Lawrence Berkeley National Laboratory

Recent Work

Title

EXTREME VALUES OF THE SPIN POLARIZATION ANALYZING POWER IN NUCLEAR REACTIONS

Permalink

<https://escholarship.org/uc/item/91b7192t>

Author

Seiler, F.

Publication Date

1977-09-01

Submitted to Nuclear Physics

UC-34d
LBL-5851
Preprint c.1

RECEIVED
LAWRENCE
BERKELEY LABORATORY

DEC 1 1977

LIBRARY AND
DOCUMENTS SECTION

EXTREME VALUES OF THE SPIN POLARIZATION
ANALYZING POWER IN NUCLEAR REACTIONS

F. Seiler, F. N. Rad, H. E. Conzett, and R. Roy

September 1977

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

For Reference

Not to be taken from this room

LBL-5851
c.1

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Extreme Values of the Spin Polarization Analyzing
Power in Nuclear Reactions*

F. Seiler⁺, F.N. Rad[‡], H.E. Conzett and R. Roy[†]

Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, U.S.A.

*Work performed under the auspices of the U.S. Energy Research and Development Administration.

⁺On leave of absence from the Physics Department of the University of Basel, Switzerland.

[‡]Present address: Laboratory for Nuclear Science, Department of Physics, MIT, Cambridge, Mass., USA.

[†]Research Council of Canada, Postdoctoral fellow. Present address: Université Laval, Québec, Canada.

Abstract

States of maximum possible polarization are investigated for ensembles of spin- $\frac{1}{2}$ and spin-1 particles in nuclear reactions. General criteria for extreme values of some components of the analyzing power are given and applied to the formulae for the observables of reactions with various spin spaces. Experimental data relevant to possible extreme values are discussed.

1. Introduction

Large values of the polarization have been known in $p-\alpha$ and $n-\alpha$ scattering for some time. Analytically the existence of absolute values of unity was first proven by Plattner and Bacher¹⁾ for several elastic scattering processes with the particularly simple spin space $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$. Subsequently, Grüebler and collaborators²⁾ proved the existence of several extreme values for the spin-1 polarization in $\vec{d}-\alpha$ elastic scattering ($\vec{1} + 0 \rightarrow 1 + 0$). In both cases a single linear relation between transition amplitudes must be satisfied. Proof is therefore simple if an analysis of the process is available.

In a nuclear reaction, as opposed to elastic scattering, the possibility of an extreme value of a polarization observable was first proposed³⁾ for the process ${}^3\text{He}(\vec{d}, p){}^4\text{He}$. The proof of existence used above for the cases of elastic scattering with a simple spin space can no longer be applied, since two conditions have to be satisfied simultaneously. Recently, these linear conditions for elements of the transition matrix were derived quite generally⁴⁾ for the analyzing power of the four-particle reaction $\vec{A} + B \rightarrow C + D$ with spins $\vec{a} + b \rightarrow c + d$. Even for multiple conditions, however, experimentally verifiable criteria in the form of "required" values of other polarization observables can be given. At the 4th Polarization Symposium 1975 in Zürich⁵⁾, several contributions discussed such criteria for various processes, and Grüebler⁶⁾ reviewed the establishment of several known points of extreme analyzing power and described their use in the absolute calibration of polarization measurements.

In this paper, which encompasses several preliminary reports⁷⁻¹⁰⁾, we provide a more general discussion of extreme values of analyzing powers. States of maximum polarization (analyzing power) are described in sect.2, and their usefulness in the calibration of polarization measurements and in the analysis of data for the determination of the M-matrix amplitudes is discussed in

sect.3. Known and proposed points of extreme values in elastic scattering and in reactions, respectively, are reviewed in sects. 4 and 5, and the required values of the other polarization observables are listed. Sect.6 then contains a summarizing discussion.

2. States of maximum polarization

2.1 DESCRIPTION OF SPIN-1 POLARIZATION

The description of a state of maximum polarization for an ensemble of spin- $\frac{1}{2}$ particles is straightforward. Therefore consideration will be given here only to the less obvious case of spin-1 polarization. It is usually described in terms of tensor moments of an irreducible set of spherical tensors τ_{kq} or of the set of corresponding cartesian tensors p_i, p_{ij} as defined by the Madison convention¹¹⁾. Since the cartesian tensors do not form an irreducible set, there exists a condition requiring that

$$p_{xx} + p_{yy} + p_{zz} = 0. \quad (1)$$

Therefore the second rank quantities usually given are p_{zz} , p_{xz} and $p_{xx} - p_{yy}$, in direct correspondence to the spherical tensors t_{2q} . An equivalent situation exists for the analyzing power T_{kq} or A_{ij} , so that again the quantity A_{yy} often does not appear directly, but only in the combination $A_{xx} - A_{yy}$. This probably is the reason why extreme values of the analyzing power A_{yy} went unnoticed in some instances even though experimental data indicating such values were available.

The limits of the values for the usual polarization parameters are

$$\frac{1}{2}\sqrt{3} \leq t_{11} \leq \frac{1}{2}\sqrt{3}, \quad -1 \leq p_y \leq 1 \quad (2a)$$

$$-\sqrt{2} \leq t_{20} \leq \frac{1}{2}\sqrt{2}, \quad -2 \leq p_{zz} \leq 1 \quad (2b)$$

$$-\frac{1}{2}\sqrt{3} \leq t_{21} \leq \frac{1}{2}\sqrt{3}, \quad -\frac{3}{2} \leq p_{xz} \leq \frac{3}{2} \quad (2c)$$

$$-\frac{1}{2}\sqrt{3} \leq t_{22} \leq \frac{1}{2}\sqrt{3}, \quad -3 \leq p_{xx} - p_{yy} \leq 3. \quad (2d)$$

For the polarization efficiencies T_{kq} and A_i, A_{ij} , also defined according to the Madison convention, the same range of values apply. For the components p_{xx} and p_{yy} that appear only in combination in eq.(2d), the individual limits are the same as for p_{zz} .

2.2 THE LAKIN CONE

Limits for the polarization attainable by an ensemble of spin-1 particles produced in a nuclear reaction were discussed very early by Lakin¹²⁾ and later by Minnaert¹³⁾. These maximum possible values were derived using the fact that the density matrix of the ensemble is positive semi-definite. This imposes restrictions on the tensor moments t_{kq} , derived from the expansion of the density matrix of the outgoing particles in terms of the set of irreducible operators τ_{kq} . Invoking time reversal invariance, identical limitations can be shown to apply to the polarization efficiencies T_{kq} . Alternately, they can be derived^{14,15)} by expanding the initial density matrix in terms of the tensor operators τ_{kq} and defining the appropriate efficiency tensors ϵ_{kq} . In the following the discussion will be restricted to the efficiencies A_i , A_{ij} or T_{kq} , but all statements apply equally to the corresponding polarizations.

Since the conditions due to parity conservation assume a particularly simple form in a transverse coordinate system S' , this was the frame of reference chosen by both Lakin and Minnaert. The z' -axis of this coordinate system lies perpendicular to the reaction plane, along $\mathbf{k}_{in} \times \mathbf{k}_{out}$, while the y' -axis is chosen along the direction $\mathbf{k}_{out}$ of the polarized particles. The relations between quantities in the transverse frame S' and the system S used in the Madison convention are given in table 1.

The positivity conditions turn out to be considerably more simple in the transverse system S' than in the Madison frame. They yield the inequalities^{12,13)}

$$\delta^2 \equiv \frac{1}{2} \{ (T'_{10})^2 + (T'_{20})^2 + 2|T'_{22}|^2 \} \leq 1 \quad (3)$$

and

$$\epsilon^2 \equiv (T'_{20} + \sqrt{2})^2 - 3(T'_{10})^2 - 6|T'_{22}|^2 \geq 0. \quad (4)$$

In the space $(T'_{10}, T'_{20}, \sqrt{2}|T'_{22}|)$ the condition $\delta^2 \leq 1$ defines the interior of a sphere, and the inequality $\epsilon^2 \geq 0$ the interior of

an inscribed cone. Clearly, condition (4) is more restrictive than inequality (3). This situation is shown graphically in fig.1, where the coordinate axes have been relabelled with the cartesian observables according to the Madison convention.

Extreme points $A_{yy} = 1$ lie anywhere on the base of the cone, and condition (4) yields the restriction

$$1 \geq (A_y)^2 + \frac{4}{9}(A_{xz})^2 + \frac{1}{9}(A_{zz} - A_{xx})^2. \quad (5)$$

Extreme values $A_y = \pm 1$ are only possible at the points A and B of fig.1, respectively. Using table 1 the necessary conditions are¹⁰⁾

$$A_{yy} = 1, \quad A_{xx} = A_{zz} = -\frac{1}{2}, \quad A_{xz} = 0. \quad (6)$$

They are not sufficient, however, since they only define a point somewhere on the line AB. Eqs.(6) thus show that the condition $A_{yy} = 1$ is a prerequisite for an extreme value $A_y = \pm 1$, and that all the other efficiencies are numerically determined. This was demonstrated for ${}^4\text{He}(\vec{d}, d){}^4\text{He}$ elastic scattering by Gruebler et al.²⁾, using the properties of that particular M-matrix. From the derivation given here, it is obvious that eqs.(6) are valid for any process¹⁰⁾

$$\vec{1} + b \neq c + d, \quad (7)$$

where b, c and d are arbitrary spins and $\vec{1}$ stands for the only polarization (spin-1) determined in or by the experiment. The requirement that both A_y and A_{yy} reach their peak values at the same point (θ_0, E_0) is therefore also a general condition, which may be useful in experimental and theoretical work. Near the point $A_y = \pm 1$, the relations between the values of A_{yy} and A_y may be of use. Noting that the slope of the Lakin cone is $\pm \frac{1}{3}$, it follows that

$$A_y = 1 - \eta \rightarrow A_{yy} \geq 1 - 3\eta. \quad (8)$$

Despite the singular nature of points A and B, a curve representing

the values of the polarization as a function of angle for a given energy may touch these points, with the trajectory starting and ending somewhere within the Lakin cone.

A point somewhat more singular is the apex of the cone (point C in fig.1), for which the tensor moment A_{yy} has the extreme value $A_{yy} = -2$. The necessary but not sufficient conditions for this value of the polarization are easily read from fig.1, which yields together with eq.(1)

$$A_y = 0, \quad A_{xx} = A_{zz} = 1, \quad A_{xz} = 0. \quad (9)$$

Note that both A_{xx} and A_{zz} must also assume their maximum possible values.

Any points on the lateral surface of the Lakin cone do not correspond to extreme values of an observable, except for the vertex and the periphery of the base of the cone. For all other points the observables lie well within the range allowed and it is only that particular combination of values that constitutes a maximum possible state of polarization. Seiler and Roser¹⁶⁾ have discussed these polarization states and have shown that, even though eq.(4) is of fourth order in the transition amplitudes, the conditions necessary and sufficient for a point on the cone are again linear in the amplitudes.

2.3 MAXIMUM VALUE OF AN OBSERVABLE

The conditions on the transition matrix M for an extreme value of an observable of the general process $\vec{A} + B \rightarrow C + D$ with spins $\vec{a} + b \rightarrow c + d$ have been discussed by Conzett and Seiler⁴). For this case the transition matrix M , defined by

$$\chi_f = M \chi_i, \quad (10)$$

where χ_i and χ_f are the initial and final state spin functions, has the dimensions $(2c+1)(2d+1)$ by $(2a+1)(2b+1)$. With α, β, γ and δ labelling the magnetic substates, the matrix M is given by

$$M \equiv [M_{\gamma, \delta; \alpha, \beta}], \quad (11)$$

and its elements obey the parity condition¹⁴)

$$M_{\gamma, \delta; \alpha, \beta} = \pi_{\text{tot}} (-)^{\sum s_i} (-)^{\alpha + \beta + \gamma + \delta} M_{-\gamma, -\delta; -\alpha, -\beta}. \quad (12)$$

where π_{tot} is the product of the intrinsic parities of the reaction partners and the quantities s_i are the spins a, b, c and d . For convenient reference in the applications that follow, the sets of relations among these elements are given below for extreme values of the components A_y and A_{yy} of the analyzing power⁴).

a) For spin $a = \frac{1}{2}$,

$$A_y = \pm 1: \quad M_{\gamma, \delta; \frac{1}{2}, \beta} \mp i M_{\gamma, \delta; -\frac{1}{2}, \beta} = 0 \quad (13)$$

b) For spin $a = 1$,

$$A_{yy} = +1: \quad M_{\gamma, \delta; 1, \beta} + M_{\gamma, \delta; -1, \beta} = 0 \quad (14)$$

$$A_y = \pm 1: \quad M_{\gamma, \delta; 1, \beta} \pm M_{\gamma, \delta; -1, \beta} = 0 \quad (15a)$$

$$\text{and} \quad M_{\gamma, \delta; 0, \beta} \pm i\sqrt{2} M_{\gamma, \delta; 1, \beta} = 0 \quad (15b)$$

$$A_{yy} = -2: \quad M_{\gamma, \delta; 1, \beta} - M_{\gamma, \delta; -1, \beta} = 0 \quad (16a)$$

$$\text{and} \quad M_{\gamma, \delta; 0, \beta} = 0. \quad (16b)$$

These conditions hold for all independent combinations of the indices β, γ and δ .

In reactions with the simple spin structure $a = c = \frac{1}{2}$ or 1 and $b = d = 0$ where there is only one independent condition, the left side of eqs. (13) and (14) can be labeled $F_{lin}(M_{ij}, \theta, E)$ and the requirement is

$$F_{lin}(M_{ij}, \theta_0, E_0) = 0 . \quad (17)$$

Thus extreme values of the analyzing power must exist where the complex function $F_{lin}(M_{ij}, \theta, E)$ crosses the origin^{1,2}. If there are several conditions, they have to be fulfilled simultaneously, that is at the same point (θ_0, E_0) , an event with a low a priori probability. In these cases the existence of an extreme point cannot be proven by the method sketched above. It is possible, however, to demonstrate the existence of an extreme-value point within a certain experimental error. This can be done either by a direct measurement with an absolute calibration or by using the fact that eq. (17) defines the values of other observables, which can be determined with relatively simple experiments. Of particular importance are those measurements that should yield a value of zero, since they are independent of the calibration of any particle polarization³).

3. Use of points of extreme polarizations

3.1 CALIBRATION OF POLARIZATIONS

In the early stages of polarization experiments, phase shift analyses were used to calibrate the measurements. A double scattering technique developed by Scott¹⁷⁾ provided the means to determine polarizations in an absolute way. This method, however, was limited in accuracy and although the precision improved over the years^{18,19)}, it was not really suitable as a primary polarization standard. Since the existence of extreme points $A_y = \pm 1$ has been proven, the latter are providing excellent primary standards for polarization measurements.

Initially the situation for deuterons was somewhat better at very low energies, where resonances induced by polarized s-wave deuterons were shown to be very good analyzers for tensor polarization²⁰⁾. However, it was realized relatively soon that admixtures of other non-resonant amplitudes^{21,22)} result in an uncertainty of the analyzing power of up to ten percent. Even at present the best values of the calibration factor are uncertain by about 1 or 2 percent²³⁾. At higher energies and especially for vector polarization, double scattering techniques had to be used, resulting in uncertainties of about ten percent^{24,25)}. The first application of an extreme polarization value for calibration purposes was made by Darden et al.²⁶⁾ using the $^{16}\text{O}(\vec{d}, \alpha_1)^{15}\text{N}^*$ reaction. This process is limited by parity conservation to the apex of the Lakin cone, independent of energy and angle (see subsect. 5.1). Despite some problems with background subtraction, the reaction clearly meets the characteristics needed in a primary standard. The subsequent demonstration of extreme values of the analyzing power lying on the base of the Lakin cone²⁾ improved the situation further, because these processes have very large cross-sections as compared to the reaction $^{16}\text{O}(\vec{d}, \alpha_1)^{15}\text{N}^*$. This is all the more important, since an accurate calibration requires a very well defined geometry in order to keep the geometrical corrections small.

The usefulness of points on the lateral surface of the Lakin cone¹⁶⁾ is somewhat limited by the fact that an accurate relative calibration between vector and tensor polarization is needed. This is necessary since at best only an overall calibration factor can be determined. Alternately, the consistency of the calibrations of vector and tensor polarization can be checked if an absolute calibration for only one of them is available. These extreme states of polarization are therefore more in the nature of secondary standards.

3.2 USE IN AN ANALYSIS

The conditions necessary and sufficient for an extreme point of the analyzing power are linear in the transition amplitudes. An analysis which includes a data point at an extreme value, searches for minima in a solution space, which is very strongly restricted by the linear condition thus imposed on the usual set of bilinear equations. This restriction of the solution space can be compared to the effects of parity conservation {eq.(12)} and time-reversal invariance which are also linear in the amplitudes. The important difference is that these conditions hold independent of energy and angle, whereas eqs.(13), (14), (15) and (16) are only valid at one point (θ_0, E_0) . In all known cases the analyzing power near an extreme value has essentially a parabolic shape as a function of both energy and angle. Thus over a relatively large area (θ, E) the data scatter statistically around values very close to the extreme and help determine its location (θ_0, E_0) . These data points are therefore almost as helpful as a measurement precisely at the point (θ_0, E_0) . This can also be seen from the following argument. Let $F_{\text{lin}}(M_{ij}, \theta_0, E_0) = 0$ be a linear condition for some amplitudes M_{ij} such as eq.(14), necessary and sufficient for an extreme value of the analyzing power. At a point (θ, E) , somewhat removed from (θ_0, E_0) , the observable will be considerably different from the extreme value. The conditions holding there are usually highly nonlinear, in view of the quadratic relations between observables. As the two points are moved closer together, these conditions begin to degenerate and upon close approach they can be written as

$$F_{\text{lin}}(M_{ij}, \theta_0, E_0) \{1 + \kappa \cdot f_{\text{nonlin}}(M_{k\ell}, \theta, E)\} = 0, \quad (18)$$

where κ is a small number and f_{nonlin} a nonlinear function of some amplitudes $M_{k\ell}$. At such a point (θ, E) the linear condition is therefore almost fulfilled and the solution space is already considerably restricted.

For a spin space more complex than that of the elastic scattering $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$ or $\frac{1}{2} + 0 \rightarrow 1 + 0$, there are generally more than one linear condition to be fulfilled simultaneously at the point (θ_0, E_0) . In these cases all that can be proven experimentally is that the analyzing power is equal to the extreme value within a certain error. However, using the argument in the preceeding paragraph, it is obvious that with eq.(18) the restriction of solution space is again of importance.

4. Extreme values in elastic scattering

4.1 PROCESSES IN THE SPIN SPACE $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$

The transition matrix M in this simple spin space is well known²⁷⁾

$$\underline{M} = A + \sigma_Y \cdot B = \begin{pmatrix} A & -iB \\ iB & A \end{pmatrix} \quad (19)$$

A point (θ_0, E_0) with an extreme value of

$$A_Y = \frac{2\text{Re}(AB^*)}{|A|^2 + |B|^2} = \pm 1 \quad (20)$$

requires¹⁾

$$M_{\frac{1}{2}, 0; \frac{1}{2}, 0} = \pm i M_{\frac{1}{2}, 0; -\frac{1}{2}, 0} \quad (21)$$

or

$$A = \pm B \quad (22)$$

This can also be derived easily from eqs.(13) and (19). The various formulae found instead of eqs.(21) and (22) in the literature depend on the definitions made. The requirement simply means that, except for a phase, the spin-flip and non-flip amplitudes have to be equal.

This criterion can be verified easily if a phase shift analysis is available¹⁾. In this manner several extreme points have been identified in different processes such as $p+{}^4\text{He}$, $n+{}^4\text{He}$, ${}^3\text{He}+{}^4\text{He}$, $t+{}^4\text{He}$, and $p+{}^{12}\text{C}$ elastic scattering^{1,6,28,29)}. If no phase shift analysis has been done, or if it is too uncertain, quadratic relations between various observables can be used to determine the existence of an extreme value³⁰⁾. This technique has been used

recently to calibrate the first polarized tritium source in Los Alamos³¹⁾.

In $p+^{12}\text{C}$ elastic scattering Meyer et al.²⁹⁾ investigated the phase shifts available in the energy region $5 \text{ MeV} < E_p < 12 \text{ MeV}$ and found between 8 and 10 extreme values. Meyer³²⁾ provided the authors with fig.2, which uses the definitions of Plattner and Bacher¹⁾. Plotted are the functions $d^{\min}_{\pm i, E_p}$, which give the closest approach of the amplitude g' to the points $\pm i$. A zero in these functions corresponds to an extreme value of the analyzing power. Clearly there are at least 8 such points. One or two more are possible for $A_y = +1$ at 8 MeV, where $d^{\min}_{+i, E_p}$ is zero within the error. This large number of extreme values corresponds closely to the number of levels in the corresponding range of excitation energies in ^{13}N . Ajzenberg-Selove³³⁾ lists 11 excited states in the same range, which confirms a correlation of extreme values and levels in the intermediate system which has been noted before 2,3,34).

4.2 PROCESSES IN THE SPIN SPACE $\vec{1} + 0 \rightarrow 1 + 0$

Three points of maximum possible analyzing power $A_{yy} = 1$ have been identified in $\vec{d} + {}^4\text{He}$ elastic scattering²⁾ below 12 MeV. An indication of an extreme value of the vector analyzing power $A_y = -1$ at an energy between 12 and 15 MeV could not be confirmed³⁵⁾. The three maxima together with measurements of the reaction ${}^{16}\text{O}(\vec{d}, \alpha_1){}^{15}\text{N}^*$, have been used for a very precise calibration of $T_{20}(0^\circ)$ of the ${}^3\text{He}(\vec{d}, p){}^4\text{He}$ reaction and of the vector analyzing power in ${}^4\text{He}(d, d){}^4\text{He}$ elastic scattering³⁶⁾. Near 30 MeV measurements by Conzett et al.³⁷⁾ yielded values of $A_y \approx 1$ at backward angles. Additional measurements confirmed the possibility³⁸⁾. However, due to the lack of a polarization standard at cyclotron energies, no precise values could be given. Further measurements covering the energy range from 20 to 40 MeV are being undertaken at present both at Berkeley³⁹⁾ and at SIN⁴⁰⁾. Performing a phase shift analysis may then solve the question. If an extreme point $A_{yy} = 1$ or $A_y = 1$ can be found near 30 MeV, the resulting calibration point would be very important for future measurements at cyclotron energies. Thus a study of the consequences of extreme points of the analyzing power are of considerable interest.

Using the parametrization of Ohlsen et al.⁴¹⁾ the transition matrix $\underline{M}$ has the structure

$$\underline{M} = \frac{1}{2} \begin{pmatrix} A - B & \sqrt{2}D & -A - B \\ -\sqrt{2}D & 2C & \sqrt{2}D \\ -A - B & -\sqrt{2}D & A - B \end{pmatrix}. \quad (23)$$

The condition for $A_{yy} = 1$ has been given earlier^{2,42)}. It follows immediately from eq.(14)

$$M_{1,0;1,0} = -M_{1,0;-1,0} \quad (24)$$

which yields

$$B = 0 \quad (25)$$

in the parametrization used for eq.(23). If this condition is introduced into the formulae for the observables⁴¹⁾, the result is

$$A_{YY} = p^{Y'Y'} = K_{YY}^{Y'Y'} = 1 \quad (26a)$$

$$A_{XX} = p^{X'X'} = K_{XX}^{X'X'} = K_{XX}^{Y'Y'} \quad (26b)$$

$$A_{ZZ} = p^{Z'Z'} = K_{ZZ}^{Z'Z'} = K_{ZZ}^{Y'Y'} \quad (26c)$$

$$K_x^{X'} = K_z^{X'} = K_{xy}^{X'} = K_{yz}^{X'} = 0 \quad (26d)$$

$$K_x^{Z'} = K_z^{Z'} = K_{xy}^{Z'} = K_{yz}^{Z'} = 0 \quad (26e)$$

$$K_x^{X'Y'} = K_z^{X'Y'} = K_{xy}^{X'Y'} = K_{yz}^{X'Y'} = 0 \quad (26f)$$

$$K_x^{Y'Z'} = K_z^{Y'Z'} = K_{xy}^{Y'Z'} = K_{yz}^{Y'Z'} = 0. \quad (26g)$$

The quantities p, A and K denote the deuteron polarization, the deuteron analyzing power and the polarization transfer coefficients, respectively.

Eqs.(26d)-(26g) are of particular interest, since a value of zero at the extreme point (θ_0, E_0) can be demonstrated independently of any calibration of polarizations. In this manner an extreme point can be established by a carefully selected series of measurements instead of a phase shift analysis³⁾.

For an extreme value $A_y = \pm 1$ the requirements on the M-matrix are derived from eq.(15)

$$M_{1,0;0,0} = \mp i \sqrt{2} M_{1,0;1,0} \quad (27)$$

$$M_{0,0;0,0} = \mp i \sqrt{2} M_{0,0;1,0} ,$$

in addition to eq.(24). In the parametrization of Ohlsen⁴¹⁾ this gives

$$\begin{aligned} A &= C = \pm iD \\ B &= 0 . \end{aligned} \quad (28)$$

Introduction into the M-matrix results in

$$\underline{M} = \frac{A}{2} \begin{pmatrix} 1 & \mp i\sqrt{2} & -1 \\ \pm i\sqrt{2} & 2 & \pm i\sqrt{2} \\ -1 & \pm i\sqrt{2} & 1 \end{pmatrix}. \quad (29)$$

Thus only the cross-section $\sigma_0 = \text{tr}(\underline{M} \underline{M}^\dagger)$ is a free parameter, while all 51 polarization observables are numerically determined⁸).

$$A_Y = p^{Y'} = K_Y^{Y'Y'} = K_{YY}^{Y'} = \pm 1 \quad (30a)$$

$$A_{YY} = p^{Y'Y'} = K_Y^{Y'} = K_{YY}^{Y'Y'} = 1 \quad (30b)$$

$$A_{xz} = p^{x'z'} = K_{xz}^{x'z'} = 0 \quad (30c)$$

$$K_x^{x'} = K_z^{x'} = K_{xy}^{x'} = K_{yz}^{x'} = 0 \quad (30d)$$

$$K_x^{z'} = K_z^{z'} = K_{xy}^{z'} = K_{yz}^{z'} = 0 \quad (30e)$$

$$K_x^{x'Y'} = K_z^{x'Y'} = K_{xy}^{x'Y'} = K_{yz}^{x'Y'} = 0 \quad (30f)$$

$$K_x^{Y'z'} = K_z^{Y'z'} = K_{xy}^{Y'z'} = K_{yz}^{Y'z'} = 0 \quad (30g)$$

$$K_{xz}^{Y'} = K_{xz}^{x'x'} = K_{xz}^{Y'Y'} = K_{xz}^{z'z'} = 0 \quad (30h)$$

$$K_y^{x'z'} = K_{xx}^{x'z'} = K_{yy}^{x'z'} = K_{zz}^{x'z'} = 0 \quad (30i)$$

$$K_y^{z'z'} = K_{zz}^{Y'} = K_y^{x'x'} = K_{xx}^{Y'} = \mp \frac{1}{2} \quad (30k)$$

$$K_{zz}^{z'z'} = K_{xx}^{z'z'} = K_{zz}^{x'x'} = K_{xx}^{x'x'} = \frac{1}{4} \quad (30l)$$

$$A_{xx} = A_{zz} = K_{xx}^{Y'Y'} = K_{zz}^{Y'Y'} = -\frac{1}{2} \quad (30m)$$

$$p^{x'x'} = p^{z'z'} = K_{yy}^{x'x'} = K_{yy}^{z'z'} = -\frac{1}{2}. \quad (30n)$$

With these conditions a straight-forward identification of an extreme point with $A_Y = \pm 1$ is possible. Care should be taken to verify each condition in several ways, choosing the observables by an inspection of the general formulae and taking into account

the relations between various observables required by other symmetries⁴¹). If such an extreme point $A_y = \pm 1$ can be found, the effort to determine precisely the limits within which $|A_y| = 1$, is well worthwhile in view of the fact that at such a point any observable can be calibrated.

5. Extreme values in reactions

5.1 PROCESSES IN THE SPIN SPACE $\vec{1} + 0 \rightarrow 0 + 0$

Polarization in this spin space is highly restricted. For the case of a positive product π_{tot} of all intrinsic parities ($1^+ + 0^+ \rightarrow 0^+ + 0^+$), Jacobsohn and Ryndin⁴³) have pointed out that the polarization observables must have the values

$$T_{20} = \frac{1}{2}\sqrt{2}, \quad T_{22} = \frac{1}{2}\sqrt{3}, \quad T_{11} = T_{21} = 0, \quad (31a)$$

or

$$\begin{aligned} A_{yy} &= -2, & A_{xx} &= A_{zz} = 1, & A_y &= A_{xz} = 0, \\ A_{xx} - A_{yy} &= 3. \end{aligned} \quad (31b)$$

Since eq.(31) is a consequence of parity conservation, it holds independent of energy and angle. This can also be seen from a discussion of the M-matrix. It has the dimensions 3×1 , and eq. (12) leads directly to

$$\underline{M} = (A \ 0 \ A) = A(1 \ 0 \ 1). \quad (32)$$

Consequently, all polarization observables are fixed, the cross-section $\sigma_0(\theta)$ being the only free parameter. This situation is similar to the one in $^4\text{He}+d$ elastic scattering for $A_y = \pm 1$ (subsect. 4.2). It is also obvious that eqs.(31) are identical to eqs.(9) for the apex of the Lakin cone (point C in fig.1). Thus the polarization efficiency in this reaction is restricted to this highly singular point, independent of energy and angle.

In using such a reaction for a calibration it should be noted that the tensor component A_{xx} and A_{yy} are usually measured directly, while A_{zz} and $A_{xx} - A_{yy}$ or T_{20} and T_{22} are quantities derived from these two measurements. An exception is an arrangement using four counters, 90 degrees apart in azimuthal angles. Of the quantities directly available, A_{yy} is four times as ef-

ficient an analyzer as A_{xx} , or A_{zz} in a four counter geometry, although all of these observables are also at limiting values. This component of the analyzing power was consequently measured by Darden and collaborators²⁶⁾ in the first absolute calibration of deuteron tensor polarization. They used the reaction $^{16}\text{O}(\vec{d}, \alpha_1) ^{14}\text{N}_0^{*+}$, an isospin-forbidden process with a relatively low cross-section. Selecting the observable with the best overall efficiency is therefore important. Another attractive feature is the insensitivity to vector polarization while all the tensor components are exactly known.

In recent years several groups^{36,44,45)} have used this process as a primary polarization standard and have calibrated other reactions as secondary standards. Several other processes, some of them isospin conserving, have been proposed by Gruebler⁶⁾, but their importance has been diminished somewhat by the extreme points $A_{yy} = 1$ found in $^4\text{He}(\vec{d}, d) ^4\text{He}$ elastic scattering, a process with large cross-sections. Even so, a very recent calibration of secondary standards and deuteron vector polarization used both processes³⁶⁾.

For the case of a negative product π_{tot} of the intrinsic parities ($1^+ + 0^+ \rightarrow 0^+ + 0^-$), the analyzing power A_{yy} assumes its maximum possible value of unity^{14,30)}. In addition a quadratic condition

$$(A_y)^2 + \left(\frac{2}{3}A_{xz}\right)^2 + \left[\frac{1}{3}(A_{zz} - A_{xx})\right]^2 = 1 \quad (33)$$

has been given³⁰⁾. It is identical with the limiting value of inequality (5). Thus the point $A_{yy} = 1$ has to lie somewhere on the periphery of the base of the Lakin cone (fig.1).

The reaction $^{16}\text{O}(\vec{d}, \alpha_3) ^{14}\text{N}_0^{*-}$ is such a process. It has been used by the Wisconsin group^{46,47)} as the first absolute standard for deuteron vector polarization, by demonstrating the existence of points $A_y = \pm 1$ with a procedure analogous to that of Plattner and Bacher¹⁾. The M-matrix in this extremely simple spin-space

has the structure

$$\underline{M} = (A \ B \ -A), \quad (34)$$

which follows directly from the parity condition (12). It therefore automatically fulfills the requirement for $A_{yy} = 1$, which is

$$M_{0,0;1,0} = -M_{0,0;-1,0} \quad (35)$$

and condition (33) limits the states of polarization to the periphery of the base of the Lakin cone. Since the points A and B for $A_y = \pm 1$ also lie on that circle, these extreme values of the vector analyzing power are possible at some point (θ_0, E_0) . Eq. (15) yields for that case⁴⁶⁾

$$M_{0,0;0,0} = \mp i\sqrt{2} \ M_{0,0;1,0} \quad (36a)$$

or

$$B = \mp i\sqrt{2} \ A. \quad (36b)$$

For the M-matrix this leads to

$$\underline{M} = A(1 \ \mp i\sqrt{2} \ -1), \quad (37)$$

which means that again only the cross-section $\sigma_0(\theta)$ is a parameter variable with angle and energy. The other observables are given by eq.(6). It is therefore obvious that a direct verification of eq.(36), which requires an analysis of the process, is not necessary. Eq.(6) yields the simultaneous conditions⁴⁷⁾

$$A_{zz} + \frac{1}{2} = 0, \quad A_{xz} = 0, \quad (38)$$

which can be verified by plotting $A_{xx} + \frac{1}{2}$ versus A_{xz} with angles θ and energy E as parameters and demonstrating the existence of a point (θ_0, E_0) at the origin by a continuity argument¹⁾.

With this type of process, the vector polarization can also be determined accurately at points on the circle other than points

A or B ⁴⁷⁾ by making use of eq.(33). The accuracy of such a measurement is limited by the fact that at the point selected the tensors $T_{2q}(\theta)$ are not known absolutely and have to be determined separately. At the extreme points A and B on the other hand the effects of the second rank moments can be corrected for accurately.

5.2 PROCESSES IN THE SPIN SPACE $\vec{1} + \frac{1}{2} \rightarrow \frac{1}{2} + 0$

The best known reaction of this type, ${}^3\text{He}(\vec{d}, p){}^4\text{He}$, was the first process other than elastic scattering for which a point (θ_0, E_0) with maximum possible tensor analyzing power $A_{yy} = 1$ was proposed³⁾. This extreme value has recently been confirmed within the experimental error³⁴⁾. In addition, an inspection of the complete polarization data of Gruebler et al.⁴⁸⁾ at 11.5 MeV and of the measurements of iT_{11} at 15 MeV by Roy et al.⁴⁹⁾ indicates the possibility of large absolute values near $\theta_{\text{cm}} = 120^\circ$ for both components A_y and A_{yy} of the analyzing power. As the energy increases, so does A_{yy} near 120° (see fig.3) and the rest of conditions (6) are nearly fulfilled at 11.5 MeV, (see fig.4) a prerequisite for values of A_y close to unity. Thus a point (θ_0, E_0) between 12 and 15 MeV with $A_y = A_{yy} = 1$ is a possibility. It should be noted that for large values of A_y the component A_{yy} has to be large also due to restriction (8). Also conditions (6) have to be more or less fulfilled. However, a circle is not very restrictive for lateral movement away from a diameter, even close to the periphery. Thus a close approach of the measured data to the required values may be indicative of an extreme point $A_y = \pm 1$.

Using the representation of Keaton et al.⁵⁰⁾, the M-matrix is

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} -iA-D \sqrt{2}F & -iA+D & -B-C & \sqrt{2}E & -B+C \\ B-C \sqrt{2}E & B+C & -iA+D & \sqrt{2}F & -iA-D \end{pmatrix}. \quad (39)$$

The conditions (15) yield³⁾

$$M_{\frac{1}{2},0;1,\frac{1}{2}} = -M_{\frac{1}{2},0;-1,\frac{1}{2}}$$

(40)

$$M_{\frac{1}{2},0;1,-\frac{1}{2}} = -M_{\frac{1}{2},0;-1,-\frac{1}{2}},$$

for $A_{yy} = 1$, which reduce to

$$A = B = 0$$

(41)

in the parametrization of Keaton et al. Introducing eq.(41) into their formulae for the observables gives the following conditions for an extreme value $A_{yy} = 1$,

$$p^{y'} = -A_{o,y} = K_{yy}^{y'} = -C_{yy,y} \quad (42a)$$

$$K_{xx}^{y'} = -C_{xx,y} \quad (42b)$$

$$K_{zz}^{y'} = -C_{zz,y} \quad (42c)$$

$$K_x^{x'} = K_z^{x'} = K_x^{z'} = K_z^{z'} = 0 \quad (42d)$$

$$K_{xy}^{x'} = K_{yz}^{x'} = K_{xy}^{z'} = K_{yz}^{z'} = 0 \quad (42e)$$

$$C_{x,x} = C_{z,x} = C_{x,z} = C_{z,z} = 0 \quad (42f)$$

$$C_{xy,x} = C_{yz,x} = C_{xy,z} = C_{yz,z} = 0. \quad (42g)$$

Here $p^{y'}$ denotes the proton polarization, $A_{o,y}$ the ^3He analyzing power, K_i^j and K_{ij}^k the polarization transfer coefficients and $C_{i,j}$ and $C_{ij,k}$ the efficiency correlation coefficients. Again eqs.(42d) to (42g) are of special importance in an experimental verification of conditions (40) or (41), because zero crossings are independent of any calibrations.

For an extreme value $A_y = \pm 1$, the conditions (15) reduce to

$$M_{\frac{1}{2},0;0,\frac{1}{2}} = \mp i\sqrt{2} M_{\frac{1}{2},0;1,\frac{1}{2}}$$

$$M_{\frac{1}{2},0;0,-\frac{1}{2}} = \mp i\sqrt{2} M_{\frac{1}{2},0;1,-\frac{1}{2}},$$
(43)

in addition to eqs.(40). In the parametrization of Keaton this yields

$$A = B = 0, \quad C = \mp iE, \quad D = \mp iF,$$
(44)

and the conditions

$$A_y = \pm 1$$
(45a)

$$A_{yy} = -K_{o,y}^{y'} = 1$$
(45b)

$$A_{xx} = A_{zz} = -\sqrt{2}$$
(45c)

$$A_{xz} = K_{xz}^{y'} = C_{xz,y} = 0$$
(45d)

$$K_x^{x'} = K_z^{x'} = K_{xy}^{x'} = K_{yz}^{x'} = 0$$
(45e)

$$K_x^{z'} = K_z^{z'} = K_{xy}^{z'} = K_{yz}^{z'} = 0$$
(45f)

$$C_{x,x} = C_{z,x} = C_{xy,z} = C_{y,zx} = 0$$
(45g)

$$C_{x,z} = C_{z,z} = C_{xy,z} = C_{yz,z} = 0$$
(45h)

$$p^{y'} = -A_{o,y} = K_{yy}^{y'} = -C_{yy,y} = t$$
(45i)

$$K_y^{y'} = -C_{y,y} = \pm t$$
(45k)

$$C_{xx,y} = -K_{xx}^{y'} = C_{zz,y} = K_{zz}^{y'} = \sqrt{2} t$$
(45l)

$$K_{o,x}^{x'} = -K_{o,z}^{z'} = u$$
(45m)

$$K_{o,z}^{x'} = K_{o,x}^{z'} = v.$$
(45n)

In addition to the cross-section σ_0 there are thus 3 free polarization parameters t , u and v . An extreme point $A_y = \pm 1$ is therefore not as valuable as in ${}^4\text{He}(d,d){}^4\text{He}$ elastic scattering. But

with the attractive features of this reaction, such as the large Q-value and cross-section, it would prove very useful.

5.3 PROCESSES IN THE SPIN SPACE $\vec{1} + 1 \rightarrow 0 + 0$

The ${}^6\text{Li}(\vec{d}, \alpha){}^4\text{He}$ reaction belongs to this set. An inspection of the data for $T_{2q}(\theta)$ of Risler et al.⁵¹⁾ shows that the values for $A_{yy}(\theta)$ come close to unity at two points⁹⁾, near 6 MeV and forward angles and near 8 MeV and $\theta_{\text{cm}} = 90^\circ$ (fig.5). Between 5 and 7 MeV the angular distributions of all observables measured change drastically and the vector analyzing power becomes positive and large. Thus also an extreme point $A_y = 1$ is possible⁹⁾.

The M-matrix for the process ${}^6\text{Li}(\vec{d}, \alpha){}^4\text{He}$ has been discussed by Köhler and Fick⁵²⁾

$$\underline{M} = -\frac{1}{2} (a-e, \sqrt{2}f, a+e; \sqrt{2}h, -2k, -\sqrt{2}h; a+e, -\sqrt{2}f, a-e). \quad (46)$$

For an extreme value $A_{yy} = 1$, the condition is⁹⁾

$$M_{0,0;1,1} = -M_{0,0;-1,1} \quad (47a)$$

or

$$a = 0. \quad (47b)$$

In this case therefore the type of proof used in the similar spin space $\vec{1} + 0 \rightarrow 1 + 0$ of ${}^4\text{He}(\vec{d}, d){}^4\text{He}$ can be used. While an analysis is not yet available, the required values of certain observables can be tested. Beside the analyzing power $A_{ij}^{(1)}$ for deuteron polarization and the efficiency $A_{ij}^{(2)}$ for ${}^6\text{Li}$ polarization, only the efficiency correlation coefficients $C_{ij,kl}$ can be measured. Such data will become available in the near future, since both vector and tensor polarized ${}^6\text{Li}$ beams are available^{53,54)} and highly polarized ${}^6\text{Li}$ targets have also been produced⁵⁵⁾.

The conditions (47) imposed on the 25 linearly independent observables of the reaction⁵²⁾, yield the values

$$A_{YY}^{(1)} = A_{YY}^{(2)} = 1 \quad (48a)$$

$$C_{x,x} = C_{x,z} = C_{z,x} = C_{z,z} = 0 \quad (48b)$$

$$C_{xy,x} = C_{xy,z} = C_{yz,x} = C_{yz,z} = 0 \quad (48c)$$

Eq.(48a) contains the result that an extreme value for the component $A_{YY}^{(i)}$ automatically implies an extreme value of the other analyzing power $A_{YY}^{(k)}$ as well, since $A_{YY}^{(1)} = A_{YY}^{(2)}$ {ref.⁵⁶}. This is of interest for the calibration of the tensor polarization of ${}^6\text{Li}$ beams in view of the two possible extreme points in the ${}^6\text{Li}(d,\alpha){}^4\text{He}$ reaction. Also it means that an extreme value $A_{YY}^{(i)} = 1$ of the tensor polarization may indicate a point where the vector polarization $A_Y^{(k)} = \pm 1$.

For a point $A_Y^{(1)} = \pm 1$, the conditions (15) give

$$M_{0,0;0,1} = \mp i\sqrt{2} M_{0,0;1,1} \quad (49a)$$

$$M_{0,0;0,0} = \mp i\sqrt{2} M_{0,0;1,0}$$

together with eqs.(47a). In the parametrization of Köhler and Fick, this means

$$a = 0, \quad h = \pm ie, \quad k = \pm if. \quad (49b)$$

For the observables the requirement is therefore

$$A_Y^{(1)} = \pm 1 \quad (50a)$$

$$A_{YY}^{(1)} = A_{YY}^{(2)} = 1 \quad (50b)$$

$$A_{xx}^{(1)} = A_{zz}^{(1)} = -\frac{1}{2} \quad (50c)$$

$$A_{xz}^{(1)} = C_{xz,y} = C_{xz,xz} = C_{xz,zz} = 0 \quad (50d)$$

$$C_{x,z} = C_{z,x} = C_{x,x} = C_{z,z} = 0 \quad (50e)$$

$$C_{xy,x} = C_{xy,z} = C_{yz,x} = C_{yz,z} = 0 \quad (50f)$$

$$A_Y^{(2)} = -2C_{zz,y} = \pm C_{y,y} \quad (50g)$$

$$A_{xz}^{(2)} = -2C_{zz,xz} = \pm C_{y,xz} \quad (50h)$$

$$A_{zz}^{(2)} = -2C_{zz,zz} = \pm C_{y,zz} , \quad (50i)$$

while the parameters not listed can vary freely.

For ${}^6\vec{\text{Li}}$ -polarization the conditions (15) give

$$M_{0,0;1,0} = \mp i\sqrt{2} M_{0,0;1,1} \quad (51a)$$

$$M_{0,0;0,0} = \mp i\sqrt{2} M_{0,0;0,1}$$

for a point $A_Y^{(2)} \pm 1$, together with eq. (47). In the parametrization of Köhler and Fick this means that

$$a = 0, f = \pm ie, k = \pm ih . \quad (51b)$$

For the observables the following conditions are obtained.

$$A_Y^{(2)} = \pm 1 \quad (52a)$$

$$A_{YY}^{(1)} = A_{YY}^{(2)} = 1 \quad (52b)$$

$$A_{xx}^{(2)} = A_{zz}^{(2)} = -\frac{1}{2} \quad (52c)$$

$$A_{xz}^{(2)} = C_{y,xz} = C_{xz,xz} = C_{zz,xz} = 0 \quad (52d)$$

$$C_{x,z} = C_{z,x} = C_{x,x} = C_{z,z} = 0 \quad (52e)$$

$$C_{xy,x} = C_{xy,z} = C_{yz,x} = C_{yz,z} = 0 \quad (52f)$$

$$A_Y^{(1)} = -2C_{y,zz} = \pm C_{y,y} \quad (52g)$$

$$A_{xz}^{(1)} = -2C_{xz,zz} = \pm C_{xz,y} \quad (52h)$$

$$A_{zz}^{(1)} = -2C_{zz,zz} = \pm C_{zz,y} . \quad (52i)$$

6. Discussion

Extreme values of the analyzing power have been used very successfully for the calibration of polarizations. In practice, however, problems have arisen with the transfer of the calibration to energies different from that of the extreme point. Systematic errors due to the increase in the spread of energy and angle as well as multiple scattering effects make a calibration to better than 1 percent a difficult undertaking. For this reason the polarization group at the ETH in Zürich advocates the use of several standards in the calibration of secondary standards³⁶). Clearly, the identification of as many extreme values as possible is therefore of importance. Of particular interest are those points that have in addition near-extreme values over a large range of energy and angle.

In this paper the conditions have been discussed in detail for all those processes which show immediate promise of having points of extreme analyzing power. Except for the simplest spin spaces, a direct analytical proof for the existence of an extreme value cannot be given. Even in the cases where it would be possible it may not be feasible because an analysis is not yet available. One solution is a careful measurement with a calibrated beam. However, all that can be shown is equality with the extreme value within a certain error. It will probably be very difficult to improve significantly on that accuracy due to the systematic errors discussed above. More stringent restrictions on the difference between the experimental and the limiting value can be derived from the measurement of other observables. These can be selected on the basis of quadratic relations among observables that are generally valid and are evaluated at the extreme point. It has been demonstrated that small deviations from the extreme value can be detected with a large sensitivity³¹). Conversely, a value slightly different from the limit can also be established quite accurately. Another method directly imposes the conditions for an extreme value on the observables³) and selects particularly those which do not involve any calibrations, such as observables

that must be zero at the extreme point. Clearly, an error in the calibration will only affect the slope of the zero-crossing but not the fact of its existence at the exact point (θ_0, E_0) . Using these methods it should be possible to establish to a very good accuracy the existence of an extreme value.

For spin spaces other than those discussed in this paper, the effect of conditions (13)-(15) on the observables can be computed by obtaining the relevant M-matrix using the methods of Csonka et al.^{57,58}). Care should always be taken to check the fulfillment of these conditions in several independent ways. In this context polarized targets and beams of polarized particles such as t , ${}^3\text{He}$, ${}^6\text{Li}$ and ${}^7\text{Li}$ assume a particular importance. In reactions between light nuclei, they often allow the determination of all first order polarization quantities, i.e. those that involve only the determination of one particle polarization. Purely from the point of view of counting statistics, they are the quantities most easily measured. Of the polarization observables of second order, i.e. those involving the measurement of two particle polarizations, the efficiency correlation coefficients are certainly the easiest to determine. Not only because sizeable beam and target polarizations can be obtained together with large count-rates, but also because the problems encountered in an accurate measurement are no more complex than those of the usual measurement of a relative cross-section. The other two kinds of second order polarization data involve a second reaction at a target geometrically removed from the first one, leading to problems with energy and angular spread in the incident beam of the second reaction. Thus, the count rates are lower and the corrections harder to evaluate. The selection of the observables used to check on conditions (13) to (15) should take these considerations into account and make use of the simpler experiments whenever possible.

In processes with more complex spin spaces, where several linear conditions must hold, the a priori probability of an extreme value is very low. For n conditions $2n$ real equations must be ful-

filled simultaneously. With only two free variables, energy E and angle θ , this is very improbable unless there is a correlation between certain amplitudes. Such correlations are possible in the neighbourhood of strong levels in the intermediate system of the reaction. The contribution of these resonances to the various transition amplitudes will cause a partly correlated variation of these quantities with energy. Depending on the background contributions to the relevant amplitudes, the conditions necessary for an extreme value may then be fulfilled at some point (θ_0, E_0) near the level energy. This proximity of intermediate levels and extreme values is quite noticeable in $\vec{d} + {}^4\text{He}$ and $\vec{p} + {}^{12}\text{C}$ elastic scattering^{2,29)} and in the reaction ${}^6\text{Li}(\vec{d}, \alpha){}^4\text{He}$ {refs.^{59,60)}}, even though the correlation is not needed due to the simple spin space. For the process ${}^3\text{He}(\vec{d}, p){}^4\text{He}$, however, two conditions must be met and a correlation is necessary. As expected, the extreme value at 9 MeV lies directly on top of the proposed $J^\pi = 7/2^+$ level^{23,59)}. Although a direct analytical proof for the existence of an extreme point is impossible, the measurements of Gruebler et al.³⁴⁾ show that at least the correlations necessary for $A_{yy} = 1$ are present. In this context the analytical proof for the existence of a point on the lateral surface of the Lakin cone in $\vec{d} + {}^4\text{He}$ elastic scattering¹⁶⁾ is of interest. Since it proves the simultaneous fulfillment of two conditions, it also demonstrates that the correlations needed do indeed occur.

While extreme values of polarization observables are immediately obvious for spin- $\frac{1}{2}$ particles, the discussion in section 2 shows that this is not necessarily so for polarizations of spin $I = 1$. Such data should therefore be discussed routinely in the context of the polarization sphere and the Lakin cone. Extreme or near extreme states of polarization may show up even though the individual observables are well within their limits.

Perhaps the most important application of extreme values of polarization observables may be found in the analyses of the processes in which they occur. Similar to the invariance conditions of parity and time reversal³⁾, the conditions (13) to (15) are linear in the elements of the M -matrix. Unlike the former, they are not valid independent of energy and angle, but hold only for the extreme point (θ_0, E_0) . There, however, they lead to severe

restrictions on the solution space. Thus the measurements made to establish the presence of an extreme value have another important application. Introduced into the data base of an analysis, they directly impose these linear conditions on the set of bilinear equations. This may be of importance even in those cases where the existence of an extreme point can be proven analytically, since the quality of the solution required for that kind of proof is not very high. With a few additional measurements, it can thus be improved considerably. Measurements on or near points of extreme values of polarization observables, together with criteria that determine the dominant amplitudes of the process^{23,59}) should help to reduce decisively the effort necessary to analyze processes with a non-trivial spin-space. This may be demonstrated in the near future, when the data on the ${}^3\text{He}(\vec{d}, p){}^4\text{He}$ reaction near the extreme value³⁴) are introduced into a full-scale analysis such as the one carried out by Dodder and Hale⁶¹). Similarly the analysis of the process ${}^6\text{Li}(\vec{d}, \alpha){}^4\text{He}$ will profit both by the preliminary evaluations^{23,60}) and by the data being taken at present in order to confirm the two extreme values proposed⁶²). It should be noted that there again these points lie in an energy region with several overlapping resonances, where the analysis is very uncertain.

The authors would like to thank Dr. H.O. Meyer for providing them with fig.2. One of us (F.S.) gratefully acknowledges the hospitality he enjoyed at LBL in 1974/75 and again in 1976. In both instances he was supported with travel grants by the Max Geldner Foundation in Basel, and for part of the work also by the Schweizerischer Nationalfonds.

References

- 1) G.R. Plattner and A.D. Bacher, Phys. Lett. 36B (1971) 211
- 2) W. Grüebler, R.A. Schmelzbach, V. König, R. Risler, B. Jenny and D. Boerma, Nucl. Phys. A242 (1975) 285
- 3) F. Seiler, Phys. Lett. 61B (1976) 144
- 4) H.E. Conzett and F. Seiler, Nucl. Phys. in print
- 5) Proc. of the 4th Int. Symp. on Polarization Phenomena in Nuclear Reactions, W. Grüebler and V. König, eds. (Birkhäuser, Basel, 1976)
- 6) W. Grüebler, ref.⁵⁾ p.307
- 7) F. Seiler, R. Roy, H.E. Conzett and F.N. Rad, ref.⁵⁾ p.550
- 8) F. Seiler, H.E. Conzett, F.N. Rad and R. Roy, ref.⁵⁾ p.571
- 9) F. Seiler, F.N. Rad, H.E. Conzett and R. Roy, ref.⁵⁾ p.587
- 10) F. Seiler, F.N. Rad and H.E. Conzett, ref.⁵⁾ p.897
- 11) Polarization Phenomena in Nuclear Reactions, H.H. Barschall and W. Haeberli, eds. (University of Wisconsin Press, 1971)
- 12) W. Lakin, Phys. Rev. 98 (1955) 139
- 13) P. Minnaert, Phys. Rev. Lett. 16 (1966) 672
- 14) M. Simonius, Polarization Nuclear Physics, Lecture Notes in Physics, Vol.30, D. Fick, ed. (Springer, 1974) p.38
- 15) S.E. Darden, ref.¹¹⁾ p.39
- 16) F. Seiler and H.W. Roser, Phys. Lett. 69B (1977) 411
- 17) M.J. Scott, Phys. Rev. 110 (1958) 1398
- 18) R.I. Brown, W. Haeberli and J.X. Saladin, Nucl. Phys. 47 (1963) 212
- 19) M.F. Johns and E.M. Bernstein, Phys. Rev. 162 (1967) 871
- 20) A. Galonsky, H.B. Willard and T.A. Welton, Phys. Rev. Lett. 2 (1959) 349
- 21) L.C. McIntyre and W. Haeberli, Nucl. Phys. A91 (1967) 369
- 22) F. Seiler and E. Baumgartner, Nucl. Phys. A153 (1970) 193
- 23) F. Seiler, Nucl. Phys. A244 (1975) 236
- 24) E.M. Bernstein, G.G. Ohlsen, V.S. Starkovich and W.G. Simon, Phys. Rev. Lett. 18 (1967) 966
- 25) G.G. Ohlsen, V.S. Starkovich, W.G. Simon and E.M. Bernstein, Phys. Lett. 28B (1969) 404

- 26) S.E. Darden, R.M. Prior and K.W. Corrigan, Phys. Rev. Lett. 25 (1970) 1673
- 27) L. Wolfenstein and I. Ashkin, Phys. Rev. 85 (1952) 947
- 28) D.C. Dodder, G.M. Hale, Nelson Jarmie, J.H. Jett, P.W. Keaton jr., R.A. Nisley and K. Witte, Phys. Rev. C15 (1977) 518
- 29) H.O. Meyer, T.P. Cleary and W. Haeberli, Nucl. Inst. Meth. 140 (1977) 85
- 30) P.W. Keaton, D.D. Armstrong, R.A. Hardekopf, P.M. Kurjan and Y.K. Lee, Phys. Rev. Lett. 29 (1972) 880
- 31) R.A. Hardekopf, G.G. Ohlsen, V. Poore and N. Jarmie, Phys. Rev. C13 (1976) 2127
- 32) H.O. Meyer, University of Basel, private communication
- 33) F. Ajzenberg-Selove, Nucl. Phys. A268 (1976) 1
- 34) W. Grüebler, P.A. Schmelzbach, V. König, B. Jenny, R. Risler, H.R. Bürgi and J. Nurzynski, Phys. Lett. 63B (1976) 273 and Nucl. Phys. A271 (1976) 29
- 35) W. Grüebler, P.A. Schmelzbach, V. König, R. Risler, B. Jenny, D.O. Boerma and W.G. Weitcamp, ref.⁵) p.564
- 36) P.A. Schmelzbach, W. Grüebler, V. König, R. Risler, D.O. Boerma and B. Jenny, Nucl. Phys. A264 (1976) 45
- 37) H.E. Conzett, W. Dahme, Ch. Leemann, J.A. McDonald and J.P. Meulders, ref.⁵) p.566
- 38) H.E. Conzett, F. Seiler, F.N. Rad, R. Roy and R.M. Larimer, ref.⁵) p.568
- 39) E.J. Stephenson, H.E. Conzett, R.M. Larimer, B.T. Leemann and R. Roy, to be published
- 40) H.R. Bürgi, W. Grüebler, R. Hardekopf, B. Jenny, V. König, J. Nurzynski, W. Reichart, R. Risler, H.W. Roser, P.A. Schmelzbach and F. Seiler, SIN Jahresbericht 1976, p.E90 and to be published
- 41) G.G. Ohlsen, J.L. Gammel and P.W. Keaton, Phys. Rev. C5 (1972) 1205
- 42) G.G. Ohlsen, P.A. Lovoi, G.C. Salzmann, A. Meyer-Berkhout, C.K. Mitchell and W. Grüebler, Phys. Rev. C8 (1973) 1262
- 43) B.A. Jacobsohn and R.M. Ryndin, Nucl. Phys. 24 (1961) 505
- 44) V. König, W. Grüebler, H. Ruh, R.E. White, P.A. Schmelzbach, R. Risler and P. Marmier, Nucl. Phys. A166 (1971) 393

- 45) T.A. Trainor, T.B. Clegg and P.W. Lisowski, Nucl. Phys. A220 (1974) 533
- 46) K. Stephenson, L.D. Knutson, W. Haeberli, R.P. Goddard and R.R. Cadmus jr., Bull. Am. Phys. Soc. 21 (1976) 637
- 47) K. Stephenson, L.D. Knutson ^{and} W. Haeberli, Nucl. Phys. A277 (1977) 365
- 48) W. Grüebler, V. König, A. Ruh, P.A. Schmelzbach, R.E. White and P. Marmier, Nucl. Phys. A176 (1971) 631
- 49) R. Roy, F. Seiler, H.E. Conzett, F.N. Rad and R.M. Larimer, ref.⁵⁾ p.548 and to be published
- 50) P.W. Keaton jr., J.L. Gammel and G.G. Ohlsen, Ann. Phys. 85 (1974) 152
- 51) R. Risler, W. Grüebler, A.A. Debenham, V. König, P.A. Schmelzbach and D.O. Boerma, Nucl. Phys., A286 (1977) 115
- 52) W.E. Köhler and D. Fick, Z. Physik 215 (1968) 408
- 53) E. Steffens, H. Ebinghaus, F. Fiedler, K. Bethge, G. Engelhardt, R. Schäfer, W. Weiss and D. Fick, Nucl. Instr. and Meth. 124 (1975) 601
- 54) D. Fick, private communication
- 55) F. Wittchow, J. Ulbricht, U. Holm, K.D. Stahl and H. Ebinghaus, Phys. Lett. 59B (1975) 29
- 56) D. Fick, ref.¹¹⁾ p.861
- 57) P.L. Csonka, M.J. Moravcsik and M.D. Scadron, Ann. Phys. 40 (1966) 100
- 58) P.L. Csonka, M.J. Moravcsik and M.D. Scadron, Ann. Phys. 41 (1967) 1
- 59) F. Seiler, Nucl. Phys. A187 (1972) 379
- 60) R. Risler, W. Grüebler, V. König and P.A. Schmelzbach, Nucl. Phys. A286 (1977) 131
- 61) D.C. Dodder and G.M. Hale, private communication and ref.⁵⁾ p.167
- 62) W. Grüebler and V. König, private communication

TABLE 1

Relations between observables p_{ij} and t_{kq} in the transversity frame S' and the coordinate system defined by the Madison convention. These relations are equally valid for the polarization efficiencies A_{ij} and T_{kq} .

Cartesian tensors		Spherical tensors	
Transverse System	Madison System	Transverse System	Madison System
$\sqrt{\frac{3}{2}} p'_z$	$\sqrt{\frac{3}{2}} p_y$	t'_{10}	$2 i t_{11}$
$\frac{1}{\sqrt{2}} p'_{zz}$	$\frac{1}{\sqrt{2}} p_{yy}$	t'_{20}	$-\frac{1}{2}(t_{20} + \sqrt{6} t_{22})$
$\frac{1}{2\sqrt{3}} (p'_{xx} - p'_{yy})$	$\frac{1}{2\sqrt{3}} (p_{zz} - p_{xx})$	$\text{Re}(t'_{22})$	$-\frac{1}{4}(\sqrt{6} t_{20} - 2t_{22})$
$\frac{1}{\sqrt{3}} p'_{xy}$	$\frac{1}{\sqrt{3}} p_{xz}$	$\text{Im}(t'_{22})$	t_{21}

0000470358727

LBL-5851

Figure captions

- Fig.1) The polarization sphere with the inscribed Lakin cone. Points A and B correspond to $A_y = \pm 1$, point C to $A_{yy} = -2$ and the base of the cone to $A_{yy} = 1$. The axes of the transverse polarization quantities T_{kq} have been partly labeled with the usual cartesian tensors.
- Fig.2) Extreme points of the analyzing power in $^{12}\text{C}(p,p)^{12}\text{C}$ elastic scattering. In the notation of Plattner and Bacher¹⁾ the minimum distance of the amplitude g' from the points $\pm i$ is given as a function of the proton energy. An extreme point will be found for $d^{\min} = 0$. At least 8 and possibly 10 such points can be seen in the figure taken from the work of Meyer³²⁾.
- Fig.3) The component A_{yy} of the analyzing power of the $^3\text{He}(\vec{d},p)^4\text{He}$ reaction, calculated from the data obtained by Gruebler et al.⁴⁸⁾. Clearly an extreme point is reached near 9 MeV and $\theta_{\text{cm}} = 27^\circ$. At angles near 120° another point might be found at energies above 12 MeV.
- Fig.4) Values of the components A_{xx} and A_{xz} of the analyzing power of the $^3\text{He}(\vec{d},p)^4\text{He}$ reaction near 10 MeV. For a point $A_y = \pm 1$ the values $A_{xx} = -\frac{1}{2}$ and $A_{xz} = 0$ are needed. With increasing energy these values may be obtained near 120 degrees. Unfortunately the data available⁴⁸⁾ reach only to 11.5 MeV.
- Fig.5) The component A_{yy} of the analyzing power of the $^6\text{Li}(\vec{d},\alpha)^4\text{He}$ reaction near 6 MeV at forward angles and near 8 MeV at $\theta_{\text{cm}} = 90^\circ$ show evidence for two extreme points $A_{yy} = 1$. The data of Risler et al.⁵¹⁾ have been used to calculate the quantity A_{yy} .

Fig. 1

Fig. 2.

Fig. 3

XBL753-2495

LBL-5851

41

00004705529

Fig. 4

XBL 754-2663

XBL753-2614

Fig. 5

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720