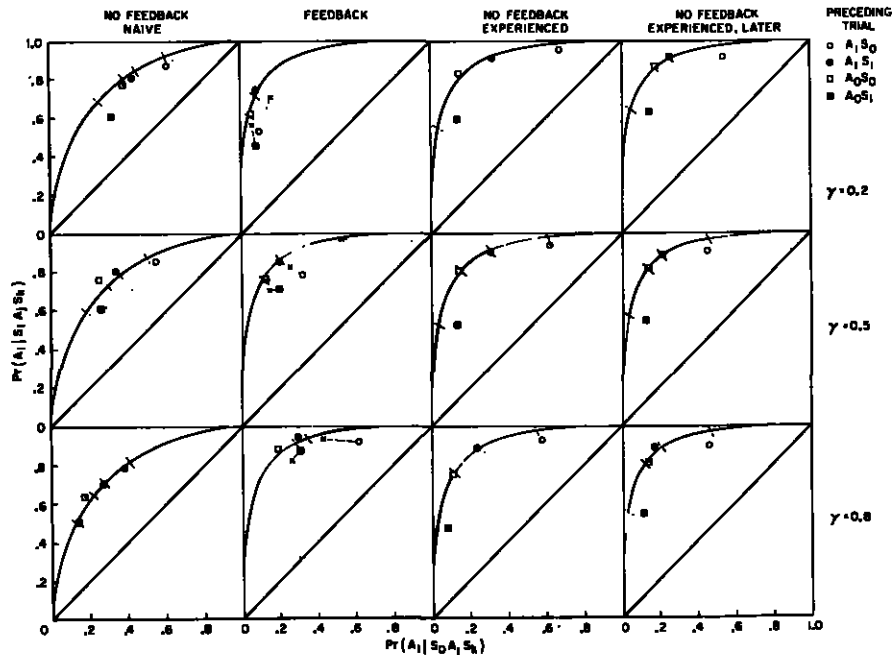


FIG. 2. ROC graphs of sequential response probabilities.

The intersections of the ROC curves and the short lines drawn perpendicularly to them plot the predicted points in Fig. 2. For the no-feedback (NF) conditions, the order of the predicted and observed points along the ROC curve is the same, and this order is independent of the presentation schedule. Both hit and false alarm rates increased as a function of the signal and response on the preceding trial as follows: an A_0 made to an S_1 , an A_0 made to an S_0 , an A_1 made to an S_1 , an A_1 made to an S_0 . Thus there was a general tendency to repeat the response made on the preceding trial but not to report the signal that occurred on the preceding trial. Also, there was a strong tendency to repeat a response that was incorrect on the preceding trial. The order of the observed points for all three NF conditions is the same as that reported by Kinchla (1966) and by Tanner *et al.* (1967) for their no-feedback conditions. Also, the accuracy of the MR-Model for predicting the sequential probabilities in the present study appears comparable to the accuracy reported by Tanner *et al.*

As noted above, for Condition F the model predicts that only two of the points for the sequential probabilities, *viz.*, $[\Pr(A_1 | S_0, A_0, S_0), \Pr(A_1 | S_1, A_0, S_0)]$ and $[\Pr(A_1 | S_0, A_1, S_1), \Pr(A_1 | S_1, A_1, S_1)]$ will fall on the ROC curve. The predicted values for these two points are indicated by the intersection of short lines with the ROC curve; the predicted and observed values for these two points have the same order in the ROC space. For the two points that are predicted to lie below the curve, *viz.*,

$\{\Pr(A_1 | S_0 A_1 S_0), \Pr(A_1 | S_1 A_1 S_0)\}$ and $\{\Pr(A_1 | S_0 A_0 S_1), \Pr(A_1 | S_1 A_0 S_1)\}$, the predicted values are designated by small crosses and a line connects these crosses with the corresponding observed points.

For Condition F (in contrast to the NF conditions) the order of the four observed points along the ROC curve is not consistent over presentation schedules. However, the order within each of the two pairs of points (the pair predicted to fall on the ROC curve, and the pair predicted to fall below the curve) is the same for each presentation schedule as the corresponding order in the three NF conditions; i.e., both hit and false alarm rates were greater when an A_1 was made to an S_1 than when an A_0 was made to an S_0 on the preceding trial, and were greater when an A_1 was made to an S_0 than when an A_0 was made to an S_1 . Inspection of Kinchla's data shows these same relationships when observers received trial-by-trial feedback.

Kinchla reported that the influence of the preceding trial's signal and response on hit and false alarm rates was much stronger when observers did not receive feedback than when they did. Similarly, in Fig. 1 of the present study, the sequential effects appear greater for the NF conditions than for Condition F; i.e., the spread of the four points in the ROC space is generally greater for the NF conditions than for Condition F. In both studies, however, the two points $\{\Pr(A_1 | S_0 A_0 S_0), \Pr(A_1 | S_1 A_0 S_0)\}$ and $\{\Pr(A_1 | S_0 A_1 S_1), \Pr(A_1 | S_1 A_1 S_1)\}$ were spread about as far apart when feedback was given as when it was not. The decrease in the overall spread for Condition F was due specifically to a decrease in the sequential effects when an error ($A_0 S_1$ or $A_1 S_0$) was made on the preceding trial. Thus in Condition F (in contrast to the NF conditions) there was not a consistent tendency to repeat a response that was incorrect on the preceding trial, but (similar to the NF conditions) there was a tendency to repeat a response made on the preceding trial whether or not it was correct.

The two points that are predicted to lie below the ROC curves in Condition F are not as well fit by the model as are the points (in all four conditions) that are predicted to lie on the curves. However, in all conditions the theoretical fits for the points that are predicted to lie on the ROC curves appear reasonably accurate; and, as noted previously, even for the points in Condition F that are predicted to lie below the curve, the relative location in the ROC space, with respect to the direction of shifts along the curve, is predicted.

Figure 3 presents observed values for hit and false alarm rates plotted on ROC graphs. As would be expected from previous research, hit and false alarm rates varied systematically as a function of γ in all experimental conditions. For all three NF conditions both the hit and false alarm probabilities decreased, as γ increased, the same relation reported by Tanner *et al.* (1967). Qualitatively then, the relationship between γ and hit and false alarm rates was the same for the three conditions. However, note that the spread of the points is greater in Condition NF/N than in Conditions NF/E and NF/EL. This evidence indicates that the influence of γ on hit and false alarm rates is reduced in a no-feedback condition that is presented after an observer

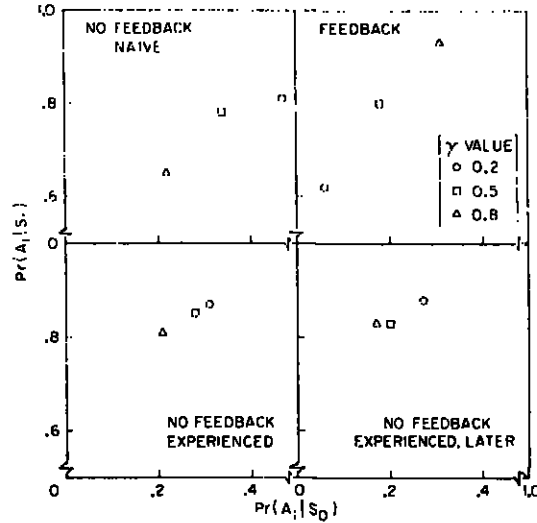


FIG. 3. ROC graphs of hit and false alarm probabilities.

has previously experienced feedback. This reduced influence apparently was not affected by the elapsed month between Conditions NF/E and NF/EL.

For Condition F both hit and false alarm rates increased as γ increased. This relationship is the same as that reported by Kinchla for his feedback condition. The influence of γ on hit and false alarm rates appears to have been stronger in Condition F than in Condition NF/N and this difference between the two conditions also is consistent with the results of previous research. Inspection of Kinchla's feedback data and Tanner and co-worker's no-feedback data suggests that the influence of γ on hit and false alarm rates was stronger in the former even though the range of γ values (0.25 to 0.75 and 0.1 to 0.9, respectively) was greater in the latter study.

Predicted values for $\Pr(A_1 | S_1)$ and $\Pr(A_1 | S_0)$ were obtained as weighted averages of appropriate sequential probabilities:

$$\Pr(A_1 | S_j) = \sum_{k=1}^2 \sum_{j=1}^2 [\Pr(A_1 | S_j A_j S_k) \Pr(A_j | S_k) \Pr(S_k)]. \quad (10)$$

The predictions for $\Pr(A_1 | S_1)$ and $\Pr(A_1 | S_0)$ are presented in Table 1, where they can be compared with observed values. For each of the four conditions the model predicts the observed values quite accurately. The largest discrepancy between observed and predicted values is 0.04, and the discrepancy for 17 of the 24 pairs of values is less than or equal to 0.01.

Figure 4 presents the observed values for $\Pr(A_1)$ as a function of γ . It is clear that feedback had a marked influence on $\Pr(A_1)$. While $\Pr(A_1)$ remained virtually constant

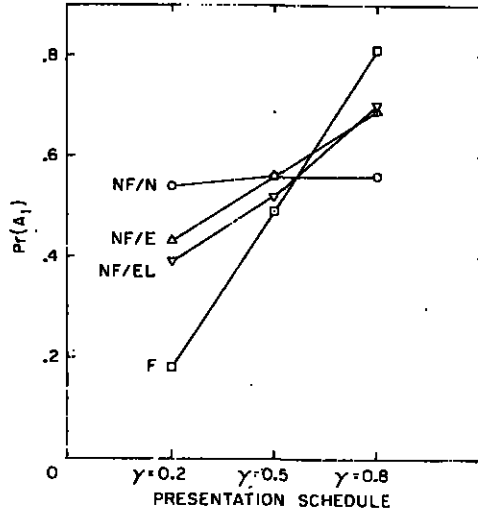


FIG. 4. Unconditional probability for reporting the loud signal.

over the presentation schedules in Condition NF/N, it approximately matched the value of γ in Condition F. These results are consistent with those of Tanner and co-worker's no-feedback condition and Kinchla's feedback condition. For Conditions NF/E and NF/EL, $\Pr(A_1)$ increased as γ increased but less markedly than in Condition F. Thus, Conditions NF/E and NF/EL lie between Conditions NF/N and F in their influence on the relationship between γ and the A_1 response probability, just as they did for the relationship between γ and the hit and false alarm rates. As in the case of hit and false alarm rates and sequential probabilities, the elapsed month between Conditions NF/E and NF/EL did not appear to influence $\Pr(A_1)$.

The predictions for $\Pr(A_1)$ and $\Pr(C)$ were obtained as weighted averages of the predicted hit and false alarm rates as follows:

$$\Pr(A_1) = \Pr(A_1 | S_1) \gamma + \Pr(A_1 | S_0)(1 - \gamma), \quad (11)$$

$$\Pr(C) = \Pr(A_1 | S_1) \gamma + [1 - \Pr(A_1 | S_0)](1 - \gamma). \quad (12)$$

The values of $\Pr(A_1)$ and $\Pr(C)$ are presented in Table 1; note that 23 of the 24 predictions are within 0.01 of the observed values.

In the application of the MIR-Model to Conditions F, NF/E, and NF/EL, certain assumptions were added to the basic model. To evaluate these additional assumptions some alternatives were considered.

For Condition F the predictions generated by Eq. 8 (shown in Table 1) were compared with those generated by Eq. 7. The estimation procedure used for Eq. 8 (Method B) was repeated for Eq. 7, since δ_0 and δ_1 obviously were dependent on γ (note in

Fig. 5 that for Condition F both δ_0 and δ_1 decreased markedly as γ increased). For $\Pr(A_1 | S_1)$, $\Pr(A_1 | S_0)$, $\Pr(A_1)$, and $\Pr(C)$, Eqs. 7 and 8 yielded essentially equivalent results; the predicted values for the two equations are all within 0.01 of each other. However, for the sequential probabilities, the predictions of Eq. 8 provided a far better fit to the data than those of Eq. 7; the respective minimum values of $\xi(\alpha, \sigma_D, \delta_0, \delta_1, \omega)$ are 10.8 and 27.9.

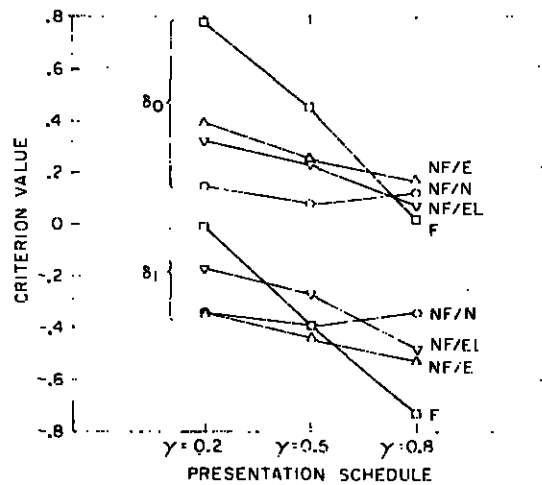


FIG. 5. Estimates of the criterion values.

Method B was used originally to estimate parameters for Conditions NF/E and NF/EL, since it was assumed that an observer might guess the values of the signal presentation probabilities and adjust his criterion values appropriately. Figure 5 shows that the estimates δ_0 and δ_1 decreased as γ increased in these two conditions, but the relationship was not as strong as it was for Condition F. Therefore, a new set of predictions was generated for Conditions NF/E and NF/EL using Method A, which required δ_0 and δ_1 to be constant over γ . Method B proved to be more accurate for all of the probabilities shown in Table 1 for the two conditions. For Conditions NF/E and NF/EL, respectively, the minimum values of $\xi(\alpha, \sigma_D, \delta_0, \delta_1, \omega)$ obtained by Method A are 75.6 and 40.2; these values are much larger than those obtained by Method B (see Table 1).

The data appear to support the assumption that δ_0 and δ_1 vary as a function of γ in Conditions F, NF/E, and NF/EL. As an additional test of the MR-Model another set of predictions was generated for Condition NF/N using Method B, i.e., allowing δ_0 and δ_1 to vary with γ . However, the estimation of additional parameters in Method B did not substantially improve the predictions for Condition NF/N. For all the response probabilities in Condition NF/N, the predictions generated by Methods A

and B are nearly identical. The values of δ_0 and δ_1 for Condition NF/N obtained using Method B are shown in Fig. 5, and it is apparent that they are virtually constant over γ . Thus the assumption that observers do not adjust their criterion values from one presentation schedule to the next in Condition NF/N appears to be supported.

CONCLUSION

The findings of this study, we believe, justify the following conclusions:

1. The results of Kinchla's (1966) feedback condition and Tanner and co-worker's (1967) no-feedback condition have been replicated in Condition F and NF/N, respectively. It has been verified that in a signal recognition task the relationship between γ and hit and false alarm probabilities depends on whether or not an observer is given information about the signal presentation probabilities.

2. The results of this study considered in relation to those of Kinchla (1966) and Tanner *et al.* (1967) suggest that the information an observer receives about the signal presentation probabilities and the influence this information has on his decisions are ordered along a dimension from (a) to (d) as follows: (a) At one end is Kinchla's feedback condition and Condition F of the present study; the observer is told the signal presentation probabilities and is given trial-by-trial feedback. As a result, the hit and false alarm rates clearly increase as γ increases. (b) Next on the dimension is Kinchla's no-feedback condition; the observer is told the signal presentation probabilities, but is not given trial-by-trial feedback. In this condition hit and false alarm rates also increase as γ increases but the effect is weaker than when trial-by-trial feedback is given. (c) Further along the dimension lie Conditions NF/E and NF/EL of the present study; the observer is not told of the signal presentation probabilities and is not given trial-by-trial feedback, but as a result of previous experience, he may realize that the signal presentation probabilities vary from session to session. Under these conditions hit and false alarm rates decrease slightly as γ increases. (d) At the other end of the dimension is Tanner and co-worker's no-feedback condition and Condition NF/N of the present study; the observer is not told that the signal presentation probabilities may change from session to session and is not given trial-by-trial feedback. With no information about the signal probabilities, the observer's hit and false alarm rates decrease markedly as γ increases.

3. The sequential effects appear to be stronger when trial-by-trial feedback is omitted than when it is given. The influence of the preceding trial's signal and response on hit and false alarm rates appears to have been equally strong in Kinchla's and Tanner and co-worker's no-feedback conditions, and the three NF conditions of the present study. The relationship was weaker in both Kinchla's feedback condition and Condition F of the present study.

4. The Memory-Recognition Model provided accurate predictions for $\Pr(A_1)$, $\Pr(A_1 | S_1)$, and $\Pr(A_1 | S_0)$ in all conditions of the present study. For the sequential probabilities, $\Pr(A_1 | S_i A_j S_k)$, the predictions are quite accurate for the three NF conditions. For Condition F, however, the predictions for two of the points in the ROC space, the points predicted to fall below the ROC curve, are less accurate; for the two points that are predicted to fall on the curve, the accuracy of the predictions is comparable to that of the NF conditions.

REFERENCES

- ATKINSON, R. C., BOWER, G. H., AND CROTHERS, E. J. *An introduction to mathematical learning theory*. New York: Wiley, 1965.
- ATKINSON, R. C., AND KINCHLA, R. A. A learning model for forced-choice detection experiments. *British Journal of Mathematical and Statistical Psychology*, 1965, **18**, 183-206.
- GREEN, D. M., AND SWETS, J. A. *Signal detection theory and psychophysics*. New York: Wiley, 1966.
- HALLER, R. W. The role of information feedback in signal recognition. Unpublished doctoral dissertation, Stanford University, 1969.
- KINCHLA, R. A. A comparison of sequential effects in detection and recognition. Experimental Psychology Series, Psychology Department, New York University, Tech. Rep., 1, 1966.
- KINCHLA, R. A. AND ALLAN, LORRAINE G. A theory of visual movement perception. *Psychological Review*, 1969, **76**, 537-558.
- KINCHLA, R. A. AND SMYZER, F. A diffusion model of perceptual memory. *Perception and Psychophysics*, 1967, **2**, 219-229.
- PARDUCCI, A., AND SANDUSKY, A. Distribution and sequence effects in judgment. *Journal of Experimental Psychology*, 1965, **69**, 450-459.
- TANNER, T. A., HALLER, R. W., AND ATKINSON, R. C. Signal recognition as influenced by presentation schedules. *Perception and Psychophysics*, 1967, **2**, 349-358.

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