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UNIVERSITY OF CALIFORNIA SAN DIEGO

Modeling, analysis, and optimization of MIMO systems with reconfigurable antennas — hybrid
parasitic arrays and dynamic metasurface antennas

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy

in

Electrical Engineering (Communication Theory and Systems)

by

Nitish Vikas Deshpande

Committee in charge:

Professor Robert W. Heath Jr., Chair
Professor Nuria Gonzalez Prelcic
Professor Ryan Kastner
Professor Florian Meyer

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University of California San Diego

2026

DEDICATION

जननी जन्मभूमिश्च स्वर्गादपि गरीयसी॥

Dedicated to Aai and Baba

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PUBLICATIONS

Journal and Magazine Publications

- [1] **N. V. Deshpande**, M. R. Castellanos, S. R. Khosravirad, J. Du, H. Viswanathan, and R. W. Heath, “A Wideband Generalization of the Near-Field Region for Extremely Large Phased-Arrays,” *IEEE Wireless Commun. Lett.*, vol. 12, no. 3, pp. 515–519, Mar. 2023.
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FIELDS OF STUDY

Major Field: Electrical Engineering

Studies in Wireless Communication and MIMO Systems

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ABSTRACT OF THE DISSERTATION

Modeling, analysis, and optimization of MIMO systems with reconfigurable antennas — hybrid parasitic arrays and dynamic metasurface antennas

by

Nitish Vikas Deshpande

Doctor of Philosophy in Electrical Engineering (Communication Theory and Systems)

University of California San Diego, 2026

Professor Robert W. Heath Jr., Chair

As antenna array apertures in the next-generation multiple-input-multiple-output (MIMO) wireless systems scale up, it is necessary to ensure reasonable scaling in the hardware cost and power consumption. In the past decade, a hybrid architecture was proposed which balanced the beamforming functionality across digital precoding and analog precoding with phase shifters, switches, or true time delays. This evolution from a fully digital architecture to a hybrid architecture enabled expanding the array aperture beyond the number of radio frequency chains. For next-generation cellular, to ensure scalability to even larger apertures, we propose the tri-hybrid MIMO paradigm which replaces the conventional antennas in MIMO with more

capable reconfigurable antennas. This dissertation investigates two types of reconfigurable antennas, namely hybrid parasitic arrays and dynamic metasurface antennas (DMAs).

In the first contribution, we use a circuit-theoretic model for parasitic arrays which accounts for the mutual coupling phenomenon, a key property that governs its beamforming mechanism. The unique challenge with parasitic arrays is the coupled magnitude and phase constraint on the beamforming weight. We propose a low-complexity technique with a closed-form solution for the reconfigurable parasitic reactance and the active antenna current in the hybrid parasitic array. We show that the proposed hybrid array outperforms the conventional fully-active array in terms of energy efficiency.

In the second contribution, we generalize the concept of hybrid parasitic arrays to a tri-hybrid MIMO system which introduces an analog switch to dynamically configure the assignment of active and parasitic elements. We propose a joint optimization algorithm to configure the digital precoder, analog switch, and parasitic reactances. Through numerical simulations, we show that the tri-hybrid architecture outperforms the benchmark hybrid architectures where one or more precoders are fixed.

In the third contribution, we investigate DMAs which are a leaky-waveguide based frequency-reconfigurable antenna. We exploit its unique frequency-reconfigurability property to develop frequency-selective beamforming and beam training algorithms. We show that there is an inherent coupling between the dominant beam direction and the operating frequency. We leverage this observation to perform single-shot beam training that enables probing different angular directions with a single OFDM symbol.

Chapter 1

Introduction

Multiple-input-multiple-output (MIMO) has been a key technology in wireless systems to enable higher data rates using multiple antennas at both transmitter and receiver [1]. The MIMO dimension has been increasing to support higher data rates with every generation of cellular and Wi-Fi systems. The hardware complexity and power consumption of the base station with a fully digital architecture as shown in Fig. 1.1 increases commensurately with the number of antennas. The push to mmWave frequencies in 5G motivated the hybrid MIMO architecture to address this scaling bottleneck [2]. This architecture enabled increasing the number of antennas beyond the number of radio frequency (RF) chains by introducing the analog beamforming network as shown in Fig. 1.2. It led to a new sub-field in MIMO research focusing on developing new models, analysis, and optimization algorithms for hybrid MIMO [3].

With new spectra such as the upper mid-band opening up for communication and the need for cellular operators to reuse the existing cellular deployment density, there is a push to further scale the number of antennas. This motivated the need to explore alternate paradigms to ensure that scaling without drastic increase in power consumption and hardware cost. We propose to incorporate reconfigurable antennas in MIMO to address this scaling concern for next-generation MIMO [4]. Although reconfigurable antennas have been studied in the antenna community for several decades, their integration into MIMO systems brings in new challenges in terms of physically-consistent modeling and optimization for system-level metrics.



Figure 1.1. Block diagram of a conventional multiple-input-multiple-output (MIMO) architecture, where each antenna at both receiver and transmitter is connected to a radio frequency (RF) chain and data converters. The precoding and combining operations are performed entirely in the digital domain. While scaling up the array aperture, the fully digital architecture leads to a high power consumption as the number of power-hungry components in the RF chain increase commensurately with the number of antennas.

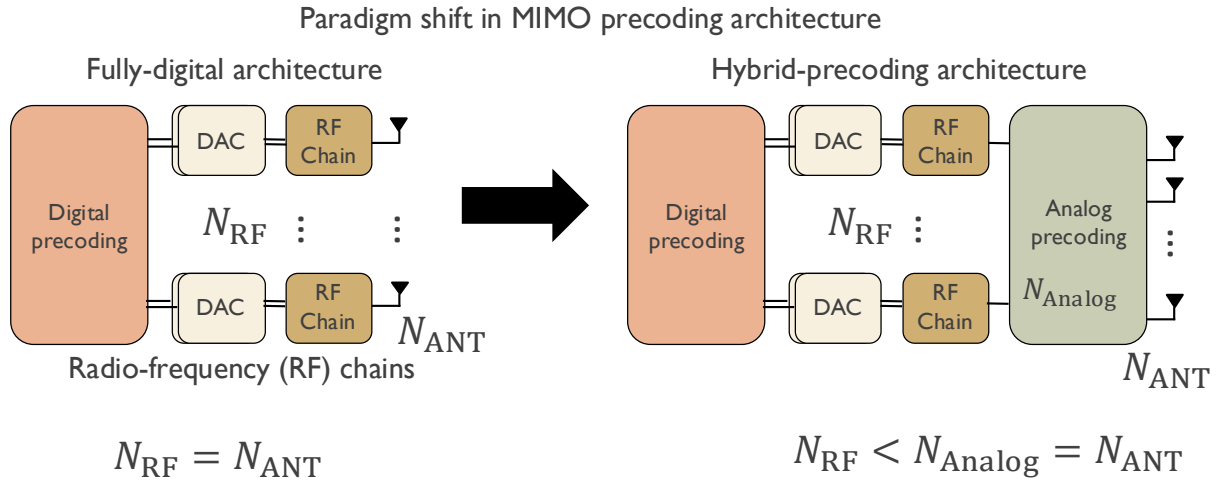


Figure 1.2. Block diagram showing the paradigm shift to a hybrid precoding architecture where the complexity of precoding is shared across the digital and analog domain. This paradigm shift enables scaling the number of antennas beyond the number of RF chains.

In Section 1.1, we summarize the conventional MIMO background. In Section 1.2, we introduce the paradigm of replacing conventional antennas with more capable reconfigurable antennas, and discuss the key challenges. We envision the tri-hybrid MIMO architecture with reconfigurable antennas that act as a third precoding layer as shown in Fig. 1.3. Specifically, in this dissertation, we investigate two types of reconfigurable antennas, parasitic hybrid arrays and dynamic metasurface antennas. In Section 1.3, we provide a summary of the dissertation contributions. In Section 1.4, we provide an outline for the chapters in this dissertation.

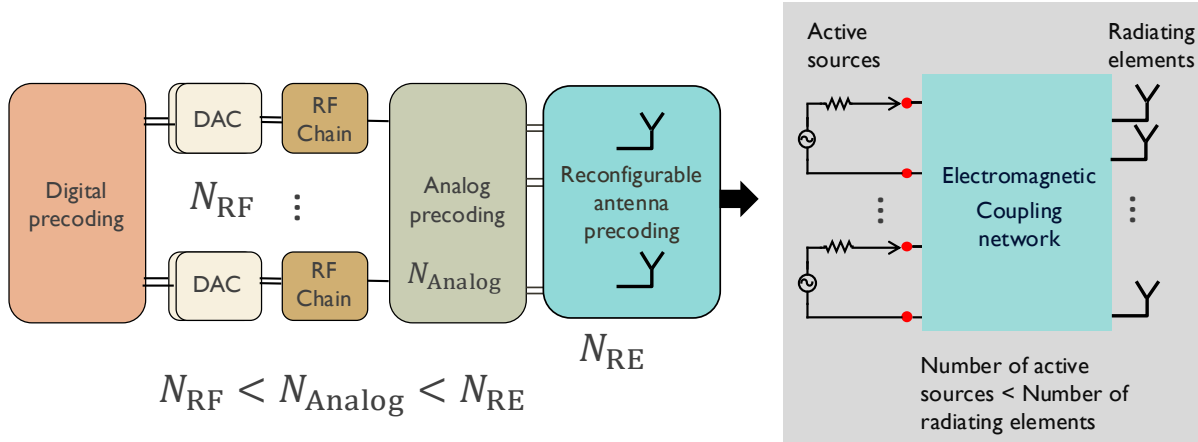


Figure 1.3. Block diagram of a tri-hybrid MIMO architecture which uses reconfigurable antennas as a third layer for transmit precoding. The shift to the reconfigurable antenna paradigm enables scaling the number of radiating elements beyond the number of active sources by using the phenomenon of electromagnetic coupling. The reconfigurable antenna precoding layer is also known as “EM” precoding in the literature.

1.1 Background on MIMO

Multiple-antenna wireless links first emerged as arrays for shaping transmitted and received waves in the propagation domain, well before multi-antenna capacity results became the usual analysis framework. Early in the twentieth century, Karl Ferdinand Braun showed how to combine radiators with controlled phase so that radiation could be steered, work recognized with the Nobel Prize in Physics and now viewed as early phased-array behavior [5]. Subsequent systems scanned beams electronically, and adaptive element weighting enabled steering toward desired arrivals while suppressing strong interferers. Fixed phased arrays and adaptive arrays are commonly grouped under the umbrella term “smart antennas”.

Once multiple antennas are available at one or both ends of a link, system designs use either diversity for reliability or spatial multiplexing for throughput. Receive diversity combines paths that see distinct fading to improve robustness. Transmit diversity provides gains from multiple transmit antennas via space-time coding. Spatial multiplexing increases data rate by sending parallel streams when scattering yields a favorable high rank channel matrix with sufficiently independent paths.

Statistical channel models and capacity analyses related the antenna dimensions to spectral efficiency in multipath. Saleh and Valenzuela introduced an indoor multipath model with clustered arrivals [6]. Foschini and Gans linked antenna count to roughly linear spectral-efficiency growth in rich scattering [7], while Telatar gave a Gaussian vector-channel capacity [8]. Paulraj's group at Stanford university proposed early spatial multiplexing with phased-array-style terminals [9], and the group at Bell Labs led by Valenzuela demonstrated layered architectures showing feasibility at laboratory scale [10].

Subsequent research focused on cellular deployments with many simultaneous users rather than an isolated point-to-point link. Multi-user MIMO focused on interference-aware precoding, scheduling, and limited-feedback protocols so users can be separated in space, time, and frequency. Base-station with large arrays further motivated massive MIMO operating regimes. Marzetta pioneered cellular massive MIMO with unlimited numbers of base station antennas and analyzed favorable propagation scaling [11]. Heath and Lozano consolidate the different flavors of MIMO from an information-theoretic capacity perspective in their textbook [1].

Millimeter-wave operation pushed for larger apertures to achieve higher beamforming gain. Rappaport's group at New York University advocated for millimeter-wave cellular propagation [12]. The hybrid MIMO architecture, pioneered by Heath's former group at UT Austin, introduced the paradigm of splitting beamforming between low-dimensional digital processing and analog beamforming networks built from phase shifters, switches, or true-time-delay lines. That split enabled commercial mmWave products and motivates ongoing work on beam training and precoding under RF hardware constraints [2, 3, 13]. The remainder of this introduction treats this conventional MIMO evolution as established background for incorporating reconfigurable antennas [4].

1.2 Reconfigurable antennas for next-generation MIMO

The preceding section reviewed conventional MIMO from phased and adaptive arrays through diversity, spatial multiplexing, multi-user operation, and hybrid mmWave architectures. In implementation, each RF chain still terminates in a physical antenna that converts guided currents to radiated fields and collects impinging waves for reception, while geometry and materials set the spatial and frequency-domain response [14]. A reconfigurable antenna adds low-power tunable elements such as semiconductor switches or varactor diodes so properties including pattern shape, resonance, or polarization can be adjusted electronically without rebuilding the mechanical layout for each mode [15]. Historical uses include missiles, aircraft, and radar [16], and the antenna literature classifies these mechanisms as pattern-, frequency-, and polarization-reconfigurable designs that shift part of the beam-shaping burden from feed networks into the radiator [15]. Section 1.2.1 explains why moving such tuning to the air interface is attractive when cellular operators scale aperture faster than RF-chain hardware [4].

1.2.1 Motivation for reconfigurable antennas in next-generation MIMO

A key driving factor in MIMO technology has been the increase in the number of antennas to support higher data rates. In a conventional fully digital architecture, each antenna is associated with one RF chain, which comprises power-hungry components such as power amplifiers, mixers, local oscillators, and data converters. For 5G, there was a trend to move to mmWave frequencies which led to the need to drastically increase the number of antennas for achieving higher beamforming gain. To enable this scaling of number of antennas without increasing the number of RF chains and power consumption, the paradigm of hybrid architecture was proposed in 5G, by introducing an analog network between the RF chains and the antennas. Several variations of this hybrid architecture were studied in literature depending on the type of analog network and the topology [13]. For example, the analog beamforming can be done through phase shifters, switches, true time delays, etc. In terms of topology, there can be fully-connected

or sub-connected architectures that lead to different tradeoffs for spectral efficiency and energy efficiency. In the past decade, there has been extensive research on hybrid MIMO to investigate several tradeoffs and develop practical algorithms for beamforming and beam training [2, 3, 13].

For next-generation cellular systems, the FR3 spectrum spanning 6 GHz to 24 GHz has been a topic of interest in the community [17]. Compared to mmWave frequencies and sub-6 GHz frequencies, FR3 is a sweet spot for both bandwidth and coverage. Cellular operators, however, are interested in reusing the current sub-6 GHz deployment density of base stations and upgrade the existing infrastructure to larger arrays for improving the throughput. This necessitates research in the direction of achieving higher spectral efficiency with larger apertures without a significant power consumption overhead and hardware cost due to RF chain components.

Introducing reconfigurable antennas in MIMO architectures is a potential solution to address the hardware and power consumption bottleneck [4]. Unlike conventional antennas which require an external active source to tune the current distribution, the reconfigurable antennas can make use of low-power electronics like varactor diodes or PIN diodes to reconfigure their intrinsic properties in addition to the external reconfiguration. A single reconfigurable antenna can have multiple tightly packed radiating elements where few elements are controlled through active feed and others are tuned passively. This naturally enables expansion of the physical aperture without scaling up in the number of RF chains. These unique features make the adoption of reconfigurable antennas suitable for MIMO systems with large apertures.

1.2.2 Challenges of integrating reconfigurable antennas in MIMO systems

Conventional MIMO communication theory assumed antennas as point sources for developing analyses and optimization algorithms. While abstracting out the antenna and array properties for developing MIMO channel models is reasonable for conventional antennas, it is inappropriate to make this assumption for reconfigurable antennas. Within a reconfigurable antenna, there are several densely packed radiating elements. The sub-wavelength spacing among

consecutive elements leads to mutual coupling that makes the point source assumption invalid. For realistic modeling of radiated power, we need to incorporate mutual coupling between different radiating elements of a reconfigurable antenna.

Each reconfigurable antenna type has different functioning mechanisms which lead to different models and methods for configuration. With conventional antennas that have fixed radiation patterns, the MIMO beamforming problem can be standardized across the different antenna types like dipoles or patches. Solutions like maximum ratio transmission (MRT), zero-forcing (ZF) precoding, or minimum mean squared error (MMSE) precoding can be applied in the digital domain that are agnostic of the antenna type. For a reconfigurable antenna, the mathematical models vary across different types depending on what intrinsic property of the antenna is reconfigured. The unique constraints for each antenna type need to be carefully incorporated when configuring MIMO systems with reconfigurable antennas. We next discuss the specific challenges associated with parasitic antennas and dynamic metasurface antennas.

1.2.3 Parasitic hybrid arrays and dynamic metasurface antennas

In this dissertation, we investigate two specific reconfigurable antennas, parasitic hybrid arrays and dynamic metasurface antennas, as shown in Fig. 1.4. Both antennas have unique functioning mechanisms which lead to different models, analysis, and optimization algorithms. Parasitic hybrid arrays consist of several active antennas that are mutually coupled to parasitic elements tuned through reconfigurable reactances [18]. Dynamic metasurface antennas (DMAs) are a type of leaky-waveguide based frequency reconfigurable antenna with dynamic resonant frequency control of the radiating slots on the waveguide [19].

From a modeling perspective, hybrid parasitic arrays and DMAs are fundamentally different. We use a multi-port circuit theory approach to develop a MIMO model incorporating parasitic elements. In this approach, the input-output relationship of the MIMO wireless link is expressed in terms of voltages, currents, and impedances. It enables realistic modeling of the mutual coupling between the parasitic and active antenna elements. For modeling DMAs,

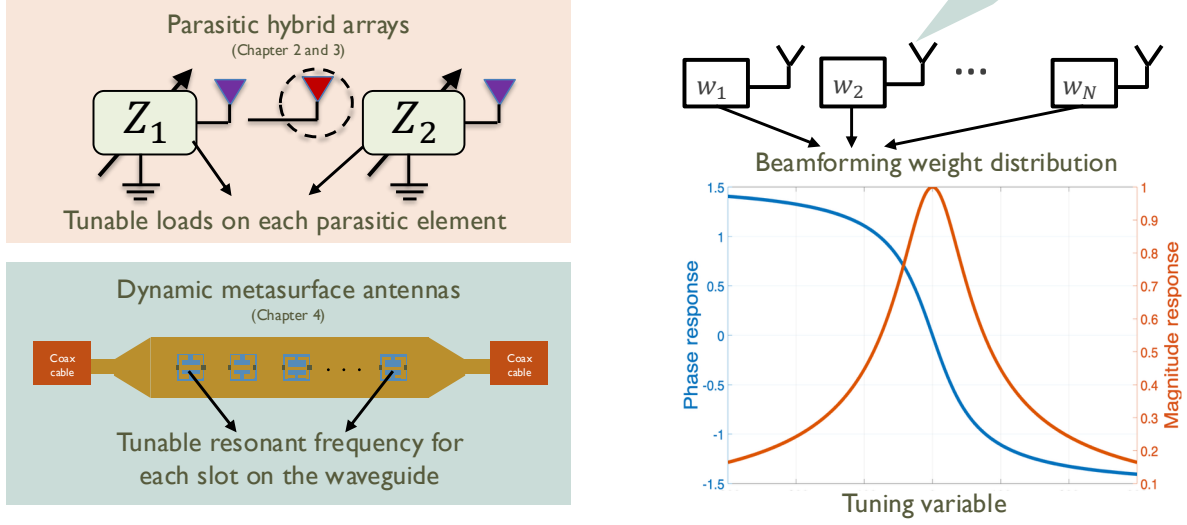


Figure 1.4. In this dissertation, we investigate two types of reconfigurable antennas, parasitic hybrid arrays and dynamic metasurface antennas. While both antennas have different functioning mechanisms, we observed a similarity in terms of the beamforming challenge associated with both antennas. From an array processing perspective, the key challenge in the beamforming is that there is an intrinsic coupling between the phase and magnitude of the beamforming weight for each radiating element.

an electromagnetic (EM) theory approach is used where each radiating slot on the waveguide is modeled as a magnetic dipole. Using this EM theory model, we express the input-output relationship for a DMA based system similar to other architectures like phased-arrays or true time delays for a consistent comparison of the beamforming gain.

Although both antennas have different functioning mechanisms, we observed some common traits in the two antenna types from an array processing and a MIMO perspective. Specifically, we analyzed the beamforming weight distribution for these antennas and observed an inherent coupling between the phase and the magnitude as shown in Fig. 1.4. Unlike a phased-array with unit modulus beamforming weight distribution, the coupled magnitude and phase constraint for both DMAs and parasitic elements creates complications in configuring these antennas. For a DMA, this beamforming weight varies by tuning the resonant frequency of each slot. For a parasitic element, it varies by tuning the reconfigurable reactance. From a MIMO perspective, a key challenge in configuring both DMAs and hybrid parasitic arrays is that

the total radiated power depends on the reconfiguration state of the antenna. This dependence complicates the problem of maximizing achievable rate subject to the transmit power constraints.

1.3 Thesis statement and a summary of contributions

The thesis statement of this dissertation is

Integrating reconfigurable antennas in MIMO systems provides a better tradeoff between spectral efficiency and energy efficiency compared to the MIMO architectures with conventional antennas.

In this dissertation, our focus is on modeling, analysis, and optimization of MIMO systems based on two reconfigurable antennas, namely hybrid parasitic arrays and dynamic metasurface antennas. Chapter 2 and Chapter 3 are based on parasitic arrays and Chapter 4 is on dynamic metasurface antennas. The main contributions of this dissertation are summarized as follows.

- In the second chapter, we formulate the model for the hybrid parasitic array with multiple active elements mutually coupled with multiple parasitic elements using a multi-port circuit theory approach. The model is validated by comparing the radiation patterns from theory and EM simulations using the Feko software. We address the coupled magnitude-phase constraint in beamforming by proposing a relaxed objective function and develop a low-complexity closed-form solution for the reconfigurable parasitic reactance. Through numerical simulations, we show that the proposed hybrid parasitic array outperforms the fully digital benchmark in terms of energy efficiency.
- In the third chapter, we extend the formulation of a hybrid parasitic array to a tri-hybrid architecture with parasitic elements and an analog switch for dynamic antenna assignment. We propose an optimization algorithm to jointly configure the digital precoders, analog switch, and the reconfigurable parasitic reactances. Through numerical simulations, we show that the proposed tri-hybrid architecture has a better tradeoff between spectral

efficiency and energy efficiency compared to the benchmarks where one or more precoders are fixed.

- In the fourth chapter, we formulate the frequency-selective DMA beamforming gain optimization problem for a line-of-sight (LOS) channel. We show that there is an inherent coupling between the main lobe direction and the operating frequency of the DMA. We use this observation to develop a closed-form expression for configuring the DMA's resonant frequency and operating frequency in terms of the receiver's angular direction. We also propose a single-shot beam training procedure for a DMA array that enables the system to probe multiple angles simultaneously with a single OFDM symbol.

1.4 Organization

This dissertation is organized as follows. Chapter 2 studies beamforming with hybrid reconfigurable parasitic antenna arrays. It develops a multi-port circuit-theoretic model for hybrid arrays with active and parasitic elements, derives closed-form beamforming solutions, and evaluates the resulting spectral-efficiency and energy-efficiency tradeoffs. Chapter 3 extends this framework to the tri-hybrid MIMO architecture with parasitic antennas and dynamic switching. It introduces a tri-hybrid system model with digital precoding, analog switching, and parasitic reconfiguration, and studies the corresponding optimization and performance tradeoffs. Chapter 4 focuses on dynamic metasurface antennas. It develops a frequency-selective beamforming framework, reveals the coupling between beam direction and operating frequency, and proposes a single-shot beam training approach. Finally, Chapter 5 concludes the dissertation and discusses several directions for future research. Appendices corresponding to Chapters 2, 3, and 4 provide supplementary derivations and supporting technical details.

1.5 Common notation

Unless stated otherwise, a bold lowercase letter such as $\mathbf{z}$ or $\mathbf{h}$ denotes a column vector and a bold uppercase letter such as $\mathbf{Z}$ or $\mathbf{H}$ denotes a matrix. The notation $|\cdot|$ indicates absolute

value, $\angle(z)$ denotes the argument of a complex number z , $(\cdot)^T$ denotes transpose, $(\cdot)^*$ denotes conjugate transpose, and $(\cdot)^c$ denotes conjugate. The notation $\|\cdot\|$ denotes the Euclidean norm and $\|\cdot\|_F$ denotes the Frobenius norm. Moreover, $\mathcal{R}\{z\}$ and $\mathcal{I}\{z\}$ denote the real and imaginary parts of a complex number z , respectively, $\odot$ denotes the element-wise product, $\text{blkdiag}(\cdot)$ denotes the block-diagonal matrix operator, $\text{sgn}(\cdot)$ denotes the signum function, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix, $\mathbf{1}$ denotes the all-ones vector, $\lfloor x \rfloor$ and $\lceil x \rceil$ denote the floor and ceiling functions, respectively, $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$, and $\mathcal{N}_{\mathbb{C}}(0, 1)$ denotes a circularly symmetric complex Gaussian random variable with zero mean and unit variance. In Chapter 4, we also use the universal constants speed of light $c = 3 \times 10^8$ [m/sec], intrinsic impedance of free space $\eta_0 \approx 377 \Omega$, and Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K.

1.6 Common abbreviations

3GPP: Third Generation Partnership Project
 AI-RAN: artificial intelligence radio access network
 AoD: angle of departure
 AO-SBA: alternating optimization stochastic beamforming algorithm
 CSI: channel state information
 CSI-RS: channel state information reference signal
 DMA: dynamic metasurface antenna
 EE: energy efficiency
 EM: electromagnetic
 ESPAR: electronically steerable parasitic array radiator
 FD-ULA: fully digital uniform linear array
 FD-UPA: fully digital uniform planar array
 FH-CDMA: frequency-hopping code-division multiple access
 FR1: frequency range 1

FR2: frequency range 2

FR3: frequency range 3

HRP-UPA: hybrid reconfigurable parasitic uniform planar array

HPS-UPA: hybrid phase-shifter sub-connected uniform planar array

ISAC: integrated sensing and communication

LOS: line of sight

MEMS: micro-electro-mechanical systems

MIMO: multiple input multiple output

MISO: multiple input single output

MMSE: minimum mean squared error

MRT: maximum ratio transmission

NLOS: non-line of sight

NR: New Radio

OFDM: orthogonal frequency-division multiplexing

PASS: pinching antenna system

PIN: positive-intrinsic-negative

REF: reconfigurability efficiency factor

RF: radio frequency

RHS: reconfigurable holographic surface

RIS: reflecting intelligent surface

SE: spectral efficiency

SNR: signal-to-noise ratio

SRS: sounding reference signal

TTD: true time delay

ULA: uniform linear array

UPA: uniform planar array

ZF: zero forcing

Chapter 2

Beamforming with hybrid reconfigurable parasitic antenna arrays

In this chapter¹, we develop a beamforming framework for hybrid reconfigurable parasitic antenna arrays. We formulate a multi-port circuit-theoretic model, derive closed-form solutions for the active currents and parasitic reactances, and compare the resulting spectral-efficiency and energy-efficiency tradeoffs with conventional benchmark architectures.

2.1 Introduction

2.1.1 Motivation and challenges

Reconfigurable parasitic antennas have the potential to enable low-cost, energy-efficient beamforming for larger arrays [15, 18, 21–24]. Unlike conventional arrays which adjust the excitation signal of each element, parasitic arrays use a combination of active and passive components that reduces the overall power consumption [25]. Parasitic elements are tuned using reconfigurable components such as varactor or PIN diodes. These elements interact with active currents through mutual coupling. The current on a parasitic element depends on the active antenna current and the reconfigurable load, complicating the beamforming gain optimization by creating non-linearity in the objective function. We focus on developing a low-complexity approach for beamforming optimization, analyze the spectral efficiency and energy efficiency of

¹This chapter is based on the published journal article [20].

a parasitic array system, and compare with conventional architectures.

2.1.2 Prior work

There are several types of reconfigurable antennas, categorized by what property of the antenna is configured [15]. Each type of reconfiguration has its own benefits and drawbacks. Reconfiguration at the antenna is a key part of the recently proposed tri-hybrid MIMO architecture [4]. In this chapter, we focus on configuring the beam pattern through parasitic elements because of their low-cost and low-powered reconfiguration mechanism. Other examples of reconfigurable antennas include dynamic metasurface antennas based on resonant frequency configuration [26], movable antennas based on dynamically changing element spacing [27], and polarization reconfigurable antennas [28].

Parasitic arrays, also popularly known as electronically steerable parasitic/ passive array radiator (ESPAR), can be used in different ways. In [29–32], ESPARs are used for direct symbol modulation and spatial multiplexing to obtain the benefits of a large array with fewer active antennas. In [33, 34], they are used for reducing mutual coupling in MIMO arrays. In [35], parasitic arrays in full-duplex transceivers were shown to reduce the self-interference. In this chapter, we focus on using parasitics to improve the beamforming gain. We envision use of parasitic elements to realize large arrays through a hybrid architecture comprising of multiple active antennas controlled through RF chains and several parasitic antennas controlled through reconfigurable reactances. This hybrid reconfigurable parasitic array has reduced power consumption and hardware cost compared to conventional architectures. Although ESPARs have versatile applications, we use the terminology “hybrid parasitic array” to be consistent with the recent evolution of beamforming architectures like the tri-hybrid MIMO [4].

Parasitic antennas have been widely studied by the antenna community. While extensive research has focused on designing and prototyping reconfigurable parasitic antennas [36–40], [41], the configuration methods predominantly rely on look-up tables that map target beamforming angles to PIN diode or varactor diode states. These prototype results are limited to a specific

design with fixed number of parasitic elements. The optimization metrics in [36–38] are limited to radiation pattern and realized gain valid only for a line-of-sight (LOS) channel. Although [39] and [40] provided system level throughput simulations for realistic channel models, the methods for configuring parasitic elements are not scalable to large arrays because they require a brute-force search across all configurations. These limitations highlight the need for scalable configuration methods that generalize beyond fixed designs and enable efficient operation in diverse propagation environments.

Prior work on configuring parasitic antennas in the wireless literature relies on ideal abstractions of the radiation patterns without providing any insight on how to practically realize those patterns by configuring active antenna currents and parasitic element loads [42–44]. A key missing component in these studies is an understanding of how parasitic configurations influence the beamforming weights of active antennas—an effect driven by mutual coupling, which alters the input impedance of the active antenna as the parasitic load states change. This coupling not only affects the current distribution but also the total radiated power, necessitating dynamic adjustment of active antenna currents to maintain power constraints. Another limitation of prior work is the lack of analytical insight into optimal parasitic configurations. Most prior work on configuring parasitic antennas is limited to iterative and heuristic methods such as steepest gradient descent in [45], stochastic optimization in [25, 46], genetic algorithm in [47], quasi-Newton method in [48], greedy algorithm in [49], iterative mode selection in [42], and multi-armed bandit theory-based radiation mode selection in [43, 44]. To summarize, [25, 42–49] fail to provide insights into how to optimally configure parasitic elements under realistic constraints on power and mutual coupling, limiting their scalability and applicability in practical systems.

Multi-port circuit theory is a powerful tool for developing tractable and physically consistent models that incorporate the mutual coupling effect, thus providing a realistic definition of total radiated power. In prior work, this tool has been used for analyzing dense MIMO arrays [50, 51]. The mutual coupling is modeled in a mathematically tractable way with impedance matrices [52]. There is limited prior work, however, on using this circuit-theory

approach for analyzing parasitic arrays. In [53], a signal model was proposed for a parasitic array system with a single RF chain. This signal model was used to optimize a multi-user system in [54]. Their approach focused on minimizing the normalized mean squared error between the desired precoder and the actual parasitic element precoder. In [55,56], [57], [58], the parasitic reactances were optimized using the correlation coefficient metric that aligns the actual radiation pattern to the desired one from maximum ratio transmission. These approaches in [55,56] [57], [58] do not guarantee optimality from a spectral efficiency perspective because the radiated power constraint is not explicitly included. In [59], closed-form design guidelines were proposed for single-fed parasitic arrays to support arbitrary precoding. These guidelines were generalized to multi-active and multi-parasitic arrays in [60]. Although [59,60] provide closed-form expressions for the parasitic impedance, they are limited to design inequalities that the parasitic array should satisfy for the array to radiate. The circuit model has not been used to derive closed-form solutions that optimize a communication-theoretic metric under realistic power constraints. We focus on deriving a closed-form solution not only because it has low-complexity but also to extract design insights from the solution and analyze scaling trends.

2.1.3 Contributions

We use the multi-port circuit theory approach from [51] to formulate the multiple-input-single-output (MISO) model for a reconfigurable parasitic array with multiple active and parasitic elements operating in a multipath wireless channel. The derived theoretical beamforming gain expression is validated through electromagnetic simulations which enables its use in formulating an optimization problem. We solve the LOS channel beamforming optimization problem for the single active and multiple parasitic antenna case. Since the beamforming optimization problem is mathematically intractable in its original form, we propose a relaxed objective function that leads to a tractable closed-form expression for the reconfigurable parasitic reactance. The accuracy of this solution is validated by comparing it with the numerical approach from the MATLAB optimization toolbox. The proposed optimization approach is also valid for different channel

models and antenna types including heterogeneity in radiation patterns of individual antenna elements. In this chapter, we show the generalization for the beamforming optimization problem with a hybrid multi-active multi-parasitic array operating in a multipath channel. We propose a near-optimal closed-form solution for the active antenna current and the parasitic reactances for beamforming. We compare the spectral and energy efficiency of the hybrid parasitic array beamforming with that of the fully digital array without any parasitic elements. Our results show that the parasitic elements improve the spectral efficiency without any additional power overhead. This demonstrates that beamforming with hybrid parasitic arrays is an energy-efficient approach.

We highlight the novelty of this chapter by comparing it to prior studies. The communication-theoretic approaches in [42–44] employ realistic system models, such as multipath channels, but rely on generic abstractions of reconfigurable antenna radiation patterns and numerical optimization methods. The circuit-theoretic approaches in [59, 60] utilize multi-port models for parasitic arrays, but are limited to providing closed-form design guidelines for parasitic impedances in the form of inequalities. They do not derive optimal solutions and are also restricted to LOS channels. In this chapter, we formulate a beamforming optimization problem for a parasitic reconfigurable hybrid array operating in a multipath propagation environment. We derive closed-form solutions for both the parasitic reactances and the active antenna currents, providing a comprehensive and tractable framework for system analysis.

Organization: In Section 2.2, we use the circuit-theory approach to formulate the system model for a parasitic array with single active antenna and multiple parasitic elements. We define the SNR and beamforming gain for a LOS channel. In Section 2.3, we formulate the beamforming optimization problem for the LOS channel and propose a closed-form expression for the parasitic reactance using an approximate objective function. The results from Section 2.3 are generalized to the case of multi-active, multi-parasitic hybrid array in a multipath channel in Section 2.4. In Section 2.5, we present numerical results on spectral efficiency and energy efficiency for the parasitic hybrid array, the fully active array, and hybrid phase-shifter sub-connected array using the dipole antenna planar array implemented in Feko software. In Section 2.6, we summarize

the key takeaways and discuss directions for future research. The MATLAB simulation code to generate the results is made publicly available to facilitate reproducibility [61].

2.2 System model

We use the multi-port circuit theory approach from [50–52] to model the reconfigurable parasitic array. In Section 2.2.1, we review the multi-port circuit modeling approach for an array with all active antennas similar to [51]. In Section 2.2.2, we formulate the model for a parasitic array with only one active antenna. In Section 2.2.3, we formulate the beamforming pattern in terms of the reconfigurable parasitic reactance.

2.2.1 Review of multi-port circuit theory model of a wireless communication system

We first review the model for a conventional antenna array with all active antennas. We assume a narrowband MISO system with N antennas at transmitter and single antenna at receiver as shown in Fig. 2.1(a). Each antenna at the transmitter is connected to an RF chain. All signals and multi-port parameters are a function of frequency and evaluated at the center frequency f_c (or center wavelength λ_c). The MISO system is modeled as an $N + 1$ port network, as shown in Fig. 2.2(a). In the circuit-theoretic formulation, there are several frameworks to capture the EM interaction between different antennas like the impedance parameter, admittance parameter, or scattering parameter framework which depends on the choice of the input source like the voltage source, current source, or power wave source. In this chapter we choose to use the impedance matrix framework to be consistent with other prior work on ESPAR [25] [30] [53] [56], [57], [58]. The voltage at each antenna port is related to the currents of all antenna ports through the impedance matrix. For a transmit array with N elements, let the impedance matrix of the transmit array be $\mathbf{Z}_{\text{TX}} \in \mathbb{C}^{N \times N}$. Let the voltage vector evaluated at f_c be $\mathbf{v}_{\text{TX}} = [v_0, v_1, \dots, v_{N-1}]^T \in \mathbb{C}^{N \times 1}$. Here, each voltage source is an abstraction of the signal at the output of the transmit RF chain. Each RF chain is comprised

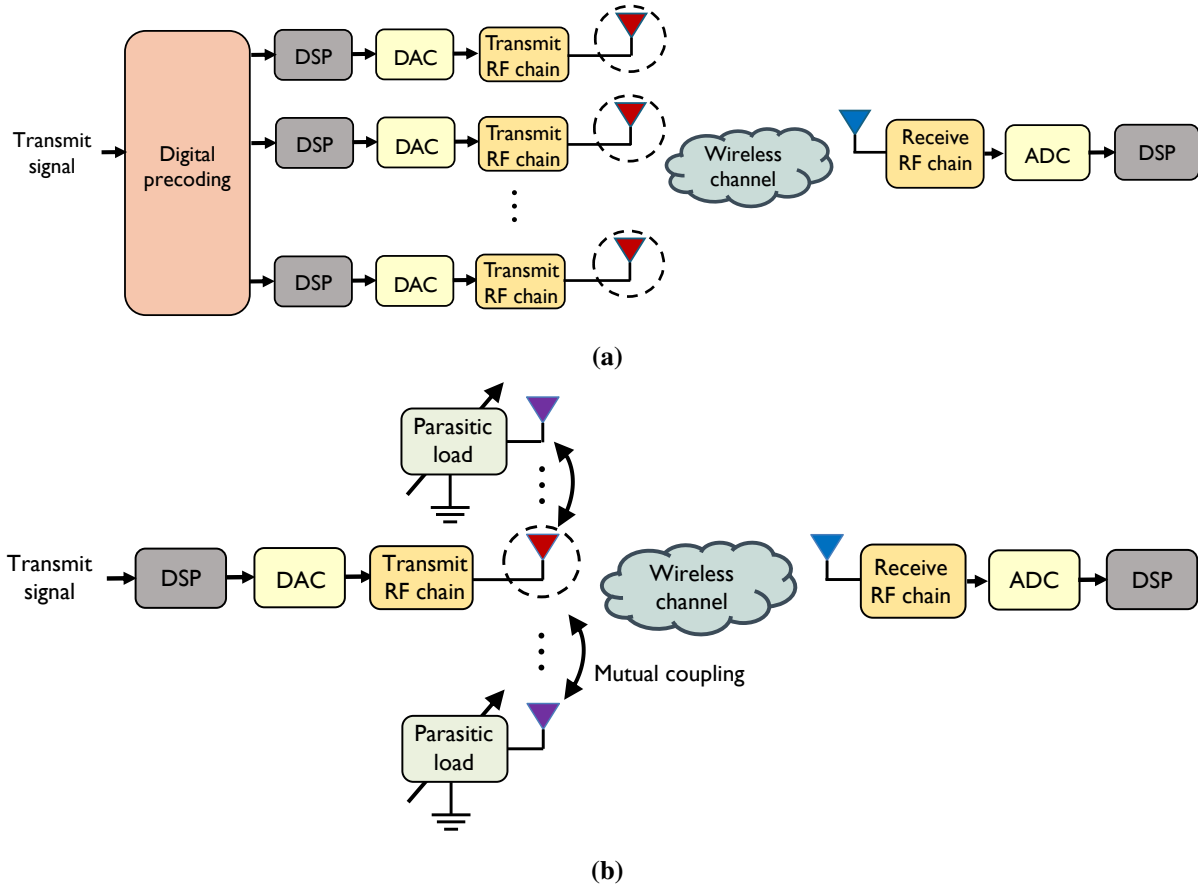


Figure 2.1. (a) In a conventional array, each antenna is connected to an RF chain (b) In a parasitic array, only one antenna is connected to an RF chain and the remaining antennas are tuned by passive loads.

of components like a mixer, a local oscillator, filters, and amplifiers that we do not show in Fig. 2.1(a) for brevity. Let the transmit information symbol in frequency domain be s and we assume it has unit variance. The excitation signal at each antenna is controlled using the digital beamformer. We use the antenna current vector to represent the beamforming vector and denote it as $\mathbf{i}_{\text{TX}} = [i_0, i_1, \dots, i_{N-1}]^T \in \mathbb{C}^{N \times 1}$. Hence, the transmit signal is $\mathbf{i}_{\text{TX}} s$.

The circuit model relates the receive voltage to the transmit current. The impedance parameter of the single receive antenna is $Z_{\text{RX}} \in \mathbb{C}$. The voltage on the receiver antenna port is $v_{\text{RX}} \in \mathbb{C}$ and the current is $i_{\text{RX}} \in \mathbb{C}$. The wireless propagation channel impedance vector $\mathbf{z}_{\text{RT}} = \mathbf{z}_{\text{TR}} \in \mathbb{C}^{N \times 1}$ accounts for the small-scale and large-scale fading in the wireless channel.

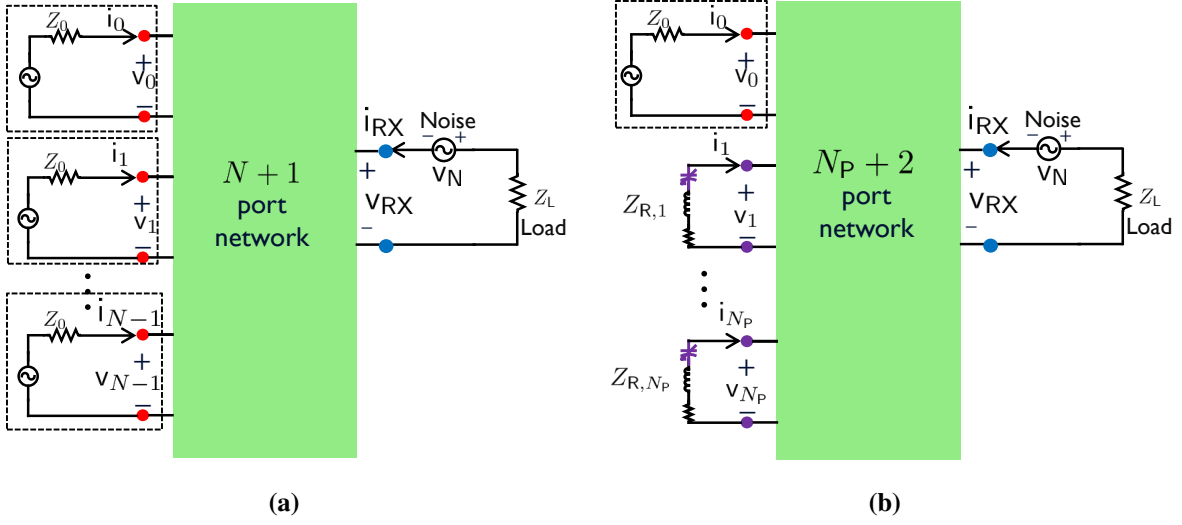


Figure 2.2. (a) Circuit model of a conventional array, where each antenna is modeled as a voltage source (b) Circuit model for a parasitic array, where the parasitic elements are tuned using variable loads.

From the multi-port model, we have [51]

$$\mathbf{v}_{TX} = \mathbf{Z}_{TX}\mathbf{i}_{TX}s + \mathbf{z}_{TR}\mathbf{i}_{RX}, \quad (2.1a)$$

$$\mathbf{v}_{RX} = \mathbf{z}_{RT}^T\mathbf{i}_{TX}s + Z_{RX}\mathbf{i}_{RX}. \quad (2.1b)$$

To simplify the system model by decoupling $\mathbf{v}_{TX}$ from $\mathbf{i}_{RX}$, we apply the unilateral approximation which results in $\mathbf{v}_{TX} \approx \mathbf{Z}_{TX}\mathbf{i}_{TX}s$. The unilateral approximation is reasonable because the transmit array is in the far-field from the receive antenna such that the signal attenuation between them is large [51]. This means that the transmitter is unaffected by the electromagnetic fields at the receiver [51].

The circuit theory approach also provides a physically consistent definition for the total transmit power radiated from the antenna array. The total transmit power is [51]

$$P_{TX} = \mathbb{E}[\mathcal{R}\{(\mathbf{i}_{TX}s)^*\mathbf{v}_{TX}\}], \quad (2.2)$$

where the expectation is over the random information symbol. Using the unilateral approximation

and the assumption of the unit variance transmit information symbol, we obtain $P_{\text{TX}} = \mathbf{i}_{\text{TX}}^* \mathcal{R}\{\mathbf{Z}_{\text{TX}}\} \mathbf{i}_{\text{TX}}$. The diagonal elements of the $\mathbf{Z}_{\text{TX}}$ matrix are called self-impedances while the non-diagonal elements are called mutual impedances. Much prior work on MIMO assumes that $\mathbf{Z}_{\text{TX}}$ is a diagonal matrix, i.e., mutual impedance is negligible. This leads to the power being proportional to norm squared of the current vector. In this chapter, we use the physically consistent definition of radiated power from (2.2) that accounts for mutual coupling. We do not incorporate a matching network in the circuit model. In practice, a matching network is used between the voltage source and the antennas. We have investigated the impact of matching networks in our prior work [62] [63], where we show that the supplied power constraint is useful to analyze the impact of matching network. In this chapter, as we are dealing with the beamforming problem, we use the total radiated power constraint. The study of impact of matching networks on a hybrid reconfigurable parasitic array is deferred to future work.

We establish the input-output signal model by solving for the received voltage. Let the load impedance on the receive antenna be Z_L . At the receiver, the voltage source v_N models the noise due to background radiation. Applying Kirchhoff's voltage law at the receive port, we have $v_{\text{RX}} = -v_N - i_{\text{RX}} Z_L$. Substituting v_{RX} in (2.1b), we have $i_{\text{RX}} = -\frac{\mathbf{z}_{\text{RT}}^T \mathbf{i}_{\text{TX}} s + v_N}{Z_{\text{RX}} + Z_L}$. Let the voltage on the load impedance v_L be expressed in terms of the input $\mathbf{i}_{\text{TX}} s$ as $v_L = -i_{\text{RX}} Z_L = \frac{Z_L}{Z_{\text{RX}} + Z_L} (\mathbf{z}_{\text{RT}}^T \mathbf{i}_{\text{TX}} s + v_N)$. Assuming that the noise voltage source v_N has a variance of σ^2 and $\mathbb{E}[|s|^2] = 1$, the SNR at the receiver is

$$\text{SNR} = \frac{|\mathbf{i}_{\text{TX}}^T \mathbf{z}_{\text{RT}}|^2}{\sigma^2}. \quad (2.3)$$

The SNR definition in (2.3) is general and applies to both conventional arrays and parasitic arrays.

2.2.2 Parasitic antenna array model with one active element

The array implementation dictates the constraints on the current vector. Beamforming in an antenna array is achieved by configuring the current vector $\mathbf{i}_{\text{TX}}$. In conventional active

arrays, the magnitude and phase of each element in $\mathbf{i}_{\text{TX}}$ can be adjusted independently. As a result, the active antenna current vector is constrained only by maximum power. In contrast, parasitic arrays create beam patterns using a combination of active phase control and mutual coupling with parasitic elements that have variable loads. These antennas, known as parasitic elements, are reconfigured by altering their variable loads. Changing the termination of these elements dynamically modifies their radiative properties, enabling beamforming but imposing additional constraints on $\mathbf{i}_{\text{TX}}$.

In this section, we describe the model for a parasitic antenna array with one active antenna connected to an RF chain and $N_{\text{P}} = N - 1$ parasitic elements that are tuned using variable loads as shown in Fig. 2.1(b). The parasitic antenna array is characterized as an $N_{\text{P}} + 2$ port network as shown in Fig. 2.2(b). This representation is independent of the physical array geometry. The beam pattern, however, depends on the array structure described in Section 2.2.3. Let the voltage at the port of the active antenna in the parasitic array be denoted as v_0 and the current as i_0 . The vector of voltages at the parasitic element ports is $\mathbf{v}_{\text{P}} = [v_1, \dots, v_{N_{\text{P}}}]^T$. We define $\mathbf{v}_{\text{TX}} = [v_0, \mathbf{v}_{\text{P}}^T]^T$. Similarly, the current vector is $\mathbf{i}_{\text{P}} = [i_1, \dots, i_{N_{\text{P}}}]^T$. Thus, $\mathbf{i}_{\text{TX}} = [i_0, \mathbf{i}_{\text{P}}^T]^T$. The first element of these vectors $\mathbf{i}_{\text{TX}}$ and $\mathbf{v}_{\text{TX}}$ corresponds to the active antenna.

We relate the currents and voltages using impedance matrices. Let the self-impedance of the active antenna be z_{00} . The vector of mutual impedances between the active antenna and N_{P} parasitic elements is $\mathbf{z}_{\text{m}}$. The mutual impedance matrix of the N_{P} parasitic elements is $\mathbf{Z}_{\text{P}}$ and is fixed based on the array design. We use the block matrix notation to express $\mathbf{Z}_{\text{TX}} = \begin{bmatrix} z_{00} & \mathbf{z}_{\text{m}}^T \\ \mathbf{z}_{\text{m}} & \mathbf{Z}_{\text{P}} \end{bmatrix}$. Using the unilateral approximation on (2.1a), we have

$$\begin{bmatrix} v_0 \\ \mathbf{v}_{\text{P}} \end{bmatrix} = \mathbf{Z}_{\text{TX}} \begin{bmatrix} i_0 s \\ \mathbf{i}_{\text{P}} s \end{bmatrix}. \quad (2.4)$$

The magnitude and phase of i_0 can be controlled independently through the RF chain. Of note, however, $\mathbf{i}_{\text{P}}$ cannot be tuned independently as the current on each parasitic element is induced

from i_0 through coupling.

We now establish the dependence of $\mathbf{i}_P$ on i_0 . Let the variable load at the port of the ℓ th parasitic element be $Z_{R,\ell}$. We define the parasitic reconfigurable load matrix as $\mathbf{Z}_R = \text{diag}\{[Z_{R,1}, \dots, Z_{R,N_P}]^T\}$. The voltage vector at the parasitic ports is $\mathbf{v}_P = -\mathbf{Z}_R \mathbf{i}_P s$. From (2.4) and expression of $\mathbf{v}_P$, the current vector on the parasitic ports in terms of the active antenna current, parasitic element mutual impedance matrix, and the reconfigurable load matrix is

$$\mathbf{i}_P = -(\mathbf{Z}_P + \mathbf{Z}_R)^{-1} \mathbf{z}_m i_0. \quad (2.5)$$

For a parasitic reconfigurable array, only current i_0 is an independent variable. All elements in the vector $\mathbf{i}_P$ are dependent on the reconfigurable load matrix $\mathbf{Z}_R$ which can be tuned independently, but constrains the set of possible values of $\mathbf{i}_P$.

2.2.3 Parasitic array beamforming gain formulation for a line-of-sight channel

The mathematical formulation in Section 2.2.1 and Section 2.2.2 is applicable for any general array with arbitrary array geometry. In this section, we make some specific assumptions on the array geometry similar to the prior work for ease of comparison. We assume a linear antenna array oriented along the x axis with inter-element spacing of d as shown in Fig. 2.3. The active antenna element is placed at the origin and the parasitic elements are arranged on both sides of the active element. The receive antenna is in the xy plane at an angle θ from the y axis. We assume a LOS channel between the transmit array and receive antenna. Let γ be the coefficient that captures large-scale fading and $\mathbf{a}_P(\theta)$ be the parasitic antenna array steering vector

$$\mathbf{a}_P(\theta) = \begin{bmatrix} e^{-j2\pi \left\lfloor \frac{N_P}{2} \right\rfloor \frac{d \sin(\theta)}{\lambda_c}}, \dots, e^{-j2\pi \frac{d \sin(\theta)}{\lambda_c}}, e^{j2\pi \frac{d \sin(\theta)}{\lambda_c}}, \\ \dots, e^{j2\pi \left\lceil \frac{N_P}{2} \right\rceil \frac{d \sin(\theta)}{\lambda_c}} \end{bmatrix}^T. \quad (2.6)$$

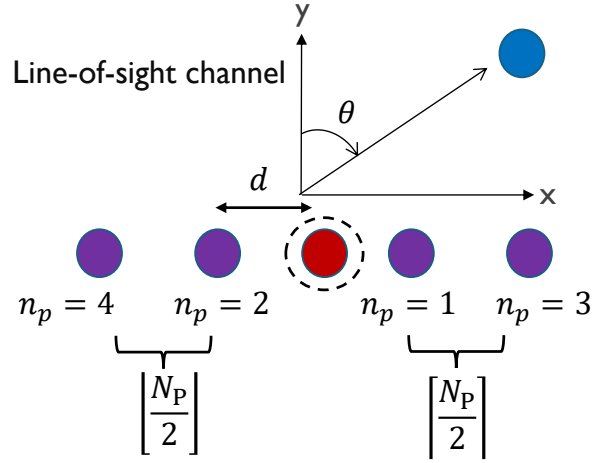


Figure 2.3. Linear array consisting of one active antenna and N_P parasitic elements with a LOS channel to the receive antenna. The array geometry is symmetric when N_P is an even number and asymmetric when N_P is an odd number. This distinction also reflects in the beamforming pattern as shown in Fig. 2.6 for different values of N_P .

The phase offsets in $\mathbf{a}_P(\theta)$ are calculated relative to the active antenna which is located at the center of the linear array as shown in Fig. 2.3. The wireless propagation channel is $\mathbf{z}_{RT} = \gamma[1, \mathbf{a}_P^T(\theta)]^T$ [51]. We express the SNR of a parasitic array for a LOS channel by using (2.3) and (2.5) to obtain

$$\text{SNR} = \frac{\gamma^2}{\sigma^2} |i_0|^2 |1 - \mathbf{a}_P^T(\theta)(\mathbf{Z}_P + \mathbf{Z}_R)^{-1} \mathbf{z}_m|^2. \quad (2.7)$$

For a parasitic array, we define the far-field beam pattern as

$$\mathcal{G}(\theta, \mathbf{Z}_R) = |1 - \mathbf{a}_P^T(\theta)(\mathbf{Z}_P + \mathbf{Z}_R)^{-1} \mathbf{z}_m|^2. \quad (2.8)$$

The beam pattern is a complicated non-linear function of the reconfigurable load matrix $\mathbf{Z}_R$, unlike a conventional array where the beam pattern $|\mathbf{i}_{TX}^T \mathbf{z}_{RT}|^2$ is based on a quadratic expression of the active antenna current vector.

2.3 Line-of-sight beamforming for a parasitic array with single active element

We formulate the parasitic array beamforming problem and propose a simplification to the objective for ease of optimization in Section 2.3.1. Using the proposed simplification, we analyze the parasitic beamforming weights and highlight the optimization challenge due to a coupled magnitude and phase relation in Section 2.3.2. We propose a closed-form solution for the parasitic reactance in Section 2.3.3 and validate the analytical solution by comparing it with MATLAB based numerical approach in Section 2.3.4.

2.3.1 Parasitic array beamforming problem formulation

We develop an algorithm to adjust the weights to maximize the beamforming gain in the direction of θ_1 . This is optimal for a LOS channel. Reconfiguring with varactor or PIN diodes is generally abstracted as terminating the parasitic element with reconfigurable reactive components [38]. We assume that the real part of each parasitic load is fixed and identical, i.e., $\mathcal{R}\{\mathbf{Z}_R\} = \mathcal{R}\{Z_R\}\mathbf{I}_{N_p}$, and that the imaginary part $\mathcal{I}\{\mathbf{Z}_R\}$ is reconfigurable. These reactive components can be either inductors or capacitors [25]. Hence, the elements of $\mathcal{I}\{\mathbf{Z}_R\}$ can be positive or negative. In practice, the elements of $\mathcal{I}\{\mathbf{Z}_R\}$ are confined to a discrete set due to hardware limitations. Specifically, for varactor diodes, the reactive load values depend on the DC biasing voltage. These constraints result in a finite and discrete set of achievable reactance values. For theoretical analysis, we simplify this by assuming $\mathcal{I}\{\mathbf{Z}_R\}$ can take continuous values. This approximation closely mimics what is possible with high-resolution tuning hardware while offering analytical tractability. The transmitter maximizes (2.8) at θ_1 by configuring the parasitic reactances as

$$\mathbf{P1:} \mathcal{I}\{\mathbf{Z}_R^*\} = \underset{\mathcal{I}\{\mathbf{Z}_R\} \in \mathbb{R}}{\operatorname{argmax}} \mathcal{G}(\theta_1, \mathbf{Z}_R). \quad (2.9)$$

Prior approaches to solve optimizations similar to **P1** involve using iterative approaches [25, 60]. We propose a closed-form solution that yields design insights by analyzing the dependence of the parasitic reactance on parameters like the mutual impedance vector, array steering vector, and self-impedance of the parasitic element.

The beam pattern has a complicated dependence on the reconfigurable load $\mathbf{Z}_R$. The matrix $\mathbf{Z}_R$ is inside an inverse term in (2.8) which makes it difficult to solve **P1** in its current form. In a conventional array beamsteering problem, the pattern depends on a linear combination of beamforming weights. To obtain a linear relation in (2.8), we simplify the matrix inverse via a diagonal approximation. From antenna array literature, we know that for most practical designs with moderate to high element-spacing, the self-impedance is higher in magnitude compared to the mutual impedance [14]. With this observation, we separate all self-impedances of the parasitic array by defining $\mathbf{D}_P = \text{diag}\{\mathbf{Z}_P\}$ and the hollow matrix $\mathbf{E}_P = \mathbf{Z}_P - \mathbf{D}_P$ which includes all mutual impedances. The matrix $\mathbf{Z}_R$ is diagonal based on the parasitic array design defined in Section 2.2.2. We combine the two diagonal matrices $\mathbf{Z}_R$ and $\mathbf{D}_P$ by defining the parasitic beamforming weight matrix $\mathbf{W}$ which is also diagonal as

$$\mathbf{W}^{-1} = \mathbf{D}_P + \mathbf{Z}_R. \quad (2.10)$$

With these definitions, we write the inverse term in (2.8) as $(\mathbf{Z}_P + \mathbf{Z}_R)^{-1} = (\mathbf{E}_P + \mathbf{D}_P + \mathbf{Z}_R)^{-1} = (\mathbf{E}_P + \mathbf{W}^{-1})^{-1}$. For small N_P , as $\|\mathbf{E}_P\|_F$ is small relative to $\|\mathbf{W}^{-1}\|_F$, we further expand the term $(\mathbf{E}_P + \mathbf{W}^{-1})^{-1}$ using perturbation theory as

$$(\mathbf{E}_P + \mathbf{W}^{-1})^{-1} = \mathbf{W} - \mathbf{W}\mathbf{E}_P\mathbf{W} + \mathcal{O}(\|\mathbf{E}_P\|_F^2). \quad (2.11)$$

The perturbation theory is a useful tool to develop a first-order approximation of the objective function. It is valid when the second order terms are negligible and the error between the actual objective and approximate objective is bounded. Using the first order term from (2.11), we

approximate the objective $\mathcal{G}(\theta_1, \mathbf{Z}_R)$ by defining $\widehat{\mathcal{G}}(\theta_1, \mathbf{W})$ in terms of $\mathbf{W}$ as

$$\widehat{\mathcal{G}}(\theta_1, \mathbf{W}) = |1 - \mathbf{a}_P^T(\theta_1) \mathbf{W} \mathbf{z}_m|^2. \quad (2.12)$$

The approximate objective $\widehat{\mathcal{G}}(\theta_1, \mathbf{W})$ is easier to optimize because $\mathbf{W}$ is a diagonal matrix. This motivates us to use $\widehat{\mathcal{G}}(\theta_1, \mathbf{W})$ as the new objective function instead of $\mathcal{G}(\theta_1, \mathbf{Z}_R)$. In Appendix A.1, we show that $\mathcal{G}(\theta_1, \mathbf{Z}_R) \rightarrow \widehat{\mathcal{G}}(\theta_1, \mathbf{W})$ for small $\|\mathbf{E}_P\|_F$.

2.3.2 Coupled magnitude and phase constraint

In a conventional active array beamforming problem, the magnitude and phase of the beamforming weight can be tuned independently. For parasitic arrays, we analyze the beamforming weight matrix $\mathbf{W}$ and show that the phase and magnitude are coupled that makes the optimization problem challenging. The i th diagonal element of the matrix $\mathbf{W}$ is $[\mathbf{W}]_{ii} = \frac{1}{[\mathbf{D}_P]_{ii} + [\mathbf{Z}_R]_{ii}}$. The phase of $[\mathbf{W}]_{ii}$ is

$$\begin{aligned} \angle([\mathbf{W}]_{ii}) &= \text{atan2}(\mathcal{I}\{[\mathbf{W}]_{ii}\}, \mathcal{R}\{[\mathbf{W}]_{ii}\}), \\ &\stackrel{(a)}{=} \tan^{-1} \left(\frac{-(\mathcal{I}\{[\mathbf{D}_P]_{ii}\} + \mathcal{I}\{[\mathbf{Z}_R]_{ii}\})}{(\mathcal{R}\{[\mathbf{D}_P]_{ii}\} + \mathcal{R}\{[\mathbf{Z}_R]_{ii}\})} \right), \end{aligned} \quad (2.13)$$

where (a) follows from the fact that the real part is always positive as it is a resistance term. We define

$$\zeta_i = \frac{1}{(\mathcal{R}\{[\mathbf{D}_P]_{ii}\} + \mathcal{R}\{[\mathbf{Z}_R]_{ii}\})}, \quad (2.14)$$

and $\varphi_i = \angle([\mathbf{W}]_{ii})$. As shown in Appendix A.2, the beamforming weight $[\mathbf{W}]_{ii}$ is expressed in terms of φ_i and ζ_i as

$$[\mathbf{W}]_{ii} = \zeta_i \cos(\varphi_i) e^{j\varphi_i}. \quad (2.15)$$

From (2.15), the magnitude of $[\mathbf{W}]_{ii}$ varies as a sinusoidal function of the phase angle, which complicates the beamforming problem.

We visualize the beamforming weight $[\mathbf{W}]_{ii}$ in the complex domain by plotting its locus. We assume that the parasitic elements are identical. Hence, the self-impedances of all parasitic elements are the same, i.e., $[\mathbf{Z}_P]_{ii} = [\mathbf{D}_P]_{ii} = Z_P$. We define $\zeta = (\mathcal{R}\{Z_P\} + \mathcal{R}\{Z_R\})^{-1}$ and set $\zeta_i = \zeta \forall i$. The general case of unique self-impedances for all parasitic impedances is discussed later in Appendix A.4. In Fig. 2.4, we plot the magnitude and phase of the normalized beamforming weight $\frac{[\mathbf{W}]_{ii}}{\zeta}$ as a function of the reconfigurable parasitic reactance $\mathcal{I}\{[\mathbf{Z}_R]_{ii}\}$ and for different values of the self reactance of parasitic elements $\mathcal{I}\{Z_P\} = \{40, 0, -40\}$. We observe that the phase angle φ_i has a limited phase range $[-\pi/2, \pi/2]$ compared to the complete phase range $[-\pi, \pi]$ for a phased-array. Another observation from Fig. 2.4 is that for values of reconfigurable parasitic reactances around 0, i.e., for $\mathcal{I}\{[\mathbf{Z}_R]_{ii}\} \in [-100, 100]$, the magnitude and phase response curves have more variation compared to the reactance values outside this range. This has implications also for the beamforming gain variation with the reconfigurable reactance as we show later in Section 2.3.4. We plot the locus of the normalized beamforming weight $[\mathbf{W}]_{ii}/\zeta$ in complex domain as a function of the phase angle φ_i in Fig. 2.5. The locus of $[\mathbf{W}]_{ii}/\zeta$ is a circle with center at $(0.5, 0)$ and a radius of 0.5. This constraint is known as the Lorentzian constraint. It also appears in the literature on dynamic metasurface antennas (DMA) [64–66].

We reformulate the optimization problem based on the Lorentzian constraint observation to leverage the solution approach developed in [64–66]. From (2.13) and the definition of ζ , we can express the optimization variable $\mathcal{I}\{[\mathbf{Z}_R]_{ii}\}$ in terms of the phase angle $\angle([\mathbf{W}]_{ii})$ as

$$\mathcal{I}\{[\mathbf{Z}_R]_{ii}\} = -\mathcal{I}\{Z_P\} - \frac{\tan(\angle([\mathbf{W}]_{ii}))}{\zeta}. \quad (2.16)$$

In (2.16), we see that there is a one-to-one mapping between $\mathcal{I}\{[\mathbf{Z}_R]_{ii}\}$ and the approximate beamforming weight phase angle. We formulate the beamforming problem using the approximate

objective $\widehat{\mathcal{G}}(\theta, \mathbf{W})$ as

$$\mathbf{P2:} \quad \max_{\mathbf{W}} \widehat{\mathcal{G}}(\theta_1, \mathbf{W}), \quad (2.17a)$$

$$\text{s.t. } [\mathbf{W}]_{ii}/\zeta = \cos(\varphi_i)e^{j\varphi_i}, \quad (2.17b)$$

$$\varphi_i \in [-\pi/2, \pi/2]. \quad (2.17c)$$

The constraints of problem **P2** are similar to the beamforming constraints of a DMA [64–66]. This inspires us to use an optimization solution approach similar to [66].

In [66], the DMA beamforming weight is transformed from a unit-modulus beamforming weight to simplify the optimization problem. We express the normalized parasitic beamforming weight in terms of a unit-modulus beamforming weight $e^{j\phi_i}$ by applying a shift-of-origin transformation as

$$\frac{[\mathbf{W}]_{ii}}{\zeta} = \frac{1}{2}(1 + e^{j\phi_i}). \quad (2.18)$$

Equating the magnitude and phase of the right hand side of (2.17b) and (2.18), we obtain the relation between ϕ_i and φ_i as

$$\phi_i = 2\varphi_i = 2\angle([\mathbf{W}]_{ii}). \quad (2.19)$$

The geometric interpretation of (2.18) and (2.19) is shown in Fig. 2.5.

As ϕ_i has range $[-\pi, \pi]$, it is easier to solve problem **P2** in terms of ϕ_i instead of φ_i . Let $\Phi = \text{diag}\{[e^{j\phi_1}, \dots, e^{j\phi_{N_p}}]^T\}$. We express $\mathbf{W}$ in terms of Φ as

$$\mathbf{W} = \frac{\zeta}{2}(\mathbf{I}_{N_p} + \Phi). \quad (2.20)$$

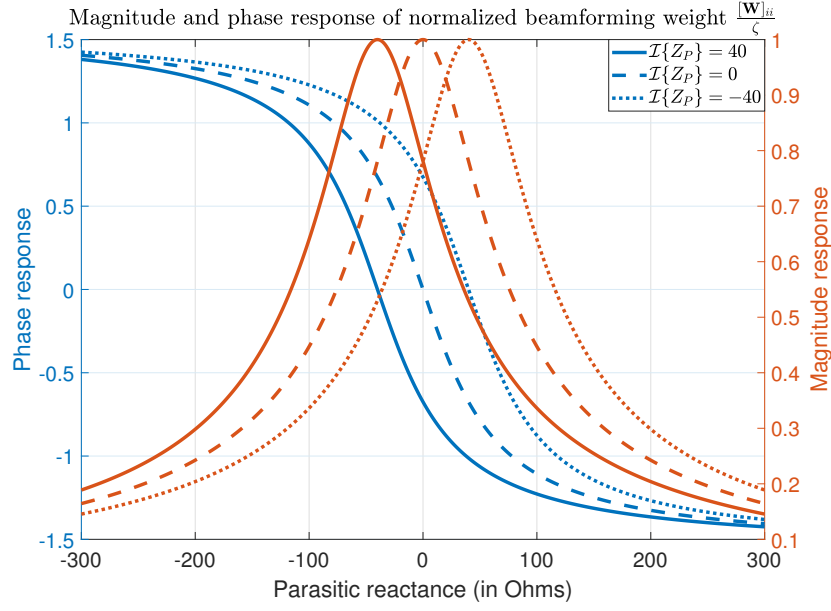


Figure 2.4. The normalized beamforming weight magnitude and phase are reconfigurable based on the parasitic reactance. The responses also depend on the self reactance of the parasitic elements which are not reconfigurable but depend only on the antenna design.

By substituting $\mathbf{W}$ from (2.20) in (2.12), we obtain

$$\hat{\mathcal{G}}(\theta_1, \Phi) = \left| 1 - \frac{\zeta}{2} \mathbf{a}_P^T(\theta_1) (\mathbf{I}_{N_P} + \Phi) \mathbf{z}_m \right|^2. \quad (2.21)$$

The problem **P2** is then equivalent to

$$\mathbf{P3:} \quad \max_{\Phi} \hat{\mathcal{G}}(\theta_1, \Phi), \quad (2.22a)$$

$$\text{s.t. } \angle([\Phi]_{ii}) \in [-\pi, \pi]. \quad (2.22b)$$

We solve **P3** in terms of Φ and then obtain the closed-form expression of parasitic reactance using (2.16) and (2.19).

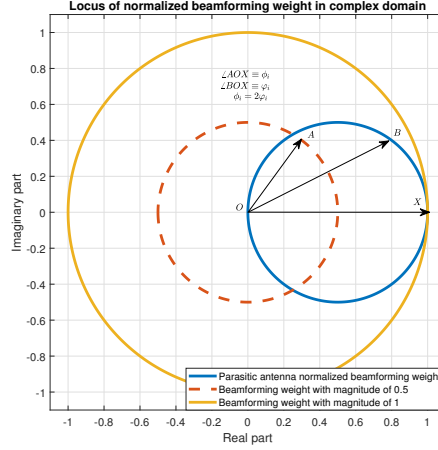


Figure 2.5. The locus of the parasitic beamforming weight in the complex plane is a circle centered at $(0.5, 0)$ with radius 0.5. We use a shift-of-origin transformation that reformulates the optimization in terms of the unit magnitude beamforming weight to avoid dealing with the coupled magnitude and phase constraint.

2.3.3 Closed-form solution for the reconfigurable parasitic load reactance

We follow a mathematical approach similar to [66] to derive the closed-form solution for Φ . In the following Theorem, we provide the closed-form solution to **P3**.

Theorem 1 *The closed-form solution to problem **P3** is*

$$\angle([\Phi]_{ii}) = \angle\left(1 - \frac{\zeta \mathbf{a}_P^T(\theta_1) \mathbf{z}_m}{2}\right) - \angle([\mathbf{a}_P(\theta_1)]_i [\mathbf{z}_m]_i) + (2m+1)\pi, \quad (2.23)$$

where m is any integer.

Proof: See Appendix A.3 for the proof. □

From the closed-form expression of Φ in (2.23), the term $\angle(1 - \zeta \mathbf{a}_P^T(\theta_1) \mathbf{z}_m / 2)$ is common across all parasitic elements. In standard beamforming problems, the optimal solution is unaffected by common phase shifts [14]. For beamforming architectures like parasitic antennas or DMAs, which have coupled magnitude and phase constraints as in (2.17b), the reformulation introduces an offset term in the objective function that is independent of the beamforming weights, as shown in (2.21). This parasitic constraint alters the problem, causing the optimal phase angle

solution to depend on common phase rotations.

We derive a closed-form expression that explicitly shows the dependence of the i th element's load reactance on the array steering vector, mutual impedance vector, self-impedance of the parasitic element, and real part of the parasitic load.

Theorem 2 *The closed-form solution for the optimal reactance obtained from the solution to **P3** is*

$$\mathcal{I}\{[\mathbf{Z}_R]_{ii}\} = -\mathcal{I}\{Z_P\} - \frac{\cot\left(\frac{\angle([\mathbf{a}_P(\theta_1)]_i[\mathbf{z}_m]_i) - \angle(1 - \frac{\zeta}{2}\mathbf{a}_P^T(\theta_1)\mathbf{z}_m)}{2}\right)}{\zeta}. \quad (2.24)$$

Proof: Using (2.16), (2.19), and (2.23), and $\tan\left(\frac{2m+1}{2}\pi - \theta\right) = \cot(\theta) \forall m \in \mathbb{Z}$, we obtain the desired result. $\square$

Using the closed-form expression from (2.24), we provide some design insights on the circuit of the reconfigurable load. We observe that the first term in the expression of $\mathcal{I}\{[\mathbf{Z}_R]_{ii}\}$ is negative of the self reactance of the parasitic element. Also, this first term is the same for all parasitic elements. This means the proposed solution can be implemented with a reconfigurable load that has a common fixed reactance to cancel out the parasitic antenna self reactance connected in series with a tunable reactance given by the second term in (2.24) which depends on the steering vector. In this section, as we assumed a LOS channel, the optimal reactance depends on the array factor. The optimization framework is generalizable to a multipath channel model by replacing the steering vector in (2.24) with a multipath channel vector as defined in (2.30).

The closed-form parasitic reactance expression is optimal for $N_P = 1$ because the approximation error $\tilde{\mathcal{G}}(\theta, Z_R) = 0$. For $N_P > 1$, the closed-form expression in (2.24) is sub-optimal for problem **P1** because the optimization is based on the approximate objective $\hat{\mathcal{G}}(\theta, \mathbf{W})$. In the numerical results section, we show that the proposed closed-form solution is nearly-optimal for $N_P = 2$ by comparing it with the solution obtained from the MATLAB Global Search toolbox. As the value of N_P increases beyond 2, the approximation is worse. For practical

designs, however, it suffices to use $N_P = 2$ as shown in Section 2.5.

2.3.4 Numerical results for LOS beamforming optimization

In this section, we evaluate the performance of the proposed parasitic beamforming approach for a dipole array for different values of N_P . The simulation parameters are as follows: The center frequency $f_c = 7$ GHz, the radius of the dipole antenna is $\frac{\lambda_c}{500}$, and length is $\frac{\lambda_c}{2}$. The dipole array impedance matrix is obtained similar to our prior work [62] which used expressions of mutual and self impedance from [14].

In Fig. 2.6, we plot the maximum beamforming pattern $\max_{\mathcal{I}\{\mathbf{z}_R\}} \mathcal{G}(\theta_1, \mathbf{z}_R)$ as a function of the angle θ_1 for $N_P \in \{1, 2, 4, 6\}$. The inter-element spacing is $0.4\lambda_c$. We compare the proposed closed-form approach in (2.24) to that obtained by numerically solving problem **P1** using the MATLAB Global Search function. For $N_P = 1$, both solutions coincide as shown in Fig. 2.6. For $N_P = 2$, $N_P = 4$, and $N_P = 6$, the performance of the proposed approach closely follows the numerical approach. The main benefit of the proposed approach is that it is easier to compute compared to the solution obtained from the iterative Global Search. An interesting feature of the parasitic array architecture is that an increase in the beamforming pattern maximum is obtained by only increasing N_P and not adding any extra active antenna. This gain also translates to spectral efficiency gain which we demonstrate in Section 2.5.

The parasitic array exhibits some unique characteristics because of its unconventional beamforming approach and array configuration. The array geometry, i.e., location of the parasitic element with respect to the active antenna and the number of parasitic elements impacts the beamforming pattern characteristics. For example, the symmetricity of the maximum beamforming pattern plot depends on whether N_P is even or odd. From Fig. 2.6, we observe that for $N_P = 4$, the beamforming pattern is higher for angles close to 90° compared to angles close to 0° . This is because of the superdirectivity effect observed at endfire angle for a dipole array [62]. We leverage this observation in the design of a planar array with multiple active and parasitic elements in Section 2.4.

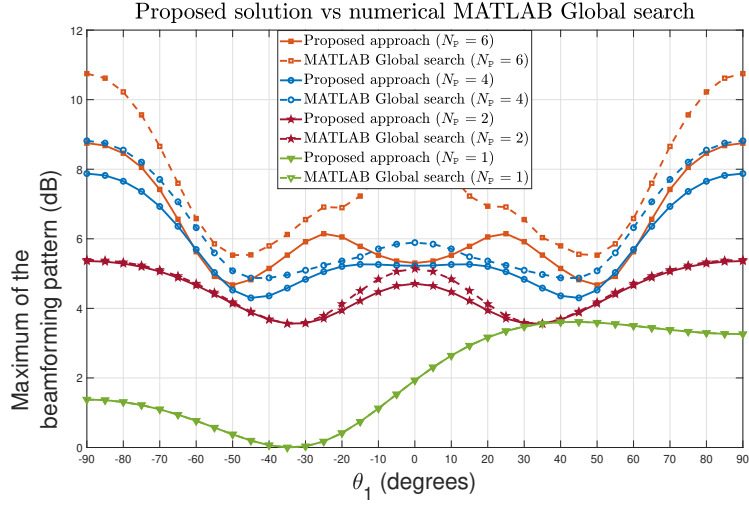


Figure 2.6. The maximum value of the beamforming pattern at each angle is shown for different values of N_P . The proposed closed-form approach is exactly the same as the MATLAB Global Search for $N_P = 1$ and closely matches the MATLAB Global Search for $N_P = \{2, 4, 6\}$.

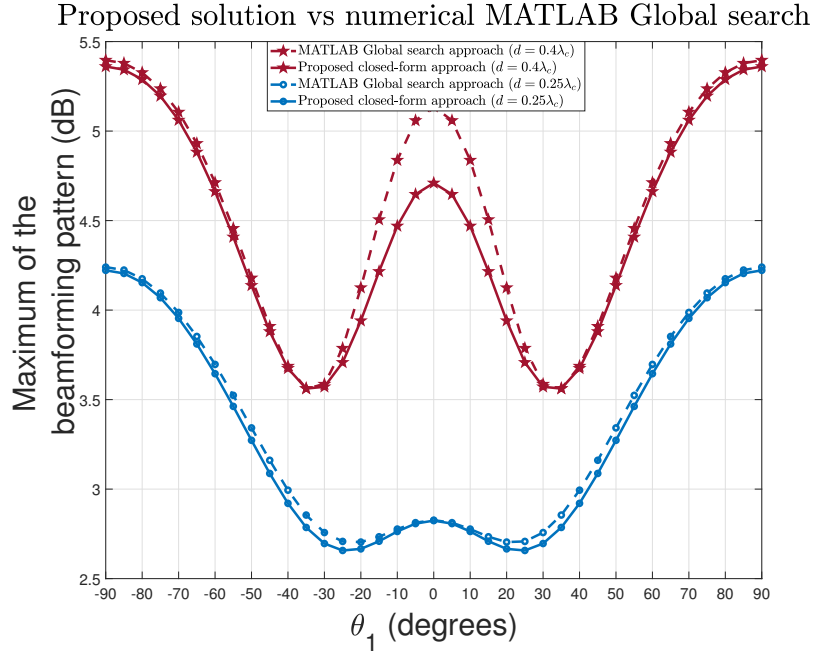
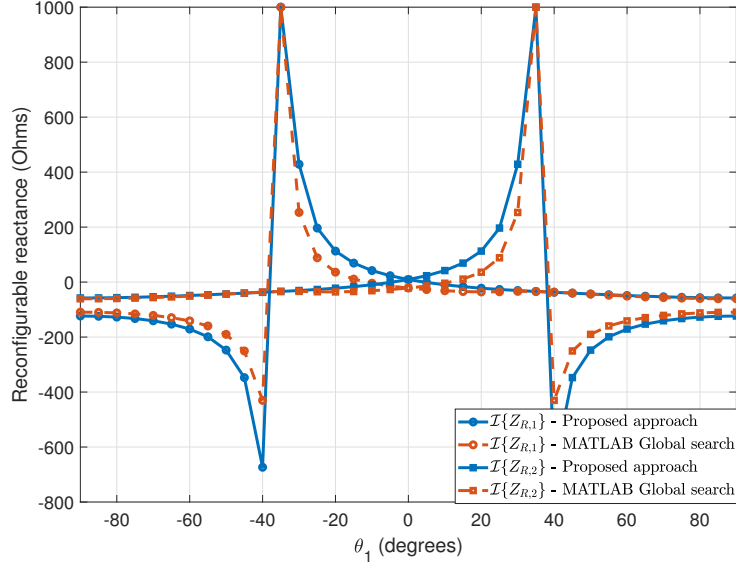


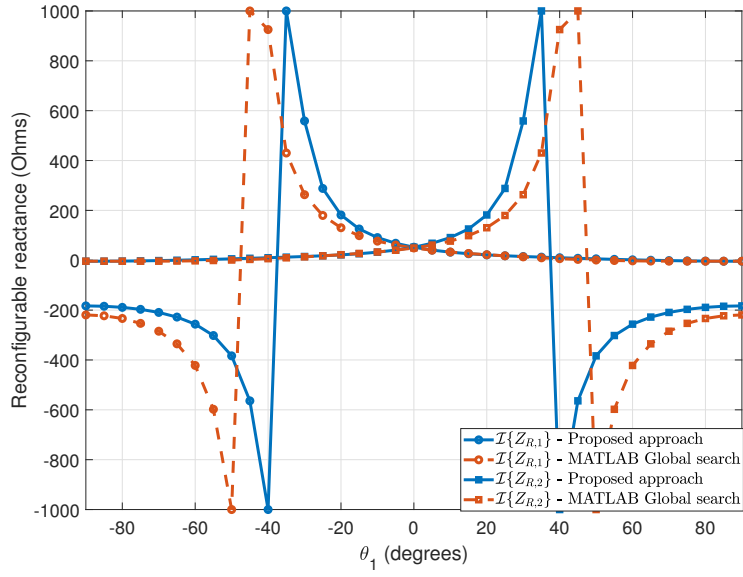
Figure 2.7. The maximum value of the beamforming pattern at each angle is shown for different values of inter-element spacing. The proposed closed-form approach closely matches the MATLAB Global Search for $N_P = 2$ for different values of inter-element spacing.

In Fig. 2.7, we set $N_p = 2$ and plot the maximum beamforming pattern as a function of the angle θ_1 for inter-element spacing $d \in \{0.25, 0.4\}\lambda_c$. Although the approximation in (2.12) is expected to worsen at smaller inter-element spacing, we observe that, similar to Fig. 2.6, the proposed closed-form approach in (2.24) performs almost the same as the MATLAB Global Search function. We observe that the relative gain of the endfire angle over broadside becomes more prominent at spacing $0.25\lambda_c$ compared to $0.4\lambda_c$ because of higher mutual coupling. We also compare the optimal reactances ($\mathcal{I}\{Z_{R,1}\}, \mathcal{I}\{Z_{R,2}\}$) required to obtain this maximum value for each θ_1 in Fig. 2.8. In Fig. 2.8(a), we plot the reactances value for $d = 0.4\lambda_c$. We observe that the reactances from the proposed closed-form approach in (2.24) closely match that from the MATLAB Global Search function. Another observation is that the curves for $\mathcal{I}\{Z_{R,1}\}$ and $\mathcal{I}\{Z_{R,2}\}$ are mirror images of each other around $\theta_1 = 0^\circ$. This is because of the array symmetry for $N_p = 2$.

Comparing Fig. 2.8(a) and the corresponding maximum beamforming pattern value in Fig. 2.7, we observe that $\mathcal{G}(\theta_1, \mathbf{Z}_R)$ is more sensitive to the reactance values when the reactance values are close to 0 compared to higher reactance values. This result can be explained through Fig. 2.4 where we see that when reactance value is close to 0, there is more variation in the phase response of the normalized beamforming weight. In Fig. 2.8(b), we observe that there is a significant deviation in the values of the reconfigurable reactances computed from the MATLAB Global Search and our proposed approach for $\theta_1 \in [-60^\circ, -20^\circ]$ and $\theta_1 \in [20^\circ, 60^\circ]$. Despite this significant deviation, there is not much difference in the two solutions for the corresponding angles in the maximum beamforming pattern value in Fig. 2.7. This result is also explained through Fig. 2.4 where we see that for reactance values outside the range $\mathcal{I}\{[\mathbf{Z}_R]_{ii}\} \in [-100, 100]$, there is no significant change in the magnitude and phase response curves of the normalized beamforming weight. Through these observations, we suggest that while designing a finite and discrete set of achievable reconfigurable reactance values, it is beneficial to design for higher number of discrete reactance values in the range $\mathcal{I}\{[\mathbf{Z}_R]_{ii}\} \in [-100, 100]$ and fewer values outside this range. The practical design of quantized reactance value set is for future research.



(a) Inter-element spacing is $d = 0.4\lambda_c$.



(b) Inter-element spacing is $d = 0.25\lambda_c$.

Figure 2.8. The reconfigurable reactances required to obtain the maximum value of the beamforming pattern from Fig. 2.7 are shown for each θ_1 . The reactance curves from the MATLAB Global Search function closely resemble our proposed closed-form solution, thus validating the accuracy of our approach.

2.4 Beamforming with hybrid multi-active and multi-parasitic array in a multipath channel

A hybrid architecture combining multiple active antennas with parasitic elements offers a practical solution to adding parasitics. It overcomes the problem that as inter-element spacing grows, the mutual coupling effect diminishes, leading to reduced performance gains. This architecture leverages the benefits of active antennas for improved control and parasitic elements for cost efficiency, making them well-suited for large-scale deployments. In this section, we generalize the circuit model derived previously to the case of multi-active and multi-parasitic array. Then, we formulate the SNR maximization problem under radiated power constraint. Finally, we derive a low-complexity closed-form solution for jointly optimizing active antenna currents and parasitic reactances in Section 2.4.3.

2.4.1 Multi-port circuit model and channel model generalization

We extend the multi-port circuit model from Fig. 2.2, to now accommodate multiple active antennas. Let the array consist of N_A active antennas, each paired with N_P parasitic elements, resulting in a total of $N_A N_P$ parasitic elements. Let the $N_A \times 1$ active antenna current vector be $\mathbf{i}_A = [i_{A,1}, \dots, i_{A,N_A}]^T$ and $N_A N_P \times 1$ parasitic element current vector be $\mathbf{i}_P = [i_{1,1}, \dots, i_{N_P,1}, \dots, i_{1,N_A}, \dots, i_{N_P,N_A}]^T$. The current beamforming vector across all active and parasitic elements is denoted as $\mathbf{i}_{TX} = [\mathbf{i}_A^T, \mathbf{i}_P^T]^T$.

The mutual impedance matrix of dimension $N_P N_A \times N_A$ between the parasitic elements and active antennas is $\mathbf{Z}_m$. We denote the mutual coupling vector between the j th active antenna and the N_P parasitic elements belonging to the row of the i th active antenna as $\mathbf{z}_{m,i,j} \in \mathbb{C}^{N_P \times 1}$.

We express $\mathbf{Z}_m$ in terms of the block-matrix notation as

$$\mathbf{Z}_m = \begin{bmatrix} \mathbf{z}_{m,1,1} & \cdots & \mathbf{z}_{m,1,N_A} \\ \vdots & \ddots & \vdots \\ \mathbf{z}_{m,N_A,1} & \cdots & \mathbf{z}_{m,N_A,N_A} \end{bmatrix}. \quad (2.25)$$

The matrix $\mathbf{Z}_A$ of dimension $N_A \times N_A$ is the mutual impedance between all active antennas and matrix $\mathbf{Z}_P$ of dimension $N_P N_A \times N_P N_A$ is the mutual impedance between all parasitic antennas. We combine these matrices into a single mutual impedance matrix for the hybrid array and denote as $\mathbf{Z}_{TX}$. Using block matrix notation, we have $\mathbf{Z}_{TX} = \begin{bmatrix} \mathbf{Z}_A & \mathbf{Z}_m^T \\ \mathbf{Z}_m & \mathbf{Z}_P \end{bmatrix}$. Let us denote the variable load at the port of the i th parasitic element corresponding to the j th active antenna as $Z_{R,i,j}$. We define $\mathbf{Z}_{R,j,j} = \text{diag}\{[Z_{R,1,j}, \dots, Z_{R,N_P,j}]\}$. The parasitic reconfigurable load matrix is generalized as

$$\mathbf{Z}_R = \text{diag}\left\{ \underbrace{[Z_{R,1,1}, \dots, Z_{R,N_P,1}]}_{\text{diag}\{\mathbf{Z}_{R,1,1}\}}, \dots, \underbrace{[Z_{R,1,N_A}, \dots, Z_{R,N_P,N_A}]}_{\text{diag}\{\mathbf{Z}_{R,N_A,N_A}\}} \right\}^T. \quad (2.26)$$

Following the steps similar to (2.4), we express the parasitic beamforming current vector as

$$\mathbf{i}_P = -(\mathbf{Z}_P + \mathbf{Z}_R)^{-1} \mathbf{Z}_m \mathbf{i}_A. \quad (2.27)$$

From (2.27), we see that the parasitic array beamforming vector depends on the reconfigurable load matrix $\mathbf{Z}_R$ and the active antenna current beamforming vector $\mathbf{i}_A$, both of which can be independently configured.

We now generalize to the geometric multipath channel with a planar array from Fig. 2.9. In this chapter, we only consider steering in the xy plane. To define the channel model with planar array, we use the following definition of array steering vectors along x and y directions.

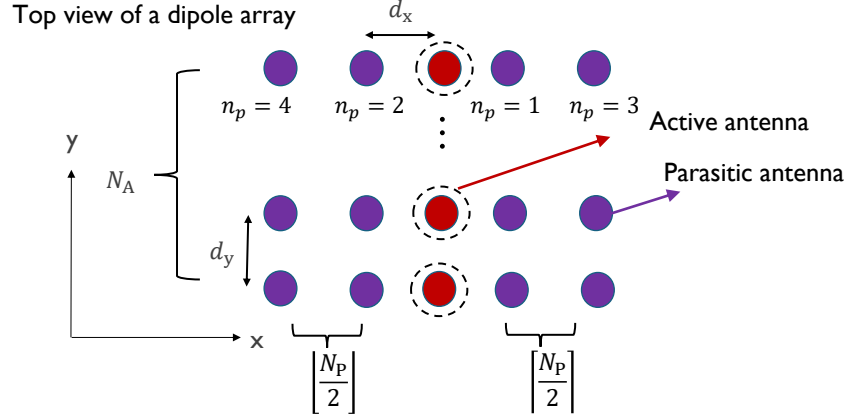


Figure 2.9. Planar array with N_A active antennas (red) and $N_P N_A$ parasitic elements (purple).

The definition of $\mathbf{a}_x(\theta)$ is the same as in (2.6), and $\mathbf{a}_y(\theta)$ is defined as

$$\mathbf{a}_y(\theta) = \left[1, e^{-j2\pi \frac{d_y \cos(\theta)}{\lambda_c}}, \dots, e^{-j2\pi \frac{(N_A-1)d_y \cos(\theta)}{\lambda_c}} \right]^T. \quad (2.28)$$

The geometric narrowband multipath channel has L paths where the complex gain of the ℓ th path is α_ℓ and the angle of incidence is θ_ℓ . Let $\mathbf{h}_A = [h_{A,1}, \dots, h_{A,N_A}]^T$ be the channel vector corresponding to the active antennas which are along y axis. We define

$$\mathbf{h}_A = \frac{1}{\sqrt{L}} \sum_{\ell=1}^L \alpha_\ell \mathbf{a}_y(\theta_\ell). \quad (2.29)$$

Similarly, let $\mathbf{h}_P = [\mathbf{h}_{P,1}^T, \dots, \mathbf{h}_{P,N_A}^T]^T$ be the channel vector corresponding to the parasitic elements. We define

$$\mathbf{h}_P = \frac{1}{\sqrt{L}} \sum_{\ell=1}^L \alpha_\ell (\mathbf{a}_y(\theta_\ell) \otimes \mathbf{a}_x(\theta_\ell)). \quad (2.30)$$

We define the channel vector for the hybrid parasitic array as $\mathbf{h} = [\mathbf{h}_A^T, \mathbf{h}_P^T]^T$. Including large-scale fading, we have $\mathbf{z}_{RT} = \gamma \mathbf{h}$. For $L = 1$ and $N_A = 1$, we can verify that $\mathbf{h}$ simplifies to the case of LOS channel with single active and multiple parasitic elements.

2.4.2 SNR maximization under radiated power constraints

In this section, we express the SNR defined in (2.3) for the case of multi-active and multi-parasitic array with a multipath channel in terms of the active antenna current vector $\mathbf{i}_A$ and the reconfigurable load matrix $\mathbf{Z}_R$. Using $\mathbf{i}_{TX}$ and $\mathbf{z}_{RT}$, we have

$$\text{SNR}(\mathbf{i}_A, \mathbf{Z}_R) = \frac{|\mathbf{i}_{TX}^T \mathbf{z}_{RT}|^2}{\sigma^2}, \quad (2.31)$$

$$\stackrel{(a)}{=} \frac{\gamma^2}{\sigma^2} |\mathbf{i}_A^T \underbrace{(\mathbf{h}_A - \mathbf{Z}_m^T (\mathbf{Z}_P + \mathbf{Z}_R)^{-1} \mathbf{h}_P)}_{\mathbf{h}_{\text{eff}}}|^2, \quad (2.32)$$

where equality (a) follows from (2.27). The effective wireless channel $\mathbf{h}_{\text{eff}} = [h_{\text{eff},1}, \dots, h_{\text{eff},N_A}]^T$ experienced by the active antenna array depends on the multipath channel, mutual impedance matrices, and reconfigurable parasitic load matrix. This effective channel is dynamically tuned through the matrix $\mathbf{Z}_R$.

The transmit current vector is constrained to limit the total radiated power. The total transmit power radiated from the antenna array is expressed using (2.2) as

$$P_{TX} = \begin{bmatrix} \mathbf{i}_A^* & \mathbf{i}_P^* \end{bmatrix} \begin{bmatrix} \mathcal{R}\{\mathbf{Z}_A\} & \mathcal{R}\{\mathbf{Z}_m^T\} \\ \mathcal{R}\{\mathbf{Z}_m\} & \mathcal{R}\{\mathbf{Z}_P\} \end{bmatrix} \begin{bmatrix} \mathbf{i}_A \\ \mathbf{i}_P \end{bmatrix}. \quad (2.33)$$

To express the power in terms of $\mathbf{i}_A$ only, we define an effective impedance matrix $\mathbf{Z}_{\text{eff}}$ as

$$\begin{aligned} \mathbf{Z}_{\text{eff}} &= \mathcal{R}\{\mathbf{Z}_A\} + \mathbf{Z}_m^* (\mathbf{Z}_P + \mathbf{Z}_R)^{-*} \mathcal{R}\{\mathbf{Z}_P\} (\mathbf{Z}_P + \mathbf{Z}_R)^{-1} \mathbf{Z}_m \\ &\quad - \mathcal{R}\{\mathbf{Z}_m^T\} (\mathbf{Z}_P + \mathbf{Z}_R)^{-1} \mathbf{Z}_m - \mathbf{Z}_m^* (\mathbf{Z}_P + \mathbf{Z}_R)^{-*} \mathcal{R}\{\mathbf{Z}_m\}. \end{aligned} \quad (2.34)$$

Using (2.27), (2.33), and (2.34), we express P_{TX} in terms of $\mathbf{i}_A$ and $\mathbf{Z}_{\text{eff}}$ as $P_{TX} = \mathbf{i}_A^* \mathbf{Z}_{\text{eff}} \mathbf{i}_A$. The effective impedance matrix $\mathbf{Z}_{\text{eff}}$ is a function of the reconfigurable load matrix and mutual impedance matrices. This effective matrix is dynamically tuned through $\mathbf{Z}_R$. For a conventional array with active elements only, $\mathbf{Z}_{\text{eff}} = \mathcal{R}\{\mathbf{Z}_A\}$.

We now formulate the optimization problem with the objective as the SNR and a constraint of $P_{\max}$ on the total radiated power from the transmit array. The optimization variables include the active antenna beamforming vector $\mathbf{i}_A$ and the imaginary part of the parasitic reconfigurable load matrix $\mathcal{I}\{\mathbf{Z}_R\}$. The beamforming optimization problem for the hybrid architecture is mathematically expressed as

$$\mathbf{P4}: \mathbf{i}_A^*, \mathcal{I}\{\mathbf{Z}_R^*\} = \underset{\mathbf{i}_A \in \mathbb{C}, \mathcal{I}\{\mathbf{Z}_R\} \in \mathbb{R}}{\operatorname{argmax}} \operatorname{SNR}(\mathbf{i}_A, \mathbf{Z}_R), \quad (2.35a)$$

$$\text{s.t. } \mathbf{i}_A^* \mathbf{Z}_{\text{eff}} \mathbf{i}_A \leq P_{\max}. \quad (2.35b)$$

The problem **P4** is challenging to solve directly because the optimization variables are tightly coupled in both the objective function and the constraints.

2.4.3 Low-complexity closed-form solution for P4

In this section, we propose a low-complexity closed-form approach for configuring $\mathbf{i}_A$ and $\mathcal{I}\{\mathbf{Z}_R\}$ that uses an approximation similar to Section 2.3. We define an upper bound on the original objective as $\operatorname{SNR}_{\text{up}}(\mathbf{i}_A, \mathbf{Z}_R)$ using the triangle inequality as

$$\operatorname{SNR}_{\text{up}}(\mathbf{i}_A, \mathbf{Z}_R) = \frac{\gamma^2}{\sigma^2} \left(\sum_{j=1}^{N_A} |\mathbf{i}_{A,j}| |h_{\text{eff},j}| \right)^2. \quad (2.36)$$

The optimization in **P4** with respect to $\mathcal{I}\{\mathbf{Z}_R\}$ is approximately solved by maximizing the upper bound $\operatorname{SNR}_{\text{up}}(\mathbf{i}_A, \mathbf{Z}_R)$. The term $h_{\text{eff},j}$ depends on the parasitic load reactances of all elements as defined in (2.32). We use a diagonalization approximation similar to Section 2.3.1. By defining $\mathbf{D}_P = \operatorname{diag}\{\mathbf{Z}_P\}$, we compute an approximate expression for the j th element of the effective channel $\hat{h}_{\text{eff},j}$ expressed as

$$\hat{h}_{\text{eff},j} = h_{A,j} \left(1 - \frac{\mathbf{z}_{m,j,j}^T (\mathbf{D}_{P,j,j} + \mathbf{Z}_{R,j,j})^{-1} \mathbf{h}_{P,j}}{h_{A,j}} \right). \quad (2.37)$$

By using the approximation $\hat{h}_{\text{eff},j}$ in (2.36), we decouple the parasitic reactance optimization problem into N_A sub-problems as maximizing the upper bound is obtained by maximizing each summand. The solution to the j th sub-problem $\mathcal{I}\{\mathbf{Z}_{R,j,j}^*\}$ is mathematically expressed as

$$\mathbf{P5}: \mathcal{I}\{\mathbf{Z}_{R,j,j}^*\} = \underset{\mathcal{I}\{\mathbf{Z}_{R,j,j}\}}{\operatorname{argmax}} |\hat{h}_{\text{eff},j}|^2, \forall j = \{1, \dots, N_A\}. \quad (2.38)$$

Each sub-problem in **P5** can be expressed in the form similar to problem **P2** because the matrix $(\mathbf{D}_{P,j,j} + \mathbf{Z}_{R,j,j})^{-1}$ is a diagonal matrix. This enables us to apply the closed-form solution proposed in Theorem 2 to this problem as well.

The mathematical form of each sub-problem in **P5** resembles that of problem **P1**. Hence, the closed-form solution to problem **P1** proposed in Theorem 2 is directly applied to each sub-problem in **P5**. After substituting the closed-form solution of $\mathcal{I}\{Z_{R,i,j}^*\}$ in the objective and constraints of problem **P4**, we optimally solve for the active antenna beamforming vector $\mathbf{i}_A$. The closed-form solution to problem **P4** based on the solution of the approximate sub-problems in **P5** is denoted as $\mathbf{i}_A^*$ and $Z_{R,i,j}^*$. The imaginary part of $Z_{R,i,j}^*$ is

$$\begin{aligned} \mathcal{I}\{Z_{R,i,j}^*\} &= -\mathcal{I}\{Z_P\} \\ &= -\frac{\cot\left(\frac{\angle\left(\frac{[\mathbf{h}_{P,j}]_i [\mathbf{z}_{m,j,j}]_i}{h_{A,j}}\right) - \angle\left(1 - \frac{\zeta}{2h_{A,j}} \mathbf{h}_{P,j}^T \mathbf{z}_{m,j,j}\right)}{2}\right)}{\zeta}. \end{aligned} \quad (2.39)$$

By substituting $\mathcal{I}\{Z_{R,i,j}^*\}$ in (2.26), (2.32), and (2.34), we denote $\mathbf{Z}_R^*$, $\mathbf{h}_{\text{eff}}^*$, and $\mathbf{Z}_{\text{eff}}^*$. The closed-form expression for $\mathbf{i}_A^*$ is

$$\mathbf{i}_A^* = \sqrt{P_{\max}} \frac{(\mathbf{Z}_{\text{eff}}^*)^{-1} \mathbf{h}_{\text{eff}}^{*c}}{\|(\mathbf{Z}_{\text{eff}}^*)^{-1/2} \mathbf{h}_{\text{eff}}^{*c}\|}. \quad (2.40)$$

The closed-form expression in (2.39) and (2.40) depends on the channel vector. Although we have used a multipath channel model in Section 2.4.1, the derived optimization closed-form

result is valid for any general channel model. A relevant generalization for future study would be to incorporate different radiation patterns for each antenna element. Such a generalization would be useful when parasitic antenna element type is different from the active antenna type.

The solution for the active antenna beamforming current in (2.40) is optimal for the problem **P4** assuming that $\mathbf{Z}_R$, $\mathbf{Z}_{\text{eff}}$, and $\mathbf{h}_{\text{eff}}$ are fixed. While deriving a closed-form expression for the parasitic reactance in (2.39), several approximations are made, leading to a sub-optimal solution. Specifically, the expression of the reconfigurable parasitic reactance $\mathcal{I}\{Z_{R,i,j}^*\}$ depends only on $\mathbf{z}_{m,j,j}$ and not on the coupling with other active antennas. This solution is reasonable when the active antennas are placed sufficiently apart ($d_y \geq 0.5\lambda_c$) such that the mutual impedance between each sub-array is negligible. The solution for the active antenna beamforming vector, however, incorporates the entire coupling matrix $\mathbf{Z}_m$ and it compensates for the sub-optimality of (2.39) by leveraging its optimal closed-form solution in (2.40). This solution strategy is consistent with prior work on hybrid beamforming using conventional antennas, as seen in [2]. In the next section, we compare the proposed beamforming approach with various benchmarks.

2.5 Spectral efficiency and energy efficiency numerical results

2.5.1 Radiation pattern validation of parasitic array with Feko

In this section, we validate the theoretical expression of the parasitic array SNR from (2.32) for a LOS channel based on the circuit theory approach through computational electromagnetic simulations in the software Feko. In our simulations, we use thin wire dipole antenna for both active and parasitic elements. The planar array configuration in Fig. 2.9 is implemented with $N_A = 6$, $N_P = 2$, $d_x = 0.4\lambda_c$, and $d_y = 0.5\lambda_c$. The simulation is performed at $f_c = 7$ GHz. We follow the strategy similar to [67] to validate the theoretical radiation pattern by comparing with Feko simulations based on arbitrary configurations of the parasitic load and excitation voltages. In Feko, we use active voltage sources for the six antennas placed on the y axis.

Each active antenna is connected to a voltage source with unit magnitude and zero phase angle, i.e., $\mathbf{v}_A = [1, 1, 1, 1, 1, 1]^T$. For the remaining 12 dipole antennas, we attach reconfigurable loads. The resistance of each load is fixed at a small value of 0.05Ω to avoid significant losses. The reactances are chosen arbitrarily from the set of real numbers. We implement two different configurations for the parasitic reactances. For configuration 1, the reactances are set to $\mathcal{I}\{\text{diag}\{\mathbf{Z}_R\}\} = [-10, 20, 40, 100, 300, 0, 5, 15, 45, 70, -60, -90]^T \Omega$ and for configuration 2, they are set to $\mathcal{I}\{\text{diag}\{\mathbf{Z}_R\}\} = [0, 200, -150, 40, -10, -50, 60, 120, -30, 40, 0, 10]^T \Omega$.

For the theoretical calculations, we extract the 18×18 impedance matrix $\mathbf{Z}_{TX}$ at 7 GHz from Feko. The matrices $\mathbf{Z}_A$, $\mathbf{Z}_m$ and $\mathbf{Z}_P$ are obtained through $\mathbf{Z}_{TX}$. For validating beam pattern in the xy plane, we use $\mathbf{h}$ with $L = 1$ and compute $\mathbf{h}_P$ and $\mathbf{h}_A$ as described in Section 2.4.1. We compute $\mathbf{i}_A$ using $\mathbf{v}_A$, $\mathbf{Z}_A$, $\mathbf{Z}_m$, $\mathbf{Z}_P$, and $\mathbf{Z}_R$ as $\mathbf{i}_A = (\mathbf{Z}_A - \mathbf{Z}_m^T(\mathbf{Z}_P + \mathbf{Z}_R)^{-1}\mathbf{Z}_m)^{-1}\mathbf{v}_A$. Substituting $\mathbf{i}_A$, $\mathbf{Z}_m$, $\mathbf{Z}_P$, $\mathbf{Z}_R$, $\mathbf{h}_P$, and $\mathbf{h}_A$ in (2.32), we obtain the beamforming pattern. For comparison purposes, we normalize this pattern such that the maximum value across the angular domain corresponds to 0 dB. In Fig. 2.10(a) and Fig. 2.10(b), we show the comparison of the theoretical pattern and the simulated pattern from Feko evaluated at 7 GHz for two different parasitic reactance configurations. For both configurations, we see that the simulated and theoretical beam patterns are well aligned. This validates the theoretical expression derived in (2.32). A benefit of using the proposed model is that it does not require updated simulation results each time the parasitic reactances change. The matrices $\mathbf{Z}_A$, $\mathbf{Z}_m$ and $\mathbf{Z}_P$ are independent of the load configurations and depend only on the antenna and array geometry. For the further simulations, we will use the impedance matrices $\mathbf{Z}_A$, $\mathbf{Z}_m$ and $\mathbf{Z}_P$ obtained from Feko and use the expression in (2.32) to optimize $\mathbf{i}_A$ and $\mathcal{I}\{\mathbf{Z}_R\}$. In this chapter, we are evaluating the impedance parameters and the beam pattern at a single frequency. The frequency response of these parameters could also be used, especially for analyzing wideband systems. This is deferred to future study.

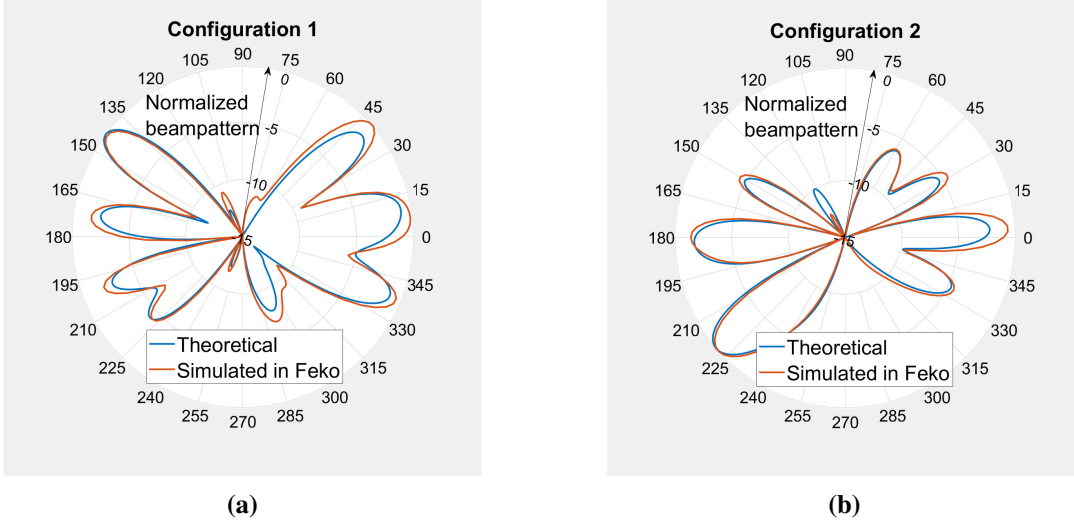


Figure 2.10. The normalized beamforming pattern is shown as a function of the azimuth angle for two different parasitic configurations. The theoretical beamforming pattern closely matches that of the Feko simulations and thus demonstrates the validity of the circuit theory approach for modeling parasitic arrays.

2.5.2 Spectral efficiency and energy efficiency

In this section, we present the spectral efficiency and energy efficiency results for our proposed hybrid reconfigurable parasitic uniform planar array (HRP-UPA) architecture. We also compare with the following three architectural benchmarks. Comparing with these benchmarks will provide us a good understanding of the tradeoff between array aperture, number of RF chains, and transmit power that are required to attain the same spectral efficiency performance.

- Fully digital uniform linear array (FD-ULA): This array is obtained by excluding all parasitic antennas from the HRP-UPA, i.e., $N_P = 0$. The purpose of this comparison is to investigate the gain in spectral and energy efficiency achieved by adding parasitic reconfigurable antenna elements to an FD-ULA architecture, thus converting it to an HRP-UPA. Although an HRP-UPA will have higher aperture compared to FD-ULA, we are interested in analyzing how much reduction in number of RF chains is possible by expanding the aperture through the use of parasitic elements.
- Fully digital uniform planar array (FD-UPA): This array is obtained by using digitally controlled sources instead of the reconfigurable reactances on the parasitic elements of the HRP-UPA.

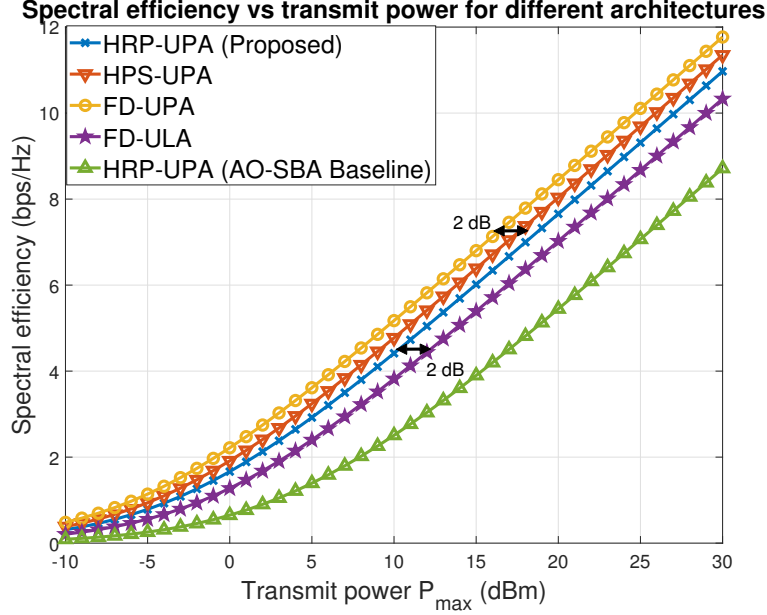


Figure 2.11. The spectral efficiency is shown as a function of the transmit power $P_{\max}$ for the proposed method and different benchmarks. We see that for our proposed architecture, a 2 dB transmit power reduction is possible compared to an FD-ULA with 6 active elements by adding two parasitic elements per active antenna, thus converting it to an HRP-UPA.

The purpose of this comparison is to have a theoretical upper bound on the spectral efficiency for the planar array which is attained when all antennas are configured with digitally controlled sources.

- Hybrid phase-shifter sub-connected uniform planar array (HPS-UPA): Each antenna is tuned by a phase shifter. All antennas in each row (parallel to x axis) are connected together to a single active source through a power-splitter. Each row has an active digitally controlled source. So, the total number of active sources equals number of rows in the planar array. The purpose of this comparison is to analyze the spectral efficiency and energy efficiency tradeoff between conventional phased-array based hybrid beamforming and the proposed hybrid parasitic-array.

For a given SNR, the spectral efficiency (in bps/Hz) is defined as $SE = \log_2(1 + \text{SNR})$. The SNR for HRP-UPA, FD-ULA, FD-UPA, and HPS-UPA are computed using the same angle of departure and path gains for the wireless channel and fixed value of total radiated transmit power $P_{\max}$. For HRP-UPA, we use the proposed solution for the parasitic reactance and active antenna

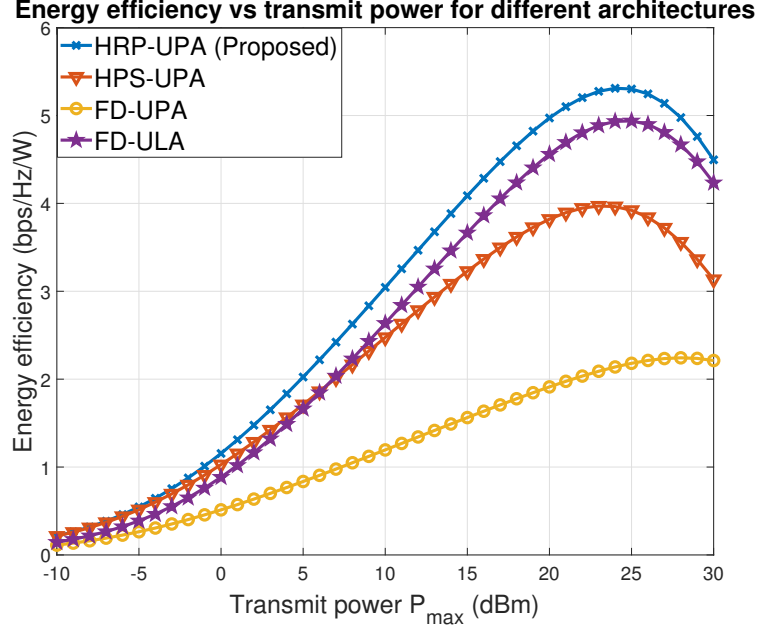


Figure 2.12. The energy efficiency is shown as a function of the transmit power $P_{\max}$ for the proposed method and different benchmarks. Our proposed HRP-UPA outperforms all benchmarks in terms of energy efficiency. The transmit power at which the energy efficiency peaks is also the lowest for our proposed architecture.

current from (2.39) and (2.40). For FD-ULA and FD-UPA, the optimal active antenna current solution is used from our prior work [62]. For HPS-UPA, we configure the phase shifters such that they apply the conjugate phase of the channel coefficient. For the sub-connected array, we use the precoding matrix $\mathbf{F}_{\text{PS}} \in \mathbb{C}^{N_A \times N_A(N_P+1)}$ where each row has $(N_P + 1)$ non-zero complex phases. We define an effective channel for HPS-UPA similar to (2.32) as $\mathbf{h}_{\text{eff,PS}} = \mathbf{F}_{\text{PS}} \mathbf{h}$ and an effective impedance matrix similar to (2.34) as $\mathbf{Z}_{\text{eff,PS}} = \mathbf{F}_{\text{PS}}^c \mathcal{R}\{\mathbf{Z}_{\text{TX}}\} \mathbf{F}_{\text{PS}}^T$. The active currents for HPS-UPA are set similar to (2.40) using $\mathbf{h}_{\text{eff,PS}}$ and $\mathbf{Z}_{\text{eff,PS}}$. The SNR expressions for these different architectures are summarized in Table 2.1. We also compare the proposed closed-form solution for the parasitic reactance and active antenna current to a baseline approach called alternating optimization stochastic beamforming algorithm (AO-SBA) from [25]. The baseline is based on minimizing a metric dependent on the cosine similarity between the desired and the actual current vector.

For the simulations, we set the distance between transmit array and receive antenna as $r = 250$ m, bandwidth $\text{BW} = 20$ MHz, antenna noise temperature $T_A = 300$ K, Boltzmann's

Table 2.1. Comparing SNR and P_{total} of different architectures.

Architecture	SNR	P_{total}
HRP-UPA	$\frac{P_{\text{max}}\gamma^2}{\sigma^2}(\mathbf{h}_{\text{eff}}^*)^*(\mathbf{Z}_{\text{eff}}^*)^{-1}\mathbf{h}_{\text{eff}}^*$	$P_{\text{max}} + N_A P_{\text{RFC}} + N_A N_P P_{\text{VAR}}$
FD-ULA	$\frac{P_{\text{max}}\gamma^2}{\sigma^2}\mathbf{h}_A^*(\mathcal{R}\{\mathbf{Z}_A\})^{-1}\mathbf{h}_A$	$P_{\text{max}} + N_A P_{\text{RFC}}$
FD-UPA	$\frac{P_{\text{max}}\gamma^2}{\sigma^2}\mathbf{h}^*(\mathcal{R}\{\mathbf{Z}_{\text{TX}}\})^{-1}\mathbf{h}$	$P_{\text{max}} + N_A(N_P + 1)P_{\text{RFC}}$
HPS-UPA	$\frac{P_{\text{max}}\gamma^2}{\sigma^2}\mathbf{h}_{\text{eff,PS}}^*(\mathbf{Z}_{\text{eff,PS}})^{-1}\mathbf{h}_{\text{eff,PS}}$	$\epsilon P_{\text{max}} + N_A P_{\text{RFC}} + N_A(N_P + 1)P_{\text{PS}}$

constant $k_B = 1.38 \times 10^{-23}$ J/K, and radiation resistance $R_r = 95.5 \Omega$. These values are used to compute the constant $\frac{\gamma^2}{\sigma^2} = \left(\frac{\lambda_c}{4\pi r}\right)^2 \frac{R_r}{4k_B T_A \text{BW}}$. In our simulations, we vary the value of P_{max} from -10 dBm to 30 dBm. Assuming total power consumption is P_{total} , the energy efficiency (in bps/Hz/W) is defined as $\text{EE} = \frac{\text{SE}}{P_{\text{total}}}$. We use the power consumption values from the conventional hybrid beamforming literature [13]. For FD-ULA and FD-UPA, the number of RF chain equals the number of antennas. We assume that each RF chain which consists of a data converter which consumes 200 mW, and mixer, local oscillator, filter, and amplifier, which consume approximately 40 mW, resulting in a total of $P_{\text{RFC}} = 240$ mW [13]. For HPS-UPA, there are N_A active RF chains and each antenna is connected to a phase shifter which consumes a power of $P_{\text{PS}} = 30$ mW. A 1-to-3 splitter with insertion loss of 1.6 dB is used to connect the RF chain to the phase shifters of each antenna in the sub-panel [68]. Each phase shifter has 0.7 dB insertion loss [68]. The total insertion loss in HPS-UPA is 2.3 dB. We multiply the factor $\epsilon = 10^{2.3/10}$ to P_{max} to account for the insertion losses. We assume that the power required to control the reconfigurable reactance through varactor diodes is negligible, i.e., $P_{\text{VAR}} \approx 0$ similar to [65]. For all architectures, we summarize the power consumption expressions in Table 2.1.

Simulation is performed and averaged over 5000 realizations of the wireless channel $\mathbf{h}$ defined in Section 2.4.1 with $L = 4$, $\theta_\ell \sim \mathcal{U}[-\pi, \pi]$, and $\alpha_\ell \sim \mathcal{N}_{\mathbb{C}}(0, 1)$. We plot the spectral efficiency in Fig. 2.11 and energy efficiency in Fig. 2.12 as a function of the transmit power P_{max} . We observe from Fig. 2.11 that the proposed approach for configuring the HRP-UPA outperforms the AO-SBA baseline by a huge margin. The AO-SBA is sub-optimal because it is

optimized for aligning the beam patterns using cosine similarity effectively discarding information about the absolute magnitude of the beamforming weights. Unlike cosine-similarity [25] and correlation-based formulations [56, 57], [58], our model incorporates a radiated-power constraint that directly links mutual coupling to the spectral efficiency, validated by closed-form results, and electromagnetic simulations. The HRP-UPA with 6 active antennas and 12 parasitic antennas outperforms the FD-ULA with 6 active antennas in terms of spectral efficiency. The 12 parasitic antennas provide additional reconfigurability, improving spectral efficiency without any additional power overhead compared to the FD-ULA. The spectral efficiency performance of HRP-UPA is slightly lower than HPS-UPA because of the coupled magnitude and phase constraint in parasitic beamforming weight unlike the unit modulus phase shifts. Compared to the FD-UPA, which has the highest spectral efficiency, the HRP-UPA requires an additional 2 dB of transmit power to achieve the same spectral efficiency. Despite needing more transmit power than FD-UPA and HPS-UPA for the same spectral efficiency, the proposed HRP-UPA has the highest energy efficiency. Our results demonstrate that using parasitic reconfigurable elements in a hybrid architecture is beneficial from an energy efficiency perspective.

2.5.3 Variation in the number of parasitic elements, active antennas, and parasitic element spacing

We now vary the number of parasitic elements corresponding to each active antenna. We generate the planar array for $N_A = 4$ and vary N_P from 0 to 6. In Fig. 2.13, we plot the spectral efficiency as a function of N_P . We observe that the spectral efficiency increases up to $N_P = 2$ and then saturates. This shows that increasing the number of parasitic elements beyond two per active antenna leads to diminishing gains. This is because beyond $N_P = 2$, the distance between parasitic elements and active antenna increases which leads to lower mutual coupling and reduces the beamforming capability. Hence, placing fewer parasitic elements closer to the active antenna is sufficient to attain reasonable spectral efficiency improvement.

We now vary the number of active antennas N_A in the y direction. For this simulation, we

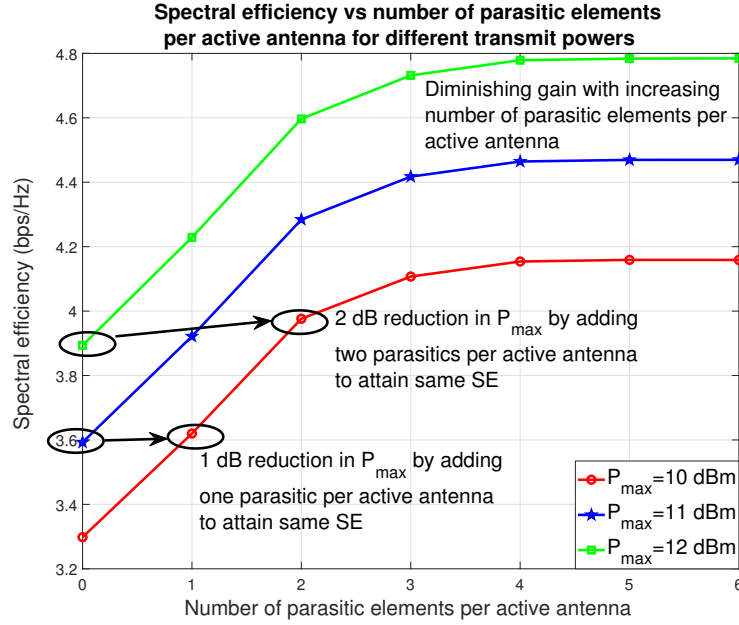


Figure 2.13. The spectral efficiency is shown as a function of the number of parasitic elements per active antenna. We observe that it is sufficient to have just two parasitic elements corresponding to each active antenna as diminishing gains are observed beyond $N_P = 2$. Moreover, we observe a 1 dB power reduction for $N_P = 1$ and 2 dB power reduction for $N_P = 2$ compared to an FD-ULA with $N_P = 0$.

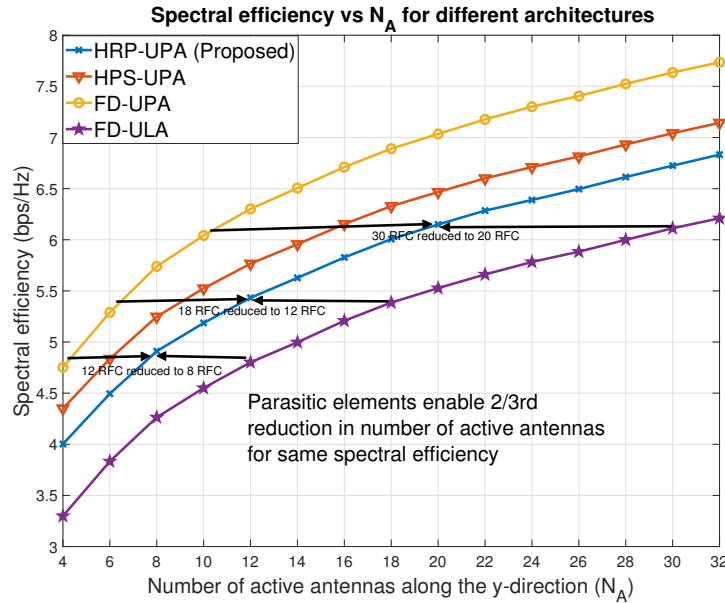


Figure 2.14. The spectral efficiency is shown as a function of N_A . The spectral efficiency of our proposed HRP-UPA with $N_P = 2$ and $N_A = 8/12/20$ is same as the FD-ULA with $N_A = 12/18/30$ antennas and FD-UPA with $N_A(N_P + 1) = 12/18/30$ antennas respectively. The reconfigurability of parasitic elements is leveraged to reduce the number of active antennas. This enables cost reduction in the RF hardware.

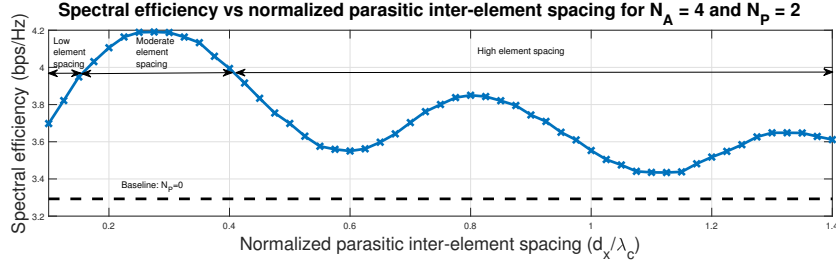


Figure 2.15. The spectral efficiency is shown as a function of the normalized parasitic inter-element spacing. We observe that for a parasitic array with moderate inter-element spacing, the spectral efficiency is the highest.

use the impedance matrices for the planar dipole array using theoretical closed-form expressions similar to that used in Section 2.3.4 and our prior work [62]. We fix $P_{\max} = 10$ dBm and $N_P = 2$. In Fig. 2.14, we plot the spectral efficiency for the proposed hybrid parasitic reconfigurable array and compare it with the three benchmarks as a function of N_A . For all architectures, the spectral efficiency increases with N_A . We observe that in comparison with FD-U LA and FD-U PA, the number of active antennas required reduce by a factor of $\frac{2}{3}$ in an HRP-U PA. Hence, by including two parasitic elements for each active antenna, the system is able to enhance the spectral efficiency which allows reduction in the number of active antennas to attain the same spectral efficiency performance. This observation can be useful to improve the spectral efficiency of power constrained devices with limited number of active RF chains by leveraging the additional reconfigurability offered by parasitic elements. To conclude, Fig. 2.13 and Fig. 2.14 show that parasitic elements enable a low-power and low-cost array architecture because of the reduction in transmit power and number of RF chains.

The mutual coupling plays a significant role in the performance of the parasitic array. The mutual coupling increases with decrease in spacing but not monotonically [14]. Its effect on the beamforming gain and spectral efficiency is also not monotonic. It was shown in [51] that the transmit beamforming gain trend with spacing varies with the angle. Specifically, the highest gain in the endfire direction occurs when inter-element spacing tends to 0, and for broadside when it is between 0.5 and 1. The spectral efficiency metric captures the average effect over different multipath channel realizations. This observation motivates us to simulate the spectral

efficiency trend of a parasitic array by varying the normalized inter-element spacing along the x direction, $\frac{d_x}{\lambda_c}$. For this simulation as well, we use the theoretical closed-form expressions for the impedance matrices. From Fig. 2.15, we observe that the highest spectral efficiency is observed for moderate inter-element spacing, i.e., $0.15 < \frac{d_x}{\lambda_c} < 0.4$. The mutual coupling within this range is sufficient for good spectral efficiency performance of a parasitic array.

2.6 Conclusions and future work

In this chapter, we applied a circuit theory model to a hybrid reconfigurable parasitic array with multiple active antennas and several parasitic elements controlled through tunable reactance. As validated through electromagnetic simulations, the circuit theory approach is useful because we can use the proposed model for different parasitic configurations instead of doing repetitive simulations. We found that the proposed HRP-UPA architecture under the same total radiated power constraint, requires only about 2 dB more transmit power to achieve the same spectral efficiency as FD-UPA, yet has around 2.5 times improvement in energy efficiency. This confirms that hybrid parasitic arrays are an energy-efficient solution for MIMO communication. These arrays have the potential to be an important component of the next-generation tri-hybrid MIMO architecture [4]. We derived closed-form expressions for configuring the parasitic reactance and active antenna currents. From the solution for parasitic reactance, we identified a specific reactance range which has a significant impact on the beamforming gain. This provides a direction to design realistic hardware solutions for configuring parasitic arrays.

In future work, we propose extending the MISO parasitic array to the MIMO case with reconfigurable parasitic antennas on both the transmitter and receiver. Analyzing reconfigurable parasitic arrays with realistic matching network constraints is an important future direction [69]. Another open challenge is a wideband generalization with frequency-selective models for parasitic antennas and matching networks [63]. In this chapter, we assumed a rectangular uniform planar array. A possible direction for future research is to explore different array geometries with

arbitrary placement of parasitic elements around active antennas. Finally, to integrate parasitic arrays in the tri-hybrid MIMO architecture [4], a future challenge is to incorporate beamforming with analog phase shifters in addition to the reconfigurability provided by parasitic elements.

Acknowledgements

Chapter 2, in full, is a reprint of the material as it appears in **N. V. Deshpande**, M. R. Castellanos, S. R. Khosravirad, J. Du, H. Viswanathan and R. W. Heath, “Beamforming With Hybrid Reconfigurable Parasitic Antenna Arrays,” in *IEEE Trans. Wireless Commun.*, vol. 25, pp. 7048-7064, 2026. The dissertation author was the primary investigator and author of this paper.

Chapter 3

The tri-hybrid MIMO architecture with parasitic antennas and dynamic switching

In this chapter¹, we present the tri-hybrid MIMO architecture with parasitic antennas and dynamic switching. We formulate a circuit-theoretic system model, develop an alternating-optimization framework for digital precoding, parasitic loading, and antenna assignment, and evaluate the resulting spectral-efficiency and energy-efficiency tradeoffs.

3.1 Introduction

3.1.1 Motivation and background

Parasitic reconfigurable hybrid arrays enable a low-cost and low-power alternative to conventional multiple-input multiple-output (MIMO) beamforming architectures [20, 25]. To implement very large array apertures without a significant increase in the number of radio frequency (RF) chains, hybrid arrays with multiple active feeds coupled to several parasitic elements offer a feasible approach. The design and configuration of such hybrid arrays brings additional challenges because of the heterogeneity in the array. Traditional array processing and MIMO beamforming techniques cannot directly be applied to hybrid parasitic arrays because of the non-linear dependence of the beamforming gain on the reconfigurable parasitic loads [20].

In the past decade, hybrid beamforming with analog phase shifters [2], switches [13], and

¹This chapter is related to the overview article [4] and the journal manuscript [70].

true-time-delay elements [71] has been extensively investigated to address the tradeoff between large apertures, power consumption, and communication performance. The tri-hybrid MIMO architecture is an evolving paradigm that combines the merits of the hybrid MIMO beamforming architecture with the unique features of reconfigurable antennas [4]. The number of radiating elements can be increased beyond the existing aperture by replacing conventional antennas in a hybrid MIMO architecture with more capable reconfigurable antennas like parasitic antennas [20], dynamic metasurface antennas [72], and pixel antennas [73]. This expansion requires no increase in the dimension of the analog or digital precoding.

3.1.2 Prior work

There are different types of reconfigurable antennas that can be classified based on their function, architecture, and reconfiguration mechanism. In terms of function, the reconfigurable antenna can be used to configure the pattern [74], frequency [75], polarization [76], or a combination of these. In terms of architecture, there can be several types depending on the propagation medium between the active feed and the radiating elements. There exists diverse reconfiguration mechanisms like control through varactor diodes, PIN diodes, micro-electro-mechanical systems (MEMS), etc. While the antenna community has been working on different kinds of reconfigurable antennas for several decades, their integration into MIMO systems has been recent [77].

Recent prior work has incorporated reconfigurable antennas in a tri-hybrid MIMO architecture. There are several variations of a tri-hybrid MIMO architecture that depend on the choice of analog beamforming and the type of reconfigurable antenna. Dynamic metasurface antennas (DMAs) were introduced in [26, 65, 78] and subsequently integrated into tri-hybrid MIMO systems in [72, 79]. Another example includes radiation-center reconfigurable array implemented using reconfigurable pixel antennas from [73]. In terms of the analog block of the tri-hybrid MIMO architecture, [72] and [73] used analog phase-shifters, while [80] has a network of switches and low resolution phase shifters with the electromagnetic (EM) precoder modeled

as a selection vector across different radiation patterns. The tri-hybrid MIMO formulation offers a unified framework to investigate different reconfigurable antennas as a third precoding layer on top of the legacy hybrid MIMO architecture [4].

The prior work on parasitic hybrid arrays is limited to specific arrangements of parasitic elements around active antennas [20, 25]. In a fixed design of active and parasitic elements, there is limited beamforming flexibility because the current on parasitic elements depends on how tightly they are coupled to the active antennas. The analog switch networks used for conventional antennas in hybrid MIMO enable dynamic connection between RF chains and antennas [13]. We propose to combine the beamforming merits of the analog switch network and the parasitic antennas in a tri-hybrid MIMO architecture as shown in Fig. 3.1(a). The analog switch will configure the assignment of each radiating element to either an active feed or a reconfigurable parasitic reactance. This combination is favorable because it can provide additional beamforming capability without extra power overhead compared to both the existing benchmarks —a hybrid architecture with analog switch for conventional antennas (Fig. 3.2(b)) and a static hybrid parasitic array (Fig. 3.2(c)).

Most prior work simply models the analog switching component in MIMO as an antenna selection problem [13] [73, 80]. A critical assumption in this formulation is that the antennas which are not selected are treated as uncoupled with the antennas that are selected. For conventional arrays, this assumption would hold if each antenna element is sufficiently far apart and appropriately loaded. In hybrid parasitic arrays, the mutual coupling is significant because of closely-spaced elements. Hence, we need to carefully account for the coupling in the model for a tri-hybrid MIMO system with parasitic elements.

A similar limitation is also observed in the prior work on modeling reconfigurable antennas in tri-hybrid MIMO. For example, [81] models the reconfigurability of each antenna as a selection from a discrete set of predefined radiation patterns. This abstraction overlooks the more complex, non-linear problem of how a specific antenna state is physically synthesized. In practice, these patterns are obtained as a result of tuning underlying circuit components like varactor

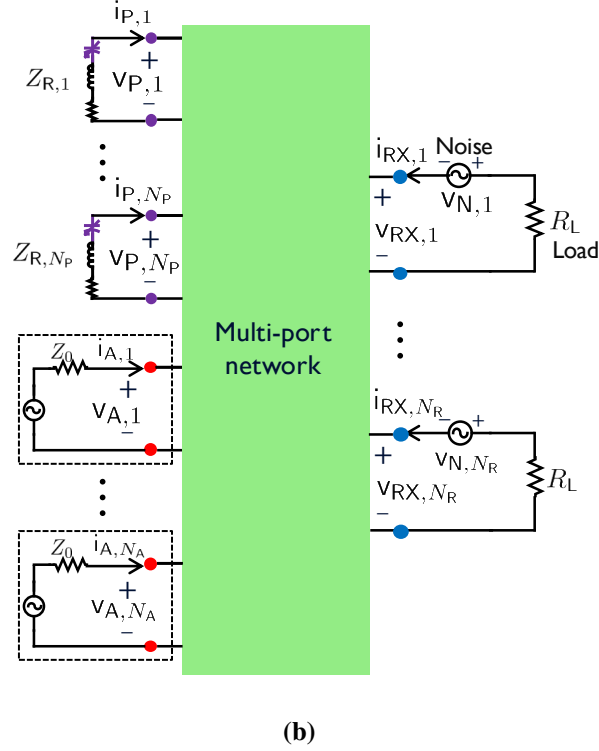
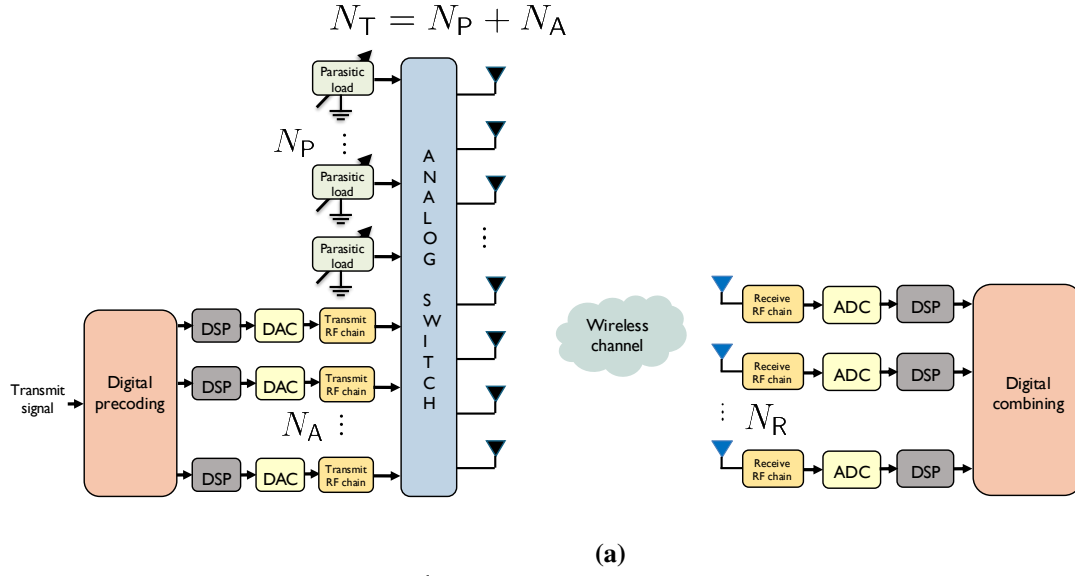


Figure 3.1. (a) The tri-hybrid MIMO architecture integrates an analog switch with reconfigurable parasitic elements. The analog switch controls the dynamic assignment of each radiating element to an active RF chain or a parasitic load. (b) The circuit model of the parasitic architecture where parasitic antenna ports are connected to reconfigurable reactances and active antenna ports are connected to voltage sources.

diodes or PIN diodes, which can be abstracted in terms of resistance, inductance, and capacitance. A circuit-theoretic modeling approach is useful to formulate physically consistent constraints for reconfigurable antennas. In our prior work [20], we modeled the hybrid parasitic array of multiple active elements and parasitic elements using the impedance based circuit framework. The active antenna current vector is expressed in terms of the mutual coupling between active and parasitic elements, the parasitic self-impedance matrix, and the reconfigurable loads. This formulation was also validated through electromagnetic simulations in [20] by comparing the theoretical and simulated radiation patterns. Related recent work in [82] also studies tunable-load reconfigurable antenna arrays through coupling-aware design. Here, we focus on the multi-port circuit-theoretic modeling of parasitic arrays in a tri-hybrid MIMO architecture with dynamic active-parasitic assignment.

Sparse arrays [83], thinned arrays [84], or reconfigurable thinned arrays [85] are alternative techniques of reducing hardware complexity and power consumption while maintaining the large aperture benefit. The focus of this chapter, however, differs in a key respect. Unlike reducing the number of physical elements in a sparse array, the technique used in a reconfigurable parasitic hybrid array is to reduce the number of active feeds by introducing reconfigurable loads on the remaining antennas. Our approach enhances the benefit of sparse arrays (Fig. 3.2(d)) by introducing reconfigurable parasitic elements without any additional power consumption overhead. We show the benefit of hybrid parasitic arrays over a sparse array through numerical simulations of spectral efficiency.

3.1.3 Contributions

In this chapter, we propose a system model for the tri-hybrid MIMO architecture which is based on parasitic reconfigurable elements, analog switch, and digital precoding. Using the multi-port circuit theoretic approach, we formulate the signal and power constraints as a function of the mutual impedance matrix, parasitic element loads, and active antenna currents. We validate the circuit-theoretic model by comparing the radiation patterns obtained for the parasitic array

designed in the Feko software. We assume that each antenna in the tri-hybrid architecture can be either driven by an active RF chain or be configured through a reconfigurable load. This selection between active feed or parasitic element is controlled dynamically through an analog switch network.

We validate the proposed architecture through an ablation study and two numerical experiments. We compare against four benchmark architectures as shown in Fig. 3.2: a fully-active array, a hybrid array with analog switch and fixed loads, a hybrid array with parasitic loads, and a sparse array with fewer active antennas. We analyze the tradeoff between energy efficiency and spectral efficiency through two experiments. First, we vary the parasitic-to-active element ratio for a fixed inter-element spacing and aperture. Second, we vary this ratio by densifying a sparse active array for a fixed aperture and number of active antennas. In the densification experiment (Section 3.4.5), we quantify gains with the reconfigurability efficiency factor (REF) of [86] and show that the benefit of parasitic elements is higher when the parasitics are densely packed between the active antennas.

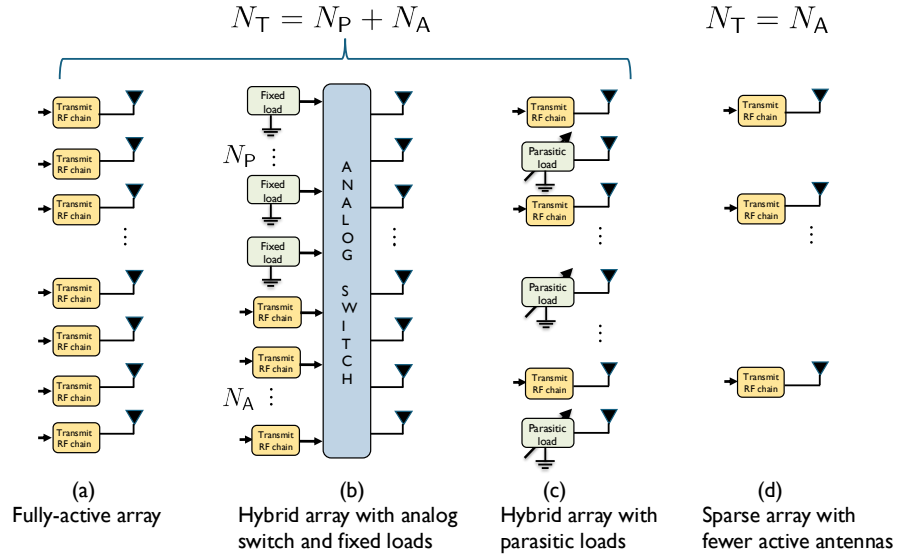


Figure 3.2. An ablation study is performed by simplifying the tri-hybrid MIMO architecture in Fig. 3.1(a) to obtain (b) and (c). The architecture (a) is shown as an upper bound benchmark in terms of spectral efficiency, whereas (d) is a lower bound.

Organization: In Section 3.2, we present the tri-hybrid MIMO system model based on

the circuit-theoretic approach, including the architecture, circuit model, validation against EM simulations, and the MIMO achievable rate and transmit power constraint. In Section 3.3, we formulate the optimization problem and describe the alternating maximization approach for digital precoding, parasitic loads, and antenna assignment via simulated annealing. In Section 3.4, we present numerical results on spectral efficiency and energy efficiency for channel fading, transmit power, active-parasitic tradeoff, and densification of sparse arrays. In Section 3.5, we present the key takeaways and discuss future research directions. The appendix derives the partial derivative used in the parasitic load optimization.

3.2 Tri-hybrid MIMO with parasitic arrays

In this section, we present the parasitic antenna based tri-hybrid MIMO system by expanding on our previous work on hybrid parasitic reconfigurable antenna arrays in [20] which uses the circuit modeling approach [51]. In Section 3.2.1, we describe the tri-hybrid MIMO architecture. In Section 3.2.2, we describe the circuit-theoretic model for the tri-hybrid architecture and establish the key input-output signal model. In Section 3.2.3, we validate the circuit model against Feko EM simulations for both hybrid and sparse arrays. In Section 3.2.4, we formulate the MIMO achievable rate and the transmit power constraint.

3.2.1 Model assumptions and notation

The tri-hybrid architecture is at the transmitter as shown in Fig. 3.1(a). We assume a single user downlink communication system where a transmitter has N_T transmit antennas and the receiver has N_R receive antennas. The number of data streams for communication is assumed to be N_S . We assume an orthogonal frequency-division multiplexing (OFDM) system with K subcarriers, where f_k denotes the k th subcarrier frequency.

Let there be N_A number of active antennas that are connected to an RF chain and $N_P = N_T - N_A$ number of parasitic antennas that are connected to reconfigurable loads. The connection between each antenna element and the type of feed is configured through an analog

switching network. Unlike our previous work [20] where the hybrid array was static, i.e., the assignment of active antennas and parasitic elements was fixed, in this chapter, we assume a dynamic assignment for each antenna element.

Let the dynamic antenna assignment vector controlled through the analog switch be denoted as $\mathbf{r}$, where $[\mathbf{r}]_i = 1$ indicates that the i th antenna is connected to an RF chain, and $[\mathbf{r}]_i = 0$ indicates that the i th antenna is connected to a parasitic load. Let $\mathcal{E} = \{1, 2, \dots, N_T\}$ be the set of all antenna element indices. We define $\mathcal{E}_A = \{i \in \mathcal{E} | [\mathbf{r}]_i = 1\}$ as the set of active element indices and $\mathcal{E}_P = \{i \in \mathcal{E} | [\mathbf{r}]_i = 0\}$ as the parasitic element indices. For example, for an array with $N_P = 5$ and $N_A = 3$, if $\mathbf{r} = [0, 1, 0, 0, 1, 0, 0, 1]$, then it indicates that $\mathcal{E}_A = \{2, 5, 8\}$, and $\mathcal{E}_P = \{1, 3, 4, 6, 7\}$.

We compare the tri-hybrid MIMO architecture with the following benchmarks.

- Fully-active array: Each transmit antenna is connected to an RF chain as shown in Fig. 3.2(a). This architecture is the most power-hungry and provides the highest spectral efficiency.
- Hybrid array with analog switch and fixed loads: Each transmit antenna is either connected to a fixed load or an RF chain. The assignment is controlled dynamically through an analog switch as shown in Fig. 3.2(b).
- Hybrid array with parasitic loads: Each transmit antenna is either connected to a parasitic load or an RF chain. The assignment is static and depends on the design of the hybrid array as shown in Fig. 3.2(c).
- Sparse array with fewer active antennas: Each transmit antenna is connected to an RF chain. The sparse array is obtained by removing the N_P parasitic elements and only preserving the N_A active elements as shown in Fig. 3.2(d).

Each benchmark corresponds to a different hardware structure. The parasitic-load architecture uses tunable reactances implemented using varactor diodes that can be optimized for the wireless

channel. The fixed-load architecture terminates antennas with a passive fixed impedance and requires no tunability. We compare these benchmarks to quantify the spectral-efficiency gains and the relevance of different hardware choices.

3.2.2 Circuit-based system model and input-output relationship

We model the MIMO system as a multi-port network with $N_R + N_T$ ports, as shown in Fig. 3.1(b). On the transmit side, the array impedance matrix $\mathbf{Z}_{TX}(f_k) \in \mathbb{C}^{N_T \times N_T}$ at subcarrier f_k is partitioned into blocks that capture self- and mutual impedance: $\mathbf{Z}_A(f_k) = \mathbf{Z}_{TX}(f_k)[\mathcal{E}_A, \mathcal{E}_A]$ for the active elements, $\mathbf{Z}_P(f_k) = \mathbf{Z}_{TX}(f_k)[\mathcal{E}_P, \mathcal{E}_P]$ for the parasitic elements, and $\mathbf{Z}_m(f_k) = \mathbf{Z}_{TX}(f_k)[\mathcal{E}_P, \mathcal{E}_A]$ for the mutual impedance between active and parasitic elements. On the receive side, $\mathbf{Z}_{RX}(f_k) \in \mathbb{C}^{N_R \times N_R}$ is the impedance matrix of the receive array. The wireless propagation between transmit and receive arrays is captured by the channel impedance matrix $\mathbf{Z}_{RT}(f_k) = \mathbf{Z}_{TR}^T(f_k) \in \mathbb{C}^{N_R \times N_T}$. We relate $\mathbf{Z}_{RT}(f_k)$ to the dimensionless MIMO propagation matrix $\mathbf{H}(f_k) \in \mathbb{C}^{N_R \times N_T}$ from Section 3.4 and a reference load resistance R_L through $\mathbf{Z}_{RT}(f_k) = R_L \mathbf{H}(f_k)$, so that $\mathbf{Z}_{RA}(f_k) = \mathbf{Z}_{RT}(f_k)[\mathcal{E}_A]$ and $\mathbf{Z}_{RP}(f_k) = \mathbf{Z}_{RT}(f_k)[\mathcal{E}_P]$. The couplings $\mathbf{Z}_{RA}(f_k) \in \mathbb{C}^{N_R \times N_A}$ and $\mathbf{Z}_{RP}(f_k) \in \mathbb{C}^{N_R \times N_P}$ are the receive-to-active and receive-to-parasitic column blocks of $\mathbf{Z}_{RT}(f_k)$, respectively. We now partition the voltage and current vectors into active and parasitic components. Let the voltage vector at the transmit antenna array ports be $\mathbf{v}_{TX}(f_k)$ and at the receive antenna array ports be $\mathbf{v}_{RX}(f_k)$. Let $\mathbf{v}_A(f_k) = \mathbf{v}_{TX}(f_k)[\mathcal{E}_A]$ and $\mathbf{v}_P(f_k) = \mathbf{v}_{TX}(f_k)[\mathcal{E}_P]$. Similarly, the currents are denoted as $\mathbf{i}_{TX}(f_k)$, $\mathbf{i}_A(f_k)$, $\mathbf{i}_P(f_k)$, and $\mathbf{i}_{RX}(f_k)$. Using these definitions, we apply Ohm's law to obtain

$$\begin{bmatrix} \mathbf{v}_A(f_k) \\ \mathbf{v}_P(f_k) \\ \mathbf{v}_{RX}(f_k) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_A(f_k) & \mathbf{Z}_m^T(f_k) & \mathbf{Z}_{RA}^T(f_k) \\ \mathbf{Z}_m(f_k) & \mathbf{Z}_P(f_k) & \mathbf{Z}_{RP}^T(f_k) \\ \mathbf{Z}_{RA}(f_k) & \mathbf{Z}_{RP}(f_k) & \mathbf{Z}_{RX}(f_k) \end{bmatrix} \begin{bmatrix} \mathbf{i}_A(f_k) \\ \mathbf{i}_P(f_k) \\ \mathbf{i}_{RX}(f_k) \end{bmatrix}. \quad (3.1)$$

We have established the relation between the currents and voltages at the ports of the parasitic elements, active antennas, and receive antennas.

As the transmit array is in the far-field of the receive array, we use the unilateral approximation, i.e., $\mathbf{Z}_{\text{RA}}^{\text{T}}(f_k) \approx \mathbf{0}$ and $\mathbf{Z}_{\text{RP}}^{\text{T}}(f_k) \approx \mathbf{0}$, because the signal attenuation between them is large. With this approximation,

$$\mathbf{v}_{\text{A}}(f_k) = \mathbf{Z}_{\text{A}}(f_k)\mathbf{i}_{\text{A}}(f_k) + \mathbf{Z}_{\text{m}}^{\text{T}}(f_k)\mathbf{i}_{\text{P}}(f_k), \quad (3.2)$$

$$\mathbf{v}_{\text{P}}(f_k) = \mathbf{Z}_{\text{m}}(f_k)\mathbf{i}_{\text{A}}(f_k) + \mathbf{Z}_{\text{P}}(f_k)\mathbf{i}_{\text{P}}(f_k). \quad (3.3)$$

Defining the variable load at the port of the ℓ th parasitic element as $Z_{\text{R},\ell}$, the parasitic reconfigurable load matrix is then $\mathbf{Z}_{\text{R}} = \text{diag}\{[Z_{\text{R},1}, \dots, Z_{\text{R},N_{\text{P}}}]^{\text{T}}\}$. The voltage vector at the parasitic ports is $\mathbf{v}_{\text{P}}(f_k) = -\mathbf{Z}_{\text{R}}\mathbf{i}_{\text{P}}(f_k)$, where the negative sign appears because the current $\mathbf{i}_{\text{P}}(f_k)$ is assumed to flow into the multi-port network, as shown in Fig. 3.1(b).

Injecting the expression of $\mathbf{v}_{\text{P}}(f_k)$ in (3.3), we obtain the current on the parasitic elements as

$$\mathbf{i}_{\text{P}}(f_k) = -(\mathbf{Z}_{\text{P}}(f_k) + \mathbf{Z}_{\text{R}})^{-1}\mathbf{Z}_{\text{m}}(f_k)\mathbf{i}_{\text{A}}(f_k). \quad (3.4)$$

From (3.1) and (3.4), the receive voltage is

$$\begin{aligned} \mathbf{v}_{\text{RX}}(f_k) &= (\mathbf{Z}_{\text{RA}}(f_k) - \mathbf{Z}_{\text{RP}}(f_k)(\mathbf{Z}_{\text{P}}(f_k) + \mathbf{Z}_{\text{R}})^{-1}\mathbf{Z}_{\text{m}}(f_k))\mathbf{i}_{\text{A}}(f_k) \\ &\quad + \mathbf{Z}_{\text{RX}}(f_k)\mathbf{i}_{\text{RX}}(f_k). \end{aligned} \quad (3.5)$$

At the receiver array, let the noise vector be modeled as the open-circuit voltage $\mathbf{v}_{\text{N}}(f_k) \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$. We denote the effective channel matrix that models the wireless propagation, mutual coupling, and the reconfigurable load as

$$\mathbf{H}_{\text{eff}}(f_k) = \mathbf{Z}_{\text{RA}}(f_k) - \mathbf{Z}_{\text{RP}}(f_k)(\mathbf{Z}_{\text{P}}(f_k) + \mathbf{Z}_{\text{R}})^{-1}\mathbf{Z}_{\text{m}}(f_k). \quad (3.6)$$

Assuming that the receive termination is a pure resistance R_{L} identical on each port and that the

receive array is uncoupled, i.e., $\mathbf{Z}_{\text{RX}}(f_k) = Z_{\text{RX}}\mathbf{I}$, the receive current is

$$\mathbf{i}_{\text{RX}}(f_k) = -\frac{\mathbf{H}_{\text{eff}}(f_k)\mathbf{i}_{\text{A}}(f_k) + \mathbf{v}_{\text{N}}(f_k)}{Z_{\text{RX}} + R_{\text{L}}}. \quad (3.7)$$

We have established the relation between the receiver current vector and the active antenna current vector.

Let the voltage vector across the load resistance be denoted as $\mathbf{v}_{\text{L}}(f_k)$ and expressed in terms of the active antenna current vector as

$$\mathbf{v}_{\text{L}}(f_k) = -\mathbf{i}_{\text{RX}}(f_k)R_{\text{L}} = \frac{R_{\text{L}}}{Z_{\text{RX}} + R_{\text{L}}}(\mathbf{H}_{\text{eff}}(f_k)\mathbf{i}_{\text{A}}(f_k) + \mathbf{v}_{\text{N}}(f_k)). \quad (3.8)$$

Let the transmit information symbol vector of dimension N_{S} be $\mathbf{s}(f_k) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. We denote the digital precoding matrix for the k th subcarrier as $\mathbf{F}_{\text{dig},k} \in \mathbb{C}^{N_{\text{A}} \times N_{\text{S}}}$. The active antenna current vector is $\mathbf{i}_{\text{A}}(f_k) = \mathbf{F}_{\text{dig},k}\mathbf{s}(f_k)$. Finally, we express the received signal vector $\mathbf{y}(f_k) = \frac{Z_{\text{RX}} + R_{\text{L}}}{R_{\text{L}}}\mathbf{v}_{\text{L}}(f_k)$ in terms of the transmit symbol vector as

$$\mathbf{y}(f_k) = \mathbf{H}_{\text{eff}}(f_k)\mathbf{F}_{\text{dig},k}\mathbf{s}(f_k) + \mathbf{v}_{\text{N}}(f_k). \quad (3.9)$$

The parasitic element control through $\mathbf{Z}_{\text{R}}$ and the analog switch controlling the antenna assignment through $\mathbf{r}$ impacts the effective channel matrix $\mathbf{H}_{\text{eff}}(f_k)$. The dependence of the effective channel matrix on the reconfigurable load is non-linear which makes the parasitic beamforming a challenging research problem.

3.2.3 Validation of circuit model via EM simulation

We validate the circuit-theoretic model in Section 3.2.2 by comparing the radiation pattern predicted by the model in (3.4) with electromagnetic (EM) simulations in the Feko software, similar to our prior work [20]. The objective is to validate the theoretical circuit model and compare the radiation patterns for both the sparse array and the hybrid parasitic array, while

keeping the overall physical aperture identical.

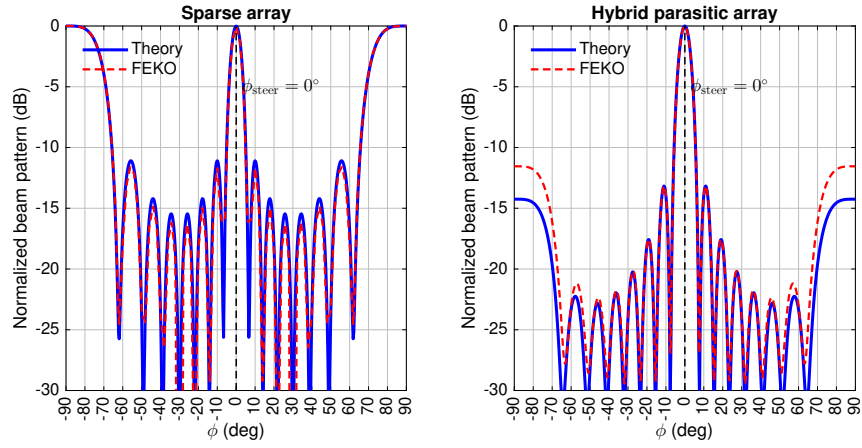
Assuming the center wavelength as λ_c , we compare two uniform linear array (ULA) designs with identical physical aperture $L \approx 7\lambda_c$.

Sparse array: We assume $N_A = 8$ active elements with inter-element spacing $d = \lambda_c$. The large spacing maintains a wide aperture for high directivity but introduces significant grating lobes in the radiation pattern.

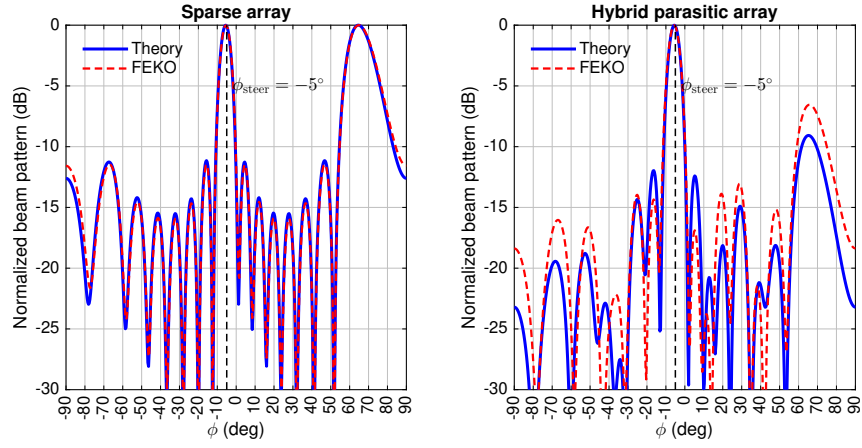
Hybrid parasitic array: We assume $N_A = 8$ active elements and $N_P = 7$ parasitic elements arranged in an alternate fashion with inter-element spacing between a parasitic and active element be $d = 0.5\lambda_c$. The physical aperture, $L \approx 7\lambda_c$, remains identical to the sparse array.

For each design, the circuit model predicts the radiation pattern from the impedance matrices and load configuration, whereas the EM simulation computes the pattern using the method of moments. In Fig. 3.3, we show the radiation pattern comparison between theory and simulations. In each subfigure, we overlay the solid circuit-model pattern with the dashed Feko-based EM-simulated pattern. Fig. 3.3(a) corresponds to broadside excitation, while Fig. 3.3(b) corresponds to beam steering to -5° . The close agreement between the circuit model and EM simulation validates the impedance-matrix formulation for both architectures. The validation of the circuit model enables us to directly use this model without performing computationally expensive EM simulations for different load configurations. By halving the spacing and filling the gaps with reactively loaded parasitic elements, the parasitic elements successfully mitigate the grating lobes observed in the sparse array.

Beyond this validation, a key design question is whether dynamically reconfiguring the assignment of active and parasitic elements via the analog switch improves grating-lobe suppression. We fix $N_A = 8$ and $N_P = 7$ to compare the three benchmarks using the theoretical circuit model: (i) the sparse array, (ii) the hybrid array with parasitic loads and static assignment, and (iii) the tri-hybrid array with dynamic switching. As shown in Fig. 3.4, the tri-hybrid architecture with optimized loads and dynamic assignment achieves substantially better grating-



(a) Broadside.



(b) Steering to -5° .

Figure 3.3. Comparison of theoretical circuit model and Feko radiation patterns for sparse and hybrid parasitic arrays: subfigure (a) broadside excitation, subfigure (b) beam steered to -5° .

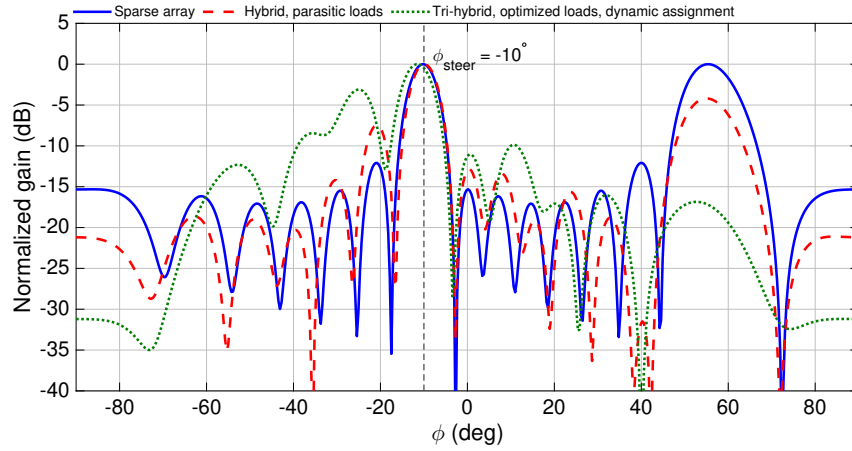


Figure 3.4. Parasitic loads with static assignment mitigate grating lobes slightly; dynamic assignment improves suppression further, at the trade-off of an emerging sidelobe.

lobe suppression than the hybrid with only optimized loads and fixed assignment. This indicates that the dynamic assignment has a higher impact on the beam pattern than parasitic load optimization.

For the static assignment, $\mathcal{E}_A = \{1, 3, 5, 7, 9, 11, 13, 15\}$ and $\mathcal{E}_P = \{2, 4, 6, 8, 10, 12, 14\}$ in an interleaved layout. For the dynamic assignment shown, we have

$$\mathcal{E}_A = \{3, 4, 6, 7, 8, 12, 13, 14\}, \quad \mathcal{E}_P = \{1, 2, 5, 9, 10, 11, 15\}.$$

Creating non-uniformity in the active antenna arrangement, rather than a uniform interleaved layout, yields an effectively aperiodic active subarray and thus contributes to grating-lobe suppression [83, 87]. Unlike prior work on non-uniform sparse arrays, here the array remains fully populated and the sparsity is only in the active feed control, with parasitic elements occupying the remaining positions.

3.2.4 MIMO achievable rate and transmit power constraint

The circuit-based approach is useful for formulating a realistic transmit power constraint accounting for mutual coupling and reconfigurable loads. Mathematically, the transmit power at the k th subcarrier is $P_{TX,k} = \mathbb{E}[\mathcal{R}\{\mathbf{i}_{TX}^*(f_k)\mathbf{v}_{TX}(f_k)\}]$. To express the transmit power in terms of the active antenna current vector, we define $\mathbf{A}_k = \mathbf{Z}_P(f_k) + \mathbf{Z}_R$ and the effective impedance as

$$\begin{aligned} \mathbf{Z}_{\text{eff}}(f_k) &= \mathcal{R}\{\mathbf{Z}_A(f_k)\} + \mathbf{Z}_m^*(f_k)\mathbf{A}_k^{-*}\mathcal{R}\{\mathbf{Z}_P(f_k)\}\mathbf{A}_k^{-1}\mathbf{Z}_m(f_k) \\ &\quad - \mathcal{R}\{\mathbf{Z}_m^T(f_k)\}(\mathbf{A}_k^{-1}\mathbf{Z}_m(f_k)) \\ &\quad - (\mathbf{Z}_m^*(f_k)\mathbf{A}_k^{-*})\mathcal{R}\{\mathbf{Z}_m(f_k)\}. \end{aligned} \tag{3.10}$$

The first term is the real part of the active-element impedance matrix $\mathcal{R}\{\mathbf{Z}_A\}$, which captures direct power dissipation in the active antennas. The second term models power coupled through the parasitic elements and their reconfigurable loads, with the mutual impedance $\mathbf{Z}_m$ between

actives and parasitics governing this coupling. The third and fourth terms capture cross-coupling between active and parasitic ports, where currents induced in the parasitics by the actives (and vice versa) contribute to the effective real power seen at the active ports. As $\mathbb{E}\{\mathbf{s}(f_k)\mathbf{s}^*(f_k)\} = \mathbf{I}$, we express $P_{\text{TX},k} = \text{Tr}(\mathbf{F}_{\text{dig},k}^* \mathbf{Z}_{\text{eff}}(f_k) \mathbf{F}_{\text{dig},k})$. Let the maximum power across all subcarriers be constrained by P_{max} . This results in $\sum_{k=1}^K P_{\text{TX},k} \leq P_{\text{max}}$. The circuit approach is required for accurately modeling the transmit power constraint for a hybrid reconfigurable parasitic array as it captures the dependence on the mutual coupling and the reconfigurable load through $\mathbf{Z}_{\text{eff}}(f_k)$. This is in contrast to a conventional array with only active antennas without coupling, the transmit power is proportional to the square of the Frobenius norm of the digital precoder [1].

We formulate the achievable rate for the MIMO-OFDM tri-hybrid system whose input-output model is expressed as (3.9). The transmitter designs the digital precoder, parasitic element loads, and the analog switch for antenna assignment based on maximization of the mutual information under the transmit power constraint across all subcarriers. Let the communication bandwidth be B . With antenna noise temperature T_A , Boltzmann constant k_B , and reference resistance R_L , thermal noise is modeled by $\sigma^2 = 4k_B T_A B R_L$ [88]. The achievable rate is expressed in terms of the maximization of the mutual information as

$\mathcal{P}_1$: Optimization over digital, analog, and parasitic loads

$$R = \max_{\mathbf{F}_{\text{dig},k}, \mathbf{Z}_R, \mathbf{r}} \frac{B}{K} \sum_{k=1}^K \log_2 \left| \mathbf{I} + \frac{\mathbf{F}_{\text{dig},k}^* \mathbf{H}_{\text{eff}}^*(f_k) \mathbf{H}_{\text{eff}}(f_k) \mathbf{F}_{\text{dig},k}}{\sigma^2} \right|, \quad (3.11a)$$

$$\text{s.t.} \quad \sum_{k=1}^K \text{Tr}(\mathbf{F}_{\text{dig},k}^* \mathbf{Z}_{\text{eff}}(f_k) \mathbf{F}_{\text{dig},k}) \leq P_{\text{max}}, \quad (3.11b)$$

$$[\mathbf{r}]_n \in \{0, 1\} \text{ and } \mathbf{1}^T \mathbf{r} = N_A, \quad (3.11c)$$

$$\mathcal{R}\{\mathbf{Z}_R\} = \mathcal{R}\{Z_R\} \mathbf{I}. \quad (3.11d)$$

The constraint (3.11b) represents the transmit power constraint across all subcarriers. The constraint (3.11c) follows from the definition of the antenna assignment vector. The constraint

(3.11d) fixes the real part of the reconfigurable load across all parasitic elements. This transforms the optimization over the reconfigurable parasitic reactance only [20]. Table 3.1 summarizes which of the variables in $\mathcal{P}_1$ are optimized for each benchmark architecture, along with the corresponding $\mathbf{H}_{\text{eff}}$ and $\mathbf{Z}_{\text{eff}}$. For the fully-active and sparse array architectures where $N_{\text{P}} = 0$, we have $\mathbf{H}_{\text{eff}} = \mathbf{Z}_{\text{RA}}$ and $\mathbf{Z}_{\text{eff}} = \mathcal{R}\{\mathbf{Z}_{\text{A}}\}$. With these definitions in place, we now turn to optimizing the digital precoder, parasitic loads, and antenna assignment.

Table 3.1. Benchmarks: optimization variables, $\mathbf{H}_{\text{eff}}$, $\mathbf{Z}_{\text{eff}}$.

Architecture	Variables	$\mathbf{H}_{\text{eff}}$	$\mathbf{Z}_{\text{eff}}$
Tri-hybrid	$\mathbf{F}_{\text{dig},k}, \mathbf{Z}_{\text{R}}, \mathbf{r}$	(3.6)	(3.10)
Fully-active	$\mathbf{F}_{\text{dig},k}$	$\mathbf{Z}_{\text{RA}}$	$\mathcal{R}\{\mathbf{Z}_{\text{A}}\}$
Hybrid, switch, fixed loads	$\mathbf{F}_{\text{dig},k}, \mathbf{r}$	(3.6)	(3.10), $\mathbf{Z}_{\text{R}}$ fixed
Hybrid, parasitic loads	$\mathbf{F}_{\text{dig},k}, \mathbf{Z}_{\text{R}}$	(3.6)	(3.10)
Sparse array	$\mathbf{F}_{\text{dig},k}$	$\mathbf{Z}_{\text{RA}}$	$\mathcal{R}\{\mathbf{Z}_{\text{A}}\}$

3.3 Tri-hybrid MIMO optimization

In this section, we solve the optimization problem in (3.11) to configure the digital precoder, parasitic loads, and the analog switch for the antenna assignment. The key challenge in configuring the tri-hybrid MIMO architecture is the mixed set of constraints (3.11b), (3.11c), and (3.11d). The variable for the reconfigurable reactance $\mathbf{Z}_{\text{R}}$ is continuous whereas each element of the assignment vector $\mathbf{r}$ has binary states. Moreover, the digital precoder is a frequency-selective precoder, whereas the parasitic loads and analog switch are frequency-flat, i.e., the same configuration applies across all subcarriers. Due to the challenging set of constraints, we propose reformulation to the original maximization problem and solve it with an alternating maximization approach. Fig. 3.5 summarizes the problem hierarchy and solution flow.

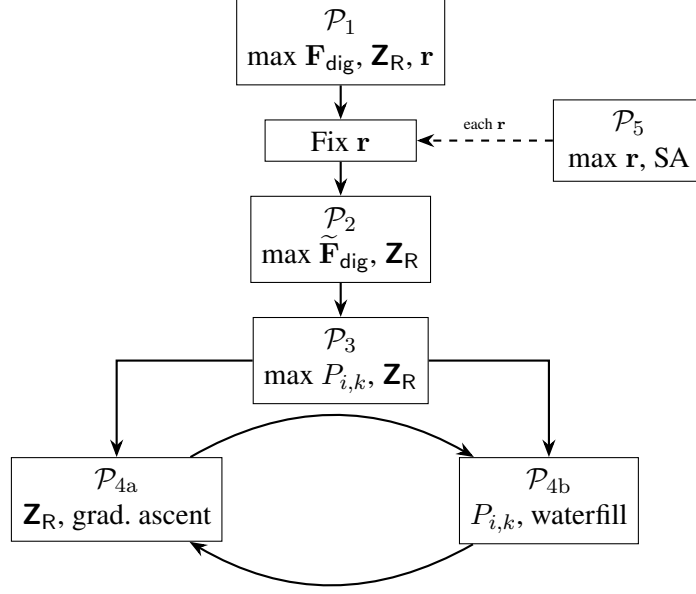


Figure 3.5. Optimization steps: $\mathcal{P}_1$ is reformulated by fixing $\mathbf{r}$ to yield $\mathcal{P}_2$. Reformulation in power terms gives $\mathcal{P}_3$, solved by alternating $\mathcal{P}_{4a}$ for parasitic loads via gradient ascent and $\mathcal{P}_{4b}$ for power via waterfilling. $\mathcal{P}_5$ optimizes $\mathbf{r}$ via simulated annealing, evaluating $\mathcal{P}_3$ for each candidate.

3.3.1 Optimization of digital precoding and parasitic loads

To formulate the joint optimization problem across the digital precoder at each subcarrier and the parasitic element loads, we assume an arbitrary assignment of active and parasitic elements. Let us first express the power constraint in the conventional form by defining $\tilde{\mathbf{F}}_{\text{dig},k} = \mathbf{Z}_{\text{eff}}^{1/2}(f_k) \mathbf{F}_{\text{dig},k}$. We also define $\tilde{\mathbf{H}}_{\text{eff}}(f_k) = \mathbf{H}_{\text{eff}}(f_k) \mathbf{Z}_{\text{eff}}^{-1/2}(f_k)$. With this definition, we provide the reformulation of the mutual information expression for a fixed assignment $\mathbf{r}$ in terms of $\tilde{\mathbf{F}}_{\text{dig},k}$ as

$\mathcal{P}_2$: Optimization over digital precoder and parasitic loads

$$\max_{\tilde{\mathbf{F}}_{\text{dig},k}, \mathbf{Z}_R} \frac{B}{K} \sum_{k=1}^K \log_2 \left| \mathbf{I} + \frac{\tilde{\mathbf{F}}_{\text{dig},k}^* \tilde{\mathbf{H}}_{\text{eff}}^*(f_k) \tilde{\mathbf{H}}_{\text{eff}}(f_k) \tilde{\mathbf{F}}_{\text{dig},k}}{\sigma^2} \right|, \quad (3.12a)$$

$$\text{s.t.} \quad \sum_{k=1}^K \|\tilde{\mathbf{F}}_{\text{dig},k}\|_{\text{F}}^2 \leq P_{\text{max}}, \quad (3.12b)$$

$$\mathcal{R}\{\mathbf{Z}_R\} = \mathcal{R}\{Z_R\} \mathbf{I}. \quad (3.12c)$$

The constraint (3.12b) is now in the standard form of a conventional MIMO system.

We solve the digital precoding problem in terms of $\tilde{\mathbf{F}}_{\text{dig},k}$. Using the singular value decomposition on $\tilde{\mathbf{H}}_{\text{eff}}(f_k)$, we express $\tilde{\mathbf{H}}_{\text{eff}}(f_k) = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^*$ and we let $\mathbf{P}_k = \text{diag}\{[P_{1,k}, \dots, P_{N_S,k}]\}$ and $\tilde{\mathbf{F}}_{\text{dig},k} = \mathbf{V}_k \begin{bmatrix} \mathbf{P}_k^{1/2} \\ \mathbf{0}_{(N_A - N_S) \times N_S} \end{bmatrix}$. Using these definitions, we express the problem $\mathcal{P}_2$ as optimization over the power terms $P_{i,k} \forall i = \{1, \dots, N_S\}, k = \{1, \dots, K\}$ and the reconfigurable load $\mathbf{Z}_R$ as

$\mathcal{P}_3$: Optimization over power terms and parasitic loads

$$\max_{P_{i,k}, \mathbf{Z}_R} \frac{B}{K} \sum_{k=1}^K \sum_{i=1}^{N_S} \log_2 \left(1 + \frac{P_{i,k} \lambda_i(\tilde{\mathbf{H}}_{\text{eff}}^*(f_k) \tilde{\mathbf{H}}_{\text{eff}}(f_k))}{\sigma^2} \right), \quad (3.13a)$$

$$\text{s.t. } \sum_{k=1}^K \sum_{i=1}^{N_S} P_{i,k} \leq P_{\max}, \text{ and } P_{i,k} \geq 0, \quad (3.13b)$$

$$\mathcal{R}\{\mathbf{Z}_R\} = \mathcal{R}\{Z_R\} \mathbf{I}. \quad (3.13c)$$

Unlike a conventional fully-digital MIMO problem with power allocation across streams and frequency, the problem is challenging because of the reconfigurable parasitic load embedded inside the term $\tilde{\mathbf{H}}_{\text{eff}}(f_k)$.

We decouple the optimization problem into two sub-problems, the first optimizing the parasitic loads for fixed power terms and the second optimizing the power terms for fixed parasitic loads. We denote the first as $\mathcal{P}_{4a}$ and the second as $\mathcal{P}_{4b}$, and they are solved in an alternating manner. With the power terms $P_{i,k}$ held fixed, we maximize the achievable rate over the parasitic loads. Let $\mathbf{R}_k = \tilde{\mathbf{H}}_{\text{eff}}^*(f_k) \tilde{\mathbf{H}}_{\text{eff}}(f_k) = \mathbf{H}_{\text{eff}}(f_k) \mathbf{Z}_{\text{eff}}^{-1}(f_k) \mathbf{H}_{\text{eff}}^*(f_k)$, and let $\lambda_i^{(k)}(\mathbf{Z}_R)$ denote the i -th eigenvalue of $\mathbf{R}_k$ sorted in descending order. The achievable rate maximized in $\mathcal{P}_{4a}$ is

$$\mathcal{R}(\mathbf{Z}_R) = \frac{B}{K} \sum_{k=1}^K \sum_{i=1}^{N_S} \log_2 \left(1 + \frac{P_{i,k} \lambda_i^{(k)}(\mathbf{Z}_R)}{\sigma^2} \right). \quad (3.14)$$

The eigenvalues and eigenvectors of $\mathbf{R}_k$ are computed when solving $\mathcal{P}_{4b}$ via waterfilling and are

reused in $\mathcal{P}_{4a}$, so no additional eigendecomposition is required. We formulate $\mathcal{P}_{4a}$ as follows.

$\mathcal{P}_{4a}$: Optimization of parasitic loads for fixed power terms

$$\mathbf{Z}_R^* = \underset{\mathbf{Z}_R}{\operatorname{argmax}} R(\mathbf{Z}_R), \quad (3.15a)$$

$$\text{s.t. } \mathcal{R}\{\mathbf{Z}_R\} = \mathcal{R}\{Z_R\}\mathbf{I}. \quad (3.15b)$$

To solve for problem $\mathcal{P}_{4a}$, we adopt the gradient ascent approach. We derive the partial derivative of $R(\mathbf{Z}_R)$ with respect to the imaginary part of the parasitic load $X_{R,i} \equiv \mathcal{I}\{\mathbf{Z}_R[i, i]\}$ in Appendix B.1. Using Hellmann–Feynman theorem [89], we express the derivative of an eigenvalue with respect to the reactance without requiring the derivative of the corresponding eigenvector. Let $\mu > 0$ denote the step size. The gradient ascent update rule for the i -th parasitic reactance at iteration $t + 1$ is

$$X_{R,i}^{(t+1)} = X_{R,i}^{(t)} + \mu \left. \frac{\partial R(\mathbf{Z}_R)}{\partial X_{R,i}} \right|_{X_{R,i} = X_{R,i}^{(t)}}, \quad (3.16)$$

where the partial derivative is evaluated at the current iterate $X_{R,i}^{(t)}$. The update is performed for all $i \in \{1, \dots, N_P\}$ in parallel, and the algorithm iterates until convergence.

$\mathcal{P}_{4b}$: Optimization of power terms for fixed parasitic loads

$$\max_{P_{i,k}} \frac{B}{K} \sum_{k=1}^K \sum_{i=1}^{N_S} \log_2 \left(1 + \frac{P_{i,k} \lambda_i (\tilde{\mathbf{H}}_{\text{eff}}^*(f_k) \tilde{\mathbf{H}}_{\text{eff}}(f_k))}{\sigma^2} \right), \quad (3.17a)$$

$$\text{s.t. } \sum_{k=1}^K \sum_{i=1}^{N_S} P_{i,k} \leq P_{\max}, \text{ and } P_{i,k} \geq 0. \quad (3.17b)$$

Problem $\mathcal{P}_{4b}$ is a standard waterfilling optimization problem. Let ν denote the water level chosen such that the power constraint $\sum_{k=1}^K \sum_{i=1}^{N_S} P_{i,k}^* = P_{\max}$ is satisfied. The optimal power

allocation is given by [1]

$$P_{i,k}^* = \max \left(0, \nu - \frac{\sigma^2}{\lambda_i(\tilde{\mathbf{H}}_{\text{eff}}^*(f_k)\tilde{\mathbf{H}}_{\text{eff}}(f_k))} \right), \quad (3.18)$$

where ν can be found by sorting the inverse eigenvalues $\frac{\sigma^2}{\lambda_i(\tilde{\mathbf{H}}_{\text{eff}}^*(f_k)\tilde{\mathbf{H}}_{\text{eff}}(f_k))}$ in ascending order and solving for ν such that the total allocated power equals P_{max} [1].

Problems $\mathcal{P}_{4a}$ and $\mathcal{P}_{4b}$ can be solved in an alternating maximization manner. Starting from an initial configuration of parasitic loads $\mathbf{Z}_R^{(0)}$, we iteratively first solve $\mathcal{P}_{4b}$ to obtain the optimal power allocation $P_{i,k}^*$ for the current parasitic loads, then update the parasitic loads by solving $\mathcal{P}_{4a}$ using gradient ascent with the current power allocation, and repeat until convergence. This alternating approach decouples the joint optimization into two tractable sub-problems.

Complexity analysis of the alternating maximization approach

We analyze the computational complexity of the alternating maximization approach for solving $\mathcal{P}_3$. In each iteration, solving $\mathcal{P}_{4b}$ via waterfilling requires: (1) computing the eigenvalues of $\tilde{\mathbf{H}}_{\text{eff}}^*(f_k)\tilde{\mathbf{H}}_{\text{eff}}(f_k)$ for all K subcarriers, which has complexity $\mathcal{O}(KN_A^3)$ for eigenvalue decomposition, and (2) sorting and solving for the water level, which has complexity $\mathcal{O}(KN_S \log(KN_S))$. For $\mathcal{P}_{4a}$, each gradient ascent step requires computing the partial derivatives derived in Appendix B.1 for all N_P parasitic elements. The dominant operations are matrix inversions and multiplications involving matrices of size $N_P \times N_P$, resulting in complexity $\mathcal{O}(N_P^3)$ per gradient evaluation. With I_G gradient ascent iterations per alternating step and I_A alternating iterations until convergence, the total complexity is $\mathcal{O}(I_A(KN_A^3 + KN_S \log(KN_S)) + I_GKN_P^3)$. For typical system parameters where $N_P \gg N_A$ and $N_P \gg N_S$, the complexity is dominated by the gradient computation in $\mathcal{P}_{4a}$, yielding an overall complexity of $\mathcal{O}(I_AI_GKN_P^3)$.

3.3.2 Optimization of parasitic assignment through simulated annealing

We now focus on optimizing the antenna assignment vector $\mathbf{r}$ for a fixed digital precoder and parasitic loads. For a given assignment vector $\mathbf{r}$, we denote $R(\mathbf{r})$ as the maximum objective value obtained after solving $\mathcal{P}_3$ through alternating maximization of $\mathcal{P}_{4a}$ and $\mathcal{P}_{4b}$. We formulate the antenna assignment optimization problem as

$\mathcal{P}_5$: Optimization of antenna assignment

$$\mathbf{r}^* = \underset{\mathbf{r}}{\operatorname{argmax}} \quad R(\mathbf{r}), \quad (3.19a)$$

$$\text{s.t. } [\mathbf{r}]_n \in \{0, 1\} \text{ and } \mathbf{1}^T \mathbf{r} = N_A, \quad (3.19b)$$

where the constraint (3.19b) ensures that exactly N_A elements are assigned as active antennas.

Problem $\mathcal{P}_5$ is a combinatorial optimization problem with a binary constraint, making it computationally challenging. We solve it using simulated annealing (SA), which is well-suited for discrete optimization [90]. Our approach extends the SA framework for antenna assignment developed in [91], which optimizes parasitic positioning for a line-of-sight cost function. Here we maximize instead the achievable rate $R(\mathbf{r})$ from $\mathcal{P}_3$ over frequency-selective channels.

The SA algorithm proceeds as follows. The algorithm initializes with a random assignment $\mathbf{r}_0$ satisfying (3.19b) and sets temperature T to an initial value T_0 . At each iteration, a candidate assignment $\mathbf{r}_{\text{cand}}$ is generated from the current $\mathbf{r}_t$ by the SWAP operation. The algorithm randomly selects one active element and one parasitic element and swaps their roles. Evaluating $R(\mathbf{r}_{\text{cand}})$ requires solving $\mathcal{P}_3$ via alternating maximization of $\mathcal{P}_{4a}$ and $\mathcal{P}_{4b}$ for that assignment. The candidate is accepted with probability

$$p_{\text{acc}} = \begin{cases} 1, & \text{if } R(\mathbf{r}_{\text{cand}}) \geq R(\mathbf{r}_t), \\ \exp((R(\mathbf{r}_{\text{cand}}) - R(\mathbf{r}_t))/T), & \text{otherwise.} \end{cases} \quad (3.20)$$

Algorithm 1. Simulated annealing for antenna assignment ($\mathcal{P}_5$)

```
1: Input: Initial temperature  $T_0$ , cooling rate  $\alpha$ , min. temperature  $T_{\min}$ , max. iterations  $I_{\max}$ ,  
   cost  $R(\cdot)$  via  $\mathcal{P}_3$   
2: Initialize: Random assignment  $\mathbf{r}_0$  with  $\mathbf{1}^T \mathbf{r}_0 = N_A$ ;  $\mathbf{r} \leftarrow \mathbf{r}_0$ ;  $R_{\text{best}} \leftarrow R(\mathbf{r}_0)$ ;  $T \leftarrow T_0$ ;  
    $t \leftarrow 0$   
3: while  $T > T_{\min}$  and  $t < I_{\max}$  do  
4:    $\mathbf{r}_{\text{cand}} \leftarrow \text{SWAP}(\mathbf{r})$  swap one active and one parasitic element  
5:   Evaluate  $R_{\text{cand}} \leftarrow R(\mathbf{r}_{\text{cand}})$  by solving  $\mathcal{P}_3$   
6:    $\Delta \leftarrow R_{\text{cand}} - R(\mathbf{r})$   
7:   if  $\Delta \geq 0$  or  $\text{rand}(0, 1) < \exp(\Delta/T)$  then  
8:      $\mathbf{r} \leftarrow \mathbf{r}_{\text{cand}}$   
9:     if  $R_{\text{cand}} > R_{\text{best}}$  then  
10:       $R_{\text{best}} \leftarrow R_{\text{cand}}$ ;  $\mathbf{r}_{\text{best}} \leftarrow \mathbf{r}_{\text{cand}}$   
11:     end if  
12:   end if  
13:    $T \leftarrow \alpha T$ ;  $t \leftarrow t + 1$   
14: end while  
15: Output: Best assignment  $\mathbf{r}_{\text{best}}$  with rate  $R_{\text{best}}$ 
```

Thus improving solutions are always accepted; worsening solutions may be accepted with probability that decreases as the rate gap increases or as temperature decreases. The temperature is reduced each iteration using an exponential schedule $T \leftarrow \alpha T$ with $\alpha \in (0, 1)$. The algorithm terminates when T falls below a minimum threshold or after a preset number of iterations. The SA procedure is summarized in Algorithm 1.

3.4 Numerical results

We evaluate the tri-hybrid architecture through Monte Carlo simulations of spectral efficiency and energy efficiency (EE). We first summarize the simulation setup and the channel model. We then study how fading statistics, transmit power limits, the active-to-parasitic ratio, and densification of sparse arrays affect spectral efficiency, energy efficiency, and their tradeoffs relative to all benchmarks in the figures.

3.4.1 Simulation setup

We use the tri-hybrid ULA architecture from Section 3.2.1 with $d = \lambda_c/2$. For fixed-assignment benchmarks, active indices $\mathcal{E}_A$ are uniformly spaced over $\{1, \dots, N_T\}$ with $\mathcal{E}_P = \mathcal{E} \setminus \mathcal{E}_A$. The transmit impedance matrix $\mathbf{Z}_{TX}(f_k)$ is generated as in [20] using the closed-form expressions for the mutual impedance and self impedance of dipole antennas. Subcarrier frequencies f_k follow the OFDM model in Section 3.2.1.

We use the four tri-hybrid ablation benchmarks from Section 3.2 (fully-active array; hybrid with analog switch and fixed loads; hybrid with parasitic loads; sparse array) and include two architectures from the conventional fully connected hybrid beamforming literature with analog phase shifters [2]. They use N_A RF chains and all N_T antennas driven through a phase-shifter (PS) network. *Hybrid PS (freq-sel)* optimizes a subcarrier-dependent analog precoder $\mathbf{F}_{RF,k}$ at each f_k . *Hybrid PS (freq-flat)* uses one analog precoder $\mathbf{F}_{RF}$ shared across subcarriers, designed from a band-averaged channel. Both references use the same wideband $\mathbf{H}(f_k)$ and $\mathbf{Z}_{TX}(f_k)$ as the other baselines, the standard digital–analog cascade [2], and waterfilling on the effective channel.

The wireless channel is modeled as a frequency-selective Rician channel. For each Monte Carlo realization, the angle of arrival ϕ_r and angle of departure ϕ_t are drawn independently and uniformly over $[-\pi/2, \pi/2]$. Let $\lambda_k = c/f_k$ denote the wavelength at subcarrier f_k , d_{tx-rx} the link distance, and K_r the Rician K -factor (the ratio of line-of-sight (LOS) to non-line-of-sight (NLOS) power). The wireless channel $\mathbf{H}(f_k) \in \mathbb{C}^{N_R \times N_T}$ at subcarrier f_k is built from the array steering vectors at the transmitter and receiver,

$$\mathbf{a}_t(\phi_t) = \left[1, e^{-j2\pi \frac{d \sin \phi_t}{\lambda_k}}, \dots, e^{-j2\pi \frac{d \sin \phi_t (N_T-1)}{\lambda_k}} \right]^T \in \mathbb{C}^{N_T \times 1}, \quad (3.21)$$

$$\mathbf{a}_r(\phi_r) = \left[1, e^{-j2\pi \frac{d \sin \phi_r}{\lambda_k}}, \dots, e^{-j2\pi \frac{d \sin \phi_r (N_R-1)}{\lambda_k}} \right]^T \in \mathbb{C}^{N_R \times 1}. \quad (3.22)$$

The non-line-of-sight component $\mathbf{H}_{NLOS}(f_k) \in \mathbb{C}^{N_R \times N_T}$ has each entry drawn independently from $\mathcal{N}_{\mathbb{C}}(0, 1)$. The channel matrix is the product of large-scale path loss and Rician small-scale

fading,

$$\mathbf{H}(f_k) = \frac{\lambda_k}{4\pi d_{\text{tx-rx}}} \left(\sqrt{\frac{1}{1+K_r}} \mathbf{H}_{\text{NLOS}}(f_k) + \sqrt{\frac{K_r}{1+K_r}} \mathbf{a}_r(\phi_r) \mathbf{a}_t^*(\phi_t) \right). \quad (3.23)$$

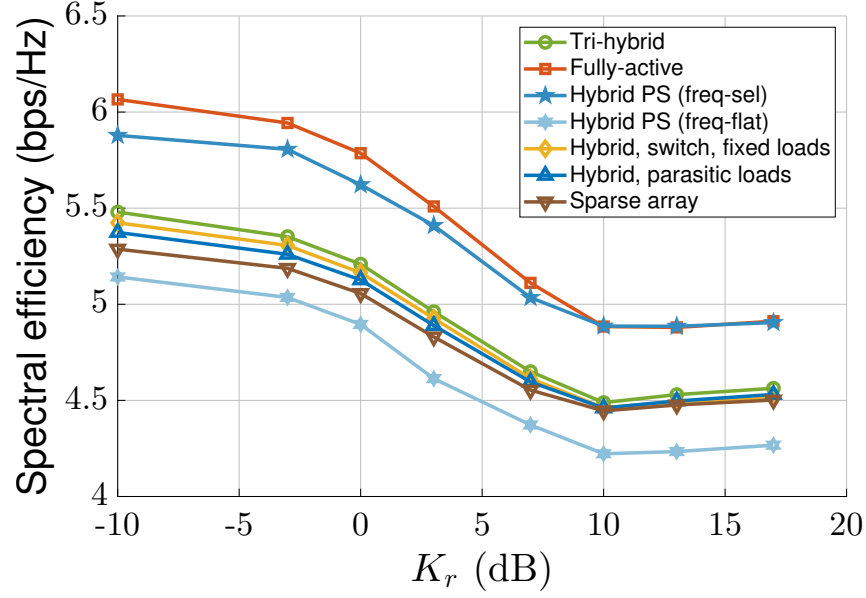
The channel is frequency-selective because λ_k varies across subcarriers. The angles ϕ_r and ϕ_t are drawn once per realization and held fixed across subcarriers. We generate 50 different realizations of the channel.

We compute $\mathbf{H}(f_k)$ from (3.23), set $\mathbf{Z}_{\text{RT}}(f_k) = R_L \mathbf{H}(f_k)$, and obtain $\mathbf{H}_{\text{eff}}(f_k)$ from (3.6) together with the parasitic-network blocks from $\mathbf{Z}_{\text{TX}}(f_k)$. Unless stated otherwise, we fix $N_R = 2$ receive antennas, center frequency $f_c = 10$ GHz, bandwidth $B = 100$ MHz, $K = 64$ OFDM subcarriers, link distance $d_{\text{tx-rx}} = 250$ m, antenna noise temperature $T_A = 300$ K, and reference resistance $R_L = 50 \Omega$. Parameters that vary across experiments (N_T , N_A , N_P , $P_{\max}$, K_r) are specified in each subsection. The achievable rate is computed using the alternating maximization of $\mathcal{P}_{4a}$ and $\mathcal{P}_{4b}$ for load and power optimization, and simulated annealing for antenna assignment optimization $\mathcal{P}_5$. In the figures below, spectral efficiency (SE) is R/B (bit/s/Hz), where R is the maximized achievable rate in (3.11) for each benchmark.

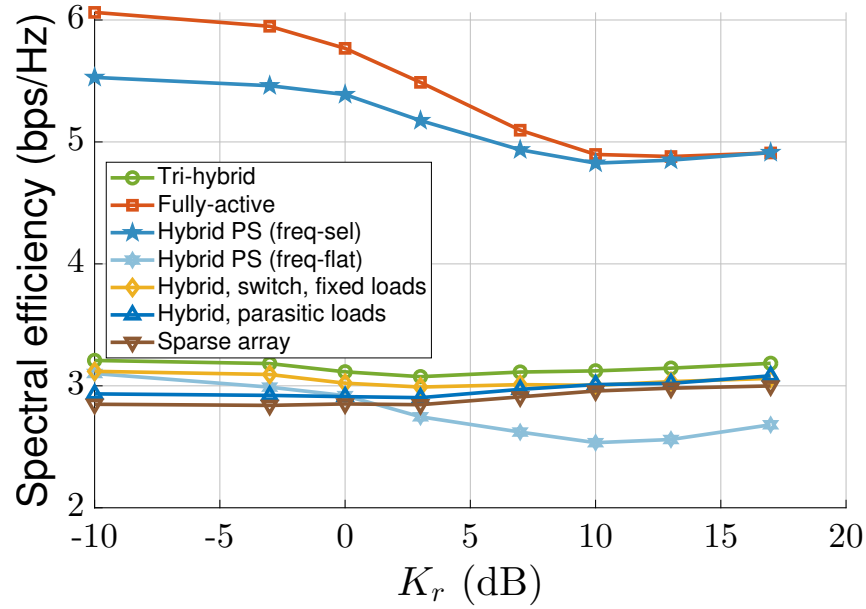
3.4.2 How does channel fading affect the tri-hybrid architecture?

We fix the maximum transmit power $P_{\max} = 1$ W and sweep the Rician factor K_r from low (Rayleigh-like, NLOS-dominated) to high (strong LOS). Two hybrid configurations are compared: (a) fewer parasitics, $N_A = 18$, $N_P = 6$, and (b) more parasitics, $N_A = 6$, $N_P = 18$. Fig. 3.6 shows spectral efficiency (SE) versus K_r for the tri-hybrid architecture and benchmarks.

At low K_r , SE is high and decreases as K_r increases. This reflects the well-known MIMO effect: rich scattering (low K_r) yields higher channel rank and thus greater multiplexing gain, whereas strong LOS (high K_r) reduces effective rank [1]. The tri-hybrid architecture achieves the highest SE among every comparison curve in Fig. 3.6, including the phase-shifter hybrid



(a) Fewer parasitics ($N_A = 18, N_P = 6$).



(b) More parasitics ($N_A = 6, N_P = 18$).

Figure 3.6. Spectral efficiency (SE) versus Rician factor K_r for two hybrid configurations.

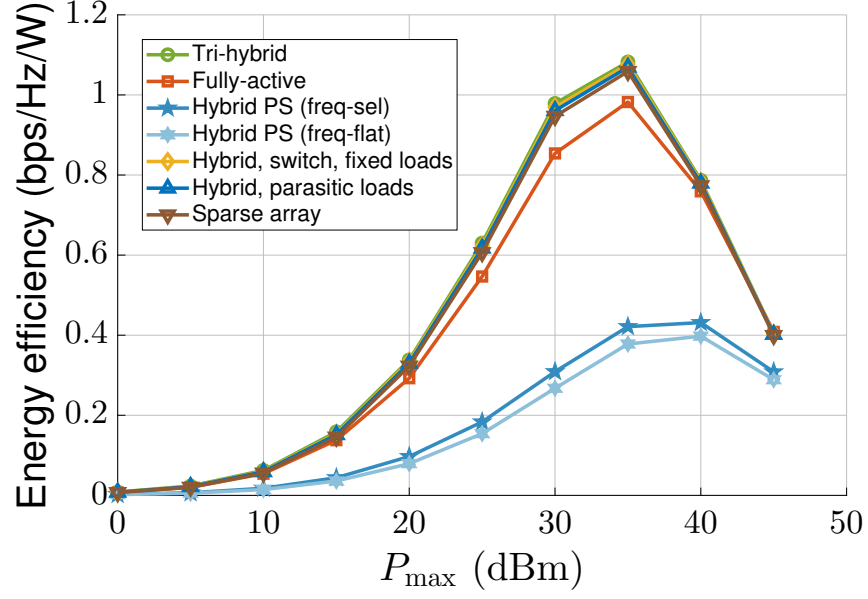
references in Section 3.4.1. The trends differ between the two configurations. In the case of fewer parasitics ($N_A = 18$, $N_P = 6$), the tri-hybrid closely tracks the fully-active array because the large number of RF chains allows it to exploit the channel rank similarly. With more parasitics ($N_A = 6$, $N_P = 18$), the tri-hybrid SE remains relatively flat across K_r , and the gap versus fully-active widens at low K_r . The reason is that with fewer active elements, the effective spatial degrees of freedom are limited by N_A . The parasitic reconfiguration improves the effective channel but cannot fully recover the multiplexing gain that additional RF chains would provide when the channel rank is high. Having characterized spectral efficiency versus fading, we next examine how transmit power shapes the tradeoff between EE and SE.

3.4.3 How does transmit power affect the tradeoff between EE and SE?

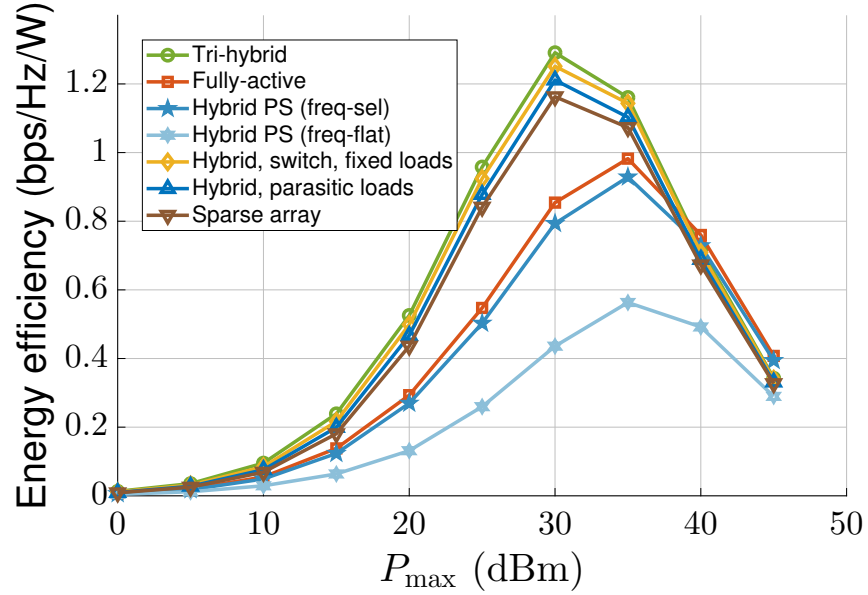
We fix the Rician factor $K_r = 1$ and sweep the maximum transmit power $P_{\max}$. Let the total power consumption be $P_{\text{total}} = P_{\max} + N_A P_{\text{RFC}}$, where P_{RFC} includes RF chain power. We compute energy efficiency $\text{EE} = \text{SE}/P_{\text{total}}$. We use $P_{\text{RFC}} = 0.24$ W per RF chain, accounting for data converters, mixers, filters, and amplifiers [20]. The power consumed by varactor diodes in the reconfigurable parasitic loads is negligible. The same two configurations as in Section 3.4.2 are used. Fig. 3.7 shows EE versus $P_{\max}$, and Fig. 3.8 shows the tradeoff between EE and SE with curves parameterized by $P_{\max}$.

Fig. 3.7 reveals an optimal $P_{\max}$ at which EE peaks. The reason is that SE increases logarithmically with $P_{\max}$, whereas P_{total} increases linearly. Thus, EE attains a maximum when the marginal SE gain from higher power is balanced by the marginal power cost. The tri-hybrid architecture achieves the highest EE among every comparison curve in Fig. 3.7, including the phase-shifter hybrid references in Section 3.4.1. The fully-active array achieves the highest SE but the lowest EE, because its total power P_{total} includes $N_T = 24$ RF chains, yielding high fixed power consumption that dominates at moderate $P_{\max}$.

Fig. 3.8 plots the tradeoff between EE and SE as $P_{\max}$ varies. Each point on a curve corresponds to a different $P_{\max}$, ranging from low SE (low power) to high SE (high power). The

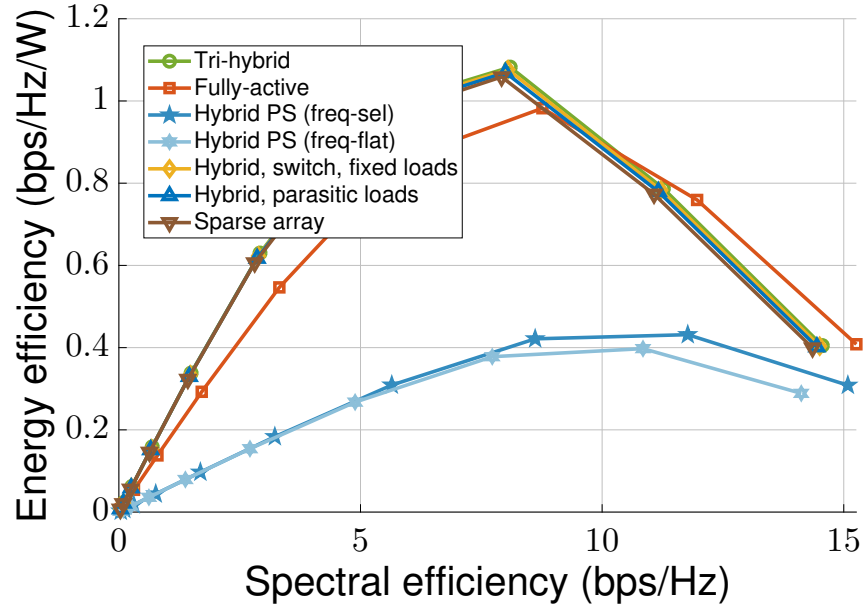


(a) EE vs $P_{\max}$ ($N_A = 18$, $N_P = 6$).

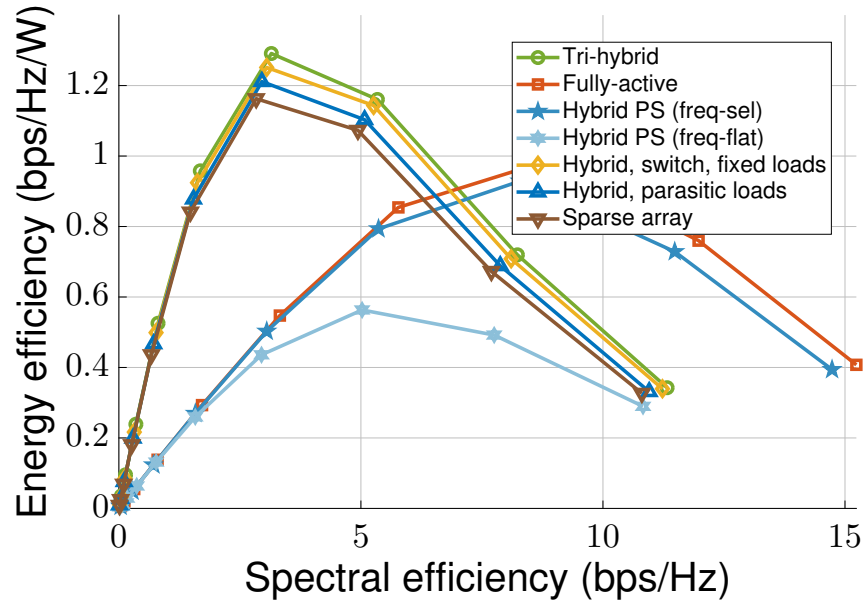


(b) EE vs $P_{\max}$ ($N_A = 6$, $N_P = 18$).

Figure 3.7. Energy efficiency (EE) versus maximum transmit power $P_{\max}$ for two hybrid configurations.



(a) EE vs SE ($N_A = 18$, $N_P = 6$).



(b) EE vs SE ($N_A = 6$, $N_P = 18$).

Figure 3.8. Energy efficiency (EE) versus spectral efficiency (SE) tradeoff for two hybrid configurations. Curves are parameterized by $P_{\max}$.

tri-hybrid curves lie above every other curve, including the phase-shifter hybrid references in Section 3.4.1, indicating that for a given SE, the tri-hybrid achieves higher EE, or equivalently, a higher SE for a given EE. This improvement reflects the benefit of replacing RF chains with reconfigurable parasitic elements while retaining reasonable communication performance. We next fix the total number of antennas N_T and the aperture, and study how the active-to-parasitic split affects EE and SE when active elements are traded for parasitic elements.

3.4.4 How should we trade active elements for parasitic elements in a fixed-size array?

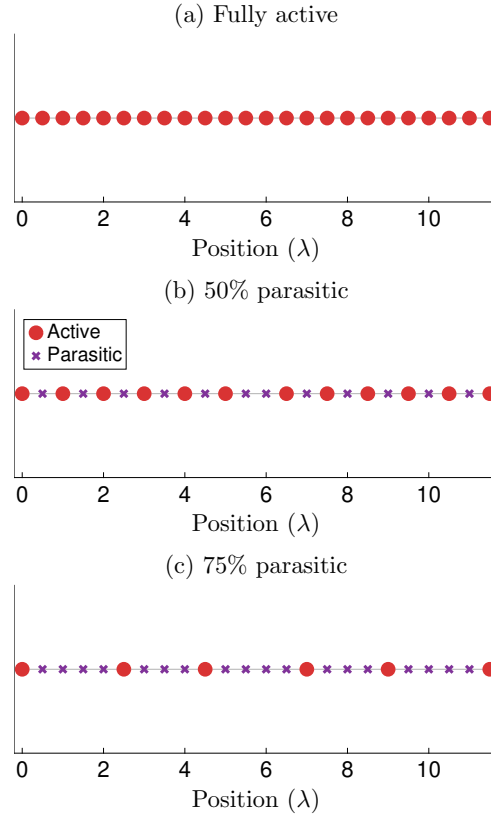


Figure 3.9. Illustration of varying the parasitic fraction at fixed $N_T = 24$. Aperture and spacing are fixed. As N_P increases, active elements are replaced by parasitics. (a) Fully active, (b) 50% parasitic, (c) 75% parasitic. Red circles indicate active elements and purple crosses indicate parasitic elements.

We sweep the parasitic fraction N_P/N_T with $N_T = 24$ fixed. As N_P increases, $N_A = N_T - N_P$ decreases, so we replace active antennas with parasitic antennas while keeping the aperture and spacing fixed. Fig. 3.9 illustrates a 24 element linear array at half-wavelength

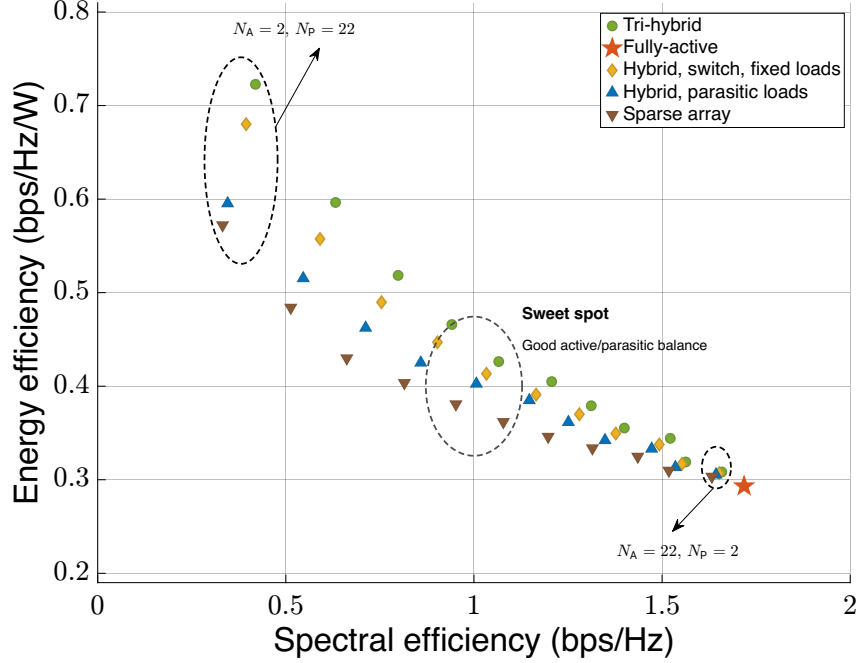


Figure 3.10. Energy efficiency (EE) versus spectral efficiency (SE) at fixed $N_T = 24$, $P_{\max} = 20$ dBm. Each benchmark is plotted as markers at the swept N_P/N_T values.

spacing with (a) fully active, (b) 50% active and 50% parasitic, and (c) 25% active and 75% parasitic. At $P_{\max} = 20$ dBm, Fig. 3.10 shows the tradeoff between EE and SE for the tri-hybrid architecture and the other benchmarks. Each benchmark appears as markers over the swept N_P/N_T values, and the fully-active array is shown as a single star point.

Fig. 3.10 reveals the tradeoff between EE and SE, as EE decreases when SE increases. Replacing active elements with parasitic elements reduces RF chain power (raising EE) but also reduces spatial degrees of freedom (lowering SE), which makes the tradeoff inherent. Moreover, one can identify different regimes. Low SE with high EE corresponds to a few active elements and many parasitic elements. High SE with low EE corresponds to many active elements and few parasitic elements. Moderate EE with moderate SE corresponds to a balanced active-parasitic ratio, i.e., a sweet spot between the two extremes. From a system design perspective, the choice of N_A and N_P can be tailored to the desired metric priority. A designer who prioritizes energy efficiency may favor a higher parasitic fraction, whereas one who prioritizes spectral efficiency may favor more active elements. A balanced design targets the intermediate regime. The

tri-hybrid markers in Fig. 3.10 lie above every other benchmark shown across the full range of N_P/N_T .

The above experiment fixes the total element count N_T and trades active elements for parasitic elements. The next subsection turns to a complementary setting where the aperture and N_A are held fixed and parasitic elements are inserted between active elements to densify a sparse array, so N_T grows with densification rather than staying fixed.

3.4.5 How many parasitic elements are needed when densifying a sparse array?

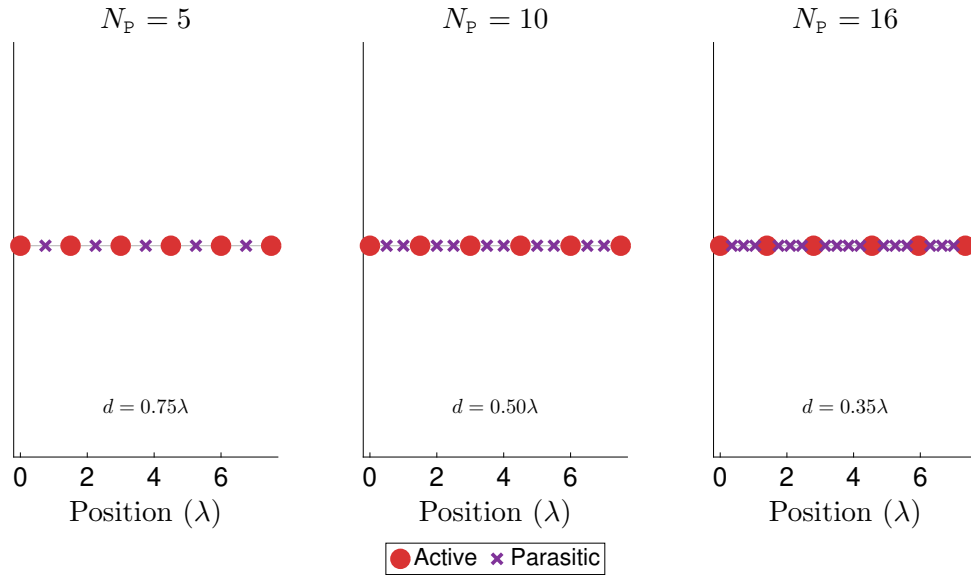


Figure 3.11. Illustration of densifying a sparse array. Baseline is $N_A = 6$ active elements at $d_{\text{init}} = 1.5\lambda_c$. Reducing spacing to d_{hybrid} inserts parasitic elements between active elements. (a) Few parasitic elements, (b) moderate, (c) many parasitic elements. Red circles indicate active elements and purple crosses indicate parasitic elements.

Sparse arrays are attractive for power-constrained systems because they use fewer active elements and thus fewer RF chains. A natural question is whether one can improve spectral or energy efficiency by densifying them with parasitic elements. Since parasitic elements are driven by reconfigurable loads rather than RF chains, adding them incurs little additional power consumption. This densification approach inserts parasitic elements in the empty space between the active elements while keeping N_A and the aperture fixed. Starting from a sparse array with

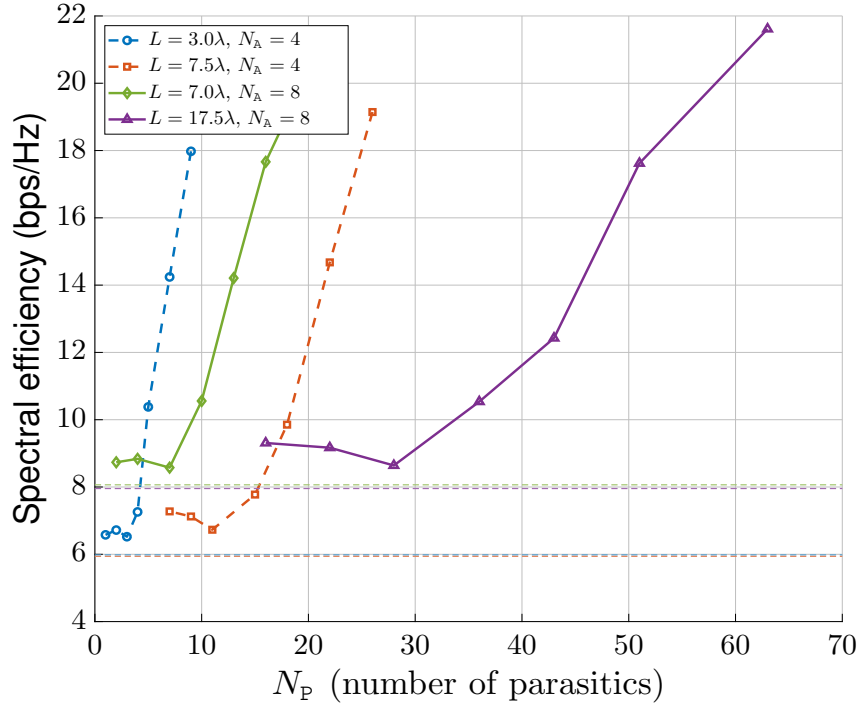


Figure 3.12. Spectral efficiency (SE) versus number of parasitics N_P when densifying a sparse array. Four designs with $d_{\text{init}} \in \{1.0\lambda_c, 2.5\lambda_c\}$ and $N_A \in \{4, 8\}$ (dashed = $N_A = 4$, solid = $N_A = 8$). Legend shows aperture length L and N_A . Dashed horizontal lines indicate the sparse baseline SE. The first few parasitics yield little or no gain. A threshold in N_P must be crossed before SE jumps.

N_A active elements at spacing d_{init} , we reduce spacing to d_{hybrid} and insert parasitic elements between the active elements as depicted in Fig. 3.11.

We choose two initial spacings $d_{\text{init}} \in \{1.0\lambda_c, 2.5\lambda_c\}$ and two active element counts $N_A \in \{4, 8\}$, giving four combinations of aperture length L and N_A . We vary the hybrid spacing d_{hybrid} to sweep N_P . We use $P_{\text{max}} = 1$ W per design. Fig. 3.12 shows SE versus N_P . The dashed lines indicate the sparse baseline SE for each design. The first few parasitic elements added to a sparse array do not improve SE. A slight degradation is seen initially. Beyond a certain N_P , a sharp jump in SE occurs. The denser spacing (smaller d_{hybrid}) yields the expected gains. Thus, densification must be sufficiently aggressive to be effective.

We use the *reconfigurability efficiency factor* (REF) metric, denoted as Υ , from our work [86] to quantify the benefit of the densification of a sparse array with parasitic elements. Let $\text{SE}_{\text{baseline}}$ denote the spectral efficiency of the sparse baseline with $N_P = 0$. Let $\text{SE}_{\text{hybrid}}$ denote the spectral efficiency of the densified tri-hybrid configuration under the same N_A and the same physical aperture. The REF is defined as the ratio of the marginal spectral-efficiency gain per parasitic element, $(\text{SE}_{\text{hybrid}} - \text{SE}_{\text{baseline}})/N_P$, to the baseline spectral efficiency per active element, $\text{SE}_{\text{baseline}}/N_A$. Mathematically, we have

$$\Upsilon = \frac{(\text{SE}_{\text{hybrid}} - \text{SE}_{\text{baseline}}) N_A}{\text{SE}_{\text{baseline}} N_P}. \quad (3.24)$$

Thus Υ is a normalized figure of merit for the SE increment due to parasitic insertion relative to the sparse reference, and it enables comparison across different array configurations.

Within each design in Fig. 3.13, Υ initially decreases with N_P and beyond a certain value of N_P , it increases. Across designs, smaller aperture at fixed N_A yields higher Υ . This is because, for a fixed number of active elements, the mutual coupling effect is stronger in smaller apertures which leads to higher benefit of introducing parasitic elements for improving spectral efficiency. Similarly, more active elements at fixed L yield higher Υ . Thus, a system designer can use the REF metric to design hybrid arrays based on the constraints on the aperture and number of RF

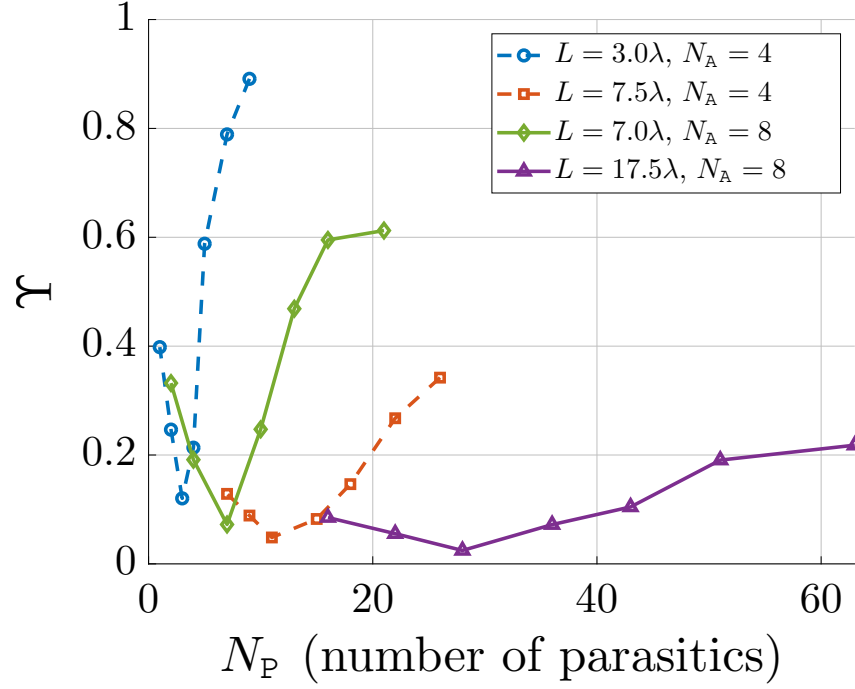


Figure 3.13. Reconfigurability efficiency factor (REF) Υ in (3.24) versus number of parasitic elements N_P . Same four designs as Fig. 3.12. Along each densification trajectory, Υ initially decreases and then increases as more parasitic elements are added. For fixed N_A , smaller aperture yields higher Υ . For fixed aperture, more active elements yield higher Υ .

chains.

Fig. 3.14 shows the tradeoff between EE and SE for the same four designs. As N_P increases, both EE and SE rise along each curve. Stars mark the sparse baseline ($N_P = 0$). This contrasts with Fig. 3.10, where N_T is fixed and increasing N_P replaces active elements with parasitic elements.

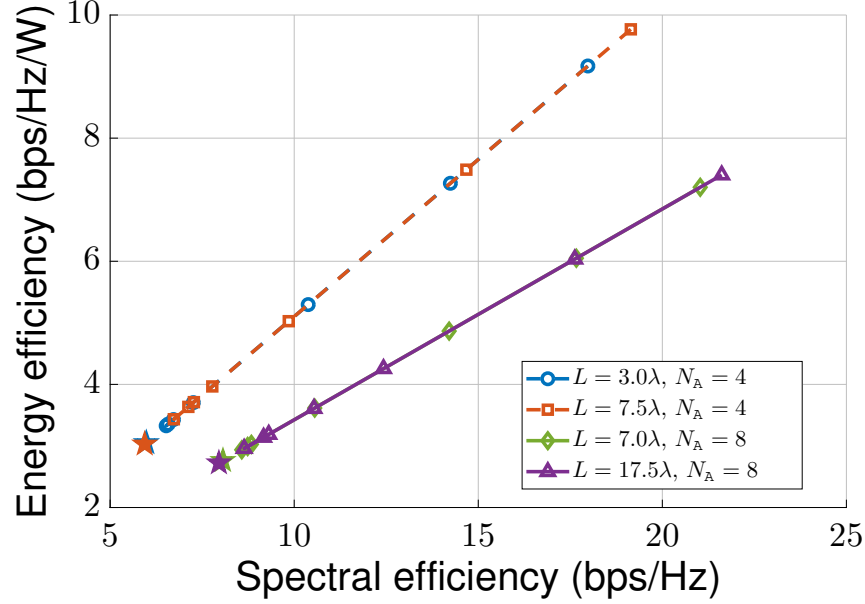


Figure 3.14. Energy efficiency (EE) versus spectral efficiency (SE) when densifying a sparse array. Same four designs as Fig. 3.12 with N_A fixed and N_T increasing as parasitic elements are added. Curves trace (SE, EE) as N_P increases.

3.5 Conclusions and future work

We propose the tri-hybrid MIMO architecture with parasitic reconfigurable antennas and an analog switch to dynamically reconfigure the assignment across active and parasitic elements. The spectral efficiency and energy efficiency performance for the proposed tri-hybrid architecture is higher than that of the hybrid benchmarks. Specifically, from the ablation study, we learn that the dynamic assignment feature provides higher performance improvement compared to the dynamic parasitic reactance reconfiguration feature. This is because active antenna selection plays a crucial role in beam shaping and the parasitic antennas further enhance the performance by leveraging the mutual coupling effect. Our experiments highlight that the introduction of parasitic elements in an active linear sparse array helps boost spectral-efficiency performance when several parasitic elements are tightly packed between the active antennas.

The current approach is limited to a single user MIMO system. An immediate next step is to extend to a multi-user MIMO system to understand the impact of interference. Future research should study other practical implementations for the tri-hybrid MIMO parasitic architecture to

identify a sweet spot between spectral efficiency and energy efficiency. For example, lowering the hardware complexity by keeping few parasitics connected to fixed loads and dynamically reconfiguring the other parasitics. While prior work has shown the relevance of matching networks in dense arrays [62], it is important to investigate its role in a tri-hybrid MIMO architecture too.

Acknowledgements

Chapter 3, in full, is a reprint of the material as it appears in **N. V. Deshpande**, M. Akrou, and R. W. Heath Jr., “The Tri-hybrid MIMO Architecture with Parasitic Antennas and Dynamic Switching,” in preparation for submission to *IEEE Trans. Wireless Commun.* The dissertation author was the primary investigator and author of this paper.

Chapter 4

Frequency-selective beamforming and single-shot beam training with dynamic metasurface antennas

In this chapter¹, we study frequency-selective beamforming and single-shot beam training with dynamic metasurface antennas. We develop a frequency-dependent beamforming model, derive design insights linking beam direction and operating frequency, and propose a single-shot training procedure for DMA arrays.

4.1 Introduction

A dynamic metasurface antenna (DMA) consists of a waveguide with multiple radiating slots that can be independently tuned to resonate at different frequencies [19, 26]. Unlike conventional antenna arrays that require additional components for beamforming, DMAs have the innate capability to beamform through low-powered varactor or PIN diodes embedded into the antennas [26]. Analog components for frequency-selective beamforming such as true time delay (TTD) elements [93] and active phase shifters [94] consume more power than DMAs [95]. The low cost and low power consumption of DMAs make them versatile for next-generation multiple-input-multiple-output (MIMO) systems. DMAs have potential applications like large array implementations in gNodeBs or use in battery-limited user equipment handsets. Despite

¹This chapter is based on the journal article [66] and the conference paper [92].

these benefits, the existing approaches to configure DMAs in a narrowband setting lead to a lower LOS beamforming gain compared to TTDs or phased-arrays. One method to improve the performance of the DMA is to take advantage of their ability to be reconfigured over a wide frequency range [96, 97]. We leverage this unique feature to propose frequency-selective beamforming with DMAs.

4.1.1 Prior work on beamforming with DMAs

Prior work on DMA beamforming focuses on the narrowband operating regime, where frequency-selective DMA effects are ignored [65, 72, 78, 98–102]. An approach to configure DMA for a single user system was proposed in [26] [65]. Other system models in prior work limited to narrowband operation include: multi-user downlink system [78], uplink system [99], [100], hybrid beamforming with the DMA [101], [72], and near-field setting [102]. The DMA beamforming approaches from [65, 72, 78, 98–102] treat the DMA beamforming weights as frequency-flat, i.e., the phase and magnitude response of the beamforming weight as a function of frequency is not explicitly modeled. Based on this frequency-flat model, the DMA configuration optimized for a fixed operating frequency may produce an unintended grating lobe in the DMA beam pattern. This effect is undesirable as it not only reduces the peak gain of the main lobe but also leads to interference in an unintended direction [103]. Further, the maximum beamforming gain may vary across the desired angular range, which is not desirable for beamforming [98]. As DMAs are frequency-reconfigurable over a large bandwidth, the methods for configuring them should be revisited using a frequency-selective model to yield higher beamforming gain, instead of a frequency-flat model. While frequency-selective beamforming has been studied for TTD arrays [104, 105] and leaky waveguide antennas [106, 107], it is yet to be explored with DMAs.

There is limited prior work on wideband systems exploiting the frequency-selective property of DMAs. Related work in [108] considers wideband DMA beamforming under realistic constraints. In [109], the system model is based on orthogonal frequency division multiplexing (OFDM), which incorporates a frequency-selective DMA model in the optimization problem.

The DMA operating band in [109], however, is fixed to a narrow fraction of the DMA's resonant frequency tuning range. DMAs can be reconfigured in a wider frequency range. For example, the DMA proof-of-concept in [19] demonstrates beam pattern measurement results from 8.5 GHz to 10.7 GHz, and those in [110] from 17.5 GHz to 20.3 GHz. To summarize, the prior work on configuring DMAs is based on optimization at a single operating frequency as opposed to a range of feasible frequencies [65, 72, 78, 98–102, 109]. Unlike prior work, we propose a new paradigm of dynamically reconfiguring the operating frequency over the entire feasible DMA tuning range, which enhances the beamforming gain.

DMAs are one of the many uses of metasurfaces in wireless communications. Reflecting intelligent surfaces (RIS), a type of reflect array, have received a significant amount of attention. In the case of RIS, the unit cell properties of a surface are controlled in a similar way as the slots on a DMA. Recently, there has also been interest in studying wideband aspects of RIS [111] [112]. The applications of RIS, though, not to mention the related mathematical optimizations, are fundamentally different. Other examples of reconfigurable antennas based on the leaky-wave concept include reconfigurable holographic surfaces (RHS) [113] and pinching antenna systems (PASS) [114]. The operation mechanism and design challenges for RHS and PASS, however, are different because they are envisioned to operate in near-field channels. In this chapter, we focus on DMAs which are potential solutions for antennas in base stations and handsets designed to operate in the far-field.

4.1.2 Approach and summary of contributions

In this chapter, we leverage the dynamic frequency-reconfiguration ability of the DMA to mitigate some limitations like the unintended grating lobe and non-uniformity in main lobe peak gain that arise when the DMA configuration is optimized for a fixed operating frequency. We envision a wireless system with DMAs that has the ability to be reconfigured to different operating frequencies. This paradigm also borrows motivation from the dynamic frequency hopping technique from wireless standards like frequency-hopping code-division-multiple-access

(FH-CDMA), dynamic spectrum sharing in 5G New Radio [115], and adaptive frequency hopping in Bluetooth [116]. Our proposed method for DMA beamforming enables the system to dynamically choose the operating frequency within a feasible band. In the first contribution, we propose an analytical solution to maximize LOS beamforming gain using a single DMA with the dynamic frequency-reconfigurability. In the second contribution, we develop a system architecture with multiple DMAs that leverages the analytical solution to enable efficient beam training.

In our first contribution, we propose a joint optimization of LOS beamforming gain over the DMA's operating frequency and per-slot resonant frequencies. We first express the optimal resonant frequencies as a closed-form parametric function of the operating frequency and the beamforming angle. Our novel contribution is the second stage of optimizing the operating frequency in a feasible tuning range of the DMA. We show that the dominant beam direction and the operating frequency of the DMA are intertwined through the DMA's frequency response. Our proposed approach for configuring the DMA is suitable for systems with available spectrum similar to the DMA tuning range, allowing for the selection of operating frequencies for communication. It also eliminates the grating lobe thus improving the peak beamforming gain, and hence achievable rate compared to prior work [26, 65, 78, 98–100].

While the first contribution analytically examines the DMA's beamforming performance, it assumes that the receiver angle is known to the transmitter. The beam training process is essential, prior to data transmission, for finding the beamforming weights that maximize the gain in the receiver's direction, which is unknown to the transmitter. For larger arrays, beam training by sequential search is inefficient. Our second contribution is an efficient beam training procedure for DMAs. Taking inspiration from the rainbow beams approach used in TTD arrays [104, 105], we demonstrate single-shot beam training with DMAs, i.e., probing different angular directions using different frequencies. The frequency reconfigurability of DMAs enables single-shot beam training. This further motivates the use of DMAs for larger arrays as beam training overhead is minimized by this approach.

We summarize the key novelties of this chapter with respect to prior work. While many studies have investigated DMAs using multi-user system models, they are based on the frequency-flat model assumption. We take a different approach by dynamically tuning the operating frequency of DMAs over a wider frequency range. To explore this new paradigm, we analyze the DMA performance using a single-user system model. We formulate the frequency-selective DMA beamforming gain optimization for a LOS channel. We derive a closed-form expression that relates the operating frequency to the receiver's angular direction. This closed-form expression reveals key design tradeoffs and reduces the algorithmic complexity of optimizing DMA weights, thus ensuring scalability to massive arrays. Part of this first contribution appears in the conference version [92].

In this chapter, we expand on those insights to propose a single-shot beam training procedure for a DMA array that enables the system to probe multiple angles simultaneously with a single OFDM symbol. We compare the beamforming gain for (i) perfectly known receiver's angular direction, and (ii) angular direction estimated via the proposed training. We show that the optimal frequency approach outperforms the benchmarks based on the fixed frequency approach for all tuning ranges.

Organization: We assume a communication system where the transmitter is equipped with a DMA and the receiver has a single isotropic antenna. We discuss the single DMA transmitter model in Section 4.2.1, formulate the propagation channel model and the received signal model in Section 4.2.2, and define the signal-to-noise ratio (SNR) and achievable rate in Section 4.2.3. In Section 4.3, we analyze the beamforming gain for the single DMA transmitter. In Section 4.4, we generalize the results of Section 4.3 to multiple DMAs arranged as a planar array, propose the single-shot beam training procedure, and present a beam training codebook design. In Section 4.5, we evaluate the DMA achievable rate performance and compare with existing approaches. The MATLAB code to generate the results is publicly available to facilitate reproducibility [61].

4.2 System model

In Section 4.2.1, we review the antenna model of a single DMA at the transmitter from [26] and state the key assumptions. Based on the antenna model derived using electromagnetic theory concepts, the beamforming gain expression is formulated in a form similar to the conventional TTD beamforming in Section 4.2.2. The achievable rate of the proposed communication system with dynamically reconfigurable operating frequency is mathematically formulated in Section 4.2.3.

4.2.1 Model of a single DMA transmitter

We review the beamforming mechanism of a DMA from [19]. A DMA has reconfigurable radiating slots on the upper surface of a conducting waveguide. Let the DMA be in the yz plane as illustrated in Fig. 4.1. The transmit signal enters the DMA from the waveguide input and propagates along its length. We assume the DMA has N equally spaced radiating slots with inter-element spacing d_y along the y axis. DMAs function as leaky-wave antennas with reconfigurable components like varactor or PIN diodes attached to the radiating slots to tune resonant frequencies. As the signal travels along the waveguide, each slot experiences a different magnitude and phase of the input signal. The input signal radiates out of each slot, and the phase and magnitude of the transmit signal can be tuned by adjusting the resonant frequency. Through this mechanism, DMAs have the capability to adjust the transmit signal without external phase shifters or delay elements. In practice, the resonant frequency is tuned digitally by adjusting the varactor diode bias voltage; this alters the intrinsic impedance, resulting in the desired frequency shift [19].

The collection of all slots on a DMA can be treated as an array which enables a mathematical formulation of the beamforming problem similar to a conventional antenna array. Using concepts from electromagnetic theory, the resonant frequency of each slot on the DMA is related to the beamforming weight through the Lorentzian model in [26]. Each slot on the DMA

is modeled as a reconfigurable magnetic dipole whose response is governed by the magnetic polarizability. We use the passband frequency f for frequency-domain representation. Let $\alpha_{M,n}(f)$ be the frequency response of the magnetic polarizability of the dipole corresponding to the n th slot on the DMA [64]. Let F be the coupling factor and Γ be the damping factor in the Lorentzian model. Let $f_{r,n}$ be the resonant frequency for the n th dipole that is dynamically reconfigurable. The magnetic polarizability expression is [26]

$$\alpha_{M,n}(f) = \frac{F(2\pi f)^2}{(2\pi f_{r,n})^2 - (2\pi f)^2 + j\Gamma 2\pi f}. \quad (4.1)$$

We assume that F and Γ are fixed and dependent on the DMA unit cell design.

To understand the coupled magnitude and phase constraint for the beamforming weight, an alternate expression for the magnetic polarizability is provided in terms of the complex argument of the polarizability denoted as $\psi_n(f)$ and defined as

$$\psi_n(f) = \text{atan2}(-\Gamma 2\pi f, (2\pi f_{r,n})^2 - (2\pi f)^2). \quad (4.2)$$

Using (4.2), we express the magnetic polarizability from (4.1) as

$$\alpha_{M,n}(f) = -F \frac{2\pi f}{\Gamma} \sin(\psi_n(f)) e^{j\psi_n(f)}. \quad (4.3)$$

We see that the reconfigurable resonant frequency $f_{r,n}$ modifies both the phase and magnitude of the magnetic polarizability. From (4.3), we observe that the phase and magnitude of the magnetic polarizability are coupled and cannot be tuned independently.

The signal propagation within the waveguide is modeled as a relative phase difference between the excitation of each polarizable dipole. For a simplified analysis, we assume that there is no attenuation of the field inside the waveguide so that the excitation amplitude is uniform across all DMA elements [26]. Including the attenuation in the model leads to a loss in the DMA beamforming gain which we quantify in Section 4.3.5 and also include in the achievable rate

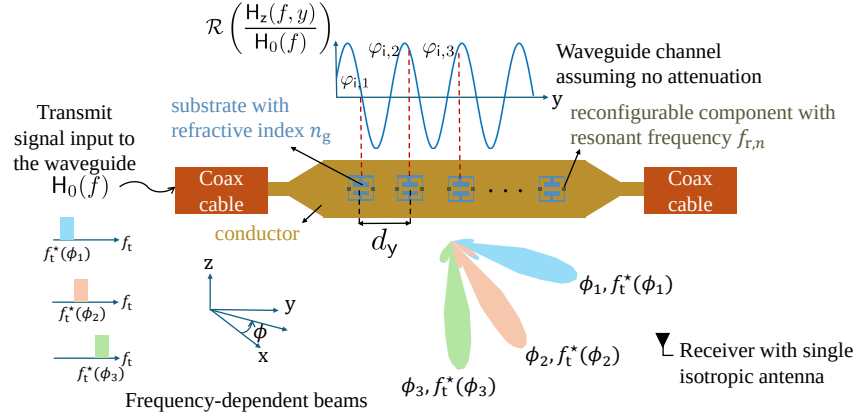


Figure 4.1. A DMA has radiating slots which are tuned using low-power reconfigurable components with signal incident from one end of the waveguide. We use the frequency reconfigurability property of DMA for frequency-selective beamforming.

simulation results in Section 4.5. Without loss of generality, let the phase-shift at the first element placed at the origin be 0. The relative phase of the excitation at the n th dipole is $e^{j\varphi_{i,n}(f)}$, which we term as the intrinsic phase shift and denote by the subscript i . We denote the refractive index by n_g , where subscript g is used to indicate the waveguide. We express the intrinsic phase-shift as

$$\varphi_{i,n}(f) = -n_g 2\pi \frac{f}{c} (n-1) d_y, \quad (4.4)$$

which is dependent on the waveguide refractive index and inter-element spacing of the DMA radiating slots.

The data signal is contained in the EM field entering the waveguide. Let $H_0(f)$ be the magnetic field at the waveguide input. The induced magnetic dipole moment $m_n(f)$ for the n th radiating slot is

$$m_n(f) = H_0(f) e^{j\varphi_{i,n}(f)} \alpha_{M,n}(f), \quad (4.5)$$

which can be configured dynamically via the magnetic polarizability. The superposition of dipole moments of each slot impacts the electric field and the received signal power which we discuss next.

4.2.2 Received signal model

Now, we describe the wireless propagation channel between the DMA transmitter and the single antenna receiver. The phase-shift due to wireless propagation at the n th slot on the transmit DMA is denoted as $e^{j\varphi_{e,n}(\phi, f)}$. Assuming LOS and far-field propagation where the receiver is at an azimuth angle ϕ from the broadside direction (x axis in Fig. 4.1), the extrinsic phase-shift angle at the n th element is $\varphi_{e,n}(\phi, f) = -2\pi \frac{f}{c} (n - 1) d_y \sin(\phi)$. Let the far-field array response vector be $\mathbf{a}(\phi, f) = [e^{j\varphi_{e,1}(\phi, f)}, \dots, e^{j\varphi_{e,N}(\phi, f)}]^T$. Note that the extrinsic phase-shift angle $\varphi_{e,n}(\phi, f)$ depends on the transmit angle of departure (AoD) ϕ which can vary unlike the intrinsic phase-shift angle $\varphi_{i,n}(f)$ that depends only on the fixed DMA waveguide design.

The overall phase difference at each slot is due to contribution from the intrinsic waveguide propagation and the extrinsic wireless propagation. We define the waveguide phase-shift vector as $\mathbf{h}_{\text{dma}}(f) = [e^{j\varphi_{i,1}(f)}, \dots, e^{j\varphi_{i,N}(f)}]^T$. The effective propagation channel is $\mathbf{h}(\phi, f) = \mathbf{h}_{\text{dma}}(f) \odot \mathbf{a}(\phi, f)$. The phase of the n th element in $\mathbf{h}(\phi, f)$ is

$$\varphi_n(\phi, f) = -2\pi \frac{f}{c} (n - 1) d_y (n_g + \sin(\phi)). \quad (4.6)$$

For a conventional phased-array, $\varphi_{i,n}(f) = 0$. Hence, $\varphi_n(\phi, f) = \varphi_{e,n}(\phi, f)$. For a matched filter strategy, this implies applying a phase-shift $\varphi_{r,n}(\phi, f) = -\varphi_{e,n}(\phi, f)$ to each antenna to beamform toward ϕ . A key difference between intrinsic and extrinsic phase shift is that the intrinsic phase shift is always non zero for a DMA because the refractive index is a positive number whereas extrinsic phase shift becomes zero when $\phi = 0^\circ$. For a DMA, the transmit beamforming should account for the additional term $\varphi_{i,n}(f)$ in the overall phase even for broadside incidence.

We define the received power in terms of the transmit power and beamforming gain. We assume that mutual coupling is negligible between each radiating slot similar to [26]. Note that the mutual coupling can also be exploited to enhance gain through alternative DMA design [117].

Based on this assumption, the electric field far-field component at a distance r is [26]

$$\mathbf{E}_\phi(f) = \eta_0 \left(\frac{2\pi f}{c} \right)^2 \sum_{n=1}^N \mathbf{m}_n(f) \frac{e^{j\varphi_{e,n}(\phi,f)}}{4\pi r}. \quad (4.7)$$

The receive electric field expression depends on the induced magnetic dipole moment which is a function of the magnetic polarizability at each slot. This shows how the polarizability terms beamform the signal from each slot.

To make the analysis consistent with traditional beamforming, we define the DMA beamforming vector as

$$\mathbf{f}_{\text{DMA}}(f) = [-\sin(\psi_1(f)) e^{j\psi_1(f)}, \dots, -\sin(\psi_N(f)) e^{j\psi_N(f)}]^T. \quad (4.8)$$

Using (4.3)-(4.8), we express the electric field in terms of the inner product of the effective propagation channel vector and the DMA beamforming vector as

$$\mathbf{E}_\phi(f) = \frac{\eta_0}{4\pi r} \left(\frac{2\pi f}{c} \right)^2 \mathbf{H}_0(f) \mathbf{F} \frac{2\pi f}{\Gamma} \mathbf{f}_{\text{DMA}}^T(f) \mathbf{h}(\phi, f). \quad (4.9)$$

We assume that the data signal contained in $\mathbf{H}_0(f)$ is a Gaussian wide-sense stationary random process. Let the wavelength be $\lambda = \frac{c}{f}$. The received power is $P_R(\phi, f) = \frac{\lambda^2}{4\pi} \frac{\mathbb{E}[|\mathbf{E}_\phi(f)|^2]}{2\eta_0}$ and the transmit power is $P_T(f) = \frac{\eta_0}{8\pi} \left(\frac{8\pi^3 f^3}{c^2} \frac{\mathbf{F}}{\Gamma} \right)^2 \mathbb{E}[|\mathbf{H}_0(f)|^2]$. Using these definitions, we have [14]

$$P_R(\phi, f) = \left(\frac{\lambda}{4\pi r} \right)^2 P_T(f) |\mathbf{f}_{\text{DMA}}^T(f) \mathbf{h}(\phi, f)|^2. \quad (4.10)$$

In our analysis, we fix the transmit power and optimize the frequency-selective beamforming gain term defined as

$$\mathcal{G}_{\text{DMA}}(\phi, f) = |\mathbf{f}_{\text{DMA}}^T(f) \mathbf{h}(\phi, f)|^2. \quad (4.11)$$

Table 4.1. Parameters used in DMA beamforming optimization

Parameter	Notation	Description
Resonant frequency	$f_{r,n}$	Dynamically reconfigurable (4.15)
Operating frequency	f_t	Dynamically reconfigurable (4.18)
Refractive index	n_g	Designed to maximize angular coverage (4.22)
Inter-antenna spacing	d_y	Designed to maximize angular coverage (4.23)
Beamforming vector	$\mathbf{f}_{\text{DMA}}(f_t)$	Dependent on $f_{r,n}$ and f_t (4.8)
Effective channel	$\mathbf{h}(\phi, f_t)$	Dependent on f_t , n_g , and d_y (4.6)
Beamforming gain	$\mathcal{G}_{\text{DMA}}(\phi, f_t)$	Dependent on $f_{r,n}$, f_t , n_g , and d_y (4.11)

The DMA configuration affects the received signal power through the frequency-selective beamforming gain.

Finally, we are interested in comparing the DMA-based transmitter with a TTD architecture [93]. We provide a review of the TTD beamforming gain definition and the optimization solution in Appendix D.1. Prior work has compared the beamforming gain of the TTD and phased-array but comparison with DMAs is missing [118]. DMA beamforming is energy-efficient but is complicated by the coupled phase and magnitude constraint. In Section 4.3, we discuss how to design the DMA beamformer despite these limitations.

4.2.3 Signal-to-noise ratio and achievable rate

We describe the communication system equipped with the DMA at the transmitter with a dynamically reconfigurable operating frequency. We measure the system performance in terms of the achievable rate. Let k_B be the Boltzmann constant in J/K and T be the noise temperature of the antenna in K. Let the communication bandwidth be fixed as B . The noise power at the receiver is $\sigma^2 = k_B T B$. The received SNR for a line-of-sight receiver at angle ϕ operating at frequency f is defined as $\text{SNR}(\phi, f) = \frac{P_R(\phi, f)}{\sigma^2}$. The received SNR varies with frequency.

We model a communication system where the center frequency of the data transmission band can be dynamically reconfigured as the DMA has the ability to resonate at different frequencies. Let the operating frequency allocated to the user by the transmitter be f_t . We assume that the operating frequency can be dynamically tuned in the range $[f_{t,\min}, f_{t,\max}]$. This

tuning range is generally larger compared to the communication bandwidth B . We assume the DMA transmitter transfers data in the band $[f_t - \frac{B}{2}, f_t + \frac{B}{2}]$. Assuming an OFDM system with K_d subcarriers uniformly spaced over $[f_t - \frac{B}{2}, f_t + \frac{B}{2}]$, the achievable rate for a receiver with AoD as ϕ and operating frequency f_t is [1]

$$R(\phi, f_t) = \frac{B}{K_d} \sum_{k=1}^{K_d} \log_2 (1 + \text{SNR}(\phi, f_k)) \text{ [bits/s]}. \quad (4.12)$$

Here, we have defined the achievable rate between a single transmitter and receiver for bandwidth B centered around f_t . In this chapter, we focus on configuring the resonant frequencies $f_{r,n}$ and operating frequency f_t of a single transmit DMA to maximize the frequency-selective beamforming gain. Optimizing the beamforming gain will result in higher received SNR, which leads to higher achievable rate. We summarize the key DMA parameters involved in the beamforming optimization in Table 4.1.

4.3 Frequency-selective beamforming gain optimization and analysis

The main focus of this section is to find the DMA configuration that achieves the highest beamforming gain over a line-of-sight channel. The AoD is assumed to be perfectly known in this section; we propose a method for AoD estimation in Section 4.4. We propose a two-stage optimization approach to maximize the gain at a specific AoD. In Section 4.3.1, we derive a closed-form expression for the DMA resonant frequencies to maximize the beamforming gain at a particular operating frequency and the desired AoD. We use this closed-form expression and further optimize the gain over the operating frequency in Section 4.3.2. The prior work on DMA beamforming is limited only to the resonant frequency optimization [65, 78], which we show is sub-optimal compared to our proposed joint optimization over resonant frequencies and operating frequency. The closed-form solution in terms of resonant frequency helps unravel the interdependence of the refractive index, AoD, inter-antenna spacing, and frequency range which

we use in Section 4.3.3 for improving the DMA angular coverage.

4.3.1 Beamforming gain optimization over resonant frequencies

We formulate the problem of maximizing the beamforming gain for a particular angle ϕ_{AoD} and operating frequency f_t in terms of the reconfigurable resonant frequency of each DMA element. We assume that the resonant frequencies can attain any positive real value $\{f_{r,n}\}_{n=1}^N$. We mathematically formulate the problem as follows:

$$\mathbf{P1} : \max_{\{f_{r,n}\}_{n=1}^N} \mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f_t), \quad (4.13a)$$

$$\text{s.t. } \{f_{r,n}\}_{n=1}^N > 0. \quad (4.13b)$$

The optimization variable $f_{r,n}$ in **P1** appears in both the phase and magnitude of the DMA beamforming weight as shown through (4.2) and (4.8), which makes it challenging to solve the problem directly in terms of $f_{r,n}$. To address this challenge, we use a shift-of-origin transformation, similar to [65], for the DMA weight as $[\mathbf{f}_{\text{DMA}}(f)]_n = \frac{-j + e^{j\tilde{\psi}_n(f)}}{2}$. The relation between $\psi_n(f)$ and $\tilde{\psi}_n(f)$ is

$$\psi_n(f) = \frac{\tilde{\psi}_n(f)}{2} - \frac{\pi}{4}, \text{ where } \tilde{\psi}_n(f) \in \left[\frac{-3\pi}{2}, \frac{\pi}{2} \right]. \quad (4.14)$$

The shift-of-origin transformation simplifies the beamforming gain optimization in **P1** by reformulating the problem in terms of $\tilde{\psi}_n(f)$ because it has an entire 2π range. We then compute the optimal $f_{r,n}$ using (4.14) and (4.2). The reformulated version of this problem has been solved in prior work [65] [78]. We review the reformulated problem **P1a** and its optimal solution in Appendix D.2.

Let the optimal solution to problem **P1** be $\{f_{r,n}^*\}_{n=1}^N$. We express it in closed-form, using

the solution to **P1a** from Appendix D.2, and substituting it in (4.14) and (4.2) to obtain

$$f_{r,n}^* = \frac{1}{2\pi} \left((2\pi f_t)^2 + \Gamma 2\pi f_t \tan \left(\frac{\pi}{4} (1 - \text{sgn}(\mathcal{S}(\phi_{\text{AoD}}, f_t))) \right) + \frac{\pi f_t d_y (n_g + \sin(\phi_{\text{AoD}}))}{c} (n - 1 - (N - 1)/2) \right)^{\frac{1}{2}}, \quad (4.15)$$

where $\mathcal{S}(\phi_{\text{AoD}}, f_t)$ is a Dirichlet-kernel-like ratio involving N , d_y , n_g and defined in (D.3) in Appendix D.2. The maximum value of the beamforming gain at ϕ_{AoD} and f_t is

$$\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_t) = \frac{1}{4} (N + |\mathcal{S}(\phi_{\text{AoD}}, f_t)|)^2. \quad (4.16)$$

Although we assume a line-of-sight channel and an ideal, lossless waveguide to derive (4.15), unlike [65], this leads to a closed-form solution that reveals how different DMA parameters are interrelated with frequency and AoD. This closed-form solution is used in Section 4.3.2 for optimizing the operating frequency for the desired angle and in Section 4.3.3 to maximize the angular coverage by optimally designing parameters like refractive index and inter-antenna spacing.

We now have a mapping between the DMA resonant frequencies to the receiver angle as a parametric function of the operating frequency. For a fixed f_t , (4.15) is the optimal solution to configure the DMA. In Section 4.3.2, we go one step further and optimize the communication system with a flexible choice of operating frequency that can leverage the DMA frequency reconfigurability to further improve the beamforming gain.

4.3.2 Beamforming gain joint optimization over resonant frequencies and operating frequency

The maximum value of $\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_t)$ varies with ϕ_{AoD} and f_t as proved in (4.16). This variation in the peak gain is significant across ϕ_{AoD} as shown by plotting (4.16) for $\phi_{\text{AoD}} \in [-90^\circ, 90^\circ]$, $f_t = 15$ GHz, and $N = 8$ in Fig. 4.2. Another issue is the unintended grating

lobe in the DMA beamforming pattern as shown in Fig. 4.3. For a TTD array, the maximum beamforming gain is independent of ϕ_{AoD} and f_t and equals N^2 as shown in Appendix D.1. This begs the question whether it is possible to attain the maximum value N^2 for a DMA too?

For a DMA operating in a system with wider spectrum availability, we formulate a problem of optimizing the operating frequency f_t to maximize the beamforming gain at a particular ϕ_{AoD} . We assume that the operating frequency range of the DMA is $[f_{t,\min}, f_{t,\max}]$. Mathematically, **P1** is generalized to include the operating frequency optimization as

$$\mathbf{P2} : \max_{f_t} \left(\max_{\{f_{r,n}\}_{n=1}^N} \mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f_t) \right), \quad (4.17a)$$

$$\text{s.t. } \{f_{r,n}\}_{n=1}^N > 0, \quad (4.17b)$$

$$f_t \in [f_{t,\min}, f_{t,\max}]. \quad (4.17c)$$

In the following lemma, we prove that it is possible to attain the same maximum value as that of a TTD array by optimizing over the operating frequency.

Lemma 1 *Let $p_{\min} = \frac{f_{t,\min} d_y (n_g + \sin(\phi_{\text{AoD}}))}{c}$ and $p_{\max} = \frac{f_{t,\max} d_y (n_g + \sin(\phi_{\text{AoD}}))}{c}$. The optimal solution to problem **P2**, f_t^* , is expressed in closed-form as*

$$f_t^*(\phi_{\text{AoD}}) = \frac{p^* c}{d_y (\sin(\phi_{\text{AoD}}) + n_g)}, \quad (4.18)$$

where $p^* \in [p_{\min}, p_{\max}]$ is the solution to the problem

$$p^* = \underset{p \in [p_{\min}, p_{\max}]}{\operatorname{argmax}} \left| \frac{\sin(\pi N p)}{\sin(\pi p)} \right|. \quad (4.19)$$

Proof: Refer Appendix D.3 for the proof. □

Appendix D.3 shows that the beamforming gain at ϕ_{AoD} satisfies

$$\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_t^*(\phi_{\text{AoD}})) \leq N^2. \quad (4.20)$$

Equality holds when $\exists p^* \in \mathbb{N}$. In that case, the optimal resonant frequencies of the DMA elements satisfy $f_{r,n}^*(\phi_{\text{AoD}}) = f_t^*(\phi_{\text{AoD}})$ for all n . When $\nexists p^* \in \mathbb{N}$, the optimal resonant frequencies are obtained by solving (4.19) numerically and substituting $f_t^*(\phi_{\text{AoD}})$ into (4.15).

From Lemma 1, we see that the maximum possible DMA beamforming gain is N^2 only when p^* is a natural number. This maximum gain equals that of a TTD array, but only at the optimal frequency of the DMA derived in (4.18). Lemma 1 also demonstrates that all slots should have the same optimal resonant frequency for the DMA to attain the maximum gain. The gain is less than N^2 when there does not exist any natural number in the range $[p_{\min}, p_{\max}]$. In this case, the optimal resonant frequencies are distinct for each DMA slot.

The DMA angular range for beamforming is limited by the constraint on the operating frequency in (4.17c). For the unconstrained DMA case, i.e., without constraint (4.17c), the maximum beamforming gain is N^2 for all AoDs. Imposing constraint (4.17c) limits the angular range for which the maximum gain N^2 is attainable. The limits $p_{\min}$ and $p_{\max}$ play a critical role in defining the regime in which the maximum gain of N^2 can be attained. These limits depend on the DMA parameters like the refractive index n_g and the inter-antenna spacing d_y . This dependence motivates us to design n_g and d_y to improve the beamforming coverage.

4.3.3 Design guidelines for improving angular coverage

In this section, we provide guidelines on optimizing the refractive index and inter-antenna spacing of existing DMA designs for improving their angular coverage. We first define the maximum gain angular range of the DMA. The maximum gain angular range is the set of angles over which the DMA beamforming gain is N^2 for some frequency $f_t^*(\phi_{\text{AoD}}) \in [f_{t,\min}, f_{t,\max}]$. Let this set be denoted as Φ . Note that the frequency at which the maximum gain is achieved is dependent on the angle as proved in Lemma 1. We define the beamforming criterion as

$$\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_t^*(\phi_{\text{AoD}})) = N^2 \quad (4.21)$$

$$\forall \phi_{\text{AoD}} \in \Phi, f_t^*(\phi_{\text{AoD}}) \in [f_{t,\min}, f_{t,\max}].$$

In other words, for each AoD $\phi_{\text{AoD}} \in \Phi$, we want to obtain the optimal operating frequency solution corresponding to the case when p^* is a natural number from Lemma 1. We provide the closed-form design solution for n_g^* and d_y^* to satisfy the beamforming criterion from (4.21) in the following lemma.

Lemma 2 *For the set $\Phi = \{\phi_{\text{AoD}} : -\pi < \phi_{\text{lw}} \leq \phi_{\text{AoD}} \leq \phi_{\text{up}} < \pi\}$, the closed-form design solution for n_g^* that satisfies (4.21) is*

$$n_g^* = \frac{\sin(\phi_{\text{up}}) - \sin(\phi_{\text{lw}})}{2} \left(\frac{f_{\text{t,max}} + f_{\text{t,min}}}{f_{\text{t,max}} - f_{\text{t,min}}} \right) - \frac{\sin(\phi_{\text{up}}) + \sin(\phi_{\text{lw}})}{2}, \quad (4.22)$$

and the closed-form design solution for d_y^* is

$$d_y^* = \frac{cp^*(f_{\text{t,max}} - f_{\text{t,min}})}{(\sin(\phi_{\text{up}}) - \sin(\phi_{\text{lw}}))f_{\text{t,min}}f_{\text{t,max}}}, \text{ where } p^* \in \mathbb{N}. \quad (4.23)$$

Proof: Refer Appendix D.4 for the proof. $\square$

The beamforming criterion in (4.21) can be satisfied by an optimal design choice of the refractive index and element spacing for the DMA. For the special case of a symmetric angular range with ϕ_{max} as the maximum desired beamforming angle from broadside ($\phi_{\text{AoD}} = 0$), let $\phi_{\text{lw}} = -\phi_{\text{max}}$ and $\phi_{\text{up}} = \phi_{\text{max}}$. This leads to $n_g^* = \sin(\phi_{\text{max}}) \left(\frac{f_{\text{t,max}} + f_{\text{t,min}}}{f_{\text{t,max}} - f_{\text{t,min}}} \right)$ and $d_y^* = \frac{cp^*(f_{\text{t,max}} - f_{\text{t,min}})}{2 \sin(\phi_{\text{max}}) f_{\text{t,min}} f_{\text{t,max}}}$. From **Case 1** of Lemma 1, we see that the maximum value of beamforming gain N^2 is obtained when the value of p^* is chosen to be a natural number. As d_y^* depends on p^* , a reasonable choice of p^* should be made such that the spacing is not too large to avoid grating lobes and not too small to avoid mutual coupling between different elements. We provide some examples of reasonable values of p^* , d_y^* , and n_g^* for different choices of ϕ_{max} , $f_{\text{t,min}}$, and $f_{\text{t,max}}$ in Table 4.2 for a fixed center frequency of the tuning range $f_c = 15$ GHz. The spacing d_y^* is expressed in terms of the wavelength $\lambda_c = \frac{c}{f_c}$.

The angular coverage is limited by the available choices of the waveguide substrate. We derive the maximum feasible beamforming angle by assuming a value for the maximum

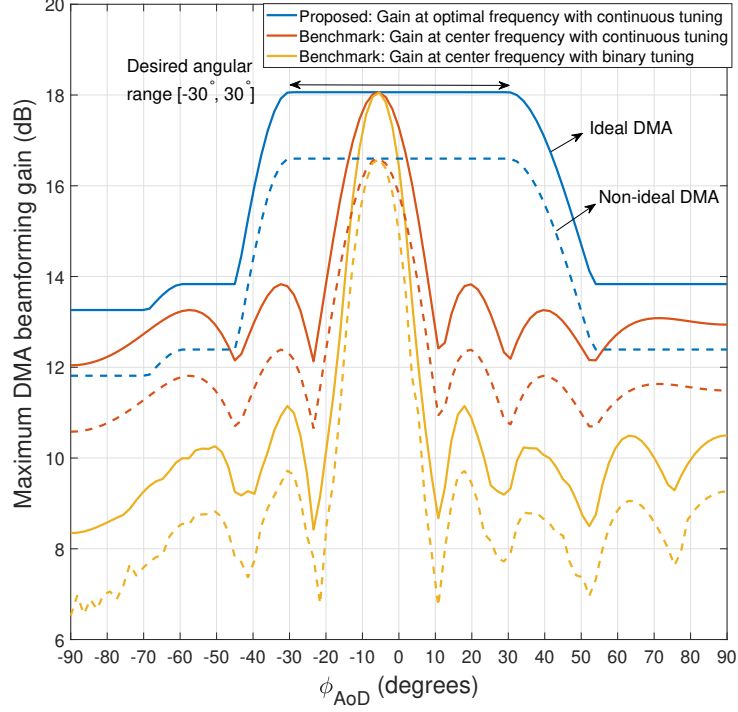


Figure 4.2. The DMA beamforming gain as a function of the AoD is shown for both ideal and non-ideal DMA for different benchmarks. The proposed approach outperforms the benchmarks with a fixed operating frequency for all angles. Beyond $\pm 30^\circ$, the beamforming gain decreases because of the operating frequency constraint (4.17c).

Table 4.2. Designing d_y^* and n_g^* for the desired maximum angular range and DMA tuning frequency range.

$\phi_{\max}$	$f_{t,\min}$	$f_{t,\max}$	T_r	p^*	d_y^*	n_g^*
20°	12.5 GHz	17.5 GHz	5 GHz	1	$0.5\lambda_c$	2
30°	12 GHz	18 GHz	6 GHz	1	$0.42\lambda_c$	2.5
45°	12.25 GHz	17.75 GHz	5.5 GHz	2	$0.53\lambda_c$	3.85
60°	11.75 GHz	18.25 GHz	6.5 GHz	2	$0.52\lambda_c$	4

practically feasible waveguide refractive index as $n_g^{\max}$. This imposes the constraint

$$n_g^* = \sin(\phi_{\max}) \left(\frac{f_{t,\max} + f_{t,\min}}{f_{t,\max} - f_{t,\min}} \right) \leq n_g^{\max}. \quad (4.24)$$

Let the frequency range over which the DMA slots are tuned be $T_r = f_{t,\max} - f_{t,\min}$. Let the maximum feasible operating frequency be $f_{t,\max} = f_c + \frac{T_r}{2}$ and the minimum feasible operating frequency be $f_{t,\min} = f_c - \frac{T_r}{2}$. From (4.24), we compute $\phi_{\max}$ as

$$\phi_{\max} = \sin^{-1} \left(\frac{n_g^{\max} T_r}{2f_c} \right). \quad (4.25)$$

From (4.25), we observe that to maximize the DMA beamforming angular coverage, the DMA should use the highest feasible waveguide refractive index and the maximum possible operating range T_r for the given f_c .

4.3.4 DMA beamforming gain frequency response analysis

So far, we have demonstrated how the DMA can achieve a gain of N^2 at AoD ϕ_{AoD} only at a particular frequency $f_t^*(\phi_{\text{AoD}})$. The data transmission, however, is for a specific bandwidth B . A phased-array experiences degradation in the beamforming gain for frequencies away from the center frequency due to beam-squint [118]. While DMA beamforming also exhibits beam-squint, the DMA beamforming gain experiences further wideband losses because of the per-element frequency-selective Lorentzian response shown in (4.1).

We analyze the frequency response of the DMA at an AoD inside the desired angular range. For a DMA designed using n_g^* and d_y^* from Lemma 2, the maximum beamforming gain N^2 is attained at $f_t^*(\phi_{\text{AoD}})$ for any AoD ϕ_{AoD} in the desired angular range $[\phi_{\text{lw}}, \phi_{\text{up}}]$. To attain the maximum beamforming gain, the resonant frequencies are tuned based on Lemma 1. Substituting

the solution from Lemma 1, we obtain

$$\mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f) = \frac{|\Gamma 2\pi f|^2 \times |\mathbf{1}^T \mathbf{h}(\phi_{\text{AoD}}, f)|^2}{|(2\pi f_t^*(\phi_{\text{AoD}}))^2 - (2\pi f)^2 + j\Gamma 2\pi f|^2}. \quad (4.26)$$

To analyze the overall DMA beamforming response, we write $\mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f)$ as product of the per-element frequency response $\mathcal{G}_{\text{element}}(\phi_{\text{AoD}}, f) = \left| \frac{\Gamma 2\pi f}{(2\pi f_t^*(\phi_{\text{AoD}}))^2 - (2\pi f)^2 + j\Gamma 2\pi f} \right|^2$ and the frequency response of the array of DMA slots $\mathcal{G}_{\text{array}}(\phi_{\text{AoD}}, f) = |\mathbf{1}^T \mathbf{h}(\phi_{\text{AoD}}, f)|^2$.

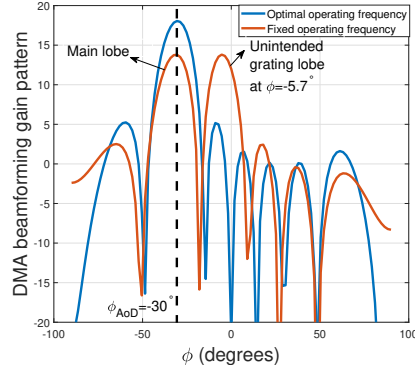
To analyze the DMA bandwidth for which the beamforming gain does not fall below a certain threshold of ν times the maximum, we define the cutoff frequencies. Let the upper cutoff frequency be the highest frequency for which $\mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f) \geq \nu N^2$ and be denoted as f_ν^u . Let the lower cutoff frequency be the lowest frequency for which $\mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f) \geq \nu N^2$ and be denoted as f_ν^l . Let us define the ν -bandwidth as $B_\nu = f_\nu^u - f_\nu^l$. We first analyze the bandwidth for the element response $\mathcal{G}_{\text{element}}(\phi_{\text{AoD}}, f)$. Let $B_{e,\nu}$ be the bandwidth for the term $\mathcal{G}_{\text{element}}(\phi_{\text{AoD}}, f)$. Let $\rho(\nu) = \frac{\nu}{1-\nu}$. For $\mathcal{G}_{\text{element}}(\phi_{\text{AoD}}, f)$, the ν -bandwidth is expressed as

$$B_{e,\nu} = \frac{\Gamma}{2\pi\sqrt{\rho(\nu)}} + \mathcal{O}(\Gamma^3). \quad (4.27)$$

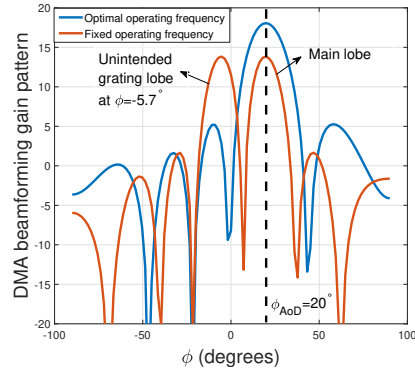
Refer Appendix D.5 for the proof of (4.27). The frequency response of $\mathcal{G}_{\text{element}}(\phi_{\text{AoD}}, f)$ has sharp degradation with frequency compared to $\mathcal{G}_{\text{array}}(\phi_{\text{AoD}}, f)$. Hence, we use the result from (4.27) to obtain $B_\nu \approx B_{e,\nu}$. The key design insight is that B_ν is directly proportional to the damping factor Γ . For example, for 3 dB bandwidth, i.e, $\nu = 0.5$, we have $B_{0.5} \approx \frac{\Gamma}{2\pi}$.

4.3.5 Numerical results for DMA beamforming gain

We compare our proposed optimal operating frequency approach with methods used in the prior work. Let the number of elements on the DMA be $N = 8$. Let the center frequency of the DMA operating band be $f_c = 15$ GHz and the center wavelength $\lambda_c = \frac{c}{f_c}$. Let the DMA operating range be $f_{t,\min} = 12$ GHz and $f_{t,\max} = 18$ GHz, and let the desired angular range be



(a) $\phi_{AoD} = -30^\circ$



(b) $\phi_{AoD} = 20^\circ$

Figure 4.3. DMA beamforming gain pattern is shown for $\phi_{AoD} = -30^\circ$ and $\phi_{AoD} = 20^\circ$. When DMA operating frequency is fixed to f_c , we observe an unintended grating lobe at $\phi_c = -5.7^\circ$. The optimal frequency approach mitigates this issue by aligning the grating lobe with the desired AoD creating a main lobe with higher gain.

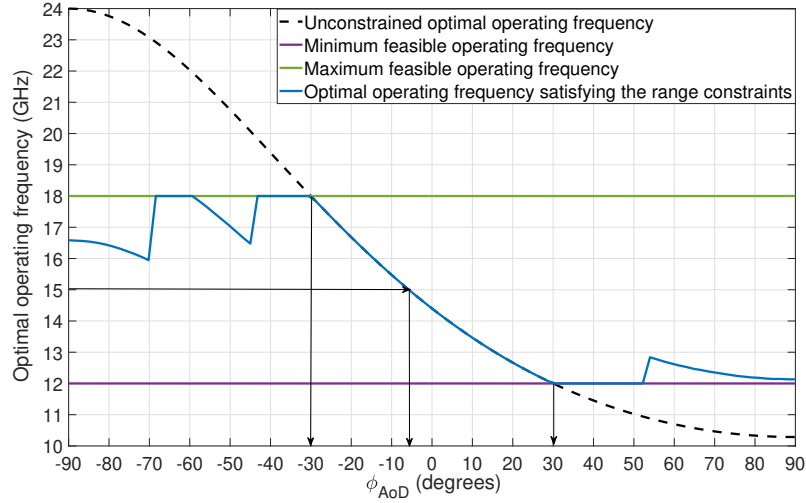


Figure 4.4. The optimal operating frequency from our proposed approach is shown as a function of the AoD. A monotonic behavior for the desired angular range $\pm 30^\circ$ is observed which corresponds to the maximum gain in Fig. 4.2.

$[-30^\circ, 30^\circ]$. From Lemma 2, we have the optimal spacing as $d_y^* = 0.42\lambda_c$ with choice of $p^* = 1$ and the optimal refractive index $n_g^* = 2.5$. Let $\Gamma = \frac{2\pi f_c}{50}$.

Fixed operating frequency approach

Prior work on DMA beamforming is based on optimizing the beamforming gain for a specific AoD and fixed operating frequency [65]. Let the operating frequency be fixed to the center frequency f_c . The DMA resonant frequencies are configured using the closed-form expression in (4.15) where $f_t = f_c$. In Fig. 4.2, we plot the optimal DMA beamforming gain at f_c as a function of the angle ϕ_{AoD} , mathematically denoted as $\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_c)$ and computed using (4.16). We observe that $\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_c) \leq \mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_t^*(\phi_{\text{AoD}}))$, $\forall \phi_{\text{AoD}}$. The equality holds only for a single angle ϕ_c . Mathematically, we can calculate ϕ_c by substituting $f_t^*(\phi_c) = f_c$ in (4.18). In Fig. 4.2, we see that $\mathcal{G}_{\text{DMA}}^*(\phi_c, f_c) = N^2$. For all angles $\phi_{\text{AoD}} \neq \phi_c$, we see that $\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_c) < N^2$.

In Fig. 4.4, we plot the optimal frequency $f_t^*(\phi_{\text{AoD}})$ from (4.18) as function of ϕ_{AoD} . By optimally choosing the operating frequency based on the angle as shown in Fig. 4.4, we significantly outperform the fixed operating frequency approach as shown in Fig. 4.2 with a gain of N^2 in the desired angular range. The maximum gain angular range depends on the DMA upper and lower tuning frequency limits. As $f_{t,\min} = 12$ GHz and $f_{t,\max} = 18$ GHz, the maximum beamforming angle for $n_g = 2.5$ is $\pm 30^\circ$. Beyond $\pm 30^\circ$, we observe a clipping in the optimal operating frequency curve in Fig. 4.4 to satisfy the operating frequency constraint (4.17c). This also reflects in the beamforming gain plot in Fig. 4.2, where we observe a decrease in the beamforming gain outside $\pm 30^\circ$. Without the DMA constraint (4.17c), the beamforming gain will be N^2 for the whole angular range. This is practically infeasible as the DMA would need to operate over a very wide frequency range of 10 GHz to 24 GHz. This is demonstrated in Fig. 4.4, where the monotonic curve extends beyond the feasible frequency range.

A drawback of beamforming by fixing the operating frequency is an unintended grating lobe. In Fig. 4.3, we plot the DMA beamforming gain pattern optimized for two different AoDs.

In both cases, we observe a grating lobe at ϕ_c when the operating frequency is fixed to f_c . The grating lobe in DMA beamforming is also pointed out in [103]. The angle at which the grating lobe occurs depends on the operating frequency. Our proposed optimal frequency tuning approach mitigates this issue by aligning the grating lobe with the desired AoD which creates a main lobe with higher gain.

Comparison between binary and continuous weights

In the analysis so far, we have assumed that DMA resonant frequencies are tunable to a continuous frequency value. Another option for configuring the DMA elements is a binary tuning mechanism leveraging PIN diodes instead of varactor diodes [119]. PIN diodes can be used as RF switches with on and off states. The binary weight beamforming problem and its solution is described in Appendix D.6. The optimal beamforming gain at the center frequency obtained after solving the binary weight beamforming problem is shown in Fig. 4.2. We see that when $\phi_{\text{AoD}} = \phi_c$, the DMA beamforming gain for binary and continuous weights are equal to N^2 . This observation is consistent with Lemma 1 that at the optimal operating frequency, the resonant frequencies on all slots are the same.

For all angles $\phi_{\text{AoD}} \neq \phi_c$, we see that the beamforming gain from binary weights is decreased compared to the continuous weights. For angles outside $[-14^\circ, 2^\circ]$, the decrease is higher than 2 dB. This is because some of the antenna elements have beamforming weight as zero, which reduces the effective number of antennas and hence the beamforming gain.

Non-ideal DMA

In the simulations and analysis so far, we have ignored the attenuation within the waveguide. For a practical DMA, the waveguide attenuation can degrade the beamforming gain [97]. To analyze the impact of waveguide attenuation, we denote the attenuation factor as α and assume an exponentially decaying attenuation vector as $\mathbf{g} = [1, e^{-\alpha d_y}, \dots, e^{-\alpha(N-1)d_y}]^T$. The effective propagation channel is modified as $\mathbf{h}(\phi_{\text{AoD}}, f) = \mathbf{h}_{\text{dma}}(f) \odot \mathbf{a}(\phi_{\text{AoD}}, f) \odot \mathbf{g}$. For simulations,

we use $\alpha = 6$ [26] and use the resonant frequency solution corresponding to the ideal DMA case.

In Fig. 4.2, we compare the beamforming gain of the ideal DMA and the non-ideal DMA. It can be shown that after including attenuation in the channel model, the beamforming gain in the desired angular range is $\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_t^*(\phi_{\text{AoD}})) = \left(\frac{1-e^{-N\alpha dy}}{1-e^{-\alpha dy}} \right)^2 < N^2$. The non-ideal DMA with attenuation exhibits a beamforming gain degradation compared to the ideal DMA by a factor of $\chi(N) = \frac{1}{N^2} \left(\frac{1-e^{-N\alpha dy}}{1-e^{-\alpha dy}} \right)^2$. For $N = 8$, this degradation is 1.47 dB and for $N = 16$, it is 3.05 dB. As $\chi(N)$ is a decreasing function of N , it is not suitable to have a large number of slots on a single DMA. To enhance beamforming gain, it is recommended to have multiple DMAs where each DMA has a moderate number of slots. For example, the peak gain of an array of 2 DMAs each with 8 slots is higher than that of a single DMA with 16 slots by 1.58 dB as shown in Fig. 4.5. We also see that the proposed optimal frequency approach outperforms the benchmark fixed frequency approach for the non-ideal DMA too. Although inclusion of attenuation in the model leads to beamforming gain degradation, the overall comparison trend remains the same. Hence, we continue with the use of the ideal DMA model for the analysis in Section 4.4 and use the DMA model with attenuation for achievable rate simulations in Section 4.5.

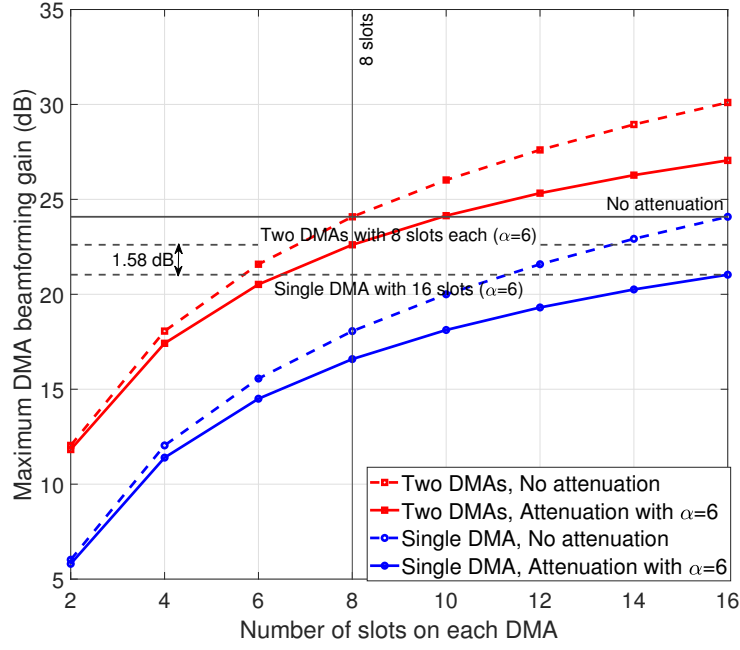


Figure 4.5. With no attenuation in the model, a two DMA array with 8 slots each has the same peak beamforming gain of 24.08 dB as a single DMA with 16 slots. With an attenuation factor of $\alpha = 6$, we observe that the two DMA array with 8 slots each performs better by 1.58 dB compared to the single DMA with 16 slots.

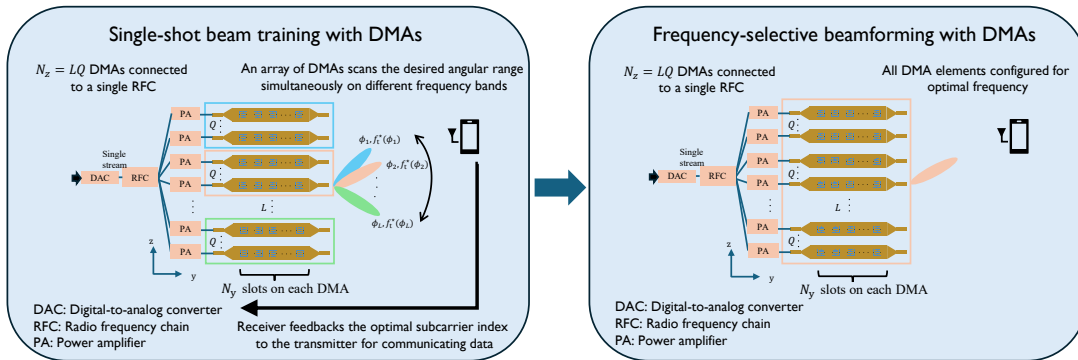


Figure 4.6. An array of N_z DMAs where each DMA has N_y radiating slots. The sub-array approach enables simultaneous probing of different directions using different frequencies to estimate the optimal resonant frequency. Post beam training, all elements of the DMA array are configured for the optimal frequency.

4.4 Single-shot beam training with DMA array

While the previous section focused on beamforming optimization for a single DMA, in this section, we extend the analysis to a system with multiple DMAs and address the beam training problem. The optimal solutions in Lemma 1 and Lemma 2 within the desired angular range do not depend on the number of slots on each DMA. The beamforming approach and insights developed for a single DMA are also applicable to a DMA planar array composed of identical DMAs.

In conventional exhaustive search beam training, the beamforming vector is determined by sequentially configuring the array for different AoDs and choosing the angle with the highest beamforming gain to compute the optimal beamforming vector for the desired user. This process is inefficient because the number of time slots required to perform beam training increases with the number of antennas [120].

The exhaustive search in time domain leads to a larger overhead in the pre-log factor of the achievable rate. In this section, we leverage the frequency reconfigurability of the DMA and propose an alternate paradigm for beam training in the frequency domain. Our proposed approach estimates the optimal DMA configuration in a single time-slot by probing different angular directions simultaneously on different subcarriers using a sub-array based approach. This single-shot approach exploiting frequency reconfigurability indicates that DMAs have the potential to enable large arrays without creating any bottleneck due to beam training overhead. DMAs replace the need for multiple antennas dedicated to different frequency bands, providing a more hardware and power-efficient solution.

4.4.1 System model for an array of DMAs

Let an array of N_z DMAs be arranged in the yz plane as shown in Fig. 4.6, where each DMA has N_y radiating slots. We assume that each DMA has the same intrinsic phase-shift. For a receiver on the xy plane, the effective propagation far-field channel for the array of DMAs is

denoted as $\mathbf{h}_A(\phi, f) \in \mathbb{C}^{N_z N_y \times 1}$ and expressed in terms of $\mathbf{h}(\phi, f) \in \mathbb{C}^{N_y \times 1}$ as

$$\mathbf{h}_A(\phi, f) = \underbrace{[\mathbf{h}^T(\phi, f), \mathbf{h}^T(\phi, f), \dots, \mathbf{h}^T(\phi, f)]^T}_{N_z \text{ times}}. \quad (4.28)$$

As the elevation angle $\theta = 90^\circ$, the propagation channel from each DMA to the receiver in the xy plane is the same. The extension to do beam training in both azimuth and elevation directions is a topic of future study.

Similar to [65], we assume the single-stream beamforming case with only one RF chain connected to all DMAs as shown in Fig. 4.6. The same signal is input to each DMA. The beamforming vector for the array of DMAs is denoted as $\mathbf{f}_{\text{ADMA}}(f) = [\mathbf{f}_{\text{DMA},1}^T(f), \mathbf{f}_{\text{DMA},2}^T(f), \dots, \mathbf{f}_{\text{DMA},N_z}^T(f)]^T$. The frequency-selective beamforming gain for the DMA array is defined similar to that of a single DMA as

$$\mathcal{G}_{\text{ADMA}}(\phi, f) = |\mathbf{f}_{\text{ADMA}}^T(f) \mathbf{h}_A(\phi, f)|^2. \quad (4.29)$$

The received power is obtained from (4.10) by replacing with $\mathcal{G}_{\text{ADMA}}(\phi, f)$.

Let the resonant frequency of the n th element on the m th DMA in the array be denoted as $f_{r,n,m}$. Similar to the analysis in Section 4.3, the maximum beamforming gain $\mathcal{G}_{\text{ADMA}}^*(\phi_{\text{AoD}}, f_{\text{t}}^*(\phi_{\text{AoD}})) = N_z^2 N_y^2$ at a specific angle ϕ_{AoD} is attained when resonant frequencies of all elements on all DMAs are set to the optimal operating frequency $f_{r,n,m} = f_{\text{t}}^*(\phi_{\text{AoD}})$. In practice, this optimal resonant frequency needs to be estimated using the beam training procedure as the AoD is unknown. We discuss the beam training procedure to estimate the optimal resonant frequency for the DMA.

4.4.2 Beam training with a single OFDM symbol

The goal of beam training is to estimate the optimal resonant frequency configuration for the DMA array to serve a user in a specific direction. To avoid training overhead, we

propose a single-shot beam training procedure which enables probing different angular directions simultaneously with a single OFDM symbol by leveraging the frequency reconfigurability of the DMA. To achieve this goal, we use the sub-array approach for beam training. Let the array of N_z DMAs be grouped into L sub-groups where each sub-group has $Q = \frac{N_z}{L}$ DMAs as shown in Fig. 4.6. The assignment of DMAs to the same subgroup is arbitrary, as all DMAs appear identical to a far-field receiver in the azimuth plane. We assume that the angle ϕ_{AoD} lies inside the angular range $[-\phi_{\text{max}}, \phi_{\text{max}}]$. We divide this angular range into L sectors, where the ℓ th sector has its peak gain at angle ϕ_ℓ . The DMA array is configured such that the ℓ th DMA sub-group focuses at ϕ_ℓ at the corresponding optimal frequency $f_t^*(\phi_\ell)$. The DMA resonant frequencies for beam training are configured as

$$f_{r,n,m} = f_t^*(\phi_\ell), \quad \forall n, m \in \{Q(\ell - 1) + 1, \dots, Q\ell\}. \quad (4.30)$$

This approach enables scanning different angles simultaneously with a single OFDM signal.

The DMA array transmits an OFDM pilot signal with K_{tr} subcarriers for beam training. Let the k th subcarrier frequency be f_k . The DMA beamforming vector configured according to (4.30) has the response $\mathbf{f}_{\text{ADMA}}(f_k)$ for the k th subcarrier. Let the actual AoD be ϕ_{AoD} . The beamforming gain at the receiver at the k th subcarrier is $\mathcal{G}_{\text{ADMA}}(\phi_{\text{AoD}}, f_k)$. The receiver computes the subcarrier index with the highest beamforming gain as

$$k^* = \underset{k}{\operatorname{argmax}} \quad \mathcal{G}_{\text{ADMA}}(\phi_{\text{AoD}}, f_k). \quad (4.31)$$

The receiver feeds back the optimal subcarrier index to the transmitter for data communication. For the data transmission stage, the transmit DMA array is configured to operate for a specific bandwidth B centered around f_{k^*} . All DMA resonant frequencies are set to $f_{r,n,m} = f_{k^*}$. Substituting this solution in (4.29), we obtain $\mathcal{G}_{\text{ADMA}}(\phi_{\text{AoD}}, f_{k^*}) = N_z^2 |\mathcal{S}(\phi_{\text{AoD}}, f_{k^*})|^2$. With this configuration, the maximum beamforming gain of $N_z^2 N_y^2$ is attained at f_{k^*} and $\phi_{\text{AoD}} = \hat{\phi}_{\text{AoD}} =$

$$\sin^{-1} \left(\frac{c}{d_y f_{k^*}} - n_g \right).$$

4.4.3 Beam training codebook design

In this section, we propose a codebook for the single-shot beam training approach. We first analyze the performance loss in beamforming when $f_{r,n,m} = f_{k^*}$. We write $\mathcal{G}_{\text{ADMA}}(\phi_{\text{AoD}}, f_{k^*}) = N_z^2 |\mathcal{S}(\phi_{\text{AoD}}, f_{k^*})|^2$ in terms of $\hat{\phi}_{\text{AoD}}$ as

$$\mathcal{G}_{\text{ADMA}}(\phi_{\text{AoD}}, f_{k^*}) = N_z^2 \left(\frac{\sin \left(\pi N_y \frac{(n_g^* + \sin(\phi_{\text{AoD}}))}{(n_g^* + \sin(\hat{\phi}_{\text{AoD}}))} \right)}{\sin \left(\pi \frac{(n_g^* + \sin(\phi_{\text{AoD}}))}{(n_g^* + \sin(\hat{\phi}_{\text{AoD}}))} \right)} \right)^2. \quad (4.32)$$

From (4.32), we see that the maximum beamforming gain of $N_z^2 N_y^2$ is attained when $\hat{\phi}_{\text{AoD}} = \phi_{\text{AoD}}$. As the error between the actual and estimated angle increases, the beamforming gain decreases. In the following lemma, we quantify the allowable range of angle estimates for which the beamforming gain is above a certain threshold of δ times the maximum gain. Let $\Psi = \frac{(n_g^* + \sin(\phi))}{(n_g^* + \sin(\hat{\phi}))}$. The maximum value of $\left(\frac{\sin(\pi N_y \Psi)}{\sin(\pi \Psi)} \right)^2$ is N_y^2 . We define $\Psi_\delta(N_y)$ such that for $\Psi \in [-\Psi_\delta(N_y), \Psi_\delta(N_y)]$, the term $\left(\frac{\sin(\pi N_y \Psi)}{\sin(\pi \Psi)} \right)^2 \geq \delta N_y^2$ for $\delta < 1$. We can compute $\Psi_\delta(N_y)$ numerically. For example, for $\delta = 0.5$ (3 dB from maximum), we have $\Psi_\delta(N_y) \approx \frac{0.448}{N_y}$.

Lemma 3 *The inequality $\mathcal{G}_{\text{ADMA}}(\phi_{\text{AoD}}, f_{k^*}) \geq \delta N_y^2 N_z^2$, is satisfied when the angle estimate $\hat{\phi}_{\text{AoD}}$ satisfies*

$$\begin{aligned} \sin^{-1} \left(\frac{n_g^* + \sin(\phi_{\text{AoD}})}{1 + \Psi_\delta(N_y)} - n_g^* \right) &\leq \hat{\phi}_{\text{AoD}} \\ &\leq \sin^{-1} \left(\frac{n_g^* + \sin(\phi_{\text{AoD}})}{1 - \Psi_\delta(N_y)} - n_g^* \right). \end{aligned} \quad (4.33)$$

Proof: When $\hat{\phi}_{\text{AoD}} = \phi_{\text{AoD}}$, we have $\Psi = 1$. To satisfy the gain threshold, we have $1 - \Psi_\delta(N_y) \leq \frac{(n_g^* + \sin(\phi_{\text{AoD}}))}{(n_g^* + \sin(\hat{\phi}_{\text{AoD}}))} \leq 1 + \Psi_\delta(N_y)$. Simplifying, we obtain the bounds on $\hat{\phi}_{\text{AoD}}$. $\square$

Beam codebook design: The goal of beam codebook design is to choose the angles where

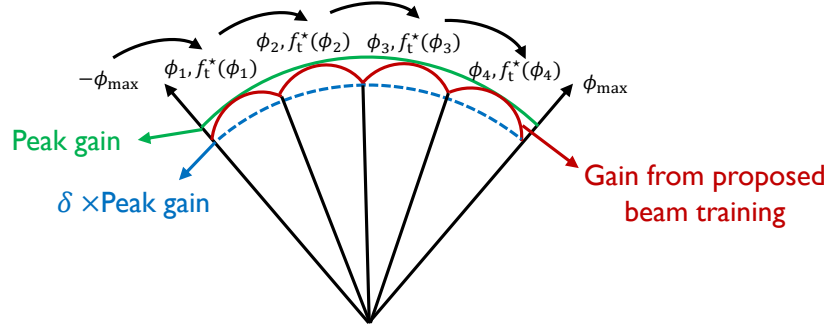


Figure 4.7. The proposed beam codebook design ensures that the beamforming gain for all angles in the desired range is within a factor of δ from the maximum where δ is a design parameter.

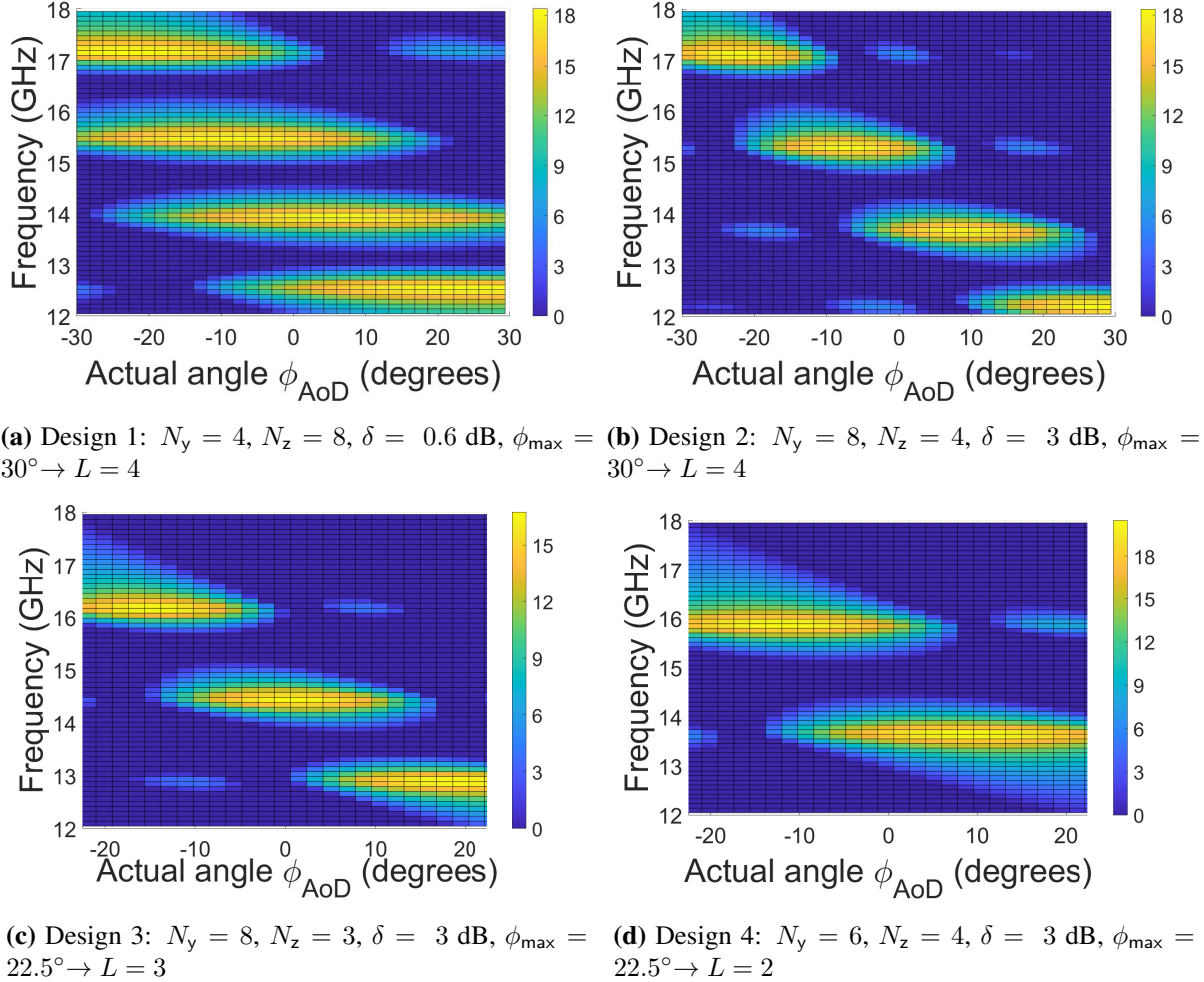
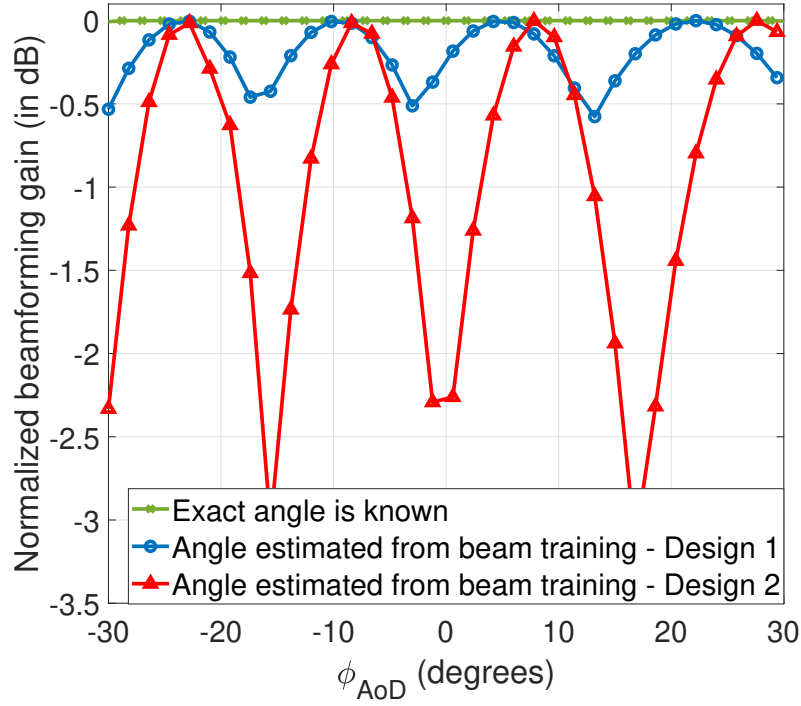
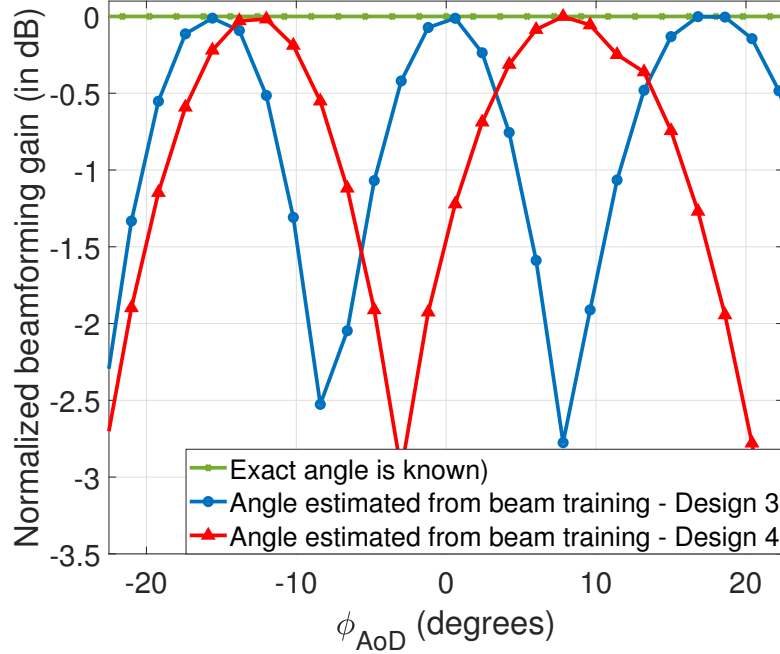


Figure 4.8. Beamforming gain response for the proposed single-shot sub-array beam training approach is shown as a function of frequency and angle. For design 1 and design 2, the optimal operating frequency decreases in $L = 4$ steps as a function of the actual angle similar to the monotonic decreasing trend obtained for the continuous curve in Fig. 4.4. The angular beam width is narrower for $N_y = 8$ compared to $N_y = 4$. For design 3 and design 4, the value of L differs even though $N_y N_z$ is fixed because of the difference in the angular beam width.



(a) Normalized beamforming gain for $N_y N_z = 32$



(b) Normalized beamforming gain for $N_y N_z = 24$

Figure 4.9. Normalized beamforming gain is shown as a function of the AoD. The beamforming gain response obtained after using the angle estimate from beam training depends on the beam codebook design and the array geometry. For DMA array with larger N_y and fixed number of beams L , the beams are narrower which reduces the beamforming gain for angles away from ϕ_ℓ .

peak gain occurs for the L sectors in the beam training procedure to ensure that the beamforming gain $\mathcal{G}_{\text{ADMA}}(\phi_{\text{AoD}}, f_{k^*})$ for any ϕ_{AoD} in the desired range does not drop below a certain threshold of δ from the peak gain. We use the result from Lemma 3 to design the beam codebook. For the ℓ th sector, let ϕ_ℓ be the angle with the peak gain, i.e., $\mathcal{G}_{\text{ADMA}}(\phi_\ell, f_{k^*}) = N_y^2 N_z^2 \forall \ell = 1, \dots, L$. The main objective of the beam codebook design is to ensure that the beamforming gain for all angles in the range $[-\phi_{\max}, \phi_{\max}]$ is above $\delta N_y^2 N_z^2$. The peak angle of the first sector is designed such that the gain at $-\phi_{\max}$ is $\delta N_y^2 N_z^2$. Equating the lower limit of $\hat{\phi}$ when $\phi = \phi_1$ in Lemma 3 to $-\phi_{\max}$, we obtain

$$\phi_1 = \sin^{-1} \left((\sin(-\phi_{\max}) + n_g^*)(1 + \Psi_\delta(N_y)) - n_g^* \right). \quad (4.34)$$

For the remaining sectors, we equate the lower limit of $\hat{\phi}$ when $\phi = \phi_\ell$ to the upper limit when $\phi = \phi_{\ell-1}$. With this procedure, the remaining $L - 1$ angles are obtained sequentially as

$$\phi_\ell = \sin^{-1} \left((\sin(\phi_{\ell-1}) + n_g^*) \frac{(1 + \Psi_\delta(N_y))}{(1 - \Psi_\delta(N_y))} - n_g^* \right), \forall \ell > 1. \quad (4.35)$$

The beam codebook design procedure is shown in Fig. 4.7. To compare differences in DMA array geometry, and the impact on beam training, we describe four examples. For each example, we fix $n_g^* = 2.5$ and vary $N_y, N_z, \phi_{\max}, \delta$ which also results in a different value of L for each case. For design 1 and design 2, we fix the total number of radiating slots $N_y N_z = 32$. For design 3 and design 4, we fix the total number of radiating slots $N_y N_z = 24$.

- Design 1: For $N_y = 4, N_z = 8, \delta \equiv 0.6$ dB, $\phi_{\max} = 30^\circ$, we have $L = 4$ with the design angles as $\phi_1 = -23.33^\circ, \phi_2 = -9.51^\circ, \phi_3 = 5.22^\circ, \phi_4 = 22.04^\circ$.
- Design 2: For $N_y = 8, N_z = 4, \delta \equiv 3$ dB, $\phi_{\max} = 30^\circ$, we have $L = 4$ with the design angles as $\phi_1 = -22.83^\circ, \phi_2 = -7.9^\circ, \phi_3 = 8.21^\circ, \phi_4 = 27.16^\circ$.
- Design 3: For $N_y = 8, N_z = 3, \delta \equiv 3$ dB, $\phi_{\max} = 22.5^\circ$, we have $L = 3$ with the design

angles as $\phi_1 = -15.31^\circ$, $\phi_2 = 0.06^\circ$, $\phi_3 = 17.33^\circ$.

- Design 4: For $N_y = 6$, $N_z = 4$, $\delta \equiv 3$ dB, $\phi_{\max} = 22.5^\circ$, we have $L = 2$ with the design angles as $\phi_1 = -12.94^\circ$, $\phi_2 = 8.35^\circ$.

The DMA resonant frequencies are set using (4.30). The beamforming gain response as a function of frequency and angle for these four examples is shown in Fig. 4.8(a)-(d). For $N_y = 8$, the beams are narrower compared to $N_y = 4$ as expected from (4.32). The optimal subcarrier index at the receiver is computed using (4.31). Using the optimal resonant configuration, we evaluate (4.32) and plot the normalized beamforming gain at the optimal operating frequency as a function of the actual AoD in Fig. 4.9. From the plot, we verify that the beamforming gain variation is within the desired threshold, thus validating Lemma 3. The comparison between design 1 and design 2 shows the effect of varying N_y while maintaining $L = 4$. The design 2 with $N_y = 8$ has narrower beams and the angles away from ϕ_ℓ experience lower beamforming gain as shown in Fig. 4.9. For design 1 and design 2, using only four distinct resonant frequencies, the DMA array is configured such that the beamforming gain is within a certain threshold of the maximum. The comparison between design 3 and design 4 shows that the number of sectors L is different even though $N_y N_z$ is fixed. For design 3, $L = 3$ and for design 4, $L = 2$. To summarize, the value of L depends on the array geometry, the threshold δ , and the maximum desired angle from the broadside $\phi_{\max}$.

4.5 Achievable rate results

We simulate the achievable rate of the DMA array for the single stream case in Section 4.5.1 and for the MIMO case with multiple streams in Section 4.5.2.

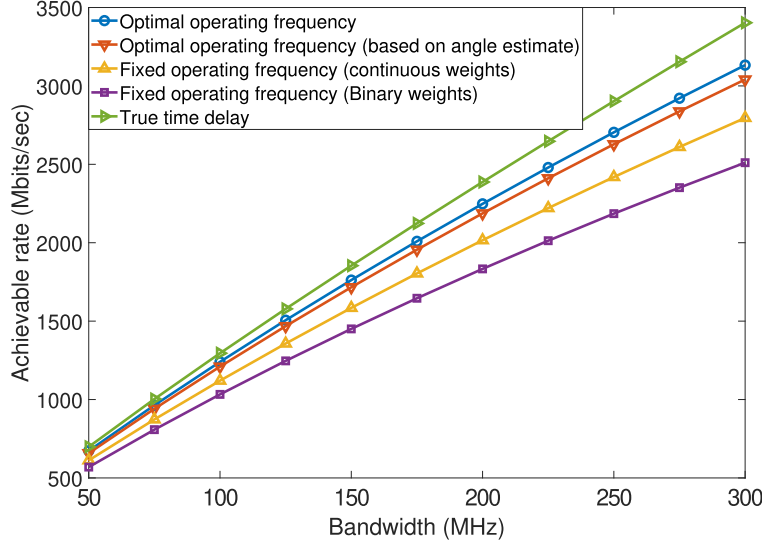


Figure 4.10. Achievable rate DMA performance is shown as a function of the communication bandwidth. The proposed optimal frequency tuning approach outperforms the fixed operating frequency approach.

4.5.1 Single stream results

Simulation setup

We evaluate the achievable rate of the DMA array using the results from Section 4.3 and Section 4.4. In our proposed approach, the center frequency f_t is dynamically reconfigured depending on the AoD. We assume two cases for comparison: **Case 1-** When the AoD is perfectly known, the center frequency is set using Lemma 1; **Case 2-** When the AoD is unknown, we use the single-shot beam training procedure from Section 4.4 and set $f_t = f_{k^*}$ obtained from (4.31). We also compare these cases from our proposed approach to that of the benchmark DMA approach based on a fixed operating frequency. For the benchmark, we set $f_t = f_c$, which is the center frequency of the feasible operating frequency range. We also compare our proposed approach with TTD.

We set the transmit power $P_T = 250$ mW and the distance between transmitter and receiver $r = 500$ m. Let $n_g = 2.5$, $f_c = 15$ GHz, $d_y = 0.42\lambda_c$, $\Gamma = \frac{2\pi f_c}{50}$, $N_y = 8$, $N_z = 4$, and $T_r = 6$ GHz. We use the non-ideal DMA model from Section 4.3.5 with attenuation factor $\alpha = 6$ [26] to ensure a fair comparison with the TTD array. For beam training, we use $L = 4$ and

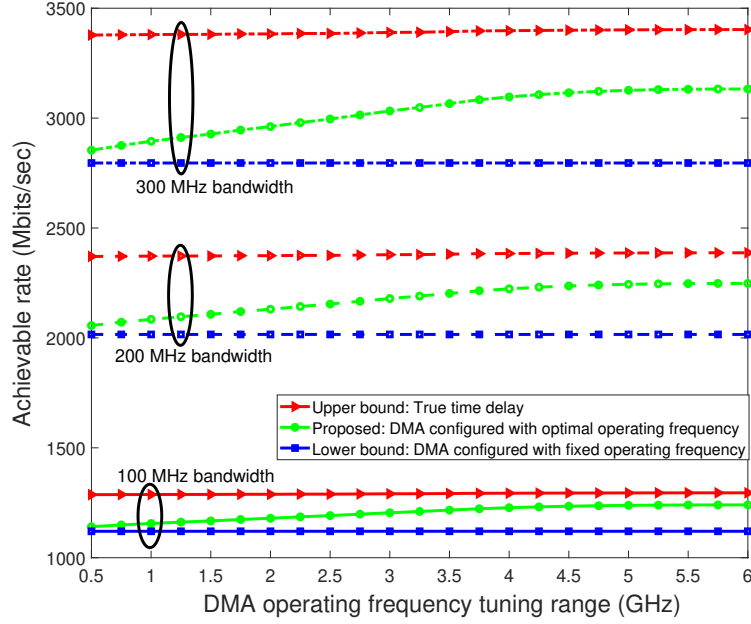


Figure 4.11. Achievable rate is shown as a function of the DMA operating frequency tuning range. DMA rate based on optimal operating frequency is lower bounded by the DMA rate based on fixed frequency and upper bounded by the TTD rate for all tuning ranges.

the angles with peak gain for each sector are chosen corresponding to the threshold $\delta \equiv 3$ dB.

Achievable rate vs communication bandwidth for different benchmarks

In Fig. 4.10, we plot the achievable rate averaged over angles $\phi \sim \mathcal{U}(-30^\circ, 30^\circ)$. In the OFDM system, we have $K_d = 64$ subcarriers. On the x axis of Fig. 4.10, we vary the communication bandwidth B . We observe that the achievable rate of the DMA configured based on our proposed optimal operating frequency approach outperforms the benchmark approach based on the fixed operating frequency. We also observe that the rate performance of the DMA when resonant frequencies are configured based on the single-shot beam training procedure is comparable to the rate obtained using perfect knowledge of the AoD. This validates the efficacy of the proposed single-shot beam training approach. We show in Fig. 4.9 that with $L = 4$ sectors, the beamforming gain varies within 3 dB of the maximum. Using only four resonant frequency configurations, it is possible to do both beam training and beamforming. In Fig. 4.10, we see that the achievable rate increases with bandwidth. The DMA achievable rate is comparable to that of

TTD up to 150 MHz bandwidth beyond which the performance gap increases. This shows that for low to moderate communication bandwidths, DMAs are better than TTDs because of similar achievable rate with lower power consumption. The power consumption analysis for a DMA from [65] shows that the power required to tune the DMA slots is orders of magnitude lower than the conventional analog beamforming.

Achievable rate vs DMA operating frequency tuning range

In Fig. 4.10, we assumed that the DMA operating range T_r is 6 GHz. In Fig. 4.11, we vary T_r by keeping all parameters same as Fig. 4.10 and plot the achievable rate obtained with three different values of the bandwidth B . The baseline approach performance does not change with T_r as the DMA operating frequency is always fixed. We observe that the proposed approach of dynamically configuring the DMA operating frequency outperforms the baseline fixed operating frequency approach for all tuning ranges. As T_r increases, the DMA achievable rate from the proposed approach tends to the TTD rate which acts as an upper bound. This implies that a DMA with high tuning range can be an alternative to the power-hungry TTD architecture.

4.5.2 MIMO with multiple streams

MIMO system model

In this section, we extend the single stream DMA array system model to the multi-stream MIMO case where each DMA waveguide is connected to an RF chain and the receiver has multiple antennas as shown in Fig. 4.12. Assuming a receiver with a uniform linear array (ULA) with N_r isotropic antennas and inter-element spacing of d_r , we denote the array steering vector associated with the receiver as $\mathbf{a}(\phi_r, f)$. Let the azimuth angle associated with the DMA array at the transmitter be ϕ_t . Let the elevation angle be $\theta = 90^\circ$. The steering vector for the DMA array is defined as $\mathbf{a}_A(\phi_t, f)$ similar to (4.28). Let the line-of-sight component of the MIMO channel be $\mathbf{H}_{\text{LOS}}(\phi_r, \phi_t, f) = \mathbf{a}(\phi_r, f)\mathbf{a}_A^T(\phi_t, f)$ and the Rayleigh-faded channel component be $\mathbf{H}_w \in \mathbb{C}^{N_r \times N_y N_z}$. With a Rician factor K_r , the MIMO channel is

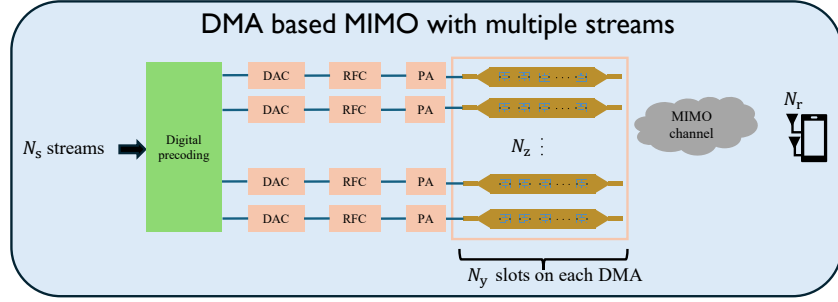


Figure 4.12. MIMO system with digital precoding and DMA-based frequency-selective beamforming, where each waveguide is connected to an RF chain.

$\mathbf{H}_{\text{Rice}}(f) = \left(\frac{\lambda}{4\pi r}\right) \left(\sqrt{\frac{K_r}{K_r+1}} \mathbf{H}_{\text{LOS}}(\phi_r, \phi_t, f) + \sqrt{\frac{1}{K_r+1}} \mathbf{H}_w\right)$. We assume that the intrinsic DMA channel with waveguide attenuation is the same for all DMAs in the array and denote it as $\tilde{\mathbf{h}}_{\text{dma}}(f) = \mathbf{h}_{\text{dma}}(f) \odot \mathbf{g}$. As the same intrinsic DMA channel applies to all DMA waveguide and receive antenna pairs, we define $\tilde{\mathbf{H}}_{\text{dma}}(f) \in \mathbb{C}^{N_r \times N_y N_z}$ which is obtained by repeating $\tilde{\mathbf{h}}_{\text{dma}}^T(f)$ in the row dimension N_r times and in the column dimension N_z times. The effective MIMO channel is obtained by taking the element-wise product of the Rician-faded wireless channel and the intrinsic DMA waveguide channel as $\mathbf{H}(f) = \mathbf{H}_{\text{Rice}}(f) \odot \tilde{\mathbf{H}}_{\text{dma}}(f)$.

MIMO achievable rate formulation

We express the multi-stream precoding for the array of DMAs as a $N_y N_z \times N_s$ matrix

$$\mathbf{F}_{\text{ADMA}}(f) = \text{blkdiag}(\mathbf{f}_{\text{DMA},1}(f), \mathbf{f}_{\text{DMA},2}(f), \dots, \mathbf{f}_{\text{DMA},N_z}(f)). \quad (4.36)$$

Let the fully digital precoder with N_s streams be $\mathbf{F}_{\text{BB}}(f) \in \mathbb{C}^{N_z \times N_s}$. We assume $N_s = \min\{N_z, N_r\}$. As joint optimization is challenging, we separately configure the DMA precoder and fully digital precoder similar to [79]. Each element of the DMA precoding matrix is configured using the DMA operating frequency closed-form expression dependent on the line-of-sight angle ϕ_t . The fully digital precoder for each subcarrier frequency f_k is then obtained by assuming an effective channel $\mathbf{H}_{\text{eff}}(f_k) = \mathbf{H}(f_k) \mathbf{F}_{\text{ADMA}}(f_k)$ and applying singular value decomposition. The power allocation for each subcarrier and stream is obtained through a water-filling algorithm to

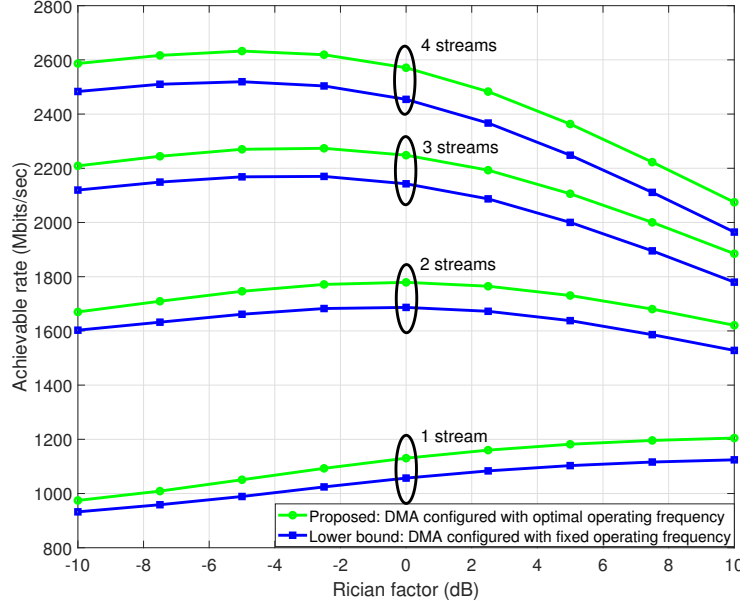


Figure 4.13. MIMO achievable rate is shown for different number of streams and by varying the Rician factor. Multi-stream MIMO works best for channels with rich scattering whereas single-stream case is better for channels with stronger line-of-sight component.

satisfy the transmit power constraint $\sum_{k=1}^{K_d} \|\mathbf{F}_{BB}(f_k)\|_F^2 \leq P$. The achievable rate for the MIMO channel is defined as

$$R = \frac{B}{K_d} \sum_{k=1}^{K_d} \log_2 \left(\left| \mathbf{I}_{N_r} + \frac{\mathbf{H}_{\text{eff}}(f_k) \mathbf{F}_{BB}(f_k) \mathbf{F}_{BB}^*(f_k) \mathbf{H}_{\text{eff}}^*(f_k)}{\sigma^2} \right| \right). \quad (4.37)$$

We compute the achievable rate of the MIMO channel averaged over different values of ϕ_r , ϕ_t , and several realizations of $\mathbf{H}_w$.

Achievable rate vs Rician factor for different number of streams

We vary the number of streams by varying the number of receive antennas, $N_r = \{1, 2, 3, 4\}$ and vary the MIMO channel Rician factor K_r in Fig. 4.13. Let $N_y = 8$ and $N_z = 4$. We set $P = K_d N_z P_T$, where $P_T = 250$ mW. The achievable rate is averaged over 500 realizations with Rayleigh-faded $\mathbf{H}_w$, $\phi_r \sim \mathcal{U}(-180^\circ, 180^\circ)$, and $\phi_t \sim \mathcal{U}(-30^\circ, 30^\circ)$. Other simulation parameters are the same as in the single-stream case described in Section 4.5.1.

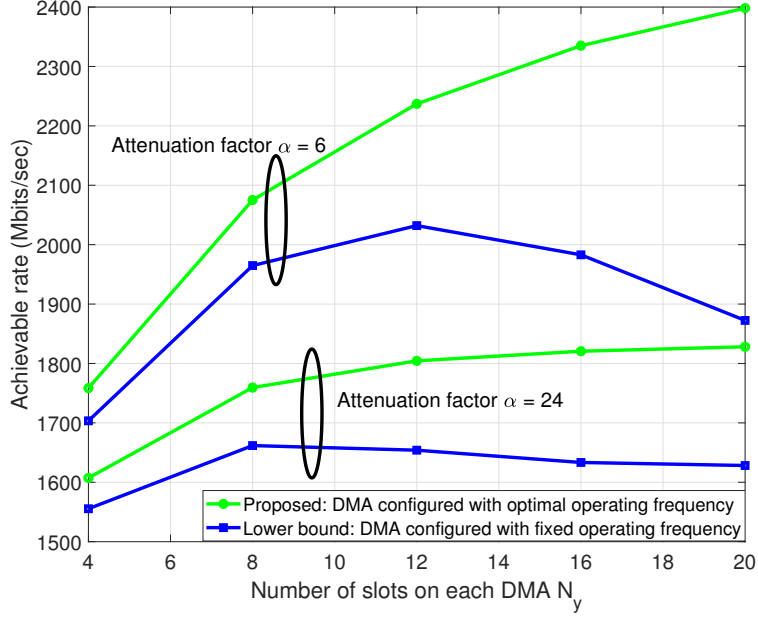


Figure 4.14. MIMO achievable rate is shown for different number of slots N_y and for different attenuation factors α . The optimal frequency tuning approach provides a better scalability with N_y compared to the fixed operating frequency approach. With higher attenuation factor, the achievable rate saturates beyond a certain value of N_y .

In Fig. 4.13, we observe that the proposed optimal frequency tuning approach outperforms the fixed frequency approach for all values of K_r . Although the optimal frequency approach is designed with the assumption of a line-of-sight channel, it is robust with rich scattering as well. When comparing the achievable rate trend for different number of streams, we observe that for the multi-stream case, lower K_r values perform better, whereas the trend is opposite for the single stream case. This aligns with the conventional MIMO paradigm.

Achievable rate vs number of DMA slots for different attenuation factors

To test the robustness of the proposed approach with respect to the attenuation factor, we plot the achievable rate as a function of the number of slots N_y on each DMA for two different values of the attenuation factor, $\alpha = 6$ and $\alpha = 24$. We set $N_s = 4$ streams. In Fig. 4.14, we observe that the fixed operating frequency approach does not scale with N_y , especially for a higher attenuation factor where the achievable rate does not increase beyond $N_y = 8$. By tuning the DMA with our proposed optimal frequency approach, we obtain a better scalability of the

achievable rate in terms of N_y . For both values of attenuation factor, the proposed approach outperforms the baseline fixed operating frequency approach.

4.6 Conclusions and future work

In this chapter, we proposed a novel approach where the system configures the DMA resonant frequencies by optimally choosing the operating frequency based on the AoD. We found that this approach outperforms the fixed operating frequency approach for all tuning ranges. By optimally designing the waveguide refractive index and inter-element spacing on DMA, the maximum beamforming gain is equivalent to the TTD array. Based on the single-shot beam training approach for the DMA to estimate the optimal resonant frequency configuration, we showed that by using a limited number of discrete resonant frequency states for both beam training and beamforming, we obtain good achievable rate performance.

Investigating the performance of the proposed approach with impedance matching impairments is suggested for future work [69, 121]. The current work is based on the assumption that mutual coupling between radiating slots is negligible. Incorporating mutual coupling in future DMA models would be useful [117] [122]. Another future direction is to extend the proposed approach to support multiple pairs of receivers and transmitters on different bands within the feasible frequency range by dynamically allocating the operating frequency. One approach to fully utilize the available spectrum is frequency-reconfigurable ad hoc networks with multiple simultaneous links [123]. Using the theoretical design insights from this chapter and validating them with hardware prototypes is an important next step. To meet the needs of future communication standards, integrating the proposed approach in a tri-hybrid MIMO architecture to support multiple data streams is essential [4]. Integration of DMAs in reflect arrays is another research direction.

Acknowledgements

Chapter 4, in full, is a reprint of the material as it appears in **N. V. Deshpande**, J. Carlson,

M. R. Castellanos, and R. W. Heath Jr., “Frequency-selective beamforming and single-shot beam training with dynamic metasurface antennas,” in *IEEE Trans. Commun.*, pp. 1–1, 2026. The dissertation author was the primary investigator and author of this paper.

Chapter 5

Concluding remarks

This dissertation presents novel approaches for integrating reconfigurable antennas in the next-generation MIMO systems. The key highlights include developing low complexity algorithms for jointly configuring active and parasitic elements in a hybrid array, proposing a new tri-hybrid architecture with analog switch and parasitic elements, and exploiting frequency-selectivity property of dynamic metasurface antennas for beamforming and single-shot beam training. In the following sections, we present the summary and future research directions related to the topics discussed in the dissertation.

5.1 Summary

In Chapter 2, we applied a circuit theory model to a hybrid reconfigurable parasitic array with multiple active antennas and several parasitic elements controlled through tunable reactance. As validated through electromagnetic simulations, the circuit theory approach is useful because we can use the proposed model for different parasitic configurations instead of doing repetitive simulations. We found that the proposed HRP-UPA architecture under the same total radiated power constraint, requires only about 2 dB more transmit power to achieve the same spectral efficiency as FD-UPA, yet has around 2.5 times improvement in energy efficiency. This confirms that hybrid parasitic arrays are an energy-efficient solution for MIMO communication. These arrays have the potential to be an important component of the next-generation tri-hybrid MIMO

architecture [4]. We derived closed-form expressions for configuring the parasitic reactance and active antenna currents. From the solution for parasitic reactance, we identified a specific reactance range which has a significant impact on the beamforming gain. This provides a direction to design realistic hardware solutions for configuring parasitic arrays.

In Chapter 3, we propose the tri-hybrid MIMO architecture with parasitic reconfigurable antennas and an analog switch to dynamically reconfigure the assignment across active and parasitic elements. The spectral efficiency and energy efficiency performance for the proposed tri-hybrid architecture is higher than that of the hybrid benchmarks. Specifically, from the ablation study, we learn that the dynamic assignment feature provides higher performance improvement compared to the dynamic parasitic reactance reconfiguration feature. This is because active antenna selection plays a crucial role in beam shaping and the parasitic antennas further enhance the performance by leveraging the mutual coupling effect. Our experiments highlight that the introduction of parasitic elements in an active linear sparse array helps boost spectral-efficiency performance when several parasitic elements are tightly packed between the active antennas.

In Chapter 4, we proposed a novel approach where the system configures the DMA resonant frequencies by optimally choosing the operating frequency based on the AoD. We found that this approach outperforms the fixed operating frequency approach for all tuning ranges. By optimally designing the waveguide refractive index and inter-element spacing on DMA, the maximum beamforming gain is equivalent to the TTD array. Based on the single-shot beam training approach for the DMA to estimate the optimal resonant frequency configuration, we showed that by using a limited number of discrete resonant frequency states for both beam training and beamforming, we obtain good achievable rate performance.

5.2 Future work

The focus of this dissertation is limited to the beamforming problem with two types of reconfigurable antennas, namely hybrid parasitic arrays and DMAs. The contributions

of this dissertation open new opportunities for the research community, including exploring other reconfigurable antenna types in the context of tri-hybrid MIMO, investigating the channel estimation problem, extending the work to multi-user settings, experimentally validating tri-hybrid MIMO, and integrating with other 6G technologies. In this section, we discuss these potential research directions and highlight key challenges that remain to be addressed but fall outside the scope of this dissertation.

5.2.1 Extension to other reconfigurable antenna types

This dissertation focused on two types of reconfigurable antennas: DMAs and hybrid parasitic arrays. More broadly, the tri-hybrid MIMO architecture can be extended to other reconfigurable apertures through a common input-output formulation. In [86], we developed a general perspective in which the effect of the reconfigurable antenna is represented through an effective channel term, similar to the formulation used in Chapter 3. This viewpoint creates opportunities to study a wider range of reconfigurable antennas and to transfer modeling and algorithmic insights across different antenna architectures.

In particular, pixel antenna arrays are a promising class of reconfigurable apertures that are closely related to the hybrid parasitic architectures in Chapters 2 and 3. In this design, sub-wavelength radiating elements, or pixels, are switched and electromagnetically coupled to the active feed, and the same circuit-theoretic viewpoint provides a natural modeling starting point. In [124], we show that the input–output model aligns with the formulations in Chapters 2 and 3, with the key difference that the design dimension is determined by the number of interconnecting switches, not the number of radiating elements, and that a graph-based linear map connects the switch-as-port model to the antenna-as-port model without re-running full-wave EM simulations for every switch configuration. Important next steps are scalable algorithms for designing the higher-dimensional switch network, and use of the reconfigurability efficiency factor (REF) from [86] to study tradeoffs between communication performance and switch count across pixel array designs.

5.2.2 Channel estimation for the tri-hybrid MIMO architecture

This dissertation focused on the beamforming problem for the tri-hybrid MIMO architecture and assumed that the channel state information is perfectly known at the transmitter. In practice, however, this information must be acquired through channel estimation. This problem is particularly challenging for tri-hybrid MIMO because several elements are controlled by reconfigurable reactances, and the corresponding channels cannot be observed directly. Building on prior work in channel estimation for hybrid MIMO architectures [13] and the tri-hybrid framework [4], an important future direction is to develop training and estimation methods that first infer configuration-dependent effective channels and then recover the underlying wireless channel, or a sufficient low-dimensional representation of it, for beamforming.

5.2.3 Extension to multi-user MIMO setting with tri-hybrid architecture

The contributions of this dissertation are limited to a single-user MIMO system. A natural next step is to extend the system model to the multi-user setting. While the circuit-theory formulation can, in principle, be generalized to multiple users, the main challenge lies in the associated optimization problem. In the single-user case, metrics such as beamforming gain or signal-to-noise ratio are directly related to the achievable rate. In the multi-user case, however, inter-user interference must be incorporated into the tri-hybrid MIMO optimization objective, making the problem substantially more difficult. Recent work has begun to examine multi-user tri-hybrid precoding [81], but the benefit of the antenna-precoding layer in the proposed architecture still needs to be understood more carefully. The key questions to address include:

- How does the active-to-parasitic element ratio in the tri-hybrid array impact the number of users that can be supported to meet specific achievable rate requirements?
- How does the tri-hybrid MIMO architecture compare with other hybrid benchmarks in a multi-user setting?

5.2.4 Experimental validation of the tri-hybrid MIMO architecture

This dissertation focused on the modeling, analysis, and optimization of reconfigurable antennas in MIMO systems and established theoretical foundations for the tri-hybrid MIMO architecture. To make this solution more viable in practice, an important next step is to develop hardware proof-of-concept platforms for tri-hybrid MIMO. While prior work has demonstrated hardware prototypes for dynamic metasurface antennas and reconfigurable antenna arrays [125, 126], a fully integrated tri-hybrid MIMO testbed that can support multiple forms of reconfigurable antennas remains an open problem. Such an experimental platform would create new opportunities to validate channel estimation, beamforming, calibration, and energy-efficiency algorithms for the tri-hybrid architecture under realistic hardware impairments.

5.2.5 Integration of the tri-hybrid MIMO architecture with other 6G technologies

Several emerging 6G paradigms, including integrated sensing and communication (ISAC), coexistence with satellite and terrestrial networks, and AI-RAN, provide promising directions for extending the tri-hybrid MIMO architecture.

Tri-hybrid MIMO for integrated sensing and communication

Integrated sensing and communication (ISAC) is an important 6G paradigm in which the same wireless infrastructure is used for both data transmission and environmental sensing [127]. The tri-hybrid MIMO architecture is a promising fit for ISAC because its digital precoding, analog switching, and antenna-level reconfiguration provide additional design degrees of freedom to realize communication and sensing beams with fewer RF chains than a fully digital array [4]. Recent work on reconfigurable and dynamic metasurface antennas has already demonstrated the potential of such apertures for communication, sensing, and imaging [110, 125]. A key future direction is therefore to develop joint tri-hybrid ISAC designs that optimize the digital precoder, switch states, and antenna configuration using both communication and sensing metrics, while

addressing practical challenges such as configuration-dependent channel and target estimation, calibration, mutual coupling, and hardware losses.

Co-existence of satellite and terrestrial networks

As 6G expands into the upper mid-band, coexistence between terrestrial systems and satellite or non-terrestrial networks will become increasingly important [17, 128]. Dynamic metasurface antennas (DMAs) are particularly relevant in this setting because their frequency-selective radiation characteristics couple the operating frequency with the dominant beam direction, creating new opportunities for interference management and spectrum sharing [66]; this potential is also reflected in recent work on frequency-reconfigurable networks with DMAs [123] and interference-resistant DMA arrays [129]. Within the tri-hybrid MIMO architecture, such DMA behavior can be combined with digital precoding and analog switching to support ground users while steering nulls or reducing interference toward satellites and other protected directions, which has already been done for the fully digital [130] and the hybrid architectures [131]. A key future direction is therefore to develop DMA-aware tri-hybrid beamforming and resource allocation schemes for integrated satellite-terrestrial networks under practical constraints such as mobility, Doppler, calibration, and hardware losses.

Tri-hybrid MIMO for AI-RAN

AI-RAN has emerged as an important 6G direction in which artificial intelligence is used to optimize the radio access network while the RAN infrastructure also evolves to support AI-native services and computing workloads [132]. In this context, tri-hybrid MIMO is relevant in two complementary ways. First, AI can be used to configure the tri-hybrid architecture by learning low-complexity mappings from channel, traffic, and hardware state information to the digital precoder, switch states, and antenna reconfiguration, thereby reducing the optimization overhead associated with real-time control of reconfigurable arrays [4, 72]. Second, the energy-efficient and scalable nature of tri-hybrid MIMO can help AI-RAN base stations support higher data-rate

and compute-intensive applications with lower RF-chain count and power consumption. A key future direction is therefore to develop AI-native control, beam management [133], and resource allocation methods for tri-hybrid MIMO under practical constraints such as limited training data, changing propagation environments, hardware impairments, and inference latency.

5.2.6 A path toward tri-hybrid MIMO in future cellular standards

5G NR supports large antenna arrays with fewer RF chains than radiating elements by standardizing interoperable procedures, defined in Third Generation Partnership Project NR technical specifications [115, 134], that coordinate digital precoding with analog beamforming in phased-array radio front ends. Important elements of this framework include beam management with synchronization and discovery signals, CSI acquisition with CSI-RS and SRS under sub-array or RF-chain constraints, and standardized codebook-based CSI reporting, including Type I and Type II formats [135], for frequency-selective wideband transmission. Together, these tools let commercial systems operate many antenna elements while keeping baseband dimension and pilot overhead bounded at both FR1 and FR2.

Looking ahead, tri-hybrid MIMO could matter for NR-style cellular standards toward 6G if digital precoding, analog switching states, and reconfigurable antenna parameters remain practical without pilot loads and CSI feedback costs that escalate far beyond what phased-array hybrids use today. Today's phased-array hybrids rely on the NR split highlighted above: calibrated CSI-RS and SRS measurements on logical antenna ports pair with standardized feedback codebooks, and most phased-array drift is absorbed by quasi-static calibration at the radio. Hybrid parasitic arrays and DMAs upset that arrangement because patterns move with tuner choices and coupling, which makes it harder for a feedback index to reliably identify the transmitted beam unless vendors align on common hardware classes. Scheduling must also contend with settling time and switching losses alongside digital updates, unlike phased-array hybrids that often bury such imperfections in coarse calibration corrections.

Taken together, these points reconnect to the topics already highlighted in this section:

configuration-dependent channel estimation and calibration, multi-user setups in which coupling among users depends on aperture configuration, portable aperture modeling beyond parasitic overlays and DMAs, and hardware demonstrations that expose residual calibration and quantization limits rather than idealized analyses alone. Progress along those lines supports the 6G directions summarized above, including integrated sensing and communication, coexistence between satellite and terrestrial networks, and AI-native RAN control, because practical radios still need to coordinate digital, switching, and antenna-level knobs under mobility and coexistence requirements comparable to NR.

Appendix A

Appendix to Chapter 2: Hybrid Reconfigurable Parasitic Antenna Arrays

A.1 Proof of the approximation result

Let $\Psi_1 = 1 - \mathbf{a}_P^T(\theta) \mathbf{W} \mathbf{z}_m$ and $\Psi_2 = \mathbf{a}_P^T(\theta) \mathbf{W} \mathbf{E}_P \mathbf{W} \mathbf{z}_m$. We have $\tilde{\mathcal{G}}(\theta, \mathbf{Z}_R) = \mathcal{G}(\theta, \mathbf{Z}_R) - \hat{\mathcal{G}}(\theta, \mathbf{W}) = |\Psi_2|^2 + 2\mathcal{R}\{\Psi_1^c \Psi_2\} - \mathcal{O}(\|\mathbf{E}_P\|_F^2)$. Using matrix norm inequality, we have $|\Psi_2| < \|\mathbf{a}_P(\theta)\| \|\mathbf{W}\|_F^2 \|\mathbf{z}_m\| \|\mathbf{E}_P\|_F = \kappa \|\mathbf{E}_P\|_F$ where κ is a constant depending on $\|\mathbf{a}_P(\theta)\|$, $\|\mathbf{W}\|_F$, and $\|\mathbf{z}_m\|$. As $\|\mathbf{E}_P\|_F \rightarrow 0$, we get $\tilde{\mathcal{G}}(\theta, \mathbf{Z}_R) \rightarrow 0$, i.e., $\mathcal{G}(\theta, \mathbf{Z}_R) \rightarrow \hat{\mathcal{G}}(\theta, \mathbf{W})$.

A.2 Proof of the beamforming weight expression

In this section, we provide a proof of (2.15). We write $[\mathbf{W}]_{ii}$ as

$$[\mathbf{W}]_{ii} = \frac{1}{[\mathbf{D}_P]_{ii} + [\mathbf{Z}_R]_{ii}}, \quad (\text{A.1})$$

$$\stackrel{(a)}{=} \frac{(\mathcal{R}\{[\mathbf{D}_P]_{ii}\} + \mathcal{R}\{[\mathbf{Z}_R]_{ii}\}) - j(\mathcal{I}\{[\mathbf{D}_P]_{ii}\} + \mathcal{I}\{[\mathbf{Z}_R]_{ii}\})}{(\mathcal{R}\{[\mathbf{D}_P]_{ii}\} + \mathcal{R}\{[\mathbf{Z}_R]_{ii}\})^2 + (\mathcal{I}\{[\mathbf{D}_P]_{ii}\} + \mathcal{I}\{[\mathbf{Z}_R]_{ii}\})^2}, \quad (\text{A.2})$$

$$\stackrel{(b)}{=} \frac{e^{j\varphi_i}}{\sqrt{(\mathcal{R}\{[\mathbf{D}_P]_{ii}\} + \mathcal{R}\{[\mathbf{Z}_R]_{ii}\})^2 + (\mathcal{I}\{[\mathbf{D}_P]_{ii}\} + \mathcal{I}\{[\mathbf{Z}_R]_{ii}\})^2}}, \quad (\text{A.3})$$

$$\stackrel{(c)}{=} \zeta_i \cos(\varphi_i) e^{j\varphi_i}, \quad (\text{A.4})$$

where (a) follows by multiplying both numerator and denominator by the complex conjugate of $[\mathbf{D}_P]_{ii} + [\mathbf{Z}_R]_{ii}$, (b) follows by the definition of $\varphi_i = \angle([\mathbf{W}]_{ii})$ in (2.13), and (c) follows by

multiplying both numerator and denominator by ζ_i as defined in (2.14).

A.3 Proof of the closed-form phase theorem

Let $\chi_1 = 1 - \frac{\zeta}{2} \mathbf{a}_P^T(\theta) \mathbf{z}_m$ and $\chi_2 = -\frac{\zeta}{2} \mathbf{a}_P^T(\theta) \Phi \mathbf{z}_m$. We expand the objective in (2.21) as $\widehat{\mathcal{G}}(\theta, \Phi) = |\chi_1 + \chi_2|^2$. The term χ_1 is a constant complex number which does not depend on the beamforming matrix Φ . The term χ_2 depends on Φ . Let $\chi_{2,i} = -\frac{\zeta}{2} e^{j\phi_i} [\mathbf{a}_P(\theta)]_i [\mathbf{z}_m]_i$. We express $\chi_2 = \sum_{i=1}^{N_P} \chi_{2,i}$. Using this expression, we expand $\widehat{\mathcal{G}}(\theta, \Phi)$ as

$$\widehat{\mathcal{G}}(\theta, \Phi) = |\chi_1|^2 + 2\mathcal{R} \left\{ \chi_1^c \sum_{i=1}^{N_P} \chi_{2,i} \right\} + \left| \sum_{i=1}^{N_P} \chi_{2,i} \right|^2. \quad (\text{A.5})$$

The second term in this expansion is maximized when the argument of the complex number $\chi_1^c \sum_{i=1}^{N_P} \chi_{2,i}$ is zero. This is obtained by setting $\angle(\chi_{2,i}) = 2m\pi + \angle(\chi_1)$ for any integer m . This setting also maximizes the third term in the expansion as all phases are aligned. Simplifying, we get the result in (2.23).

A.4 Generalization to unique self-impedances and element radiation patterns

In Theorem 2, we assumed that the self-impedances and element radiation patterns of all the parasitic elements are the same. In this section, we generalize that result to the case when self-impedances and radiation element patterns of all parasitic elements are different. This case arises when there are element pattern distortions due to closely-spaced elements, or if the antenna type of parasitic elements may differ, thus resulting in different self-impedances and element radiation patterns. We define $\zeta = \text{diag}\{[\zeta_1, \zeta_2, \dots, \zeta_{N_P}]^T\}$. In Section 2.2.3, we used the array steering vector with omnidirectional radiation pattern. We generalize by defining the element radiation pattern vector as $\mathbf{e}_P(\theta) = [e_1(\theta), \dots, e_{N_P}(\theta)]^T$. Thus, the effective channel vector is

$\mathbf{h}_P(\theta) = \mathbf{a}_P(\theta) \odot \mathbf{e}_P(\theta)$. The objective in **P3** is generalized as

$$\widehat{\mathcal{G}}(\theta_1, \Phi) = \left| 1 - \frac{1}{2} \mathbf{h}_P^T(\theta_1) \zeta (\mathbf{I}_{N_P} + \Phi) \mathbf{z}_m \right|^2. \quad (\text{A.6})$$

Following the mathematical strategy similar to the special case in Theorem 2, we obtain the general expression of the optimal reactance as

$$\mathcal{I}\{[\mathbf{Z}_R]_{ii}\} = -\mathcal{I}\{Z_P\} - \frac{\cot\left(\frac{\angle([\mathbf{h}_P(\theta_1)]_i [\mathbf{z}_m]_i) - \angle(1 - \frac{\mathbf{h}_P^T(\theta_1) \zeta \mathbf{z}_m}{2})}{2}\right)}{\zeta_i}. \quad (\text{A.7})$$

From this general result, we see that the optimal reactance of the i th parasitic element depends on the self-impedances of all parasitic elements through the matrix ζ .

Appendix B

Appendix to Chapter 3: The tri-hybrid MIMO architecture with parasitic antennas and dynamic switching

B.1 Derivation for partial derivative

We derive the partial derivative of the achievable rate $R(\mathbf{Z}_R)$ in $\mathcal{P}_{4a}$ with respect to the imaginary part of the parasitic load $X_{R,i} \equiv \mathcal{I}\{\mathbf{Z}_R[i, i]\}$. Let $\mathbf{R}_k = \mathbf{H}_{\text{eff}}(f_k) \mathbf{Z}_{\text{eff}}^{-1}(f_k) \mathbf{H}_{\text{eff}}^*(f_k)$ and let $\lambda_j^{(k)}$ denote the j -th eigenvalue of $\mathbf{R}_k$, sorted in descending order (as in (3.14)). The objective is

$$R(\mathbf{Z}_R) = \frac{B}{K} \sum_{k=1}^K \sum_{j=1}^{N_S} \log_2 \left(1 + \frac{P_{j,k} \lambda_j^{(k)}(\mathbf{Z}_R)}{\sigma^2} \right), \quad (\text{B.1})$$

We use the following identities in the derivation:

- Hellmann–Feynman: $\frac{\partial \lambda}{\partial X} = \mathbf{u}^* \frac{\partial \mathbf{R}}{\partial X} \mathbf{u}$ for Hermitian $\mathbf{R}$;
- Inverse differential: $\frac{\partial \mathbf{A}^{-1}}{\partial X} = -\mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial X} \mathbf{A}^{-1}$;
- Diagonal load: $\frac{\partial \mathbf{Z}_R}{\partial X_{R,i}} = \mathbf{j} \mathbf{E}_{ii}$.

B.1.1 Step 1: Gradient via chain rule and Hellmann–Feynman

Since $P_{j,k}$ are fixed when optimizing over $\mathbf{Z}_R$, the chain rule gives

$$\frac{\partial R}{\partial X_{R,i}} = \frac{B}{K \ln 2} \sum_{k,j} \frac{P_{j,k}}{\sigma^2 + P_{j,k} \lambda_j^{(k)}} \frac{\partial \lambda_j^{(k)}}{\partial X_{R,i}}.$$

By the Hellmann–Feynman theorem [89],

$$\frac{\partial \lambda_j^{(k)}}{\partial X_{R,i}} = (\mathbf{u}_j^{(k)})^* \frac{\partial \mathbf{R}_k}{\partial X_{R,i}} \mathbf{u}_j^{(k)},$$

where $\mathbf{u}_j^{(k)}$ is the eigenvector of $\mathbf{R}_k$ for $\lambda_j^{(k)}$. Thus

$$\frac{\partial R}{\partial X_{R,i}} = \frac{B}{K \ln 2} \sum_{k=1}^K \sum_{j=1}^{N_S} \frac{P_{j,k}}{\sigma^2 + P_{j,k} \lambda_j^{(k)}} (\mathbf{u}_j^{(k)})^* \frac{\partial \mathbf{R}_k}{\partial X_{R,i}} \mathbf{u}_j^{(k)}. \quad (\text{B.2})$$

B.1.2 Step 2: Partial derivative of the rate matrix

By the product rule,

$$\begin{aligned} \frac{\partial \mathbf{R}_k}{\partial X_{R,i}} &= \mathbf{H}_{\text{eff}}(f_k) \mathbf{Z}_{\text{eff}}^{-1}(f_k) \left(\frac{\partial \mathbf{H}_{\text{eff}}(f_k)}{\partial X_{R,i}} \right)^* \\ &+ \mathbf{H}_{\text{eff}}(f_k) \frac{\partial \mathbf{Z}_{\text{eff}}^{-1}(f_k)}{\partial X_{R,i}} \mathbf{H}_{\text{eff}}^*(f_k) + \frac{\partial \mathbf{H}_{\text{eff}}(f_k)}{\partial X_{R,i}} \mathbf{Z}_{\text{eff}}^{-1}(f_k) \mathbf{H}_{\text{eff}}^*(f_k). \end{aligned} \quad (\text{B.3})$$

B.1.3 Step 3: Partial derivative of the effective channel

From (3.6), the first term is independent of $\mathbf{Z}_R$. Let $\mathbf{A}_k = \mathbf{Z}_P(f_k) + \mathbf{Z}_R$. Since $\mathbf{Z}_R$ is diagonal with fixed real part, $\frac{\partial \mathbf{Z}_R}{\partial X_{R,i}} = j\mathbf{E}_{ii}$ where $\mathbf{E}_{ii}$ has 1 at (i, i) and zeros elsewhere. Using the inverse differential identity,

$$\frac{\partial \mathbf{H}_{\text{eff}}(f_k)}{\partial X_{R,i}} = j\mathbf{Z}_{\text{RP}}(f_k) \mathbf{A}_k^{-1} \mathbf{E}_{ii} \mathbf{A}_k^{-1} \mathbf{Z}_m(f_k). \quad (\text{B.4})$$

B.1.4 Step 4: Partial derivative of the inverse effective impedance

From (3.10), the first term is constant. Let $\mathbf{M}_k = \mathbf{A}_k^{-1} \mathbf{E}_{ii} \mathbf{A}_k^{-1}$ and $\mathbf{N}_k = \mathbf{A}_k^{-*} \mathbf{E}_{ii} \mathbf{A}_k^{-*}$.

With $\frac{\partial \mathbf{A}_k^{-1}}{\partial X_{R,i}} = -j\mathbf{M}_k$, the derivatives of the second, third, and fourth terms are

$$\begin{aligned} & \frac{\partial}{\partial X_{R,i}} [\mathbf{Z}_m^* \mathbf{A}_k^{-*} \mathcal{R}\{\mathbf{Z}_P\} \mathbf{A}_k^{-1} \mathbf{Z}_m] \\ &= -j\mathbf{Z}_m^* \mathbf{A}_k^{-*} \mathcal{R}\{\mathbf{Z}_P\} \mathbf{M}_k \mathbf{Z}_m - j\mathbf{Z}_m^* \mathbf{N}_k \mathcal{R}\{\mathbf{Z}_P\} \mathbf{A}_k^{-1} \mathbf{Z}_m, \end{aligned} \quad (\text{B.5})$$

$$\frac{\partial}{\partial X_{R,i}} [-\mathcal{R}\{\mathbf{Z}_m^T\} \mathbf{A}_k^{-1} \mathbf{Z}_m] = j\mathcal{R}\{\mathbf{Z}_m^T\} \mathbf{M}_k \mathbf{Z}_m, \quad (\text{B.6})$$

$$\frac{\partial}{\partial X_{R,i}} [-\mathbf{Z}_m^* \mathbf{A}_k^{-*} \mathcal{R}\{\mathbf{Z}_m\}] = j\mathbf{Z}_m^* \mathbf{N}_k \mathcal{R}\{\mathbf{Z}_m\}. \quad (\text{B.7})$$

Their sum gives $\frac{\partial \mathbf{Z}_{\text{eff}}^{-1}(f_k)}{\partial X_{R,i}}$. Then

$$\frac{\partial \mathbf{Z}_{\text{eff}}^{-1}(f_k)}{\partial X_{R,i}} = -\mathbf{Z}_{\text{eff}}^{-1}(f_k) \frac{\partial \mathbf{Z}_{\text{eff}}(f_k)}{\partial X_{R,i}} \times \mathbf{Z}_{\text{eff}}^{-1}(f_k). \quad (\text{B.8})$$

Appendix C

Additional validation with Feko

The Feko validations in Section 2.5.1 (Chapter 2) and Section 3.2.3 (Chapter 3) assume dipoles in free space. While that setting is adequate for validating the circuit-theoretic approach, deployed arrays are often backed by a ground plane so that most radiation lies on one side of the aperture. This appendix summarizes an additional computational validation for a hybrid linear array placed above an *infinite perfect electric conductor* (PEC).

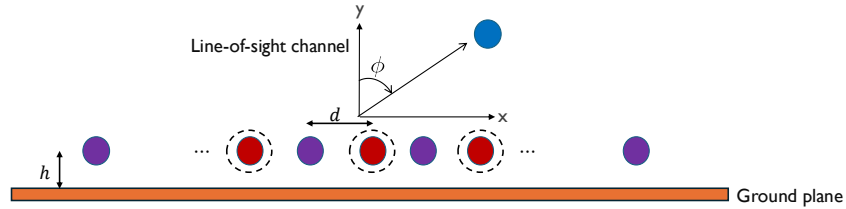


Figure C.1. Hybrid parasitic uniform linear array (ULA) backed by an infinite horizontal PEC ground. Active elements appear in red; parasitic elements appear in purple, in an alternating layout.

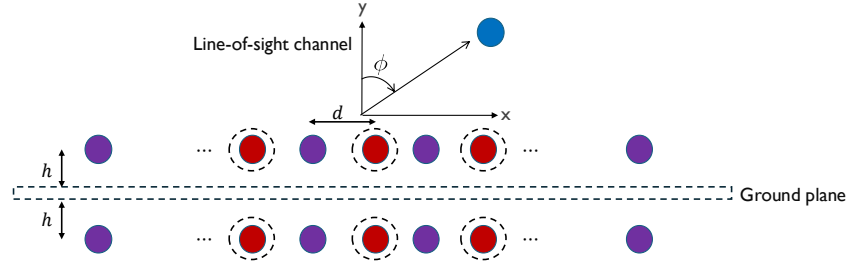


Figure C.2. Image equivalent of Fig. C.1: each physical dipole is mirrored below the infinite PEC, with image currents opposite to those on the corresponding real dipoles.

C.1 Simulation setup with a ground plane

The geometry in Fig. C.1 follows the alternating active–parasitic ULA layout from Section 3.2.3, with an infinite horizontal PEC a distance h below the plane of the dipole centers. The dipole currents are parallel to the ground (horizontal electric sources). The structure is simulated in Feko.

C.2 Model with a ground plane

For an infinite PEC, the standard image construction replaces the conductor by virtual dipoles located at the mirror positions, as in Fig. C.2, with currents opposite those on the real horizontal sources.

The impedance-domain construction is unchanged from Section 2.5.1: extract the open-circuit impedance matrix $\mathbf{Z}_{\text{TX}}$ from the full-wave simulator, partition into $\mathbf{Z}_{\text{A}}$, $\mathbf{Z}_{\text{P}}$, and $\mathbf{Z}_{\text{m}}$, impose the diagonal parasitic-load matrix $\mathbf{Z}_{\text{R}}$, and solve for $\mathbf{i}_{\text{A}}$, with parasitic currents given by (3.4) for the hybrid layout in Chapter 3 (see (2.5) for the analogous single-active exposition in Chapter 2). After forming the free-space directional trend from these currents as in those sections, multiply by the scalar interference factor from image theory [14]. Let ϕ denote the polar angle measured from the $+z$ axis normal to the PEC into the upper half-space containing the array, with $k = 2\pi/\lambda_{\text{c}}$ as throughout the thesis. Up to the same peak normalization used in

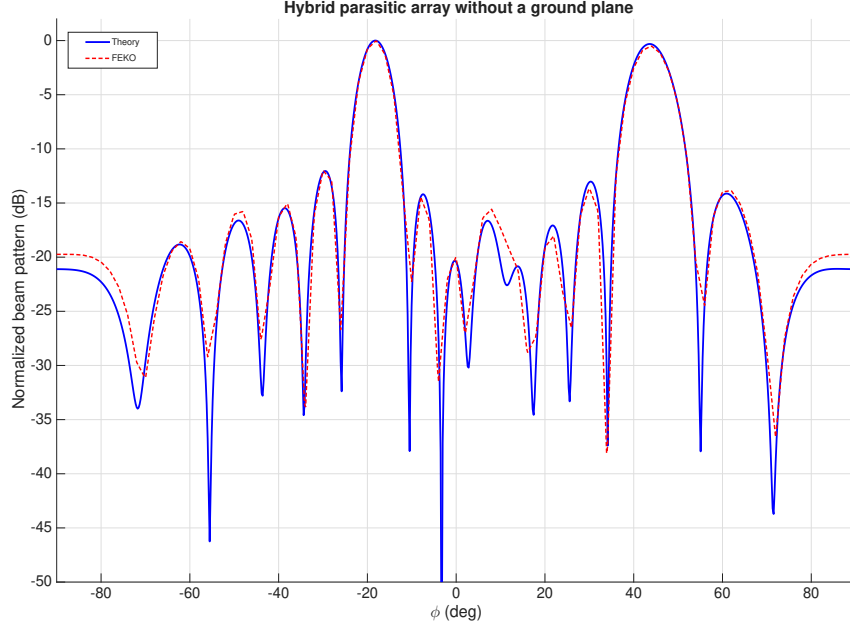


Figure C.3. Comparison of theoretical circuit model and Feko radiation patterns for a hybrid array without a ground plane.

Section 2.5.1,

$$|P(\phi)|^2 \propto |P_{f.s.}(\phi)|^2 \sin^2(k h \cos \phi), \quad (\text{C.1})$$

where $|P_{f.s.}(\cdot)|^2$ is the power-based pattern from the partitioned circuit model prior to applying the backing factor. Except for amplitude scaling, $|P_{f.s.}|^2 \sin^2(k h \cos \phi)$ matches classical image-theory expressions.

This scalar refinement omits several full-wave details of the backed array (cross-polarization, conductor losses, and impedance effects not captured by a single global factor); it applies the textbook elevation factor to currents already obtained via $\mathbf{Z}_{TX}$.

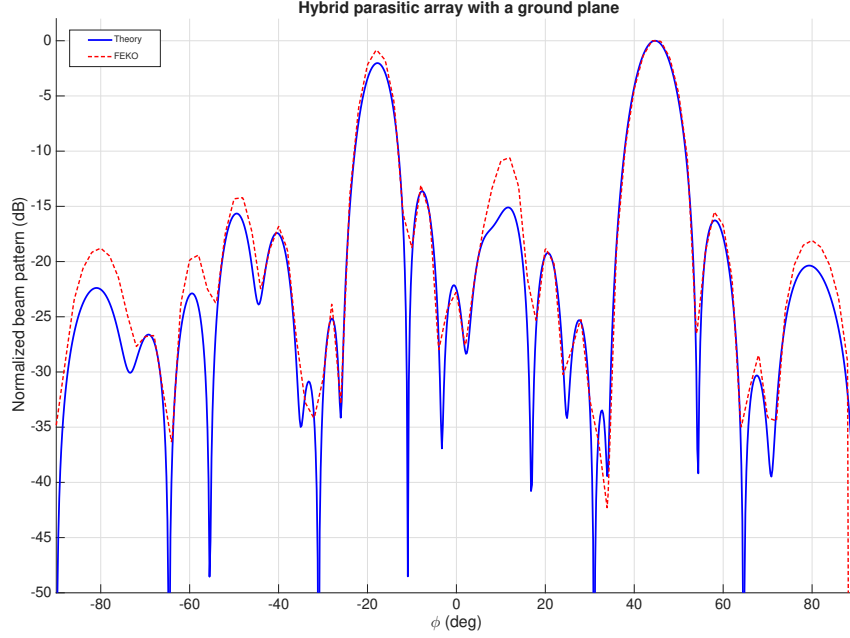


Figure C.4. Comparison of theoretical circuit model and Feko radiation patterns for a hybrid array with a ground plane as shown in Fig. C.1.

C.3 Radiation pattern comparison

Figure C.3 compares the circuit-based pattern to Feko without a ground plane; Fig. C.4 repeats the comparison with the geometry of Fig. C.1. In both cases, the theoretical pattern tracks the Feko result, which continues to support the impedance-based hybrid-array model from Chapters 2–3.

With the ground plane, the idealized factor (C.1) predicts deep nulls at zeros of $\sin(kh \cos \phi)$. In particular, along the aperture horizon ($\phi = \pi/2$, propagation parallel to the conductor), $\cos \phi = 0$, so the analytic overlay vanishes, which matches the grazing suppression in Fig. C.4.

Appendix D

Appendix to Chapter 4: Dynamic Meta-surface Antennas

D.1 Beamforming gain optimization for TTD

The TTD beamforming architecture for a conventional array with τ_n delay at n th element produces a frequency-selective beamforming vector as $\mathbf{f}_{\text{TTD}}(f) = [e^{j2\pi f\tau_1}, \dots, e^{j2\pi f\tau_N}]^T$ and the corresponding beamforming gain is $\mathcal{G}_{\text{TTD}}(\phi, f) = |\mathbf{f}_{\text{TTD}}^T(f)\mathbf{h}(\phi, f)|^2$ where the intrinsic phase vector is an all ones vector. For a TTD array, the formulation for maximizing the beamforming gain at ϕ_{AoD} and f_t is

$$\max_{\{\tau_n\}_{n=1}^N} \mathcal{G}_{\text{TTD}}(\phi_{\text{AoD}}, f_t), \quad (\text{D.1a})$$

$$\text{s.t. } \{\tau_n\}_{n=1}^N \geq 0. \quad (\text{D.1b})$$

The optimal solution is $\tau_n^* = \frac{dy}{c}(n-1)\sin(\phi_{\text{AoD}})$ for $\phi_{\text{AoD}} \geq 0$ and $\tau_n^* = \frac{dy}{c}(n-N)\sin(\phi_{\text{AoD}})$ for $\phi_{\text{AoD}} < 0$. The maximum value of the beamforming gain is $\mathcal{G}_{\text{TTD}}^*(\phi_{\text{AoD}}, f_t) = N^2$.

D.2 Reformulated version of problem P1

We reformulate **P1** in terms of $\{\tilde{\psi}_n(f_t)\}_{n=1}^N$ as

$$\mathbf{P1a} : \max_{\{\tilde{\psi}_n(f_t)\}_{n=1}^N} \mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f_t), \quad (\text{D.2a})$$

$$\text{s.t. } \{\tilde{\psi}_n(f_t)\}_{n=1}^N \in [-3\pi/2, \pi/2]. \quad (\text{D.2b})$$

We follow the solution approach similar to [65] [78]. Let

$$\mathcal{S}(\phi, f_t) = \frac{\sin\left(\pi N \frac{f_t d_y(n_g + \sin(\phi))}{c}\right)}{\sin\left(\pi \frac{f_t d_y(n_g + \sin(\phi))}{c}\right)}. \quad (\text{D.3})$$

The optimal solution to problem **P1a** $\{\tilde{\psi}_n^*(f_t)\}_{n=1}^N$ is

$$\begin{aligned} \tilde{\psi}_n^*(f_t) = & -\frac{\pi}{2} \text{sgn}(\mathcal{S}(\phi_{\text{AoD}}, f_t)) \\ & + \frac{2\pi f_t d_y(n_g + \sin(\phi_{\text{AoD}}))}{c} \left(n - 1 - \frac{N-1}{2}\right) + 2m\pi, \end{aligned} \quad (\text{D.4})$$

where m is an integer such that $\tilde{\psi}_n^*(f_t) \in [-\frac{3\pi}{2}, \frac{\pi}{2}]$.

D.3 Proof of Lemma 1

The inner maximization in problem **P2** is solved in (D.4) and the closed-form resonant frequency solution is expressed in (4.15). We leverage these solutions to solve problem **P2**. The optimal operating frequency is expressed as $f_t^*(\phi_{\text{AoD}}) = \underset{f_t \in [f_{t,\min}, f_{t,\max}]}{\text{argmax}} |\mathcal{S}(\phi_{\text{AoD}}, f_t)|$. There are two cases for expressing the solution of **P2** depending on whether p^* is a natural number or not.

Case 1: The global maximum of $|\mathcal{S}(\phi_{\text{AoD}}, f_t)|$ is N and is attained when $p^* = \frac{f_t^* d_y(n_g + \sin(\phi_{\text{AoD}}))}{c}$ is a natural number. Using (4.15), $f_{r,n}^*(\phi_{\text{AoD}}) = f_t^*(\phi_{\text{AoD}})$ when p^* is a natural number.

Case 2: The global maximum N cannot be attained if there is no integer $p^* \in [p_{\min}, p_{\max}]$. Hence, p^* is obtained numerically by solving (4.19).

D.4 Proof of Lemma 2

From **Case 1** of Lemma 1, we see that to satisfy the requirement $\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_{\text{t}}^*) = N^2$, the value of p^* has to be a natural number which lies inside $\left[\frac{f_{\text{t},\min} d_y (n_{\text{g}} + \sin(\phi_{\text{AoD}}))}{c}, \frac{f_{\text{t},\max} d_y (n_{\text{g}} + \sin(\phi_{\text{AoD}}))}{c} \right]$. To satisfy $\mathcal{G}_{\text{DMA}}^*(\phi_{\text{AoD}}, f_{\text{t}}^*) = N^2 \forall \phi_{\text{AoD}} \in [\phi_{\text{lw}}, \phi_{\text{up}}]$, the upper limit computed at $\phi \in [\phi_{\text{lw}}, \phi_{\text{up}}]$ should be greater than or equal to a natural number p^* and the lower limit computed at $\phi_{\text{AoD}} \in [\phi_{\text{lw}}, \phi_{\text{up}}]$ should be less than or equal to the same number p^* . Mathematically, $\forall \phi_{\text{AoD}} \in [\phi_{\text{lw}}, \phi_{\text{up}}]$, we get

$$\frac{f_{\text{t},\min} d_y}{c} (n_{\text{g}} + \sin(\phi_{\text{AoD}})) \leq p^* \leq \frac{f_{\text{t},\max} d_y}{c} (n_{\text{g}} + \sin(\phi_{\text{AoD}})). \quad (\text{D.5})$$

Both the upper and lower limit on p^* are an increasing function of ϕ_{AoD} in the desired angular range. Hence, we have

$$\frac{f_{\text{t},\min} d_y (n_{\text{g}} + \sin(\phi_{\text{up}}))}{c} \leq p^* \leq \frac{f_{\text{t},\max} d_y (n_{\text{g}} + \sin(\phi_{\text{lw}}))}{c}. \quad (\text{D.6})$$

By equating the upper and lower limit of p^* in (D.6), we get the desired results in (4.22) and (4.23) where equality holds for $n_{\text{g}} = n_{\text{g}}^*$ and $d_y = d_y^*$.

D.5 Proof of the cutoff-frequency expression

Mathematically, the closed-form expression for the cutoff frequencies are obtained as a solution to $\frac{\Gamma^2 4\pi^2 f^2}{(4\pi^2 (f_{\text{t}}^*(\phi_{\text{AoD}}))^2 - 4\pi^2 f^2)^2 + \Gamma^2 4\pi^2 f^2} = \nu$. We express this equation as a quadratic in f^2 . Solving the quadratic, we obtain the desired cutoff frequencies as

$$f_{\text{e},\nu}^{\text{l}} = \sqrt{(f_{\text{t}}^*(\phi_{\text{AoD}}))^2 + \frac{\Gamma^2 - \sqrt{\Gamma^4 + 16\Gamma^2\pi^2(f_{\text{t}}^*(\phi_{\text{AoD}}))^2\rho(\nu)}}{8\pi^2\rho(\nu)}}, \quad (\text{D.7})$$

$$f_{\text{e},\nu}^{\text{u}} = \sqrt{(f_{\text{t}}^*(\phi_{\text{AoD}}))^2 + \frac{\Gamma^2 + \sqrt{\Gamma^4 + 16\Gamma^2\pi^2(f_{\text{t}}^*(\phi_{\text{AoD}}))^2\rho(\nu)}}{8\pi^2\rho(\nu)}}. \quad (\text{D.8})$$

The ν -bandwidth closed-form expression is obtained by applying binomial expansion to (D.7) and (D.8).

D.6 Binary weight beamforming problem

The binary weight beamforming problem is

$$\mathbf{P3} : \max_{\mathbf{f}_{\text{DMA}}(f_c)} \mathcal{G}_{\text{DMA}}(\phi_{\text{AoD}}, f_c), \quad (\text{D.9a})$$

$$\text{s.t. } [\mathbf{f}_{\text{DMA}}(f_c)]_n \in \{0, 1\}. \quad (\text{D.9b})$$

To solve **P3**, we leverage the fact that each element in $\mathbf{h}(\phi_{\text{AoD}}, f)$ lies on a unit circle in the complex plane. We interpret the problem as selection of points on a unit circle that maximizes the resultant sum of the complex numbers. From this interpretation, an optimal solution with non-zero weights will always exist. The optimal configuration has all points on a complex semicircle. Using this intuition, we search over $2N$ possible semicircles to compute the optimal configuration.

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